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Decentralized Leadership Meets Soft Budget

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Decentralized leadership meets soft budget ^{*}

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Abstract

The objectives of the present paper are twofold. First, we aim to synthesize the two strands of the literature on the incentive effect of intergovernmental transfers, the decentralized leadership and the soft budget problem both of which address the discretionary nature of the central transfer policy. We develop a simple decentralized leadership model in which the local governments move first and the central government transfer scheme is decided ex post. The ex post discretion of transfer by the central government pursuing social welfare distorts the ex ante incentive of the local governments, inducing the strategic action of the latter government. This paper also shows that the direction of the ex ante distortion moral hazard problem relies on what decision is made ex ante by type of authority is given to the local government, namely public expenditure or tax collection ex ante. Second we examine the robustness of the incentive problem. The benchmark model incorporates spillovers and is extended in several directions, including tax competition and distortionary taxes, and two period setting. The essence of the incentive problem remains the same.

Key Words: decentralized leadership, soft budget,
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1. Introduction

In the federalism literature, intergovernmental transfers have been discussed from the normative standpoint as device to cope with inefficiency and inequity in a decentralized fiscal system in which local level governments are granted autonomy to decide their public spending and taxes within their jurisdictions (Boadway and Hobson (1996)). To be more specific, if properly designed the transfers serve to internalize fiscal externalities/ spillovers and assure fiscal equity equalizing net fiscal benefits across regions. The political economy consideration accounting for the incentive of the central authority pursuing own interest may change implications of the central transfer policy, however as being addressed by the public choice literature. Not only self interested nature of the central government, but its commitment ability has been increasingly concerned as well. The time consistency literature has raised the pervasive incentive consequences due to lack of the commitment of the central government despite its benevolent objective (Fisher (1980)).

There are two strands of the literature on the commitment problem in the context of intergovernmental transfers. The soft budget literature has addressed the ex ante moral hazard or adverse incentive consequences on the local governments in the anticipation of the ex post bailing out by the central government in the pursuit of the ex post objective. The soft budget problem describes “the situation when an entity (say a province) can manipulate its access to funds in undesirable way “(Rodden et al (2003)) and is formulated in the context of the sequential game the local government moving first and the central government deciding transfer policy after the local fiscal status is revealed (See Inman (2003)). Dewatripont and Maskin (1995) establish the soft budget

as the incentive problem due to time inconsistency in the context of relationship between lender and borrower in credit market as well. The modeling may be analogous to the Samaritan's dilemma with the grant recipient acting as a Stakelberg leader accounting for the ex post behavior of the bailing out/grant providing principle. The benevolency of the latter is not necessarily needed, however, for this problem to arise. Goodspeed (2002) models political economy of the soft budget. Von Hagen and Dahlbeng (2002) address political motive of the center in bailing out indebted regions as well ²

The equilibrium consequence is mixed. The local government may become too large, overspending and/or over-borrowing, or may be too small exerting little tax effort and thus raising less own revenue. Wildasin (1997) for instance, establishes that in the presence of inter-regional spillovers, there arises "under-provision" of local own expense a large size jurisdiction being bailed out more frequently which is known as "Too big to fail" principle.

The second strand is the literature on decentralized leadership that has addressed the ex ante horizontal and reciprocal externalities with the central government acting as a Stakerberg follower and local governments as leaders but established different implications from the soft budget problem. Caplan et al (2000) argue that efficient allocation of locally provided public services is achieved when inter-regional spillovers are present. The ex post transfer serves to internalize the spillover effect, the transfer being lump-sum ex post but being perceived as matching form by the regions ex ante.

²As is well known, the concept of the soft budget was first proposed by Kornai (1986) in the context of socialist economy. For a comprehensive survey on the theory of the soft budget, see Kornai, Maskin and Roland (2003), Qian and Roland (1998), Dewatripont, Maskin, Roland (2000), Dewatripont and Roland (2000). Interestingly, in the federalism literature, the soft budget has often characterized a feature of "decentralized fiscal system" but a close fiscal tie between governments remaining and/or task assignment being ambiguous.

Köthenbürger (2004) introduces the horizontal tax competition into the decentralized leadership model and shows that whereas inefficiency created by tax competition is internalized, inefficiency is created due to transfer so exhibiting the trade-off. In Caplan et al (2000), the ex ante horizontal interaction is through the spillover generating expenditure, cost of which being shared nation wide by the ex post intergovernmental transfer, whereas Köthenbürger (2004) considers the horizontal externality on the revenue side.

The present paper aims to synthesize the decentralized leadership and the soft budget problem. We develop a simple decentralized leadership model in which the local governments move first and the transfer scheme is decided ex post. The difference between soft budget and decentralized leadership lies that the former as formulated by Dewatripont and Maskin (1995) is basically partial equilibrium model addressing ex post fiscal tie between principal and his agent whereas the latter accounts for general equilibrium effect that gives rise to ex ante horizontal externalities among local governments. In both, the ex post discretion by the central government pursuing social welfare distorts the ex ante incentive of the local governments that induces the strategic reaction of the latter. In this respect, the mechanism of decentralized leadership is identical to the soft budget, both of which addresses commitment problem.

We establish that the direction of the ex ante distortion relies on what decision is made ex ante by the local government, namely public expenditure or tax collection. In the fiscal competition literature, it is well-known that expenditure competition and tax one exhibit different equilibrium consequences, but in both the result is “under-provision” or “under-taxation” relative to the full cooperation outcome (Wildasin (1989)). With ex post discretion on the intergovernmental transfers, the ex ante horizontal interaction

through tax collection effort give rise to qualitatively different results from the one through expenditure, with “under-taxation” being the case in the former and local governments over-spending in the latter. That is, it is not straightforward to see whether the soft budget/ decentralized leadership causes too large or too small local government in terms of per capital expense.

We examine the robustness of the incentive problem as well. In the benchmark mode, we incorporate spillover effects of public expenditure financed by lump-sum taxation. Later we extend the model to the case of distortional taxes. We have efficient outcomes only in some polar cases such as when locally provided public good is pure in nature as is assumed in Caplan et al (2000) and when the central and local tax bases are perfectly overlapped leading to the vertical tax externality. The Pareto efficient outcome in a decentralized leadership and the too big to fail principle will be shown to be model specific relying on timing of decision making and on degree of spillovers.

At this point, we would like to address the empirical relevancy of our problem. The soft budget problem is not a theoretical artifact but its empirical evidence is abundant. Dillenger et al (2001) note experiences of Latin America that rapid decentralization coming with separation of taxing and expenditure decisions put stress on the central budget and ultimately macro economic stability because of ex post rescues of indebted local governments. Von Hagen and Dahlbeng (2002) address the practice of bailing out local governments in Sweden. Shleifer and Treisman (2000) give an ad-hoc nature of federal transfers in Russia in the 90s, in which enhancing tax collection/mobilization in a region is followed by lower allocation of the transfer to that region. Martinez-Vazquez and Boix (2001) also raise the evidence that FFSR indeed discourages the tax effort at the regional level. In Germany federalism, Baretta et al (2002) present the evidence that

the horizontal equalization discourages the tax collection effect of the states. Applying both non-parametric and parametric methodology, Boger and Kerstens (1996) estimate the inefficiency, namely deviation of the actual cost from estimated minimum cost due to the dependency on intergovernmental transfers among Belgium local governments. Akai et al (2002) address bailing out nature of Japanese intergovernmental transfers (Local Allocation Tax) that have the gap filling feature. Rodden et al(2002) provides comprehensive case studies on the soft budget.

The rest of this paper is organized as follows. Section 2 illustrates the general model with decentralized leadership and ex post transfer. In Section 3, we consider two scenarios where expenditure level is selected ex ante and tax level is selected ex ante and characterize the interesting results that two cases create moral hazard problems with opposite directions. We extend the model by introducing capital tax competition in Section 4. In Section 5, we analyze the general model in which two taxes by the central and the local governments are levied on the various types of tax bases. In Section 6, we also analyze the two period model to address the local government's incentive to borrow and enhance own tax base or regional economy. Section 7 considers other extensions, namely non-separable utility. Section 8 concludes this paper.

2. Model with decentralized leadership and ex post transfer

2.1 Environment

The economy contains I regions. There are the central and local governments. Each region consists of the representative resident. Denote a size of population in region i

by n_i with the total population given by $\sum_{i=1}^I n_i \equiv N$. The residents in region i are endowed with a fixed amount of per capita income y_i . Later we turn to the case that y_i is variable and income tax is distortional. The total income in this economy then becomes $\sum_{i=1}^I n_i y_i \equiv Y$. We abstract away intra-regional preference heterogeneity here to focus on inter-regional conflicts of interest, but account for the case where either n_i or y_i (or both) may be different across regions.

Public services

There are two public goods/services, denoted by g_i and G in terms of per capita consumption. We assume that g_i is locally provided which may generate inter-regional spillover, the degree of which is represented by λ . G is a per capita national public service and is uniformly provided by the central government. We can allow for G to be pure, however, and thus there is scale economy in the consumption without altering the essence of our argument.

Resident's utility

The residents benefit from the private and public consumption. We assume that their preference is separable so that the residential utility in region i is expressed by:

$$U(c_i, g_i, G) = u((1-\tau)y_i - t_i) + \left\{ (1-\lambda)v(g_i) + \lambda E\left(\sum_{j=1}^I n_j g_j\right) \right\} + \Phi(G) \quad (1)$$

where $c_i = (1-\tau)y_i - t_i$ and λ represents degree of the spillover with $0 \leq \lambda \leq 1$.

$\lambda E(\sum_{j=1}^I n_j g_j)$ gives the spillover effect from all regions and g_i is pure when $\lambda=1$. In

the above, the central tax rate on income y_j is denoted by τ . We suppose that the local government levies the lump sum tax t_i : since y_j is assumed to be fixed, local income tax gives the same result.

Government's budget constraint and Intergovernmental transfer

The budget constraint of the local government is written as:

$$n_i t_i + S_i = n_i g_i, \quad (2)$$

where S_i denotes the subsidy from the central government to the region. We suppose that S_i can go to either sign allowing maximal discretion in the grants policy. The negative transfer implies that the central government taxes local government. Turning to the central budget, it becomes:

$$NG + \sum_{i=1}^I S_i = \tau \sum_{i=1}^I n_i y_i = \tau Y. \quad (3)$$

The central government possesses full control over S_i so as to pursue own objective. We suppose that it cannot commit to the transfer policy, however, implying that S_i is optimized from the ex post standpoint taking as given the ex ante local decisions as fully explored later.

In the following benchmark model, we assume that $\tau = 0$ to address the horizontal equalization nature of the transfers unless explicitly stated. This may appear ad hoc, but it reflects institutional features in a country where the sub-national governments are in charge of collecting the central and local taxes as is the case in Germany and in

the former socialist countries. In section 6, we re-introduce the central tax to see the robustness of our argument.

For the later use, we also give overall resource constraint as follows:

$$\sum_{i=1}^I n_i c_i + \sum_{i=1}^I n_i g_i + \sum_{i=1}^I n_i G = Y. \quad (4)$$

The objectives of Central and Local governments

The central and local governments are assumed to be benevolent so as to abstract political economy consideration and address the commitment problem. To be precise, the central government decides the transfers to maximize the utilitarian objective, i.e., the sum of regional utilities:

$$W = \sum_{i=1}^I n_i U(c_i, g_i) = \sum_{i=1}^I \left[u((1-\tau)y_i - t_i) + \left\{ (1-\lambda)v(g_i) + \lambda E\left(\sum_{j=1}^J n_j g_j\right) \right\} + \Phi(G) \right] \quad (5.1)$$

On the other hand, the local government aims to maximize the welfare of own region:

$$V_i = u((1-\tau)y_i - t_i) + \left\{ (1-\lambda)v(g_i) + \lambda E\left(\sum_{j=1}^J n_j g_j\right) \right\} + \Phi(G). \quad (5.2)$$

Timeline

Timing is very important in our model, in which the decision making is divided into several stages. We always assume that S_i is decided ex post in the sense of the decentralized leadership. We consider the two scenarios depending on whether the local governments ex ante chooses g_i or t_i . The remaining policy instruments including G_i are determined ex post. To be precise, timeline in each scenario is as follows.

	Scenario A	Scenario B
Stage 1 (Ex ante)	g_i is decided by the local government.	t_i is decided by the local government

Stage 2 (Ex post)	The central government optimizes S_i and G	
	t_i is determined so fulfill the local budget.	g_i is determined so fulfill the local budget.
Stage 3:	Given all policies implemented, residents enjoy consumption and finally resident's utility is determined.	

In stage 2, the central government acts taking as given the ex ante decisions by local government: in this regard, the central government is the Stackelberg follower. In stage 1, the local government accounts for how their ex ante choices (of g_i or t_i) affects the ex post central policy, especially ex post design of the intergovernmental transfers, as the Stackelberg leader, but behave in Nash manner toward the other local governments in the same stage.

In the literature, either scenario has been supposed. Caplan et al (2000) follows our first scenario. In the two period setting, Goodspeed (2002) considers that sub-national governments borrow to expand their first period spending and raise taxes in the second period to make repayment. His case may be closer to Scenario A as well. On the other hand, Wildasin (1997) and Köthenbürger (2004) adopt the second scenario supposing that local tax collection effects are sunk ex ante. The present paper does not aim to examine which scenario is empirically plausible but to see how the timing structure affects the equilibrium consequences.

2.2 First best optimal allocation

Before illustrating the subgame perfect equilibrium, as a reference, let us consider the

first best allocation that is determined by maximizing the social welfare W subject to the resource constraint:

$$\underset{c_i, g_i, G_i}{MAX} W \quad \text{subject to} \quad \sum_{i=1}^I n_i c_i + \sum_{i=1}^I n_i g_i + \sum_{i=1}^I n_i G_i = \sum_{i=1}^I n_i y_i .$$

The first best allocation is characterized by

$$u'(c_i) = (1-\lambda)v'(g_i) + \lambda NE'(\sum_{i=1}^I n_i g_i) = \Phi'(G_i) = \gamma \quad (6)$$

alongside with the resource constraint. The implication of the above first-order conditions is straightforward:

$$c_i = c^*, \quad g_i = g^*, \quad G_i = G^* \quad \text{for all } i \text{ and } j$$

In the case of $\lambda = 1$, g^* is coincident with the Samuelson condition:

$$\frac{NE'(\sum_{i=1}^I n_i g^*)}{u'(c^*)} = 1 \quad (7)$$

If the central government were able to commit, it could replicate the first best allocation optimizing the grants from the ex ante standpoint. To be more specific, S_i including matching component to internalize the spillovers can be set so that

$$S_i/n_i = (1-m_i)g^* + c^* - y_i + m_i g_i \quad \text{where} \quad m_i = \lambda(N-n_j)E'(Ng^*)/u'(c^*)$$

m_j is reduced to $(N-n_j)/N$ when $\lambda = 1$. Therefore, the inefficiency observed in the following is due to the lack of the commitment of the central government and the ex ante strategic decisions taken at the local level.

3. Benchmark case with a spillover effect of public good

In this section, we analyze the basic model with various degrees of spillovers effect of the local public service. Under this basic setting, ex post subsidy in the decentralized leadership model creates moral hazard problem corresponding to the soft budget problem unless the degree of spillover is perfect. We also show that the direction of the moral hazard, i.e., whether local government is too large or too small in terms of public services provided, depends on which policy instrument, g_i (Scenario A) or t_i (Scenario B), is decided ex ante by the local governments.

In the following unless explicitly stated, we assume that $\tau = 0$ to address the horizontal equalization nature of the transfers. This may appear ad hoc, but it reflects institutional features in a country where the sub-national governments are in charge of collecting the central and local taxes as are cases in Germany and in the former socialist countries. The central tax rate turns to be redundant in Scenario A but it can restore the first best under Scenario B if optimized as noted in 3.2.

3.1 Scenario A: Expenditure level is selected ex ante

In this section, we consider that g_i is decided ex ante and t_i is adjusted after the ex post transfer to balance the local budget with various degrees of spillovers effect of the local public service. Under this scenario, we establish ex post subsidy creates overspending at the local level unless the degree of spillover is perfect. In the following, we proceed backward ways starting from the second stage.

Stage 2 Ex post behavior of the central government

Since g_i is already decided ex ante, t_i is adjusted ex post such as to balance the

budget, $t_i = g_i - S_i/n_i$ with S_i being transferred from the central government. Then the central government chooses S_i and G to solve the following problem:

$$\underset{S_i, G}{MAX} W = \sum_{i=1}^I \left[u(y_i - \bar{g}_i - S_i/n_i) + (1-\lambda)v(g_i) + \lambda E\left(\sum_{i=1}^I n_i g_i\right) + \Phi(G) \right]$$

subject to $NG + \sum_{i=1}^I S_i = 0$ where $\tau = 0$ is assumed. The first order conditions become:

$$u'(y_i - g_i - S_i/n_i) = u'(y_j - g_j - S_j/n_j) = \Phi'(G), \quad (8)$$

implying that the consumption level is perfectly equalized so that

$$c_i = c_j = \frac{1}{N} \sum_i (Y_i - g_i) \equiv \bar{c}. \quad (9)$$

Denote the ex post optimal level of the central public service by \bar{G} . The ex post optimal subsidy level is described as

$$S_i = n_i \bar{c} + n_i g_i - (1-\tau)n_i y_i. \quad (10)$$

Inserting this into the central budget constraint and rearranging, we have

$$N\bar{G} + N\bar{c} = Y - \sum_{i=1}^I n_i g_i. \quad (11)$$

Note that \bar{c} and \bar{G} is determined by solving (8) and (11). Both \bar{c} and \bar{G} are given as the increasing function of $M \equiv \frac{1}{N}(Y - \sum_{i=1}^I n_i g_i)$, namely, $\bar{c}(M)$ and $\bar{G}(M)$. For

the latter use, we state the following lemma:

Lemma 1:

Both \bar{c} and \bar{G} are increasing in M with

$$1 = \frac{d\bar{c}}{dM} + \frac{d\bar{G}}{dM} \quad (12)$$

At this point, let us illustrate the features of the ex post optimal transfer function. Given that c_i is equalized, combining the local budget and the resident's budget constraints, we have

$$S_i/n_i = g_i + \bar{c}(M) - y_i. \quad (13)$$

Accounting for Lemma 1, we can establish that ex post per capita transfer to region i is increasing in own expense and decreases when other regions expend more:

$$\frac{\partial s_i}{\partial g_i} = 1 + \frac{\partial \bar{c}}{\partial M} \frac{n_i}{N} > 0 \quad (14.1)$$

$$\frac{\partial s_i}{\partial g_j} = \frac{\partial \bar{c}}{\partial M} \frac{n_j}{N} < 0 \quad \text{with } j \neq i \quad (14.2)$$

Goodspeeds (2002) in the two period setting raises the possibility that (14.2) becomes positive. The present model reveals that his case is unlikely, but that the central government responds to increase of g_i in one region by decreasing transfers to others.

Substituting (13) into (2) yields $t_i = y_i - \bar{c}(M)$. This is the resident's budget, so we can interpret that the local government of region i is concerned with not own budget but the one of its residents whose consumption is ex post determined by the central authority. t_i does not depend upon own expenditure directly. The latter affects the local tax rate

only through $M \equiv \frac{1}{N}(Y - \sum_{i=1}^I n_i g_i)$. That is t_i is ex post adjusted not by own expenses

but by the remaining resource for consumption in the economy. To make our point clear, imagine a small region i so that $n_i/N \approx 0$. Then t_i become taken to be constant regardless of g_i .

Stage 1 Ex ante Behavior of the local government

Accounting for the ex post central policy, which is summarized by $\bar{c}(M)$ and $\bar{G}(M)$, the local governments independently select g_i to maximize the local utility in region i . Their optimization problem is expressed by:

$$\begin{aligned} \text{MAX}_{g_i} \quad & V_i = u(\bar{c}(M)) + (1-\lambda)v(g_i) + \lambda E\left(\sum_{i=1}^I n_i g_i\right) + \Phi(\bar{G}(M)) \\ \text{subject to} \quad & M = \frac{1}{N}\left(Y - \sum_{i=1}^I n_i g_i\right) \end{aligned}$$

Applying Lemma 1, the first order condition is reduced to:

$$\frac{n_i}{N}u'(\tilde{c}) = (1-\lambda)v'(\tilde{g}_i) + \lambda n_i E'\left(\sum_{i=1}^I n_i \tilde{g}_i\right). \quad (15)$$

Tilde designates solution to the ex ante problem. The above has straightforward interpretation. The right hand side is the regionally perceived benefit of the local public service at margin whereas the left hand side represents the marginal cost from the regional perspective. $1 - \frac{n_i}{N}$ is the portion of the cost accruing to the other regions.

The Sub-Game Equilibrium:

The time consistent (subgame perfect) equilibrium is characterized by

$$u'(\tilde{c}) = \Phi'(\tilde{G}) \quad (16.1)$$

$$\frac{n_i}{N}u'(\tilde{c}) = (1-\lambda)v'(\tilde{g}_i) + \lambda n_i E'\left(\sum_{i=1}^I n_i \tilde{g}_i\right) \quad (16.2)$$

$$N(\tilde{c} + \tilde{G}) + \sum_{i=1}^I n_i \tilde{g}_i = Y \quad (16.3)$$

We know that the Samuelson condition is satisfied and thus the equilibrium becomes the first best only if $\lambda = 1$. Otherwise, there is tendency of over-spending of the public service as stated in Proposition 1:

Proposition 1

- (a) When $\lambda < 1, \bar{g} > g^*, \tilde{c} < c^*, \tilde{G} < G^*$ where $\bar{g} \equiv \sum_{i=1}^J n_i \tilde{g}_i / N$.
- (b) When $\lambda < 1, \tilde{g}_i$ takes a larger value for smaller region, namely, $\tilde{g}_1 > \tilde{g}_2$ if $n_1 < n_2$.
- (c) \tilde{g}_i is over-provided in the sense that $\frac{d}{dg_i} W|_{g_i=\tilde{g}_i} < 0$, given $g_j = \tilde{g}_j$ ($j \neq i$)

For the proof, see Appendix 1. Proposition 1 is sharply in contract with Caplan et al (2000). They argue that the decentralized leadership achieves the efficient allocation but this applies only to a polar case of $\lambda = 1$. The intuition is the following. With $\lambda = 1$, the local public service has a perfect spillover effect. Therefore the level of public service becomes too small at the degree of n_j/N . On the other hand, the ex post subsidy from the central government tends to make g_i too large at the extent of n_j/N . The two opposing effects perfectly offset, leading to the first best allocation. In the case of $\lambda < 1$, however, the moral hazard motive due to the ex post cost sharing dominates the free riding one associated with spillover effect. The over-spending in per capita term is exacerbated in a less populated region. g_i is excessive from our social welfare standpoint in that the marginal reduction enhances it. The comparison with the first best value g^* is not straightforward, however The average value of g_i in the

equilibrium exceeds g^* . Along with (b) of the Proposition, we have $\tilde{g}_i > g^*$ for regions with relatively smaller population. We can say that $\tilde{g}_i > g^*$ holds for all regions either when $u(c)$ is close to linear or when regions are relatively homogeneous in terms of population. It is conceivable, however, that the inequality is reversed in large regions when the utility is relatively concave. In Appendix 2, we suppose $u(c)$ takes log-form and find the condition for $\tilde{g}_i > g^*$ to hold. It should be noted that $\tilde{g}_i > g^*$ for some large regions does not contradict (c) of Proposition 1. The latter is local analysis addressing the marginal change while $\tilde{g}_j \gg g^*$ is global comparison.

It is also noteworthy that the national public service G is underprovided relative to the first best. Moreover, S_i/n_i per capita grant is larger in a smaller region due to that $S_i/n_i = \tilde{g}_i + \tilde{c} \cdot y_i$ and Proposition 1(b). In this regard, smaller regions are treated favorably.

This situation may resemble pork barrel politics model by Inman and Rubinfeld (1996) and Weingast et al(1981). In scenario A, both the soft budget and the pork barrel politics give rise to over-provisions of locally benefiting public goods. They reflect different institutional settings however. In the former, the ex ante inefficiency is due to lack of commitment of the central government whereas the latter supposes that the decision making within legislature is fragmented with the universal norm being adopted. In addition, the present model presumes a strong central authority possessing maximal discretion in its grants policy. On the other hand, the government is weak and susceptible to the regional demand in the model of pork barrel politics.

3.2 Scenario B; Tax level is selected ex ante

Stage 2 Ex post behavior of the central government

We again begin with the ex post decision making. In this alternative scenario B, t_i is decided ex ante, whereas g_i is ex post adjusted to balance the budget ; $g_i = t_i + S_i / n_i$.

Taking t_i as given, the central government chooses S_i and G to maximize W subject to the budget constraint:

$$MAX_{S_i, G} W = \sum_{i=1}^I n_i \left[u(y_i - t_i) + (1 - \lambda)v(t_i + S_i / n_i) + \lambda E(\sum_{i=1}^I n_i t_i + S_i) + \Phi(G) \right]$$

$$\text{subject to } R = (1/N)(\sum_{i=1}^I n_i t_i)$$

leading to the first order conditions such that

$$(1 - \lambda)v'(g_i) + \lambda NB'(\sum_{i=1}^I n_i g_i) = (1 - \lambda)v'(g_j) + \lambda NE'(\sum_{i=1}^I n_i g_i) = \Phi'(G). \quad (17)$$

The above implies that the expenditure level is perfectly equalized, that is, $g_i = g_j \equiv \bar{g}$.

In the present context, S_i works as horizontal equalization of local public services.

Substituting $S_i = n_i(\bar{g} - t_i)$ into the central budget constraint and re-arranging establish

$$N(\bar{G} + \bar{g}) = \sum_{i=1}^I n_i t_i. \quad (18)$$

\bar{g} and \bar{G} are determined by (17) and (18), and thus becomes the function of

$R \equiv (1/N)(\sum_{i=1}^I n_i t_i)$, namely, $\bar{g}(R)$ and $\bar{G}(R)$. Similar to Lemma 1, we have the following lemma.

Lemma 2

$\bar{g}(R)$ and $\bar{G}(R)$ are increasing in R with

$$1 = \frac{d\bar{g}}{dR} + \frac{d\bar{G}}{dR}. \quad (19)$$

Stage 1 Ex ante Behavior of the local government

As in the previous scenario, we suppose that the local governments act strategically toward the ex post central policy summarized by $\bar{g}(R)$ and $\bar{G}(R)$ but they are Nash players toward one another. To be precise, each local government solves the following optimization with respect to its own tax rate t_i taking t_j ($j \neq i$) as given:

$$\text{MAX}_{t_i} V_i = u(y_i - t_i) + (1 - \lambda)v(\bar{g}(R)) + \lambda E(N\bar{g}(R)) + \Phi(\bar{G}(R))$$

The first order condition becomes

$$\frac{\partial V_i}{\partial t_i} = \frac{n_i}{N} \left\{ (1 - \lambda)v'(\bar{g}(R)) + \lambda NE'(N\bar{g}(R)) \right\} \frac{d\bar{g}}{dR} + \Phi'(\bar{G}(R)) \frac{d\bar{G}}{dR} - u'(y_i - t_i) = 0. \quad (20)$$

Applying lemma 2, (20) reduces to

$$u'(\hat{c}_i) = \frac{n_i}{N} \{ (1 - \lambda)v'(\hat{g}) + \lambda NE'(N\hat{g}) \} \quad (21)$$

The right hand side is the marginal benefit of raising tax from the local standpoint that is discounted at the rate of n_j/N whereas the left hand side is regionally born cost of taxation.

Sub-Game Perfect Equilibrium:

Then the time consistent (subgame perfect) equilibrium is characterized by the system of equations derived from the ex ante and ex post decision makings.

$$(1 - \lambda)v'(\hat{g}) + \lambda NE'(N\hat{g}) = \Phi'(\hat{G}) \quad (22.1)$$

$$u'(\hat{c}_i) = \frac{n_i}{N} \{(1 - \lambda)v'(\hat{g}) + \lambda NE'(N\hat{g})\}; \quad (22.2)$$

$$\sum_{i=1}^J n_i \hat{c}_i + N(\hat{g} + \hat{G}) = Y \quad (22.3)$$

In contrast with Scenario A, we have under-provision of the local public goods compared to the first best as stated in Proposition 2. (The proof is similar to the one for Proposition 1)

Proposition 2

Irrespective of the degree of spillover, we have

- (a) $\hat{g} < g^*, \bar{c} > c^*, \hat{G} < G^*$ where $\bar{c} = \sum_{i=1}^J n_i \hat{c}_i / N$
- (b) \hat{c}_i takes a larger value for smaller region, namely, $\hat{c}_1 > \hat{c}_2$ if $n_1 < n_2$.
- (c) \hat{c}_i is excessive in the sense that $\left. \frac{dW}{dc_i} \right|_{c_i=\hat{c}_i} < 0$, given $c_j = \hat{c}_j$ ($j \neq i$)

The proof of the above proposition is essentially the same as Proposition 1. The intuition is straightforward. Ex post equalization of the fiscal capacities give rise to ex ante free riding motive among the regions lowering the tax collection efforts. Such free riding incentive is exacerbated for less populated regions as stated in Proposition 2(b). The upshot is that all regions end up with being unfunded in the

sense that both local and national public services are under-provided relative to the first best. We see that even with $\lambda = 1$, the under-provision is not solved.

Noting that $\hat{g} = \hat{t}_i + \hat{S}_i/n_i$ and $\hat{c}_i = y_i - \hat{t}_i$, per capita transfer to region i can be calculated by $\hat{S}_i/n_i = \hat{g} - \hat{t}_i = \hat{g} + \hat{c}_i(n_i) - y_i$. From Proposition 2(b), per capita transfer is larger for less populated and/or less wealthy regions. This result is different from Wildasin (1997) that raises the case of the “too big to fail” with larger regions being more likely to be bailed out. The difference between the present paper and Wildasin lies on the following. First, the latter assumes that the central government represents interest of the non-bailing out regions ex post, namely pursuing the sum of these regions’ welfare. On the other hand, in the present model, the central government is ex post concerned with social welfare W with positive weights being placed on all regions. Second, we consider that the central authority possesses maximal discretion ex post in allocating the grants across regions, the local per capita expenses being fully equalized, whereas in Wildasin (1997), the central government is allowed only to increase the transfer to the bailing out region adjusting G to balance the central budget, with S_j to other (non-bailing out) regions being kept at the first best value. We do not intend to discuss which model is more plausible but it is noteworthy that the too big to fail principle is model specific and lacks robustness. Our result that smaller regions are more easily rescued or treated favorably is consistent with the observation by Von Hagen and Dahlberg (2002) in the context of Swedish local public finance although they have addressed political economy consideration.

Ex Post Optimization of the Central Tax:

So far we have assumed that $\tau = 0$. This assumption is irrelevant in Scenario A since

the ex post central tax is redundant. For Scenario B, however, the ex post optimization of τ makes difference. The first order condition for the central tax rate is given by

$$\sum_i n_i u'((1-\tau)y_i - t_i)y_i / Y = \Phi'(G)$$

(23)

Suppose that regions are identical in all aspects. Then the ex post optimization gives $u'(c) = \Phi'(\bar{G})$, which is the first best allocation alongside with the resource constraint.

Corollary to Proposition 2

Assume that all regions are homogeneous. Then, independent of the ex ante choice of t_i , the central policy leads to the first best outcome, namely, $c_i = c^*$, $g_i = g^*$, $G_i = G^*$ for all i .

Along with the lack of the commitment, the absence of the ex post discretion of the central authority raising the tax revenue which is supposed in Wildasin (1997) and Köthenbürger (2004) as well contributes to our incentive problem in Scenario B. This corollary can be extended to the heterogeneous regions if the central tax rate can be differentiated among regions, τ_i .

It is wrong to conclude that Proposition 2 is implausible, however. In section 6, we discuss the situation where both the central and local taxes are distortionary and the central tax can be optimized ex post. It can be seen that the above Corollary is a polar case that holds only when the taxes are lump-sum. Otherwise, the sub-game perfect equilibrium under Scenario B is featured by local taxes being too low and local public goods being under-provided.

3.3 Intuition and Discussion

We have considered the two scenarios when the central government designs the transfer policy from the ex post standpoint and possesses the discretion to pursue the ex post discretion. It is revealed that the timing of decision making is critical. Under Scenario A with g_i being decided ex ante, the consequence is that the local governments expand excessively, whereas under Scenario B in which t_i is chosen ex ante, the local governments' fiscal capacities are too small. The difference is due to the nature of the ex post intergovernmental transfers. In Scenario A, they lead to "cost sharing", allowing each local government to export cost of its own expense to others generating the situation analogous to the common pool problem and thus encouraging over-spending ex ante. Scenario B yields the case of the ex post "Revenue sharing" which ex ante motivate the local government to "free-ride" on the tax collection efforts of the others. Given that $\tilde{G} < G^*$, $\hat{G} < G^*$ and $\sum_i S_i + G = 0$ with $\tau = 0$, total amount of the intergovernmental transfers become excessive relative to the first best.

The soft budget literature has focused on ex post vertical fiscal tie between the upper and lower governments. The present model shows, however, that such ex post vertical interaction brings about the horizontal externalities once we incorporate general equilibrium effect, namely ex post increase in the transfer being born nation-wide. The ex ante regional decisions of expanding g_j gives rise to "negative" externalities to others in Scenario A whereas Scenario B gives the case that the ex ante tax collection efforts create positive externalities contributing to the ex post shared fund. Such horizontal externalities have been addressed by the decentralization leadership literature in which the externalities associated with the ex post cost sharing are exactly

matched with those arising from the free riding motive when g_i is a pure public good. The present model synthesizing the two literature establishes that the ex ante inefficiency raised by the soft budget literature is robust with the Pareto optimality in the decentralized leadership being a polar case although the direction of the ex ante distortions relies on the timeline of decisions.

Our model formulated closer to the decentralized leadership one differs from the standard setting of the soft budget problem in a few aspects. First, we do not account for uncertainty associated with local public projects and with the central government's commitment ability. Such uncertainty can be easily incorporated. With the uncertainty of the first sort, namely project costs, intergovernmental transfers serve as insurance device, but introduce some moral hazard behavior taking excessive risk at the local level that must be incorporated from the second best standpoint. In the absence of commitment, however, the central government will allocate grants based upon cost realizations, leading to the cost sharing as described under Scenario A, which induces local governments to undertake too risky projects. We can also consider two types of the center with and without commitment, and then the ex ante decisions of local governments rely on their prospect for central government type.

Second, the soft budget literature supposes that the ex post decision of bailing out indebted or overspending regions is occasional and explicit involving policy change from the ex ante announcement, whereas such deviation is not obvious in the present model. Our model allows that the ex post rescue can be frequent and implicit with grants formula being manipulated in a way to reflect the ex post optimum; the formula of intergovernmental transfers could be math to rationalize an intended allocation (Bird(1994)). More generous transfers can be made to compensate overspending regions

but in the name of internalizing spillovers and/or accounting for region specific fiscal needs.

To see it more closely, note that in the sub-game perfect equilibrium, the ex post transfers can be expressed in terms of regional population and income. Under Scenario A, for instance, we have $S_i/n_i \equiv \tilde{s}_i = \tilde{g}(n_i) + \tilde{c} - y_i$ where $\tilde{g}_i = \tilde{g}(n_i)$ and $\tilde{g}(n_i)$ is decreasing in n_i . The central government may then announce that \tilde{s}_i reflects the regional fiscal needs as function of n_i alongside with regional income/fiscal capacity y_i and lump-sum component \tilde{c} , although of course, it is $s_i = g_i + \bar{c}(M) - y_i$ that is anticipated by the local governments. The ex post formula based transfer may explicitly contain the cost sharing component, say $\tilde{m}_i = 1 - n_i/N$. But this ex post optimal matching rate differs from the prospective rate to internalize the spillover $m_i = \lambda(N - n_j)E'/u'$. The soft or hard budget does not reply on the presence or absence of cost sharing but upon whether the matching rate is optimized from ex ante or ex post standpoints. We can discuss likewise under Scenario B as well.

One may claim that ex post the local governments are still constrained by own budgets given that t_i is adjusted ex post after the transfer so that $t_i = g_i - \tilde{S}_i/n_i$ for instance under Scenario A. As illustrated in Section 3, however, if we substitute $s_i = g_i + \bar{c}(M) - y_i$ into this local budget constraint, we obtain $t_i = \bar{c}(M) - y_i$, which is the constraint perceived by the local governments ex ante. Along with the central optimization ex post, $\bar{c}(M)$ is determined dependent upon (11). This in turn implies

that the ex ante decisions of the local governments are constrained by the economic wide resource constraint with the governments' budgets being integrated ex post through the transfers, but not by own budget . In this regard, the local budgets are softened (from the ex ante perspective).

We have supposed that the central government is benevolent and thus the soft budget problem is akin to the Samaritan's Dilemma. The two are not synonymous, however. The former could arise even when the incumbent central government is politically motivated, say to assure re-election as formulated in Goodspeed (2002). The ex post grants allocation will then be favorable to politically influential regions. Ex ante politically favored regions will shark and/or the local governments may act strategically to enhance the ex post favor say undertaking lobbying activities.

A possible objection against the strategic behavior of the local governments may be that it is informationally demanding for individual local governments to foresee how the central grants policy responds to their ex ante choices especially when the grants are determined upon a complicated formula and especially when there are a large number of regions. All we need to establish our argument is, however, the local governments' prospect of the central authority ultimately bearing the fiscal burden to pursue the inter-regional equity rather than their detailed knowledge of the computation of ex post transfers.

4 Capital Tax Competition

In the benchmark model, we have assumed that the local government can levy lump-sum tax. In the following, we turn to the case where local governments finances

their expenses with their tax base being inter-regionally mobile giving rise to tax competition among regions.

The regions faces identical production function per person, $f(K_i)$, where K_i is a capital level per person and $f(K_i)$ is strictly concave. Then income per person becomes $y_i = f(K_i) - f'(K_i)K_i + \rho\bar{K}$, where ρ is net of tax return on capital and \bar{K} is initial endowment per person and the capital endowment is assumed to be equally distributed among regions. In the present model, therefore, inter-regional heterogeneity arises solely due to difference in regional population.

Given the tax rate on capital, t_i , the profit maximizing company selects the level of capital according to $f'(K_i) = \rho + t_i$. This determines the capital demand, which is described as $K_i = k(\rho + t_i)$ with $k' = 1/f'' < 0$.

Capital market equilibrium is given by $\sum_i n_i k(\rho + t_i) = \sum_i n_i \bar{K}$, which determines the level of capital return per unit as a function of capital tax rates in all regions, namely, $\rho = \rho(t_1, \dots, t_I)$ with $\partial\rho/\partial t_i < 0$. It is well known that the absolute value of $\partial\rho/\partial t_i$ is larger for more populated regions. Accounting for the equilibrium condition, per capita capital and income in each region are described as $K_i = k(\rho(t_1, \dots, t_I) + t_i)$, $y_i = y(\rho(t_1, \dots, t_I), t_i)$.

4.1 Scenario A; Expenditure level is selected ex ante

Stage 2 Ex post behavior of the central government

Given that g_i is decided ex ante and S_i is transferred, t_i is adjusted such as to balance the budget, $t_i n_i K_i (\rho(t_1, \dots, t_i) + t_i) = n_i g_i - S_i$, implying that if $n_i g_i - S_i$ is same in all regions, the tax rates become identical among them.

Noting $c_i = y_i$ in this section, the central government aim to maximize W subject to its budget constraint with respect to S_i and G :

$$\underset{S_i, G}{MAX} W = \sum_{i=1}^I n_i [u(y(\rho(t_1, \dots, t_N), t_i)) + v(g_i) + \Phi(G)]$$

subject to $NG + \sum_{i=1}^I S_i = 0$.

It is straightforward to see that $t_i = t$ achieves efficient allocation of capital across regions, equalizing the marginal productivities and thus maximizing the national output. In addition, $t_i = t$ leads to the equalization of $c_i = y_i$. The technology or per capital production, which is identical among regions, leads to $K_i = K_j = \bar{K}$, which in turn equalize the wages. The consumption equalization is desirable from the equity or social welfare maximizing standpoint. Given this situation, the ex post optimum is to set the subsidy so as to realize $t_i = t$.

To be more specific, the ex post subsidy is determined to fulfill $NG + \sum_{i=1}^I S_i = 0$ and

$tn_i\bar{K} = n_i\bar{g}_i - S_i$. Then we have $t = \frac{1}{\bar{K}N}(NG + \sum_{i=1}^l n_i g_i)$. On the other hand, income per

person is given by $y_i = \bar{y} = f(\bar{K}) - f'(\bar{K})\bar{K} + \rho\bar{K}$, which reduces to:

$$y_i = \bar{y} = f(\bar{K}) - t\bar{K} = f(\bar{K}) - \frac{1}{N}(NG + \sum_{i=1}^l n_i g_i) = \bar{c} \quad (24)$$

Therefore, the central government's problem becomes

$$\underset{G}{MAX} W = \sum_{i=1}^l n_i U(c_i, g_i, G) = \sum_{i=1}^l n_i \left[u\left(f(\bar{K}) - \frac{1}{N}(NG + \sum_{i=1}^l n_i g_i)\right) + v(g_i) + \Phi(G) \right]$$

leading to the first order condition of

$$u'(\bar{c}) = \Phi'(G) \quad (25).$$

Denote equalized consumption and the central public service by \bar{c} and \bar{G} , which are

determined by (25) and $\bar{c} + \bar{G} = f(\bar{K}) - \frac{1}{N} \sum_{i=1}^l n_i g_i \equiv M$ and given as the function of

M , namely, $\bar{c}(M)$ and $\bar{G}(M)$ to which Lemma 1 applies. .

Stage 1 Ex ante Behavior of the local government

Turn to Stage 1. Ex ante, each local government solves the following:

$$\underset{g_i}{MAX} V_i = u(\bar{c}(M) + v(g_i) + \Phi(\bar{G}(M)))$$

The first order condition becomes

$$\frac{\partial V_i}{\partial g_i} = -\frac{n_i}{N} u'(c) \left(\frac{d\bar{c}}{dM} + \frac{d\bar{G}}{dM} \right) + v'(g_i) = 0. \quad (26)$$

Noting Lemma 1, the above reduces to

$$\frac{n_i}{N}u'(\tilde{c}) = v'(\tilde{g}_i). \quad (27)$$

The time consistent (subgame perfect) equilibrium is characterized by

$$u'(\tilde{c}) = \Phi'(\tilde{G}), \quad \frac{n_i}{N}u'(\tilde{c}) = v'(\tilde{g}_i), \quad N(\tilde{c} + \tilde{G}) + \sum_{i=1}^I n_i \tilde{g}_i = Nf(\bar{K}). \quad (28)$$

Comparing with the condition of the first best allocation, we can establish Proposition 3:

Proposition 3

- (a) $\bar{g} > g^*, \tilde{c} < c^*, \tilde{G} < G^*$ where $\bar{g} = \sum_{i=1}^J n_i \tilde{g}_i / N$.
- (b) \tilde{g}_i takes a larger value for smaller region, namely, $\tilde{g}_1 > \tilde{g}_2$ if $n_1 < n_2$.
- (c) \tilde{g}_i is over-provided in the sense that $\left. \frac{d}{dg_i} W \right|_{g_i = \tilde{g}_i} < 0$

In the presence of capital tax competition, we have the same results with the local public service being excessive whereas the national public services being under-provided. It is noteworthy that this result is as opposed to the case of standard tax competition.

4.2 Scenario B; Tax level is selected ex ante

This is the scenario considered in Köthenbürger (2004). Our model is different in that we account for the heterogeneity across regions in terms of regional population whereas Köthenbürger (2004) focuses on the symmetric equilibrium with identical regions.

Stage 2 Ex post behavior of the central government

Given that t_i is decided ex ante and S_i is transferred, g_i is adjusted to balance the budget. Then ex post total welfare is expressed by:

$$W = \sum_{i=1}^I n_i U(c_i, g_i, G) = \sum_{i=1}^I n_i \left[u(y_i) + v(t_i K_i(\rho(t_1, \dots, t_I) + t_i) + \frac{S_i}{n_i}) + \Phi(G) \right]$$

where

$$g_i = t_i K_i(\rho(t_1, \dots, t_I) + t_i) + \frac{S_i}{n_i}$$

The central government chooses S_i and G so as to maximize W subject to

$NG + \sum_{i=1}^I S_i = 0$, giving rise to the following first order conditions for G and S_i .

$$v'(t_i K_i(\rho + t_i) + \frac{S_i}{n_i}) = v'(t_j K_j(\rho + t_j) + \frac{S_j}{n_j}) = \Phi'(G). \quad (29)$$

Again the expenditure level is perfectly equalized, that is, $g_i = g_j \equiv \bar{g}$. S_i works as horizontal equalization of local public services.

Substituting $\frac{S_i}{n_i} = t_i K_i(\rho + t_i) - \bar{g}$ into the central budget constraint and re-arranging

yield $NG + \sum_{i=1}^I n_i (t_i K_i(\rho + t_i) - \bar{g}) = 0$, which is rewritten as

$$N(\bar{G} + \bar{g}) = \sum_{i=1}^I n_i t_i K_i(\rho + t_i) \quad (30)$$

\bar{g} and \bar{G} are determined by solving the above (29) and (30), so that they are given as the function of

$$R \equiv (1/N) \left(\sum_{i=1}^I n_i t_i K_i(\rho(t_1, \dots, t_I) + t_i) \right) \quad (31)$$

namely, $\bar{g}(R)$ and $\bar{G}(R)$. We have

$$1 = \frac{d\bar{g}}{dR} + \frac{d\bar{G}}{dR}, \quad (32)$$

namely., Lemma 2 holds.

Stage 1 Ex ante Behavior of the local government

Accounting for the ex post central policy, which is summarized by $\bar{g}(R)$ and $\bar{G}(R)$, at stage 1, the local governments chooses t_i to maximize the local utility in region i :

$$\text{MAX}_{t_i} V_i = u(y_i) + v(\bar{g}(R)) + \Phi(\bar{G}(R)) .$$

The first order condition becomes

$$\frac{\partial V_i}{\partial t_i} = v'(g) \left(\frac{d\bar{g}}{dR} + \frac{d\bar{G}}{dR} \right) \frac{dR}{dt_i} + u'(y_i) \frac{dy_i}{dt_i} = 0. \quad (33)$$

Inserting $\frac{dy_i}{dt_i} = -K_i + \frac{d\rho}{dt_i}(\bar{K}_i - K_i)$ and noting (32), it reduces to:

$$u'(y_i) \left(K_i - \frac{d\rho}{dt_i}(\bar{K}_i - K_i) \right) = v'(g_i) \frac{n_i}{N} \left(K_i + t_i \frac{1}{f''(k_i)} + \left(\frac{d\rho}{dt_i} \right) \sum_i t_i \frac{1}{f''(k_i)} \right) \quad (34)$$

Let us focus on the symmetric equilibrium such that $K_i = K_j = \bar{K}$. Then $t_i = t$ and we

have $y_i = y$. We also have $\frac{d\rho}{dt_i} = -\frac{1}{N}$. Therefore we have $u'(\hat{c}_i) = \frac{n_i}{N} v'(\hat{g}_i)$ implying

that $\hat{g}_i < g_i^*$. Then in the symmetric equilibrium, the time consistent (subgame perfect)

equilibrium is characterized by

$$v'(\hat{g}) = \Phi'(\hat{G}), \quad u'(\hat{c}_i) = \frac{n_i}{N} v'(\hat{g}) \quad \text{and} \quad \sum_{i=1}^I \hat{c}_i + I(\hat{g} + \hat{G}) = Y \quad (35)$$

The comparison with the first best condition establishes that $\hat{g} < g^*$, i.e., under-provision of the local public service. This Scenario is the one of Köthenbürger (2004) addressing that the ex post discretionary transfer may not resolve the problem of under-taxation due to capital tax competition.

The model can be easily extended to the case of heterogeneous regions with respect to the productivity. For the sake of simplicity, assume that the production function is quadratic so that f'' is constant and $\frac{d\rho}{dt_i} = -\frac{1}{N}$. Then the equilibrium condition for the ex ante choice of the tax rate is written as:

$$u'(y_i) \left(1 + \frac{1}{NK_i} (\bar{K}_i - K_i) \right) = v'(g_i) \frac{n_i}{N}$$

If region i is exporting capital and thus $K_i < \bar{K}$, the parenthesis on the left hand side is larger than unity, and along with the right hand side representing the free riding motive due to the ex post revenue sharing, we have the under-provision of g . In the case that the region imports the capital, on the other hand, the strategic motive of exporting capital tax burden to the non-residents through lowering the net of tax return leads such a region to excessively increase t_i , which must be compared with the free riding motive.

Proposition 4:

[1] Köthenbürger (2004); Assume the symmetry so that $K_i = K_j = \bar{K}$. Then, even in the case with distortionary taxation, we have the same results, which are

(a) $\hat{g} < g^*, \hat{c}_i > c^*, \hat{G} < G^*$

(b) \hat{c}_i takes a larger value for smaller region, namely, $\hat{c}_1 > \hat{c}_2$ if $n_1 < n_2$.

[2] Consider that regions differ in terms of productivity, but assume that the production technology is quadratic. Then more productive region importing more capital from the outside or region with less endowment of capital levies a lower tax rate, exacerbating the free riding due to the ex post revenue sharing.

5 Distortionary Central and Local Taxes

Now we allow the central tax rate to be optimized ex post but suppose that the central and local taxes are distortionary. We consider the equilibrium consequence under Scenario B in which local tax rates are decided ex ante. For the sake of simplicity, we assume that all regions are identical so that we can focus on the symmetric equilibrium. In Corollary to Proposition 2, it is stated that the first best can be achieved once τ is optimized ex post. It is established however that this is not valid in the present context, but the equilibrium is characterized by under taxation.

Distortionary Taxes:

We consider that the per capita central and local tax bases denoted by b_i and B_i are elastic with respect to the tax rates so that

$$b_i \equiv b(t_i, \tau) \quad \text{and} \quad B_i \equiv B(t_i, \tau). \quad (36)$$

with

$$\frac{\partial b}{\partial t_i} \leq 0 \quad \text{and} \quad \frac{\partial B}{\partial \tau} \leq 0$$

where t_i and τ are respectively local and central tax rates. If τ is wage income tax

rate, labor supply may be declining with it being raised and thus lower wage income B_i .

We can imagine other margins of response to taxation. Instead of discouraging working incentive, the tax may induce tax planning activities such as rearranging their income to tax favorable forms, which in turn decreases taxable income. The present model incorporates general behavioral response as has been formulated in Slemrod and Kopczuk (2002). We can interpret elasticity of the local tax base likewise.

Tax Externalities:

The distortionary nature does not only give rise to economic cost of taxation but also

may lead to vertical tax externalities that may be positive or negative. Beside the vertical one, we may have horizontal tax externality among local governments. Köthenbürger (2004) consider capital tax competition in the context of decentralized leadership. In the following we abstract inter-regional competition to highlight our point.

Suppose that the central and local governments share the same tax base. It is known that unilateral tax increase by one government imposes negative externality on tax revenue to another level government tax base being decreased (Boadway and Keen (1996)). If this is so, we can write $b(t_i + \tau) = B(t_i + \tau)$ with $b' = B' < 0$. In more general context, their tax bases may be imperfectly overlapped. For instance, the central government levies comprehensive income tax while local income taxation is limited to payroll. Alternatively, wage income taxation may be exclusive to the center and local governments may rely on consumption taxes. Even so, the tax externalities are present. They disappear when the central and local tax bases are perfectly separated, i.e., the two level government levy different goods that are independent one another.

Resident's utility

Given the tax parameters, the residents maximize own utilities that give arise to the indirect utility by $u(t_i, \tau)$ with:

$$\frac{\partial u}{\partial t_i} = -\alpha_i b_i \quad \text{and} \quad \frac{\partial u}{\partial \tau} = -\alpha_i B_i, \quad (37)$$

where λ_i is marginal utility of income. The total utility including benefits of the public goods can be expressed by

$$U = u(t_i, \tau) + v(g_i) + \Phi(G). \quad (38)$$

Government's budget constraint

The budget constraint of the local government is written as:

$$n_i t_i b_i + S_i = n_i g_i \quad (39)$$

Turning to the central budget, it becomes:

$$NG + \sum_{i=1}^I S_i = \tau \sum_{i=1}^I n_i B_i . \quad (40)$$

Ex post, the central government decides S_i that effectively integrates the central and local budgets to yield:

$$NG + \sum_{i=1}^I n_i g_i = \tau \sum_{i=1}^I n_i B_i + \sum_{i=1}^I n_i t_i b_i \quad (41)$$

Stage 2 Ex post behavior of governments

Given that t_i is decided by the local government ex ante, the behavior of the central government becomes to select g_i , G and τ , subject to the combined budget constraint:

$$\underset{g_i, G, \tau}{MAX} W = \sum_{i=1}^I n_i \left[\tilde{u}(t_i, \tau) + \left\{ (1-\lambda)v(g_i) + \lambda E \left(\sum_{j=1}^I n_j g_j \right) \right\} + \Phi(G) \right]$$

subject to (41). The first order conditions for G and g_i become

$$(1-\lambda)v'(g_i) + \lambda NB' \left(\sum_{i=1}^I n_i g_i \right) = (1-\lambda)v'(g_j) + \lambda NE' \left(\sum_{i=1}^I n_i g_i \right) = \Phi'(G) = \mu \quad (42)$$

where μ is the Lagrangian multiplier, implying that the expenditure level is perfectly equalized so that $g_i = g_j \equiv \bar{g}$. The first order condition for τ is given by:

$$\mu \left\{ \sum_{i=1}^I n_i B_i + \tau \sum_{i=1}^I n_i \frac{\partial B_i}{\partial \tau} + \sum_{i=1}^I n_i t_i \frac{\partial b_i}{\partial \tau} \right\} = \sum_{i=1}^I n_i \alpha_i B_i . \quad (43)$$

In the symmetric equilibrium, we have

$$\mu(B + \tau B_\tau + t b_\tau) = \alpha B \quad (44)$$

Denoting by \bar{G} and \bar{g} respectively ex post optimums of the central and local public services, the resource allocation has to fulfill

$$N(\bar{G} + \bar{g}) = \tau \sum_{i=1}^I n_j B_i + \sum_{i=1}^I n_j t_i b_i \quad (45)$$

Stage 1 Ex ante Behavior of the local government

Accounting for the ex post central policy, the local governments chooses t_i to maximize own residents' utility, that is,

$$\underset{t_i}{MAX} V_i = \left[u(t_i, \tau) + \{(1-\lambda)v(\bar{g}) + \lambda E(N\bar{g})\} + \Phi(\bar{G}) \right]$$

By using (42) and (44), in the symmetry, we have³

$$\begin{aligned} \frac{dV_i}{dt_i} &= \frac{n}{N} \left\{ \mu(b + \tau B_i + t b_i) \right\} - \alpha b . \\ &= \frac{n}{N} \mu \left\{ t \left(b_i - \frac{b}{B} b_\tau \right) + \tau \left(B_i - \frac{b}{B} B_\tau \right) \right\} - \left(1 - \frac{n}{N} \right) \alpha b \end{aligned} \quad (46)$$

³ This is derived as follows. First, using (37) and (42), we have

$$\begin{aligned} \frac{dV_i}{dt_i} &= -\alpha_i \left(b_i + B_i \frac{d\tau}{dt_i} \right) + \left\{ (1-\lambda)v'(\bar{g}) + \lambda N B'(N\bar{g}) \right\} \frac{d\bar{g}}{dt_i} + \Phi'(G) \frac{d\bar{G}}{dt_i} \\ &= -\alpha_i \left(b_i + B_i \frac{d\tau}{dt_i} \right) + \mu \left\{ \frac{d\bar{g}}{dt_i} + \frac{d\bar{G}}{dt_i} \right\} \end{aligned}$$

Totally differentiating (45) with respect to t_i , we have

$$\frac{d}{dt_j} (\bar{G} + \bar{g}) = \frac{n_j}{N} \left\{ b_j + \left(t \frac{\partial b_j}{\partial t_j} + \tau \frac{\partial B_j}{\partial t_j} \right) \right\} + \frac{1}{N} \left\{ \left(\sum_{i=1}^I n_i \left(B_i + \tau \frac{\partial B_i}{\partial \tau} + t_i \frac{\partial b_i}{\partial \tau} \right) \right) \frac{d\tau}{dt_j} \right\} .$$

In the symmetric equilibrium, we have

$$\frac{d}{dt_j} (\bar{G} + \bar{g}) = \frac{n}{N} (b + t b_i + \tau B_i) + (B + \tau B_\tau + t b_\tau) \frac{d\tau}{dt_j} .$$

Therefore, using (44), we have (46) as follows.

$$\begin{aligned} \frac{dV_i}{dt_i} &= -\alpha b - \mu (B + \tau B_\tau + t b_\tau) \frac{d\tau}{dt_i} + \mu \left\{ \frac{n}{N} (b + t b_i + \tau B_i) + (B + \tau B_\tau + t b_\tau) \frac{d\tau}{dt_i} \right\} \\ &= \mu \frac{n}{N} (b + t b_i + \tau B_i) - \alpha b \end{aligned}$$

The second equality is for the later use. Denote by \hat{t} the symmetric equilibrium value of the local tax rate. If the solution is interior, i.e., $\hat{t} > 0$, we establish:

$$\frac{n}{N}\mu(b + \tau B_t + tb_t) = \alpha b \quad (47)$$

(47) is not always the case. At this point, let us consider two polar cases. First assume that neither t nor τ is distortionary, so that $b_t = b_\tau = B_t = B_\tau = 0$. Then the last equality in (46) reduces to

$$\frac{dV_i}{dt_i} = -\left(1 - \frac{n}{N}\right)\alpha b < 0. \quad (48)$$

This implies that we establish $\hat{t} = 0$ given that t is restricted to non-negative. Second, both t and τ are levied on the completely overlapped tax base, $b_t = b_\tau = B_t = B_\tau$. Again, (46) becomes coincident with (48). Therefore, we have $\hat{t} = 0$ in this case as well.

Welfare implication

The welfare implication of the equilibrium can be examined by simultaneously differentiating $V = \tilde{u}(t, \tau) + v(\bar{g}) + \Phi(\bar{G})$ with respect to t accounting for $\bar{G} + \bar{g} = \tau B + tb$ and evaluating the derivative at $t = \hat{t}$:

$$\left. \frac{dV}{dt} \right|_{t=\hat{t}} = -\alpha b + \mu(b + \tau B_t + tb_t) \quad (49)$$

First consider the case of $\hat{t} > 0$. Then substituting (47), the above reduces to

$$\left. \frac{dV}{dt} \right|_{t=\hat{t}} = -\alpha b + \frac{N}{n}\alpha b > 0, \quad (50)$$

which implies that \hat{t} is too small. Second, suppose that we have $\hat{t}=0$ in the symmetric equilibrium. Then making use of (44), we can establish:

$$\left. \frac{dV}{dt} \right|_{t=\hat{t}=0} = \mu(b + \tau B_t) - \frac{b}{B} \mu(B + \tau B_\tau) = \mu \tau \left(B_t - \frac{b}{B} B_\tau \right). \quad (51)$$

Now we apply the Slutsky decomposition as $B_t = B_t^c - bB_M$ and $B_\tau = B_\tau^c - BB_M$, where B_t^c and B_τ^c represent the compensated term of B_t and B_τ . B_M is the income effect term. Then we can establish:

$$\left. \frac{dV}{dt} \right|_{t=\hat{t}=0} = \mu \tau \left(B_t^c - bB_M - \frac{b}{B} (B_\tau^c - BB_M) \right) = \mu \tau \left(B_t^c - \frac{b}{B} B_\tau^c \right) = \mu b (\varepsilon_\tau^b - \varepsilon_\tau^B), \quad (52)$$

where $\varepsilon_\tau^b = \tau \frac{b^c}{b}$ and $\varepsilon_\tau^B = \tau \frac{B_\tau^c}{B}$. The last equality comes from $b^c = B_t^c$. ε_τ^b and

ε_τ^B are the compensated elasticities of the local and the central tax bases, respectively with respect to τ with $\varepsilon_\tau^B > 0$. It is plausible to assume $\varepsilon_\tau^B < \varepsilon_\tau^b$ where the sign of the latter depends on whether the central tax base is substitute or complementary with the local one.

We have $\varepsilon_\tau^B = \varepsilon_\tau^b$ so (52) is zero implying that the welfare is maximized in the equilibrium if the tax bases are completely overlapped or the tax bases are not elastic i.e., the elasticities are zero. In so far as $\varepsilon_\tau^b > \varepsilon_\tau^B$, (52) takes positive value so the equilibrium local tax is too low. Then the following proposition is established.

Proposition 5

Suppose that the central tax is optimized ex post.

- (a) When the central and local tax bases are completely overlapped so that $\varepsilon_r^b = \varepsilon_r^B$ or both taxes are non-distortionary, we have $\hat{t} = 0$, which is the second best optimal in the former and the first best in the latter.
- (b) Insofar as $\varepsilon_r^b > \varepsilon_r^B$, \hat{t} is too low, compared with the social optimal level.

Except polar cases in Proposition 5(a), which corresponds to the case discussed in Corollary to Proposition 2, we can conclude that the under-taxation under Scenario B is relevant characteristic when the central tax and transfer policies are optimized from ex post standpoint which is foreseen ex ante by the local governments.

6 Two Period Model: Investment for enhancing Tax Base

Finally we consider another type dynamic model with two periods and investment. We show that the similar inefficiency by soft budget constraint is created. This inefficiency is derived by the ex ante decision of investment in the first period and ex post bailout in the second period, different from the previous models. Instead, we examine two cases with and without local government borrowing.

6.1 Basic Setting

Consider that the economy lasts two periods. In the first period, each local government spends I^g , public investment, that enhances the regional production in the second period and g , the first period public consumption. In the following, G_i represents the local public service for the consumption purpose in the second period. We assume that the regional production, that is, the regional income, in the second period is endogenous

and produced by the first period public investment as follows: $y_i = y(I_i^g)$.

Resident's budget and utility

Write by c_i^t and t_i^t , private consumption and local tax level in region i in time t .

Initial endowment in each region is denoted by z_i . Then consumption in each period can be written as

$$c_i^1 = z_i - t_i^1, \quad c_i^2 = y(I_i^g) - t_i^2 \quad (53)$$

Assuming that resident's utility is from consumption and public good in each period and is separable, utility in region i is given by:

$$U(c_i^1, c_i^2, g_i, G_i) = u_1(z_i - t_i^1) + v(g_i) + u_2(y(I_i^g) - t_i^2) + \Phi(G_i), \quad (54)$$

where the discount rate is assumed to be zero.

Government's budget constraint and Intergovernmental transfer

The budget constraint of the local government in period 1 becomes

$$t_i^1 + b_i = g_i + I_i^g, \quad (55)$$

where b_i represents local borrowing per level capita. The budget constraint in period 2 becomes

$$t_i^2 + S_i / n_i - b_i = G_i, \quad (56)$$

where, for simplicity, the interest rate for the borrowing is assumed to be zero and S_i denotes the subsidy from the central government to the region, similar to the former sections. We again assume that S_i can go to either sign subject to $\sum_{i=1}^I S_i = 0$; the

negative transfer implies that the central government taxes local government. The central government possesses full discretion over S_i so as to maximize own objective.

The objectives of Central and Local governments

The central government decides the transfer level such as to maximize the total utility of regions, that is, $\sum_i n_i U(c_i^1, c_i^2, g, G_i)$. The local government, on the other hand, decides the level of public services such as to maximize the utility of own region, that is, $U(c_i^1, c_i^2, g, G_i)$, which is,

$$U(c_i^1, c_i^2, g_i, G_i) = u_1(z_i - t_i^1) + v(g_i) + u_2(y(I_i^g) - t_i^2) + \Phi(G_i). \quad (57)$$

Accounting for the local budget constraint, it reduces to:

$$U(c_i^1, c_i^2, g_i, G_i) = u_1(z_i - g_i - I_i^g + b_i) + v(g_i) + u_2(y(I_i^g) - t_i^2) + \Phi(t_i^2 + S_i / n_i - b_i). \quad (58)$$

Given this basic setting, we analyze the effect of ex post transfer by the central government on the local government ex ante decision of I_i^g and b_i . We examine two cases where (i) b_i is ex ante regulated by the central government and (ii) b_i is freely issued.

6.2 First best optimal allocation

As a benchmark, let us consider the first best allocation that is determined by maximizing the social welfare W subject to the resource constraint:

$$\underset{c_i^1, c_i^2, g_i, G_i, I_i^g}{MAX} W = \sum_i n_i U(c_i^1, c_i^2, g_i, G_i)$$

$$\text{subject to } \sum_{i=1}^I n_i c_i^1 + \sum_{i=1}^I n_i g_i + \sum_{i=1}^I n_i I_i^g + \sum_{i=1}^I n_i c_i^2 + \sum_{i=1}^I n_i G_i = \sum_{i=1}^I n_i y(I_i^g) + \sum_{i=1}^I n_i z_i .$$

The first best allocation is characterized by

$$u_1'(c^{1**}) = v'(g^{**}) = u_2'(c^{2**}) = \Phi'(G^{**}) \quad \text{and} \quad y'(I_i^{g**}) = 1 \quad (59)$$

6.3 Ex Post Behavior of the central and local governments

Given that $y(I_i^g)$ and b_i are decided ex ante and S_i is transferred, the local government decides t_i^2 to maximize the ex post, second period regional utility,

$$\text{MAX}_{t_i^2} u_2(y(\bar{I}_i^g) - t_i^2) + \Phi(t_i^2 + \bar{S}_i/n_i - \bar{b}_i),$$

The first order condition becomes

$$u_2'(y(\bar{I}_i^g) - t_i^2) = \Phi'(t_i^2 + S_i/n_i - \bar{b}_i). \quad (60)$$

The central government chooses S_i to maximize ex post social welfare subject to the budget constraint, which is

$$\text{MAX}_{S_i} \sum_{i=1}^I n_i \{u_2(y(\bar{I}_i^g) - \bar{t}_i^2) + \Phi(\bar{t}_i^2 + S_i/n_i - \bar{b}_i)\} \quad \text{subject to} \quad \sum_{i=1}^I S_i = 0.$$

The first order condition becomes

$$\Phi'(G_i) = \Phi'(G_j). \quad (61)$$

The second period public service is perfectly equalized ex post, $G_i = \bar{G}$. Noting that (60), we show that the second period consumption in each region is also perfectly equalized ex post, that is, $c_i^2 = \bar{c}^2$.

The ex post optimal subsidy level is described as

$$S_i = n_i \bar{c}^2 + n_i \bar{G} - n_i y(I_i^g) + n_i b_i. \quad (62)$$

Inserting this into the central budget, we have

$$N\bar{G} + N\bar{c}^2 = \sum_{i=1}^I n_i (y(I_i^g) - b_i). \quad (63)$$

As in the benchmark case, \bar{c}^2 and \bar{G} are determined by solving (60) and (63), and can be written as $\bar{c}^2(Z)$ and $\bar{G}(Z)$ where $Z \equiv \frac{1}{N} \sum_{i=1}^I n_i (y(I_i^g) - b_i)$. We have

$$1 = \frac{d\bar{c}^2}{dZ} + \frac{d\bar{G}}{dZ}, \quad (64)$$

similar to Lemma 1.

Now we can consider the effect of the borrowing in the first period on the ex post transfer in the second period. Since $S_i = n_i \left(\frac{1}{N} \sum_{i=1}^I n_i (y(I_i^g) - b_i) \right) - y(I_i^g) + b_i$, we have

$$\frac{\partial S_i / n_i}{\partial b_i} = 1 - \frac{n_i}{N} > 0 \quad \text{and} \quad \frac{\partial S_i / n_i}{\partial b_j} = -\frac{n_j}{N} < 0 \quad (65)$$

Again the result is different from Goodspeed (2003) that addresses the possibility that increasing one region's debt could raise the ex post transfer to another region.

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6.4 Ex ante Behavior of the local government

Case (i): b_i is centrally regulated:

Accounting for the ex post central policy, which is summarized by $\bar{c}^2(Z)$ and $\bar{G}(Z)$.

The local governments chooses g_i and I_i^g and maximize the local utility in region i , that is,

$$\text{MAX}_{g_i, I_i^g} V_i = u_1(z_i - g_i - I_i^g + b_i) + v(g_i) + u_2(\bar{c}^2(Z)) + \Phi(\bar{G}(Z)) \text{ given } b_i.$$

The first order conditions become

$$v'(g_i) = u_1'(c_i^1) = \frac{n_i}{N} u_2'(\bar{c}^2(Z)) \left(\frac{d\bar{c}^2}{dZ} + \frac{d\bar{G}}{dZ} \right) y'(I_i^g) \quad (66)$$

The central government regulates b_i to maximize the social welfare W but taking as given the local policy decisions; i.e. it acts in Nash manner as the local governments do one another. Then we have another condition

$$u_1'(\tilde{c}_i^1) = u_2'(\tilde{c}^2). \quad (67)$$

Then, we can establish

$$1 = \frac{n_i}{N} y'(\tilde{I}_i^g). \quad (68)$$

Comparing with the first best allocation, the resource allocation of c_i^1 and c_i^2 is efficient but the investment level is inefficient and too small, which is $\tilde{I}_i^g < I_i^{g**}$.

Proposition 6

- (a) $\tilde{I}_i^g < I_i^{g**}$
- (b) \tilde{I}_i^g takes a smaller value for smaller region, namely, $\tilde{I}_1^g < \tilde{I}_2^g$ if $n_1 < n_2$.

Case (ii): b_i is freely issued

Now the local government is granted free hand to borrow ex ante. The regional optimization leads to the following first order conditions:

$$v'(g_i) = u_1'(c_i^1) = \frac{n_i}{N} u_2'(\bar{c}^2(Z)), \quad u_2'(\bar{c}^2(Z)) = \Phi'(\bar{G}(Z)) \text{ and } y'(I_i^g) = 1. \quad (69)$$

Then the time consistent (subgame perfect) equilibrium is characterized by

$$u_2'(\tilde{c}^2) = \Phi'(\tilde{G}), \quad u_1'(\tilde{c}_i^1) = \frac{n_i}{N} u_2'(\tilde{c}^2), \quad v'(\tilde{g}_i) = u_1'(\tilde{c}_i^1), \quad y'(I_i^g) = 1 \quad (70)$$

alongside with the resource constraint. Comparing with the condition of the first best allocation, we have Proposition 7:

Proposition 7

- (a) $\tilde{I}_i^g = I_i^{g**}$, $\bar{b} > b^{**}$, $\tilde{c}^2 < c^{2**}$, $\tilde{G} < G^{**}$ where $\bar{b} \equiv \sum_{i=1}^J n_i \tilde{b}_i / N$.
- (b) \tilde{g}_i^1 takes a smaller value for smaller region, namely, $\tilde{g}_1^1 < \tilde{g}_2^2$ if $n_1 < n_2$.
- (c) \tilde{b}_i is over-provided in the sense that $\frac{d}{db_i} W|_{b_i=\tilde{b}_i} < 0$

Proof

Suppose $\bar{b} \leq b^{**}$. Then, similar to Lemma 1, this implies that $\tilde{c}^2 \geq c^{2**}$, $\tilde{G} \geq G^{**}$.

Then the time consistent (subgame perfect) equilibrium conditions implies $\tilde{c}^1 > c^{1**}$, $\tilde{g}_i > g_i^{**}$. On the other hand, the total budget constraint in period 1 for all

regions becomes $N\bar{b} = \sum_{i=1}^J n_i (\tilde{g}_i + \tilde{c}_i^1 - z_i + \tilde{I}_i^g)$, which leads to $\bar{b} > b^{**}$. This

contradicts however $\bar{b} \leq b^{**}$.

The social welfare in the equilibrium is given by:

$$W = \sum_{i=1}^I n_i \{u_1(c_i) + v(g_i)\} + Nu_2(\bar{c}^2(Z)) + N\Phi(\bar{G}(Z)), \quad (71)$$

where $Z \equiv \frac{1}{N} \sum_{i=1}^I n_i (y(I_i^g) - b_i)$. Differentiating above with respect to \tilde{b}_i and

evaluating the equilibrium establish:

$$\frac{1}{n_i} \frac{dW}{db_i} = v'(\tilde{g}_i) - u'(\tilde{c}^2) = \left(\frac{n_i}{N} - 1 \right) u'(\tilde{c}^2) < 0 \quad (72)$$

In the first equality, we use $\frac{d\tilde{c}_i^1}{db_i} + \frac{d\tilde{g}_i}{db_i} = 1$ and $1 = \frac{d\tilde{c}^2}{dZ} + \frac{d\tilde{G}}{dZ}$, and the second equality

comes from equilibrium conditions. QED

The above two propositions reveal the trade off associated with the restriction on the local borrowing when intergovernmental transfer is discretionary being optimized from the ex post stand point. The central regulation on b_i prevents the over-borrowing at the local level but discourages the investment to enhance the tax base due to the ex post revenue sharing. On the other hand, the ex ante local discretion on b_i leads to the over-borrowing because of the ex post cost sharing whereas the public investment turns to be optimal.

7 Other Extension: Non-Separable Utility

We have supposed that the private consumption, c , is separable from the local public service, g , which leads to ex post equalization of the private consumption under Scenario A and of the local public spending under Scenario B. In the following, we are back to the benchmark model but abstract inter-regional spillovers for simplicity. Instead we drop this assumption and consider a more general form of the utility function, that is, $U(c_i, g_i, G) = u(c_i, g_i) + \Phi(G)$, where $c_i = y_i - t_i$. Our focus is on Scenario A.

Stage 2 Ex post behavior of governments

At stage 2, given g_i , the central government chooses S_i and G to maximize W subject to the budget constraint:

$$\underset{S_i, G}{MAX} W = \sum_{i=1}^I [u(y_i - g_i - S_i/n_i, g_i) + \Phi(G)] \text{ subject to } NG + \sum_{i=1}^I S_i = 0.$$

The first order conditions are

$$u_c(y_i - g_i - S_i/n_i, g_i) = u_c(y_j - g_j - S_j/n_j, g_j) = \Phi'(G) = \mu \quad (73)$$

where μ is the Lagrange multiplier of the central budget.

The private consumption is not necessarily equalized among the regions and its extent which the consumption level is equalized depends on the relative level of g_i among regions and the degree of complementarity or substitutability between consumption and the local public service. It is immediate to see that

$$c_i \geq c_j \text{ for all } i, j \text{ such that } g_i \geq g_j \text{ if } u_{cg} > 0, \quad (74.1)$$

$$c_i \geq c_j \text{ for all } i, j \text{ such that } g_i \leq g_j \text{ if } u_{cg} < 0. \quad (74.2)$$

Inserting the ex post optimal subsidy, $S_i = n_i c_i + n_i g_i - n_i y_i$, into the central budget and rearranging, we have $NG + \sum_{i=1}^I n_i c_i = Y - \sum_{i=1}^I n_i g_i$. Making use of this resource constraint and $u_c(c_i, g_i) = u_c(c_j, g_j) = \Phi'(\bar{G}) = \mu$, we can obtain c_i and \bar{G} . Totally differentiating each of the first order conditions gives:

$$d\mu = \Phi''(G)dG, \quad d\mu = u_{cc}(c_i, g_i)dc_i + u_{cg}(c_i, g_i)dg_i \quad d\mu = u_{cc}(c_j, g_j)dc_j, \quad (75)$$

Inserting to $NdG + n_i dc_i + \sum_{j \neq i}^I n_j dc_j = n_i dg_i$, we have

$$\frac{N}{\Phi''(G)}d\mu + \frac{n_i}{u_{cc}(c_i, g_i)}(d\mu - u_{cg}(c_i, g_i)dg_i) + \sum_{j \neq i}^I n_j \frac{1}{u_{cc}(c_j, g_j)}d\mu = -n_i dg_i \quad (76)$$

Equation (76) becomes

$$\left(\frac{N}{\Phi''(G)} + \sum_{i=1}^I n_i \frac{1}{u_{cc}(c_i, g_i)} \right) d\mu = n_i \left(\frac{u_{cg}(c_i, g_i)}{u_{cc}(c_i, g_i)} - 1 \right) dg_i \quad (77)$$

Therefore we establish

$$\frac{dG}{dg_i} = \frac{dG}{d\mu} \frac{d\mu}{dg_i} = \frac{1}{\Phi''(G)} n_i \frac{\left(\frac{u_{cg}(c_i, g_i)}{u_{cc}(c_i, g_i)} - 1 \right)}{\left(\frac{N}{\Phi''(G)} + \sum_{i=1}^I n_i \frac{1}{u_{cc}(c_i, g_i)} \right)} \quad (78.1)$$

$$\frac{dc_i}{dg_i} = \frac{dc_i}{d\mu} \frac{d\mu}{dg_i} + \frac{dc_i}{dg_i} = \frac{1}{u_{cc}(c_i, g_i)} n_i \frac{\left(\frac{u_{cg}(c_i, g_i)}{u_{cc}(c_i, g_i)} - 1 \right)}{\left(\frac{N}{\Phi''(G)} + \sum_{i=1}^I n_i \frac{1}{u_{cc}(c_i, g_i)} \right)} - \frac{u_{cg}(c_i, g_i)}{u_{cc}(c_i, g_i)} \quad (78.2)$$

Stage 1 Ex ante Behavior of the local government

At stage 1, the local government solves the following:

$$\text{MAX}_{g_i} V_i = u(c_i(g_i)) + v(g_i) + \Phi(\bar{G}(g_i))$$

The first order condition is given by $\frac{\partial V_i}{\partial g_i} = u'(c) \left(\frac{dc_i + d\bar{G}}{dg_i} \right) + v'(g_i) = 0$. Therefore we

have

$$u'(\tilde{c}) + u'(\tilde{c}) \left\{ \left(\frac{u_{cg}(c_i, g_i)}{u_{cc}(c_i, g_i)} - 1 \right) \left(1 - \frac{n_i}{N} J_i \right) \right\} = v'(\tilde{g}_i) \quad (79)$$

where $J_i \equiv \frac{\left(\frac{1}{u_{cc}(c_i, g_i)} + \frac{1}{\Phi''(G)} \right)}{\left(\frac{1}{\Phi''(G)} + \sum_{i=1}^I \frac{n_i}{N} \frac{1}{u_{cc}(c_i, g_i)} \right)}$

Noting that $1 - \frac{n_i}{N} J_i > 0$, we can derive the following proposition.

Proposition 8

- (a) $\tilde{g}_i > g^*$ if $u_{cg} > 0$.
- (b) If $u_{cg} < 0$, $\tilde{g}_i < g^*$ could be the case.

Therefore, our argument of excessively large local spending under Scenario A can be extended to the case of non-separable preferences when public and private consumption are complementary. If the two are substitute, however, there could arise the under-provision of g_i , reversing the direction of the ex ante distortion.

8. Conclusion

The decentralized leadership literature has noted that ex post optimized transfers serve to internalize fiscal externalities associated with local spending (Caplan et al

(2004)) or local taxation (Köthenbürger (2004)), whereas *the soft budget literature* raises distortion on the regional ex ante incentives in the anticipation of the ex post bailing out or cost/revenue sharing arrangement. The two literatures address the commitment issue of the central transfer policy which is characterized by a sequential game with the local level governments as Stakelberg leader to the central authority. The present paper aims to synthesize them, both of which address the commitment problem. Our major findings are that (i) direction of the ex ante distortion relies on what policy instrument is decided ex ante at the local level, i.e., tax revenue raising effort or local spending, and (ii) except the extreme situations, the lack of the central government commitment to own transfer policy leads to inefficiency, either under taxation or over-spending relative to the first best or the commitment solution.

In the federalism literature, however, it is only in the last decade that more attention has been paid on the incentive problem arising from the lack of commitment or ex post discretion in the intergovernmental transfers. We should not take for granted the commitment ability of the central authority, i.e., its ability to design transfers from the ex ante standpoint. With local level governments gaining more autonomy and discretion within their jurisdictions through fiscal decentralization in many countries, the soft budget problem will become real not just a theoretical artifact as long as the fiscal tie between governments remains discretionary, so serious consideration is needed on how to assure the hard budget at the local level.

Appendix 1: Proof of Proposition 1:

(a) Suppose $\bar{g} \leq g^*$. Then Lemma 1 implies that . Then comparing (6) and (15) leads to:

$$(1-\lambda)\frac{N}{n_i}v'(\tilde{g}_i)+\lambda NB'(N\bar{g})=u'(\tilde{c}) \quad u'(c^*)=(1-\lambda)v'(g^*)+\lambda NE'(Ng^*) \quad (\text{A.1.1})$$

Note that $\bar{g} \leq g^*$ and $N/n_i > 1$. Thus when $\lambda < 1$, the above equation holds only if $\tilde{g}_i > g^*$ for all i . This contradicts however $\bar{g} \leq g^*$.

(b) It is immediate from (11).

(c) The social welfare in the equilibrium is given by:

$$W = Nu(\bar{c}(\tilde{M})) + (1-\lambda)\sum_{i=1}^I n_i v(\tilde{g}_i) + \lambda NE\left(\sum_{i=1}^I n_i \tilde{g}_i\right) + N\Phi(\bar{G}(\tilde{M})) \quad (\text{A.1.2})$$

where $\tilde{M} = \frac{1}{N}\left(Y - \sum_{i=1}^I n_i \tilde{g}_i\right)$. Differentiating above with respect to \tilde{g}_i and evaluating

the equilibrium establish:

$$\frac{1}{n_i} \frac{dW}{dg_i} = (1-\lambda)v'(\tilde{g}_i) + \lambda NE'(N\bar{g}) - u'(\tilde{c}) = \left(\frac{n_i}{N} - 1\right)u'(\tilde{c}) < 0 \quad (\text{A.1.3})$$

In the first equality, we use Lemma 1 and the second equality comes from (15). QED

Appendix 2: Example:

In this appendix, we provide the example to compare the sub-game perfect solution with the first best one. For simplicity, assume that there is no spillover, namely $\lambda = 0$. We specify the utility function as follows.

$$U(c_i, g_i, G) = \log(c_i) + \log(g_i) + \log(G) \quad (\text{A.2.1})$$

The first best allocation is characterized by $c^* = g^* = G^*$. Equation (4), overall resource constraint, implies

$$N(c^* + g^* + G^*) = Y \quad (\text{A.2.2})$$

which derives

$$c^* = g^* = G^* = \frac{Y}{3N} \quad (\text{A.2.3})$$

A.2.1 Scenario A: Expenditure level is selected ex ante

From ex post behavior of the central government at Stage 2, Equation (8) implies

$$\bar{c} = \bar{G}.$$

Therefore Equation (4) implies

$$\bar{c} = \frac{1}{2N} \left(Y - \sum_{i=1}^I n_i g_i \right). \quad (\text{A.2.4})$$

From ex ante behavior of the local government in Stage 1, Equation (15) implies

$$\frac{1}{g_i} = \frac{n_i}{N} \frac{1}{\bar{c}}, \text{ which reduces to}$$

$$g_i = \bar{c} \frac{N}{n_i}. \quad (\text{A.2.5})$$

Inserting (A.2.5) into (A.2.4), we have the equilibrium levels as follows.

$$\tilde{c} = \frac{1}{2+I} \frac{Y}{N} \quad (\text{A.2.6})$$

and

$$\tilde{g}_i = \frac{1}{2+I} \frac{Y}{N} \left(\frac{n_i}{N} \right)^{-1}. \quad (\text{A.2.7})$$

Result 1

Comparing with the first best level of $g^* = \frac{Y}{3N}$, we have the following result.

$$\tilde{g}_i \begin{cases} < \\ > \end{cases} g^* \text{ if and only if } \frac{n_i}{N} \begin{cases} > \\ < \end{cases} \frac{3}{2+I},$$

where I represents the number of regions.

The local public good is under provided in a larger region. In addition we have

$$\tilde{c} = \tilde{G} = \frac{1}{2+I} \frac{Y}{N} < \frac{1}{3} \frac{Y}{N} = c^* = G^*,$$

which means that the consumption level and the central public good is larger than the optimal levels.

A.2.2 Scenario B; Tax level is selected ex ante

From ex post behavior of the central government at Stage 2, Equation (20) implies

$$\bar{g} = \bar{G}.$$

Therefore Equation (4) implies

$$\bar{g} = \frac{1}{2N} \left(Y - \sum_{i=1}^I n_i c_i \right). \quad (\text{A.2.8})$$

From ex ante behavior of the local government in Stage 1, Equation (24) implies

$\frac{1}{c_i} = \frac{n_i}{N} \frac{1}{\bar{g}}$, which reduces to

$$c_i = \bar{g} \frac{N}{n_i}. \quad (\text{A.2.9})$$

Inserting (A.2.9) into (A.2.8), we have the equilibrium levels as follows.

$$\hat{g} = \frac{1}{2+I} \frac{Y}{N} \quad (\text{A.2.10})$$

and

$$\hat{c}_i = \frac{1}{2+I} \frac{Y}{N} \left(\frac{n_i}{N} \right)^{-1} .$$

$\frac{n_i}{N} \approx \frac{1}{I} < \frac{3}{2+I}$, means that the local public goods in Scenario A and the consumption level in Scenario B are too large. On the other hand, if population is concentrated relatively into some urban areas, g is too little in these urban areas and too large in other areas.

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