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Triple Implementation by Sharing Mechanisms in Production Economies with Unequal Labor Skills*

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Abstract

Under production economies with unequal labor skills, we study axiomatic characterizations of Pareto subsolutions which are implementable by *sharing mechanisms* in Nash, strong Nash, and subgame-perfect equilibria. The sharing mechanism allows agents to work freely while asking them to give information concerning their demands for outputs, their labor skills, and the prices of goods. Then, the mechanism distributes the produced output to the agents, according to the given information and the profile of their labor hours. Based on the characterizations, we may see most of fair allocation rules, which embody the ethical principles of *responsibility and compensation,* cannot be implementable when individuals' labor skills are private information.

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Keywords: labor sovereignty, triple implementation, different labor skills

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1 Introduction

In this paper, we consider implementation problems of allocation rules in production economies with possibly unequal labor skills among individuals. On this problem, a number of works such as Varian (1994), Hurwicz et al. (1995), Hong (1995), Suh (1995), Tian (1999, 2000), Yoshihara (1999), and Kaplan and Wettstein (2000) have proposed simple or natural mechanisms (game forms) for implementing some particular allocation rules like the Walrasian solution and the proportional solution (Roemer and Silvestre (1993)). In contrast, a few works such as Shin and Suh (1997) and Yoshihara (2000) have discussed characterizations of allocation rules implementable by such simple or natural mechanisms. In these works, however, there are two implicit assumptions about the basic information structure among individuals and the social planner (mechanism coordinator).

The first implicit assumption is that the planner can know every individual's level of labor skill, or otherwise, every individual has the same labor skill. Thus, the main problem of asymmetric information in this structure is reduced to the possibility of misrepresenting each individual's preference ordering,[1] and at most, the possibility of understating each individual's endowment of material goods.[2] However, if every individual is possibly endowed with a different level of labor skill, it is more natural to consider the informational structure such that the planner cannot know each individual's true level of labor skill, and so the individual may have an incentive to *overstate,* as well as to understate, his own labor skill. Note that the possibility of overstating individual labor skill is an essential feature of production economies with asymmetric information, since the planner cannot require individuals to "place the claimed endowments on the table" (Hurwicz et al. (1995)) in advance of production. Thus, taking this feature of the problem into consideration, our concern in this paper is to characterize the class of allocation rules, each of which assigns a subset of Pareto efficient allocations to each economic environment, and is implementable by one type of *natural mechanism* even when individuals' labor skills are unknown to the planner.

What kind of game form should we take as a *natural mechanism* in this context? This issue is relevant to our discussion on the second im-

[1] For instance, Varian (1994), Suh (1995), Shin and Suh (1997), Yoshihara (1999, 2000), and Kaplan and Wettstein (2000) discussed this type of problem.

[2] For instance, Hurwicz et al. (1995), Hong (1995), and Tian (1999, 2000) discussed this type of problem.

plicit assumption in the present literature on implementation in production economies. Although Shin and Suh (1997) and Yoshihara (2000) defined the conditions for characterizing "natural mechanisms" in production economies, the list of those conditions[3] is not yet satisfactory, since they omit another important feature of production economies with asymmetric information. Usually, the mechanisms in the implementation literature consist of pairs of strategy spaces and outcome functions, where each agent is required to announce some information, and the outcome function assigns an allocation to each profile of individuals' strategies. So, in production economies where one of the main productive factors is labor, it seems to be implicitly assumed that the planner is authorized to force individuals to provide the amount of labor time assigned by the outcome function of the mechanism.[4] However, the planner may not necessarily be able to exert such authority.

To solve this problem, we introduce another condition, *labor sovereignty* (Kranich (1994)), for characterizing "natural mechanisms" in production economies, and propose *sharing mechanisms* as a type of game form satisfying labor sovereignty. Labor sovereignty requires that every individual should have a right to choose his own labor time. Under sharing mechanisms, each individual can freely supply his labor time, and he is asked to give information concerning his demand for consumption goods and his labor skill. After that, the outcome function only distributes the produced output to agents, according to the information they gave and the record of their labor hours done.

Thus, the underlying question this paper attempts to solve is summarized as follows: what kinds of allocation rules which assign some Pareto efficient allocations are implementable by sharing mechanisms, even when individuals' labor skills are unknown to the planner? As the following section discusses in detail, we will take three equilibrium notions, *Nash*, *strong Nash*, and *subgame-perfect Nash*, for the non-cooperative games defined by sharing mechanisms.[5] We will identify two axioms which characterize allo-

[3] Those conditions are *feasibility*, *forthrightness*, *best response property*, and *simple strategy spaces*, which were originally proposed by Dutta, Sen, and Vohra (1995) and Saijo, Tatamitani, and Yamato (1996) to characterize "natural mechanisms" in pure exchange economies.

[4] Roemer (1989) pointed out this implicit assumption explicitly.

[5] Yamada and Yoshihara (2002) proposed a sharing mechanism which triply implements the proportional solution in these three equilibria, when the production function is differentiable.

cation rules *triply implementable by sharing mechanisms* in the above three equilibria. The two axioms are respectively relevant to the ethical principles of *responsibility and compensation* (Fleurbaey (1998)) in fair allocation problems. Thus, by using the characterization results in this paper, we will have some insight on implementability of fair allocation rules in terms of responsibility and compensation.

In the following discussion, the model is defined in Section 2. Section 3 provides a characterization of triple implementation by sharing mechanisms. Section 4 gives some samples of implementable and unimplementable Pareto subsolutions respectively. Some concluding remarks appear in Section 5. All the proofs of the theorems will be relegated to the Appendix.

2 The Basic Model

There are two goods, one of which is an input good (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[6] The population in the society is given by the set $N = \{1, \ldots, n\}$, where $2 \leq n < +\infty$. Each agent i's consumption vector is denoted by $z_i = (x_i, y_i)$, where x_i denotes his labor time, and y_i denotes his assigned amount of output consumption. It is assumed that all the agents face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, so that they have the same consumption set $[0, \bar{x}] \times \mathbb{R}_+$. Each agent i's preference is defined on $[0, \bar{x}] \times \mathbb{R}_+$ and represented by a utility function $u_i : [0, \bar{x}] \times \mathbb{R}_+ \to \mathbb{R}$ which is continuous and quasi-concave on $[0, \bar{x}] \times \mathbb{R}_+$, and strictly monotonic (decreasing in labor time and increasing in the share of output) on $[0, \bar{x}) \times \mathbb{R}_{++}$.[7] We denote by $\mathcal{U}$ the class of such utility functions. Each agent i is also characterized by a *labor skill* which is represented by a positive real number $s_i^t \in \mathbb{R}_{++}$. The superscript t on s_i^t indicates "true," so that s_i^t denotes *agent i's true labor skill.* The universal set of labor skills for all agents is denoted by $\mathcal{S} = \mathbb{R}_{++}$.[8] The labor skill $s_i^t \in \mathcal{S}$ implies i's *labor endowment* per unit of labor time. It can also be interpreted as i's *labor intensity* which would be exercised in the production

[6]The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.

[7]The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

[8]For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

process: that is, i's labor input per unit of labor time.[9] Thus, if his *supply of labor time* is $x_i \in [0, \bar{x}]$ and his labor intensity is $s_i^t \in \mathcal{S}$, then it is $s_i^t x_i \in \mathbb{R}_+$ which implies his *labor contribution* to the production process measured in efficiency units. The production technology is described by a production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ which is assumed to be continuous, strictly increasing, concave, and $f(0) = 0$, but not necessarily differentiable. For simplicity, we fix a production function f for all economies. Thus, the economy is characterized by a pair of profiles $\mathbf{e} \equiv (\mathbf{u}, \mathbf{s}^t)$ with $\mathbf{u} = (u_1, \ldots, u_n) \in \mathcal{U}^n$ and $\mathbf{s}^t = (s_1^t, \ldots, s_n^t) \in \mathcal{S}^n$. Denote the class of such economies by $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$.

Given $\mathbf{s}^t = (s_1^t, \ldots, s_n^t) \in \mathcal{S}^n$, an allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *feasible for* $\mathbf{s}^t$ if $\sum y_i \leq f\left(\sum s_i^t x_i\right)$. We denote by $Z(\mathbf{s}^t)$ the set of feasible allocations for $\mathbf{s}^t \in \mathcal{S}^n$. An allocation $\mathbf{z} = (z_1, \ldots, z_n) \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *Pareto efficient for* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ if $\mathbf{z} \in Z(\mathbf{s}^t)$ and there does not exist $\mathbf{z}' = (z_1', \ldots, z_n') \in Z(\mathbf{s}^t)$ such that for all $i \in N$, $u_i(z_i') \geq u_i(z_i)$, and for some $i \in N$, $u_i(z_i') > u_i(z_i)$. A *solution* is a correspondence $\varphi : \mathcal{E} \twoheadrightarrow ([0, \bar{x}] \times \mathbb{R}_+)^n$ such that for each $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\varphi(\mathbf{e}) \subseteq Z(\mathbf{s}^t)$. We particularly focus on solutions which respectively assign a subset of Pareto efficient allocations to each economy. We call such solutions *Pareto subsolutions.* Given a Pareto subsolution φ, an allocation $\mathbf{z} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is *φ-optimal for* $\mathbf{e} \in \mathcal{E}$ if $\mathbf{z} \in \varphi(\mathbf{e})$.

2.1 Sharing mechanisms

A *normal-form game form* is a pair $\Gamma = (A, h)$, where $A = A_1 \times \cdots \times A_n$, A_i being the s*trategy space of agent* $i \in N$, and $h : A \to ([0, \bar{x}] \times \mathbb{R}_+)^n$ being the *outcome function* which associates each $\mathbf{a} \in A$ with a unique element $h(\mathbf{a}) \in ([0, \bar{x}] \times \mathbb{R}_+)^n$. The i-th component of $h(\mathbf{a})$ will be denoted by $h_i(\mathbf{a}) \equiv (h_{i1}(\mathbf{a}), h_{i2}(\mathbf{a}))$, where $h_{i1}(\mathbf{a}) \in [0, \overline{x}]$ and $h_{i2}(\mathbf{a}) \in \mathbb{R}_+$. Given $\mathbf{a} \in A$, let $\mathbf{a}_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$. A normal-form game form $\Gamma = (A, h)$ is

[9] It might be more natural to define labor endowment and labor intensity in a discriminative way: for example, if $\overline{s}_i^t \in \mathcal{S}$ is i's labor endowment per unit of labor time, then i's labor intensity is a variable s_i^t, where $0 < s_i^t \leq \overline{s}_i^t$. In such a formulation, we may view the amount of s_i^t as being determined endogenously based upon the agent's characteristic of preference ordering (utility function). In spite of this more natural view, we will assume in the following discussion that the labor intensity is a constant value, $s_i^t = \overline{s}_i^t$, for the sake of simplicity. The main theorems in the following discussion would remain valid with a few changes in the settings of the economic environments even if the labor intensity were assumed to be varied.

labor sovereign if for every $i \in N$ and every $x_i \in [0, \bar{x}]$, there exists a strategy $a_i \in A_i$ such that for any $\mathbf{a}_{-i} \in A_{-i}$, $h_{i1}(a_i, \mathbf{a}_{-i}) = x_i$.

We are interested in labor sovereign game forms, and focus on *sharing mechanisms* that only distribute output among the agents according to their announcements on their private information and their supplied labor time.

Definition 1. *A* sharing mechanism *is a function $g : M \times [0, \bar{x}]^n \to \mathbb{R}^n_+$ such that for any $(\mathbf{m}, \mathbf{x}) \in M \times [0, \bar{x}]^n$, $g(\mathbf{m}, \mathbf{x}) = \mathbf{y}$ for some $\mathbf{y} \in \mathbb{R}^n_+$, where $\mathbf{m} = (m_1, \ldots, m_n)$ denotes the agents' messages.*

A sharing mechanism g is *feasible* if for any $\mathbf{s}^t \in \mathcal{S}^n$, any $\mathbf{m} \in M$, and any $\mathbf{x} \in [0, \bar{x}]^n$, $(\mathbf{x}, g(\mathbf{m}, \mathbf{x})) \in Z(\mathbf{s}^t)$. Note that a feasible sharing mechanism g needs not refer to $\mathbf{s}^t$ in dividing the total output $f\left(\sum s^t_j x_j\right)$, and is expressed as a game form $\Gamma_g = (M \times [0, \bar{x}]^n, h)$ such that for any $(\mathbf{m}, \mathbf{x}) \in M \times [0, \bar{x}]^n$, $h(\mathbf{m}, \mathbf{x}) = (\mathbf{x}, g(\mathbf{m}, \mathbf{x}))$. Clearly, Γ_g is a labor sovereign normal-form game form, which is uniquely corresponding to g. We denote by $\mathcal{G}$ the class of such feasible sharing mechanisms.

Given a feasible sharing mechanism $g \in \mathcal{G}$, a *feasible sharing game* is defined for each economy $\mathbf{e} \in \mathcal{E}$ as a non-cooperative game $(N, M \times [0, \bar{x}]^n, g, \mathbf{e})$. Fixing the set of players N and their strategy sets $M \times [0, \bar{x}]^n$, we simply denote a feasible sharing game $(N, M \times [0, \bar{x}]^n, g, \mathbf{e})$ by $(g, \mathbf{e})$.

Given a strategy profile $(\mathbf{m}, \mathbf{x}) \in M \times [0, \bar{x}]^n$, let $\left(\mathbf{m}_{m'_i}, \mathbf{x}_{x'_i}\right) \in M \times [0, \bar{x}]^n$ be another strategy profile which is obtained by replacing the i-th component (m_i, x_i) of $(\mathbf{m}, \mathbf{x})$ with (m'_i, x'_i). A strategy profile $(\mathbf{m}^*, \mathbf{x}^*) \in M \times [0, \bar{x}]^n$ is a (*pure-strategy*) *Nash equilibrium of the feasible sharing game* $(g, \mathbf{e})$ if for any $i \in N$ and any $(m_i, x_i) \in M_i \times [0, \bar{x}]$, $u_i(x^*_i, g_i(\mathbf{m}^*, \mathbf{x}^*)) \geq u_i\left(x_i, g_i\left(\mathbf{m}^*_{m_i}, \mathbf{x}^*_{x_i}\right)\right)$. Denote by $NE(g, \mathbf{e})$ the set of Nash equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *Nash equilibrium allocation of the feasible sharing game* $(g, \mathbf{e})$ if there exists $\mathbf{m} \in M$ such that $(\mathbf{m}, \mathbf{x}) \in NE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{m}, \mathbf{x})$ where $\mathbf{x} = (x_i)_{i \in N}$ and $\mathbf{y} = (y_i)_{i \in N}$. Denote by $NA(g, \mathbf{e})$ the set of Nash equilibrium allocations of $(g, \mathbf{e})$. A feasible sharing mechanism $g \in \mathcal{G}$ is said to *implement* a solution φ on $\mathcal{E}$ *in Nash equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = \varphi(\mathbf{e})$.

A strategy profile $(\mathbf{m}^*, \mathbf{x}^*) \in M \times [0, \bar{x}]^n$ is a (*pure-strategy*) *strong (Nash) equilibrium of the feasible sharing game* $(g, \mathbf{e})$ if for any $T \subseteq N$ and any $(m_i, x_i)_{i \in T} \in (M_i)_{i \in T} \times [0, \bar{x}]^{\#T}$, there exists $j \in T$ such that

$$u_j\left(x^*_j, g_j(\mathbf{m}^*, \mathbf{x}^*)\right) \geq u_j\left(x_j, g_j\left((m_i, x_i)_{i \in T}, (m^*_k, x^*_k)_{k \in T^c}\right)\right).^{10}$$

[10] For any $T \subseteq N$, $\#T$ denotes the number of agents in T. For any $T \subseteq N$, T^c denotes

Denote by $SNE(g, \mathbf{e})$ the set of strong equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *strong equilibrium allocation of the feasible sharing game* $(g, \mathbf{e})$ if there exists $\mathbf{m} \in M$ such that $(\mathbf{m}, \mathbf{x}) \in SNE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{m}, \mathbf{x})$ where $\mathbf{x} = (x_i)_{i \in N}$ and $\mathbf{y} = (y_i)_{i \in N}$. Denote by $SNA(g, \mathbf{e})$ the set of strong equilibrium allocations of $(g, \mathbf{e})$. A feasible sharing mechanism $g \in \mathcal{G}$ is said to *implement* a solution φ on $\mathcal{E}$ *in strong equilibria,* if for all $\mathbf{e} \in \mathcal{E}$, $SNA(g, \mathbf{e}) = \varphi(\mathbf{e})$.

2.2 Timing Problem with Sharing Mechanisms

We should mention here that $\mathbf{m}$ and $\mathbf{x}$ represent the agents' different kinds of strategic actions: $\mathbf{m}$ indicates the agents' announcements on their private information, while $\mathbf{x}$ their engagements in production activity by supplying those amounts of labor time. Thus, there may be a time difference between the point in time when $\mathbf{m}$ is announced and the period when $\mathbf{x}$ is exercised. It implies that there may be at least *two polar cases of time sequence of decision making*: the agents may announce $\mathbf{m}$ before they engage in production activity, or they may announce $\mathbf{m}$ after supplying $\mathbf{x}$. The former allows the case when each agent i decides his supply of labor time with the knowledge of the announcements $\mathbf{m}$, while the latter allows the case when each agent i decides his announcement m_i with the knowledge of the agents' actions $\mathbf{x}$ in production process.

So, we should additionally consider at least two types of two-stage game forms:

(1) The first type is that in the first stage, every agent i simultaneously makes an announcement, m_i, on his private information, and in the second stage, every agent i engages in the production activity and provides his labor time, x_i, according to his preference. After the production process, the outcome function assigns a distribution of the output produced.

(2) The second type has the converse sequence of strategic actions. In the first stage, every agent i engages in the production activity and provides his labor time, x_i, according to his preference, and after the production, every agent i simultaneously makes an announcement, m_i, on his private information in the second stage. Finally, the outcome function assigns a distribution of the output produced.

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the complement of T in N.

Given a feasible sharing mechanism $g \in \mathcal{G}$, the *feasible (1) type g-implicit two-stage mechanism* Γ_g^1 is a two-stage extensive game form in which the first stage consists of selecting $\mathbf{m}$ from M, the second stage consists of selecting $\mathbf{x}$ from $[0, \bar{x}]^n$, and the final stage assigns an outcome $(\mathbf{x}, g(\mathbf{m}, \mathbf{x}))$. Given $g \in \mathcal{G}$, the *feasible (2) type g-implicit two-stage mechanism* Γ_g^2 is a two-stage extensive game form in which the first stage consists of selecting $\mathbf{x}$ from $[0, \bar{x}]^n$, the second stage consists of selecting $\mathbf{m}$ from M, and the final stage assigns an outcome $(\mathbf{x}, g(\mathbf{m}, \mathbf{x}))$.

Given a feasible (1) type g-implicit two-stage game $\left(\Gamma_g^1, \mathbf{e}\right)$ and a strategy profile $\mathbf{m} \in M$ in the first stage of the game $\left(\Gamma_g^1, \mathbf{e}\right)$, let us denote its corresponding second stage subgame by $\left(\Gamma_g^1(\mathbf{m}), \mathbf{e}\right)$. Let $\mathbf{x}^{\mathbf{e}} : M \to [0, \bar{x}]^n$ be a *Nash equilibrium mapping* such that for each $\mathbf{m} \in M$, $\mathbf{x}^{\mathbf{e}}(\mathbf{m})$ is a (pure-strategy) Nash equilibrium of the subgame $\left(\Gamma_g^1(\mathbf{m}), \mathbf{e}\right)$. Denote the set of such Nash equilibrium mappings of the game $\left(\Gamma_g^1, \mathbf{e}\right)$ by $\mathbf{X}^{\mathbf{e}}$. A strategy profile $(\mathbf{m}^*, \mathbf{x}^{\mathbf{e}*}) \in M \times \mathbf{X}^{\mathbf{e}}$ is a (*pure-strategy*) *subgame-perfect (Nash) equilibrium of the feasible (1) type g-implicit two-stage game* $\left(\Gamma_g^1, \mathbf{e}\right)$ if for any $i \in N$ and any $m_i \in M_i$,

$$u_i\left(x_i^{\mathbf{e}*}(\mathbf{m}^*), g_i(\mathbf{m}^*, \mathbf{x}^{\mathbf{e}*}(\mathbf{m}^*))\right) \geq u_i\left(x_i^{\mathbf{e}*}(\mathbf{m}_{m_i}^*), g_i\left(\mathbf{m}_{m_i}^*, \mathbf{x}^{\mathbf{e}*}(\mathbf{m}_{m_i}^*)\right)\right),$$

where $x_i^{\mathbf{e}*}(\mathbf{m})$ is the i-th component of the Nash equilibrium strategy profile $\mathbf{x}^{\mathbf{e}*}(\mathbf{m})$ in the second stage subgame induced by the strategy choice $\mathbf{m}$ in the first stage.

Given a feasible (2) type g-implicit two-stage game $\left(\Gamma_g^2, \mathbf{e}\right)$ and a strategy profile $\mathbf{x} \in [0, \bar{x}]^n$ in the first stage of the game $\left(\Gamma_g^2, \mathbf{e}\right)$, let us denote its corresponding second stage subgame by $\left(\Gamma_g^2(\mathbf{x}), \mathbf{e}\right)$. Let $\mathbf{m}^{\mathbf{e}} : [0, \bar{x}]^n \to M$ be a *Nash equilibrium mapping* such that for each $\mathbf{x} \in [0, \bar{x}]^n$, $\mathbf{m}^{\mathbf{e}}(\mathbf{x})$ is a (pure-strategy) Nash equilibrium of the subgame $\left(\Gamma_g^2(\mathbf{x}), \mathbf{e}\right)$. Denote the set of such Nash equilibrium mappings of the game $\left(\Gamma_g^2, \mathbf{e}\right)$ by $\mathbf{M}^{\mathbf{e}}$. A strategy profile $(\mathbf{m}^{\mathbf{e}*}, \mathbf{x}^*) \in \mathbf{M}^{\mathbf{e}} \times [0, \bar{x}]^n$ is a (*pure-strategy*) *subgame-perfect (Nash) equilibrium of the feasible (2) type g-implicit two-stage game* $\left(\Gamma_g^2, \mathbf{e}\right)$ if for any $i \in N$ and any $x_i \in [0, \bar{x}]$,

$$u_i\left(x_i^*, g_i(\mathbf{m}^{\mathbf{e}*}(\mathbf{x}^*), \mathbf{x}^*)\right) \geq u_i\left(x_i, g_i\left(\mathbf{m}^{\mathbf{e}*}\left(\mathbf{x}_{x_i}^*\right), \mathbf{x}_{x_i}^*\right)\right).$$

Denote by $SPE\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $SPE\left(\Gamma_g^2, \mathbf{e}\right)$) the set of subgame-perfect equilibria of $\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^2, \mathbf{e}\right)$). An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *subgame-perfect equilibrium allocation of* $\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^2, \mathbf{e}\right)$) if there exists $(\mathbf{m}, \mathbf{x^e}) \in SPE\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $(\mathbf{m^e}, \mathbf{x}) \in SPE\left(\Gamma_g^2, \mathbf{e}\right)$) such that $\mathbf{x^e}(\mathbf{m}) = \mathbf{x}$ and $\mathbf{y} = g(\mathbf{m}, \mathbf{x^e}(\mathbf{m}))$ (resp. $\mathbf{y} = g(\mathbf{m^e}(\mathbf{x}), \mathbf{x})$) where $\mathbf{x} = (x_i)_{i \in N}$ and $\mathbf{y} = (y_i)_{i \in N}$. Denote by $SPA\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $SPA\left(\Gamma_g^2, \mathbf{e}\right)$) the set of subgame-perfect equilibrium allocations of $\left(\Gamma_g^1, \mathbf{e}\right)$ (resp. $\left(\Gamma_g^2, \mathbf{e}\right)$). Given $g \in \mathcal{G}$, the feasible (1) type g-implicit two-stage mechanism Γ_g^1 (resp. the feasible (2) type g-implicit two-stage mechanism Γ_g^2) is said to *implement* a solution φ on $\mathcal{E}$ *in subgame-perfect equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $SPA\left(\Gamma_g^1, \mathbf{e}\right) = \varphi(\mathbf{e})$ (resp. $SPA\left(\Gamma_g^2, \mathbf{e}\right) = \varphi(\mathbf{e})$). Given $g \in \mathcal{G}$, the feasible (1) type g-implicit two-stage mechanism Γ_g^1 (resp. the feasible (2) type g-implicit two-stage mechanism Γ_g^2) is said to *triply implement* a solution φ on $\mathcal{E}$ *in Nash, strong, and subgame-perfect equilibria* if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = SNA(g, \mathbf{e}) = SPA\left(\Gamma_g^1, \mathbf{e}\right) = \varphi(\mathbf{e})$ (resp. $NA(g, \mathbf{e}) = SNA(g, \mathbf{e}) = SPA\left(\Gamma_g^2, \mathbf{e}\right) = \varphi(\mathbf{e})$).

3 Implementation by sharing mechanisms

Throughout our discussion, we assume that each agent prefers consumption vectors with a positive amount of output and a positive amount of leisure, to consumption vectors with no output or no leisure.

Assumption 1 (boundary condition of utility functions):

$$\forall i \in N, \forall z_i \in [0, \bar{x}) \times \mathbb{R}_{++}, \forall z_i' \in \partial([0, \bar{x}] \times \mathbb{R}_+), u_i(z_i) > u_i(z_i').$$[11]

If the production function is not differentiable, it is possible that the slope of the budget line faced by the agents could not be uniquely verified by the information of the production possibility set around the production point. Thus, the mechanism would need additional information on the price.

Let us introduce a new notation here. We denote the set of price vectors by the unit simplex $\Delta \equiv \{p = (p_x, p_y) \in \mathbb{R}_+ \times \mathbb{R}_+ : p_x + p_y = 1\}$, where p_x represents the price of labor (measured in efficiency units) and p_y the price of output. The message space M of the sharing mechanism considered in the present paper is defined by $M \equiv \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n$ with generic element $(\mathbf{p}, \mathbf{s}, \mathbf{w})$, where $\mathbf{p} = (p^1, \ldots, p^n)$ in which p^i denotes i's reported price vector, $\mathbf{s} = (s_1, \ldots, s_n)$ in which s_i denotes i's reported amount of labor skill, and $\mathbf{w} = (w_1, \ldots, w_n)$ in which w_i denotes i's desired amount of output consumption. Moreover, we define efficiency prices as follows.

[11] $\partial([0, \bar{x}] \times \mathbb{R}_+) \equiv ([0, \bar{x}] \times \mathbb{R}_+) \setminus ([0, \bar{x}) \times \mathbb{R}_{++})$.

Definition 2. *A price vector* $p = (p_x, p_y) \in \Delta$ *is* an efficiency price for $\mathbf{z} = (\mathbf{x}, \mathbf{y}) \in [0, \bar{x}]^n \times \mathbb{R}^n_+$ at $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *iff*

(i) $(x_i)_{i \in N} \in \arg \max_{(x'_i)_{i \in N}} \ p_y f\left(\sum s^t_i x'_i\right) - p_x \sum s^t_i x'_i$;

(ii) *for all* $i \in N$ *and all* $z'_i \in [0, \bar{x}] \times \mathbb{R}_+$*, if* $u_i(z'_i) \geq u_i(z_i)$*, then* $p_y y'_i - p_x s^t_i x'_i \geq p_y y_i - p_x s^t_i x_i$.

The set of efficiency prices for $\mathbf{z}$ at $\mathbf{e}$ is denoted by $\Delta^P(\mathbf{e}, \mathbf{z})$.

We define implementability by such a sharing mechanism as follows.

Definition 3. *A Pareto subsolution* φ *is* (1) or (2) type triply labor sovereign-implementable, *if there exists a feasible sharing mechanism* $g \in \mathcal{G}$ *such that*:

(i) Γ^1_g (resp. Γ^2_g) triply implements φ *on* $\mathcal{E}$ in Nash, strong, and subgame-perfect equilibria;

(ii) *g meets the* forthrightness*: for all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $(\mathbf{x}, \mathbf{y}) \in \varphi(\mathbf{e})$*, there exists* $p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$ *such that* $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e})$ and $g(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$*, where* $\mathbf{p} = (p^i)_{i \in N}$ with $p^i = p$ for all $i \in N$;

(iii) *g has the following property: for all* $\mathbf{e} = (u_i, s^t_i)_{i \in N} \in \mathcal{E}$*, if* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$ *and* $g(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{y}$*, where* $\mathbf{p} = (p^i)_{i \in N}$*,* $\mathbf{x} = (x_i)_{i \in N}$*, and* $\mathbf{y} = (y_i)_{i \in N}$*, then there exists* $p \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$ *such that*

$$g_i\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s^t_i (x'_i - x_i)\right\}$$

for all $i \in N$ *and all* $(p^{i\prime}, s'_i, x'_i, w'_i) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$;

(iv) *g has the following property*: *for all* $\mathbf{e} = (u_i, s^t_i)_{i \in N} \in \mathcal{E}$*, if* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$*, then* $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$ *and* $g(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w})$ *whenever* $s_i = s'_i$ for all $i \in N$ *with* $x_i > 0$.

Definition 3 (ii) was first introduced by Dutta et al. (1995) and discussed by Saijo et al. (1996). It requires that if the agents' strategy profile is consistent with the given economy and a φ-optimal allocation, then the profile must be a Nash equilibrium and the outcome must coincide with the allocation. That is, any φ-optimal allocation should be realizable as an equilibrium outcome in a straightforward way.

Definition 3 (iii) requires a kind of informational efficiency of the mechanism. It says that in equilibrium, each agent's attainable set under the mechanism must be included in a half space which is inevitably included in the lower contour set of his utility function when the equilibrium allocation is Pareto efficient. The point is that this half space is defined by only the information on the production point and the production possibility set. Owing to this condition, the mechanism coordinator does not need to know all the information on the agents' preferences in order to get φ-optimal allocations as equilibrium allocations.

Definition 3 (iv) is another requirement of informational efficiency. It says that the distribution of output by the mechanism would not change regardless of any change in skill information announced by "non-working" agents. That is, unexercised labor skills should be equally taken into account in the determination of distribution. Owing to this condition, the mechanism coordinator needs not consider degenerative labor skills.

We introduce two axioms as necessary conditions for the labor sovereign implementation.

Supporting Price Independence (SPI) (Yoshihara (1998), Gaspart (1998))**.** *For all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *and all* $\mathbf{z} \in \varphi(\mathbf{e})$*, there exists* $p \in \Delta^P(\mathbf{e}, \mathbf{z})$ *such that for all* $\mathbf{e}' = (\mathbf{u}', \mathbf{s}^t) \in \mathcal{E}$*, if* $p \in \Delta^P(\mathbf{e}', \mathbf{z})$*, then* $\mathbf{z} \in \varphi(\mathbf{e}')$*.*

Let $\Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{z}) \equiv \{p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{z}) : \forall \mathbf{u}' \in \mathcal{U}^n \text{ s.t. } p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{z}), \mathbf{z} \in \varphi(\mathbf{u}', \mathbf{s}) \text{ holds}\}$.

Independence of Unused Skills (IUS). *For all* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) = (u_i, s_i^t)_{i \in N} \in \mathcal{E}$ *and all* $\mathbf{z} = (x_i, y_i)_{i \in N} \in \varphi(\mathbf{e})$*, there exists* $p \in \Delta^P(\mathbf{e}, \mathbf{z})$ *such that for all* $\mathbf{e}' = (\mathbf{u}, \mathbf{s}^{t\prime}) = (u_i, s_i^{t\prime})_{i \in N} \in \mathcal{E}$ *where* $s_i^t = s_i^{t\prime}$ *for all* $i \in N$ *with* $x_i > 0$*, if* $p \in \Delta^P(\mathbf{e}', \mathbf{z})$*, then* $\mathbf{z} \in \varphi(\mathbf{e}')$*.*

Let $\Delta^{IUS}(\mathbf{u}, \mathbf{s}, \mathbf{z}) \equiv \{p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{z}) : \forall \mathbf{s}' \in \mathcal{S}^n \text{ s.t. } p \in \Delta^P(\mathbf{u}, \mathbf{s}', \mathbf{z}), \mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}'),$ where $s_i' = s_i$ for all $i \in N$ with $x_i > 0\}$, where $\mathbf{z} = (x_i, y_i)_{i \in N}$.

The axiom SPI requires that any φ-optimal allocation should remain to be φ-optimal if the profile of utility functions changes, but still keeping the Pareto efficiency of this allocation. This implies that SPI is a condition for informational efficiency, since it only requires a local information of individuals' preference orderings. As for the second axiom, IUS, it requires that any φ-optimal allocation should remain to be φ-optimal if the labor skills of non-working agents in this allocation change, but still keeping the Pareto efficiency of this allocation. This implies that IUS is also a condition for informational efficiency, since it admits ignorance of the skill information on non-working agents.

The two axioms can also have some implications of *responsibility* and *compensation* (Fleurbaey (1998)) in fair allocation problems. The axiom SPI represents a "stronger" condition of responsibility, since it requires independence of the *particular* change of individuals' responsible factors like utility functions. It is "stronger" because SPI is stronger than *Maskin Monotonicity* (Maskin (1999)), which was taken as a relatively strong axiom of responsibility by Fleurbaey and Maniquet (1996). The axiom IUS, in contrast, can be interpreted as a weaker condition of compensation, since it requires independence of the *particular* change of individuals' non-responsible factors like labor skills. It is "weaker" because IUS is weaker than the axiom of *Independence of Skill Endowments* (Yoshihara (2003)), which was taken as a relatively weak axiom of compensation by Yoshihara (2003).

Note that a Pareto subsolution φ satisfies SPI and IUS if and only if for all $\mathbf{e} \in \mathcal{E}$, all $\mathbf{z} \in \varphi(\mathbf{e})$, there exists $p \in \Delta^{SPI}(\mathbf{e}, \mathbf{z})$ and $p' \in \Delta^{IUS}(\mathbf{e}, \mathbf{z})$. In general, $\Delta^{SPI}(\mathbf{e}, \mathbf{z}) \neq \Delta^{IUS}(\mathbf{e}, \mathbf{z})$. However, there exists some intersection between the two sets as the following lemma shows.

Lemma 0: *Let φ satisfy SPI and IUS. Then, for all $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ and all $\mathbf{z} \in \varphi(\mathbf{e})$, there exists $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z}) \cap \Delta^{IUS}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$.*

Proof. Given $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, let $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^t)$ and $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$. Without loss of generality, for this $\mathbf{z} = (x_i, y_i)_{i \in N}$, let $x_1 = 0$ and $x_i > 0$ for any $i \neq 1$. Consider any $\mathbf{s}^{t\prime} = \left(s_1^{t\prime}, \mathbf{s}_{-1}^t\right)$ such that $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{z})$. If $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^{t\prime})$ is shown, then by the definition of Δ^{IUS}, we have that $p \in \Delta^{IUS}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$.

Let us consider $\mathbf{u}^* = (u_2^*, \mathbf{u}_{-2})$ where $u_2^*(x, y) = p_y y - p_x s_2^t x$ for all $(x, y) \in [0, \bar{x}] \times \mathbb{R}_+$. Since $\Delta^P(\mathbf{u}^*, \mathbf{s}^t, \mathbf{z}) = \{p\}$ and φ satisfies SPI, $\mathbf{z} \in \varphi(\mathbf{u}^*, \mathbf{s}^t)$ and $\Delta^{SPI}(\mathbf{u}^*, \mathbf{s}^t, \mathbf{z}) = \{p\}$. Consider moving from $(\mathbf{u}^*, \mathbf{s}^t)$ to $(\mathbf{u}^*, \mathbf{s}^{t\prime})$. By the definition of $\mathbf{u}^*$, $\Delta^P(\mathbf{u}^*, \mathbf{s}^{t\prime}, \mathbf{z}) = \{p\}$. Since φ satisfies IUS, we have $\mathbf{z} \in \varphi(\mathbf{u}^*, \mathbf{s}^{t\prime})$ and $\Delta^{IUS}(\mathbf{u}^*, \mathbf{s}^{t\prime}, \mathbf{z}) = \{p\}$. Consider moving from $(\mathbf{u}^*, \mathbf{s}^{t\prime})$ to $(\mathbf{u}, \mathbf{s}^{t\prime})$. Since $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{z})$ and φ satisfies SPI, $\mathbf{z} \in \varphi(\mathbf{u}, \mathbf{s}^{t\prime})$. ■

SPI and IUS are necessary conditions for the labor sovereign triple implementation.

Theorem 1. *If a Pareto subsolution φ is* (1) (*resp.* (2)) type triply labor sovereign-implementable*, then φ satisfies SPI.*

Theorem 2. *If a Pareto subsolution φ is* (1) (*resp.* (2)) type triply labor sovereign-implementable*, then φ satisfies IUS.*

We next show that SPI and IUS together constitute a sufficient condition for the labor sovereign triple implementation. First, we propose a feasible sharing mechanism $g \in \mathcal{G}$ for which Γ_g^1 and Γ_g^2 respectively triply implement Pareto subsolutions satisfying SPI and IUS in Nash, strong, and subgame-perfect equilibria. To construct our mechanism, let us introduce two feasible sharing mechanisms defined as follows:

- $g^{\mathbf{w}}$ is such that for each $\mathbf{s}^t \in \mathcal{S}^n$ and each strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, and for all $i \in N$,
$$g_i^{\mathbf{w}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s_k^t x_k\right) & \begin{array}{l}\text{if } x_i = \mu(\mathbf{x}_{-i}) \text{ and} \\ w_i > \max\left\{f\left(\sum s_k \bar{x}\right),\ \max_{j \neq i}\{w_j\}\right\},\end{array} \\ 0 & \text{otherwise,} \end{cases}$$
where $\mu(\mathbf{x}_{-i}) \equiv \max_{x_j < \bar{x}, j \neq i} \left\{\frac{x_j + \bar{x}}{2}\right\}$.

- $g^{\mathbf{s}}$ is such that for each $\mathbf{s}^t \in \mathcal{S}^n$ and each strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, and for all $i \in N$,
$$g_i^{\mathbf{s}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s_k^t x_k\right) & \text{if } x_i = 0,\ w_i = 0, \text{ and } s_i > s_j \text{ for all } j \neq i, \\ 0 & \text{otherwise.} \end{cases}$$

The mechanism $g^{\mathbf{w}}$ assigns all of the produced output[12] to only one agent who provides the maximal *interior* amount of labor time and reports a maximal amount of demand for the output, where $\mu(\mathbf{x}_{-i})$ is a scheme to have agents find their best response strategies. The mechanism $g^{\mathbf{s}}$ also assigns all of the produced output to only one agent who reports the highest labor skill and does not work at all.

Given $p \in \Delta$ and $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, let $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} \equiv \left\{\mathbf{u} \in \mathcal{U}^n : (\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s}) \text{ and } p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})\right\}$. Given $p \in \Delta$ and $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, let $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \equiv \{i \in N : \exists (x_i', w_i') \in [0, \bar{x}) \times \mathbb{R}_{++}$ s.t. $\varphi\left(p, \mathbf{s}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)^{-1} \neq \varnothing\Big\}$.

Given a strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ such that $p^i = p$ for all i, an agent $i \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is called a "*potential deviator*." Let us discuss the meaning of "potential deviators." Consider $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} = \varnothing$ and $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \neq \varnothing$. The first equation implies that the strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is inconsistent with the solution φ. The second $N(p, \mathbf{s}, \mathbf{x}, \mathbf{w}) \neq \varnothing$ implies there is an agent

[12]Note that we implicitly assume that the mechanism coordinator can hold all of the produced output after the production process, although he may not monitor that process perfectly.

i who can change his strategy to another one (p^i, s_i, x_i', w_i') so that the new strategy profile $\left(\mathbf{p}, \mathbf{s}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ would become consistent with φ. That is, it may be this i who makes the current strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ inconsistent with φ. This is the meaning that $i \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is a "potential deviator."

We propose a feasible sharing mechanism $g^* \in \mathcal{G}$ which works in each given $\mathbf{s}^t \in \mathcal{S}^n$ as follows:

For any $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = (p^i, s_i, x_i, w_i)_{i \in N} \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$,
Rule 1: if $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$, then
1-1: if for some $p \in \Delta$, $p^i = p$ for all $i \in N$ and $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} \neq \varnothing$, then $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$,
1-2: if there exists $j \in N$ such that for some $p \in \Delta$, $p^i = p$ for all $i \neq j$, $\varphi(p^j, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} = \varnothing$, and $j \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$, then $g_j^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) =$
$$\begin{cases} \max\left\{0, \min\left\{w_j' + \frac{p_x}{p_y}\left(s_j x_j - s_j x_j'\right), f\left(\sum s_k^t x_k\right)\right\}\right\} & \text{if } w_j > f\left(\sum s_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$
and $g_i^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \neq j$,
1-3: for any other case, $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g^{\mathbf{w}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$,

Rule 2: if $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, then $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = g^{\mathbf{s}}(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$.

It is easy to see that g^* satisfies *forthrightness* (Saijo et al. (1996)) and *best response property* (Jackson et al. (1994)). Moreover, g^* is a mechanism of the *price-quantity* type, and so satisfies *self-relevancy* (Hurwicz (1960)). It is also easy to check that the mechanism g^* is feasible. Note that the total amount of output $f\left(\sum s_k^t x_k\right)$ is observable, even without the true information of labor skills, after the production process, since the coordinator can hold all of the produced output.

In the following, a strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ is called *φ-consistent* if for some $p \in \Delta$, $p^i = p$ for all $i \in N$ and $\varphi(p, \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} \neq \varnothing$. Given a strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$, g^* works as follows. First of all, g^* computes the expected amount of output $f\left(\sum s_k x_k\right)$ from the data $(\mathbf{s}, \mathbf{x})$ and compares this with the actual produced amount of output $f\left(\sum s_k^t x_k\right)$. In the case that these two values coincide, if the strategy profile is φ-consistent, then g^* distributes $f\left(\sum s_k^t x_k\right)$ in accordance with $\mathbf{w}$ under Rule 1-1. If $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is not φ-consistent, and there is a unique potential deviator, then g^* punishes him under Rule 1-2. For any other case of $f\left(\sum s_k x_k\right) =$

$f\left(\sum s_k^t x_k\right)$, g^* assigns the same value as $g^{\mathbf{w}}$ under Rule 1-3. In the case that $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, g^* assigns the same value as $g^{\mathbf{s}}$ under Rule 2.

Before we discuss the performance of g^* formally, let us briefly explain how the mechanism induces true information of labor skills at least for working agents in the following part (A), and how it attains desirable allocations in the following part (B):

(A) g^* distributes the total amount $f\left(\sum s_k^t x_k\right)$ of output among agents according to the agents' supplies of labor time $\mathbf{x}$, reported price vectors $\mathbf{p}$, reported labor skills $\mathbf{s}$, and demands for the output $\mathbf{w}$. The problem is that the agents' true labor skills are not observable and they may misrepresent their labor skills so as to increase their share of output. To solve this problem, a scheme of reward-and-punishment is set up in the mechanism as follows. First, if $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, then clearly $\mathbf{s} \neq \mathbf{s}^t$ holds, and there must be at least one agent, say $j \in N$, who has misrepresented his labor skill, $s_j \neq s_j^t$, and supplied a positive amount of labor time $x_j > 0$. Then, this agent is definitely punished under the application of Rule 2.

Second, consider the case that $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$ but $\mathbf{s} \neq \mathbf{s}^t$. Then, there are at least two agents who have misrepresented their labor skills while supplying positive amounts of labor time, or someone, say j, has chosen "non-working" while misrepresenting his labor skill. Let us put aside the latter case for the moment. In the former case, suppose one of such misrepresenting agents, say $j \in N$, changes from $x_j > 0$ to $x_j' = 0$, while reporting a sufficiently high level of labor skill. Then, the situation $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$ shifts to $f\left(s_j x_j' + \sum_{i \neq j} s_i x_i\right) \neq f\left(s_j^t x_j' + \sum_{i \neq j} s_i^t x_i\right)$, thereby j may be better off under the application of Rule 2. Thus, the case may not correspond to an equilibrium situation. The following lemma actually confirms this insight.

Lemma 1: *Let Assumption 1 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Given an economy* $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, *let a strategy profile* $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ *be a Nash equilibrium of the game* $(g^*, \mathbf{u}, \mathbf{s}^t)$ *such that* $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$. *Then, it follows that* $s_i = s_i^t$ *for all* $i \in N$ *with* $x_i > 0$.

(B) What explanation remaining is mainly how the mechanism implements the Pareto subsolution φ when all agents report their true labor skills, $\mathbf{s} = \mathbf{s}^t$. To do this, we adopt a scheme developed by Yoshihara (2000a).

Since $\mathbf{s} = \mathbf{s}^t$, the strategy profile $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in \triangle^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ corresponds only to Rule 1. Note that among the three subrules of Rule

1, only Rule 1-1 can realize a desirable allocation in equilibrium, while the other two are to punish agents who have deviated from the situation of Rule 1-1. Suppose $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ is φ-consistent. Then, $(\mathbf{x}, \mathbf{w})$ becomes the outcome by Rule 1-1, which is a φ-optimal allocation for some economy with $\mathbf{s} = \mathbf{s}^t$. However, this does not necessarily imply that $(\mathbf{x}, \mathbf{w})$ is φ-optimal for the actual economy. If $(\mathbf{x}, \mathbf{w})$ is not Pareto efficient for the actual economy, $(\mathbf{x}, \mathbf{w})$ should not be an equilibrium allocation. Rule 1-2 is necessary for solving this problem: if $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-1 and results in the allocation $(\mathbf{x}, \mathbf{w})$, any agent is able to benefit from another consumption vector on the budget line, determined by a supporting price at $(\mathbf{x}, \mathbf{w})$ by deviating to induce Rule 1-2. Therefore, if $(\mathbf{x}, \mathbf{w})$ is an equilibrium allocation, then $(\mathbf{x}, \mathbf{w})$ must be Pareto efficient.

We are ready to discuss full characterizations of labor sovereign triple implementation by examining the performance of g^*.

Theorem 3. *Let Assumption 1 hold. Then, if a Pareto subsolution φ satisfies SPI and IUS, then φ is* (1) (*resp.* (2)) type triply labor sovereign-implementable by g^*.

Note that this result does not depend upon the number of agents: any Pareto subsolution satisfying SPI and IUS can be triply implementable by g^* even in economies of *two agents*.

Corollary 1. *Let Assumption 1 hold. Then, a Pareto subsolution φ is* (1) (*resp.* (2)) type triply labor sovereign-implementable *if and only if φ satisfies SPI and IUS.*

By Corollary 1, we can have two new insights on the implementability of Pareto subsolutions in production economies with unequal skills. First, we can classify what solutions are implementable or not, if the profile of labor skills becomes unknown to the coordinator, from among the Pareto subsolutions which were implementable when the profile was known to him. Note that it is easy to see any Pareto subsolution is labor sovereign-implementable if and only if it satisfies SPI, whenever the labor skills are known to the coordinator. Secondly, since the two axioms, SPI and IUS, can be regarded as the axioms of responsibility and compensation as we have discussed above, Corollary 1 indicates that the implementable solutions in this problem should have a rather strong property on responsibility, as well as only a rather weak property on compensation.

4 Characterization Results

Based upon the characterization in the previous section, let us examine which Pareto subsolutions are implementable or not, when the production skills are private information. In the first place, let us discuss the three variations of the *Walrasian solution*, as follows:

Definition 4. *Given a profit share* $\theta = (\theta_1, \ldots, \theta_n) \in [0,1]^n$ *with* $\sum \theta_i = 1$, *a solution* φ^W *is* Walrasian *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^W(\mathbf{e})$ *implies that there exists* $p = (p_x, p_y) \in \Delta$ *such that:*
(i) *for all* $\mathbf{z}' \in Z(\mathbf{s}^t)$, $\sum (p_y y_i' - p_x s_i^t x_i') \leq \sum (p_y y_i - p_x s_i^t x_i)$;
(ii) *for all* $i \in N$, $z_i \in \arg\max_{(x,y)\in B(p, s_i^t, z_i, \theta_i)} u_i(x,y)$ *where* $B(p, s_i^t, z_i, \theta_i) \equiv \{(x,y) \in [0,\bar{x}] \times \mathbb{R}_+ : p_y y - p_x s_i^t x \leq \theta_i \sum (p_y y_i - p_x s_i^t x_i)\}$.

Definition 5 (Roemer and Silvestre (1989, 1993))**.** *A solution* φ^{PR} *is the* proportional solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{PR}(\mathbf{e})$ *implies that:*

(i) $\mathbf{z}$ *is Pareto efficient for* $\mathbf{e}$;
(ii) *for all* $i \in N$, $y_i = \frac{s_i^t x_i}{\sum s_j^t x_j} \sum y_j$.

Definition 6 (Roemer and Silvestre (1989))**.** *A solution* φ^{EB} *is the* equal benefit solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{EB}(\mathbf{e})$ *implies that:*

(i) $\mathbf{z}$ *is Pareto efficient for* $\mathbf{e}$;
(ii) *there exists an efficiency price* $p = (p_x, p_y) \in \Delta$ *for* $\mathbf{z}$ *at* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *such that for all* $i \in N$, $p_y y_i - p_x s_i^t x_i = \frac{1}{n} \sum \left(p_y y_j - p_x s_j^t x_j\right)$.

As Yoshihara (2000) already showed, all the above three solutions respectively satisfy SPI. Thus, to confirm the implementability of each of the three solutions, it suffices to examine on IUS. Then:

Lemma 6. *The Walrasian solution* φ^W *satisfies IUS.*

Lemma 7. *The proportional solution* φ^{PR} *satisfies IUS.*

Lemma 8. *The equal benefit solution* φ^{EB} *satisfies IUS.*[13]

Thus, the Walrasian solution, the proportional solution, and the equal benefit solution are implementable by sharing mechanisms.

[13] The proof of **Lemma 8** would be a variation of that of **Lemma 6**.

Corollary 2. *Let Assumption 1 hold. Then, the Walrasian solution* φ^W *is (1) (resp. (2)) type triply labor sovereign-implementable.*

Corollary 3. *Let Assumption 1 hold. Then, the proportional solution* φ^{PR} *is (1) (resp. (2)) type triply labor sovereign-implementable.*

Corollary 4. *Let Assumption 1 hold. Then, the equal benefit solution* φ^{EB} *is (1) (resp. (2)) type triply labor sovereign-implementable.*

By Corollary 1, there may exist some Pareto subsolutions which are implementable by sharing mechanisms whenever the profile of labor skills is public information, but fails to be implementable once it becomes private information. As a sample of such solutions, we introduce the following*:*

Definition 7 (Fleurbaey and Maniquet (1996))**.** *A solution* $\varphi^{\widetilde{u}RWEB}$ *is the* $\widetilde{u}$-reference welfare equivalent budget solution *if for any* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, $\mathbf{z} \in \varphi^{\widetilde{u}RWEB}(\mathbf{e})$ *implies that there exists an efficiency price* $p = (p_x, p_y) \in \Delta$ *for* $\mathbf{z}$ *at* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ *such that for all* $i, j \in N$, $\max_{(x,y)\in B\left(p,s_i^t,z_i\right)} \widetilde{u}(x,y) = \max_{(x',y')\in B\left(p,s_j^t,z_j\right)} \widetilde{u}(x',y')$ *where*

$$B\left(p, s_k^t, z_k\right) \equiv \left\{(x,y) \in [0,\bar{x}] \times \mathbb{R}_+ : p_y y - p_x s_k^t x \leq p_y y_k - p_x s_k^t x_k\right\}.$$

This solution requires the agents' budget sets to be of equal value in terms of the reference utility $\widetilde{u}$. It satisfies SPI, but not IUS.

Lemma 9. *The* $\widetilde{u}$*-reference welfare equivalent budget solution* $\varphi^{\widetilde{u}RWEB}$ *satisfies SPI.*

Lemma 10. *The* $\widetilde{u}$*-reference welfare equivalent budget solution* $\varphi^{\widetilde{u}RWEB}$ *does not satisfy IUS.*

Corollary 5. *The* $\widetilde{u}$*-reference welfare equivalent budget solution* $\varphi^{\widetilde{u}RWEB}$ *is not (1) (resp. (2)) type triply labor sovereign-implementable.*

In summary, the above characterization results give us a new insight on implementation of Pareto subsolutions in production economies. It has been already shown by Dutta, et. al (1995), Saijo, et. al (1999), and Yoshihara (2000), that the three variations of the Walrasian types are implementable by natural mechanisms, while the *no-envy and efficient solution* is not, under an implicit assumption that the production skills of agents are known to the coordinator. In this paper, in contrast, we have seen that the three variations of the Walrasian types are implementable by sharing mechanisms even if the skills of agents are private information. However, some type of fair allocation rule like the $\widetilde{u}$-reference welfare equivalent budget solution can be implementable by sharing mechanisms whenever the skills are known to the coordinator, while it fails to be implementable once the skills are private information.

5 Concluding remarks

We characterized implementation by sharing mechanisms in production economies with unequal labor skills. The class of Pareto subsolutions implementable by sharing mechanisms is characterized by two axioms, Supporting Price Independence and Independence of Unused Skills. Based upon this characterization, we examined that the Walrasian, the proportional, and the equal benefit solutions are respectively implementable, while the $\widetilde{u}$-reference welfare equivalent budget solution fails to be implementable if individuals' labor skills become unknown to the planner. This result may indicate impossibility of implementing a Pareto subsolution which compensates the relatively lower skilled individuals, whenever the labor skills are private information. This is because such a solution may change the shares of produced outputs, corresponding to changes in the profile of labor skills, regardless of individuals' labor hours, which indicates such a solution might violate Independence of Unused Skills.

The workability of our proposed feasible sharing mechanism depends on the following two implicit but reasonable assumptions: first, although every individual i's labor performance, $s_i x_i$, measured in efficiency units is imperfectly observable and unverifiable by the planner, his working hour, x_i, is perfectly observable. Second, in spite of such imperfect observability, the planner can observe the real amount of outputs produced in the economy, so that he can compare this amount with the expected amount of outputs which were obtained by the announcements of individuals. We believe that these implicit assumptions are reasonable enough to formulate the essential aspect of informational asymmetry in production economies. However, it is an open question to discuss, even without the above two implicit assumptions, implementation of Pareto subsolutions by natural mechanisms in production economies with possibly unequal labor skills.

6 Appendix

6.1 Proofs of Theorem 1 and Theorem 2

Proof of Theorem 1. Suppose a Pareto subsolution φ is triply labor sovereign-implementable. Then, there exists a feasible sharing mechanism $g \in \mathcal{G}$ which satisfies the conditions (i)-(iv) in Definition 3. For any $\mathbf{z} = (x_i, y_i)_{i \in N} \in [0, \bar{x}]^n \times \mathbb{R}^n_+$ and any $\mathbf{e} = (u_i, s_i^t)_{i \in N}$, $\mathbf{e}' = (u_i', s_i^{t\prime})_{i \in N} \in \mathcal{E}$ where $s_i^t = s_i^{t\prime}$ $(\forall i \in N)$, suppose that $\mathbf{z} \in \varphi(\mathbf{e})$ and there exists a price $p \in \Delta^P(\mathbf{e}, \mathbf{z}) \cap \Delta^P(\mathbf{e}', \mathbf{z})$. By (ii), $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e})$ and $g(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$, where $\mathbf{p} = (p)_{i \in N}$, $\mathbf{s}^t = (s_i^t)_{i \in N}$, $\mathbf{x} = (x_i)_{i \in N}$, and $\mathbf{y} = (y_i)_{i \in N}$, so by (iii),

$$g_i\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}^t_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{y}_{w_i'}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i^t (x_i' - x_i)\right\}$$

for all $i \in N$ and all $(p^{i\prime}, s_i', x_i', w_i') \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. Since $p \in \Delta^P(\mathbf{e}', \mathbf{z})$, this implies $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e}')$ and $(\mathbf{x}, \mathbf{y}) \in NA(g, \mathbf{e}')$. Hence, $\mathbf{z} \in \varphi(\mathbf{e}')$ by (i). ■

Proof of Theorem 2. Suppose a Pareto subsolution φ is triply labor sovereign-implementable. Then, there exists a feasible sharing mechanism $g \in \mathcal{G}$ which satisfies the conditions (i)-(iv) in Definition 3. For any $\mathbf{z} = (x_i, y_i)_{i \in N} \in [0, \bar{x}]^n \times \mathbb{R}^n_+$ and any $\mathbf{e} = (u_i, s_i^t)_{i \in N}$, $\mathbf{e}' = (u_i', s_i^{t\prime})_{i \in N} \in \mathcal{E}$ where $u_i = u_i'$ for all $i \in N$ and $s_i^t = s_i^{t\prime}$ for all $i \in N$ with $x_i > 0$, suppose that $\mathbf{z} \in \varphi(\mathbf{e})$ and there exists a price $p \in \Delta^P(\mathbf{e}, \mathbf{z}) \cap \Delta^P(\mathbf{e}', \mathbf{z})$. By (ii), $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e})$ and $g(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$, where $\mathbf{p} = (p)_{i \in N}$, $\mathbf{s}^t = (s_i^t)_{i \in N}$, $\mathbf{x} = (x_i)_{i \in N}$, and $\mathbf{y} = (y_i)_{i \in N}$, which implies $(\mathbf{p}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e})$ and $g(\mathbf{p}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y}) = g(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$ by (iv). Then, by (iii),

$$g_i\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}^{t\prime}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{y}_{w_i'}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i^t (x_i' - x_i)\right\}$$

for all $i \in N$ and all $(p^{i\prime}, s_i', x_i', w_i') \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. Since $p \in \Delta^P(\mathbf{e}', \mathbf{z})$, this implies $(\mathbf{p}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y}) \in NE(g, \mathbf{e}')$ and $(\mathbf{x}, \mathbf{y}) \in NA(g, \mathbf{e}')$. Hence, $\mathbf{z} \in \varphi(\mathbf{e}')$ by (i). ■

6.2 Proof of Theorem 3

Proof of Lemma 1. Suppose there exists $j \in N$ with $s_j \neq s_j^t$ and $x_j > 0$. Let $N(\mathbf{s}, \mathbf{x})$ be the set of such j. Since $f\left(\sum s_i x_i\right) = f\left(\sum s_i^t x_i\right)$, $N(\mathbf{s}, \mathbf{x})$ is not a singleton. Moreover, any $j \in N(\mathbf{s}, \mathbf{x})$ can obtain $y_j' = f\left(\sum_{i \neq j} s_i^t x_i\right) > 0$ with $s_j' > \max_{i \neq j}\{s_i\}$, $x_j' = 0$, and $w_j' = 0$ under Rule 2. Note that

$$\begin{aligned}
\sum_{j \in N(\mathbf{s},\mathbf{x})} y_j' &= \sum_{j \in N(\mathbf{s},\mathbf{x})} f\left(\sum_{i \neq j} s_i^t x_i\right) = \sum_{j \in N(\mathbf{s},\mathbf{x})} f\left(\sum_{i \in N(\mathbf{s},\mathbf{x}) \setminus \{j\}} s_i^t x_i + \sum_{k \notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right) \\
&\geq \sum_{j \in N(\mathbf{s},\mathbf{x})} f\left(s_j^t x_j + \sum_{k \notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right) \text{ (since } N(\mathbf{s},\mathbf{x}) \text{ is not a singleton)} \\
&\geq f\left(\sum_{j \in N(\mathbf{s},\mathbf{x})} \left(s_j^t x_j + \sum_{k \notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right)\right) \text{ (since } f \text{ is concave and } f(0) \geq 0) \\
&\geq f\left(\sum_{j \in N(\mathbf{s},\mathbf{x})} s_j^t x_j + \sum_{k \notin N(\mathbf{s},\mathbf{x})} s_k^t x_k\right) \\
&= f\left(\sum s_k^t x_k\right) \geq \sum_{j \in N(\mathbf{s},\mathbf{x})} y_j \equiv \sum_{j \in N(\mathbf{s},\mathbf{x})} g_j^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}).
\end{aligned}$$

Hence, there must be $j \in N(\mathbf{s}, \mathbf{x})$ with $y_j' \geq y_j$. This j has an incentive to change x_j to $x_j' = 0$ to obtain y_j', for $u_j(x_j', y_j') \geq u_j(x_j', y_j) \geq u_j(x_j, y_j)$, where $u_j(x_j', y_j') > u_j(x_j', y_j)$ if $y_j = 0$ by Assumption 1, while $u_j(x_j', y_j) > u_j(x_j, y_j)$ if $y_j > 0$ by strict monotonicity of utility functions. Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ does not constitute a Nash equilibrium. ■

Lemma 2: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then, g^** implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in Nash equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be any given.

(1) First, we show that $\varphi(\mathbf{e}) \subseteq NA(g^*, \mathbf{e})$. Let $\mathbf{z} = (\mathbf{x}, \mathbf{y}) \in \varphi(\mathbf{e})$. Let the strategy profile of agents be $(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) = (p^i, s_i^t, x_i, y_i)_{i \in N} \in (\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$ such that $p^i = p$ for all $i \in N$ where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \mathbf{z})$. Then $g^*(\mathbf{p}, \mathbf{s}^t, \mathbf{x}, \mathbf{y}) =$

$\mathbf{y}$ by Rule 1-1. Suppose that an individual $j \in N$ deviates from $\left(p^j, s_j^t, x_j, y_j\right)$ to $\left(p^{j\prime}, s_j', x_j', w_j'\right) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. Note by Assumption 1 and the continuity of utility functions, if $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = 0$, then the deviation gives no reward to j.

Since every $i \neq j$ truly reports his skill, $s_j' = s_j^t$ is necessary to induce some subrule of Rule 1 with $x_j' > 0$. That is, the deviation cannot induce Rule 1-3 as long as $x_j' > 0$, which is a necessary condition for the deviator to consume a positive output under Rule 1-3. If the deviation induces Rule 2, then $x_j' > 0$, so that $g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = 0$. In fact, if $x_j' = 0$, then $\sum_{i \neq j} s_i^t x_i + s_j' x_j' = \sum_{i \neq j} s_i^t x_i + s_j^t x_j'$, so that $f\left(\sum_{i \neq j} s_i^t x_i + s_j' x_j'\right) = f\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j'\right)$, which contradicts the fact that Rule 2 is induced.

Suppose the deviation induces Rule 1-2. If $x_j' > 0$, then $s_j' = s_j^t$ and

$$g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) \leq \max\left\{0, y_j + \frac{p_x}{p_y}\left(s_j^t x_j' - s_j^t x_j\right)\right\},$$

which implies j cannot gain from his deviation. Let us consider the case $x_j' = 0$. The application of Rule 1-2 implies that there exist x_j'' and w_j'' such that $\varphi\left(p, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j''}, \mathbf{y}_{w_j''}\right)^{-1} \neq \varnothing$. Note that the information of $\mathbf{p}$ induces $w_j'' + \frac{p_x}{p_y}\left(s_j' x_j' - s_j' x_j''\right) = y_j + \frac{p_x}{p_y}\left(s_j' x_j' - s_j^t x_j\right)$. Thus,

$$g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) \leq \max\left\{0, y_j + \frac{p_x}{p_y}\left(s_j' x_j' - s_j^t x_j\right)\right\},$$

which implies again j cannot gain from his deviation.

Finally, if the deviation induces Rule 1-1, then

$$g_j^*\left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j'}^t, \mathbf{x}_{x_j'}, \mathbf{y}_{w_j'}\right) = w_j' = f\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j'\right) - \sum_{i \neq j} y_i.$$

The Pareto efficiency of $\mathbf{z}$ implies no additional benefit for j.

(2) Second, we will show that $NA\left(g^*, \mathbf{e}\right) \subseteq \varphi\left(\mathbf{e}\right)$. Let $\left(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}\right) = \left(p^i, s_i, x_i, w_i\right)_{i \in N} \in \left(\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+\right)^n$ be a pure-strategy Nash equilibrium of the feasible sharing game $\left(g^*, \mathbf{e}\right)$.

Suppose that $\left(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}\right)$ induces Rule 2. If $N^0\left(\mathbf{x}\right) \equiv \{i \in N : x_i = 0\} = \varnothing$, then $g_i^*\left(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}\right) = 0$ for all $i \in N$. Note here that there exists at least an individual $j \in N$ such that $\sum_{i \neq j} s_i x_i \neq \sum_{i \neq j} s_i^t x_i$. In fact, if not, then $(n-1) \cdot (\sum s_i x_i) = (n-1) \cdot (\sum s_i^t x_i)$, which contradicts the fact that Rule 2 is induced. Thus, there exists an individual $j \in N$ in this case, who can enjoy $g_j^* \left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j^{\prime}}, \mathbf{x}_{x_j^{\prime}}, \mathbf{w}_{w_j^{\prime}} \right) > 0$ with $s_j^{\prime} > \max \{s_i\}_{i \in N}$, $x_j^{\prime} = 0$, and $w_j^{\prime} = 0$ under Rule 2.

If $N^0(\mathbf{x}) \neq \varnothing$, then any $j \in N^0(\mathbf{x})$ can monopolize all $f(\sum s_k^t x_k)$ by reporting his labor skill as $s_j^{\prime}$ so that $s_j^{\prime} > s_i$ for all $i \neq j$, while supplying $x_j^{\prime} = 0$ and reporting $w_j^{\prime} = 0$ under Rule 2. Thus, if $\# N^0(\mathbf{x}) \geq 2$, then no profile of agents' strategies can constitute a Nash equilibrium under Rule 2.

If $\# N^0(\mathbf{x}) = 1$ and $\# N \backslash N^0(\mathbf{x}) \geq 2$, then there exists at least an individual $j \in N \backslash N^0(\mathbf{x})$ such that $\sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i x_i \neq \sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i^t x_i$ under Rule 2. In fact, if not, then $(n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i x_i \right) = (n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i^t x_i \right)$, which contradicts the fact that Rule 2 is induced. Thus, there exists an individual $j \in N \backslash N^0(\mathbf{x})$, who can enjoy $g_j^* \left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j^{\prime}}, \mathbf{x}_{x_j^{\prime}}, \mathbf{w}_{w_j^{\prime}} \right) > 0$ with $s_j^{\prime} > \max \{s_i\}_{i \in N}$, $x_j^{\prime} = 0$, and $w_j^{\prime} = 0$ under Rule 2. Suppose $\# N^0(\mathbf{x}) = 1$ with $N^0(\mathbf{x}) = \{i\}$ and $\# N \backslash N^0(\mathbf{x}) = 1$ with $N \backslash N^0(\mathbf{x}) = \{j\}$. If $w_i > 0$, then i can enjoy $g_i^* \left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i^{\prime}}, \mathbf{x}_{x_i^{\prime}}, \mathbf{w}_{w_i^{\prime}} \right) = f\left(s_j^t x_j\right)$ with $x_i^{\prime} = 0$ and $w_i^{\prime} = 0$ under Rule 2. If $w_i = 0$, then j can enjoy $g_j^* \left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j^{\prime}}, \mathbf{x}_{x_j^{\prime}}, \mathbf{w}_{w_j^{\prime}} \right) = f\left(s_j^t x_j^{\prime}\right)$ with $p^{j\prime} = p^i$, $s_j^{\prime} = s_j^t$, $x_j^{\prime} = \frac{\overline{x}}{2}$, and $w_j^{\prime} > f\left(s_j^t \overline{x} + s_i \overline{x}\right)$ under Rule 1-3. Thus, if $\# N^0(\mathbf{x}) = 1$, no profile of agents' strategies can constitute a Nash equilibrium corresponding to Rule 2.

Suppose that $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ induces Rule 1-2 or 1-3. Then, there exists $j \in N$ such that $g_j^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$. For this j, $s_j = s_j^t$ or $x_j = 0$ by Lemma 1. Thus, if j deviates with supplying a positive amount of labor while truly reporting his skill, then the deviation induces either of Rule 1-1, Rule 1-2, or Rule 1-3. Suppose $p^k \neq p^l$ for some k, $l \neq j$. Then, j can enjoy $g_j^* \left(\mathbf{p}_{p^{j\prime}}, \mathbf{s}_{s_j^{\prime}}, \mathbf{x}_{x_j^{\prime}}, \mathbf{w}_{w_j^{\prime}} \right) > 0$ with $p^{j\prime}$ such that $p^{j\prime} \neq p^k$ and $p^{j\prime} \neq p^l$, $s_j^{\prime} = s_j^t$, $x_j^{\prime} = \mu(\mathbf{x}_{-j}) < \overline{x}$, and $w_j^{\prime} > \max \left\{ f\left(\sum_{i \neq j} s_i \bar{x} + s_j^{\prime} \bar{x} \right), \max_{i \neq j} \{w_i\} \right\}$ under Rule 1-3. Suppose $p^k = p^l = p$ for all k, $l \neq j$. Then, if $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-2, either $j \in N(p, \mathbf{s}, \mathbf{x}, \mathbf{w})$ or not. In any case, by choosing $p^{j\prime} = p$, $s_j^{\prime} = s_j^t$, $w_j^{\prime} > \max \left\{ f\left(\sum_{i \neq j} s_i \bar{x} + s_j^{\prime} \bar{x} \right), \max_{i \neq j} \{w_i\} \right\}$, and appropriate choice of $x_j^{\prime}$ as a deviating strategy, j can get a positive amount of output. By the same type of deviating strategy, j can also get a positive amount of output when $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-3.

Thus, $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-1, and $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{w}$. In this case, there exists $\mathbf{u}' \in \mathcal{U}^n$ such that $p \in \Delta^{SPI}(\mathbf{u}',\mathbf{s},\mathbf{x},\mathbf{w})$ where $p^i = p$ for all $i \in N$. Moreover, $p \in \Delta^P(\mathbf{u},\mathbf{s}^t,\mathbf{x},\mathbf{w})$ and $(\mathbf{x},\mathbf{w})$ is Pareto efficient for $(\mathbf{u},\mathbf{s}^t)$, for otherwise, some j has an incentive to deviate to Rule 1-2, which contradicts the fact that $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*,\mathbf{e})$. Consider another economy $(\mathbf{u}',\mathbf{s}') \in \mathcal{E}$ such that $s_i' = \min\{s_i, s_i^t\}$ for each $i \in N$ with $x_i = 0$ while $s_i' = s_i\,(= s_i^t$ by Lemma 1) for every $i \in N$ with $x_i > 0$. First, the Pareto efficiency of $(\mathbf{x},\mathbf{w})$ for $(\mathbf{u}',\mathbf{s})$ implies the efficiency of $(\mathbf{x},\mathbf{w})$ for $(\mathbf{u}',\mathbf{s}')$ since $s_i' \leq s_i$ for all $i \in N$ with $x_i = 0$ and $s_i' = s_i$ for all $i \in N$ with $x_i > 0$. Hence $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u}',\mathbf{s})$ implies $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u}',\mathbf{s}')$ by IUS. Next, the Pareto efficiency of $(\mathbf{x},\mathbf{w})$ for $(\mathbf{u},\mathbf{s}^t)$ implies the efficiency of $(\mathbf{x},\mathbf{w})$ for $(\mathbf{u},\mathbf{s}')$ since $s_i' \leq s_i^t$ for all $i \in N$ with $x_i = 0$ and $s_i' = s_i^t$ for all $i \in N$ with $x_i > 0$. Note here $p \in \Delta^{SPI}(\mathbf{u}',\mathbf{s},\mathbf{x},\mathbf{w}) \cap \Delta^P(\mathbf{u}',\mathbf{s}',\mathbf{x},\mathbf{w})$ and $p \in \Delta^P(\mathbf{u},\mathbf{s}^t,\mathbf{x},\mathbf{w}) \cap \Delta^P(\mathbf{u},\mathbf{s}',\mathbf{x},\mathbf{w})$. Thus $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u}',\mathbf{s}')$ in turn implies $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u},\mathbf{s}')$ by SPI. Finally, since $(\mathbf{x},\mathbf{w})$ is Pareto efficient for $(\mathbf{u},\mathbf{s}^t)$ and $s_i' = s_i^t$ for all $i \in N$ with $x_i > 0$, $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u},\mathbf{s}')$ implies $(\mathbf{x},\mathbf{w}) \in \varphi(\mathbf{u},\mathbf{s}^t)$ by IUS. ■

Lemma 3: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then, g^** implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in strong equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u},\mathbf{s}^t) \in \mathcal{E}$ be any given. Since $NA(g^*,\mathbf{e}) = \varphi(\mathbf{e})$, we have only to show $NA(g^*,\mathbf{e}) \subseteq SNA(g^*,\mathbf{e})$. Assume that there exists $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*,\mathbf{e})$ such that for some $T \subseteq N$ with $2 \leq \#T < n$ and some $(p^{i\prime}, s_i', x_i', w_i')_{i\in T} \in (\Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+)^{\#T}$,

$$u_j\left(x_j, g_j^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})\right) < u_j\left(x_j', g_j^*\left(\left(p^{i\prime}, s_i', x_i', w_i'\right)_{i\in T}, \left(p^k, s_k, x_k, w_k\right)_{k\in T^c}\right)\right)$$

for all $j \in T$. Note first that $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-1 and $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{w}$, as is shown in the proof of Lemma 2. Moreover, $(\mathbf{x},\mathbf{w})$ is Pareto efficient for $(\mathbf{u},\mathbf{s}^t)$.

By construction of g^*, there is at most one agent who can enjoy a positive amount of output under Rule 1-2, Rule 1-3, and Rule 2. Thus, by Assumption 1, the deviation by T should induce Rule 1-1. Then,

$$g^*\left(\left(p^{i\prime}, s_i', x_i', w_i'\right)_{i\in T}, \left(p^k, s_k, x_k, w_k\right)_{k\in T^c}\right) = \left((w_i')_{i\in T}, (w_k)_{k\in T^c}\right).$$

That is, even under the deviation, every $k \in T^c$ works for the same time and enjoys the same amount of output consumption. Hence, the assumption that the deviation is beneficial for every $i \in T$ contradicts the Pareto efficiency of $(\mathbf{x}, \mathbf{w})$. Thus, $NA(g^*, \mathbf{e}) \subseteq SNA(g^*, \mathbf{e})$. ■

Lemma 4: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then, $\Gamma^2_{g^*}$* implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in subgame-perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be any given. $NA(g^*, \mathbf{e}) = \varphi(\mathbf{e})$ by Lemma 2. Moreover, $SPA\left(\Gamma^2_{g^*}, \mathbf{e}\right) \subseteq NA(g^*, \mathbf{e})$. Hence, we have only to show $\varphi(\mathbf{e}) \subseteq SPA\left(\Gamma^2_{g^*}, \mathbf{e}\right)$.

First, we will show that in every second stage subgame, there is at least one Nash equilibrium strategy. Let us take a strategy mapping $\mathbf{m}^{\mathbf{e}*} : [0, \bar{x}]^n \to \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+$ such that for each second stage subgame $\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$, $\mathbf{m}^{\mathbf{e}*}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}, \mathbf{w})$ where for all $i \in N$

$$\left(p^i, s_i, w_i\right) = \begin{cases} \left((0,1), s^t_i, f\left(\sum s^t_k \bar{x}\right) + 1\right) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}) \\ \left((0,1), s^t_i, f\left(\sum s^t_k \bar{x}\right) + 2\right) & \text{otherwise} \end{cases}.$$

Then, we can see that $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$. Note that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-3. Since $p^i = (0,1)$ for all $i \in N$, no individual can induce Rule 1-1 by changing his strategy. Inducing Rule 1-2 with $\left(p^{j\prime}, s'_j, w'_j\right)$ would not be beneficial for any $j \in N$ since $p^i = (0,1)$ for another $i \neq j$ and there does not exist $\left(x''_j, w''_j\right) \in [0, \bar{x}] \times \mathbb{R}_{++}$ such that $\varphi\left(p^i, \mathbf{s}_{s'_j}, \mathbf{x}_{x''_j}, \mathbf{w}_{w''_j}\right)^{-1} \neq \varnothing$. Moreover, if any $i \in N$ can induce Rule 2 by some deviation, then it must be the case that $x_i > 0$, which implies this deviation is not beneficial for i. Finally, if any $i \in N$ can deviate to induce Rule 1-3 again, then such a deviation gives him no additional benefit, since the individual i who has $x_i = \mu(\mathbf{x}_{-i})$ is already fixed in the first stage game. Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$.

Now, we will show that for the given $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in \varphi(\mathbf{e})$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. Consider the following strategy profile of the extensive game $\left(\Gamma^2_{g^*}, \mathbf{e}\right)$:
(1) In the first stage, every individual i supplies $\widehat{x}_i$.
(2) In the second stage, the agents take a strategy mapping $\mathbf{m}^{\mathbf{e}} : [0, \bar{x}]^n \to \Delta^n \times \mathcal{S}^n \times \mathbb{R}^n_+$ such that $\mathbf{m}^{\mathbf{e}}(\mathbf{x}) = (\mathbf{p}, \mathbf{s}, \mathbf{w})$ as follows:

(2-1): if $\mathbf{x} = \widehat{\mathbf{x}}$ is the action profile of all individuals in the first stage, then for any $i \in N$, $m_i^{\mathbf{e}}(\mathbf{x}) = (p^i, s_i, w_i) = (p, s_i^t, \widehat{y}_i)$ where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$;
(2-2): if $\mathbf{x} = \widehat{\mathbf{x}}_{x'_j}$, where $x'_j \neq \widehat{x}_j$, is the action profile of all individuals in the first stage, then for this $j \in N$,

$$m_j^{\mathbf{e}}(\mathbf{x}) = \left(p^j, s_j, w_j\right) = \left((0,1), s_j^t, f\left(\sum s_k^t \bar{x}\right) + 1\right),$$

and for all $i \neq j$,

$$m_i^{\mathbf{e}}(\mathbf{x}) = \left(p^i, s_i, w_i\right) = \begin{cases} (p, s_i^t, \widehat{y}_i) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}), \\ ((1,0), s_i^t, f(\sum s_k^t \bar{x}) + 2) & \text{otherwise,} \end{cases}$$

where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$;
(2-3): in any other case, $\mathbf{m}^{\mathbf{e}}(\mathbf{x}) = \mathbf{m}^{\mathbf{e}*}(\mathbf{x})$.

Note that for the subgame of (2-1), $(\mathbf{p}, \mathbf{s}, \mathbf{w}) = (\mathbf{p}, \mathbf{s}^t, \widehat{\mathbf{y}}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$, since $(\mathbf{p}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in NE(g^*, \mathbf{e})$. Also, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$ for any subgame $\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$ of the case (2-3), as we have already shown. Moreover, we will show that for the subgame of (2-2), $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$. Since $\mathbf{s} = \mathbf{s}^t$, if anyone deviates to induce Rule 2, then he has to supply a positive amount of labor, which implies the deviation is not beneficial for him by the construction of Rule 2.

Consider the case that for all $i \neq j$, $x_i \neq \mu(\mathbf{x}_{-i})$ in (2-2). Then, $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-2. No $i \neq j$ can induce Rule 1-1 nor can he consume a positive output under Rule 1-2 since $p^j = (0,1)$ and $\varphi\left(p^j, \mathbf{s}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)^{-1} = \varnothing$ for any $(x'_i, w'_i) \in [0, \bar{x}] \times \mathbb{R}_{++}$. Moreover, no $i \neq j$ can consume a positive output under Rule 1-3 since $x_i \neq \mu(\mathbf{x}_{-i})$. On the other hand, j can deviate to induce Rule 1-1, or Rule 1-2, but it does not make him better off. When Rule 1-1 is induced, j's output consumption becomes $f\left(s_j^t x'_j + \sum_{k \neq j} s_k^t \widehat{x}_k\right) - \sum_{k \neq j} \widehat{y}_k$, which is no more than $\widehat{y}_j + \frac{p_x}{p_y} s_j^t \left(x'_j - \widehat{x}_j\right)$ since $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$. When Rule 1-2 is induced again, j's output consumption remains

$$\max\left\{0, \min\left\{\widehat{y}_j + \frac{p_x}{p_y}\left(s_j^t x'_j - s_j^t \widehat{x}_j\right), f\left(\sum s_k^t x_k\right)\right\}\right\}.$$

Moreover, if j deviates to induce Rule 1-3, then he has no positive output consumption. Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$.

Consider the case that there exists $i \neq j$ with $x_i = \mu(\mathbf{x}_{-i})$ in (2-2). Then $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-3. Any deviation cannot induce Rule 1-1 since $p^j = (0,1)$ and $p^i = (1,0)$. Agent j's deviation to induce Rule 1-2 results in $g_j^* (\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ since $p^i = (1,0)$ for $i \neq j$ and $\varphi((1,0), \mathbf{s}, \mathbf{x}, \mathbf{w})^{-1} = \varnothing$ for any $(\mathbf{s}, \mathbf{x}, \mathbf{w})$. The same is true for i. Moreover, if any $i \in N$ can induce Rule 1-3 by changing his strategy, then such a deviation gives him no additional benefit, since the individual i who has $x_i = \mu(\mathbf{x}_{-i})$ is already fixed in the first stage game. Thus, $(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$.

Now, let us see that the above strategy profile (1)-(2) constitutes a subgame perfect equilibrium of the extensive game $\left(\Gamma_{g^*}^2, \mathbf{e}\right)$. In accordance with (1)-(2-1), the outcome becomes $(\widehat{\mathbf{x}}, \widehat{\mathbf{y}})$. Suppose some j has an incentive to deviate from $\widehat{x}_j$ to $x_j' \neq \widehat{x}_j$ in the first stage. Then by (2-2), he only gets

$$g_j^* \left(\mathbf{m}^{\mathbf{e}}\left(\widehat{\mathbf{x}}_{x_j'}\right), \widehat{\mathbf{x}}_{x_j'}\right) \leq \max\left\{0, \min\left\{\widehat{y}_j + \frac{p_x}{p_y}\left(s_j^t x_j' - s_j^t \widehat{x}_j\right), f\left(\sum s_k^t x_k\right)\right\}\right\},$$

which contradicts the fact that $\widehat{\mathbf{z}}$ is Pareto efficient for $\mathbf{e}$. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in SPA\left(\Gamma_{g^*}^2, \mathbf{e}\right)$. ■

Lemma 5: *Let Assumption 1 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then,* $\Gamma_{g^*}^1$ implements *any Pareto subsolution φ satisfying SPI and IUS on $\mathcal{E}$* in subgame-perfect equilibria.

Proof. Let φ be a Pareto subsolution satisfying SPI and IUS. Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be any given. $NA(g^*, \mathbf{e}) = \varphi(\mathbf{e})$ by Lemma 2. Moreover, $SPA\left(\Gamma_{g^*}^1, \mathbf{e}\right) \subseteq NA(g^*, \mathbf{e})$. Hence we have only to show $\varphi(\mathbf{e}) \subseteq SPA\left(\Gamma_{g^*}^1, \mathbf{e}\right)$.

First, we will show that in every second stage subgame, there is at least one Nash equilibrium strategy. Let

$$I(p, \mathbf{s}, \mathbf{0}, \mathbf{w}) \equiv \left\{i \in N : \exists x_i', \varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}\right)^{-1} \neq \varnothing\right\}.$$

Let us take a strategy mapping $\mathbf{x}^{\mathbf{e}*} : \Delta^n \times \mathcal{S}^n \times \mathbb{R}_+^n \to [0, \bar{x}]^n$ such that for each second stage subgame $\left(\Gamma_{g^*}^1(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$: for all $i \in N$,
(i) if $s_i = s_i^t$ and $\exists p$ s.t. $p^j = p$ for all $j \in N$ and $i = \min I(p, \mathbf{s}, \mathbf{0}, \mathbf{w})$, then $x_i^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = x_i'$ such that $\varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}\right)^{-1} \neq \varnothing$;
(ii) if $s_i = s_i^t$, $w_i > f\left(\sum s_k \bar{x}\right)$, and $\exists p$ s.t. $p^j = p$ $(\forall j \neq i)$ and $i \in N(p, \mathbf{s}, \mathbf{0}, \mathbf{w})$, then

$$x_i^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \arg\max_{x_i} u_i\left(x_i, \max\left\{0, \min\left\{w_i' + \frac{p_x}{p_y} s_i^t (x_i - x_i'), f\left(s_i^t x_i\right)\right\}\right\}\right);$$

(iii) if $s_i = s_i^t, w_i > \max\left\{f\left(\sum s_k\bar{x}\right), \max_{j\neq i}\{w_j\}\right\}$, and $[\{\exists p \text{ s.t. } p^j = p\ (\forall j \neq i)\} \Rightarrow i \notin N(p, \mathbf{s}, \mathbf{0}, \mathbf{w})]$, then $x_i^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \frac{\bar{x}}{2}$;
(iv) otherwise, $x_i^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = 0$,
where (x_i', w_i') comes from $\varphi\left(p, \mathbf{s}, \mathbf{0}_{x_i'}, \mathbf{w}_{w_i'}\right)^{-1} \neq \varnothing$. Note that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{w})$ corresponds to one of the subrules of Rule 1 since $x_i = 0$ for all i with $s_i \neq s_i^t$. Then, we can see that $\mathbf{x}^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^1(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. To simplify the notation, let us denote $\mathbf{x}^* = \mathbf{x}^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w})$ in the following discussion.

Since $x_i = 0$ for all i with $s_i \neq s_i^t$, if anyone deviates to induce Rule 2, then he has to supply a positive amount of labor. This implies such a deviation is not beneficial for him by the construction of Rule 2.

Suppose $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-1. Then, since $(\mathbf{p}, \mathbf{s}, \mathbf{w})$ is already fixed, no unilateral deviation from $\mathbf{x}^*$ can induce Rule 1-1. Moreover, $w_i \leq f\left(\sum s_k\bar{x}\right)$ for any i when $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-1, which implies that no individual would gain under Rule 1-2 nor Rule 1-3.

Suppose $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-2. Then, we will first show that there exists a unique agent $j \in N$ who takes (ii) of the strategy mapping $x_j^{\mathbf{e}*}$, while any other $i \neq j$ takes (iv) of the strategy $x_i^{\mathbf{e}*}$. By the definition of Rule 1-2, there exists $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ for $p = p^i\ (\forall i \neq j)$, which implies $w_i \leq f\left(\sum s_k\bar{x}\right)$ for any $i \neq j$. Thus, no $i \neq j$ can take (ii) and (iii) of the strategy mapping under Rule 1-2. Also, when $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-2, no agent should take (i) of the strategy mapping $x_i^{\mathbf{e}*}$. Thus, any $i \neq j$ should take (iv) of the strategy $x_i^{\mathbf{e}*}$, while $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ should take (ii) of $x_j^{\mathbf{e}*}$ so as to induce Rule 1-2. In this strategy profile, any $i \neq j$ cannot gain by any deviation to induce Rule 1-2, 1-3, or Rule 2. Also, since $w_j > f\left(\sum s_k\bar{x}\right)$, every $i \neq j$ cannot induce Rule 1-1. As for $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$, he cannot induce Rule 1-1 by any deviation because of $w_j > f\left(\sum s_k\bar{x}\right)$. Also, he cannot induce Rule 1-3, since $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ implies $j \in N\left(p, \mathbf{s}, \mathbf{x}^*_{x_j'}, \mathbf{w}\right)$. Finally, j cannot gain by deviation to induce Rule 1-2. Thus, taking (ii) of the strategy mapping is the best response for $j \in N(p, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$, which implies $\mathbf{x}^{\mathbf{e}*}$ is a Nash equilibrium.

Suppose $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}^*, \mathbf{w})$ corresponds to Rule 1-3. Then, $\mathbf{x}^*$ consists of $x_j^* = \frac{\bar{x}}{2}$ and $x_i^* = 0$ for any $i \neq j$, or $x_i^* = 0$ for all $i \in N$. In both cases, $\mathbf{x}^* \in NE\left(\Gamma_{g^*}^1(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. In the latter case, any i takes (iv) of $x_i^{\mathbf{e}*}$, and there is no other better strategy for i on his labor choice, given $(\mathbf{p}, \mathbf{s}, \mathbf{w})$ and $\mathbf{x}^*_{-i} = \mathbf{0}_{-i}$. In the former case, $x_j^* = \frac{\bar{x}}{2}$ is the best response for $j \in N$ to $\mathbf{x}^*_{-j} = \mathbf{0}_{-j}$. In fact, $j \in N$ cannot induce Rule 1-1 or 1-2, while he cannot gain by any deviation to Rule 2, given $\mathbf{x}^*_{-j} = \mathbf{0}_{-j}$. In contrast, given $x_j^* = \frac{\overline{x}}{2}$ and $\mathbf{x}^*_{-\{i,j\}} = \mathbf{0}_{-\{i,j\}}$, any $i \neq j$ cannot gain by any deviation to Rule 1-3 or 2. Also, i cannot induce Rule 1-1 or 1-2, since $w_j > f\left(\sum s_k \bar{x}\right)$ implies $i \notin N\left(p, \mathbf{s}, \left(\mathbf{0}_{-j}, \frac{\overline{x}}{2}\right), \mathbf{w}\right)$ even if $p = p^k$ $(\forall k \neq i)$. Thus, $\mathbf{x}^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w}) \in NE\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

Now, we will show that for the given $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in \varphi(\mathbf{e})$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. Consider the following strategy profile of the extensive game $\left(\Gamma^1_{g^*}, \mathbf{e}\right)$:

(1) In the first stage, every individual i reports $(p^i, s_i, w_i) = (p, s_i^t, \widehat{y}_i)$ where $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}})$.

(2) In the second stage:

(2-1): if $(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \left((p^i)_{i \in N}, \mathbf{s}^t, \widehat{\mathbf{y}}\right)$ with $p^i = p$ for all $i \in N$ is the action profile of all individuals in the first stage, then any $i \in N$ supplies $x_i = \widehat{x}_i$;

(2-2): if $(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \left(\left(p^{j\prime}, (p^i)_{i \neq j}\right), \mathbf{s}^t, \widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$ and $w_j' > f\left(\sum s_k^t \bar{x}\right)$, is the action profile of all individuals in the first stage, then for this $j \in N$,

$x_j = \arg\max_{x_j'} u_j\left(x_j', \min\left\{\widehat{y}_j + \frac{p_x}{p_y} s_j^t\left(x_j' - \widehat{x}_j\right), f\left(\sum_{i \neq j} s_i^t \widehat{x}_i + s_j^t x_j'\right)\right\}\right)$

and for all $i \neq j$, $x_i = \widehat{x}_i$;

(2-3): if $(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \left(\left(p^{j\prime}, (p^i)_{i \neq j}\right), \mathbf{s}^t, \widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$, $\left(p^{j\prime}, w_j'\right) \neq (p, \widehat{y}_j)$ and $w_j' \leq f\left(\sum s_k^t \bar{x}\right)$, is the action profile of all individuals in the first stage, then $\mathbf{x} = \overline{\mathbf{x}}$;

(2-4): if $(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \left(\left(p^{j\prime}, (p^i)_{i \neq j}\right), \mathbf{s}^t_{s_j'}, \widehat{\mathbf{y}}_{w_j'}\right)$, where $p^i = p$ for all $i \neq j$ and $s_j' \neq s_j^t$, is the action profile of all individuals in the first stage, then for this $j \in N$, $x_j = \frac{\overline{x}}{2}$, and for all $i \neq j$, $x_i = 0$;

(2-5): in any other case, $\mathbf{x}^{\mathbf{e}}(\mathbf{p}, \mathbf{s}, \mathbf{w}) = \mathbf{x}^{\mathbf{e}*}(\mathbf{p}, \mathbf{s}, \mathbf{w})$.

Note that for the subgame of (2-1), $\mathbf{x} = \widehat{\mathbf{x}} \in NE\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$, since $w_i \leq f\left(\sum s_k \bar{x}\right)$ for any $i \in N$ and no deviator can enjoy a positive amount of output under Rule 1-2. Also, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ for the subgame of (2-5), as we have already shown. Moreover, we will see that for the subgames from (2-2) to (2-4), $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

Note that $g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to Rule 1-2 if $\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ corresponds to (2-2). In this case, nobody can induce Rule 1-1 nor Rule 2. Moreover, no $i \neq j$ can enjoy a positive amount of output under Rule 1-2 nor Rule 1-3 since $w_i = \widehat{y}_i \leq f\left(\sum s_k \bar{x}\right)$. Finally, j cannot induce Rule 1-3 by only changing labor supply. Thus, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{p}, \mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

If $\left(\Gamma^1_{g^*}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ corresponds to (2-3), $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 1-3 since $\mathbf{x}=\overline{\mathbf{x}}$, where nobody can enjoy positive output consumption by the definition of Rule 1-3. In this case, some unilateral deviation may induce Rule 1-2 or Rule 1-3, but it is not beneficial for any $i \in N$ since $w_i \leq f\left(\sum s_k \bar{x}\right)$. Thus, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Finally, $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to Rule 2 if $\left(\Gamma^1_{g^*}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$ corresponds to (2-4). In this case, any $x'_j > 0$ induces Rule 2 again and $x'_j = 0$ makes the total output zero. Thus, j cannot enjoy positive output consumption in any case. As for $i \neq j$, any $x'_i \neq x_i$ induces Rule 2 again, but the deviation brings no additional benefit for i since $x_i = 0$ and $x'_i > 0$. Thus, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{p},\mathbf{s},\mathbf{w}),\mathbf{e}\right)$.

Now, let us see that the above strategy profile (1)-(2) constitutes a subgame perfect equilibrium of the extensive game $\left(\Gamma^1_{g^*},\mathbf{e}\right)$. By the strategy profile (1)-(2) of the extensive game $\left(\Gamma^1_{g^*},\mathbf{e}\right)$, $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = g^*\left(\left((p)_i\right),\mathbf{s}^t,\widehat{\mathbf{x}},\widehat{\mathbf{y}}\right) = \widehat{\mathbf{y}}$. Suppose that some j has an incentive to deviate from (p^j, s_j, w_j) to $\left(p^{j\prime}, s_j, w'_j\right)$ in the first stage. Then, by (2-2) and (2-3), he only gets

$$g^*_j\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{x}\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{w}_{w'_j}\right),\mathbf{w}_{w'_j}\right) \leq \widehat{y}_j + \frac{p_x}{p_y} s^t_j\left(x_j - \widehat{x}_j\right)$$

where $x_j = x_j\left(\mathbf{p}_{p^{j\prime}},\mathbf{s},\mathbf{w}_{w'_j}\right)$. This contradicts the fact that $\widehat{\mathbf{z}}$ is Pareto efficient for $\mathbf{e}$. Suppose that some j has an incentive to deviate from (p^j, s_j, w_j) to $\left(p^{j\prime}, s'_j, w'_j\right)$ with $s_j \neq s'_j$ in the first stage. Then by (2-4), he cannot enjoy positive output consumption. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}},\widehat{\mathbf{y}}) \in SPA\left(\Gamma^1_{g^*},\mathbf{e}\right)$. ■

Proof of Theorem 3. Let Assumptions 1 hold. Let φ be a Pareto subsolution satisfying SPI and IUS. By Lemmas 2, 3, 4, and 5, $\Gamma^1_{g^*}$ (resp. $\Gamma^2_{g^*}$) triply implements φ *on* $\mathcal{E}$ in Nash, strong, and subgame-perfect equilibria. Moreover, g^* meets the forthrightness, as is shown in the former half of the proof of Lemma 2. Thus, it suffices to show g^* meets Definition 3 (iii) and (iv).

Let us show on Definition 3 (iii). The latter half of the proof of Lemma 2 shows that if $(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*,\mathbf{e})$, then it corresponds to Rule 1-1 and $g^*(\mathbf{p},\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{w}$. For any $i \in N$ and any $(p^{i\prime}, s'_i, x'_i, w'_i) \in \Delta \times \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+$, if $\left(\mathbf{p}_{p^{i\prime}},\mathbf{s}_{s'_i},\mathbf{x}_{x'_i},\mathbf{w}_{w'_i}\right)$ corresponds to Rule 1-1 or Rule 1-2, then

$$g^*_i\left(\mathbf{p}_{p^{i\prime}},\mathbf{s}_{s'_i},\mathbf{x}_{x'_i},\mathbf{w}_{w'_i}\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s^t_i\left(x'_i - x_i\right)\right\}.$$

If $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ corresponds to Rule 1-3, it implies either (i) $\varphi\left(p^{i\prime}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) \neq \varnothing$ and $i \in N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ or (ii) $i \notin N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$, where $p = p^k$ for all $k \neq i$. The case (i) implies $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$ by $w_i' \leq f\left(\sum s_k^t \overline{x}\right)$. Consider the case (ii). Since by Lemma 2, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{e})$ and $p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ hold, $i \in N\left(p, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ whenever $s_i' = s_i = s_i^t$. Thus, $s_i' \neq s_i$ which implies $x_i' = 0$ because $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ corresponds to Rule 1-3. Then, $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$. If $\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right)$ corresponds to Rule 2, then $x_i' > 0$ by Lemma 1 and $g_i^*\left(\mathbf{p}_{p^{i\prime}}, \mathbf{s}_{s_i'}, \mathbf{x}_{x_i'}, \mathbf{w}_{w_i'}\right) = 0$. Thus, g^* meets Definition 3 (iii).

For Definition 3 (iv), note again that if $(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g^*, \mathbf{e})$, then it corresponds to Rule 1-1, or there exists $\mathbf{u} \in \mathcal{U}^n$ such that $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ and for all $i \in N$, $p^i = p \in \Delta^{SPI}(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$. Moreover, if $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ for some $\mathbf{u} \in \mathcal{U}^n$, then $p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w})$ implies that for any $\mathbf{s}' \in \mathcal{S}^n$ such that $s_i' = s_i$ for all $i \in N$ with $x_i > 0$, there exists some $\mathbf{u}' \in \mathcal{U}^n$ such that $p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}', \mathbf{x}, \mathbf{w})$. By SPI, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}, \mathbf{s})$ and $p \in \Delta^P(\mathbf{u}, \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w})$ together imply $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s})$. By IUS, $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s})$ and $p \in \Delta^P(\mathbf{u}', \mathbf{s}, \mathbf{x}, \mathbf{w}) \cap \Delta^P(\mathbf{u}', \mathbf{s}', \mathbf{x}, \mathbf{w})$ together imply $(\mathbf{x}, \mathbf{w}) \in \varphi(\mathbf{u}', \mathbf{s}')$. Thus $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w})$ also corresponds to Rule 1-1. Hence $g^*(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) = g^*(\mathbf{p}, \mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{w}$ and $(\mathbf{p}, \mathbf{s}', \mathbf{x}, \mathbf{w}) \in NE(g^*, \mathbf{e})$. ■

6.3 Proofs of Lemmas in Section 4

Proof of Lemma 6. Consider an economy $(\mathbf{u}, \mathbf{s}^t)$ such that for some allocation $(\mathbf{x}, \mathbf{y})$, $(\mathbf{x}, \mathbf{y}) \in \varphi^W(\mathbf{u}, \mathbf{s}^t)$. Let us take a competitive equilibrium price $p = (p_x, p_y)$ which corresponds to $(\mathbf{x}, \mathbf{y})$ at $(\mathbf{u}, \mathbf{s}^t)$. Suppose the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}, \mathbf{s}^{t\prime})$ so that $\mathbf{s}_i^{t\prime} = \mathbf{s}_i^t$ for all $i \in N$ with $x_i > 0$ but still $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y})$. Then, by the definition of efficiency prices and the strict monotonicity of utility functions, it holds that:
(i) for all $\mathbf{z}' \in Z(\mathbf{s}^{t\prime})$, $\sum (p_y y_i' - p_x s_i^{t\prime} x_i') \leq \sum (p_y y_i - p_x s_i^{t\prime} x_i)$;
(ii) for all $i \in N$, $z_i \in \arg \max_{(x,y) \in B\left(p, s_i^{t\prime}, z_i, \theta_i\right)} u_i(x, y)$ where $B(p, s_i^{t\prime}, z_i, \theta_i) \equiv \left\{(x, y) \in [0, \bar{x}] \times \mathbb{R}_+ : p_y y - p_x s_i^{t\prime} x \leq \theta_i \sum \left(p_y y_j - p_x s_j^{t\prime} x_j\right)\right\}$ and $\theta_i = \frac{p_y y_i - p_x s_i^{t\prime} x_i}{\sum \left(p_y y_j - p_x s_j^{t\prime} x_j\right)}$.
Therefore, $(\mathbf{x}, \mathbf{y}) \in \varphi^W(\mathbf{u}, \mathbf{s}^{t\prime})$. Thus, the Walrasian solution satisfies IUS. ■

Proof of Lemma 7. Consider an economy $(\mathbf{u}, \mathbf{s}^t)$ such that for some allocation $(\mathbf{x}, \mathbf{y})$, $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^t)$. Let $p = (p_x, p_y) \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$. Suppose the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}, \mathbf{s}^{t\prime})$ so that $\mathbf{s}_i^{t\prime} = \mathbf{s}_i^t$ for all $i \in N$ with

$x_i > 0$ but still $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y})$. Then, since $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^t)$ and $\mathbf{s}_j^{t\prime} = \mathbf{s}_j^t$ for all $j \in N$ with $x_j > 0$, for each $i \in N$, $y_i = \frac{s_i^t x_i}{\sum s_j^t x_j} \sum y_j = \frac{s_i^t x_i}{\sum_{j \in N, x_j > 0} s_j^t x_j} \sum y_j = \frac{s_i^{t\prime} x_i}{\sum_{j \in N, x_j > 0} s_j^{t\prime} x_j} \sum y_j = \frac{s_i^{t\prime} x_i}{\sum s_j^{t\prime} x_j} \sum y_j$. Therefore, $(\mathbf{x}, \mathbf{y}) \in \varphi^{PR}(\mathbf{u}, \mathbf{s}^{t\prime})$. Thus, the proportional solution satisfies IUS. ■

Proof of Lemma 9. Consider an economy $(\mathbf{u}, \mathbf{s}^t)$ such that for some allocation $(\mathbf{x}, \mathbf{y})$, $(\mathbf{x}, \mathbf{y}) \in \varphi^{\widetilde{u}RWEB}(\mathbf{u}, \mathbf{s}^t)$. Let $p = (p_x, p_y) \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$ such that for all $i, j \in N$, $\max_{(x,y) \in B\left(p, s_i^t, z_i\right)} \widetilde{u}(x, y) = \max_{(x', y') \in B\left(p, s_j^t, z_j\right)} \widetilde{u}(x', y')$. Suppose the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}', \mathbf{s}^t)$ so that $p \in \Delta^P(\mathbf{u}', \mathbf{s}^t, \mathbf{x}, \mathbf{y})$. Since the skill profile remains $\mathbf{s}^t$, $(\mathbf{x}, \mathbf{y})$ is still feasible in the new economy. Moreover, for any $i, j \in N$, $\max_{(x,y) \in B\left(p, s_i^t, z_i\right)} \widetilde{u}(x, y) = \max_{(x', y') \in B\left(p, s_j^t, z_j\right)} \widetilde{u}(x', y')$ in the new economy, since the budget sets of all agents and the reference utility function $\widetilde{u}$ remain unaltered even after the change of economy. Therefore, $(\mathbf{x}, \mathbf{y}) \in \varphi^{\widetilde{u}RWEB}(\mathbf{u}', \mathbf{s}^t)$. Thus, the $\widetilde{u}$-reference welfare equivalent budget solution satisfies SPI. ■

Proof of Lemma 10. Consider an economy $(\mathbf{u}, \mathbf{s}^t)$ such that for some allocation $(\mathbf{x}, \mathbf{y})$, $(\mathbf{x}, \mathbf{y}) \in \varphi^{\widetilde{u}RWEB}(\mathbf{u}, \mathbf{s}^t)$ where $x_i = 0$ for some $i \in N$. Let $p = (p_x, p_y) \in \Delta^P(\mathbf{u}, \mathbf{s}^t, \mathbf{x}, \mathbf{y})$. Let $y = v(x)$ represent the indifference curve for $\widetilde{u}$ that is tangent to i's budget set $\{(x, y) : p_y y - p_x s_i^t x \leq p_y y_i\}$. Suppose the economy $(\mathbf{u}, \mathbf{s}^t)$ changes to $(\mathbf{u}, \mathbf{s}^{t\prime})$ such that $\mathbf{s}_i^{t\prime} > \mathbf{s}_i^t$, $\mathbf{s}_{-i}^{t\prime} = \mathbf{s}_{-i}^t$ and $p \in \Delta^P(\mathbf{u}, \mathbf{s}^{t\prime}, \mathbf{x}, \mathbf{y})$ where the agent i's new budget set $\{(x, y) : p_y y - p_x s_i^{t\prime} x \leq p_y y_i\}$ intersects with $\{(x, y) : y > v(x)\}$. Then, the maximized utility for $\widetilde{u}$ under the new budget set is greater than that under the original budget set. Therefore, $(\mathbf{x}, \mathbf{y}) \notin \varphi^{\widetilde{u}RWEB}(\mathbf{u}, \mathbf{s}^{t\prime})$. Thus, the $\widetilde{u}$-reference welfare equivalent budget solution dose not satisfy IUS. ■

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