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Abstract

An extended social choice framework is proposed for the analysis of libertarian assignment of individual rights in the primordial stage of rule selection. The crucial concepts of our framework are the *extended social states*, viz. the pairs of narrowly defined social states and the mechanisms through which narrowly defined social states are chosen, and the *extended constitution function* which aggregates each profile of individual ordering functions into a social ordering function. A set of necessary and sufficient conditions for the existence of an extended constitution function, which is uniformly rational and chooses a minimally libertarian assignment of individual rights, is identified.

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1 Introduction

Ever since Sen (1970, Chapter 6 & Chapter 6*; 1970a; 1976; 1983) acutely crystallized the logical conflict between the welfaristic outcome morality in the weak form of the Pareto principle and the non-welfaristic claim of libertarian rights into the *impossibility of a Paretian liberal*, a huge literature have evolved along several distinct avenues.[1] In the first place, some of the early literature either repudiated the importance of Sen's impossibility theorem, or tried to find an escape route from the logical impasse identified by Sen.[2] In the second place, capitalizing on the seminal observation by Nozick (1974, pp.164-166), alternative articulations of libertarian rights, which are game-theoretic in nature, were proposed by Gärdenfors (1981), Sugden (1985), Gaertner, Pattanaik and Suzumura (1992), Deb (1990; 1994), Hammond (1995; 1996) and Peleg (1998). Recollect that Sen's original articulation of libertarian rights was in terms of the preference-contingent constraints on social choice rules by means of individual decisiveness.[3] In contrast, these game-theoretic articulations captured the essence of libertarian rights by means of individual freedom of choosing admissible strategies in the game-theoretic situations where individual liberties are at stake. Unlike the first class of work, where the focus of the analyses was either to deny the essence of Sen's impossibility theorem, or to resolve it systematically, these game-theoretic articulations of libertarian rights were meant to provide more legitimate methods of capturing the essence of what libertarian rights should mean without claiming that the impossibility of a Paretian liberal would disappear if only the alternative articulations of libertarian rights

[1] Some of these literature are succinctly surveyed and evaluated by Suzumura (1996; 2003).

[2] Representative work along these lines include Bernholz (1974), Gibbard (1974), Nozick (1974, pp.164-166), Blau (1975), Osborne (1975), Seidl (1975), Farrell (1976), and Buchanan (1976/1996). Sen (1976) commented on, and in some cases rejected, these early proposals. In so doing, he developed a resolution of the impossibility of a Paretian liberal of his own, which hinges on the concept of a *liberal individual,* "who claims only those parts of his preferences that are compatible with others' preferences over their respective protected spheres to count in social choice [Suzumura (1983, p.196)]." This resolution scheme came to be known as the *Sen-Suzumura resolution scheme* after Austen-Smith (1982).

[3] Suppose that there are two social states, say x and y, which differ only in somebody's personal matters and nothing else. If the person in question prefers x to y, then Sen would confer on him the decisive power of rejecting the social choice of y from any social choice environment in which x is available.

were adopted.[4] In the third place, the crucial problem of initial conferment of libertarian rights was often mentioned in the literature without providing the fully-fledged analytical framework.[5,6] Suffice it to cite just one salient example. In his rebuttal to the game-form articulation proposed by Gaertner, Pattanaik and Suzumura (1992), Sen (1992, p.155) concluded with the following observation: "Gaertner *et al.* (1992) do, in fact, pose the question, 'How does the society decide which strategies should or should not be admissible for a specific player in a given context?' This, as they rightly note, is 'an important question'. ... [I]t is precisely on the answer to this further question that the relationship between the game-form formulations and social-choice formulations depend ... . We must not be too impressed by the 'form' of the 'game forms'. We have to examine its contents and its rationale. The correspondence with social-choice formulations becomes transparent precisely there." The purpose of this paper is to contribute to this less cultivated issue within the theory of libertarian rights.

Capitalizing on the insightful observation by Arrow (1963, pp.89-90) on the decision process as a value, Pattanaik and Suzumura (1994; 1996) developed an extended framework of social choice theory which is suitable for

[4] As a matter of fact, Pattanaik (1996a), and Deb, Pattanaik and Razzolini (1997) showed that there are several natural variants of the impossibility of a Paretian liberal even when libertarian rights are articulated in terms of game forms.

[5] Pattanaik and Suzumura (1994; 1996) and Suzumura (1996; 2003) identified three distinct issues in the analysis of libertarian rights. The first issue is the *formal structure* of rights. The second issue is the *realization* of conferred rights. The third issue is the *initial conferment* of rights. In Sen's theory of libertarian rights, the formal structure of rights was articulated in terms of the preference-contingent constraints on social choice rules, whereas the issue of the realization of conferred rights could be boiled down to the existence of a social choice rule which respects the preference-contingent constraints on social choice rules. However, Sen has never addressed himself to the issue of the initial conferment of rights. This is presumably because his interest was focussed squarely on the conflict between the non-welfaristic claim of libertarian rights and the welfaristic claim of the Pareto principle, so that it was unnecessary for him to develop a fully-fledged theory of the initial conferment of libertarian rights.

[6] Within the conceptual framework of the Arrow-Sen social choice theory, where libertarian rights are captured by means of the preference-contingent constraints on social choice rules, Austen-Smith (1979) and Gaertner (1982) tried to invoke the idea of fair or envy-free assignment of libertarian rights, whereas Harel and Nitzan (1987) tried to formulate the libertarian trade of initially conferred rights. Unfortunately, the analytical reach of the former approach seems to be severely limited, whereas the latter approach does not seem to have any qualification whatsoever to be called the "libertarian resolution of the Paretian liberal paradox." On this latter point, see Suzumura (1991).

the analysis of initial conferment of libertarian rights in the primordial stage of rule selection. Instead of asking the existence of an Arrovian social welfare function, or an Arrovian constitution function, which aggregates each and every profile of individual preference orderings on the universal set of social states, this extended framework asks the existence of an extended constitution function, which aggregates each and every profile of individual ordering functions on the universal set of extended social states, where by an extended social state is meant a pair of the conventionally defined social state and the mechanism through which the conventionally defined social state is brought about, into a social ordering function. Within this extended conceptual framework, this paper shows the existence of an extended constitution function which enables the society to decide on the initial conferment of libertarian rights without violating two essential requirements. The first requirement is procedural in nature; the extended constitution function should satisfy the essentially Arrovian requirements of the Pareto principle, non-dictatorship, and informational efficiency. The second requirement is the uniform rationalizability of the initially conferred libertarian rights in the sense that (1) the conferred libertarian rights must be rationalizable in terms of the social ordering function generated by the extended constitution function, and (2) the conferred libertarian rights should be uniformly applicable to whichever profile of individual preference orderings over conventionally defined social states that may materialize after the primordial stage of rule selection.

Apart from this introduction, the paper consists of four sections and an appendix. Section 2 explains our basic model. Section 3 formalizes the social decision procedure in the primordial stage of rule selection in terms of the Arrovian extended constitution function. Section 4 asserts the existence of an extended constitution function which enables the society to decide on the initial conferment of libertarian rights subject to the two essential requirements mentioned above. Section 5 concludes, and the Appendix gathers all the involved proofs.

2 The Basic Model

2.1 Description of Social States

The society consists of n individuals, where $2 \leq n < +\infty$, and N denotes the set of all individuals, viz. $N = \{1, \cdots, i, \cdots, n\}$. Ω denotes the set of all impersonal features of the world and, for each $i \in N$, X_i denotes the set of all personal actions of individual i. Then the set of all conventionally defined social states is given by $\Omega \times (\Pi_{i \in N} X_i)$. In other words, a conventionally defined social state is a list $(\omega, x_1, \cdots, x_i, \cdots, x_n)$, where $\omega \in \Omega$ and $x_i \in X_i$ for all $i \in N$. To simplify matters, however, we will fix an impersonal feature of the world $\omega \in \Omega$ throughout the rest of this paper, and focus on the social choice of a short-cut list of individual actions $x = (x_1, \cdots, x_i, \cdots, x_n) \in X \equiv \Pi_{i \in N} X_i$.

To motivate the concept of personal actions, let us consider the following:

Example 1: Ann, Edwin, and the Judge are single, but they are contemplating the possibility of marriage. Since they reside in a traditional town, only a male proposes and a female either accepts it, or turns it down. This situation can be described by defining the set of individuals by $N = \{\text{Ann, Edwin, Judge}\}$, and the set of actions of each and every individual by $X_{Ann} = \{s,\ m_E,\ m_J\}$, $X_{Edwin} = \{s,\ p\}$ and $X_{Judge} = \{s,\ p\}$, where s denotes the action of "remaining single", m_E denotes the action of "marrying Edwin", m_J denotes the action of "marrying the Judge", and p denotes the action of "proposing to Ann". Figure 1 describes how social interactions among individuals' actions result in social outcomes. Edwin and the Judge simultaneously choose either p or s without knowing which choice the other has made. After knowing Edwin's and the Judge's choices, Ann chooses one of her actions, viz. s, m_E , or m_J. ■

Insert Figure 1 around here.

This example illustrates the important fact that *a list of individuals' actions need not be socially feasible.* Indeed, in the presence of marriage convention prevailing in the community, a list of individuals' actions $(m_E,\ s,\ p) \in X_{Ann} \times X_{Edwin} \times X_{Judge}$ is conceivable, but not feasible, since "Ann marries Edwin" and "Edwin remains single" are incompatible actions.

Thus, the set of feasible social states is, in general, different from the set of lists of individuals' actions. Let $A \subseteq X$ be the set of *feasible social states.*

Throughout this paper, it is assumed that $3 \leq \#A < +\infty$. In the case of **Example 1**, A is defined by

$$(\{s\} \times \{s,p\} \times \{s,p\}) \cup (\{s,m_E\} \times \{p\} \times \{s,p\}) \cup (\{s,m_J\} \times \{s,p\} \times \{p\}).$$

Note that (m_E,p,p) and (m_E,p,s) are both in A, and they result in the same consequential outcome, viz. "Ann marries Edwin, leaving the Judge single."[7] Nevertheless, we should treat these two feasible social states separately, as we should not lose sight of the difference in the processes through which the same consequential outcome is brought about.[8]

With the purpose of accommodating several distinct contexts in which libertarian claim of rights may pose an issue of social-choice theoretic relevance, let us decompose the set X_i of all personal actions of each and every $i \in N$ as $X_i = X_i^1 \times X_i^2$, where X_i^1 denotes the set of i's non-controversially private actions, and X_i^2 denotes the set of i's private actions in the context of socially interactive matters. A typical example of the elements of X_i^1 are alternative sleeping postures of individual i, and a typical example of the elements of X_i^2 are alternative actions of individual i in the public space where there exists no prior social agreement concerning the priority between the smoker's right for free smoking and the non-smoker's right for clean air. The set of all logically conceivable individual actions is then defined by $X \equiv (\Pi_{i \in N} X_i^1) \times (\Pi_{i \in N} X_n^2)$. In what follows, it is assumed that the set A of all feasible social states is *decomposable* in the sense that it can be written as $A \equiv A^1 \times A^2$, where A^1 can be further decomposed as $A^1 \equiv \Pi_{i \in N} A_i^1$ and $A_i^1 \subseteq X_i^1$ for all $i \in N$, whereas we do not require any further decomposability of $A^2 \subseteq \Pi_{i \in N} X_i^2$.[9]

Let us illustrate these concepts/assumptions in terms of the following example.

Example 2: Let $N = \{\text{Ann, Edwin, Judge}\}$. We define the sets of personal actions of Ann, Edwin, and the Judge, respectively, by $X_{Ann} \equiv X_{Ann}^1 \times X_{Ann}^2$ with $X_{Ann}^1 \equiv \{b,r\}$ and $X_{Ann}^2 \equiv \{s,\ m_E,\ m_J\}$, $X_{Edwin} \equiv X_{Edwin}^1 \times X_{Edwin}^2$

[7] Likewise, $(m_J,\ p,\ p)$ and $(m_J,\ s,\ p)$ result in the same consequential outcome, viz. "Ann marries the Judge, leaving Edwin single", whereas $(s,\ p,\ p)$, $(s,\ p,\ s)$, $(s,\ s,\ p)$ and $(s,\ s,\ s)$ result in the same consequential outcome, viz. "Ann, Edwin and the Judge all remain single".

[8] Gibbard (1974), Suzumura (1978), and Hammond (1996) adopted the same approach to the description of social states.

[9] For each $i \in N$, and each $\mathsf{x}^1 \in A^1$, let $\mathsf{x}_{-i}^1 \equiv (x_1^1, \cdots, x_{i-1}^1, x_{i+1}^1, \cdots, x_n^1)$ and $A_{-i}^1 \equiv \Pi_{j \neq i} A_j^1$.

with $X^1_{Edwin} \equiv \{b, r\}$ and $X^2_{Edwin} \equiv \{s, p\}$, and $X_{Judge} \equiv X^1_{Judge} \times X^2_{Judge}$ with $X^1_{Judge} \equiv \{b, r\}$ and $X^2_{Judge} \equiv \{s, p\}$, where s, m_E, m_J and p carry the same meaning as in **Example 1**, whereas b (resp. r) means "wearing a blue shirt" (resp. "wearing a red shirt"). The set of logically conceivable social states is given by $X \equiv \left(X^1_{Ann} \times X^1_{Edwin} \times X^1_{Judge}\right) \times \left(X^2_{Ann} \times X^2_{Edwin} \times X^2_{Judge}\right)$, whereas the set of feasible social states is defined by $A \equiv A^1 \times A^2$, where A^1 is defined by

$$A^1_{Ann} \times A^1_{Edwin} \times A^1_{Judge} \equiv \{b, r\} \times \{b, r\} \times \{b, r\}$$

and A^2 is defined by

$$(\{s\} \times \{s, p\} \times \{s, p\}) \cup (\{s, m_E\} \times \{p\} \times \{s, p\}) \cup (\{s, m_J\} \times \{s, p\} \times \{p\}) .$$

Clearly, A^1 is decomposablle, whereas A^2 is indecomposable. The indecomposability of A^2 reflects the socially interactive nature of the marriage problem. Indeed, the feasibility of Ann's marriage is conditional upon some male's prior proposal, whereas the feasibility of Edwin's (resp. Judge's) marriage is conditional upon Ann's decision of not choosing "remaining single." ■

Although the socially interactive matters are characterized in **Example 2** by the indecomposability of A^2, the following example shows that socially interactive matters may still exist even when A^2 happens to be decomposable.

Example 3: Let $N = \{\text{Smoker, Non-Smoker}\}$, and let the sets of personal actions of the smoker and the non-smoker be $X^1_S \equiv \{b, r\}$ and $X^1_{NS} \equiv \{b, r\}$ for the non-controversially private matter of wearing a blue shirt (b) or a red shirt (r), and $X^2_S \equiv \{s, ns\}$ and $X^2_{NS} \equiv \{a, na\}$ for the socially interactive matter of smoking in the public space, where s (resp. ns) means that the smoker smokes (resp. doesn't smoke), and a (resp. na) means that the non-smoker admits (resp. doesn't admit) the smoker to smoke in the same public space. The set of conceivable social states is then defined by $X \equiv (X^1_S \times X^1_{NS}) \times (X^2_S \times X^2_{NS})$, which coincides with the set A of feasible social states, where $A \equiv A^1 \times A^2$, $A^1 \equiv X^1_S \times X^1_{NS}$ and $A^2 \equiv X^2_S \times X^2_{NS}$. This definition forces us to say that a social state, where the smoker smokes in the public space in neglect of the non-smoker's non-admission, is socially feasible. This seems inevitable in the absence of any established convention concerning the priority between the smoker's claim for free smoking and the non-smoker's claim for clean air. In this case, not only A^1, but also A^2

is decomposable. Nevertheless, there seems to exist a non-trivial difference between A^1 and A^2. Unlike the issue of wearing a blue shirt or a red shirt, where it seems natural, viz. non-controversial, to leave the choice between these options to both individuals separately, it is far from natural to leave the smoker to choose between smoking and non-smoking without weighing his claim against the non-smoker's claim. Put differently, there is a serious issue of social choice between the two alternative conferments of individual rights, viz. the conferment on the smoker of his right for free smoking, on the one hand, and the conferment on the non-smoker of his right for clean air, on the other. This issue of the initial conferment of individual rights in the present context will be further pursued in **Example 4** and **Example 6** below. ■

In what follows, we will be interested in the existence of a social decision procedure which is minimally libertarian in the sense that it determines the initial conferment of individual rights over A in such a way as to confer on each $i \in N$ the complete autonomy in the choice of his/her purely personal matters, viz. the issues belonging to A_i^1.[10] Before coming to this issue,

[10] In this context, it is worth recollecting Farrell's (1976, pp.8-9) criticism of Sen (1970; 1970a) to the following effect: "[T]he attempt to insert 'Liberalism' by means of individual decisiveness is ... an unnatural and artificial device, introduced as an afterthought. Suppose two states, x and y, differ only in a matter purely private to individual j. Would a ... Liberal say that individual j should be decisive between x and y, so as to have a modicum of individual liberty? He is much more likely to say that there is no social choice to be made between x and y, since they differ in a matter private to individual j." Pursuing this critical remark formally, Farrell proposed what he christened the "Liberal Partition": "To say that the choice between two elements of S [of possible social states] is not a social one may be formalized by saying that they are 'socially equivalent', where the relation of being socially equivalent is an equivalence relation on the set S. It defines a collection of subsets of S which are non-empty, disjoint and collectively exhaustive — that is, a partition P of S. ... [T]he problem of social choice is that of choosing among elements of P, not elements of S; once a socially equivalent subset has been selected, the choice of an element from this subset ... is not a social choice, but will be determined by the private decisions." In Farrell's own admission, however, "the determination of a Liberal Partition sounds purely formal, but in practice may be anything but formal. The battle between those who want a very coarse partition and those who want a very fine one — between those who wish to leave a good deal to individual decision and those who would leave very little — has been, is, and is likely to remain a major political issue. Thus, no Liberal Partition can be determined without value judgements and political disputation, perhaps on a large scale." It is with the purpose of bringing this issue of value judgements and political disputation clearly into relief, rather than assuming it away by hiding behind the *a priori* Liberal

however, we must specify what we mean by the social decision-making rules.

2.2 Social Decision-Making Rules

For each individual $i \in N$, $R_i \subseteq X \times X$ denotes i's (weak) preference ordering defined over X. For any $\mathbf{x}, \mathbf{y} \in X, (\mathbf{x}, \mathbf{y}) \in R_i$ means that $\mathbf{x}$ is at least as good as $\mathbf{y}$ according to i's judgements. $P(R_i)$ and $I(R_i)$ denote, respectively, the strict preference relation and the indifference relation corresponding to R_i, viz., $(\mathbf{x}, \mathbf{y}) \in P(R_i)$ if and only if $[(\mathbf{x}, \mathbf{y}) \in R_i \ \& \ (\mathbf{y}, \mathbf{x}) \notin R_i]$, and $(\mathbf{x}, \mathbf{y}) \in I(R_i)$ if and only if $[(\mathbf{x}, \mathbf{y}) \in R_i \ \& \ (\mathbf{y}, \mathbf{x}) \in R_i]$. $\mathcal{R}$ denotes the universal set of preference orderings defined over X. An n-tuple $\mathbf{R} = (R_1, R_2, \cdots, R_n)$ of individual preference orderings, one ordering for each individual $i \in N$, is called a *profile* of individual preference orderings over X. Thus, $\mathcal{R}^n$ denotes the universal set of logically conceivable profiles.

To accommodate two alternative articulations of libertarian rights within our conceptual framework, we introduce two articulations of social decision-making rules, viz., social choice correspondences and game forms. A *social choice correspondence* (SCC) is a correspondence $\sigma : \mathcal{R}^n \to A$ such that, for each profile $\mathbf{R} \in \mathcal{R}^n$, $\sigma(\mathbf{R})$ denotes a non-empty subset of A. The set of all logically conceivable social choice correspondences is denoted by $\mathbf{\Sigma}$. A *game form* is a pair $\gamma = (M, g)$, where $M \equiv \prod_{i \in N} M_i$ and M_i denotes a set of *permissible strategies* for individual $i \in N$, and $g : M \to A$ is an *outcome function* which specifies, for each strategy profile $\mathbf{m} \in M$, a feasible outcome $g(\mathbf{m}) \in A$. When a profile $\mathbf{R} \in \mathcal{R}^n$ and a game form $\gamma = (M, g)$ are specified, a triplet $(N, \mathbf{R}, \gamma)$ defines a fully-fledged non-cooperative game. The set N of all players being fixed throughout this paper, we may denote this non-cooperative game simply by $(\mathbf{R}, \gamma)$ without ambiguity.

An important juncture in our analysis of the performance of game forms as social decision-making rules is the specification of the equilibrium concept. Throughout this paper, we will focus on the Nash equilibrium concept. Given a game form $(\mathbf{R}, \gamma)$, a strategy profile $\mathbf{m}^* \in M$ is said to be a *Nash equilibrium in pure strategies*, Nash equilibrium for short, if $\left(g(\mathbf{m}^*), g\left(m_i, \mathbf{m}^*_{-i}\right)\right) \in R_i$ holds for all $i \in N$ and all $m_i \in M_i$.[11] A feasible social outcome $\mathbf{x} \in A$ is said to be a *Nash equilibrium outcome* of the game $(\mathbf{R}, \gamma)$ if there exists a

Partition, that our present conceptual framework is developed.

[11] For each $i \in N$, and each $\mathsf{m} \in M$, let $\mathsf{m}_{-i} \equiv (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$ and $M_{-i} \equiv \Pi_{j \neq i} M_j$.

Nash equilibrium $\mathbf{m}^*$ satisfying $\mathbf{x} = g(\mathbf{m}^*)$. The set of all Nash equilibrium outcomes of the game $(\mathbf{R}, \gamma)$ will be denoted by $\boldsymbol{\tau}_{NE}(\mathbf{R}, \gamma)$. A game form γ is said to be *Nash solvable* if $\boldsymbol{\tau}_{NE}(\mathbf{R}, \gamma)$ is non-empty for each and every profile $\mathbf{R} \in \mathcal{R}^n$.[12] In the rest of this paper, we will focus on the admissible set $\boldsymbol{\Gamma}_{\mathsf{NS}}$ of game forms which are Nash solvable. It is true that this assumption excludes many — otherwise legitimate — game forms from our arena of discourse. Nevertheless, given our purpose of identifying a class of game forms which are Nash solvable, minimally libertarian, and democratically choosable in the primordial stage of rule selection, this restriction seems to be warranted. In what follows, we define the universal set of social decision-making rules by $\Theta \equiv \boldsymbol{\Sigma} \cup \boldsymbol{\Gamma}_{\mathsf{NS}}$.

Before closing this preliminary account of the concept of social decision-making rules, it may deserve emphasis that social choice correspondences and game forms, respectively, are vehicles which enable us to articulate libertarian rights in sharply contrasting ways. Indeed, the articulation in terms of social choice correspondences enables us to capture the essence of libertarian rights by means of the preference-contingent constraints on social choice rules; in contrast, the articulation in terms of game forms enables us to capture the essence of libertarian rights by means of the individual freedom in choosing his admissible strategies in the game-theoretic interactions where individual liberties are at stake. Although we juxtapose these two methods of articulation without prejudice, it will be shown that there exists a social decision procedure which can implement only the game-form articulation of libertarian rights.

3 Social Decision Procedure for Rule Selection

Which social decision-making rule should a society choose and materialize? To make this question operational, we visualize the two-stage social decision procedure. In the first stage, which may be called the *primordial stage of rule selection*, the society chooses a social decision-making rule without knowing which profile of individual preference orderings on the set of

[12] Abdou (1998; 1998a) identified some sufficiency conditions for a game form to be Nash-solvable, whereas Peleg, Peters and Storchen (2002) succeeded in finding a necessary and sufficient condition for the Nash solvability.

conventionally defined social states will emerge; in the second stage, which may be called the *realization stage*, the society confronts the emerged profile of individual preference orderings and decides on a social outcome through the social decision-making rule chosen in the first stage.

To make the scenario of this two-stage social decision procedure precise, we invoke the extended social choice framework introduced by Pattanaik and Suzumura (1994; 1996).[13] For any $\mathbf{x} \in A$ and $\theta \in \Theta$, a pair $(\mathbf{x}, \theta) \in A \times \Theta$ is called an *extended social alternative*. The intended interpretation of this pair is that the feasible social outcome $\mathbf{x} \in A$ is attained through the social decision-making rule $\theta \in \Theta$. Note, however, that $\mathbf{x}$ need not actually be realizable through θ. Indeed, the realizability of an extended social alternative $(\mathbf{x}, \theta)$ may make sense in the first place only when a profile $\mathbf{R} \in \mathcal{R}^n$ of individual preference orderings is specified. To be more precise, given a profile $\mathbf{R} \in \mathcal{R}^n$, an extended alternative $(\mathbf{x}, \theta) \in A \times \Theta$ is said to be a *realizable pair under* $\mathbf{R}$ if and only if either (1) $\theta = \sigma \in \mathbf{\Sigma}$ and $\mathbf{x} \in \sigma(\mathbf{R})$, or (2) $\theta = \gamma \in \mathbf{\Gamma}_{\mathsf{NS}}$ and $\mathbf{x} \in \boldsymbol{\tau}_{NE}(\mathbf{R}, \gamma)$. In what follows, $\mathcal{RP}(\mathbf{R})$ denotes the set of all realizable pairs under $\mathbf{R}$.

We are now ready to formalize the two-stage social decision procedure. In the primordial stage of rule selection, it is assumed that no one is in the privileged position of knowing which profile $\mathbf{R}$ of individual preference orderings will materialize in the realization stage which is to come later. Hence, the informational basis of social choice in this stage is assumed to be each i's value judgements over the desirability of social decision-making rules, which is captured by his *ordering function* $Q_i : \mathcal{R}^n \to (A \times \Theta)^2$ such that, for each $\mathbf{R} \in \mathcal{R}^n$, $Q_i(\mathbf{R}) \subseteq \mathcal{RP}(\mathbf{R}) \times \mathcal{RP}(\mathbf{R})$ is an extended preference ordering defined over $\mathcal{RP}(\mathbf{R})$. By definition, $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q_i(\mathbf{R})$, or $(\mathbf{x}, \theta) Q_i(\mathbf{R}) (\mathbf{x}', \theta')$ for the sake of brevity, means that having a social outcome $\mathbf{x}$ through a social decision-making rule θ is at least as good for the society as having a social outcome $\mathbf{x}'$ through a social decision-making rule θ' according

[13] This extended social choice framework à la Pattanaik and Suzumura capitalizes on the insightful observation by Arrow (1963, pp.89-90) to the following effect: "Up to now, no attempt has been made to find guidance by considering the components of the vector which defines the social state. One especially interesting analysis of this sort considers that, among the variables which taken together define the social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. For example, an individual may have a positive preference for achieving a given distribution through the free market mechanism over achieving the same distribution through rationing by the government." See, also, Suzumura (1996; 1999; 2000; 2003).

to i's value judgements. Let $\mathcal{Q}$ be the set of all logically possible ordering functions.

Note that the extended preference ordering $Q_i(\mathbf{R})$ enables us to capture various types of value judgements held by i. To illustrate this crucial fact, let $P(Q_i(\mathbf{R}))$ and $I(Q_i(\mathbf{R}))$ stand for the strict preference part and the indifference part of $Q_i(\mathbf{R})$, respectively. Suppose that, for any $(\mathbf{x}, \theta), (\mathbf{x}, \theta') \in \mathcal{RP}(\mathbf{R})$, $((\mathbf{x}, \theta), (\mathbf{x}, \theta')) \in I(Q_i(\mathbf{R}))$ holds. Then $Q_i(\mathbf{R})$ embodies *consequentialist value judgements* in the sense that $(\mathbf{x}, \theta)$ and $(\mathbf{x}, \theta')$ are judged indifferent as long as the consequential outcome $\mathbf{x}$ remains the same, no matter how the social decision-making rules θ and θ' differ from each other. In the second place, suppose that, for any $(\mathbf{x}, \theta), (\mathbf{x}', \theta) \in \mathcal{RP}(\mathbf{R})$, $((\mathbf{x}, \theta), (\mathbf{x}', \theta)) \in I(Q_i(\mathbf{R}))$ holds. Then $Q_i(\mathbf{R})$ embodies *non-consequentialist value judgements* in the sense that $(\mathbf{x}, \theta)$ and $(\mathbf{x}', \theta)$ are judged indifferent as long as the social decision-making rule θ remains the same, no matter how the consequential outcomes $\mathbf{x}$ and $\mathbf{x}'$ differ from each other. In between these two polar extremes, there are many value judgements which weigh consequentialist considerations against non-consequentialist considerations and they are all representable in terms of the extended preference ordering.

We are now ready to introduce the concept of an *extended constitution function.*

Definition 1: *An* extended constitution function (**ECF**) *is a function* Ψ *which maps each and every profile of individual ordering functions* $\mathbf{Q} = (Q_i)_{i \in N}$ *in an appropriate domain* $\Delta_\Psi \subseteq \mathcal{Q}^n$ *into a social ordering function* Q*,* viz. $\Psi(\mathbf{Q}) = Q \in \mathcal{Q}$ for all $\mathbf{Q} \in \Delta_\Psi$.

The concept of extended constitution function is a natural extension of the Arrovian social welfare function or constitution function [Arrow (1963)]. Note, however, that in this extended framework, there are two types of individual preference orderings for each $i \in N$: one is an individual's preference ordering R_i over X, which represents i's subjective tastes, and the other is i's ordering function Q_i, which represents i's ethical value judgements.[14] The latter is announced in the primordial stage of rule selection, and constitutes the informational basis of the **ECF** to select a social decision-making rule,

[14]Note, however, that the individual ordering function may generate an extended preference ordering which is *selfish* in nature: Q_i expresses i's *selfish judgements* if and only if, for all $\mathsf{R} \in \mathcal{R}^n$ and all $(\mathsf{x}, \theta), (\mathsf{x}', \theta') \in \mathcal{RP}(\mathsf{R})$, $\big((\mathsf{x}, \theta), (\mathsf{x}', \theta')\big) \in Q_i(\mathsf{R})$ (resp. $P(Q_i(\mathsf{R}))$) $\Leftrightarrow (\mathsf{x}, \mathsf{x}') \in R_i$ (resp. $P(R_i)$).

whereas the former emerges after the primordial stage and, given the social decision-making rule selected through the **ECF** in the primordial stage, it constitutes the informational basis for realizing a feasible social outcome.

When an **ECF** Ψ is specified, we may define the associated *rational social choice function* as follows. For each profile of individual ordering functions $\mathbf{Q} \in \Delta_\Psi$, Ψ determines a social ordering function $Q = \Psi(\mathbf{Q})$ which, in turn, determines the set of best extended social alternatives for each $\mathbf{R} \in \mathcal{R}^n$ by

(1) $B_{\mathsf{Q}}(\mathbf{R}) \equiv \{(\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R}) \mid \forall (\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R}) {:} ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q(\mathbf{R})\}$,

where $Q = \Psi(\mathbf{Q})$. The set of social decision-making rules chosen through Ψ is then given by

(2) $C_\Psi(Q; \mathbf{R}) \equiv \{\theta \in \mathbf{\Theta} \mid \exists\ \mathbf{x} \in A : (\mathbf{x}, \theta) \in B_{\mathsf{Q}}(\mathbf{R})\}$,

where $Q = \Psi(\mathbf{Q})$. In what follows, C_Ψ will be called the *rational social choice function chosen through* Ψ.

As one of the desirable requirements on the rational social choice function chosen through Ψ, we introduce the following condition:

Uniformity of Rational Choice (URC): *For all* $\mathbf{Q} \in \Delta_\Psi$,

$$\bigcap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R}) \neq \varnothing,$$

where $Q = \Psi(\mathbf{Q})$.

If the Condition **URC** is satisfied and a social decision-making rule $\theta^* \in \cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R})$ is chosen, the rule θ^* applies *uniformly* to each and every future realization of $\mathbf{R} \in \mathcal{R}^n$. Since the social decision-making rule is nothing other than the formal method of specifying the rights-structure prevailing in the society prior to the realization of the profile of individual preference orderings, it seems desirable, if at all possible, to design the extended constitution function Ψ satisfying the condition **URC**. Note that if only we implement a $\theta^* \in \cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R})$, then θ^* will prevail as the bacic social decision-making rule no matter how frivolously the profile $\mathbf{R}$ undergoes a change.[15]

[15] In general, however, we should expect that $\theta^* \in C_\Psi(Q; \mathsf{R})$ cannot but depend on the realized profile $\mathsf{R} \in \mathcal{R}^n$. Reflecting this fact, the conditions under which **URC** will be satisfied turn out to be rather stringent.

The next requirement on Ψ is that it is *minimally democratic* in the sense that the unanimous individual value judgements must be faithfully reflected in the social value judgements. To be more precise, we introduce the following two versions of the Pareto principle.

Strong Pareto Principle (SP): *For all* $\mathbf{Q} \in \Delta_\Psi$, *all* $\mathbf{R} \in \mathcal{R}^n$, *and all* $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$,

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \left(\bigcap_{i \in N} Q_i(\mathbf{R})\right) \cap \left(\bigcup_{i \in N} P\left(Q_i(\mathbf{R})\right)\right) \Rightarrow ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P(Q(\mathbf{R})),$$

where $Q = \Psi(\mathbf{Q})$.

Pareto Indifference Principle (PI): *For all* $\mathbf{Q} \in \Delta_\Psi$, *all* $\mathbf{R} \in \mathcal{R}^n$, *and all* $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$,

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \bigcap_{i \in N} I(Q_i(\mathbf{R})) \Rightarrow ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in I(Q(\mathbf{R})),$$

where $Q = \Psi(\mathbf{Q})$.

Let $\pi : N \to N$ denote a permutation on N, and let Π denote the set of all possible permutations on N. We may formulate a weak equity requirement on Ψ as follows.

Anonymity (AN): *For all* $\mathbf{Q} \in \Delta_\Psi$ *and all* $\pi \in \Pi$, $\Psi(\mathbf{Q}) = \Psi(\pi \circ \mathbf{Q})$, *where* $\pi \circ \mathbf{Q} \equiv (Q_{\pi(i)})_{i \in N}$.

Given $\mathbf{R} \in \mathcal{R}^n$ and an **ECF** Ψ, an individual $d \in N$ is called an $\mathbf{R}$-*dictator under* Ψ if, for all $\mathbf{Q} \in \Delta_\Psi$ and all $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$, $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P(Q_d(\mathbf{R}))$ implies $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P(Q(\mathbf{R}))$, where $Q = \Psi(\mathbf{Q})$. We are now ready to introduce the last Arrovian requirement on Ψ as follows.

Non-Dictatorship (ND): *For all* $\mathbf{R} \in \mathcal{R}^n$, *there is no* $\mathbf{R}$-*dictator under* Ψ.

Although our extended social choice framework is Arrovian in the sense of aggregating individual values into social choice, there are four essential differences which deserve emphasis. In the first place, Arrow's (1963) constitution function aggregates each profile of individual preference orderings into a social preference ordering, both preferences being defined over the set of conventionally defined social states, and the social choice from each opportunity set is defined by means of the maximization of social preference ordering subject to the constraint imposed by the opportunity set prescribed from outside. Thus, once the social preference ordering is determined through social aggregation of individual values, no channel is left for individuals to exert influence on the determination of consequential social outcomes. In sharp contrast, our extended constitution function aggregates each profile of individual ordering functions into a social ordering function, which enables the society to decide on the social decision-making rule to be applied in the stage of choosing consequential social outcomes. In other words, even after the social ordering function is determined through Arrovian aggregation of individual values which are represented by individual ordering functions, there is a further stage of social choice of consequential outcomes, where individuals are able to participate in the process of social choice.

In the second place, the Pareto principle imposed on the Arrovian constitution function is nothing other than an outcome morality which implies that the socially chosen consequential outcomes must be weakly Pareto-efficient. In contrast, although the Pareto principle imposed on the extended constitution function looks exactly like the requirement imposed on the Arrovian constitution function, it does not in general guarantee that the consequential outcomes generated through the social decision-making rule, which is socially chosen in the primordial stage of rule selection, is weakly Pareto efficient. In other words, the pair of Axioms **SP** and **PI** imposed on the extended constitution function is *not* an outcome morality, *but* a procedural requirement of democratic aggregation of individual ordering functions.

In the third place, our requirement of non-dictatorship imposed on the extended constitution function excludes the existence of an $\mathbf{R}$-dictator for each and every profile $\mathbf{R} \in \mathcal{R}^n$, whereas Arrow's requirement of non-dictatorship excludes the existence of an individual who has his own way no matter which profile $\mathbf{R} \in \mathcal{R}^n$ may prevail. In our present context where we impose the Arrovian requirement on the extended constitution function rather than on the Arrovian constitution function, however, the requirement **ND,** which is stronger than the original Arrow Condition, seems to retain the natural flavor of Arrow's non-dictatorship.

In the fourth and last place, Arrow's axiom of the Independence of Irrelevant Alternatives, which plays a crucial role in establishing his impossibility theorem, is conspicuously missing from our list. The possibility theorems we are going to establish in this section have a lot to do with this fact. However, the independence axiom will reappear in the context of the uniformity of rational choice in Section 4.

3.1 Minimally Libertarian Ordering Function

Any ordering function $Q \in \mathcal{Q}$ embodies some ideas about what constitutes a desirable social decision-making rule. Such ideas can be captured in terms of some characterizing axioms, which one may impose on the admissible class of ordering functions.

As an auxiliary step in formulating the axiom which captures the intrinsic value of libertarian rights, we introduce the following:

Definition 2: *A social decision-making rule $\theta \in \Theta$ is* minimally libertarian *if*
(1) *$\theta = \sigma \in \mathbf{\Sigma}$ implies that, for all $\mathbf{R} \in \mathcal{R}^n$, all $i \in N$, and all $\mathbf{x}$, $\mathbf{x}' \in A$ such that $\mathbf{x}$ and $\mathbf{x}'$ have the same components except those in A_i^1, if $(\mathbf{x}, \mathbf{x}') \in P(R_i)$, then $\mathbf{x}' \notin \sigma(\mathbf{R})$;*
(2) *$\theta = \gamma \in \mathbf{\Gamma}_{\mathsf{NS}}$ implies that $\gamma = (M, g)$ is such that*
(i) *$M_i = M_i^1 \times M_i^2$ for all $i \in N$;*
(ii) *For all $i \in N$ and all $x_i^1 \in A_i^1$, there exists an $m_i^1 \in M_i^1$ such that $g(m_i^1, M_i^2, M_{-i}) \subseteq \{x_i^1\} \times A_{-i}^1 \times A^2$.*[16]

Let us denote the set of minimally libertarian social decision-making rules by Θ^L $(= \mathbf{\Sigma}^L \cup \mathbf{\Gamma}_{\mathsf{NS}}^{\mathsf{L}})$ with generic element θ^L $(= \sigma^L$ or $\gamma^L)$. We may assert that

Lemma 1: $\Theta^L \neq \varnothing$.

By definition, each minimally libertarian social decision-making rule θ^L respects every individual's libertarian rights over his purely personal matters. When $\theta^L = \sigma^L$, it permits each $i \in N$ to exercise his *decisive power* at least over the set of his purely personal matters A_i^1. When $\theta^L = \gamma^L$, each $i \in N$ can secure any $x_i^1 \in A_i^1$ by choosing an appropriate strategy $m_i^1 \in M_i^1$ of himself irrespective of how $m_i^2 \in M_i^2$ and $\mathbf{m}_{-i} \in M_{-i}$ are specified. Assuring these individual rights over A_i^1 for all $i \in N$ in this sense must be a minimal

[16] For the sake of notational convenience, let $g\,(m_i, M_{-i}) \equiv \cup_{\mathsf{m}_{-i} \in M_{-i}} g\,(m_i, \mathsf{m}_{-i})$ and $g(m_i^1, M_i^2, M_{-i}) \equiv \cup_{m_i^2 \in M_i^2} \cup_{\mathsf{m}_{-i} \in M_{-i}} g(m_i^1, m_i^2, \mathsf{m}_{-i})$. Furthermore, $\{x_i^1\} \times A_{-i}^1 \times A^2 \equiv (A_1^1 \times \cdots \times A_{i-1}^1 \times \{x_i^1\} \times A_{i+1}^1 \times \cdots \times A_n^1) \times A^2$.

requirement for libertarian decision-making rules, since we cannot regard a society as respecting individual liberties if it prohibits some individual actions even with respect to their purely personal matters.

If an ordering function respects the intrinsic importance of individual liberties, it should give every minimally libertarian decision-making rule priority over any other rule in evaluating extended social alternatives. To be more precise, given $\mathbf{R} \in \mathcal{R}^n$, we require the following:

Priority of Minimal Libertarianism for R (PML(R)): *For all* $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$, *if* $\theta \in \Theta^L$ *and* $\theta' \in \Theta \backslash \Theta^L$, *then* $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P(Q(\mathbf{R}))$.

Given $\mathbf{R} \in \mathcal{R}^n$, let us denote the class of ordering functions which satisfy **PML**$(\mathbf{R})$ by $\mathcal{Q}^{\mathsf{ML}(\mathbf{R})}$. We are now ready to introduce the following:

Definition 3: *An ordering function* $Q \in \mathcal{Q}$ *is* minimally libertarian *if and only if it satisfies* **PML**$(\mathbf{R})$ *for all* $\mathbf{R} \in \mathcal{R}^n$.

We denote the class of minimally libertarian ordering functions by $\mathcal{Q}^{\mathsf{ML}}$.

3.2 Generating Minimally Libertarian Ordering Functions through Extended Constitution Function

We are now in the stage of asking the following crucial question concerning the initial conferment of minimally libertarian rights. Suppose that each and every individual expresses his personal views on the intrinsic and/or instrumental values of individual liberty in the form of his ordering function. With the profile of individual ordering functions thus expressed as the informational basis for social choice, can we design an extended constitution function which is Paretian, anonymous and confers on each and every individual minimally libertarian rights?

With this question in mind, let us define, for each $\mathbf{R} \in \mathcal{R}^n$, the following binary relation on $\mathcal{RP}(\mathbf{R})$:

$$\Xi^{ML}(\mathbf{R}) \equiv [(A \times \Theta^L) \times (A \times (\Theta \setminus \Theta^L))] \cap [\mathcal{RP}(\mathbf{R}) \times \mathcal{RP}(\mathbf{R})].$$

Note that, for any $Q \in \mathcal{Q}^{\mathsf{ML}(\mathbf{R})}$, $P(Q(\mathbf{R})) \supseteq \Xi^{ML}(\mathbf{R})$ holds. Given $\mathbf{Q} \in \mathcal{Q}^n$ and $\mathbf{R} \in \mathcal{R}^n$, let us define the following family of subsets of N:

$$\mathcal{N}^{ML}(\mathbf{Q}; \mathbf{R}) \equiv \{N^* \subseteq N \mid \bigcup_{i \in N^*} P(Q_i(\mathbf{R})) \supseteq \Xi^{ML}(\mathbf{R})\},$$

which helps us to define $N^{ML}(\mathbf{Q};\mathbf{R})$ as a subset of N such that (1) $N^{ML}(\mathbf{Q};\mathbf{R}) \in \mathcal{N}^{ML}(\mathbf{Q};\mathbf{R})$; and (2) there is no $N^* \in \mathcal{N}^{ML}(\mathbf{Q};\mathbf{R})$ such that $N^* \subsetneq N^{ML}(\mathbf{Q};\mathbf{R})$. The intended interpretation of $N^{ML}(\mathbf{Q};\mathbf{R})$ is that it represents a *minimal coalition* of individuals who can jointly support the value of minimal libertarianism at $\mathbf{Q} \in \mathcal{Q}^n$ and $\mathbf{R} \in \mathcal{R}^n$. Needless to say, the configurations of $N^{ML}(\mathbf{Q};\mathbf{R})$ may vary according to the variations in $\mathbf{Q} \in \mathcal{Q}^n$ and $\mathbf{R} \in \mathcal{R}^n$.

We are now ready to state the first main result of this paper.

Theorem 1: *There exists an* **ECF** Ψ_{ML} *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle, *and* Anonymity *such that, for all* $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$, *the following two statements are equivalent*:

(α) $Q^{ML} = \Psi_{ML}(\mathbf{Q})$ *is a* minimally libertarian ordering function;

(β) $\#N^{ML}(\mathbf{Q};\mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$.

Two observations on the nature of **Theorem 1** may be in order. In the first place, the basic message of **Theorem 1** seems to have a family resemblance to that of the so-called Sen-Suzumura resolution scheme for the *impossibility of a Paretian liberal* [Sen (1976); Suzumura (1978; 1983, Chapter 7); Austen-Smith (1982)]. Indeed, they seem to suggest in common a general moral to the effect that the ultimate guarantee for a minimal amount of personal liberty should be found, not in the social choice mechanism *per se*, but in an individual′s attitude to respect each other's minimal individual liberty. Apart from this common moral, however, the Sen-Suzumura resolution scheme and the present **Theorem 1** are quite different in nature. The former approach assigns a pivotal role to the so-called "liberal" individual in constructing a social choice correspondence within the conventional social choice framework which respects minimal individual liberty in the sense of Sen and generates social outcomes which are conditionally Pareto efficient. In contrast, **Theorem 1** is concerned with the democratic aggregation of individual ordering functions which embody people′s ethical value judgements into a social ordering function which respects the priority of minimal liberty. Indeed, the two Pareto Principles in **Theorem 1** are not the outcome morality which requires the Pareto efficiency of consequential outcomes, but the procedural value which requires that the extended constitution function must endorse unanimous individual value judgements.[17]

[17] In this sense, **Theorem 1** does not conflict with the incompatibility results between the Pareto efficiency and the game-form articulations of libertarian rights discussed by Deb, Pattanaik, and Razzolini (1997) and Peleg (1998).

In the second place, note that $N^{ML}(\mathbf{Q}^1;\mathbf{R}^1)$ and $N^{ML}(\mathbf{Q}^2;\mathbf{R}^2)$ can be disjoint for $(\mathbf{Q}^1;\mathbf{R}^1) \neq (\mathbf{Q}^2;\mathbf{R}^2)$. Capitalizing on this observation, we can derive two simple, yet meaningful corollaries of **Theorem 1**. Define, for any $(\mathbf{Q};\mathbf{R}) \in \mathcal{Q}^n \times \mathcal{R}^n$, a subset $N_*^{ML}(\mathbf{Q};\mathbf{R})$ of N by

$$i \in N_*^{ML}(\mathbf{Q};\mathbf{R}) \Longleftrightarrow Q_i \in \mathcal{Q}^{\mathbf{ML(R)}}$$

and a subset $N_*^{ML}(\mathbf{Q})$ of N by

$$N_*^{ML}(\mathbf{Q}) \equiv \bigcap_{\mathbf{R}\in\mathcal{R}^n} N_*^{ML}(\mathbf{Q};\mathbf{R}).$$

By definition, an individual $i_0 \in N_*^{ML}(\mathbf{Q})$ deserves the title of a *minimal libertarian.*

We may now assert the following:

Corollary 1: *There exists an* **ECF** Ψ_{ML} *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle, *and* Anonymity *such that, for all* $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$, *if* $\#N_*^{ML}(\mathbf{Q};\mathbf{R}) \geq 1$ *for all* $\mathbf{R} \in \mathcal{R}^n$, *then* $Q^{ML} = \Psi_{ML}(\mathbf{Q})$ *is the* minimally libertarian ordering function.

Corollary 2: *There exists an* **ECF** Ψ_{ML} *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle, *and* Anonymity *such that, for all* $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$, *if* $\#N_*^{ML}(\mathbf{Q}) \geq 1$, *then* $Q^{ML} = \Psi_{ML}(\mathbf{Q})$ *is the* minimally libertarian ordering function.

The message of **Corollary 2** is simple but appealing. It says that, if there exists at least one minimal libertarian in the society, then a minimally libertarian social ordering function can be socially chosen through an anonymous and democratic social decision procedure even when all other individuals are illiberal and/or selfish.

The extended constitution function Ψ_{ML} appearing in **Theorem 1** can be axiomatically characterized. All we need in addition to the axioms which are already introduced is the following:

Respect for Libertarian Judgements (RLJ): *For all* $\mathbf{Q} \in \Delta_\Psi$, *all* $\mathbf{R} \in \mathcal{R}^n$ *and all* $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in \Xi^{ML}(\mathbf{R})$, *if there exists at least one individual* $i \in N$ *such that* $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q_i(\mathbf{R}))$, *then* $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q(\mathbf{R}))$, where $Q = \Psi(\mathbf{Q})$.

Then we have:

Theorem 2: Ψ *satisfies the* Strong Pareto Principle, *the* Pareto Indifference Principle, Anonymity, *and* Respect for Libertarian Judgements *if and only if* $\Psi = \Psi_{ML}$.

Shifting the focus of our attention from the extended constitution function Ψ to the social decision-making rule chosen through Ψ, we may assert the following:

Theorem 3: *There exists an* **ECF** Ψ_{ML} *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle *and* Anonymity *such that, for all* $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$, *the following statements are equivalent*:

(α) $C_{\Psi_{ML}}(Q^{ML};\mathbf{R}) \subseteq \Theta^L$ for all $\mathbf{R} \in \mathcal{R}^n$, *where* $Q^{ML} = \Psi_{ML}(\mathbf{Q})$;

(β) $\# N^{ML}(\mathbf{Q};\mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$.

That is to say, any minimally libertarian ordering function derived from Ψ_{ML} under $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$ always rationalizes the minimally libertarian social decision-making rules. We may also obtain the following characterization result.

Theorem 4: *For any minimally libertarian ordering function derived from* Ψ_{ML} *under* $\mathbf{Q} \in \Delta_{\Psi_{ML}} \equiv \mathcal{Q}^n$, *viz.* $Q^{ML} = \Psi_{ML}(\mathbf{Q})$, *and for all* $\mathbf{R} \in \mathcal{R}^n$, $\theta \in C_{\Psi_{ML}}(Q^{ML};\mathbf{R})$ *implies* $\theta = \gamma^L$.

In view of the controversies on the formulation of libertarian rights between the proponents of the social choice correspondence approach and those of the game-form approach, **Theorem 4** may be of particular interest. Although the concept of minimally libertarian ordering functions does not make any lopsided treatment between $\mathbf{\Sigma}^L$ and $\mathbf{\Gamma}^L_{NS}$, the possibility of choosing a social decision-making rule from $\mathbf{\Sigma}^L$ automatically disappears in the primordial stage of rule selection, once we require that every minimally libertarian rule should guarantee individuals' rights to choose at least over A^1_i for all $i \in N$. This is because $\mathbf{\Sigma}^L = \varnothing$ as shown in the proof of **Theorem 4**. Thus, as long as our requirement for minimal libertarianism is valid, the controversy on the formulation of libertarian rights may be effectively resolved by **Theorem 4**.

4 Uniformly Rational Choice through Minimally Libertarian ECF

Theorem 1 and **Theorem 3** do not guarantee by themselves that the minimally libertarian ordering function can identify a minimally libertarian game form which is uniformly applicable in the realization stage. If there are multiple minimally libertarian game forms, however, the rational choice function associated with the minimally libertarian ordering function may switch from one minimally libertarian game form to the other when the profile of individual preference orderings undergoes a change, which one may find rather perplexing.

To ensure the strong requirement of the uniform applicability of a minimally libertarian game form, we must add a new condition on the ordering function Q. It endows Q with a characteristic of non-consequentialist evaluation over extended alternatives in the sense that *the evaluation in accordance with Q is invariant with respect to the changes in the profiles of individual preference orderings* $\mathbf{R} \in \mathcal{R}^n$. It is for this purpose that we now introduce the concept of a *rights-system,* which is an assignment of ordered pairs of social alternatives to each individual, viz. an n-tuple $\mathbf{D} = (D_i)_{i \in N}$ of subsets of $A \times A$. The universal class of rights-systems is denoted by $\mathcal{D}$. Note that, since A is a finite set, $\mathcal{D}$ is also finite.

In what follows, an important role is played by the concept of the *realization of a rights-system by a decision-making rule* θ, which is defined as follows.

Definition 4: *A decision-making rule* $\theta \in \Theta$ realizes a rights-system $\mathbf{D} = (D_i)_{i \in N} \in \mathcal{D}$ *if* θ *satisfies the following*:
(*a*) *If* $\theta = \sigma \in \mathbf{\Sigma}$, *for all* $\mathbf{R} \in \mathcal{R}^n$ *and for all* $i \in N$, $(\mathbf{x}, \mathbf{x}') \in D_i \cap P(R_i)$ ensures that $\mathbf{x}' \notin \sigma(\mathbf{R})$;
(*b*) *If* $\theta = \gamma = (M, g) \in \mathbf{\Gamma}_{NS}$, *for all* $i \in N$, $(\mathbf{x}, \mathbf{x}') \in D_i$ *ensures that there exists* $m_i \in M_i$ *such that* $\mathbf{x} \in g(m_i, M_{-i})$ *and* $\mathbf{x}' \notin g(m_i, M_{-i})$.[18]

A rights-system $\mathbf{D} = (D_i)_{i \in N} \in \mathcal{D}$ is called a *θ-rights-system* if there is a decision-making rule $\theta \in \Theta$ which realizes $\mathbf{D}$. Note that, given two rights-systems $\mathbf{D} = (D_i)_{i \in N}$ and $\mathbf{D}' = (D_i')_{i \in N}$, if $D_i \supseteq D_i'$ holds for all $i \in N$ and

[18] **Definition 4**(b) is just the $\mathbf{TIV}_n$ condition discussed by Deb, Pattanaik, and Razzolini (1997). The spirit behind this condition is also close to the concept of α-effectivity developed by Deb (1990, 1994), Pattanaik (1994), and Peleg (1998).

$D_j \supsetneq D'_j$ holds for at least one $j \in N$, and $\mathbf{D}$ is a θ-rights-system for some $\theta \in \Theta$, then $\mathbf{D}'$ is also a θ-rights-system. In particular, a trivial rights-system $\mathbf{D}^{\varnothing} = (D_i^{\varnothing})_{i \in N} \in \mathcal{D}$, where $D_i^{\varnothing} = \varnothing$ for all $i \in N$, is a θ-rights-system for any $\theta \in \Theta$, according to **Definition 4**. This also implies that, for each $\theta \in \Theta$, there is at least one θ-rights-system. Thus, for each $\theta \in \Theta$, there is a unique θ-rights-system $\mathbf{D}^{\theta} = (D_i^{\theta})_{i \in N} \in \mathcal{D}$ such that, for any other θ-rights-system $\mathbf{D}' = (D'_i)_{i \in N} \in \mathcal{D}$, $D_i^{\theta} \supseteq D'_i$ holds for all $i \in N$ and $D_j^{\theta} \supsetneq D'_j$ holds for at least one $j \in N$. Let us call such a θ-rights-system the *maximal θ-rights-system*. The class of maximal θ-rights-systems for any $\theta \in \Theta$ will be denoted by $\mathcal{D}_{\Theta}$.

To illustrate **Definition 4**(b), consider the following:

Example 4: Let $N = \{\text{Smoker, Non-Smoker}\}$. They are in the same compartment, where it is requested that passengers should not smoke unless fellow passengers permit it. It follows that Smoker (resp. Non-smoker) has two personal actions, viz. $s =$ "to smoke" and $ns =$ "not to smoke" (resp. $p =$ "to permit smoking" and $np =$ "not to permit smoking"). This situation can be described by Figure 2.

Insert Figure 2 around here.

To convert this extensive game form into the strategic game form, the strategy set M_S for Smoker and the strategy set M_{NS} for Non-smoker can be defined by

$$M_S = \{(s \mid p), ns\}; M_{NS} = \{p, np\},$$

where $(s \mid p) =$ "to smoke if permitted, not to smoke if not permitted." The outcome function $g : M_S \times M_{NS} \to A$ is defined by

$$g((s \mid p), p) = (s, p); g((s \mid p), np) = (ns, np)$$
$$g(ns, p) = (ns, p); g(ns, np) = (ns, np).$$

Let $\gamma = (M_S \times M_{NS}, g)$, and let $\mathbf{D}^{\gamma} = (D_S^{\gamma}, D_{NS}^{\gamma})$ be the rights-system realized by γ in the sense of **Definition 4(b)**. It is easy to check that

$$D_S^{\gamma} = \{((ns, p), (s, p)), ((s, p), (ns, p)) ((ns, np), (s, p)), ((ns, np), (ns, p))\}$$

and

$$D_{NS}^{\gamma} = \{((ns, np), (s, p)), ((s, p), (ns, np)), ((ns, np), (ns, p)), ((ns, p), (ns, np))\}$$

hold in this case. ■

Note that the concept of *coherent rights-system* introduced by Suzumura (1978; 1983, Chapter 7) is essential in assuring the well-definedness of the associated social choice correspondence, since the social choice correspondence σ, which realizes the rights-system $\mathbf{D}^{\sigma} = (D_i^{\sigma})_{i \in N} \in \mathcal{D}_{\Theta}$, is well-defined if and only if $\mathbf{D}^{\sigma}$ is coherent [Suzumura (1978; 1983, Chapter 7)].[19] In contrast, the coherence of the rights-system is not required for the associated game form to be assured of non-empty Nash equilibrium outcomes whenever the equilibrium concept is that of Nash equilibrium. These points can be illustrated by the following:

Example 5: Let $N = \{1, 2\}$ and let $M = M_1 \times M_2$, where $M_1 = \left\{s, s'\right\}$ and $M_2 = \left\{t, t'\right\}$. The outcome function $g : M \to A \equiv \{x, y, z\}$ is defined by $g(s, t) = x$, $g\left(s, t'\right) = y$, $g\left(s', t\right) = x$, and $g\left(s', t'\right) = z$.

Insert Figure 3 around here.

Then, the γ-maximal rights system $\mathbf{D}^{\gamma} = (D_1^{\gamma}, D_2^{\gamma})$ associated with the game form γ is defined in accordance with Definition 4(b) as follows:

$$D_1^{\gamma} = \{(x, y), (x, z), (y, z), (z, y)\} \text{ and } D_2^{\gamma} = \{(x, y), (y, x), (x, z), (z, x)\}.$$

It is clear that $\mathbf{D}^{\gamma}$ is incoherent. This non-coherence of $\mathbf{D}^{\gamma}$ notwithstanding, γ has a Nash equilibrium outcome for every profile of individual preference orderings. This is because, invoking Abdou (1998, 1998a), we can verify that $\gamma \equiv (M, g)$ is *Nash-solvable*, since γ is the two-person game form and satisfies the *tightness* condition in the sense of Abdou (1998, 1998a). ■

As an auxiliary step in evaluating the desirability of alternative rights-systems, let us introduce a linear ordering relation J over $\mathcal{D}_{\Theta}$.[20] The universal

[19] A *critical loop* in the rights-system D^{σ} is defined to be a finite sequence of ordered pairs $\{(\mathsf{x}(\mu), \mathsf{y}(\mu))\}_{\mu=1}^{t}$ $(2 \leq t < +\infty)$ such that (i) $(\mathsf{x}(\mu), \mathsf{y}(\mu)) \in \cup_{i \in N} D_i^{\sigma}$ for all $\mu \in \{1, 2, ..., t\}$, (ii) there exists no $i^* \in N$ such that $(\mathsf{x}(\mu), \mathsf{y}(\mu)) \in D_{i^*}^{\sigma}$ for all $\mu \in \{1, 2, ..., t\}$, and (iii) $\mathsf{x}(1) = \mathsf{y}(t)$ and $\mathsf{x}(\mu) = \mathsf{y}(\mu - 1)$ for all $\mu \in \{2, 3, ..., t\}$. The rights-system D^{σ} is *coherent* if and only if there exists no critical loop of any order t in D^{σ}.

[20] The following discussions and analytical results using J remain valid even if the property of J is weakened to a weak ordering.

class of such linear ordering relations is denoted by $\mathcal{J}$. Each $J \in \mathcal{J}$ expresses an evaluation of rights-systems from the viewpoint of the extent of individual liberty conferred by each rights-system: each rights-system $\mathbf{D}$ represents the extent of individual liberty distributed among individuals, and each linear ordering relation J evaluates alternative distributions from the intrinsic rather than instrumental value of individual liberty.

Equipped with this concept of the extent-of-liberty judgements, we may now introduce the following condition on the admissible class of ordering functions.

Intrinsic Evaluation of Rights-Systems (IER): *The admissible ordering function $Q \in \mathcal{Q}$ should be such that there exists a linear ordering $J_Q \in \mathcal{J}$ such that, for all $\mathbf{R} \in \mathcal{R}^n$, and all $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$ satisfying either $\theta, \theta' \in \Theta^L$ or $\theta, \theta' \in \Theta \setminus \Theta^L$, we have*

$$\left[\mathbf{D}^{\theta} \neq \mathbf{D}^{\theta'} \text{ or } \mathbf{x} = \mathbf{x}'\right] \Rightarrow \left[((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q(\mathbf{R}) \Leftrightarrow (\mathbf{D}^{\theta}, \mathbf{D}^{\theta'}) \in J_Q\right].$$

The intuitive meaning of **IER** is that Q evaluates the extended social alternatives in some qualified subset of the set of all extended social alternatives on the basis of intrinsic rather than instrumental value of their associated rights-systems. In other words, Q satisfying **IER** evaluates each extended alternative $(\mathbf{x}, \theta)$ realizable under $\mathbf{R}$ by focusing only on the extent of individual liberty associated with the decision-making rule θ, thereby completely ignoring its instrumental value in bringing about the consequential social outcome $\mathbf{x}$. In fact, by connecting Q with J_Q in the way **IER** stipulates, Q is endowed with the following independence property: For all $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R})$ and all $(\mathbf{y}, \theta)$, $(\mathbf{y}', \theta') \in \mathcal{RP}(\mathbf{R}')$ with $\mathbf{D}^{\theta} \neq \mathbf{D}^{\theta'}$,

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q(\mathbf{R}) \Leftrightarrow ((\mathbf{y}, \theta), (\mathbf{y}', \theta')) \in Q(\mathbf{R}').$$

The linear ordering relation J_Q plays an important role in determining the rights-assignment over the set of socially interactive matters. To illustrate this fact, consider the following:

Example 6: Let $N = \{\text{Smoker, Non-Smoker}\}$ as in **Example 4**. Consider the game tree described in Figure 4.

Insert Figure 4 around here.

Unlike Figure 2, where Non-smoker's claim for clean air is bestowed priority over Smoker's claim for free smoking, Figure 4 describes the situation where Smoker can choose freely either to smoke (s) or not to smoke (ns) no matter whether Non-smoker perseveres (p) or does not persevere (np). To convert this extensive game form into the strategic game form, the strategy set M_S^* for Smoker and the strategy set M_{NS}^* for Non-smoker can be defined by

$$M_S^* = \{s, ns\}\,; M_{NS}^* = \{p, (np \mid s)\}\,,$$

where $(np \mid s)$ means "not persevere if Smoker smokes, persevere otherwise." The normal (strategic) game form is defined by $\gamma^* = (M^*, g^*)$, where $M^* = M_S^* \times M_{NS}^*$ and the outcome function g^* is defined by

$$g^*(s,p) = (s,p)\,; g^*(s,(np \mid s)) = (s,np)$$

$$g^*(ns,p) = (ns,p)\,; g^*(ns,(np \mid s)) = (ns,p)\,.$$

We may then check that

$$D_S^{\gamma^*} = \{((s,np),(ns,p)), ((ns,p),(s,np)), ((s,p),(ns,p)), ((ns,p),(s,p))\}$$

$$D_{NS}^{\gamma^*} = \{((s,np),(s,p)), ((s,p),(s,np)), ((ns,p),(s,np)), ((ns,p),(s,p))\}\,.$$

Thus, the comparison between the conferment of priority to Smoker's claim and that to Non-smoker's claim can be boiled down to the ordering between $\mathbf{D}^{\gamma}$ in **Example 4** and $\mathbf{D}^{\gamma^*}$ in the present **Example 6**. ■

Let $B_J(\mathcal{D}_\Theta)$ be the set of best elements in $\mathcal{D}_\Theta$ with respect to a binary relation $J \subseteq \mathcal{D}_\Theta \times \mathcal{D}_\Theta$. Since $\mathcal{D}_\Theta$ is finite, we are assured that $B_J(\mathcal{D}_\Theta)$ is non-empty whenever J is complete and acyclic. By using $B_J(\mathcal{D}_\Theta)$, we may check the characteristics of minimally libertarian ordering functions satisfying **IER**, in particular, from the viewpoint of *uniformity of rational choice*.

To begin with, let us characterize the ordering function which generates a uniformly applicable decision-making rule by using $B_J(\mathcal{D}_\Theta)$.

Theorem 5: *An ordering function Q generates a uniformly applicable decision-making rule if and only if there exist a complete and acyclic binary relation $J_Q \subseteq \mathcal{D}_\Theta \times \mathcal{D}_\Theta$ and some $\theta^* \in \Theta$* satisfying $\mathbf{D}^{\theta^*} \in B_{J_Q}(\mathcal{D}_\Theta)$ *such that, for all $\mathbf{R} \in \mathcal{R}^n$, there exists $\mathbf{x}^* \in A$ satisfying $(\mathbf{x}^*, \theta^*) \in \mathcal{RP}(\mathbf{R})$ and that, for all $(\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R})$, $((\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta)) \in Q(\mathbf{R})$ holds.*

By using this characterization result, we may verify that **IER** is a sufficient condition for **URC** to be satisfied. This result may be of interest, as **IER** has an implication which is relevant for libertarian ethics.

Corollary 3: *A minimally libertarian ordering function Q generates a uniformly applicable decision-making rule if it satisfies* **IER**.

By virtue of this result, we may assert that an ordering function satisfying **IER** is not only capable of making a consistent value judgement from the viewpoint of libertarian ethics, but also it is capable of guaranteeing the uniformity of rational choice in the primordial stage of rule selection.

What remains for us is to identify the conditions under which there exists a democratic extended constitution function which always generates a minimally libertarian ordering function satisfying **IER**. Let us denote the class of ordering functions which satisfy **IER** by $\mathcal{Q}^{\mathsf{IE}}$. Given $\mathbf{Q} \in \mathcal{Q}^n$, define a set of individuals $N^{IE}(\mathbf{Q}) \subseteq N$ by $i \in N^{IE}(\mathbf{Q}) \Leftrightarrow Q_i \in \mathcal{Q}^{\mathsf{IE}}$. Given $\mathbf{Q} \in \mathcal{Q}^n$, we may say that i is a *non-consequentialist libertarian* whenever $i \in N^{IE}(\mathbf{Q})$.

The focus of our subsequent analysis is to verify whether minimally libertarian ordering functions satisfying **IER** can be socially chosen through an **ECF** satisfying the Arrovian condition of Independence of Irrelevant Alternatives (**IIA**) along with the two versions of the Pareto Principle and Non-Dictatorship. In general, there is no **ECF** Ψ satisfying the Arrovian **IIA** as well as **SP** and **ND** if Ψ has the universal domain $\mathcal{Q}^n$. In order to circumvent this impasse, we restrict the domain of the extended constitution function Ψ to an appropriate subset $r(\mathcal{Q}^n)$ of $\mathcal{Q}^n$ and require the following:

Independence on $r(\mathcal{Q}^n)$ ($\mathbf{I}_{r(\mathcal{Q}^n)}$): *For all $\mathbf{R} \in \mathcal{R}^n$, all $\mathbf{Q}$, $\mathbf{Q}' \in r(\mathcal{Q}^n)$, and all $(\mathbf{x}, \theta)$, $(\mathbf{x}', \theta') \in A \times \Theta$, if*

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q_i(\mathbf{R}) \Leftrightarrow ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q'_i(\mathbf{R})$$

holds for all $i \in N$, then

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q(\mathbf{R}) \Leftrightarrow ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q'(\mathbf{R})$$

holds, where $Q = \Psi(\mathbf{Q})$ and $Q' = \Psi(\mathbf{Q}')$.

Concerning the richness of the restricted domain $r(\mathcal{Q}^n)$, we require that it is *sufficiently rich* in the sense that (1) $r(\mathcal{Q}^n) \supseteq \left(\mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}\right)^n$, and (2) if $\mathbf{Q} \in r(\mathcal{Q}^n)\backslash\left(\mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}\right)^n$, then $(Q_i^*, \mathbf{Q}_{-i}) \in r(\mathcal{Q}^n)$ holds for any $i \in N$ and any $Q_i^* \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$. Then, we obtain the following result.

Theorem 6: *There exists an* **ECF** Ψ *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle, *and* Non-dictatorship *such that, for some sufficiently rich domain* $r(\mathcal{Q}^n)$, *the following two statements are equivalent*:
(α) Ψ *satisfies* Independence on $r(\mathcal{Q}^n)$, *and* $Q^{ML} = \Psi(\mathbf{Q})$ *is a* minimally libertarian ordering function *satisfying* **IER** *for all* $\mathbf{Q} \in r(\mathcal{Q}^n)$;
(β) *for each and every* $\mathbf{Q} \in r(\mathcal{Q}^n)$, $\#N^{ML}(\mathbf{Q};\mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$, *and that* $\bigcap_{\mathbf{Q} \in r(\mathcal{Q}^n)} N^{IE}(\mathbf{Q}) \neq \varnothing$.

Remark 1: *The sufficiently rich domain* $r(\mathcal{Q}^n)$ *constructed in the proof of* **Theorem 6** *is a maximal domain under which the assertion of* **Theorem 6** *is valid, viz., for any* $r'(\mathcal{Q}^n) \subseteq r(\mathcal{Q}^n)$, *the assertion of* **Theorem 6** *holds, whereas for any* $r'(\mathcal{Q}^n) \supsetneq r(\mathcal{Q}^n)$, *it no longer holds.*

According to **Theorem 6**, there exists a Paretian and non-dictatorial **ECF** which can choose minimally libertarian ordering functions satisfying **IER** even with the requirement of the Arrovian **IIA** if and only if there is a group of individuals who jointly support the requirement of minimal libertarianism, and there uniformly exists at least one *non-consequentialist libertarian* over $r(\mathcal{Q}^n)$.[21] The last condition implies the existence of at least one individual who is willing to restrict his domain of ordering functions so as to be faithful to the ethics of non-consequentialist libertarianism in evaluating rights-systems. Thus, whenever such an individual exists together with a minimally libertarian group of individuals, the uniformly rational social choice of minimally libertarian rules through a democratic and informationally efficient social choice procedure is possible.

The following theorem identifies a meaningful condition for the uniformity-of-rational-choice property of Ψ_{ML}.

Theorem 7: *There exists an* **ECF** Ψ_{ML} *satisfying the* Strong Pareto Principle, *the* Pareto Indifference Principle, Non-dictatorship, *and* Independence on $r(\mathcal{Q}^n)$ *on some sufficiently rich domain* $r(\mathcal{Q}^n)$ *such that, for each* $\mathbf{Q} \in$

[21] Although **Theorem 6** imposes Non-dictatorship and Independence on $r(\mathcal{Q}^n)$ on the Paretian extended constitution function rather than Anonymity, we can obtain another characterization result similar to **Theorem 6** simply by imposing Anonymity instead of the above two axioms.

$r(\mathcal{Q}^n)$, *if* $\#N^{ML}(\mathbf{Q};\mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$, *and* $\bigcap_{\mathbf{Q}\in r(\mathcal{Q}^n)} N^{IE}(\mathbf{Q}) \neq \varnothing$, *then there exists* $\gamma^L \in \bigcap_{\mathbf{R}\in\mathcal{R}^n} C_{\Psi_{ML}}(\mathbf{Q};\mathbf{R})$.

Theorem 6 and **Theorem 7** show unambiguously that the existence of a non-consequentialist libertarian plays an essential role in guaranteeing the uniformity of rational choice of social decision-making rules. If, in contrast, all individuals evaluate the rights-systems on the sole basis of consequentialist judgements, the uniformity of rational choice will not be guaranteed in general, even when the derived social ordering function is minimally libertarian. If the social ordering function is not minimally libertarian, the difficulty of uniform rational choice is even more serious. Since the institutional framework of a society will be robust if its rights-system will not have to be switched to something else with a frivolous change in individual tastes, the uniformity-of-rational-choice may be construed to be a desirable, if stringent, requirement for the stability of constitutional democracy.

5 Concluding Remarks

Capitalizing on the tripartite classification of the issues of libertarian rights due to Pattanaik and Suzumura (1994; 1996), this paper analysed the issue of initial conferment of libertarian rights in the primordial stage of rule selection, which was left relatively uncultivated in the literature. Not only the traditional articulation of libertarian rights along the line of Sen (1970; 1970a, Chapter 6*) and Gibbard (1974) in terms of the preference-contingent constraints on social choice correspondences, but also the game-form articulation thereof along the line of Sugden (1985), and Gaertner, Pattanaik and Suzumura (1992) are accommodated within our generalized conceptual framework. In this framework, the social decision on the initial conferment of libertarian rights is modelled by means of the extended constitution function satisfying the essentially Arrovian requirements of the Pareto principles, non-dictatorship, and informational efficiency. If we add the requirement of unrestricted domain, this conceptual framework will be trapped again by the Arrovian impossibility theorem. However, a necessary and sufficient condition for the existence of a minimally libertarian extended constitution function can be identified under suitable domain restriction. It is also shown that this minimally libertarian extended constitution function enables the society to assign the minimally libertarian game-form rights that can be maintained no matter which profile of individual preference orderings over conventionally defined social states materializes in the stage of the realization of conferred rights. This uniformity of rational choice is an admittedly stringent requirement. Accordingly, the conditions under which it is satisfied are strong and clearly non-consequentialist in nature. Nevertheless, the uniformly rational conferment of rights deserves to be highlighted, as it is free from the frivolous changes in individual preferences over the consequential outcomes.

It may deserve special emphasis that the minimally libertarian game-form rights chosen through the minimally libertarian extended constitution function has a nice consequentialist property. Indeed, the non-cooperative game defined by the minimally libertarian game form always has a Pareto efficient Nash equilibrium outcome for any profile of individual preference orderings over consequential outcomes. This is because the minimally libertarian game form constructed in this paper is Nash solvable, and the Nash solvable game form always has a Pareto efficient equilibrium outcome [Peleg, Peters and Storchen (2002)]. It follows that the uniformly rational social choice of the minimally libertarian game-form rights provides us with a resolution of the strong Pareto-liberal paradox in the sense of Deb, Pattanaik and Razzolini (1997), although it does not resolve the weak Pareto-liberal paradox in their sense.[22]

In conclusion, it is hoped that our preliminary exploration of the issue of initial conferment of libertarian rights will serve as a useful pilot study of this important area of research in social choice theory.

6 Appendix

Proof of Lemma 1: Since $\Theta^L = \mathbf{\Sigma}^L \cup \mathbf{\Gamma}^L_{NS}$, it suffices to show that there is a Nash-solvable game form which is minimally libertarian.

Note that although A^1 is decomposable, A^2 is not necessarily decomposable. In the following argument, we will treat A^2 as being indecomposable.

[22] The strong Pareto-liberal paradox in the game-form articulation of rights implies that there exists a profile of individual preference orderings over consequential outcomes under which every equilibrium outcome of the induced game is Pareto inefficient. In contract, the weak Pareto-liberal paradox implies that there exists a profile of individual preference orderings over consequential outcomes under which there exists an equilibrium outcome of the induced game which is Pareto inefficient.

The case where A^2 is decomposable can be treated as a special case of this case.

Given $i \in N$, let[23]

$$\rho_i(A^k) \equiv \left\{x_i^k \in X_i^k \mid \exists\, \mathbf{x}_{-i}^k :\ (x_i^k, \mathbf{x}_{-i}^k) \in A^k\right\} \text{ for each } k = 1, 2$$

and, given $S \subseteq N$, let[24]

$$\rho_S(A^k) \equiv \left\{\mathbf{x}_S^k \in X_S^k \mid \exists\, \mathbf{x}_{N\backslash S}^k :\ (\mathbf{x}_S^k, \mathbf{x}_{N\backslash S}^k) \in A^k\right\} \text{ for each } k = 1, 2.$$

Moreover, given $i \in N$, $S \subseteq N\backslash\{i\}$, and $\mathbf{x}_S^k \in \rho_S(A^k)$, let[25]

$$\rho_i(A^k; \mathbf{x}_S^k) \equiv \left\{x_i^k \in \rho_i(A^k) \mid \exists\, \mathbf{x}_{N\backslash(S\cup\{i\})}^k :\ (x_i^k, \mathbf{x}_S^k, \mathbf{x}_{N\backslash(S\,\cup\,\{i\})}^k) \in A^k\right\}$$

for each $k = 1, 2$. In what follows, let $S^i \equiv \{1, \ldots, i-1\} \subseteq N$ and ${}^iS \equiv \{i+1, \ldots, n\} \subseteq N$ for notational convenience.

Let a minimally libertarian game form $\gamma = (M, g)$ be defined as follows:
(1) For $i = 1$, $M_i = M_i^1 \times M_i^2 \equiv A_i^1 \times \rho_i(A^2)$; and
(2) for $i = 2, \ldots, n$, $M_i = M_i^1 \times M_i^2$, where M_i^1 and M_i^2 are the sets of all possible strategy functions, each element of which being defined by

$$m_i^k : \prod_{j=1}^{i-1} M_j^k \to \rho_i(A^k) \quad (k = 1, 2)$$

such that, for each $\mathbf{m}_{S^i}^k = \left(m_j^k\right)_{j=1}^{i-1} \in \Pi_{j=1}^{i-1} M_j^k$, $m_i^k\left(\mathbf{m}_{S^i}^k\right) \in \rho_i(A^k; (x_1^k, \ldots, x_{i-1}^k))$, where $x_1^k = m_1^k \in \rho_1(A^k)$ and $x_h^k = m_h^k\left(\mathbf{m}_{S^h}^k\right) \in \rho_h(A^k; (x_1^k, \ldots, x_{h-1}^k))$ for all $h = 2, \ldots, i-1$.
(3) The outcome function $g : M \to A$ is defined as follows: for any $\mathbf{m} = (m_i^1, m_i^2)_{i\in N} \in M$, $g(\mathbf{m}) = (g^1(\mathbf{m}^1), g^2(\mathbf{m}^2)) = (\mathbf{x}^1, \mathbf{x}^2)$, where $(\mathbf{x}^1, \mathbf{x}^2) = ((x_i^1)_{i\in N}, (x_i^2)_{i\in N})$ and $(x_1^1, x_1^2) = (m_1^1, m_1^2)$ and $(x_i^1, x_i^2) = \left(m_i^1\left(\mathbf{m}_{S^i}^1\right), m_i^2\left(\mathbf{m}_{S^i}^2\right)\right)$ for all $i = 2, \ldots, n$.

Note that, since $M_i^1 \times M_i^2$ is the universal set of strategy functions satisfying the above condition **(2)**, we have the following properties on $M_i^1 \times M_i^2$ for each $i \in N\backslash\{1\}$:

[23] If $k = 1$, $\rho_i(A^1) = A_i^1$ for any $i \in N$. Moreover, if A^2 is decomposable, then $\rho_i(A^2) = A_i^2$ for any $i \in N$.

[24] If $k = 1$, then $\rho_S(A^1) = \Pi_{i\in S} A_i^1$. Moreover, if A^2 is decomposable, then $\rho_S(A^2) = \Pi_{i\in S} A_i^2$.

[25] If $k = 1$, then $\rho_i(A^1; \mathsf{x}_S^1) = A_i^1$. Moreover, if A^2 is decomposable, then $\rho_i(A^2; \mathsf{x}_S^2) = A_i^2$.

(2-1) for each $\mathbf{m}^1_{S^i} \in \prod_{j=1}^{i-1} M^1_j$, and each $x^1_i \in A^1_i$, M^1_i contains every strategy function $m^1_i \in M^1_i$ satisfying $m^1_i\left(\mathbf{m}^1_{S^i}\right) = x^1_i$; and

(2-2) for each $\mathbf{m}^2_{S^i} \in \prod_{j=1}^{i-1} M^2_j$, where $m^2_1 = x^2_1 \in \rho_1(A^2)$ and $m^2_h\left(\mathbf{m}^2_{S^h}\right) = x^2_h \in \rho_h(A^2;(x^2_1,\ldots,x^2_{h-1}))$ for all $h = 2,\ldots,i-1$, there exists a strategy function $m^2_i \in M^2_i$ satisfying $m^2_i\left(\mathbf{m}^2_{S^i}\right) = x^2_i$ for each $x^2_i \in \rho_i(A^2;(x^2_1,\ldots,x^2_{i-1}))$.

For this game form γ, the associated effectivity function E^γ can be defined by $E^\gamma(\varnothing) = \varnothing$ and, for each non-empty $S \subseteq N$,

$$E^\gamma(S) \equiv \left\{B \subseteq A \mid \exists \mathbf{m}_S = (m_i)_{i\in S} \in M_S, \forall \mathbf{m}_{N\backslash S} \in M_{N\backslash S} : g(\mathbf{m}_S, \mathbf{m}_{N\backslash S}) \in B\right\}.$$

Furthermore, the polar E^γ_* of the effectivity function E^γ can be defined by $E^\gamma_*(\varnothing) = \varnothing$ and, for each non-empty $S \subseteq N$,

$$E^\gamma_*(S) \equiv \{B \subseteq A \mid B \cap B' \neq \varnothing \text{ for all } B' \in E^\gamma(N\backslash S)\}.$$

Since E^γ is an α-effectivity function associated with the game form γ, we can easily see that E^γ satisfies *monotonicity* and *supperadditivity* (Moulin (1983); Peleg (1998)). Thus, according to Peleg, Peters and Storchen (2002), the necessary and sufficient condition for the existence of Nash-solvable representation of E^γ is given as follows:

$$\left[B^i \in E^\gamma_*(i) \text{ for all } i \in N\right] \Rightarrow \bigcap_{i\in N} B^i \neq \varnothing. \qquad (*)$$

In what follows, we will show that E^γ indeed satisfies this condition (*).

Given $i \in N$ and $\mathbf{m}_{-i} \in M_{-i}$, $g(M_i, \mathbf{m}_{-i})$ and all supersets thereof belong to $E^\gamma(N\backslash\{i\})$. Thus, $B^i \in E^\gamma_*(i)$ if and only if there exists

$$\mathcal{L}^k_i(B^i) \equiv \left\{L^k_i(\mathbf{m}^k_{-i})(\neq \varnothing) \subseteq M^k_i \mid \mathbf{m}^k_{-i} \in M^k_{-i}\right\} \quad (k = 1, 2)$$

such that

$$B^i \supseteq \left(\bigcup_{\mathbf{m}^1_{-i}\in M^1_{-i}} g^1(L^1_i(\mathbf{m}^1_{-i}), \mathbf{m}^1_{-i})\right) \times \left(\bigcup_{\mathbf{m}^2_{-i}\in M^2_{-i}} g^2(L^2_i(\mathbf{m}^2_{-i}), \mathbf{m}^2_{-i})\right). \quad (**)$$

For each $i \in N$ and each $\mathbf{m}^k_{-i} \in M^k_{-i}$ where $k = 1, 2$, we will denote the generic element of the set $L^k_i(\mathbf{m}^k_{-i})$ by $m^k_i(\cdot;\mathbf{m}^k_{-i})$.

Take any $B^i \in E^\gamma_*(i)$ for each $i \in N$, and consider $\bigcap_{i\in N} B^i$. By definition, for each $i \in N$, we can identify the two sets $\mathcal{L}^1_i(B^i)$ and $\mathcal{L}^2_i(B^i)$ satisfying the condition (**). Then, for each $i \in N$ and each $\mathbf{m}^k_{-i} \in M^k_{-i}$ where $k = 1, 2$, let us choose one $m^k_i(\cdot; \mathbf{m}^k_{-i})$ from $L^k_i(\mathbf{m}^k_{-i}) \in \mathcal{L}^k_i(B^i)$, and define $\overline{L}^k_i(\mathbf{m}^k_{-i}) \equiv \left\{m^k_i(\cdot; \mathbf{m}^k_{-i})\right\} \subseteq L^k_i(\mathbf{m}^k_{-i})$. Then, define

$$\overline{B}^i \equiv \prod_{k=1}^{2} \left[\bigcup_{\mathbf{m}^k_{-i} \in M^k_{-i}} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i}) \right] \subseteq B^i.$$

Note that $\overline{B}^i \in E^{\gamma}_*(i)$ for each $i \in N$. We will show $\bigcap_{i \in N} \overline{B}^i \neq \varnothing$, which immediately implies $\bigcap_{i \in N} B^i \neq \varnothing$, as desired.

First, consider $\overline{B}^1 \cap \overline{B}^2$. We will show that for each $\widetilde{\mathbf{m}}^k_{{}^2S} \in M^k_{{}^2S}$ where $k = 1, 2$, there exists $\left(\widetilde{x}^k_1, \widetilde{m}^k_2\right) \in M^k_1 \times M^k_2$ such that

$$g^k\left(\widetilde{x}^k_1, \widetilde{m}^k_2, \widetilde{\mathbf{m}}^k_{{}^2S}\right) \in \bigcap_{i=1,2} \left[\bigcup_{\mathbf{m}^k_{-i} \in M^k_{-i}} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i}) \right],$$

which immediately implies $\overline{B}^1 \cap \overline{B}^2 \neq \varnothing$.

Given $\widetilde{\mathbf{m}}^k_{{}^2S} \in M^k_{{}^2S}$ where $k = 1, 2$, we can find strategy functions $\widehat{m}^k_2 \in M^k_2$ and $\overline{\mathbf{m}}^k_{{}^2S} \in M^k_{{}^2S}$ such that:
(i) for all $x^k_1 \in M^k_1$, $\widehat{m}^k_2\left(x^k_1\right) = m^k_2\left(x^k_1; x^k_1, \widetilde{\mathbf{m}}^k_{{}^2S}\right) \in \rho_2(A^k)$;
(ii) for each $i \in {}^2S$, $\overline{m}^k_i\left(x^k_1, \widehat{m}^k_2, \overline{\mathbf{m}}^k_{S^i \setminus \{1,2\}}\right) = \widetilde{m}^k_i\left(x^k_1, m^k_2\left(\cdot; x^k_1, \widetilde{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{S^i \setminus \{1,2\}}\right) \in \rho_i(A^k)$ for all $x^k_1 \in M^k_1$. There exist such functions by the universality of M^k_2 and $M^k_{{}^2S}$. Then, for all $x^k_1 \in M^k_1$ where $k = 1, 2$,

$$g^k\left(x^k_1, \widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right) = g^k\left(x^k_1, m^k_2\left(\cdot; x^k_1, \widetilde{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right) \tag{a}$$

holds. Thus, given $\widetilde{\mathbf{m}}^k_{{}^2S} \in M^k_{{}^2S}$ where $k = 1, 2$,

$$\bigcup_{x^k_1 \in M^k_1} \left\{g^k\left(x^k_1, m^k_2\left(\cdot; x^k_1, \widetilde{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right)\right\} = g^k\left(M^k_1, \widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right). \tag{b}$$

Since there exists $x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{S^i}\right) \in M^k_1$ such that

$$\left\{g^k\left(x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right)\right\} = g^k\left(\overline{L}^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \tag{c}$$

we may observe in view of (b) and (c) that

$$g^k\left(x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), m^k_2\left(\cdot; x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right) \in \bigcap_{i=1,2} \left[\bigcup_{\mathbf{m}^k_{-i} \in M^k_{-i}} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i}) \right]$$

holds for each given $\widetilde{\mathbf{m}}^k_{{}^2S} \in M^k_{{}^2S}$, where

$$g^k\left(x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), m^k_2\left(\cdot; x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right), \widetilde{\mathbf{m}}^k_{{}^2S}\right) = g^k\left(x^k_1\left(\widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right), \widehat{m}^k_2, \overline{\mathbf{m}}^k_{{}^2S}\right)$$

holds by virtue of (a).

Next, for any $j \in N\backslash\{1,2\}$, let us suppose that, for each $\widetilde{\mathbf{m}}^k_{{}^jS\cup\{j\}} \in M^k_{{}^jS\cup\{j\}}$ where $k = 1, 2$, there exists $\widetilde{\mathbf{m}}^k_{S^j} \in M^k_{S^j}$ such that

$$g^k\left(\widetilde{\mathbf{m}}^k_{S^j}, \widetilde{\mathbf{m}}^k_{{}^jS\cup\{j\}}\right) \in \bigcap_{i\in S^j}\left[\bigcup_{\mathbf{m}^k_{-i}\in M^k_{-i}} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i})\right]. \quad (***)$$

Given this supposition, we will show in the following that, for each $\widetilde{\mathbf{m}}^k_{{}^jS} \in M^k_{{}^jS}$ where $k = 1, 2$, there exists $\widetilde{\mathbf{m}}^k_{S^j\cup\{j\}} \in M^k_{S^j\cup\{j\}}$ such that

$$g^k\left(\widetilde{\mathbf{m}}^k_{S^j\cup\{j\}}, \widetilde{\mathbf{m}}^k_{{}^jS}\right) \in \bigcap_{i\in S^j\cup\{j\}}\left[\bigcup_{\mathbf{m}^k_{-i}\in M^k_{-i}} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i})\right].$$

For each given $\widetilde{\mathbf{m}}^k_{{}^jS} \in M^k_{{}^jS}$ where $k = 1, 2$, we can find strategy functions $\widehat{m}^k_j \in M^k_j$ and $\overline{\mathbf{m}}^k_{{}^jS} \in M^k_{{}^jS}$ such that:
(iii) for all $\mathbf{m}^k_{S^j} \in M^k_{S^j}$, $\widehat{m}^k_j\left(\mathbf{m}^k_{S^j}\right) = m^k_j\left(\mathbf{m}^k_{S^j}; \mathbf{m}^k_{S^j}, \widetilde{\mathbf{m}}^k_{{}^jS}\right) \in \rho_j(A^k)$;
(iv) for each $i \in {}^jS$,

$$\overline{m}^k_i\left(\mathbf{m}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{S^i\backslash(S^j\cup\{j\})}\right) = \widetilde{m}^k_i\left(\mathbf{m}^k_{S^j}, m^k_j\left(\cdot; \mathbf{m}^k_{S^j}, \widetilde{\mathbf{m}}^k_{{}^jS}\right), \widetilde{\mathbf{m}}^k_{S^i\backslash(S^j\cup\{j\})}\right) \in \rho_i(A^k)$$

for all $\mathbf{m}^k_{S^j} \in M^k_{S^j}$. There exist such functions by the universality of M^k_j and $M^k_{{}^jS}$. Then, it holds that, for all $\mathbf{m}^k_{S^j} \in M^k_{S^j}$ where $k = 1, 2$,

$$g^k\left(\mathbf{m}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{{}^jS}\right) = g^k\left(\mathbf{m}^k_{S^j}, m^k_j\left(\cdot; \mathbf{m}^k_{S^j}, \widetilde{\mathbf{m}}^k_{{}^jS}\right), \widetilde{\mathbf{m}}^k_{{}^jS}\right). \quad \text{(d)}$$

Thus, given $\widetilde{\mathbf{m}}^k_{{}^jS} \in M^k_{{}^jS}$ where $k = 1, 2$,

$$\bigcup_{\mathbf{m}^k_{S^j}\in M^k_{S^j}} \left\{g^k\left(\mathbf{m}^k_{S^j}, m^k_j\left(\cdot; \mathbf{m}^k_{S^j}, \widetilde{\mathbf{m}}^k_{{}^jS}\right), \widetilde{\mathbf{m}}^k_{{}^jS}\right)\right\} = g^k\left(M^k_{{}^jS}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{{}^jS}\right). \quad \text{(e)}$$

For each $\mathbf{m}^k_{{}^jS\cup\{j\}} \in M^k_{{}^jS\cup\{j\}}$ where $k = 1, 2$, there exists $\widetilde{\mathbf{m}}^k_{S^j} \in M^k_{S^j}$ such that the condition (***) holds, so that we can find $\widehat{\mathbf{m}}^k_{S^j} \in M^k_{S^j}$ for a particular $\left(\widehat{m}^k_j, \overline{\mathbf{m}}^k_{{}^jS}\right) \in M^k_{{}^jS\cup\{j\}}$ such that $g^k\left(\widehat{\mathbf{m}}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{{}^jS}\right)$ satisfies the condition

(***). Since $g^k\left(\widehat{\mathbf{m}}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{jS}\right) \in g^k\left(M^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{jS}\right)$, it follows from (e) that $g^k\left(\widehat{\mathbf{m}}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{jS}\right) \in \underset{\mathbf{m}^k_{-j}\in M^k_{-j}}{\cup} g^k(\overline{L}^k_j(\mathbf{m}^k_{-j}), \mathbf{m}^k_{-j})$. Since

$$g^k\left(\widehat{\mathbf{m}}^k_{S^j}, m^k_j\left(\cdot; \widehat{\mathbf{m}}^k_{S^j}, \widetilde{\mathbf{m}}^k_{jS}\right), \widetilde{\mathbf{m}}^k_{jS}\right) = g^k\left(\widehat{\mathbf{m}}^k_{S^j}, \widehat{m}^k_j, \overline{\mathbf{m}}^k_{jS}\right)$$

holds by (d), we may conclude that, for each $\widetilde{\mathbf{m}}^k_{jS} \in M^k_{jS}$ where $k = 1, 2$,

$$g^k\left(\widehat{\mathbf{m}}^k_{S^j}, m^k_j\left(\cdot; \widehat{\mathbf{m}}^k_{S^j}, \widetilde{\mathbf{m}}^k_{jS}\right), \widetilde{\mathbf{m}}^k_{jS}\right) \in \underset{i\in S^j\cup\{j\}}{\cap} \left[\underset{\mathbf{m}^k_{-i}\in M^k_{-i}}{\cup} g^k(\overline{L}^k_i(\mathbf{m}^k_{-i}), \mathbf{m}^k_{-i})\right].$$

In summary, this argument implies for any $j \in N\backslash\{1\}$ that, if $\underset{i\in S^j}{\cap} \overline{B}^i \neq \varnothing$ holds, then $\left(\underset{i\in S^j}{\cap} \overline{B}^i\right) \cap \overline{B}^j \neq \varnothing$ also holds, which implies $\underset{i\in N}{\cap} B^i \neq \varnothing$, the desired result.

By the above argument, the effectivity function E^γ satisfies (*), so that E^γ has a Nash-solvable representation $\gamma^* = (M^*, g^*)$. Our remaining task is to show that γ^* is minimally libertarian. Note that for any $i \in N$ and any $x^1_i \in A^1_i$, there is a strategy function $\widehat{m}^1_i \in M^1_i$ such that, for any $\mathbf{m}^1_{S^i} \in M^1_{S^i}$, $\widehat{m}^1_i(\mathbf{m}^1_{S^i}) = x^1_i$, which follows from the universality of M^1_i. Thus, for any $i \in N$, any $x^1_i \in A^1_i$, and any $m^2_i \in M^2_i$, $g(\widehat{m}^1_i, m^2_i, M_{-i}) = \left(g^1(\widehat{m}^1_i, M^1_{-i}), g^2(m^2_i, M^2_{-i})\right) \subseteq \{x^1_i\} \times A^1_{-i} \times g^2(m^2_i, M^2_{-i})$ belongs to $E^\gamma(i)$, so that $\{x^1_i\} \times A^1_{-i} \times g^2(m^2_i, M^2_{-i}) \in E^{\gamma^*}(i)$ for each $i \in N$, each $x^1_i \in A^1_i$, and each $m^2_i \in M^2_i$. This implies that, for each $i \in N$, M^*_i can be decomposed as $M^*_i = M^{*1}_i \times M^{*2}_i$, and for each $x^1_i \in A^1_i$ and each $g^2(m^2_i, M^2_{-i}) \subseteq A^2$, there exist $\overline{m}^{*1}_i \in M^{*1}_i$ and $\overline{m}^{*2}_i \in M^{*2}_i$ such that $g^*(\overline{m}^{*1}_i, \overline{m}^{*2}_i, M^*_{-i}) \subseteq \{x^1_i\} \times A^1_{-i} \times g^2(m^2_i, M^2_{-i})$. In other words, for each $i \in N$ and each $x^1_i \in A^1_i$, there exists $\overline{m}^{*1}_i \in M^{*1}_i$ such that, for any $m^{*2}_i \in M^{*2}_i$ and any $\mathbf{m}^*_{-i} \in M^*_{-i}$, $g^*(\overline{m}^{*1}_i, m^{*2}_i, \mathbf{m}^*_{-i}) \in \{x^1_i\} \times A^1_{-i} \times A^2$ holds. Thus, γ^* is minimally libertarian. ■

Proof of Theorem 1: (*Only if* part): Suppose that $\#N^{ML}(\mathbf{Q}; \mathbf{R}) = 0$ for some $\mathbf{R} \in \mathcal{R}^n$ and some $\mathbf{Q} \in \mathcal{Q}^n$. This implies that $\underset{i\in N'}{\cup} P(Q_i(\mathbf{R})) \not\supseteq \Xi^{ML}(\mathbf{R})$ holds for any subset N' of N. Thus, there exists at least one pair $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \Xi^{ML}(\mathbf{R})$ such that $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \notin \underset{i\in N}{\cup} P(Q_i(\mathbf{R}))$. $Q_i(\mathbf{R})$ being complete, we then obtain $((\mathbf{x}', \theta'), (\mathbf{x}, \theta)) \in \underset{i\in N}{\cap} Q_i(\mathbf{R})$. Thus $((\mathbf{x}', \theta'), (\mathbf{x}, \theta)) \in P(Q(\mathbf{R})) \cup I(Q(\mathbf{R}))$ holds, where $Q = \Psi(\mathbf{Q})$, for any

ECF Ψ satisfying **SP** and **PI**, which means that any **ECF** satisfying **SP** and **PI** cannot associate a minimally libertarian ordering function to this profile $\mathbf{Q}$.

(*If* part): Given $\mathbf{Q} \in \mathcal{Q}^n$, let $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$. Given $\mathbf{R} \in \mathcal{R}^n$, let $Q^{PML}(\mathbf{R}) \equiv Q^N(\mathbf{R}) \cup \Xi^{ML}(\mathbf{R})$, where $Q^N(\mathbf{R}) \equiv \underset{i\in N}{\cap} Q_i(\mathbf{R})$. Then $I\left(Q^N(\mathbf{R})\right) = \underset{i\in N}{\cap} I\left(Q_i(\mathbf{R})\right)$ and $P\left(Q^N(\mathbf{R})\right) = \left(\underset{i\in N}{\cap} Q_i(\mathbf{R})\right) \setminus \left(\underset{i\in N}{\cap} I\left(Q_i(\mathbf{R})\right)\right)$. We show that $Q^{PML}(\mathbf{R})$ is a *consistent* binary relation in the sense of Suzumura (1983, Chapter 1),[26] whenever $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$. Suppose that $Q^{PML}(\mathbf{R})$ is not consistent. Then, there exists an incoherent cycle

$$\Lambda \equiv \{((\mathbf{x}(1), \theta(1)), (\mathbf{x}(2), \theta(2))), \cdots, ((\mathbf{x}(t), \theta(t)), (\mathbf{x}(1), \theta(1)))\} \subseteq Q^{PML}(\mathbf{R})$$

of some order t, where $2 \leq t < +\infty$. It is clear that $\Lambda \nsubseteq \Xi^{ML}(\mathbf{R})$ and $\Lambda \nsubseteq P\left(Q^N(\mathbf{R})\right)$, which is because $P\left(Q^N(\mathbf{R})\right)$ is transitive. This implies that $((\mathbf{x}(k), \theta(k)), (\mathbf{x}(k+1), \theta(k+1))) \in \Xi^{ML}(\mathbf{R})$ for some $k = 1, 2, \ldots, t$, which in turn implies that $\theta(k+1) \notin \Theta^L$.[27] Since $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$, it follows that, for each $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \Xi^{ML}(\mathbf{R})$, there exists at least one individual $i \in N$ such that $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P\left(Q_i(\mathbf{R})\right)$. Thus, since $\theta(k+1) \notin \Theta^L$, $((\mathbf{x}(k+1), \theta(k+1)), (\mathbf{x}(k+2), \theta(k+2))) \in Q^N(\mathbf{R})$ must hold, which implies that $\theta(k+2) \notin \Theta^L$. We may continue the same argument for $s = 3, \ldots, t$, and arrive at the conclusion that $\theta(k) \notin \Theta^L$, which is a desired contradiction. Thus, $Q^{PML}(\mathbf{R})$ is consistent.

Invoking Suzumura (1983, Theorem A(5)), for each $\mathbf{Q}$ and $\mathbf{R}$, $Q^{PML}(\mathbf{R})$ has an ordering extension, say $Q(\mathbf{R})$. Let us define an **ECF** Ψ_{ML} as follows: For all $\mathbf{Q} \in \mathcal{Q}^n$, and all $\pi \in \Pi$,

$$\Psi_{ML}(\mathbf{Q}) = \Psi_{ML}(\pi \circ \mathbf{Q}) = \begin{cases} Q \text{ if the condition } (\beta) \text{ is satisfied;} \\ Q^{EN} \text{ otherwise,} \end{cases}$$

where $Q^{EN}(\mathbf{R})$ is an ordering extension of $Q^N(\mathbf{R})$ for all $\mathbf{R} \in \mathbf{R}^n$. $Q^N(\mathbf{R})$ being transitive, hence consistent, $Q^{EN}(\mathbf{R})$ does exist. Since $Q_{ML}(\mathbf{R}) \supseteq$

[26] Let R be a binary relation on X. A finite subset $\{x(1), \cdots, x(t)\}$ of X, where $2 \leq t < +\infty$, satisfying $(x(1), x(2)) \in P(R)$, $(x(2), x(3)) \in R$, $\cdots$, $(x(t), x(1)) \in R$ is called an *incoherent cycle* of R of the order t. R is said to be *consistent* if there exists no incoherent cycle of any order. A binary relation R^* is called an *extension* of R if and only if $R \subseteq R^*$ and $P(R) \subseteq P(R^*)$. It is shown in Suzumura (1983, Theorem A(5)) that *there exists an ordering extension of* R *if and only if* R *is consistent.*

[27] If $k = t$, it should be understood that $\theta(t+1) = \theta(1)$.

$Q^{PML}(\mathbf{R})$, for each $\mathbf{Q}$ and $\mathbf{R}$, where $Q_{ML} = \Psi_{ML}(\mathbf{Q})$, Ψ_{ML} satisfies **SP** and **PI**. Moreover, by construction, we can conclude that Ψ_{ML} satisfies **AN**. It is easy to see that the condition (α) is satisfied whenever the condition (β) is satisfied. ■

Proof of Corollary 1: Note that, given $\mathbf{R} \in \mathcal{R}^n$ and $\mathbf{Q} \in \mathcal{Q}^n$, $N_*^{ML}(\mathbf{Q}; \mathbf{R}) \subseteq N^{ML}(\mathbf{Q}; \mathbf{R})$. Thus, by **Theorem 1**, we obtain the desired result. ■

Proof of Corollary 2: By the definition of $N_*^{ML}(\mathbf{Q})$, the result is obvious. ■

Proof of Theorem 2: Suppose an **ECF** Ψ satisfies **SP**, **PI**, and **AN**, but $\Psi \neq \Psi_{ML}$. Then, by **Thereom 1**, there is $\mathbf{Q} \in \mathcal{Q}^n$ such that $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$ holds for all $\mathbf{R} \in \mathcal{R}^n$, but $\Psi(\mathbf{Q}) \notin \mathcal{Q}^{\mathsf{ML}}$. This implies that, for some $\mathbf{R} \in \mathcal{R}^n$, there exists an ordered pair $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \Xi^{ML}(\mathbf{R})$ such that $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \notin P(Q(\mathbf{R}))$, where $Q = \Psi(\mathbf{Q})$. Since $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$, there exists an individual $i \in N$ such that $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in P(Q_i(\mathbf{R}))$. Thus, Ψ does not satisfy **RLJ**. The converse implication is obvious. ■

Proof of Theorem 3: By **Theorem 1**, $Q^{ML} = \Psi_{ML}(\mathbf{Q})$ is minimally libertarian for all $\mathbf{R} \in \mathcal{R}^n$ if and only if $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$ holds for all $\mathbf{R} \in \mathcal{R}^n$. Note that the rational choice function associated with the minimally libertarian ordering function satisfies the statement (α) if and only if $\Theta^L \neq \varnothing$. Since $\mathbf{\Gamma}_{NS}^L \neq \varnothing$, we can obtain the desired result. ■

Proof of Theorem 4: By virtue of **Theorem 3**, we have only to show that there is no social choice correspondence satisfying minimal libertarianism, viz. $\mathbf{\Sigma}^L = \varnothing$. Assume that $\sigma \in \mathbf{\Sigma}^L$. Since $A^1 = A_1^1 \times A_2^1 \times \cdots \times A_n^1$, the rights-system which σ assigns to A^1 must contain a *critical loop* in the sense of Suzumura (1978).

Consider a profile $\mathbf{R} \in \mathcal{R}^n$ which is defined as follows. For each $i \in \{1, 2\} \subseteq N$, divide A_i^1 into A_{i1}^1 and A_{i2}^1, where $A_{i1}^1 \cup A_{i2}^1 = A_i^1$ and $A_{i1}^1 \cap A_{i2}^1 = \varnothing$. Suppose that for any $\mathbf{x}, \mathbf{y} \in A$,

$$(\mathbf{x}, \mathbf{y}) \in P(R_1) \Leftrightarrow (x_1^1, x_2^1) \in A_{11}^1 \times A_{21}^1 \text{ and } (y_1^1, y_2^1) \in A_{12}^1 \times A_{21}^1,$$
$$(\mathbf{x}, \mathbf{y}) \in P(R_1) \Leftrightarrow (x_1^1, x_2^1) \in A_{12}^1 \times A_{22}^1 \text{ and } (y_1^1, y_2^1) \in A_{11}^1 \times A_{22}^1,$$
$$(\mathbf{x}, \mathbf{y}) \in I(R_1) \Leftrightarrow (x_1^1, x_2^1), (y_1^1, y_2^1) \in A_{1h}^1 \times A_{2k}^1 \text{ for all } h, k \in \{1, 2\},$$

and

$$(\mathbf{x}, \mathbf{y}) \in P(R_2) \Leftrightarrow (x_1^1, x_2^1) \in A_{11}^1 \times A_{22}^1 \text{ and } (y_1^1, y_2^1) \in A_{11}^1 \times A_{21}^1,$$

$$(\mathbf{x},\mathbf{y}) \in P(R_2) \Leftrightarrow (x_1^1, x_2^1) \in A_{12}^1 \times A_{21}^1 \text{ and } (y_1^1, y_2^1) \in A_{12}^1 \times A_{22}^1,$$
$$(\mathbf{x},\mathbf{y}) \in I(R_2) \Leftrightarrow (x_1^1, x_2^1), (y_1^1, y_2^1) \in A_{1h}^1 \times A_{2k}^1 \text{ for all } h, k \in \{1, 2\},$$

whereas $\mathbf{R}_{-\{1,2\}}$ is an arbitrary sub-profile of preferences for individuals in $N\backslash\{1, 2\}$.

For each $\mathbf{x}^2 \in A^2$ and each $(x_3^1, x_4^1, \cdots, x_n^1) \in A_3^1 \times A_4^1 \times \cdots \times A_n^1$, any social alternative $\mathbf{y} = (y_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2) \in A$ cannot be an element of $\sigma(\mathbf{R})$ whenever $(y_1^1, y_2^1) \in A_{12}^1 \times A_{21}^1$, because $\mathbf{y}$ is rejected by person 1 with some $\mathbf{x} = (x_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2)$, where $(x_1^1, y_2^1) \in A_{11}^1 \times A_{21}^1$. Likewise, any social alternative $\mathbf{y} = (y_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2) \in A$ cannot be an element of $\sigma(\mathbf{R})$ whenever $(y_1^1, y_2^1) \in A_{11}^1 \times A_{22}^1$, because $\mathbf{y}$ is rejected by person 1 with some $\mathbf{x} = (x_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2)$ where, $(x_1^1, y_2^1) \in A_{12}^1 \times A_{22}^1$. Furthermore, any social alternative $\mathbf{y} = (y_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2) \in A$ cannot be an element of $\sigma(\mathbf{R})$ whenever $(y_1^1, y_2^1) \in A_{11}^1 \times A_{21}^1$, because $\mathbf{y}$ is rejected by person 2 with some $\mathbf{x} = (y_1^1, x_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2)$, where $(y_1^1, x_2^1) \in A_{11}^1 \times A_{22}^1$. Finally, any social alternative $\mathbf{y} = (y_1^1, y_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2) \in A$ cannot be an element of $\sigma(\mathbf{R})$ whenever $(y_1^1, y_2^1) \in A_{12}^1 \times A_{22}^1$, because $\mathbf{y}$ is rejected by person 2 with some $\mathbf{x} = (y_1^1, x_2^1, x_3^1, x_4^1, \cdots, x_n^1; \mathbf{x}^2)$, where $(y_1^1, x_2^1) \in A_{12}^1 \times A_{21}^1$. Thus, there is no social alternative which is selected by σ under $\mathbf{R} \in \mathcal{R}^n$. This is a contradiction, because σ must be non-empty valued. ■

Proof of Theorem 5: (*If* part): To begin with, by Sen (1970, Lemma 1*1), $B_{J_Q}(\mathcal{D}_\Theta)$ is non-empty if and only if $J_Q \subseteq \mathcal{D}_\Theta \times \mathcal{D}_\Theta$ is complete and acyclic. Second, by virtue of the definition of $\mathcal{D}_\Theta$, if $\mathbf{D}^{\theta^*} \in B_{J_Q}(\mathcal{D}_\Theta)$, then for any $\mathbf{R} \in \mathcal{R}^n$, there is at least one $\mathbf{x}^* \in A$ such that $(\mathbf{x}^*, \theta^*) \in \mathcal{RP}(\mathbf{R})$. Let $((\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta)) \in Q(\mathbf{R})$ for all $(\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R})$. Thus, $(\mathbf{x}^*, \theta^*) \in B_Q(\mathbf{R})$, so that $\theta^* \in C_\Psi(\mathbf{Q}; \mathbf{R})$. This argument holds true for any other $\mathbf{R}' \in \mathcal{R}^n$, so that $\theta^* \in \bigcap_{\mathbf{R}' \in \mathcal{R}^n} C_\Psi(\mathbf{Q}; \mathbf{R}')$.

(*Only if* part): Suppose that, for any complete and acyclic binary relation $J_Q \subseteq \mathcal{D}_\Theta \times \mathcal{D}_\Theta$ and any $\theta^* \in \Theta$ with $\mathbf{D}^{\theta^*} \in B_{J_Q}(\mathcal{D}_\Theta)$, there exist some $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$ such that, for any $(\mathbf{x}^*, \theta^*) \in \mathcal{RP}(\mathbf{R})$, if $((\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta)) \in Q(\mathbf{R})$ for all $(\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R})$, then for any $(\mathbf{x}^{**}, \theta^*) \in \mathcal{RP}(\mathbf{R}')$, there is some $(\mathbf{x}', \theta') \in \mathcal{RP}(\mathbf{R}')$ such that $((\mathbf{x}', \theta'), (\mathbf{x}^{**}, \theta^*)) \in P(Q(\mathbf{R}'))$. This implies that $\theta^* \in C_\Psi(\mathbf{Q}; \mathbf{R})$ and $\theta^* \notin C_\Psi(\mathbf{Q}; \mathbf{R}')$. Thus, Q does not satisfy **URC**. ■

Proof of Corollary 3: Assume that $Q \in \mathcal{Q}^{\mathbf{ML}}$ satisfies **IER**. Then, there is a linear ordering $J_Q \in \mathcal{J}$ which is compatible with **IER**. By minimal libertarianism, for any $\theta^* \in \Theta^L$, any $\theta \in \Theta\backslash\Theta^L$, any $\mathbf{R} \in \mathcal{R}^n$, and any

$(\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R})$, we have $((\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta)) \in P(Q(\mathbf{R}))$. Let us define a new binary relation $J_Q^* \subseteq \mathcal{D}_\Theta \times \mathcal{D}_\Theta$ as follows:

$$J_Q^* \equiv [J_Q \cap (\mathcal{D}_{\Theta^L} \times \mathcal{D}_{\Theta^L})] \cup \left[J_Q \cap \left(\mathcal{D}_{\Theta \backslash \Theta^L} \times \mathcal{D}_{\Theta \backslash \Theta^L}\right)\right] \cup \left(\mathcal{D}_{\Theta^L} \times \mathcal{D}_{\Theta \backslash \Theta^L}\right),$$

where $\mathcal{D}_{\Theta^L}$ is the set of maximal θ-rights-systems when $\theta \in \Theta^L$, whereas $\mathcal{D}_{\Theta \backslash \Theta^L}$ is the set of maximal θ-rights-systems when $\theta \in \Theta \backslash \Theta^L$. By definition, J_Q^* is a quasi-ordering, and it is complete over $\mathcal{D}_\Theta$. Moreover, if $\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}\right), \left(\mathbf{D}^{\theta'}, \mathbf{D}^\theta\right) \in J_Q^*$, then $\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}\right) \notin \left(\mathcal{D}_{\Theta^L} \times \mathcal{D}_{\Theta \backslash \Theta^L}\right)$ and $\left(\mathbf{D}^{\theta'}, \mathbf{D}^\theta\right) \notin \left(\mathcal{D}_{\Theta^L} \times \mathcal{D}_{\Theta \backslash \Theta^L}\right)$. Thus, $\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}\right), \left(\mathbf{D}^{\theta'}, \mathbf{D}^\theta\right) \in J_Q$, which implies $\mathbf{D}^\theta = \mathbf{D}^{\theta'}$ by the antisymmetry of J_Q. This also implies J_Q^* is antisymmetric, so that it is a linear ordering. Note that if $\mathbf{D}^{\theta^*} \in B_{J_Q^*}(\mathcal{D}_\Theta)$, then $\theta^* \in \Theta^L$. By **IER**, we may observe that, whenever $\mathbf{D}^{\theta^*} \in B_{J_Q^*}(\mathcal{D}_\Theta)$, for any $\theta \in \Theta^L$, any $\mathbf{R} \in \mathcal{R}^n$, and any $(\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta) \in \mathcal{RP}(\mathbf{R})$, $((\mathbf{x}^*, \theta^*), (\mathbf{x}, \theta)) \in Q(\mathbf{R})$ must hold. Thus, noting the fact that $Q \in \mathcal{Q}^{\mathsf{ML}}$, we may observe that Q satisfies the necessary and sufficient condition for **URC** by virtue of **Theorem 4**. ■

Proof of Theorem 6: Take any $i \in N$ whose domain of ordering functions $\mathcal{Q}_i$ is restricted to $\mathcal{Q}^{\mathsf{IE}}$. For any $j \neq i$, let j's domain be given by $\mathcal{Q}$. Furthermore, let $\overline{\mathcal{Q}}^n \subseteq \mathcal{Q}^n$ be the set of profiles of ordering functions such that, for any $\mathbf{Q} \in \overline{\mathcal{Q}}^n$, $\#N^{ML}(\mathbf{Q}; \mathbf{R}) \geq 1$ for all $\mathbf{R} \in \mathcal{R}^n$. Then, let

$$r(\mathcal{Q}^n) \equiv \overline{\mathcal{Q}}^n \cap \left(\underbrace{\mathcal{Q} \times \cdots \times \mathcal{Q}}_{i-1 \text{ times}} \times \mathcal{Q}^{\mathsf{IE}} \times \underbrace{\mathcal{Q} \times \cdots \times \mathcal{Q}}_{n-i \text{ times}} \right).$$

We show that, for this $r(\mathcal{Q}^n)$, Ψ_{ML} satisfies Independence on $r(\mathcal{Q}^n)$ together with choosing minimally libertarian ordering functions satisfying **IER**. What remains to be shown is that if $\bigcap_{\mathbf{Q} \in r(\mathcal{Q}^n)} N^{IE}(\mathbf{Q}) = \varnothing$, then Ψ_{ML} does not always choose minimally libertarian ordering functions satisfying **IER** without violating Independence on $r(\mathcal{Q}^n)$.

(*If* part): Given $\mathbf{R} \in \mathcal{R}^n$, let

$$\begin{aligned} \mathcal{NCP}(\mathbf{R}) \equiv & \left[\left((A \times \Theta^L) \cap \mathcal{RP}(\mathbf{R})\right) \times \left((A \times \Theta^L) \cap \mathcal{RP}(\mathbf{R})\right)\right] \\ & \cup \left[\left((A \times (\Theta \setminus \Theta^L)) \cap \mathcal{RP}(\mathbf{R})\right) \times \left((A \times (\Theta \setminus \Theta^L)) \cap \mathcal{RP}(\mathbf{R})\right)\right]. \end{aligned}$$

Given $\mathbf{R} \in \mathcal{R}^n$, for all $\mathbf{Q} \in r(\mathcal{Q}^n)$, and all $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \mathcal{NCP}(\mathbf{R})$, define Ψ_{ML} as follows: (i) if $\left[\mathbf{D}^\theta \neq \mathbf{D}^{\theta'} \text{ or } \mathbf{x} = \mathbf{x}'\right]$, then

$$((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q_{ML}(\mathbf{R}) \Leftrightarrow ((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in Q_i(\mathbf{R}),$$

and (ii) if $\left[\mathbf{D}^{\theta}=\mathbf{D}^{\theta'} \text{ and } \mathbf{x}\neq\mathbf{x}'\right]$, then there is a unique individual $j(\mathbf{D}^{\theta})\neq i$ associated with $\mathbf{D}^{\theta}$ such that:

$$\begin{cases} ((\mathbf{x},\theta),(\mathbf{x}',\theta'))\in P\left(Q_{ML}(\mathbf{R})\right), & \text{if } ((\mathbf{x},\theta),(\mathbf{x}',\theta'))\in P\left(Q^{N}(\mathbf{R})\right); \\ ((\mathbf{x},\theta),(\mathbf{x}',\theta'))\in Q_{ML}(\mathbf{R}) \Leftrightarrow ((\mathbf{x},\theta),(\mathbf{x}',\theta'))\in Q_{j(\mathbf{D}^{\theta})}, & \text{otherwise,} \end{cases}$$

where $Q_{ML}=\Psi_{ML}(\mathbf{Q})$. By construction, $Q_{ML}(\mathbf{R})\cap\mathcal{NCP}(\mathbf{R})$ is a lexicographic order over $\mathcal{NCP}(\mathbf{R})$, which is compatible with an **IER** ordering $Q^{IE}(\mathbf{R})$, since $Q_i\in\mathcal{Q}^{\mathbf{IE}}$. Moreover, it is compatible with **SP**, **PI**, **ND**, and $\mathbf{I}_{r(\mathcal{Q}^n)}$ over $\mathcal{NCP}(\mathbf{R})$. Next, given $\mathbf{R}\in\mathcal{R}^n$, for each $\mathbf{Q}\in r(\mathcal{Q}^n)$, $Q_{ML}(\mathbf{R})\cap\Xi^{ML}(\mathbf{R})\equiv\Xi^{ML}(\mathbf{R})$. Thus,

$$Q_{ML}(\mathbf{R})\cap\left[\Xi^{ML}(\mathbf{R})\cup\mathcal{NCP}(\mathbf{R})\right]=\Xi^{ML}(\mathbf{R})\cup\left[Q^{NC}(\mathbf{R})\cap\mathcal{NCP}(\mathbf{R})\right].$$

Finally, for all $\mathbf{Q}\in\mathcal{Q}^n\backslash r(\mathcal{Q}^n)$, all $\mathbf{R}\in\mathcal{R}^n$, and all $\pi\in\Pi$, $Q_{ML}(\mathbf{R})=Q^{\pi}_{ML}(\mathbf{R})=Q^{EN}(\mathbf{R})$, where $Q^{\pi}_{ML}=\Psi_{ML}(\pi\circ\mathbf{Q})$ and $Q^{EN}(\mathbf{R})$ is an ordering extension of the Pareto quasi-ordering $Q^{N}(\mathbf{R})$.

Thus, Ψ_{ML} is well-defined, and it satisfies **SP**, **PI**, **ND**, and $\mathbf{I}_{r(\mathcal{Q}^n)}$. Moreover, $\Psi_{ML}(\mathbf{Q})$ is a minimally libertarian ordering function satisfying **IER** for all $\mathbf{Q}\in r(\mathcal{Q}^n)$. Thus, it is enough to show that $Q_{ML}(\mathbf{R})$, where $Q_{ML}=\Psi_{ML}(\mathbf{Q})$, is an ordering for any $\mathbf{Q}\in r(\mathcal{Q}^n)$. It is easy to check that $Q_{ML}(\mathbf{R})$ is a consistent relation. Moreover, it is complete over $\mathcal{RP}(\mathbf{R})$ by definition. Thus, by Suzumura (1983, Theorem A(5)), $Q_{ML}(\mathbf{R})$ must be the ordering extension of itself.

(*Only if* part): Next, let us take a sufficiently rich domain $r'(\mathcal{Q}^n)$ such that for any $\mathbf{Q}\in r'(\mathcal{Q}^n)$, $\#N^{ML}(\mathbf{Q};\mathbf{R})\geq 1$ for all $\mathbf{R}\in\mathcal{R}^n$, and $\#N^{IE}(\mathbf{Q})\geq 1$. However, $\bigcap_{\mathbf{Q}\in r'(\mathcal{Q}^n)} N^{IE}(\mathbf{Q})=\varnothing$. This implies that, for all $j\in N$, there exists $\mathbf{Q}\in r'(\mathcal{Q}^n)$ such that $j\notin N^{IE}(\mathbf{Q})$, so that for the particular individual $i\in N$, we see $i\notin N^{IE}(\mathbf{Q})$ for some $\mathbf{Q}\in r'(\mathcal{Q}^n)$. Then, it follows that $r'(\mathcal{Q}^n)\supsetneq\left(\mathcal{Q}^{\mathbf{ML}}\cap\mathcal{Q}^{\mathbf{IE}}\right)^n$.

Given $\mathbf{R}\in\mathcal{R}^n$, let $\widehat{\mathcal{Q}}^{\mathbf{IE}}(\mathbf{R})\equiv\{\widehat{Q}(\mathbf{R})=Q(\mathbf{R})\cap\mathcal{NCP}(\mathbf{R})\mid Q\in\mathcal{Q}^{\mathbf{IE}}\}$ and $\widehat{\mathcal{Q}}^{\mathbf{MLIE}}(\mathbf{R})\equiv\{\widehat{Q}(\mathbf{R})=Q(\mathbf{R})\cap\mathcal{NCP}(\mathbf{R})\mid Q\in\mathcal{Q}^{\mathbf{ML}}\cap\mathcal{Q}^{\mathbf{IE}}\}$. Furthermore, let $\widehat{\mathcal{Q}}^{\mathbf{IE}}\equiv\bigcup_{\mathbf{R}\in\mathcal{R}^n}\widehat{\mathcal{Q}}^{\mathbf{IE}}(\mathbf{R})$ and $\widehat{\mathcal{Q}}^{\mathbf{MLIE}}\equiv\bigcup_{\mathbf{R}\in\mathcal{R}^n}\widehat{\mathcal{Q}}^{\mathbf{MLIE}}(\mathbf{R})$. Let us show that $\widehat{\mathcal{Q}}^{\mathbf{IE}}=\widehat{\mathcal{Q}}^{\mathbf{MLIE}}$. First, for each $\mathbf{R}\in\mathcal{R}^n$, $\widehat{\mathcal{Q}}^{\mathbf{IE}}(\mathbf{R})\supseteq\widehat{\mathcal{Q}}^{\mathbf{MLIE}}(\mathbf{R})$ by definition. Second, let us take a $Q\in\mathcal{Q}^{\mathbf{IE}}\backslash\mathcal{Q}^{\mathbf{ML}}$. Then, there is a linear ordering $J_Q\in\mathcal{J}$ which is compatible with Q with respect to **IER**. Define a new binary relation $J^*_Q\subseteq\mathcal{D}_\Theta\times\mathcal{D}_\Theta$ as follows:

$$J^*_Q\equiv\left[J_Q\cap(\mathcal{D}_{\Theta^L}\times\mathcal{D}_{\Theta^L})\right]\cup\left[J_Q\cap\left(\mathcal{D}_{\Theta\backslash\Theta^L}\times\mathcal{D}_{\Theta\backslash\Theta^L}\right)\right]\cup\left(\mathcal{D}_{\Theta^L}\times\mathcal{D}_{\Theta\backslash\Theta^L}\right).$$

As in the proof of **Corollary 3**, we may prove that $J_Q^* \in \mathcal{J}$. Then, define an ordering function Q^* as follows: For each $\mathbf{R} \in \mathcal{R}^n$,

$$Q^*(\mathbf{R}) \equiv \Xi^{ML}(\mathbf{R}) \cup [Q(\mathbf{R}) \cap \mathcal{NCP}(\mathbf{R})].$$

Note that $Q^*(\mathbf{R})$ is an ordering over $\mathcal{RP}(\mathbf{R})$. Moreover, $Q^* \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$, since $J_Q^* \in \mathcal{J}$ is compatible with Q^* with respect to **IER**, and $Q^*(\mathbf{R}) \supseteq \Xi^{ML}(\mathbf{R})$ for any $\mathbf{R} \in \mathcal{R}^n$. By definition, $Q(\mathbf{R}) \cap \mathcal{NCP}(\mathbf{R}) = Q^*(\mathbf{R}) \cap \mathcal{NCP}(\mathbf{R})$ for any $\mathbf{R} \in \mathcal{R}^n$. This implies that, for each $\mathbf{R} \in \mathcal{R}^n$, $\widehat{\mathcal{Q}}^{\mathsf{IE}}(\mathbf{R}) = \widehat{\mathcal{Q}}^{\mathsf{MLIE}}(\mathbf{R})$ holds, as was to be verified.

The last property implies that, for any triple $\{\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\}$, any linear ordering over $\{\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\}$ is a subrelation of some $J_Q \in \mathcal{J}$, where J_Q is compatible with some $Q \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$ with respect to **IER**. Consider the case where $\{\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\} \subseteq \mathcal{D}_{\Theta^L}$ (resp. $\subseteq \mathcal{D}_{\Theta \backslash \Theta^L}$). Then, for any linear ordering $J\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\right)$ over this triple, any linear ordering extension $J_Q \in \mathcal{J}$ such that $J_Q \supseteq J\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\right)$ is compatible with some $Q \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$ with respect to **IER**. This property follows from $\widehat{\mathcal{Q}}^{\mathsf{IE}} = \widehat{\mathcal{Q}}^{\mathsf{MLIE}}$. Second, consider the case where $\{\mathbf{D}^\theta, \mathbf{D}^{\theta'}\} \subseteq \mathcal{D}_{\Theta^L}$ (resp. $\subseteq \mathcal{D}_{\Theta \backslash \Theta^L}$) and $\mathbf{D}^{\theta''} \in \mathcal{D}_{\Theta \backslash \Theta^L}$ (resp. $\in \mathcal{D}_{\Theta^L}$). Then, for any linear ordering $J\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\right)$ over this triple, there is a linear ordering extension $J_Q \in \mathcal{J}$ such that $J_Q \supseteq J\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\right)$, where J_Q is compatible with some $Q \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$ in terms of **IER**. Note that, for example, if

$$\left(\mathbf{D}^\theta, \mathbf{D}^{\theta''}\right), \left(\mathbf{D}^{\theta''}, \mathbf{D}^{\theta'}\right), \left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}\right) \in J\left(\mathbf{D}^\theta, \mathbf{D}^{\theta'}, \mathbf{D}^{\theta''}\right),$$

then for any $\mathbf{D} \in \mathcal{D}_{\Theta \backslash \Theta^L}$ (resp. $\in \mathcal{D}_{\Theta^L}$), and any $\mathbf{D}' \in \mathcal{D}_{\Theta^L} \backslash \{\mathbf{D}^\theta, \mathbf{D}^{\theta'}\}$ (resp. $\in \mathcal{D}_{\Theta \backslash \Theta^L} \backslash \{\mathbf{D}^\theta, \mathbf{D}^{\theta'}\}$)

$$\left(\mathbf{D}^\theta, \mathbf{D}\right), \left(\mathbf{D}, \mathbf{D}^{\theta'}\right) \in J_Q \; \& \; \left[\text{either } \left(\mathbf{D}', \mathbf{D}^\theta\right) \in J_Q \text{ or } \left(\mathbf{D}^{\theta'}, \mathbf{D}'\right) \in J_Q\right].$$

In this way, J_Q is compatible with some $Q \in \mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}$ in terms of **IER** by using $\widehat{\mathcal{Q}}^{\mathsf{NC}} = \widehat{\mathcal{Q}}^{\mathsf{MLNC}}$.

Thus, $\left(\mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}\right)^n$ implies that the society can utilize the *free triple property* regarding the admissible domain of $\mathbf{J_Q} \equiv (J_{Q_i})_{i \in N}$ derived from $(Q_i)_{i \in N} \in \left(\mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}\right)^n$. Therefore, by the Arrovian impossibility theorem [Arrow (1963)], there exists a (*local*) dictator $d \in N$ over $\mathcal{D}_\Theta$.

Given $\mathbf{Q} \in \left(\mathcal{Q}^{\mathsf{ML}} \cap \mathcal{Q}^{\mathsf{IE}}\right)^n$, take $((\mathbf{x}, \theta), (\mathbf{x}', \theta')) \in \mathcal{NCP}(\mathbf{R})$ with $\mathbf{D}^\theta \neq \mathbf{D}^{\theta'}$ such that

$$((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q_d(\mathbf{R})) \Leftrightarrow (\mathbf{D}^{\theta},\mathbf{D}^{\theta'}) \in P(J_{Q_d})$$
$$((\mathbf{x}',\theta'),(\mathbf{x},\theta)) \in P(Q_k(\mathbf{R})) \Leftrightarrow (\mathbf{D}^{\theta'},\mathbf{D}^{\theta}) \in P(J_{Q_k}) \ (\forall k \neq d).$$

Since d is the dictator over $\mathcal{D}_\Theta$, we must have $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q_{ML}(\mathbf{R}))$, where $Q_{ML} = \Psi_{ML}(\mathbf{Q})$. Take $\{\mathbf{y},\mathbf{y}'\} \neq \{\mathbf{x},\mathbf{x}'\}$ with $((\mathbf{y},\theta),(\mathbf{y}',\theta')) \in \mathcal{NCP}(\mathbf{R})$. Take $\mathbf{Q}' \in r'(\mathcal{Q}^n)$ such that, for all $k \neq d$, $Q'_k = Q_k$, while $Q'_d \notin \mathcal{Q}^{\mathbf{IE}}$ such that $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q'_d(\mathbf{R}))$ and $((\mathbf{y}',\theta'),(\mathbf{y},\theta)) \in Q'_d(\mathbf{R})$. By the sufficient richness of $r'(\mathcal{Q}^n)$, we can find such a profile in this domain. Note that, by the definition of **IER**, $((\mathbf{y},\theta),(\mathbf{y}',\theta')) \in P(Q_d(\mathbf{R}))$ and $((\mathbf{y}',\theta'),(\mathbf{y},\theta)) \in P(Q_k(\mathbf{R}))$ must hold for all $k \neq d$. Then, by $\mathbf{I}_{r(\mathcal{Q}^n)}$, it must be true that $((\mathbf{x},\theta),(\mathbf{x}',\theta')) \in P(Q'_{ML}(\mathbf{R}))$, while by **SP**, $((\mathbf{y}',\theta'),(\mathbf{y},\theta)) \in P(Q'_{ML}(\mathbf{R}))$, where $Q'_{ML} = \Psi_{ML}(\mathbf{Q}')$. Thus, $\Psi_{ML}(\mathbf{Q}')$ does not satisfy **IER**. ■

Proof of Theorem 7: By the definitions of $\mathbf{\Gamma}_{NS}$ and $\mathcal{D}_\Theta$, for each $\mathbf{D}^\theta \in \mathcal{D}_\Theta$, there is at least one $\sigma \in \mathbf{\Sigma}$ such that $\mathbf{D}^\sigma = \mathbf{D}^\theta$, or there is at least one Nash-solvable game form $\gamma \in \mathbf{\Gamma}_{NS}$ such that $\mathbf{D}^\gamma = \mathbf{D}^\theta$. Let $\mathbf{D}_{J_Q} \in B_{J_Q}(\mathcal{D}_\Theta)$, where $Q = \Psi_{ML}(\mathbf{Q})$. Let θ^* be a decision-making rule satisfying $\mathbf{D}^{\theta^*} = \mathbf{D}_{J_Q}$. By **Theorem 4**, without loss of generality, $\theta^* \in \mathbf{\Gamma}^L_{NS}$. By **Theorem 5**, **Theorem 6** and **Corollary 3**, we can obtain $\theta^* \in C_{\Psi_{ML}(\mathbf{Q})}(\mathbf{R})$ for all $\mathbf{R} \in \mathcal{R}^n$, whenever $\mathbf{Q} \in r(\mathcal{Q}^n)$, where $r(\mathcal{Q}^n)$ is defined in the proof of **Theorem 6**. ■

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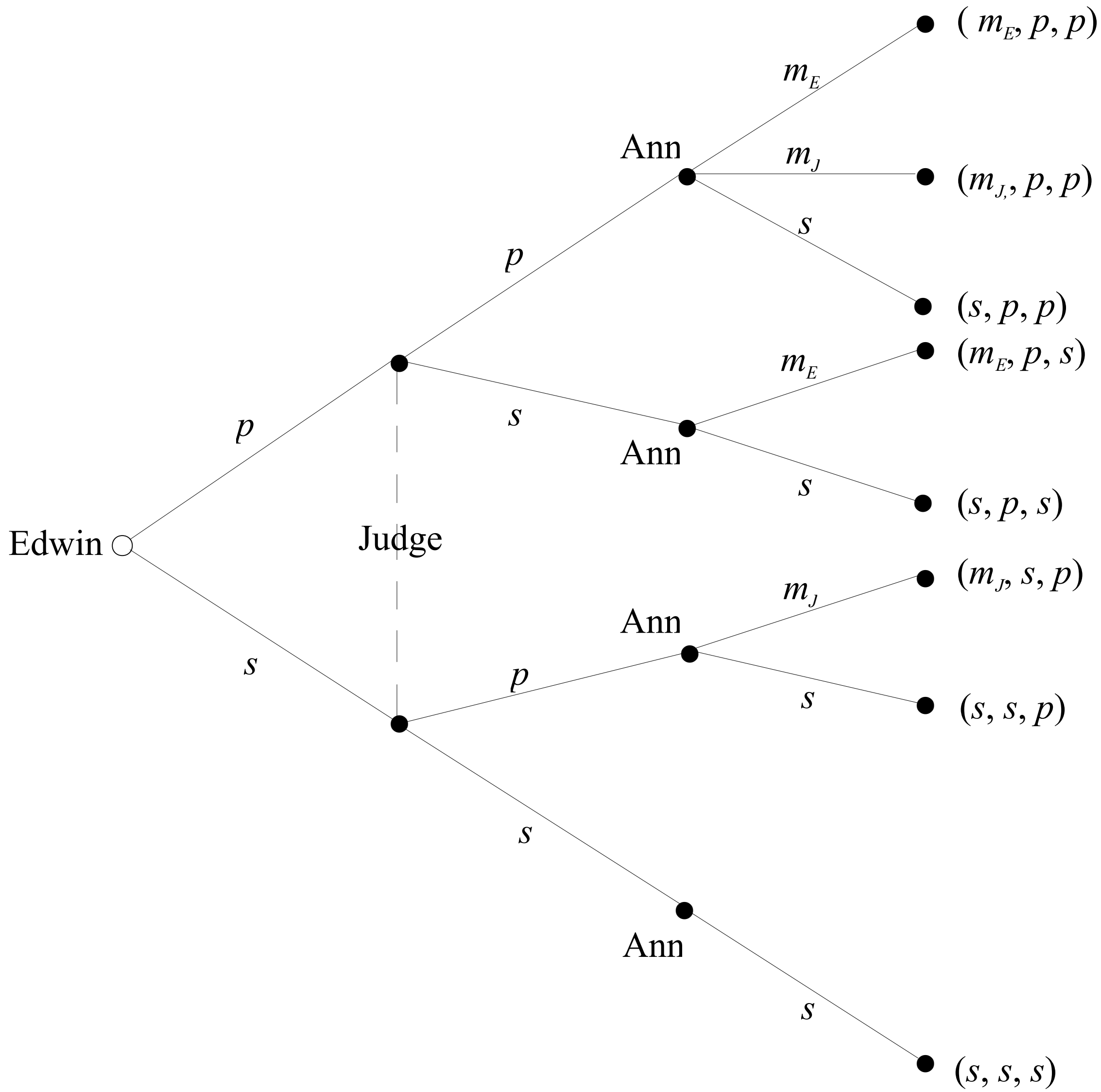

Figure 1 : Game Tree of Example 1

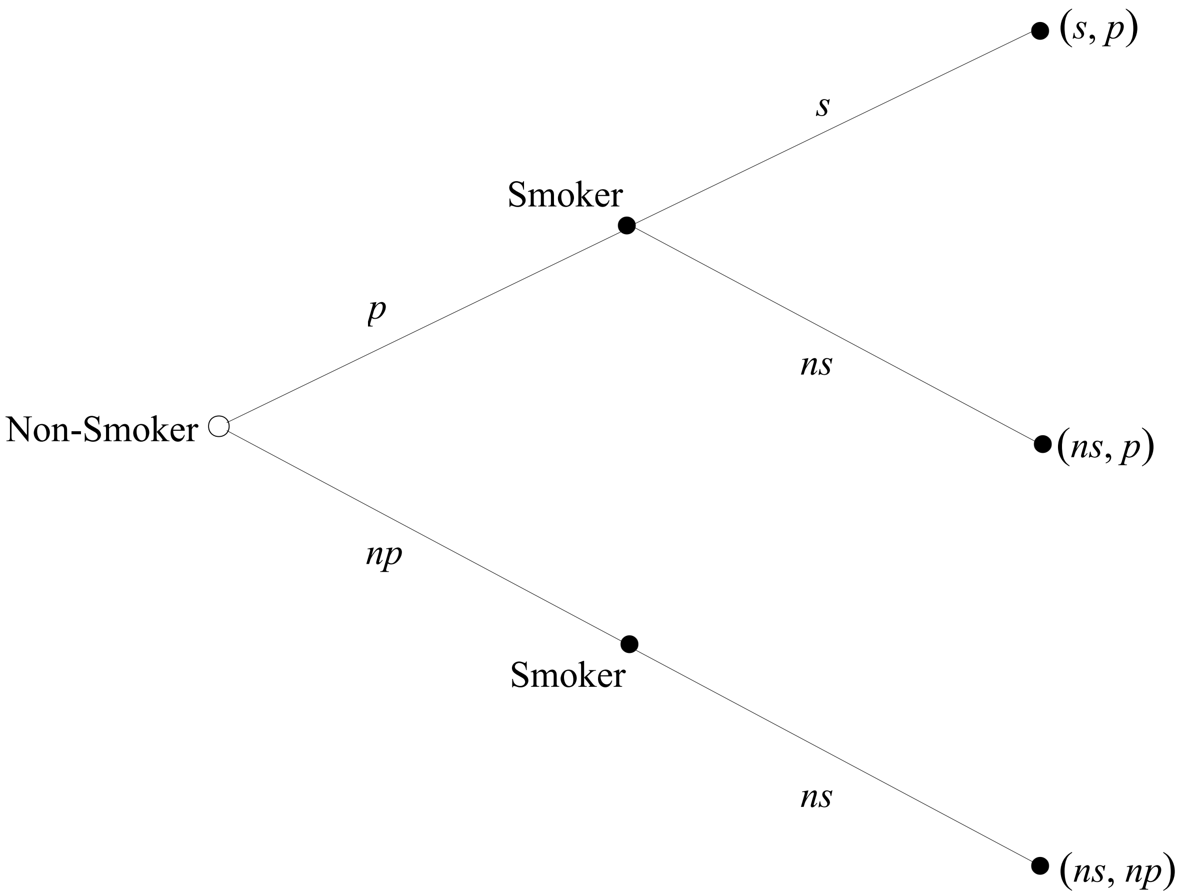

Figure 2 : Game Tree of Example 4

Player 2

		t	t'
Player 1	s	x	y
	s'	x	z

Figure 3 : Outcome Function of Example 5

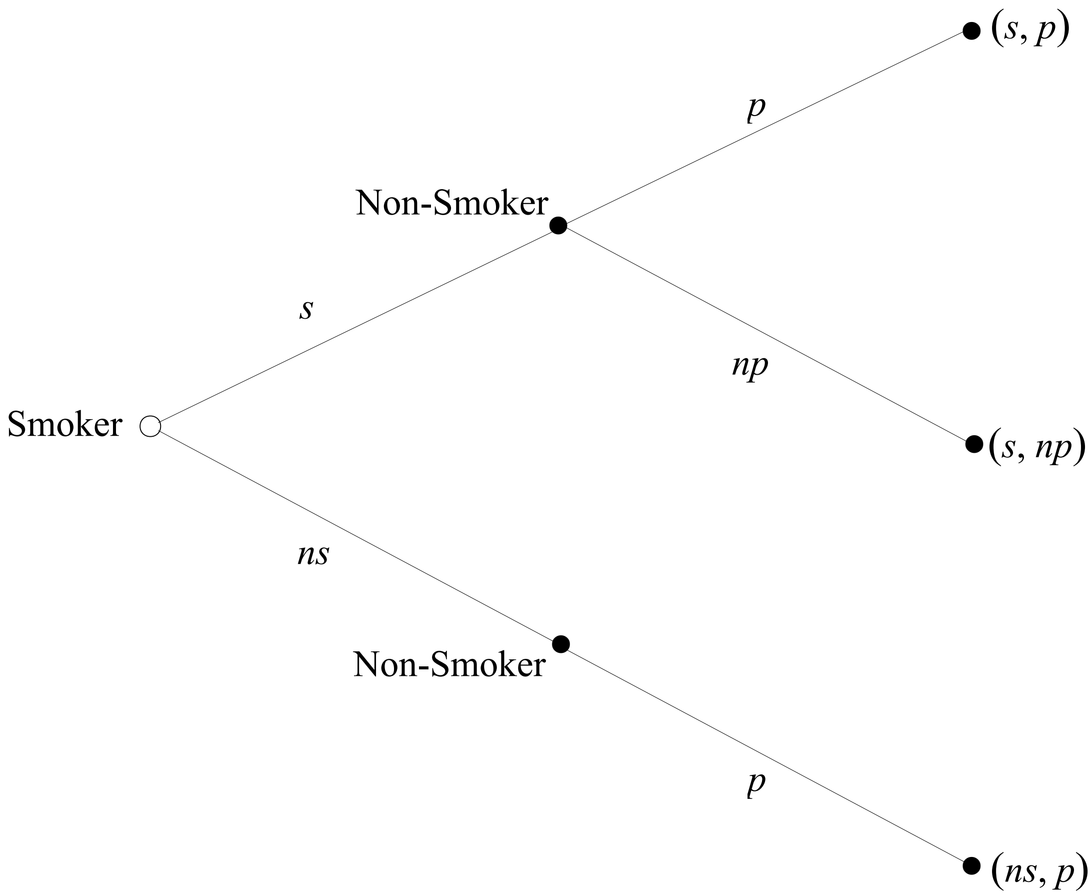

Figure 4 : Game Tree of Example 6