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Subsidizing or Limiting the Number of Charities

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Subsidizing or Limiting the Number of Charities

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Abstract

We consider a model where creating a charity costs a fixed amount and individual contributions depend on how close donors feel with respect to the charity. In that setting we show that there are an optimal number of charities and an optimal rate of subsidization that depend on the set-up cost and on the attachment of donors to charities that share the same values as theirs.

Keywords: charities, joy of giving

JEL classification: L31, H41

1 Introduction

In the winter 2004-2005, the Tsunami tragedy triggered an unprecedented spurt of generosity all over the world. Individuals contributed to this cause through a number of charities. In Belgium, e.g., they did that through charities associated with political or religious movements (Catholics, liberals, socialists, ...), and it has been shown that people were more willing to donate through charities close to their religious, political or other leanings.

In most countries, charitable funds benefit from tax breaks, but need to be licensed by public authorities. In other words, the government can control the number of charities, and at the same time grant tax exemptions to charitable contributions. Establishing a charitable fund involves fixed costs, and for that reason one does not imagine a world in which everyone would have his own charity. To put it differently, if there were no such costs

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involved to establishing a charity and if the government were in favor of the charitable cause in question, there would be no reason not to have a charity for each individual that would share the same values as his. However, because of fixed cost, their number needs to be limited. The question is how limited? And where are they to be located on the scale of individual values? There are the two questions addressed by this paper.

More concretely, we take a society consisting of a number of individuals all alike but for some specific religious, political, cultural values that are parametrized on a single dimension. Each individual has a utility function with three arguments: a composite consumption good, the amount of resources devoted to a charitable cause (e.g. the Tsunami relief) and his personal contribution. The utility for consumption and for the charitable cause is the same for all of them; the utility for the contribution itself depends on the distance between the individual's characteristic and the "nearest" charity. Take the example of Belgium and of the Tsunami relief. If there had been just one charity, e.g. *Caritas Catholica*, it is clear that the Belgian Catholics would have contributed more than the non Catholics. Within such a setting, we want to see the optimal number of charities and the amount of tax relief that a utilitarian social planner would choose. Individuals choose freely how much to contribute given the existing charities and the tax subsidy. Charities that have a licence locate so to maximize funds.

In the rest of this paper, we sketch the basic model and show that the optimal number of charities is a function of the set-up cost and of the joy of giving itself a function of the location of charities on the value scale. Then we introduce subsidies. If subsidies can be collected without distortion and income can be redistributed in a lump-sum way, the optimal number of charity is one. Introducing distortions, this number is likely to be larger than one and the subsidies larger than zero.

2 Basic model

2.1 Individual choice

We consider a society consisting of N individuals characterized by a system of values parametrized by a variable x_i that is uniformly distributed on a circle.

Each individual i has a quasi linear utility function:

$$U_i = c_i + v(Z) + g(s_i, \gamma_i)$$

where c_i is consumption, Z is the aggregate amount of contributions, s_i the individual contribution and γ_i a function to be defined below, which depends

on x_i and on the closest charity. The function $v(\cdot)$ is strictly concave and g is increasing in s and γ . The function g_i reflects the joy of giving and will take a particular specification:

$$g_i = \gamma_i s_i - \frac{1}{2} s_i^2.$$

As long as Z comes from individual contributions, we have

$$Z = \sum_i^N s_i.$$

Each individual will choose s_i given the following budget constraint.

$$c_i = (1 - \tau) y - f - (1 - \sigma) s_i$$

where τ is a flat tax rate, y , income, f , the cost of contribution that will be shown to decrease with the size of the charity, and σ is the subsidy rate, if any.

Each individual i makes a positive contribution s_i to the charitable cause acting non cooperatively, namely taking all the other contributions s_ℓ as given $\left(\sum_{\ell \neq i} s_\ell = Z_{-i} \right)$. In other words, he chooses:

$$s_i^* = \gamma_i - (1 - \sigma) + v'(s_i^* + Z_{-i}). \quad (1)$$

Reintroducing this value of s_i in the utility function, we obtain an indirect utility:

$$V_i(\tau, v, \gamma_i, Z_i) = (1 - \tau) y - f - (1 - \sigma) s_i^* + (\gamma_i s_i^* - (s_i^*)^2 / 2) + v(s_i^* + Z_{-i})$$

with

$$\frac{\partial V_i}{\partial \gamma_i} = s_i^* > 0.$$

Aggregating s_i^* we obtain the Nash equilibrium value of Z ,

$$Z^* = N(\bar{\gamma} - (1 - \sigma) + v'(Z^*))$$

where $\bar{\gamma} = \sum \gamma_i / N$ and $Z^* = Z(\bar{\gamma}, \sigma)$. The signe under an argument is the sign of the function with respect to that argument.

2.2 First-best

In this paper, we assume that the social planner is interested by the sum of individual utilities from which the joy of giving is excluded. Denoting consumption by c_i , a social planner that can control quantities directly would solve the following problem:

$$\text{Max} \quad \sum^N [c_i + v(Z)] - \lambda [\sum^N (c_i - y) + Z]$$

which leads to the Samuelson condition.

$$Nv'(Z) = 1.$$

The multiplier λ is associated with the resource constraint.

Note that another version of the first-best could impose that financing the public good has to go through charities. In that case, the social optimum is $M = 1$ and the social planner's problem is to maximize:

$$\sum^N [y - s_i + v(Z)] - \lambda [\sum^N (y - s) + F]$$

where F is the set up cost of creating a charity. We get the Samuelson condition with a loss of resources equal to F . This optimum can be decentralized in an economy with subsidized contribution financed by a lump-sum tax. The reason is simple: with a quasi-linear utility function, redistribution does not matter.

Instead of c_i we could introduce a strictly concave transformation $u(c_i)$. This would reflect some aversion to inequality. In that case, the objective function is:

$$\sum^N [u(c_i) + v(Z)],$$

and the optimal solution would be:

$$\begin{aligned} u'(c_i) &= 1 \\ Nv'(Z) &= 1 \end{aligned}$$

and

$$N \frac{v'(Z)}{u'(c)} = 1,$$

where $c_i = c$. In this case, decentralization requires individualized lump-sum taxes. Individuals whose x_i is close to the \hat{x} of the unique charity have to be compensated for a higher contribution than those who don't feel as close to \hat{x} .

3 Three stage game

We now turn to the problem of charities which have not been introduced explicitly. In this paper, the number of charities is determined by the government through some kind of licencing. In other words free entry is not allowed. In that respect, our approach is different from that used in spatial models of firms and in model of breacking up of nations. We consider a game in three stages:

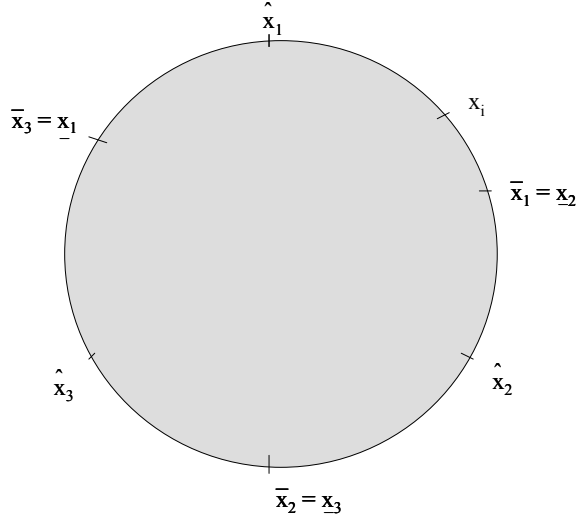
- (i) the government chooses the number of charities $M < N$,
- (ii) each charity $j (= 1, \dots, M)$ locates itself, namely chooses \hat{x}_j on a circle along which individuals values x_i are uniformly distributed,
- (iii) each individual i makes a contribution to the closest charity in terms of distance between his x_i and that of the charity \hat{x}_j .

As in a standard subgame perfect, we proceed backwards.

3.1 Choice of contribution

We consider the tax parameters τ, σ and the number of charities as given. Further, as it will be shown in the next subsection 3.2, charities are going to located equidistantly from one another on the circle, individual values being located uniformly on that circle. This is represented on Figure 1 where $M = 3$. If the perimeter of the circle is 1, each charity covers a range of potential contributors equal to $\frac{1}{3}$.

Figure 1
Individuals and charities



We can now define explicitly γ_{ij} , the index of joy of giving of an individual i to charity j . We write

$$\gamma_{ij} = a - b|x_i - \hat{x}_j|$$

and naturally an individual i will always contribute to j and not to j' if

$$|x_i - \hat{x}_j| < |x_i - \hat{x}_{j'}|.$$

Define $\bar{x}_j = \frac{\hat{x}_j + \hat{x}_{j+1}}{2}$ and $\underline{x}_j = \frac{\hat{x}_{j-1} + \hat{x}_j}{2}$ as the boundary of charity j and P_j as the number of potential contributors to j , we have:

$$P_j = (\bar{x}_j - \underline{x}_j) N = (\hat{x}_{j+1} - \hat{x}_{j-1}) \frac{N}{2}.$$

The optimal contributions to charity j denoted S_j are then:

$$S_j = P_j [\bar{\gamma}_j - (1 - \sigma) + v'(S_j + Z_{-j})]$$

where $\bar{\gamma}_j = \bar{\gamma} = a - \frac{b}{4M}$ is the mean value of γ_{ij} .

3.2 The location of M charities

We now see how the M charities, M being determined by the central planner, are to be located along the circle of values.

Following the spatial model of the circular city [see Tirole (1988)], we know that they are located equidistantly from one another and that they locate in the middle of the range of their potential contributors.

This intuitive result rests on the assumptions of our model: uniform distribution of x_i along the circle, given number M of charities, an objective function that amounts to maximize total contributions to each charity, individual contributions increasing with proximity. In particular, these assumptions imply that all charities have the same size.

Take charity j . Given Z_{-j} , \underline{x}_j and \bar{x}_j , it will choose \hat{x}_j such that it maximizes

$$S_j = P_j [\bar{\gamma}_j - (1 - \sigma) + v'(S_j + Z_{-j})]$$

where $\bar{\gamma}_j = \frac{N}{P_j} \int_{\underline{x}_j}^{\bar{x}_j} [a - b|\hat{x}_j - x_j|] dx_j$.

One can easily show that this yields the value \hat{x}_j that minimizes the aggregate distance between the $x_i \in (\underline{x}_j, \bar{x}_j)$ and \hat{x}_j , that is

$$\hat{x}_j = \frac{1}{2} [\underline{x}_j + \bar{x}_j]$$

and thus:

$$\bar{\gamma}_j = \gamma = a - \frac{b}{4M}.$$

3.3 The choice of M and σ

As observed in the previous section, with a quasi-linear utility function and non distortionary taxes, the optimal number of charities, M , is 1 and the first-best is achievable. This appears clearly from the following equation:

$$Z^* = N \left[a - \frac{b}{4M} - (1 - \sigma) + v'(Z^*) \right]. \quad (2)$$

Keeping Z^* constant, we obtain:

$$\frac{dM}{d\sigma} = \frac{b}{-4M^2} < 0.$$

In other words, σ and M are substitutable.

To depart from this trivial solution, we need either some aversion to inequality or some tax distortion. For the latter, we will use a quadratic deadweight loss parametrized by θ . This yields the revenue function:

$$N\tau \left(1 - \frac{\theta}{2}\tau \right) y = \sigma \sum s_i^* = \sigma Z^*. \quad (3)$$

As to the utility function, we adopt as an alternative to the quasi-linear function:

$$u(c_i) + v(Z).$$

As already mentioned F is the set up cost and $f = F/P$ is the share borne by each individual. One can also write $f = FM/N$.

We can now write the objective of the social planner

$$SW = \sum \{u[(1 - \tau)y - f - (1 - \sigma)s_i^*] + v(Z^*)\}. \quad (4)$$

Its problem is to maximize (4) subject to (3) and to the values of s_i^* and Z^* defined by (1) and (2).

We totally differentiate (4):

$$\begin{aligned} dSW &= v'(Z^*) dZ^* - (1 - \sigma) \sum \alpha_i ds_i^* - \alpha Ny d\tau \\ &\quad - \lambda F dM + [N \text{cov}(\alpha_i, s_i^*) + \alpha Z^*] d\sigma \end{aligned} \quad (5)$$

where $\alpha_i = u'(c_i)$ and $\alpha = \sum u'(c_i)/N$. Given the strict concavity of $u'(c_i)$, one gets $\text{cov}(\alpha_i, s_i^*) > 0$, as high contributors have a relatively low disposable income.

From (1), one writes:

$$ds_i^* = d\gamma_i + d\sigma + v''(Z^*) dZ^* \quad (6)$$

and from (3),

$$d\tau = \frac{Z^* d\sigma + \sigma dZ^*}{Ny(1 - \theta\tau)}. \quad (7)$$

Substituting (6) and (7) in (5), we obtain:

$$\begin{aligned} dSW &= (v' - \alpha) dZ^* - \alpha F dM + N \text{cov}(\alpha_i, s_i^*) d\sigma \\ &\quad + (1 - \sigma) N \text{cov}(\alpha_i, d\gamma_i^*) - \frac{\tau\theta}{1 - \theta} (Z^* d\sigma + \sigma dZ^*) \end{aligned} \quad (8)$$

where $\text{cov}(\alpha_i, d\gamma_i^*)$ cannot be easily signed. Note however one can easily show that $\text{cov}(\alpha_i, s_i) > 0$.

4 Optimal policy

We now use equation (8) to see the optimal policy under different assumptions.

4.1 No distortion ($\theta = 0$), no aversion to inequality. ($u(c) = c$)

This is the case considered in 2.2 where we were considering the decentralization of the first-best solution under the same conditions: lump-sum tax along with a subsidy on contributions and quasi-linear utility function.

We have the same solution. Using a tilde for the first-best, we have:

$$v'(\tilde{Z}) = 1$$

which defines \tilde{Z} . The value of $\tilde{\sigma}$ is given by:

$$\tilde{Z} = N \left[a + \frac{b}{4M} - 1 + \tilde{\sigma} + v'(\tilde{Z}) \right].$$

From there and from (5), we have that

$$d SW = -\alpha F dM < 0,$$

which implies that $M = 1$.

4.2 No aversion to inequality ($u'(c) = c$) and tax distortion ($\theta > 0$)

Assuming that $u''(c) = 0$ means that the covariance terms vanish from equation (8). To dispose of Z^* , we differentiate (2) so that:

$$(1 - Nv'') dZ^* = N d\sigma + \frac{bN}{4M^2} dM. \quad (9)$$

Substituting (9) in (8) and assuming interior solution (in particular $\frac{d SW}{dM} = 0$) we obtain:

$$\left[(v' - 1) - \frac{\tau\theta\sigma}{1 - \tau\theta} \right] \frac{N}{1 - Nv''} = \frac{\tau\theta}{1 - \tau\theta} Z^* \quad (10)$$

$$\left[(v' - 1) - \frac{\tau\theta\sigma}{1 - \tau\theta} \right] \frac{N}{1 - Nv''} \frac{b}{4M^2} = F. \quad (11)$$

To obtain the optimal values \hat{M} , $\hat{\sigma}$ and \hat{t} , one uses (10), (11), (2) and (3). We are here interested by the number of charities.

Combining these conditions (10) and (11) yields:

$$\hat{M} = \left[\frac{b}{4F} \frac{\hat{\tau}\theta}{1 - \hat{\tau}\theta} \hat{Z} \right]^{1/2}. \quad (12)$$

Equation (12) along with (2) and (3) indicates that the optimal number of charities is negatively related to F (which is intuitive) and is positively related to θ , τ and Z .

4.3 General case

Combining (8) and (9) and assuming again interior solutions, we have:

$$\frac{dSW}{d\sigma} = \left(v' - \alpha - \frac{\tau\theta\sigma}{1 - \tau\theta} \right) \frac{N}{1 - Nv''} - \frac{\tau\theta}{1 - \tau\theta} Z^* + N \text{cov}(\alpha_i, s_i^*) = 0 \quad (13)$$

$$\begin{aligned} \frac{dSW}{dM} &= \left(v' - \alpha - \frac{\tau\theta\sigma}{1 - \theta} \right) \frac{N}{1 - Nv''} \frac{b}{4M^2} \\ &\quad - \alpha F - (1 - \sigma) N \text{cov} \left(\alpha_i, \frac{d\gamma_i}{dM} \right) = 0 \end{aligned} \quad (14)$$

Formula (13) gives the optimal subsidy and (14) the optimal number of charities. Note that σ does not affect γ_i , which explain why one does not find $\text{cov}(\alpha_i, d\gamma_i)$ in (13).

From (13) and (14) we get:

$$M^1 = \frac{b^{1/2} \left[\frac{\tau\theta}{1 - \tau\theta} Z^* - N \text{cov}(\alpha_i, d\gamma_i) \right]^{1/2}}{[\alpha F + (1 - \sigma) N \text{cov}(\alpha_i, \frac{d\gamma_i}{dM})]^{1/2}}. \quad (15)$$

Clearly, when the covariance terms are nil we get formula (12). To interpret the role of the two covariance terms in this expression, it is important to see where they intervene in the social welfare maximization. The $\text{cov}(\alpha_i, s_i^*)$ is attached to an increase in σ ; clearly, when σ increases, the distribution of disposable income becomes more equal. As there is a trade-off between σ and M , it is not surprising to see that this covariance term pushes for less charities.

The $\text{cov} \left(\alpha_i, \frac{d\gamma_i}{dM} \right)$ is related to the formula of M . Assume it is positive. What does it mean? It means that there is a positive relation between individual consumption and the effect of an additional charity on the joy of giving. In other words adding one charity has a regressive effect and it thus not surprising that if positive this covariance implies less charities.

5 Conclusion

In this paper we dealt with the question of the optimal number of charities. This question is in the spirit of the related issue concerning the size of the nations [see Alesina and Spolaore, 1997]. The objective is to reach a certain level of aggregate contributions to some common causes. Contributions

are made through charities. If there were not set-up cost in establishing a new charity, there would be as many charities as individuals. With set-up costs, the number of charities has to be reduced. Without aversion towards inequality and without tax distortion, one charity suffices even though individuals who have values located far away from the charity would contribute little. There is always a subsidy that can induce them to contribute enough. With tax distortion, the case for a lower number gets stronger. With aversion towards inequality, the government wants to avoid too many disparities of disposable income (income net of tax and of contribution). This is achieved with relatively less charities.

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