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Risk premiums versus liquidity premiums: A simple numerical example

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Abstract. This paper presents a simple theoretical framework for differentiating the effect of the magnitude of uncertainty on ex-ante excess returns (risk premiums) from the effect of the subsequent resolution of uncertainty on ex-ante excess returns (liquidity premiums) in the presence of irreversible decisions, using an overlapping generations model. Employing Kreps-Porteus preferences, numerical examples demonstrate that liquidity premiums emerge with strong preferences for early resolution as well as large elasticities of intertemporal substitution. This numerical investigation may shed light on the fundamental advantage of Kreps-Porteus preferences in dynamic asset pricing models with time-varying investment opportunities.

JEL classification: D81, E21, G12.

Keywords: risk premium, liquidity premium, liquidity demand, Kreps-Porteus preferences, resolution of uncertainty.

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1. Introduction In the fields of macroeconomics and financial economics, the effect of risk-averse behavior on asset pricing has not always been clearly differentiated from the pricing impact driven by demand for liquidity or flexibility. In the case of money demand, for example, money is sometimes characterized as the safest asset in terms of risk-averse behavior, whereas at other times it is characterized as the most flexible asset from the viewpoint of liquidity demand. According to the former interpretation, money is priced highly relative to other risky assets by the magnitude of risk premiums. Given the latter interpretation, however, money is valuable due to its convenience or flexibility.

Citing Jones and Ostroy (1984), Hahn (1990) argues that risk premiums and liquidity premiums are fundamentally different from each other due to their theoretical natures. On the one hand, risk premiums are yielded as a consequence of aversion toward risky assets and a preference for safe assets. On the other hand, liquidity premiums are generated as a result of temporarily holding liquid assets while waiting for uncertainty to be resolved, in the presence of irreversible decisions. In other words, risk premiums are driven by the magnitude of risk involved in uncertain events, while liquidity premiums are caused by the subsequent resolution of uncertainty. In modern macroeconomics and finance, the generation of risk premiums has been explored theoretically in depth and examined empirically in detail. However, the tendency has been for the generation of liquidity premiums to be treated implicitly rather than explicitly.

This paper exploits a simple three-period setup employed by Epstein (1980) in order to systematically differentiate the effect of the subsequent resolution of uncertainty from the effect of the magnitude of underlying risks attached to uncertain events. Then, it embeds these models in an overlapping generations model, thereby numerically exploring general equilibrium implications, in particular liquidity demand and asset pricing.

The most important feature of our investigation is that the model adopts Kreps-Porteus preferences as an essential device to generate liquidity premiums. As Epstein (1980) clearly demonstrates, consumers with high elasticity of intertemporal substitution tend to refrain from committing to irreversible decisions, instead choosing to temporarily hold liquid assets when they expect uncertainty to be resolved subsequently. It immediately follows from this implication that liquidity demand has, at most, tiny impacts on premiums within an expected utility framework; risk aversion is inversely proportional to intertemporal substitution in such a framework, and premiums over risk-free rates themselves turn out to be negligible with near-risk neutral behavior.

As Miyazaki and Saito (2003) show, however, not only larger elasticity of intertemporal substitution, but also stronger preferences for early resolution help to yield liquidity demand. Under Kreps-Porteus preferences, risk aversion increases with preferences for early resolution, given intertemporal substitution, and liquidity demand thus has a chance to be reflected in larger excess returns over risk-free rates. Given the above setup, our numerical analysis demonstrates that liquidity premiums are indeed generated as a consequence of reduced commitment to current consumption and enhanced demand for liquid assets. As discussed in the conclusion, our numerical investigation may shed light on the fundamental advantage of Kreps-Porteus preferences in dynamic asset pricing models with time-varying investment opportunities.

This paper is organized as follows. Section 2 presents a simple theoretical framework where Epstein's (1980) three-period model is embedded in an overlapping generations setup. Section 3 numerically explores the asset pricing implications of this model. Section 4 discusses several theoretical and empirical implications of our numerical investigation.

2. A Theoretical Framework Let us consider a three-period overlapping generations economy (OLG economy). Each generation is referred to as young, middle-aged, or old. The population of each generation is constant over time and standardized to be one. A young consumer is endowed with $w_0(>0)$ units of goods, whereas a middle-aged consumer is endowed with $w_1(>0)$ units. There is no endowment for old consumers. In what follows, some kinds of heterogeneity with respect to either information sets or preference parameters may be considered among different generations, but heterogeneity is not at all applicable within any generation.

Each generation may have access to financial markets to allocate consumption goods over three periods as follows. A young consumer can lend or borrow in one-period risk-free assets, whereas a middle-aged consumer can invest in one-period risk-free assets and also in one-period risky assets. A middle-aged consumer is allowed to hold short positions in risk-free assets, but not in risky assets. The consumers transact in the financial markets in a competitive manner once they participate. One-period returns on risky assets R_t^x are given exogenously, whereas one-period risk-free rates R_t^f are determined endogenously as a result of transactions between young and middle-aged consumers.

A young consumer will obtain a message concerning future opportunities for risky investment when he is middle-aged. A young consumer consequently faces two alternatives: (i) consuming currently, or (ii) saving via risk-free assets. The motivation for the latter alternative is to wait for uncertainty of risky investments to be resolved later. In this setup, risk-free assets serve as liquid assets for young consumers in the sense that holdings of risk-free assets allow them to behave flexibly in response to new information. The above setup where young consumers are not allowed to participate in risky asset markets enables liquidity premiums to emerge in a significant manner. Without this kind of participation constraint, lower risk-free rates driven by liquidity demand would be cancelled out by an incentive to leverage risky investment by borrowing at such low risk-free rates. As an alternative constraint, limited borrowing in risk-free markets might also work in favor of liquidity premiums[1] .

More concretely, a one-period risky return R_t^x will take a value of either R^g or R^b ($R^g > R^b$). According to the information obtained initially by a consumer born at date i, a two-period ahead return R_{i+2}^x will take a value of R^g with probability α and a value of R^b with probability $1-\alpha$. When he is middle-aged, the consumer receives an additional message Y_{i+1} concerning a one-period ahead risky return R_{i+2}^x. This independently and identically distributed (i.i.d.) random message takes a value of Y^g with a probability of α and a value of Y^b with probability $1-\alpha$; $\text{Prob}(Y_{i+1}=Y^g)=\alpha$ and $\text{Prob}(Y_{i+1}=Y^b)=1-\alpha$.

[1] Holmström and Tirole (2001), for example, assume that consumers are not allowed to hold any short positions in risk-free assets in order to obtain positive liquidity premiums.

The probability of R^x_{i+2} conditional on this interim message is characterized as follows:

$$\begin{aligned}
\mathrm{Prob}(R^x_{i+2} = R^g | Y_{i+1} = Y^g) &= \rho + \alpha(1-\rho), \\
\mathrm{Prob}(R^x_{i+2} = R^b | Y_{i+1} = Y^g) &= 1 - \rho - \alpha(1-\rho), \\
\mathrm{Prob}(R^x_{i+2} = R^g | Y_{i+1} = Y^b) &= \alpha(1-\rho), \text{ and} \\
\mathrm{Prob}(R^x_{i+2} = R^b | Y_{i+1} = Y^b) &= 1 - \alpha(1-\rho).
\end{aligned}$$

A parameter $\rho \in [0,1]$ represents how uncertainty concerning future risky returns is resolved one period later. As ρ approaches 1, the message provides more information with respect to future risky returns, whereas it is less informative when ρ is closer to zero. Extreme cases include the perfect resolution of uncertainty when $\rho = 1$, and the absence of resolution when $\rho = 0$. The above assumption that $\mathrm{Prob}(R^x_{i+2} = R^g) = \mathrm{Prob}(Y_{i+1} = Y^g) = \alpha$ is also adopted by Jones and Ostroy (1984) as a parsimonious characterization of the resolution of uncertainty[2] . Later, we will consider not only the case where all generations experience resolution of uncertainty in an identical manner, but also the case where only a particular generation can receive the interim message.

As discussed in the introduction, we employ Kreps-Porteus preferences (see Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1990; and others). Given the above initial endowment and financial opportunities, a representative consumer born at date i thus maximizes the following objective function with respect to an investment plan (risk-free bonds a^i_i and a^i_{i+1}, and risky assets x^i_{i+1}).

$$\begin{aligned}
\max_{a^i_i} \Bigg[(w_0 - a^i_i)^{\frac{\sigma-1}{\sigma}} + \beta E_i \Bigg\{ \max_{a^i_{i+1}, x^i_{i+1}} \Big\{ (w_1 + R^f_i a^i_i - a^i_{i+1} - x^i_{i+1})^{\frac{\sigma-1}{\sigma}} \\
+ \beta \left[E_{i+1} (R^f_{i+1} a^i_{i+1} + R^x_{i+2} x^i_{i+1})^{1-\gamma} \right]^{\frac{\sigma-1}{\sigma(1-\gamma)}} \Big\} \Bigg\} \Bigg]^{\frac{\sigma}{\sigma-1}}, \qquad (1)
\end{aligned}$$

where E_t is the conditional expectation operator based on the information available at

[2] More general definitions of the resolution of uncertainty were originally proposed by Marschak and Miyasawa (1968), and subsequently introduced into economic models by Epstein (1980) and others.

date t; β (> 0) is a discount factor; σ (> 0) is the elasticity of intertemporal substitution; and γ (> 0) is a degree of relative risk aversion. As Epstein and Zin (1989) discuss, consumers prefer early resolution of uncertainty to late resolution when $\sigma\gamma > 1$, and vice versa when $\sigma\gamma < 1$. With $\sigma\gamma = 1$, the above objective function reduces to an expected utility framework, or a utility function with constant relative risk aversion (CRRA), where consumers are indifferent with respect to timings of resolution.

We make two remarks on the above optimization problem. First, the information available at date $i+1$ (Ω_{i+1}) differs from Ω_i because of the interim message Y_{i+1}. It means that young consumers have to decide between consumption (an irreversible decision) and risk-free savings before they receive the interim message. This aspect regarding resolution of uncertainty may cause young consumers to demand risk-free assets as liquidity to enable them to respond flexibly to the arrival of new information. Employing the CRRA preference, Epstein (1980) demonstrates that young consumers with strong intertemporal substitution or $\sigma > 1$ do indeed have such liquidity demand. As uncertainty is expected to be resolved more (as ρ increases in our setup), consumers allocate more resources from current consumption to risk-free savings.

Second, as a result of adopting Kreps-Porteus preferences, the third period expected utility $\left[E_{i+1}(R^f_{i+1}a^i_{i+1} + R^x_{i+2}x^i_{i+1})^{1-\gamma}\right]^{\frac{\sigma-1}{\sigma(1-\gamma)}}$ is not added in a linear manner in equation (1) unless $\sigma\gamma = 1$. This feature of preferences generates an additional motive for liquidity demand. Miyazaki and Saito (2003) prove that consumers with large elasticities of intertemporal substitution in combination with strong preferences for early resolution have motives to temporarily hold liquid assets until uncertainty is resolved to some extent. More concretely, Miyazaki and Saito establish the following sufficient conditions: when $\sigma > 1$ and $\sigma\gamma \geq 1$, young consumers save more as ρ is closer to one[3] . Thus, the introduction of Kreps-Porteus preferences helps to substantially expand the potential opportunities for liquidity demand.

An equilibrium risk-free rate is determined endogenously by the lending-borrowing pro-

[3] To be precise, the sufficient conditions for liquidity demand include the case where $\sigma < 1$ and $\sigma + \gamma < 2$.

cess between young and middle-aged consumers. Using dynamic programming techniques, we can derive the optimal asset demand a_i^i, a_{i+1}^i, and x_{i+1}^i as:

$$
\begin{aligned}
a_i^i &= f^i(\Omega_i), & (2)\\
a_{i+1}^i &= g^i(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1}), \text{ and} & (3)\\
x_{i+1}^i &= h^i(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1}), & (4)
\end{aligned}
$$

where the information set Ω_i is recursively defined as: $\Omega_i = \{\Omega_{i-1}, x_{i-1}^{i-2}, a_{i-1}^{i-2}, a_{i-1}^{i-1}, R_i^f, R_i^x, Y_i\}$. (See the appendix for more detailed descriptions of f^i, g^i and h^i.) Then, an equilibrium risk-free rate R_i^f is determined such that:

$$a_i^i + a_i^{i-1} = 0. \tag{5}$$

3. A Numerical Investigation This section presents the numerical results of several experiments in order to demonstrate how an equilibrium risk-free rate is influenced by both the riskiness of investment opportunities and the resolution of uncertainty within the framework constructed in the previous section. In this section, decreases in risk-free rates driven by risk-averse behavior are called *risk premiums*, while decrements in risk-free rates caused by resolution of uncertainty are called *liquidity premiums*. More concretely, on the one hand, risk premiums correspond to the extent to which risk-free rates are driven by mean-preserving spreads of risky returns R_{i+2}^x from the perspective of a young consumer born at date i. On the other hand, liquidity premiums are defined as the extent to which risk-free rates change due to degrees of resolution of uncertainty or changes in ρ. Note that in this OLG model, ex-ante excess returns (premiums) can be defined as the difference between exogenously-given unconditional means of risky returns and equilibrium risk-free rates, because risk-free rates are determined in equilibrium before the interim message is realized.

This section considers the following cases. In the first case, hereafter referred to as **Case 0**, there is no resolution at all, and $\rho = 0$ for all generations. In this case, only risk premiums can be examined through the effects of mean-preserving spreads of risky returns on risk-free rates. In contrast with Case 0, the second case, (**Case 1**), takes the resolution of uncertainty into consideration. That is, all generations with identical preferences receive the interim message when they are middle-aged. In principle, Case 1 can explore how liquidity premiums are determined within a general equilibrium framework. As shown below, in Case 1, liquidity demand does indeed emerge due to increases in ρ, but such demand is not reflected in risk-free rates in a significant manner. A major reason for this is that the resolution of uncertainty enhances the demand of young consumers for risk-free assets, but it also promotes a shift from safe assets to risky assets among middle-aged consumers and lowers their demand for safe assets. Consequently, liquidity impacts on risk-free rates are cancelled out by decreases in safe-asset demand from middle-aged consumers and are somewhat negligible.

We prepare an additional case to highlight liquidity effects on risk-free rates. In the third case, (**Case 2**), only a particular generation can receive the interim message, $\rho > 0$ for a particular generation, and $\rho = 0$ for the other generations. In other words, intergenerational heterogeneity is introduced into the parameter ρ. The numerical procedures of the above three cases are described briefly in the appendix.

For quantitative experiments, we choose admissible values of parameters β, R^g, R^b, α, w_0, w_1, σ, γ, and ρ. The choice of parameters here is motivated, not by attempts to mimic a real economy, but by efforts to explore the qualitative implications of the above OLG model. β is set to be $1/1.02$ throughout the experiments. Both R^g and R^b are chosen such that the unconditional mean is equal to 1.1. Our numerical procedure begins with the setup where $R^g = 1.2$, $R^b = 1.0$, and $\alpha = 0.5$ ($E(R^x) = 1.1$). In terms of endowment, w_0 and w_1 are assumed to be 30 and 100, respectively. Such an assumption concerning initial endowment would promote young consumption instead of young savings without any liquidity demand.

With respect to preference parameters, the elasticity of intertemporal substitution σ takes values between $1/3$ and 8, while γ changes from 1 to 8. Accordingly, the choice of preference parameters cover both preference for early resolution ($\sigma\gamma > 1$) and late resolution ($\sigma\gamma < 1$). The degree of resolution of uncertainty ρ takes a value of either 0.0 or 0.8. In most examples, therefore, liquidity premiums are defined as the difference in risk-free rates between cases where $\rho = 0.0$ and where $\rho = 0.8$.

3.1. **Case 0: no resolution of uncertainty** Table 1 summarizes the numerical results of Case 1 where $\rho = 0$ for all generations. A steady-state equilibrium emerges as an immediate consequence of fixed risky investment opportunities. As mentioned before, a risk premium is defined as $E(R^x) - R^f$, and decreases in risk-free rates result in increases in risk premiums.

Beginning with the assumption that $R^g = 1.2$, $R^b = 1.0$, and $\alpha = 0.5$, when a degree of risk aversion γ increases, given elasticity of intertemporal substitution σ, middle-aged consumers increase their risk-free investments, but decrease their risky investments [4] . This means that middle-aged consumers with greater risk aversion shift funds from risky to safe assets. Consequently, the risk-free rate declines.

When σ increases, given γ, dissavings of young consumers decline, and risky investment increases. A young consumer with large elasticity of intertemporal substitution tends to allocate more to future consumption, given that risk-free rates are higher than time preference rates (which are equal to 2% throughout the numerical exercises). Such consumption allocation in turn raises demand for risk-free bonds from middle-aged consumers through wealth effects. An increase in demand for safe assets from both young and middle-aged consumers jointly contributes to decreases in equilibrium risk-free rates. However, the effect of σ on risk-free rates is not so strong as that of γ.

Nevertheless, we conjecture that the above monotonic depressing (increasing) effect of risk aversion on risk-free rates (risk premiums) may be weakened when risk-free rates are below time preference rates as a result of the introduction of large risks. When the riskiness of the future investment opportunity is extremely large, young consumers may

[4] Exceptionally, risky investment increases when γ changes from one to two, given that $\sigma = 8$.

consume currently instead of transferring resources to the future. Such a tendency may be more pronounced for those with both stronger intertemporal substitution and larger risk aversion, as these individuals tend to be more interested in choosing the timing of consumption *and* are more averse to future consumption volatilities.

Figure 1 raises riskiness to $R^g = 1.3$ and $R^b = 0.9$ by mean-preserving spreads and compares it with $R^g = 1.2$ and $R^b = 1.0$. According to this figure, additional risk premiums are still monotonically increasing in risk aversion for those with relatively weak intertemporal substitution[5] . For those with $\sigma = 8$, however, additional risk premiums are decreasing when γ is above four. This kind of finding is not available from an expected utility framework where it is impossible to increase σ and γ simultaneously.

3.2. **Case 1: with resolution of uncertainty** Unlike Case 0, Case 1, where all generations receive the interim message ($\rho > 0$), generates a stationary Markov equilibrium. That is, risk-free rates change over time depending on which state of Y^g or Y^b is realized, and investment and consumption plans are influenced by the movement of risk-free rates. (See the appendix for a more detailed characterization of this stationary Markov equilibrium.)

Table 2 reports the unconditional means of risk-free rates and investment plans under $\rho = 0.8$.[6] In addition, the last column of Table 2 presents liquidity premiums, defined as differences in risk-free rates between such rates under $\rho = 0.8$ (reported in the third column of Table 3) and under $\rho = 0.0$ (reported in the third column of Table 1). Figure 2 depicts how demand functions for risk-free assets from young consumers change as resolution of uncertainty is greater under $\sigma = \gamma = 3$.

As shown by Miyazaki and Saito (2003), large elasticities of intertemporal substitution ($\sigma > 1$) and strong preferences for early resolution ($\sigma\gamma \geq 1$) jointly contribute to increases

[5] In Figure 1, additional risk premiums happen to be close to one another for various values of σ at $\gamma = 3$, but they are still different from one another in rigorous terms.

[6] For this calculation, 5200 random variables of the interim message and risky returns are generated and, given this fixed random seed, equilibrium risk-free rates and investment plans are derived numerically. The unconditional means of these variables are computed after dropping the first 200 observations.

in liquidity demand in the current setup. Figure 2 demonstrates that liquidity demand from young consumers is boosted as ρ is closer to one. Nevertheless, Table 2 documents negative liquidity premiums. In other words, although liquidity demand is generated, such demand is not reflected directly in the equilibrium behavior of risk-free rates.

A major reason for the above asymmetry between demand and risk-free rates is that a larger ρ raises liquidity demand from the young consumers of the current generation, but it promotes a shift from risk-free assets to risky assets among the middle-aged consumers of the previous generation as a result of the resolution of uncertainty. In other words, stronger liquidity demand from young consumers is largely cancelled out by weaker demand for risk-free assets from middle-aged consumers. Therefore, liquidity effects on risk-free rates are not observed clearly in the numerical result of Case 1. As previously suggested, Case 2 introduces intergenerational heterogeneity in order to highlight liquidity impacts on risk-free rates.

3.3. **Case 2: intergenerational heterogeneity in ρ** In Case 2, only one particular generation can receive the interim message, while the other generations do not. More concretely, only the generation born at date I receives the interim message Y_{I+1} with $\rho_I = 0.8$. On the other hand, preference parameters σ and γ are common among all generations. For simplicity, it is assumed that generation $i < I$ does not know that generation I receives the interim message, and that generation $i > I$ does not expect the arrival of any interim messages at all. Based on this setup, demand for risk-free assets from middle-aged consumers of generation $I-1$ is completely independent of the resolution of uncertainty. Accordingly, liquidity demand from young consumers of generation I may be translated almost straight into equilibrium risk-free rates. Note that ex-ante excess returns (premiums) are still defined as the deviation of unconditional means of risky returns from risk-free rates, because risk-free rates are determined in equilibrium before the arrival of the interim message. One possible interpretation of this setup is that a particular generation happens to face resolvable uncertainty.

Table 3 presents the numerical result of date-I risk-free rates and investment plans (R^f_I,

$a_I^I(= -a_I^{I-1})$, and x_I^{I-1}) under $\rho_I = 0.8$. In addition, the last column of Table 3 reports liquidity premiums, which are defined as differences in risk-free rates between values under $\rho = 0.8$ and under $\rho = 0.0$. Figure 3 depicts liquidity premiums for various values of σ when $1 \leq \gamma \leq 8$, while Figure 4 plots liquidity premiums when $0 < \gamma < 1$. Comparing these liquidity premiums with the demand for safe assets from young consumers (a_0) in Table 1 (also reported in parentheses in the fourth column of Table 3), it is possible to explore whether liquidity demand is indeed promoted by the resolution of uncertainty.

The numerical results are summarized as follows. First, if the elasticity of intertemporal substitution σ is equal to one, then demand for safe assets from young consumers is completely independent of the resolution of uncertainty, and there are no liquidity premiums. Second, when σ is greater than one, and early resolution is preferred ($\sigma\gamma > 1$), then liquidity demand emerges and positive liquidity premiums are generated. In particular, as shown in Figure 3, liquidity premiums increase with the degree of relative risk aversion, γ, given $\sigma > 1$. The second feature is consistent with Miyazaki and Saito's (2003) finding that liquidity demand emerges when $\sigma > 1$ and $\sigma\gamma \geq 1$ (a sufficient condition). Consistent with Epstein's (1980) result, even CRRA preferences yield positive liquidity premiums as long as σ is larger than one ($\sigma = 3, \gamma = 1/3$ and $\sigma = 8, \gamma = 1/8$ in Figure 4). However, as discussed in the introduction, only tiny premiums are generated under rather low values of γ. Third, even if σ is less than one, liquidity premiums, though extremely small, are yielded as long as both σ and γ are small (see the case where $\sigma = \frac{1}{3}$ in Figures 3 and 4). The third observation is again consistent with Miyazaki and Saito's finding that another sufficient condition for liquidity demand is that $\sigma < 1$ and $\sigma + \gamma < 2$.[7]

Inferring from the result of Case 0, we reasonably expect that large risks associated with investment opportunities may have an additional impact on risk-free rates or premiums. Given extremely risky investment, young consumers with strong intertemporal substitution and large risk aversion may consume instead of saving, thereby canceling out liquidity

[7] According to Figure 3, even if $\frac{1}{3} + \gamma > 2$, liquidity premiums are still positive as long as γ is relatively small. However, this is not necessarily inconsistent with Miyazaki and Saito (2003) because they derive only sufficient conditions for liquidity demand.

demand. Suppose that generation I faces values of $R^g = 1.3$ and $R^b = 0.9$ with $\rho = \sqrt{1/2}$, and that the other generations experience values of $R^g = 1.2$ and $R^b = 1.0$ with $\rho = 0$. $\rho = \sqrt{1/2}$ is chosen such that the conditional volatility for generation I is exactly equal to the unconditional volatility for the other generations without any interim message in terms of average absolute deviations. One possible interpretation of this setup is that a particular generation happens to face large, but resolvable, uncertainty.

Table 4 presents the numerical results of the above case. In addition, Figure 5 plots liquidity premiums, which are defined as the deviations from the risk-free rate of Case 0 with $R^g = 1.2$ and $R^b = 1.0$. As Figure 5 demonstrates, when elasticity of intertemporal substitution is large ($\sigma = 8$), liquidity demand is cancelled out largely by a disincentive for young consumers to save when γ is beyond four, and premiums (risk-free rates) are decreasing (increasing) in risk aversion.

In sum, young consumers with strong preferences for early resolution, as well as large elasticities of intertemporal substitution, generate liquidity demand, and such demand is reflected directly in equilibrium risk-free rates. Extreme riskiness of investment opportunities, on the other hand, dampens liquidity demand to some extent, and tends to raise risk-free rates for those with both strong intertemporal substitution and high risk aversion.

4. Discussion In this paper, we have presented an overlapping generations framework where an ex-ante excess return over a risk-free rate can be divided into a risk premium component and a liquidity premium component. In this framework, an incentive to postpone consumption (irreversible expenditures) until uncertainty is resolved, to some extent, triggers liquidity demand. Such demand may result in decreases in risk-free rates or increases in liquidity premiums. By nature, such liquidity premiums, which are caused by consumers waiting for uncertainty to be resolved, are fairly different from risk premiums driven by riskiness of investment opportunities. Our numerical examples have shown that consumers with large elasticities of intertemporal substitution, as well as strong preferences for early resolution of uncertainty, generate vigorous liquidity demand, thereby resulting in positive liquidity premiums.

In addition, our numerical investigation sheds light on the advantage of Kreps-Porteus preferences in dynamic asset pricing models. As investigated in Weil (1989), if investment opportunities are fixed over time, then the Kreps-Porteus preference does not play any active role in determining excess returns. This is because, in an expected utility framework, premiums are determined mainly by degrees of relative risk aversion and are almost independent of intertemporal substitution. However, Kandel and Stambaugh (1991), and others, demonstrate that not only risk aversion, but also intertemporal substitution plays an important role in determining premiums and risk-free rates when investment opportunities are time-varying in terms of first and second moments.

If changes in conditional moments of risky returns can be regarded as consequences of the arrival of new messages, then, as discussed in this paper, we may analyze the role of intertemporal substitution in determining excess returns via the economic mechanism of liquidity demand and liquidity premiums. As the additional experiments of Case 0 and Case 2 suggest, one caveat for this interpretation is that extreme riskiness of investment opportunities may weaken liquidity demand from consumers with both strong intertemporal substitution and high risk aversion.

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Appendix: In this appendix, we provide a description of optimal decision functions (2), (3), and (4) discussed in Section 2, and the numerical procedures adopted in Section 3.

Optimal decision functions This subsection briefly explains how to derive optimal decision functions from the maximization problem (1) in Section 2. As usual, these decision functions are solved by backward induction. Consider a consumer born at date i. When he is middle-aged, the consumer maximizes the following objective function contingent on Ω_{i+1}:

$$\left\{(w_1 + R_i^f a_i^i - a_{i+1}^i - x_{i+1}^i)^{\frac{\sigma-1}{\sigma}} + \beta\left[E_{i+1}(R_{i+1}^f a_{i+1}^i + R_{i+2}^x x_{i+1}^i)^{1-\gamma}\right]^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right\}^{\frac{\sigma}{\sigma-1}}, \quad (6)$$

where Ω_{i+1} contains a_i^i, R_i^f, R_{i+1}^f, and Y_{i+1}, but not a_{i+1}^i, x_{i+1}^i, or R_{i+2}^x. The first order conditions with respect to a_{i+1}^i and x_{i+1}^i yield:

$$\begin{aligned} a_{i+1}^i &= (w_1 + R_i^f a_i^i) \cdot D_0 \cdot (R^g D_b - R^b D_g), \text{ and} \\ x_{i+1}^i &= (w_1 + R_i^f a_i^i) \cdot D_0 \cdot R_{i+1}^f \cdot (D_g - D_b), \end{aligned}$$

where

$$\begin{aligned} D_0(R_{i+1}^f, Y_{i+1}) &= [D_g(R_{i+1}^f - R^b) + D_b(R^g - R_{i+1}^f) + R_{i+1}^f(R^g - R^b)]^{-1}, \\ D_g(R_{i+1}^f, Y_{i+1}) &= \left(\beta R_{i+1}^f (R^g - R^b)\right)^\sigma \left(\frac{\pi(Y_{i+1})}{R_{i+1}^f - R^b}\right)^{\frac{1}{\gamma}} \\ &\quad \times \left[\frac{\pi(Y_{i+1})^{\frac{1}{\gamma}}}{(R_{i+1}^f - R^b)^{\frac{1-\gamma}{\gamma}}} + \frac{(1-\pi(Y_{i+1}))^{\frac{1}{\gamma}}}{(R^g - R_{i+1}^f)^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{\sigma\gamma-1}{1-\gamma}}, \\ D_b(R_{i+1}^f, Y_{i+1}) &= \left(\beta R_{i+1}^f (R^g - R^b)\right)^\sigma \left(\frac{1-\pi(Y_{i+1})}{R^g - R_{i+1}^f}\right)^{\frac{1}{\gamma}} \\ &\quad \times \left[\frac{\pi(Y_{i+1})^{\frac{1}{\gamma}}}{(R_{i+1}^f - R^b)^{\frac{1-\gamma}{\gamma}}} + \frac{(1-\pi(Y_{i+1}))^{\frac{1}{\gamma}}}{(R^g - R_{i+1}^f)^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{\sigma\gamma-1}{1-\gamma}}, \\ \pi(Y_{i+1}) &= \begin{cases} \rho + \alpha(1-\rho) & \text{if} \quad Y_{i+1} = Y^g \\ \alpha(1-\rho) & \text{if} \quad Y_{i+1} = Y^b \end{cases}. \end{aligned}$$

Given a_i^i, R_i^f, R_{i+1}^f and Y_{i+1}, demand functions for safe and risky assets, $g(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1})$ and $h(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1})$, are characterized analytically. When the intergenerational heterogeneity is introduced, an upper subscript i is added, such as $g^i(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1})$ and $h^i(a_i^i, R_i^f, R_{i+1}^f, Y_{i+1})$.

Substituting these decision functions into equation (6) leads to the middle-aged value function,

$$(w_1 + R_i^f a_i^i)\tilde{V}^i(R_{i+1}^f, Y_{i+1}),$$

where

$$\tilde{V}^i(R^f_{i+1}, Y_{i+1}) = D_0 \cdot R^f_{i+1} \cdot (R^g - R^b) \left[1 + \beta \left\{\pi(Y_{i+1}) D_g^{1-\gamma} + (1 - \pi(Y_{i+1})) D_b^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Substituting the above value function into equation (1), the objective function maximized by a young consumer of generation i is as follows:

$$\left[(w_0 - a^i_i)^{\frac{\sigma-1}{\sigma}} + \beta(w_1 + R^f_i a^i_i)^{\frac{\sigma-1}{\sigma}} E_i \left\{\tilde{V}^i(R^f_{i+1}, Y_{i+1})^{\frac{\sigma-1}{\sigma}}\right\}\right]^{\frac{\sigma}{\sigma-1}}.$$

Note that Ω_i contains a^i_i and R^f_i, but not Y_{i+1} and R^f_{i+1}. The first-order condition with respect to a^i_i yields:

$$a^i_i = \frac{w_0 \left[\beta R^f_i E_i \left\{\tilde{V}^i(R^f_{i+1}, Y_{i+1})^{\frac{\sigma-1}{\sigma}}\right\}\right]^{\sigma} - w_1}{\left[\beta R^f_i E_i \left\{\tilde{V}^i(R^f_{i+1}, Y_{i+1})^{\frac{\sigma-1}{\sigma}}\right\}\right]^{\sigma} + R^f_i}.$$

The decision function of a^i_i, denoted as f or f^i, depends not only on R^f_i, but also on the conditional expectation of $\tilde{V}^i(R^f_{i+1}, Y_{i+1})^{\frac{\sigma-1}{\sigma}}$.

In the following cases, the decision functions f, g, or h may be expressed in a simpler manner. First, if $\sigma = 1$, then $E_i \left\{\tilde{V}^i(R^f_{i+1}, Y_{i+1})^{\frac{\sigma-1}{\sigma}}\right\}$ reduces to $1 + \beta$, and f is accordingly equal to $\frac{\beta(1+\beta)w_0}{1+\beta+\beta^2} - \frac{w_1}{R^f_i(1+\beta+\beta^2)}$. That is, even if the interim message is expected to arrive, the young consumer's demand for safe assets does not ever change in this case. In addition, γ is irrelevant in determining the young saving-consumption decision. Second, if $\rho = 0$, then the decision functions g and h do not depend on Y_{i+1}. As a result, f is explained solely by R^f_i and the conditional expectation of R^f_{i+1}. Third, when all generations have identical preferences and R^f is constant over time, as in Case 0, the decision functions are described as $a_0 = f(R^f)$, $a_1 = g(a_0, R^f)$, and $x_1 = h(a_0, R^f)$. Fourth, when all generations have identical preferences, and R^f_{i+1} follows a stationary transition function of R^f_i and Y_{i+1}, as in Case 1, then the decision functions are described as $a_0 = f(R^f_i)$, $a_1 = g(a_0, R^f_i, R^f_{i+1}, Y_{i+1})$, and $x_1 = h(a_0, R^f_i, R^f_{i+1}, Y_{i+1})$. These properties of the decision functions are used in calculating equilibrium risk-free rates in the numerical experiment.

4.1. **Numerical procedures** This subsection briefly explains how to obtain the numerical results reported in Section 3. A basic procedure is as follows:

1. Guess a sequence of risk-free rates.
2. Given the above guess, solve the dynamic optimization problem in the manner described in the previous subsection. Apply a grid method to the numerical derivation of the decision functions.
3. Update a new sequence of the risk-free rate using the numerically derived decision functions and the market-clearing condition (5). Use a spline interpolation to find the equilibrium risk-free rate that satisfies equation (5).
4. Iterate the above steps until a sequence of risk-free rates converges.

In terms of the market clearing condition (5), $f(R^f)+g(f(R^f),R^f)=0$ in Case 0 where a steady state equilibrium obtains, whereas $f(R^f_i)+g(f(R^f_{i-1}),R^f_{i-1},R^f_i,Y_i)=0$ in Case 1 where a stationary Markov equilibrium emerges.

In Case 2, the equilibrium is neither stationary nor in a steady state because of the intergenerational heterogeneity. Any generation where $i<I$ follows the same decision functions as in Case 0. For generations where $i>I$, an equilibrium is influenced, which results in the state of Y_{I+1} being realized, and a^i_i can be denoted as $f^i(R_i,Y_{I+1})$ for $i>I$. Given these equilibrium conditions, R^f_I is determined such that the sum of a^{I-1}_I and a^I_I is equal to zero.

Table 1: The numerical result of Case 0 with $R^g = 1.2$ and $R^b = 1.0$

σ	γ	$R^f(\%)$	$a_0 = -a_1$	x_1
1/3	1	9.373	-13.191	29.172
1/3	2	8.764	-13.252	28.148
1/3	3	8.156	-13.297	28.171
1/3	4	7.561	-13.341	28.195
1/3	5	6.982	-13.384	28.219
1/3	6	6.424	-13.425	28.243
1/3	7	5.889	-13.465	28.268
1/3	8	5.381	-13.502	28.293
1	1	9.326	-11.294	32.098
1	2	8.665	-11.483	31.844
1	3	8.022	-11.670	31.595
1	4	7.401	-11.852	31.352
1	5	6.806	-12.028	31.117
1	6	6.240	-12.198	30.892
1	7	5.704	-12.360	30.677
1	8	5.200	-12.514	30.474
3	1	9.205	-5.855	40.485
3	2	8.428	-6.479	42.676
3	3	7.702	-7.121	41.510
3	4	7.022	-7.737	40.392
3	5	6.390	-8.324	39.330
3	6	5.807	-8.880	38.328
3	7	5.271	-9.402	37.387
3	8	4.780	-9.890	36.506
8	1	8.996	5.513	57.992
8	2	8.078	4.356	68.318
8	3	7.224	2.679	64.811
8	4	6.453	1.056	61.441
8	5	5.764	-0.490	58.250
8	6	5.153	-1.942	55.265
8	7	4.615	-3.292	52.498
8	8	4.142	-4.538	49.949

Table 2: The numerical result of Case 1 with $R^g = 1.2$ and $R^b = 1.0$

σ	γ	$R^f(\%)$	$a_0 = -a_1$	x_1	liquidity premium (%)
1/3	1	9.760	-13.182	29.208	-0.387
1/3	2	9.537	-13.227	28.154	-0.773
1/3	3	9.307	-13.244	28.162	-1.151
1/3	4	9.073	-13.260	28.170	-1.512
1/3	5	8.832	-13.277	28.179	-1.850
1/3	6	8.583	-13.295	28.187	-2.159
1/3	7	8.326	-13.313	28.196	-2.437
1/3	8	8.060	-13.331	28.204	-2.680
1	1	9.743	-11.434	31.962	-0.416
1	2	9.494	-11.502	31.870	-0.829
1	3	9.240	-11.572	31.776	-1.218
1	4	8.977	-11.642	31.680	-1.576
1	5	8.706	-11.713	31.584	-1.900
1	6	8.425	-11.787	31.484	-2.186
1	7	8.134	-11.862	31.382	-2.430
1	8	7.833	-11.939	31.277	-2.633
3	1	9.697	-6.458	39.779	-0.492
3	2	9.395	-6.606	42.791	-0.967
3	3	9.081	-6.850	42.343	-1.379
3	4	8.753	-7.098	41.888	-1.732
3	5	8.411	-7.354	41.419	-2.021
3	6	8.054	-7.616	40.938	-2.248
3	7	7.686	-7.886	40.442	-2.415
3	8	7.307	-8.164	39.933	-2.527
8	1	9.623	2.824	54.282	-0.627
8	2	9.268	3.410	66.982	-1.190
8	3	8.881	2.817	65.747	-1.656
8	4	8.470	2.218	64.497	-2.017
8	5	8.037	1.608	63.227	-2.273
8	6	7.583	0.976	61.916	-2.430
8	7	7.115	0.315	60.553	-2.500
8	8	6.639	-0.371	59.141	-2.497

Table 3: The numerical result of Case 2 with $R^g = 1.2$ and $R^b = 1.0$

σ	γ	$R_I^f(\%)$	$a_I^I = -a_I^{I-1}$	x_I^{I-1}	liquidity premium (%)
1/3	1	9.372	-13.144 (-13.191)	29.219	0.001
1/3	2	8.764	-13.239 (-13.252)	28.160	0.001
1/3	3	8.157	-13.306 (-13.297)	28.162	-0.001
1/3	4	7.563	-13.371 (-13.341)	28.165	-0.002
1/3	5	6.987	-13.434 (-13.384)	28.168	-0.005
1/3	6	6.432	-13.495 (-13.425)	28.173	-0.008
1/3	7	5.901	-13.553 (-13.465)	28.178	-0.011
1/3	8	5.395	-13.608 (-13.502)	28.186	-0.015
1	1	9.326	-11.294 (-11.294)	32.098	0.000
1	2	8.665	-11.483 (-11.483)	31.844	0.000
1	3	8.022	-11.670 (-11.670)	31.595	0.000
1	4	7.401	-11.852 (-11.852)	31.352	0.000
1	5	6.806	-12.028 (-12.028)	31.117	0.000
1	6	6.240	-12.198 (-12.198)	30.892	0.000
1	7	5.704	-12.360 (-12.360)	30.677	0.000
1	8	5.200	-12.514 (-12.514)	30.474	0.000
3	1	9.204	-5.810 (-5.855)	40.529	0.001
3	2	8.423	-6.320 (-6.479)	42.835	0.006
3	3	7.690	-6.896 (-7.121)	41.733	0.012
3	4	7.002	-7.451 (-7.737)	40.677	0.019
3	5	6.362	-7.982 (-8.324)	39.671	0.028
3	6	5.770	-8.485 (-8.880)	38.720	0.037
3	7	5.225	-8.961 (-9.402)	37.824	0.046
3	8	4.726	-9.408 (-9.890)	36.984	0.054
8	1	8.994	5.674 (5.513)	58.154	0.003
8	2	8.053	5.286 (4.356)	69.251	0.025
8	3	7.178	3.853 (2.679)	65.988	0.046
8	4	6.381	2.465 (1.056)	62.853	0.072
8	5	5.664	1.139 (-0.490)	59.879	0.100
8	6	5.026	-0.115 (-1.942)	57.088	0.128
8	7	4.461	-1.293 (-3.292)	54.487	0.154
8	8	3.964	-2.394 (-4.538)	52.076	0.178

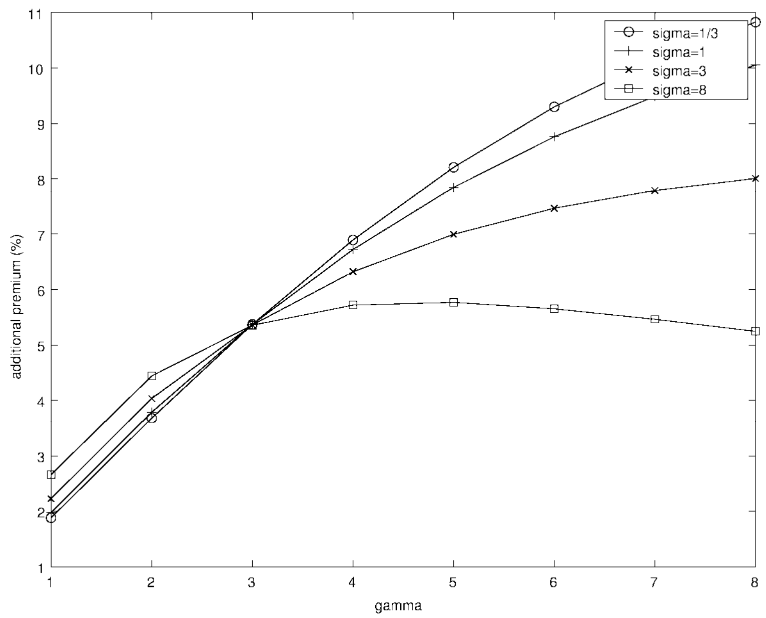
(1) The numbers in parentheses in the fourth column are the risk-free savings of young consumers reported in Table 1.

Table 4: The numerical result of the case with large, but resolvable uncertainty for generation I ($R^g = 1.3$ and $R^b = 0.9$ with $\rho = \sqrt{1/2}$ for generation I versus $R^g = 1.2$ and $R^b = 1.0$ with $\rho = 0.0$ for all generations)

σ	γ	$R^f_I(\%)$	$a^I_I = -a^{I-1}_I$	x^{I-1}_I	liquidity premium (%)
1/3	1	9.384	-13.696 (-13.191)	28.666	-0.011 (0.001)
1/3	2	8.780	-13.615 (-13.252)	27.783	-0.016 (0.001)
1/3	3	8.174	-13.577 (-13.297)	27.890	-0.018 (-0.001)
1/3	4	7.580	-13.567 (-13.341)	27.968	-0.019 (-0.002)
1/3	5	7.000	-13.565 (-13.384)	28.037	-0.018 (-0.005)
1/3	6	6.440	-13.565 (-13.425)	28.103	-0.016 (-0.008)
1/3	7	5.902	-13.562 (-13.465)	28.169	-0.012 (-0.011)
1/3	8	5.388	-13.558 (-13.502)	28.237	-0.008 (-0.015)
1	1	9.326	-11.294 (-11.294)	32.098	0.000 (0.000)
1	2	8.665	-11.483 (-11.483)	31.844	0.000 (0.000)
1	3	8.022	-11.670 (-11.670)	31.595	0.000 (0.000)
1	4	7.401	-11.852 (-11.852)	31.352	0.000 (0.000)
1	5	6.806	-12.028 (-12.028)	31.117	0.000 (0.000)
1	6	6.240	-12.198 (-12.198)	30.892	0.000 (0.000)
1	7	5.704	-12.360 (-12.360)	30.677	0.000 (0.000)
1	8	5.200	-12.514 (-12.514)	30.474	0.000 (0.000)
3	1	9.173	-4.228 (-5.855)	42.112	0.032 (0.001)
3	2	8.384	-5.252 (-6.479)	43.901	0.044 (0.006)
3	3	7.651	-6.148 (-7.121)	42.479	0.051 (0.012)
3	4	6.969	-6.953 (-7.737)	41.173	0.053 (0.019)
3	5	6.340	-7.706 (-8.324)	39.945	0.051 (0.028)
3	6	5.763	-8.417 (-8.880)	38.787	0.043 (0.037)
3	7	5.238	-9.086 (-9.402)	37.700	0.033 (0.046)
3	8	4.760	-9.711 (-9.890)	36.684	0.020 (0.054)
8	1	8.925	9.699 (5.513)	62.179	0.072 (0.003)
8	2	7.967	8.494 (4.356)	72.474	0.111 (0.025)
8	3	7.086	6.183 (2.679)	68.330	0.138 (0.046)
8	4	6.303	4.005 (1.056)	64.399	0.150 (0.072)
8	5	5.619	1.887 (-0.490)	60.629	0.145 (0.100)
8	6	5.028	-0.148 (-1.942)	57.055	0.125 (0.128)
8	7	4.520	-2.061 (-3.292)	53.721	0.095 (0.154)
8	8	4.082	-3.822 (-4.538)	50.658	0.060 (0.178)

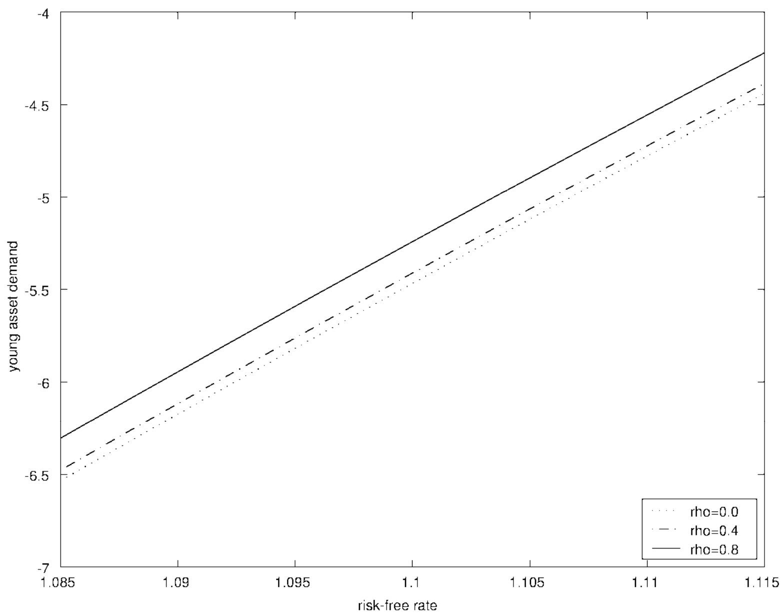
(1) The numbers in parentheses in the fourth column are the risk-free savings of the young consumers reported in Table 1.
(2) The numbers in parentheses in the sixth column are the liquidity premiums reported in Table 3.

Figure 1: Additional risk premiums due to more volatile risky returns (Case 0)



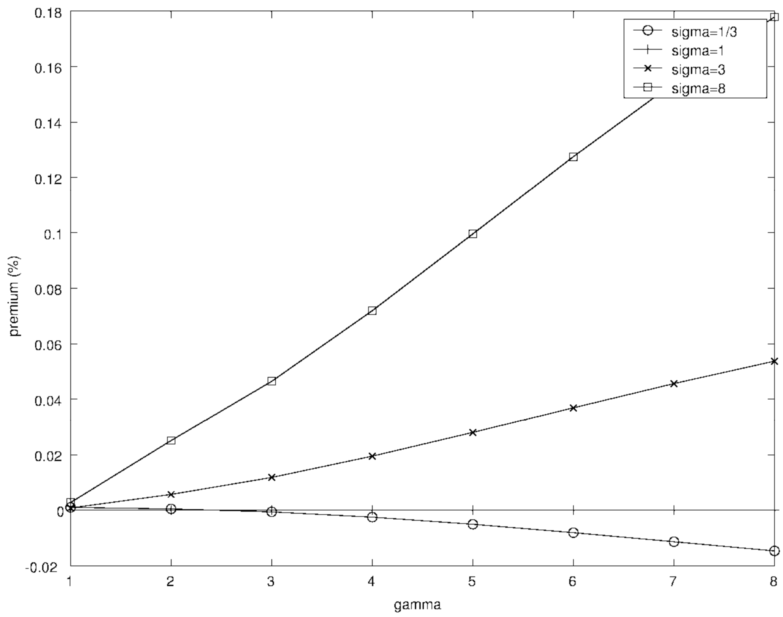
(1) The above figure plots differences in risk premiums between under $R^{\mathsf{g}} = 1.3$ and $R^{\mathsf{b}} = 0.9$ and under $R^{\mathsf{g}} = 1.2$ and $R^{\mathsf{b}} = 1.0$ in Case 0.

Figure 2: Demand functions for safe assets from young consumers and resolution of uncertainty (Case 1)



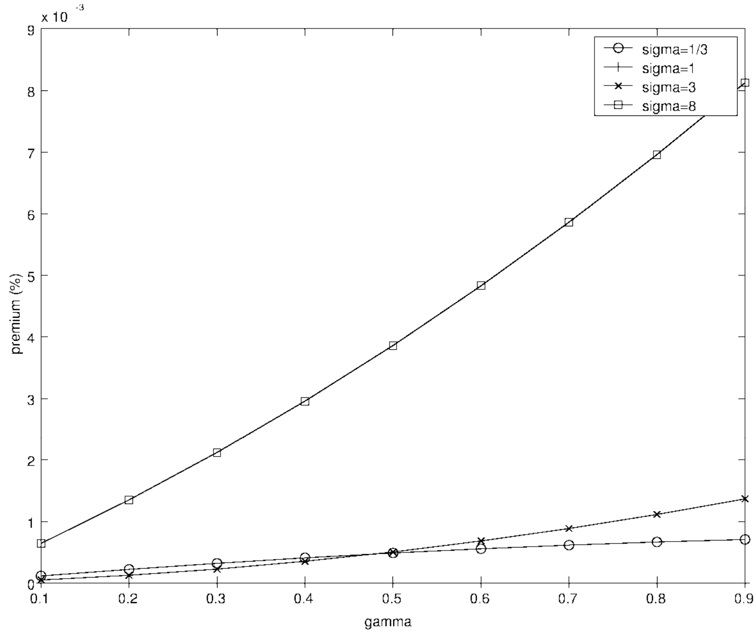
(1) The above figure plots demand functions for safe assets from young consumers with $\rho = 0.0$, 0.4, and 0.8 under $\sigma = 3$ and $\gamma = 3$.

Figure 3: Liquidity premiums based on $\rho = 0.0$ versus $\rho = 0.8$ (Case 2)



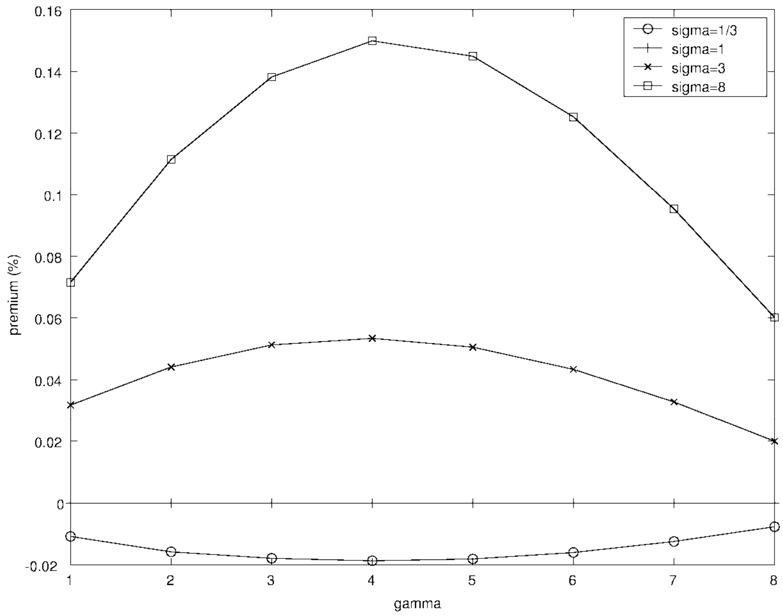
(1) The above figure plots differences in risk-free rates between with $\rho = 0.8$ and with $\rho = 0.0$ under $R^{\mathrm{g}} = 1.2$ and $R^{\mathrm{b}} = 1.0$.

Figure 4: Liquidity premiums based on $\rho = 0.0$ versus $\rho = 0.8$ with $\gamma < 1$ (Case 2)



(1) The above figure plots differences in risk-free rates between with $\rho = 0.8$ and with $\rho = 0.0$ under $R^{\mathrm{g}} = 1.2$ and $R^{\mathrm{b}} = 1.0$.

Figure 5: Liquidity premiums with large, but resolvable uncertainty



(1) The above figure plots differences in risk-free rates between under $R^{\mathsf{g}} = 1.3$ and $R^{\mathsf{b}} = 0.9$ with $\rho = \sqrt{1/2}$ for generation I, and under $R^{\mathsf{g}} = 1.2$ and $R^{\mathsf{b}} = 1.0$ with $\rho = 0.0$ for all generations.