COE-RES Discussion Paper Series
Center of Excellence Project
The Normative Evaluation and Social Choice of
Contemporary Economic Systems

Graduate School of Economics and Institute of Economic Research
Hitotsubashi University

COE/RES Discussion Paper Series, No.159
February 2006

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On the intergenerational and intertemporal sharing of cohort-specific shocks on permanent income [*†]

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February, 2006

ABSTRACT. This paper investigates the intergenerational sharing of shocks on the permanent income of new entry cohorts through capital market transactions. Even when Lucas trees only are traded among generations, procyclical cohort-specific shocks are shared to some extent by the movement of asset prices; cohorts with lower endowments might benefit much more than cohorts with higher endowments from asset pricing dynamics. However, such cohort shocks remain partially uninsured, particularly when the elasticity of intertemporal substitution is large and the frequency of investment opportunities is limited. Given a reasonable set of parameters concerning the Japanese labor market, a welfare loss would be 1–3% in terms of the certainty equivalence consumption level. The optimal policy argument suggests that a policy should reflect *not* the intertemporal transfer from cohorts with high endowments to cohorts with low endowments, *but* the intratemporal risk sharing between labor and capital income.

Keywords: cohort-specific shock, intergenerational transfer, capital income tax.
JEL Classification Number: H20, H31, J24.

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[†] We would like to thank Kenn Ariga, Toni Braun, and Yasushi Iwamoto for helpful discussions, and seminar participants at Hitotsubashi University, Kyoto University, the Cabinet Office, and the 2005 autumn meeting of the Japanese Economic Association for their useful comments. We are grateful for grants from the Economic and Social Research Institute, the Cabinet Office, Japan. The first author would like to acknowledge financial support from the Hosei Institute on Aging. The second and third authors are thankful for a grant-in-aid from the Ministry of Education and Science, Japan.

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1. Introduction This paper investigates the intergenerational sharing of shocks on the permanent income of new entry cohorts through capital market transactions. For the purpose of this research, we define cohort-specific permanent shocks as long-term risks taken by the cohort members when they enter labor markets for the first time after graduation. It is usually difficult to perfectly insure against the risks using transactions of contingent claims when cohort-specific permanent shocks have already been realized. We focus on the role of the capital market and the asset pricing mechanism in partially sharing the cohort-specific permanent shocks. In addition, we investigate which policy interventions help to restore a first-best allocation, and whether such a first-best allocation can be decentralized.

There are many empirical studies supporting the existence of cohort-specific effects on labor earnings. For example, Baker et al. (1994) studied the personnel data records of managers in a single company, and found that first-time wages after entry are positively correlated with the lifetime wages of the corresponding cohort. Using the National Longitudinal Survey of Youth (NLSY), Kahn (2005) investigated the career outcomes of college graduates who entered labor markets in the 1982 recession, and concluded that graduation in a recession brings about a substantially negative effect on wages throughout a career. Using the NLSY, Kletzer and Fairlie (2001) found that a job displacement at an early stage of a career leads to persistent wage losses. Card and Lemieux (2001) demonstrated that date of birth significantly affects the lifetime wages of each cohort using microdata from the US, the UK, and Canada.[1]

Despite the difficulties involved in accessing microdata in Japan, several papers have addressed cohort-specific effects in the Japanese labor market. Ohtake and Inoki (1997), Ohta (1999), and Okamura (2000) found that not only the cohort size, but also the economic conditions of a graduation year yield long-run effects on both wage rates and tenure. Genda (1997), whose study our numerical exercise relies upon, demonstrated that cohort-specific

[1] Von Watcher and Bender (2005) insisted that cohort effects are not permanent, but temporary. Allowing for job mobility among firms and employees, they found that an early job displacement as well as a graduation in a recession has only transitory wage effects in Germany. However, their discussion may not apply to the Japanese labor market given its low labor mobility.

economic conditions as well as an increase in the number of college graduates account for differences in wages between college and high-school graduates in Japan. These findings may reflect the fact that aggregate shocks tend to be shifted onto young workers because the Japanese labor market adjusts mainly through new entry to the labor force.

Assuming that cohort-specific effects on lifetime wages are present, rather than explaining how such effects are generated,[2] we investigate the sharing of cohort-specific shocks among different cohorts through financial transactions. To focus purely on the effects of cohort-specific shocks on lifetime wages, we adopt a theoretical framework proposed by Huffman (1987). Within Huffman's framework, on the one hand, perishable consumption goods are endowed upon the entry cohort. On the other hand, Lucas trees yield perishable consumption goods as dividends, and financial claims on Lucas trees are traded among various cohorts. As shown in detail later, given this endowment and dividend structure, the inability of entry cohorts to trade financial claims prior to birth is a fundamental source of market incompleteness regardless of the presence of endowment or dividend shocks.

In this model, a shock on entry endowments can be regarded as a cohort-specific shock on capitalized lifetime labor income or permanent income. Then, we can investigate how such uninsured entry shocks are shared among different cohorts through active transactions of Lucas trees. This setup abstracts completely from age-specific consumption patterns, the timing of labor income receipts (lifecycle wage profiles), idiosyncratic shocks on labor income, or any other type of financial constraints. Thus, any welfare loss computed under the above setup would arise as a consequence of missing prior-to-birth markets. In a case where such an estimated loss is not negligible, we will consider the extent to which welfare would be improved by fixing the above fundamental source of market incompleteness either through policy interventions or by enabling prior-to-birth transactions with possible short positions.

To evaluate welfare losses owing to uninsured cohort-specific shocks in a more gen-

[2] Kahn (2005) concluded that a theory of task-specific human capital (Gibbons and Waldman, 2003) was consistent with her empirical finding about cohort effects on wages.

eral environment, we augment Huffman's (1987) setup with nonexpected utility, known as Kreps–Porteus preferences, to separate the elasticity of intertemporal substitution from relative risk aversion. A major reason for this generalization is that existing papers, including Bohn (1998) and Krueger and Kubler (2003), have suggested that the effect arising purely from intertemporal substitution is significant in the determination of intergenerational allocation. As demonstrated in the next sections, the separation between intertemporal substitution and risk aversion is indeed useful in analyzing the welfare impact of active financial transactions among cohorts.

Our analytical focus contrasts sharply with existing papers in several respects. Many papers, including Gordon and Varian (1988), Bohn (1998), Shiller (1999), Ball and Mankiw (2001), DeMange (2002), and Krueger and Kubler (2003), have explored the intergenerational risk-sharing issue using stochastic overlapping generations models with incomplete markets. However, in contrast to our focus on permanent labor-income shocks faced by young cohorts, these authors were interested in the sharing of capital-income risks borne by old cohorts.[3] In addition, their focus was on redistribution mechanisms from young to old consumers such as a pay-as-you-go social security system, as opposed to our focus on income transfer from old to newly born generations. Further, some of the above works considered only a two-period lifetime horizon for every cohort in order to limit financial transactions between generations, whereas our setup involves a multiperiod lifetime horizon to allow for trading opportunities among generations. On the other hand, the study by Campbell and Nosbusch (2005) is similar to our investigation in that these authors explored the intergenerational risk sharing between labor and capital income in a multiperiod overlapping generations model. However, they did not focus on welfare losses associated with intertemporal substitution and cohort-specific shocks, both of which constitute a main

[3] In focusing on the intergenerational sharing of capital- income risks, some of the above authors assumed that human capital, yielding labor income, is a riskless asset. This assumption may be justified by empirical evidence that labor income has a small risk and a weak correlation with capital income (Campbell et al., 2001). However, Benzoni et al. (2005) discovered that labor and capital income are cointegrated, and that the relationship between the two variables magnifies labor-income risk in the long run. Consequently, cohort-specific shocks on permanent labor income can no longer be regarded as negligible.

interest of our paper.

Given a reasonable set of parameters concerning the Japanese labor market, a welfare loss would be 1–3% in terms of the certainty equivalence consumption level. One interesting result is that when cohort-specific shocks are procyclical in accordance with dividend shocks, generations with lower endowments would benefit much more from asset pricing dynamics than do cohorts with higher endowments. That is, although cohorts receive lower endowments when they enter an economy in a recession, they can expand their future consumption opportunities by purchasing Lucas trees at substantially cheaper prices.

In regard to the decentralization of the optimal allocation, a new cohort may reach the first-best outcome by borrowing consumption goods from existing cohorts prior to birth, and repaying outstanding debts from everyday capital income (dividends from Lucas trees) until death. This argument suggests that a policy should reflect *not* the intertemporal transfer from cohorts with higher endowments to cohorts with lower endowments, *but* the intratemporal risk sharing between endowment receivers (entry cohorts) and capital-income earners (existing cohorts).

Our paper is organized as follows. Section 2 presents our theoretical framework. Section 3 conducts numerical exercises under various assumptions with respect to the combination of preference parameters as well as the extent of intergenerational transaction opportunities. In Section 4, we derive the optimal subsidy/tax policy, and decentralize the first-best allocation. Section 5 offers concluding remarks.

2. Theoretical framework We follow Huffman (1987) as a baseline model to investigate the intergenerational sharing of cohort-specific permanent shocks through capital market transactions or by policy interventions. This section describes the basic framework in detail.

2.1. **Basic setup** We consider an overlapping generations economy. Identical consumers enter the economy at time t, and live until time $t+N-1$. The population of each cohort is constant over time, and normalized to one. A consumer aged zero at time t is endowed with y_t (> 0) units of consumption goods, and receives no endowments afterward. Here, y_t follows a first-order Markov process. There is no insurance market for this entry shock on y_t. As discussed in the Introduction, this uninsured shock on y_t is regarded as the entry shock on capitalized labor income or permanent income. As explained below, consumers use a portion of their initial endowments to obtain part of the Lucas trees, thereby allocating consumption goods over N periods.

As shown in detail in Section 4, the above structure of endowments and dividends is responsible for market incompleteness, and consequently is a fundamental source of inefficiency in this setup. More concretely, the inability of entry cohorts to trade financial claims prior to birth—in particular, to make short positions before birth—is a fundamental source of market incompleteness regardless of the presence of endowment or dividend shocks. Hence, restoration of a first-best allocation must involve fixing this type of market incompleteness or enabling prior-to-birth transactions with possible short positions.

There exist fixed K units of nondepreciable physical capital or Lucas trees. In period t, consumers sell or buy physical capital at a market price P_t, measured in terms of consumption goods. When consumers hold physical capital at the beginning of period t, they receive d_t (> 0) units of goods per capital as dividends. Again, d_t follows a first-order Markov process. As long as $N > 2$, the transaction of physical capital takes place among generations. We exclude any short position on financial instruments, including Lucas trees.

As described so far, zero-year-old consumers earn no dividends and the other consumers receive no endowments. Consumers do not hold any negative assets because they are not allowed to consume beyond their remaining capitalized labor income. Owing to the absence of bequest motives, agents exhaust physical capital up to the last period of their life.

We assume that endowments y_t and dividends d_t are generated by a four-state Markov chain z_t with state space $Z = \{z_{HH}, z_{HL}, z_{LH}, z_{LL}\}$, where the first (second) lower subscript denotes the realization of y_t (d_t). To be specific, income and dividend shocks are represented by:

$$y_t = \begin{cases} (1+\epsilon_y)\bar{y} & \text{if} \quad z_t \in \{z_{HH}, z_{HL}\} \\ (1-\epsilon_y)\bar{y} & \text{if} \quad z_t \in \{z_{LH}, z_{LL}\} \end{cases}, \tag{1}$$

$$d_t = \begin{cases} (1+\epsilon_d)\bar{d} & \text{if} \quad z_t \in \{z_{HH}, z_{LH}\} \\ (1-\epsilon_d)\bar{d} & \text{if} \quad z_t \in \{z_{HL}, z_{LL}\} \end{cases}, \tag{2}$$

where $\bar{y}$ and $\bar{d}$ are the average values, and ϵ_y and ϵ_d represent the volatility of endowments and dividends.

Following Krueger and Kubler (2003), we characterize the transition matrix of z_t by:

$$(1-\delta)\Pi + \delta I,$$

where I is a four-dimensional identity matrix, each row of Π corresponds to the stationary distribution of each state $(\pi_{HH}, \pi_{HL}, \pi_{LH}, \pi_{LL})$, and $0 \le \delta < 1$. If $\delta = 0$, then y_t and d_t are sequentially independent. An increase in δ makes shocks more serially correlated. The stationary distribution Π is assumed to be symmetric; $\pi_{HH} = \pi_{LL}$ and $\pi_{HL} = \pi_{LH}$. If $\pi_{HH} > (<)\ 0.25$, then y_t and d_t are positively (negatively) correlated with each other. When $\pi_{HH} = 0.5\ (0.0)$, y_t and d_t are perfectly positively (negatively) correlated with each other.

Let $c_{j,t+j}$ and $x_{j,t+j}$ be the consumption goods and capital holdings of an agent born in t at the age of j. A consumer maximizes a utility function characterized by the following Kreps–Porteus preference.[4] The agent's utility function at the age of j $(< N)$, denoted by $U_{j,t+j}$, is recursively defined as follows:

$$U_{j,t+j} = \left\{ c_{j,t+j}^{\frac{1-\sigma}{\sigma}} + \beta \left[E_{t+j} U_{j+1,t+j+1}{}^{1-\gamma} \right]^{\frac{1-\sigma}{\sigma(1-\gamma)}} \right\}^{\frac{\sigma}{1-\sigma}},$$

and $U_{N,t} = 0\ \forall\ t$, where E_{t+j} is the expectation operator conditional on any information

[4] See Kreps and Porteus (1978) and Epstein and Zin (1989).

available at time $t+j$. β (>0) is a discount factor, σ $(>0,\ \sigma\neq 1)$ is the elasticity of intertemporal substitution, and γ $(>0,\ \gamma\neq 1)$ is the degree of relative risk aversion.

If $\sigma\gamma = 1$, then the above specification reduces to a constant relative risk aversion preference. The maximization problem under $\sigma=\gamma=1$ (logarithmic preferences) is the same as in Huffman (1987).

A consumer born at t chooses a plan of $\{c_{j,t+j}, x_{j,t+j}\}_{j=0}^{N-1}$ to maximize $U_{0,j}$ subject to:

$$\begin{aligned} c_{0,t} + P_t x_{0,t} &= y_t, \\ c_{j,t+j} + P_{t+j} x_{j,t+j} &= \left(P_{t+j} + d_{t+j}\right) x_{j-1,t+j-1}, \text{ if } j = 1, \ldots N-1, \end{aligned}$$

$x_{j,t+j} > 0$ for $j = 0$, ..., $N-2$, and $x_{N,t+N-1} = 0$.

Let us define a competitive equilibrium for the above framework. There are two competitive markets (consumption goods and physical capital) in terms of the cross-sectional allocation. A competitive equilibrium at time t is a collection of consumers' optimal plans $\{c_{j,t}, x_{j,t}\}_{j=0}^{N-1}$ and a price P_t, such that all markets are cleared: $\sum_{j=0}^{N-1} c_{j,t} = y_t + d_t K$ for the consumption goods market, and $\sum_{j=0}^{N-2} x_{j,t} = K$ for the physical capital market at time t. By Walras's law, if the capital market is cleared, then the consumption goods market is cleared automatically.

As z_t follows a first-order Markov process, the optimal decision rules and the equilibrium price function can be represented by $c_{j,t} = c_j(\mathbf{x}_{t-1}, z_t)$, $x_{j,t} = \mathbf{x}_j(\mathbf{x}_{t-1}, z_t)$, and $P_t = P(\mathbf{x}_{t-1}, z_t)$, where $\mathbf{x}_{t-1} \equiv (x_{0,t-1}, \ldots, x_{N-1,t-1})$ is a one-period lagged capital distribution among generations. As shown later, the equilibrium process of capital prices is indeed influenced by the evolving cross-generational distribution of capital holdings. Substituting the optimal rules and the equilibrium price function into the lifetime utility function, we obtain the indirect lifetime utility of a consumer born at time t ($V(\mathbf{x}_{t-1}, z_t)$).

As shown in the next section, a shock on endowments (cohort-specific permanent shocks) as well as a shock on the dividend process may be shared partially among cohorts through the intergenerational transaction of physical capital.

2.2. Methods for evaluating risk sharing To evaluate the intergenerational allocation of cohort-specific permanent shocks, we adopt both unconditional and conditional welfare measures for the indirect lifetime utility. The conditional expected lifetime utility is the welfare evaluated after the realization of cohort-specific permanent shocks. On the other hand, the unconditional expected lifetime utility is the welfare evaluated before their realization.

As discussed in the Introduction, we make a welfare comparison under various combinations of preference parameters (σ and γ) as well as the extent of intergenerational transaction opportunities (measured in terms of N).[5] Obviously, the absolute level of welfare varies from one case to another, and comparing these absolute measures does not deliver any intuitive interpretations. For this reason, we adopt as a reference point the certainty equivalence consumption (hereafter, the CEQ consumption) for both the conditional and the unconditional lifetime welfare. Thus, the CEQ consumption level $\bar{c}$ is computed such that the computed value function, conditionally and unconditionally, may be equal to $U_0 \equiv \left\{(1+\beta+\cdots\beta^N)\bar{c}^\rho\right\}^{\frac{1}{\rho}}$.

Then, we report the ratio of the CEQ consumption for a market allocation relative to a government allocation, where government is assumed to redistribute the entire resources of consumption goods available at time t among consumers with weights according to ages:

$$\bar{c}_j\left(z_t\right) = \frac{\beta^j}{1+\beta+\cdots+\beta^{N-1}}(y_t + d_t K).$$

We refer to the above intergenerational allocation as a *simple sharing rule*. As demonstrated in Appendix A, this simple sharing rule corresponds to the first-best allocation if a preference is either logarithmic ($\sigma = \gamma = 1$), or time-additive with $\beta = 1$ ($\sigma\gamma = 1$). Although this may not be the first- best solution in more general cases, the allocation delivered by this rule always yields a substantially higher welfare than does the market allocation, as shown in the numerical examples in the next section. Consequently, the extent

[5] An increase in N expands the opportunity for physical capital transactions among generations.

to which the cohort-specific shock is shared effectively can be inferred from how close to one the unconditional CEQ consumption ratio is.

2.3. **Properties of the logarithmic preference case** In this subsection, we discuss several important properties concerning the equilibrium allocation for the logarithmic preference ($\sigma = \gamma = 1$). In this case, a closed form is available for the equilibrium characterization. That is, the optimal consumption/saving rule follows the policy below:

$$c_{0,t} = \psi_0 y_t,\ \ x_{0,t} = (1-\psi_0)\frac{y_t}{P_t}, \tag{3}$$

and

$$c_{j,t+j} = \psi_j(P_{t+j}+d_{t+j})x_{j-1,t+j-1},,\ \ x_{j,t+j} = (1-\psi_j)\frac{P_{t+j}+d_{t+j}}{P_{t+j}}x_{j-1,t+j-1}, \tag{4}$$

where $\psi_j = \frac{1}{1+\beta+\cdots+\beta^{N-1-j}}$ for $j = 0, 1, \ldots, N-2$, and $\psi_{N-1} = 1$. These optimal rules are often called 'myopic' in the sense that the rules depend only on the current state variables.

In addition, the consumption rule is written as:

$$c_{j,t+j} = \beta^j \psi_0 y_t \prod_{i=1}^{j} \frac{P_{t+i}+d_{t+i}}{P_{t+i-1}}, \quad j = 1, \ldots, N-1. \tag{5}$$

Given the capital-market-clearing condition $K = \sum_{i=0}^{N-2} x_{i,t}$, the equilibrium asset prices $(P(\mathbf{x}_{t-1}, z_t))$ is derived as:

$$P(\mathbf{x}_{t-1}, z_t) = \frac{(1-\psi_0)y_t + d_t\xi(\mathbf{x}_{t-1})}{K - \xi(\mathbf{x}_{t-1})}, \tag{6}$$

where $\mathbf{x}_{t-1} = (x_{0,t-1}, \ldots, x_{N-2,t-1})$, and $\xi(\mathbf{x}_{t-1}) = \sum_{j=1}^{N-2}(1-\psi_j)x_{j-1,t-1}$. Note that $\xi(\mathbf{x}_{t-1}) < K$ always holds.

As equation (6) implies, $P(\mathbf{x}_{t-1}, z_t)$ depends on the realization of both endowments (y_t) and dividends (d_t), as well as the one-period lagged cross-generational capital distribution $\mathbf{x}_{t-1}$. $P(\mathbf{x}_{t-1}, z_t)$ is increasing in both y_t and d_t. On the other hand, the volatility of $P(\mathbf{x}_{t-1}, z_t)$ becomes larger either when the volatility of y_t, d_t (ϵ_y, or ϵ_d) rises, or when y_t and

d_t are more positively correlated. Among the four possible states given $\mathbf{x}_{t-1}$, $P(\mathbf{x}_{t-1}, z_{HH})$ is the highest, whereas $P(\mathbf{x}_{t-1}, z_{LL})$ is the lowest. In addition, it is possible to prove that $P(\mathbf{x}_{t-1}, z_{HL}) > P(\mathbf{x}_{t-1}, z_{LH})$, if and only if $(1-\psi_0)\epsilon_y \bar{y} > \epsilon_d \bar{d} \xi(\mathbf{x}_{t-1})$.

According to equation (6), if the cross-generational distribution of capital holdings is skewed more toward younger generations, then the equilibrium price is higher.[6] The fact that equilibrium pricing is influenced by the cross-generational capital distribution $\mathbf{x}_{t-1}$ tends to generate a negative serial correlation in asset prices even if there is no serial correlation in either y_t or d_t. A lower realization of d_t, leading to a decrease in the current asset prices $P(\mathbf{x}_{t-1}, z_t)$, favors the entry generation over the older generations; that is, the entry generation can purchase capital at a lower price than can the older generations. Then, an increase in the saving of the entry generation yields stronger subsequent demand for capital, thereby sustaining asset prices in the next period. In this way, asset prices becomes higher one period after a decrease in asset prices under a lower realization of d_t. This is a major source of negative serial correlation in asset pricing.

A gross return on capital, defined as $\frac{P_{t+1}+d_{t+1}}{P_t}$, is equal to:

$$\frac{P_{t+1}+d_{t+1}}{P_t} = \frac{(1-\psi_0)y_{t+1} + d_{t+1}K}{(1-\psi_0)y_t + d_t \xi(\mathbf{x}_{t-1})},$$

whereas its average is:

$$\frac{\bar{P}+\bar{d}}{\bar{P}} = 1 + \frac{1 - \frac{\xi(\bar{\mathbf{x}})}{K}}{(1-\psi_0)\frac{\bar{y}}{\bar{d}K} + \frac{\xi(\bar{\mathbf{x}})}{K}}.$$

The above derivation implies that the average return on capital is greater than one. Thus, given zero time preferences ($\beta = 1$), the consumption profile is upward sloping on average in the market allocation,[7] whereas it is flat on average in the first-best allocation. Hence, the degree of welfare loss is associated with the extent to which the consumption profile is upward sloping at market equilibrium. In addition, it is possible to prove that as the ratio of labor income relative to capital income ($\bar{y}/\bar{d}K$) increases, the average return

[6] Note that $1-\psi_j$ is decreasing in age j.
[7] Note that the consumption profile is written as equation (5).

decreases.

The above asset pricing behavior may generate interesting properties of risk sharing among generations. That is, thanks to the asset pricing movement, a cohort with low endowments is not necessarily inferior to a cohort with high endowments in terms of welfare. Suppose that y_t and d_t are perfectly positively correlated; that is, only z_{HH} and z_{LL} emerge. In addition, there is no serial correlation in either y_t or d_t. Below, we demonstrate that the entry generation with low y_t may share risks with the entry generation with high y_t through the movement of asset pricing. Indeed, the former may attain higher lifetime welfare than the latter as a consequence of financial transactions among generations.

Given the optimal (myopic) consumption/saving rule, the unconditional indirect utility $V(\mathbf{x}_{t-1})$ $(= \sum_{j=0}^{N-1} \beta^j \ln(c_{j,t+j}))$ is expressed as:

$$
\begin{aligned}
V(\mathbf{x}_{t-1}) \quad = \quad & \psi_0 \sum_{j=0}^{N-1} \beta^j \ln(\beta^j) + (1 + \beta + \cdots + \beta^{N-1}) \ln(y_t) \\
& + (\beta + \cdots + \beta^{N-1}) \ln \frac{P_{t+1} + d_{t+1}}{P_t} + \cdots + \beta^{N-1} \ln \frac{P_{t+N-1} + d_{t+N-1}}{P_{t+N-2}}.
\end{aligned}
$$

The above equality is established by using equation (5).

In computing the conditional expectations of indirect utility $V(\mathbf{x}_{t-1}, z_t)$, y_{t+j} and d_{t+j} can be replaced by their averages $\bar{y}$ and $\bar{d}$ because there is no serial correlation in endowments or dividends. Further, it is assumed that the conditional average of P_{t+j} is constant over time. Then, $V(\mathbf{x}_{t-1}, z_t)$ may be approximated by

$$
V(\mathbf{x}_{t-1}, z_t) \approx \textit{constant} + (1 + \beta + \cdots + \beta^{N-1}) \ln(y_t) - (\beta + \cdots + \beta^{N-1}) \ln P_t.
$$

In addition, if N is sufficiently large, and β is close to one, $V(\mathbf{x}_{t-1}, z_t)$ is further approximated by $\textit{constant} + N \ln \frac{y_t}{P_t}$.

Given the above approximation, it is possible to prove that if $\epsilon_y < \epsilon_d$, then $V(\mathbf{x}_{t-1}, z_{LL}) >$

$V(\mathbf{x}_{t-1}, z_{HH})$.[8] That is, when the volatility of cohort-specific endowment shocks is small relative to that of aggregate shocks on dividends, the entry generation with low y_t can attain higher welfare than the entry generation with high y_t.

A major reason for the above welfare consequence of the market allocation is that in a recession state (z_{LL}), the entry generation suffers from a low level of human capital, but they can purchase physical capital at cheap prices as a consequence of low realization of dividends. In a boom state (z_{HH}), on the other hand, the entry generation enjoys a high level of human capital, but they are forced to purchase costly physical capital for future consumption. Given a relatively large ϵ_d, asset pricing is volatile, and P_t is low enough for the entry generation to purchase much capital during a recession period. In other words, a negative shock on permanent income borne initially by the entry cohort would be passed over to older generations (asset holders). One caveat of the above intergenerational sharing outcome is that although the welfare conditional on entry is fairly similar among all cohorts under $\epsilon_y = \epsilon_d$, the corresponding welfare remains short of the first-best allocation.

The above argument indicates that any inefficiency arising from the market allocation may not be a consequence of the failure of risk sharing between cohorts with low and high endowments. As discussed in Section 4, the optimal allocation may be attained by pooling resources among endowment receivers (entry cohorts) and capital-income earners (existing cohorts).

3. Quantitative properties of market allocation

3.1. Parameter settings Based on the theoretical framework presented in the previous section, we examine quantitative properties of the market allocation in this section, and the optimal government intervention in the next section. Our numerical exercises are based

[8] Using equation (6), we find $\frac{y_t}{P_t} = \frac{K-\xi(x_{t-1})}{(1-\psi_0)+\xi(x_{t-1})d_t/y_t}$, and

$$E(V_t|z_{LL}) > E(V_t|z_{HH}) \Leftrightarrow \frac{(1-\epsilon_y)\bar{y}}{P(\cdot, z_{LL})} > \frac{(1+\epsilon_y)\bar{y}}{P(\cdot, z_{HH})} \Leftrightarrow \frac{(1-\epsilon_y)\bar{y}}{(1-\epsilon_d)\bar{d}} > \frac{(1+\epsilon_y)\bar{y}}{(1+\epsilon_d)\bar{d}}.$$

Using simple algebra, we obtain $\epsilon_y < \epsilon_d$ as the condition under which $E(V_t|z_{LL}) > E(V_t|z_{HH})$ holds.

on the macroeconomic performance and labor market of the Japanese economy.

For this purpose, we have to determine a set of parameters including N, K, $\bar{d}$, ϵ_d, $\bar{y}$, ϵ_y, π_{HH}, δ, β, γ, and σ. It is assumed that one period corresponds to five years, and that a consumer lives for 12 periods ($N = 12$). The amount of physical capital is standardized at $K = 100$.

The average dividend ratio $\bar{d}$ is set at the average ratio of aggregate capital income relative to physical capital. Based on the capital income share of outputs and outputs per physical capital reported in Hayashi and Prescott (2004), the average ratio is $\bar{d} = 0.099$ for the period between 1974 and 2000.

According to Tauchen (1986),[9] the two-state Markov process of d_t is approximated as:

$$\begin{bmatrix} 0.89 & 0.11 \\ 0.11 & 0.89 \end{bmatrix}$$

at the annual frequency. Given the above approximated transition probability, ϵ_d is jointly approximated to be 0.1411. Hence, d_t takes either 0.08503 or 0.11297. In addition, the serial correlation coefficient in d_t is computed as $\delta = 0.289$, when one period is five years.[10]

In our framework, the capitalized labor income is endowed at the beginning of a lifetime. Hence, the average y_t/K corresponds to the average of aggregate labor income relative to physical capital. According to the labor income share of outputs and outputs per physical capital reported by Hayashi and Prescott (2004), the average of y_t is $\bar{y} = 34.9$ for the period between 1974 and 2000 given that $K = 100$.

For simplicity, we assume that the cohort-specific permanent shock is perfectly procyclical over business cycles. More concretely, the entry shock on permanent income is

[9] Tauchen (1986) proposed a method to approximate a finite number of discrete state variables and a transition probability matrix from the first-order autoregression model with normally distributed errors.

[10] Concretely, δ is chosen such that after raising the transition probability matrix to the fifth power, the first element 0.6443 may be equal to $0.5(1-\delta)+\delta$, which corresponds to the first element of the following matrix:

$$(1-\delta)\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} + \delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

completely correlated with the aggregate shock on dividends; $\pi_{HH} = 0.5$. Then, only z_{HH} and z_{LL} emerge.

The volatility of y_t (ϵ_y) is based on Genda (1997). Genda (1997) compared business-cycle-related changes in the wage profile between the age of 25 through 29 and the age of 40 through 44 for male university graduates,[11] He found that the wage profile is steeper (flatter) for workers who graduated during booms (recessions). As a concrete number, we pick up the slope (the 15-year wage growth) of the entry year 1963 (57%) as that of a boom period, and the slope of the entry year 1965 (52%) as that of a recession period.

We assume that the entry wage level is identical, that the wage profile is steep to the age of 49, and then flat to the retirement age of 64,[12] and that a discount rate is equal to the growth of wages. Then, the difference in lifetime income between the boom entry and the recession entry amounts to 5.24%. Accordingly, the volatility of y_t is equal to $\epsilon_y = 0.0262$, and y_t takes either 33.986 or 35.814. Inferred from the discussion in the previous section, our numerical setup in which ϵ_y ($= 0.0262$) is smaller than ϵ_d ($= 0.1411$) would yield strong risk sharing between the entry generation with high endowments and with low endowments.

For numerical exercises in the context of overlapping generations models, time preference β is often chosen such that the predicted lifecycle profile of asset accumulation is consistent with the observed profile among households. In our framework, however, such a lifecycle aspect is abstracted from completely. Thus, we set $\beta = 1$ for simplicity.

The above assumption about time preference yields reasonable predictions under the logarithmic preference. First, the predicted annual growth of consumption is 5.07%, and it is comparable with the observed growth between the ages of 25 and 64.[13] Second, the economy-wide consumption inequality amounts to 0.1612 in terms of the standard deviation of logarithmic consumption. Ohtake and Saito (1998) reported that the consump-

[11] Genda (1997) controlled aggregate shocks by subtracting the growth of the average wage of male university graduate workers.

[12] Such a trend in a wage profile is observed for the monthly cash earnings of male university graduates in Table 2 reported by the Ministry of Labour and Welfare (2004).

[13] The growth rate is calculated from the expenditure per household by the age group of household heads in Table 6, obtained from the Ministry of Internal Affairs and Communications (2000).

tion inequality reached 0.4911 in 1989 using the same measure. Our prediction suggests that around one-third of the economy-wide consumption inequality can be explained by business-cycle-related cohort-specific shocks on labor income; idiosyncratic shocks and population shocks, both of which are out of our consideration, may be responsible for another two-thirds of consumption inequality.

For a numerical procedure, we employ the algorithm proposed by Krusell and Smith (1998). The detailed procedure is discussed in Appendix B.

3.2. **Numerical results** We begin with the logarithmic preference case. As the first column (perfectly positive contemporaneous correlation between y_t and d_t, or $\pi_{HH} = 0.5$) of Table 1 shows, the unconditional CEQ consumption is 3.687, and the ratio relative to that of the first-best allocation is 98.9%. That is, the presence of uninsured cohort-specific shocks reduces welfare by 1.1% in terms of the certainty equivalence consumption. Such a welfare loss, relative to the optimal level of consumption, is not at all negligible.

As suggested in the previous subsection, in our numerical setup where ϵ_y ($= 0.0262$) is smaller than ϵ_d ($= 0.1411$), the CEQ consumption conditional on z_{LL} (3.745) is greater than that conditional on z_{HH} (3.630) because the entry generation with low endowments can purchase physical capital for future consumption at low prices. That is, the procyclical movement of asset pricing favors the cohort with low endowments over the cohort with high endowments. As discussed in the previous section, the serial correlation in asset pricing tends to decline in relation to the original dividend process. In our baseline case where $\delta = 0.289$, the serial correlation in asset pricing is 0.238. If δ is assumed to be zero, then it falls to -0.038.

In the case with a weaker or negative contemporaneous correlation between endowments and dividends ($\pi_{HH} = 0.25$ and 0.0 in Table 1), the movement of asset pricing does not assist sharing between the low-endowment and high-endowment entry cohorts. The CEQ consumption conditional on z_{LH} is much lower than that conditional on z_{HL} in the absence of the procyclical asset pricing. Nevertheless, the unconditional CEQ consumption is higher in the cases where $\pi_{HH} = 0.25$ or 0.0 than in the case where $\pi_{HH} = 0.5$. This is because in the latter case, the price movement excessively favors the entry cohort with low dividends as ϵ_d is much larger than ϵ_y in our numerical setup.[14]

Next, using Tables 2 through 4, we examine the impact of both intertemporal substitution (γ) and relative risk aversion (σ) on the CEQ ratio (the ratio of the unconditional CEQ consumption relative to that of the approximated first-best allocation) and asset pricing. As shown in Table 2, the unconditional CEQ consumption ratio is decreasing in σ given γ. For example, when $\gamma = 0.2$, the unconditional ratio decreases from 99.7% to 96.8% as σ increases from 0.2 to 5.0. The resulting welfare loss is serious.

Table 4 shows that the average price of physical capital is more expensive in the case of large σ than in the case of small σ. That is, stronger intertemporal motives promote the postponement of consumption, thereby boosting asset demand. Accordingly, in the case with a larger σ, it costs more to employ physical capital as a risk-sharing instrument. Accordingly, as shown in Table 3, the merit of procyclical asset pricing becomes smaller for the entry generation with low endowments, and an increase in σ narrows the difference in the conditional CEQ ratio between z_{HH} and z_{LL}.

On the other hand, risk-averse behavior would generate two opposite effects on asset pricing. First, risk-averse consumers may promote precautionary savings, which have a positive impact on asset pricing. Second, risk-averse investors may require premiums on risk assets, and discount asset pricing. According to Tables 2 through 4, when σ is less than one (weaker intertemporal motives), the former effect is dominant. That is, as γ increases, the average asset price becomes more expensive, and the unconditional and conditional CEQ ratios decrease slightly. When σ is larger than one, the former effect is almost canceled by the latter, and the risk-aversion coefficients do not have a significant effect on either asset pricing or the unconditional CEQ ratio.

Now, we evaluate the effect of the expansion of trading opportunities. In contrast to the previous setup ($N = 12$), a 60-year lifetime is divided into six periods ($N = 6$) on

[14] We numerically confirm that the unconditional CEQ consumption is highest in the case where $\pi_{HH} = 0.5$ when $\epsilon_y = \epsilon_d$.

the one hand, and into 20 periods ($N = 20$) on the other hand. Table 5 compares the unconditional CEQ ratio between the two cases. As this table clearly demonstrates, when intertemporal substitution is large, the expansion of trading opportunities enhances the CEQ ratio to some extent. For example, when $\gamma = 0.2$ and $\sigma = 5.0$, the CEQ ratio increases from 96.5% to 96.9%. That is, the expansion of trading frequency assists those with strong intertemporal saving motives.

In summary, the level and movement of asset pricing play key roles in sharing cohort-specific shocks between cohorts with high endowments and cohorts with low endowments. That is, the intergenerational risk sharing through financial transactions is effective to the extent that physical capital as a risk-sharing instrument is cheaply available. In other words, the capital market allocation is less efficient in the presence of strong demand for physical assets. In addition, the expansion of trading opportunities assists young cohorts with stronger incentives to postpone consumption.

4. On the optimal intervention and decentralized allocation

4.1. **Optimal policy** As documented in the previous section, capital market transactions among generations assist in sharing cohort-specific permanent shocks to some extent. However, the shocks remain partially uninsured, particularly under large elasticity of intertemporal substitution and limited frequency of transaction opportunities because physical capital as a risk-sharing instrument is quite costly in these cases. Therefore, there may be an opportunity for a government to directly intervene in the intertemporal or intergenerational allocation.

The preceding theoretical and numerical argument indicates that any inefficiency arising from the capital market allocation may not be a consequence of the failure of risk sharing between cohorts with low endowments and cohorts with high endowments. The current section demonstrates that the optimal allocation may be attained by pooling resources among endowment receivers (entry cohorts) and capital-income earners (existing cohorts). Concretely, when preference is logarithmic, the first-best allocation is attainable through a subsidy to the entry generation financed by a 100% levy on dividend income. In addition, this section carefully examines whether the first-best allocation may be achieved in a decentralized manner.

Suppose that a government provides a transfer to the youngest (entry) generation. To finance this transfer, the government levies taxes on the dividends of the other generations at a rate τ. In this case, we can obtain the optimal rule of consumption and saving by replacing y_t and d_t with $\tilde{y}_t = y_t + \tau d_t K$ and $\tilde{d}_t = (1-\tau)d_t$.

Given a 100% levy on dividends ($\tau = 1$), the following optimal consumption and saving rules of age j at period t are derived from equations (3) and (4):

$$\begin{aligned} c_{0,t} &= \psi_0(y_t + d_t K), \quad x_{0,t} = (1-\psi_0)\frac{y_t + d_t K}{P_t}, \\ c_{j,t} &= P_t \psi_j x_{j-1,t-1}, \quad x_{j,t} = (1-\psi_j)x_{j-1,t-1}, \quad j = 1, \ldots, N-2 \end{aligned}$$

In regard to the equilibrium price of physical capital P_t, we obtain from equation (6):

$$P_t = (1-\psi_0)\frac{y_t + d_t K}{K - \xi(\mathbf{x}_{t-1})},$$

where $\xi(\mathbf{x}_{t-1}) = \sum_{j=1}^{N-2}(1-\psi_j)x_{j-1,t-1}$.

Substituting the above pricing equation into $x_{0,t} = (1-\psi_0)\frac{y_t + d_t K}{P_t}$ yields $x_{0,t} = K - \xi(x_{t-1})$. Since $x_{j,t} = (1-\psi_j)x_{j-1,t-1}$ for $j \geq 1$, the cross-generational capital distribution $\mathbf{x}_t = (x_{0,t}, \ldots, x_{j-1,t})$ is independent of y_t and d_t, and depends only on $\mathbf{x}_{t-1}$. Hence, the capital distribution is constant over time. We denote the time-invariant capital distribution by $\bar{\mathbf{x}} = (\bar{x}_0, \ldots, \bar{x}_{N-2})$. Thus, $c_{j,t} = P_t \psi_j \bar{x}_{j-1}$ for $j \geq 1$. Note that the consumption profile as well as the asset pricing are proportional to the aggregate outcome $y_t + d_t K$. We can show that $\bar{x}_{j-1} = \beta^j \frac{1}{\psi_j} \frac{\psi_0}{1-\psi_0}[K - \xi(\bar{\mathbf{x}})]$.[15] Accordingly, we obtain:

$$c_{j,t} = \psi_0 \beta^j (y_t + d_t K), \quad j = 0, \ldots, N-1,$$

which corresponds to the consumption allocation under the optimal sharing rule (the first-best allocation).

By conducting intensive numerical calculation, we have confirmed that the above subsidy to the entry generation financed by a 100% levy on dividends would almost yield the first-best allocation even if preference is not logarithmic. For example, even in the case where $\sigma = \gamma = 5.0$, where both σ and γ are far from one, the unconditional CEQ ratio reaches 99.7% under this combination of subsidy and tax under the numerical setup discussed in the previous section.

4.2. **Decentralized allocation** Given the above optimal policy, cohort-specific shocks and dividend shocks are pooled completely, and all generations are exposed only to aggregate risks (proportional to $y_t + d_t K$) through the movement of asset prices P_t. The next question is whether the first-best allocation can be achieved in a decentralized manner.

To imitate the resource allocation delivered by the optimal policy combination of subsidies and taxes, we suppose that the entry cohort borrows an amount equivalent to the aggregate capital income prior to entry, and repays its outstanding debts using the entire capital income earned at every age until death. Is this borrowing contract arbitrage-free under the first-best allocation?

As discussed in the previous subsection, $c_{j,t} = \psi_0 \beta^j (y_t + d_t K)$ holds at the first-best allocation, and a stochastic discount factor between time t and $t+j$, $M_{t,t+j}$, can be char-

[15] Using $\bar{x}_0 = K - \xi(\bar{\mathbf{x}})$, $\bar{x}_j = (1-\psi_j)\bar{x}_{j-1}$ and $1 - \psi_{j-1} = \frac{\beta \psi_{j-1}}{\psi_j}$, we obtain:

$$\bar{x}_{j-1} = (1-\psi_{j-1})(1-\psi_{j-2}) \cdots (1-\psi_1)[K - \xi(\bar{\mathbf{x}})] = \beta^{j-1} \frac{\psi_1}{\psi_j}[K - \xi(\bar{\mathbf{x}})] = \beta^j \frac{1}{\psi_j} \frac{\psi_0}{1-\psi_0}[K - \xi(\bar{\mathbf{x}})].$$

The last equality is established by $\psi_1 = \frac{\beta \psi_0}{1-\psi_0}$.

acterized as $\beta^j \frac{c_{0,j}}{c_{j,t+j}}$ there. Consequently, we obtain as $M_{t,t+j}$:

$$M_{t,t+j} = \frac{y_t + d_t K}{y_{t+j} + d_{t+j} K}.$$

As discussed before, the cross-generational capital distribution is constant over time under the first-best allocation. A capital holding at age j is defined as $\bar{x}_j$. Thus, for the above borrowing contract, the following arbitrage condition should be satisfied prior to entry:

$$E_{t-1}\left[d_t K\right] = E_{t-1}\left[M_{t,t+1} d_{t+1} \bar{x}_0 + M_{t,t+2} d_{t+2} \bar{x}_1 + \ldots + M_{t,t+N-1} d_{t+N-1} \bar{x}_{N-2}\right].$$

After manipulation, the above condition is written as

$$E_{t-1}\left[\frac{d_t K}{y_t + d_t K}\right] = E_{t-1}\left[\frac{d_{t+1}}{y_{t+1} + d_{t+1} K}\bar{x}_0 + \frac{d_{t+2}}{y_{t+2} + d_{t+2} K}\bar{x}_1 + \ldots + \frac{d_{t+N-1}}{y_{t+N-1} + d_{t+N-1} K}\bar{x}_{N-2}\right].$$

We have $\sum_{j=0}^{N-2} \bar{x}_j = K$ from a market-clearing condition. Thus, the above equality implies that if $\frac{d_t}{y_t + d_t K}$ is sequentially independent ($\delta = 0$), then the arbitrage condition holds.[16]

Given that $\delta = 0$, the first-best allocation is attainable in a decentralized manner as long as the entry cohort can arrange the above type of borrowing contract prior to entry. The preceding argument suggests that the allocation inefficiency arising from Huffman's (1987) setup is indeed a consequence of the inability of the entry cohort to make short positions prior to entry. Viewed from a different angle, the inefficiency associated with the capital market allocation comes *not* from the failure of an intertemporal transfer from a cohort with high endowments to one with low endowments, *but* from insufficient intratemporal risk sharing by receivers of labor endowments (entry cohorts) and capital-income earners

[16] In addition, when $\epsilon_y = \epsilon_d$, $\frac{d_t}{y_t + d_t K}$ is constant over time. Consequently, we obtain

$$\frac{d_t K}{y_t + d_t K} = \frac{d_{t+1}}{y_{t+1} + d_{t+1} K}\bar{x}_0 + \frac{d_{t+2}}{y_{t+2} + d_{t+2} K}\bar{x}_1 + \ldots + \frac{d_{t+N-1}}{y_{t+N-1} + d_{t+N-1} K}\bar{x}_{N-2},$$

and the arbitrage condition can hold even after entry.

(existing cohorts).

5. Concluding remarks This paper first evaluates the intergenerational sharing of a procyclical cohort-specific shock on permanent income through the mechanism of capital market transactions, given the absence of prior-to-birth markets. The level and movement of asset pricing play a key role in sharing those shocks through active financial transactions. The market allocation is effective to the extent that physical capital is cheaply available as a risk-sharing instrument. Conversely, the capital market fails to efficiently share the cohort-specific permanent shocks in the presence of strong capital demand for intertemporal reasons, or when there is a low frequency of transaction opportunities among generations.

Our numerical investigation shows that given a reasonable set of parameters concerning the Japanese labor market, when welfare losses are evaluated prior to birth, the market incompleteness with respect to the cohort-specific permanent shock would result in welfare losses of 1–3% in terms of the certainty equivalence consumption level. That is, uninsured cohort shocks entail nonnegligible welfare costs. However, such inefficiency in the capital market transactions may not be a consequence of a failure of risk sharing between a cohort with high endowments and a cohort with low endowments; equity between the cohorts may be achieved by the movement of asset prices to some extent as long as cohort-specific shocks are procyclical.

As a possible optimal policy, we demonstrate that a subsidy to the entry generation financed by a 100% levy on dividends from Lucas trees would yield the first-best allocation as a consequence of the pooling of labor and capital income. In addition, the first-best allocation may be attained even in a decentralized manner as long as the entry cohort can make short positions prior to entry.

Given these implications from our theoretical and numerical investigation, a policy response to uninsured cohort-specific shocks should reflect *not* the intertemporal transfer from high-endowment cohorts to low-endowment cohorts, *but* the intratemporal risk sharing between labor-endowment receivers (entry cohorts) and capital-income earners (existing cohorts).

One of the theoretical limitations of our setup is that capital supply is exogenous and fixed. If this assumption is relaxed, then capital accumulation or decumulation may serve as a hedge device for cohort-specific shocks. We leave this extension to future research.

Appendix

A: Proof of the optimality of a simple sharing rule In this appendix, we prove that a simple-sharing-rule allocation $c_{j,t} = \beta^j(y_t+d_tK)/(1+\beta+\cdots+\beta^{N-1})$ for $j = 0, 1, \ldots, N-1$ is the first-best allocation if the utility function is logarithmic or time-additive ($\sigma\gamma = 1$) with $\beta = 1$.

We obtain the first-best allocation by solving a social planner's problem as of time 0. Let U_t be the lifetime utility function of a consumer born at time t. When $\sigma\gamma = 1$, W_t is represented as $W_t = \sum_{j=0}^{N-1}\beta^j u(c_{j,t+j})$ for $t \geq 0$, and $W_t = \sum_{j=-t}^{N-1}\beta^j u(c_{j,t+j})$ for $-1+N \leq t < 0$, where $u(c)$ is either $\ln(c)$ if $\gamma = 1$, or $\frac{c^{1-\gamma}}{1-\gamma}$ if $\gamma \neq 1$. The social planner maximizes a welfare function:

$$E_0[\sum_{t=-N+1}^{\infty}\lambda_t W_t],$$

subject to the feasibility constraint $\sum_{i=0}^{N-1} c_{i,t} = y_t + d_tK$ for each period t, where $\lambda_t > 0$ is a welfare weight.

The first-best consumption plan is characterized by a set of the first-order conditions:

$$\mu_t = \lambda_{t-j}\beta^j u'(c_{j,t})$$

for $j = 0, \ldots, N-1$ where μ_t is the Lagrange multiplier associated with the feasibility condition at period t.

We assume that the planner's weight on the period t entry generation (λ_t) is identical among all generations. Then:

$$u'(c_{0,t}) = \beta u'(c_{1,t}) = \ldots = \beta^{N-1}u'(c_{N-1,t})$$

holds. The allocation $c_{j,t} = \beta^j (y_t + d_t K)/(1+\beta+\cdots+\beta^{N-1})$ for $j = 0, 1, \ldots, N-1$ satisfies the above condition if preference is logarithmic or time-additive ($\sigma\gamma = 1$) with $\beta = 1$.

B: Numerical procedures In this appendix, we briefly describe the computational procedure used in our paper. Basically, we follow the algorithm proposed by Krusell and Smith (1998). In this economy, the optimal consumption rule in period t depends on the exogenous state variables z_t as well as a one-period lagged cross-generational distribution of capital holdings $\mathbf{x}_{t-1} = (x_{0,t-1}, \ldots, x_{N-2,t-1})$, while the asset pricing P_t summarizes the information concerning the wealth distribution. To ensure simplicity of calculation, we assume that the decision rules of consumers at age j in period t depend on z_t, P_t, and their own capital holding $x_{j,t-1}$. In addition, we assume that all consumers predict the capital price in period $t+1$ using the following forecasting rule:

$$\ln P_{t+1} = a\left(z_t, z_{t+1}\right) + b\left(z_t, z_{t+1}\right) \ln P_t,$$

where coefficients a and b depend on the current and future states z_t and z_{t+1}.

The Krusell–Smith algorithm proceeds as follows.

1. Make a grid of points on both individual capital holdings x and capital price P for each state of $z = z_{HH}$, z_{HL}, z_{LH}, and z_{LL}. We make 50 grid points in x, and 100 grid points in P at equal intervals. The lower bound of x is set at 0.01, whereas its upper bound is set at 40 for $N = 6$ and 20 for $N = 20$. The lower (upper) bound of P is half (1.5 times) as large as the asset price under the logarithmic preference.

2. Choose the case of the logarithmic preference without any uncertainty as the initial value of the capital distribution ($\{x_{j,0}\}_{j=0}^{N-1}$) and the forecasting rule of capital pricing.

3. Given the forecasting rule, solve the maximization problem of each age group by a backward induction at each point in the grid. Starting from $\hat{V}_N = 0$, $\hat{V}_{j+1}$ is computed

based on the following Bellman's equation:

$$\hat{V}_j(x,z,P) = \max_{x'} \left\{ f(x,x',z,P) + \beta \left[E\hat{V}_{j+1}(x',z',P')^{1-\gamma} \right]^{\frac{1-\sigma}{\sigma(1-\gamma)}} \right\}^{\frac{\sigma}{1-\sigma}},$$

where:

$$f(x,x',z,P) = \begin{cases} (y(z) - Px')^{\frac{1-\sigma}{\sigma}} & \text{if} \quad j=0 \\ ((P+d(z))x - Px')^{\frac{1-\sigma}{\sigma}} & \text{otherwise.} \end{cases}$$

$y(z)$ and $d(z)$ are determined by equations (1) and (2). The forecasting rule sets one-period ahead asset prices P'. The value of $\hat{V}_{j+1}$ for any point other than the grid points is computed by the two-dimensional piecewise linear interpolation. In this way, the decision rule of the age j cohort ($x_{j,t} = \hat{g}_j(x_{j-1,t-1}, z_t, P_t)$) is approximated.

4. Generate exogenous state variables $\{z_t\}$ for 41,000 periods. Given the initial capital distribution $\{x_{j,0}\}_{j=0}^{N-1}$ and the forecasting rule of capital prices, compute $\left\{ \{x_{j,t}\}_{j=0}^{N-1}, P_t \right\}_{t=1}^{41,000}$ using the approximated decision function $x_{j,t} = \hat{g}_j(x_{j-1,t-1}, z_t, P_t)$, and the market-clearing condition $\sum_{j=0}^{N-1} x_{j,t} = K$. Given P_t found by the bisection method, $\{x_{j,t}\}_{j=0}^{N-1}$ and $c_{j,t}$ can be calculated.

5. Discard the first 1,000 observations of the above simulated sample. Update the prediction rule by regressing P_{t+1} on P_t for a given (z_t, z_{t+1}).

6. Repeat steps 3 through 5 until the forecasting rule converges.

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Table 1: Unconditional and Conditional CEQ Consumption with Logarithmic Preferences under Different Contemporaneous Correlation Coefficients between Endowments and Dividends

Contemporaneous correlation coefficient between y and d	$+1.0$ ($\pi_{HH} = 0.5$)	0.0 ($\pi_{HH} = 0.25$)	-1.0 ($\pi_{HH} = 0.0$)
Unconditional CEQ (CEQ ratio)	3.687 (0.989)	3.688 (0.989)	3.689 (0.988)
CEQ conditional on high y and high d	3.630	3.631	-
CEQ conditional on high y and low d	-	3.801	3.803
CEQ conditional on low y and high d	-	3.575	3.578
CEQ conditional on low y and low d	3.745	3.748	-

Table 2: Unconditional CEQ Consumption Ratios under Various Combinations of Relative Risk Aversion and Elasticity of Intertemporal Substitution

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.997	0.993	0.991	0.991
$\sigma = 0.8$	0.991	0.991	0.991	0.989
$\sigma = 1.0$	-	0.989	-	-
$\sigma = 1.25$	0.986	0.986	0.986	0.986
$\sigma = 5.0$	0.968	0.968	0.968	0.969

Table 3: Conditional CEQ Consumption Ratios under Various Combinations of Relative Risk Aversion and Elasticity of Intertemporal Substitution

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.899	0.898	0.898	0.894
	1.098	1.097	1.096	1.091
$\sigma = 0.8$	0.962	0.962	0.962	0.962
	1.020	1.020	1.020	1.019
$\sigma = 1.0$	-	0.967	-	-
	-	1.011	-	-
$\sigma = 1.25$	0.972	0.972	0.972	0.972
	1.000	1.000	1.000	1.000
$\sigma = 5.0$	0.974	0.974	0.974	0.975
	0.962	0.962	0.962	0.963

(i) In each cell, the number in the above row corresponds to the CEQ consumption ratio conditional on high y and high d, whereas the number in the below row corresponds to the CEQ consumption ratio conditional on low y and low d.

Table 4: Averages and Standard Errors of Logarithmic Asset Prices

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.728 (0.144)	0.730 (0.143)	0.730 (0.143)	0.736 (0.144)
	0.863 (0.052)	0.864 (0.052)	0.864 (0.052)	0.870 (0.053)
	0.592 (0.046)	0.594 (0.045)	0.595 (0.045)	0.600 (0.046)
$\sigma = 0.8$	0.785 (0.057)	0.785 (0.057)	0.785 (0.057)	0.786 (0.057)
	0.842 (0.008)	0.842 (0.008)	0.842 (0.008)	0.842 (0.008)
	0.728 (0.008)	0.728 (0.008)	0.728 (0.008)	0.729 (0.008)
$\sigma = 1.0$	-	0.803 (0.049)	-	-
	-	0.851 (0.006)	-	-
	-	0.754 (0.005)	-	-
$\sigma = 1.25$	0.831 (0.040)	0.831 (0.040)	0.831 (0.040)	0.830 (0.040)
	0.871 (0.003)	0.870 (0.003)	0.870 (0.003)	0.870 (0.003)
	0.791 (0.003)	0.791 (0.003)	0.791 (0.003)	0.790 (0.003)
$\sigma = 5.0$	1.037 (0.017)	1.037 (0.017)	1.037 (0.017)	1.036 (0.017)
	1.054 (0.003)	1.054 (0.003)	1.054 (0.003)	1.053 (0.003)
	1.021 (0.003)	1.020 (0.003)	1.020 (0.003)	1.019 (0.003)

(i) The numbers in parentheses are the standard errors of logarithmic asset prices.
(ii) In each cell, the number in the above row corresponds to the unconditional average, whereas the numbers in the middle and below rows correspond to the averages conditional on high y and high d, and low y and low d.

Table 5: Effects of Time Diversification ($N = 6$ versus $N = 20$)

	$\gamma = 0.2$	$\gamma = 1.0$	$\gamma = 1.25$	$\gamma = 5.0$
$\sigma = 0.2$	0.996⇒0.997	0.992⇒0.993	0.991⇒0.991	0.989⇒0.993
$\sigma = 0.8$	0.989⇒0.991	0.989⇒0.991	0.989⇒0.991	0.988⇒0.989
$\sigma = 1.0$	-	0.987⇒0.989	-	-
$\sigma = 1.25$	0.984⇒0.987	0.984⇒0.987	0.984⇒0.987	0.984⇒0.987
$\sigma = 5.0$	0.965⇒0.969	0.965⇒0.969	0.965⇒0.969	0.966⇒0.970

(i) In each cell, the left-sided (right-sided) number corresponds to the unconditional CEQ consumption ratio in the case of $N = 6$ ($N = 20$).