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Abstract

In this paper we consider a simple model of an industry with network externalities, where a benefit of each consumer from network services depends on the size of the network. We first consider a single network and cover the cases with and without fixed cost of entry. We then turn to the two network industry, where the incumbent network and a new entrant network compete for the market and they may differ both in their marginal costs and demand structures. In addition, we identify several situations where public policy may play a crucial role in sustaining socially advantageous network service provision.

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1 Introduction

Telecommunications industry is one of the constant sources of fascinating theoretical as well as policy-oriented issues in the economics of industrial organization, regulation, and competition policy. One crucial feature of this industry is the presence of network externalities, whereby the benefit accruing to each user of the services it provides depends not only on the intrinsic value of these services to him/herself, but also on how many other users are involved in the same market. This feature alone can create an important policy issue, since the industry may never be started in the presence of network externalities unless some public measures can convince consumers of the persistence of this industry at a sufficiently large scale. It also changes the nature of competition in this industry substantially from that in, say, manufacturing industries with a result that the nature of regulation and competition policies for the purpose of improving the industry performance from the viewpoint of social welfare will have to be designed completely anew.[1]

Another crucial feature of telecommunications industry is that it is a typical example of network infrastructure industry with economies of vertical integration between natural monopoly segments, viz. local telecommunications services, and potentially competitive segments, viz. long-distance telecommunications services. Thus, the successful provision of telecommunications facilities should be construed as an essential part of any comprehensive program for initiating and sustaining economic development.[2] Closely connected with this second feature is an important policy issue of designing and implementing the privatization of state monopoly in natural monopoly segments and/or the introduction of actual competition in potentially competitive segments.[3] It is in view of these fascinating problems which are posed by telecommunications industry that this paper attempts to re-examine in the simplest possible way the standard theory of competition, regulation and welfare in the presence of network externalities.

Apart from this introduction, the paper consists of three sections. Section 2 is devoted to the simple prototype case of single network with the purpose of crystallizing the

[1]Seminal contributions to the analysis of network externalities include Rohlfs (1974), Oren and Smith (1981), and Katz and Shapiro (1985).

[2]See Joscow (1999) and Suzumura (1999) for alternative scenarios of regulatory reform in network infrastructure sectors in developing countries.

[3]Traditionally, telecommunications industry was construed to be natural monopoly altogether in view of the large fixed cost in the form of initial investment in fixed network facilities. It was due to the recent development of optic fibre technology, which incurs much less fixed cost in constructing long-distance networks, that long-distance telecommunications services came to be recognized as potentially competitive.

implications of network externalities in isolation. The analysis begins with the case with no fixed cost. Even in this simplest possible case, there may exist multiple fulfilled expectations equilibria where all consumers correctly foresee the network's size. It is shown that the welfare optimal fulfilled expectations equilibrium is locally stable, but it can be attained only when consumers are convicted that the network size exceeds some critical mass. These crucial features of the economies with network externalities are analyzed under the assumption of marginal cost pricing. We then proceed to the case of monopoly pricing. This is done partly for the sake of welfare comparisons between marginal cost pricing and monopoly pricing, and partly for the sake of examining the effect of fixed cost on the welfare-enhancing government policies. Section 3 turns to the case of two networks, where the incumbent network and a new entrant network, which are assumed to differ both in their marginal costs and the demand structures, compete with each other for the market. To confine the complexity of our analysis within reasonable bound, this analysis is conducted under the assumption of no fixed cost, the policy issues related to fixed costs and government policies to cope with these issues having been treated separately in Section 2. We identify stable and unstable fulfilled expectations equilibria, and analyse the welfare performance of stable fulfilled expectations equilibria. Our verdict on the welfare performance of competition between the incumbent network and a new entrant network culminates into the proposition which asserts that either the incumbent network alone, or a new entrant network alone, but never both, serve the consumers best. Section 4 concludes this paper with three final remarks on the directions to be further explored in the future.

2 Single Network Case

To begin with, let us consider the case of a single network X that sells a homogeneous service. Each consumer either purchases one unit of the service, or stays out of the market. The consumers differ with respect to their constant marginal utility of network's size. We assume that the distribution of consumers' marginal utilities of network's size is given by the uniform distribution over the closed interval $[0, 1]$ with a total mass of all potential consumers being equal to one. The network's size, i.e., the mass of consumers who decided to enter the market, is denoted by $x \in [0, 1]$. Then the gross benefit of a consumer with marginal utility α is assumed to be simply αx, which captures the phenomenon of network externalities in the simplest possible way.

Assume first the marginal cost of providing a unit of service of the network is constant and equal to C_X. Assume further that the firm uses the *marginal cost pricing* and charges the per unit price equal to C_X with $0 < C_X < 1$. We also assume that all consumers have identical expectations with respect to the network's size. Thus, for an expected size of network $x \in [0, 1]$, the set of consumers who enter the market is given by

$$S(x) = \{\alpha \in [0, 1] | \alpha x - C_X > 0\}.$$

Denote by $\alpha(x)$ the highest value of α that ensures a consumer with marginal utility α does not belong to $S(x)$. Since $C_X > 0$, $\alpha(x)$ is also positive. The function $\alpha(x)$ is decreasing in x and is given by:

$$\alpha(x) = \begin{cases} 1 & if \quad x \leq C_X \\ \frac{C_X}{x} & if \quad x > C_X. \end{cases}$$

Since the mass of consumers who enter the market is $1 - \alpha(x)$, it follows that the set $S(x)$ is empty if $0 \leq x \leq C_X$. However, if $x > C_X$, the set $S(x)$ represents a nonempty connected interval

$$S(x) = (\alpha(x), 1].$$

Now we can introduce the notion of *a fulfilled expectations equilibrium*, FEE for short, where all consumers correctly foresee the network's size.

Definition 2.1: The network's size x^* is *a fulfilled expectations equilibrium* if the mass of $S(x^*)$ is equal to x^*.

This definition immediately yields the equilibrium condition

$$\alpha(x^*) = 1 - x^*.$$

It is easy to see that a corner solution $x^* = 0$ always represents an equilibrium. Figure 1 exhibits all three possible cases, which are classified in terms of the marginal cost C_X.

[Insert Figure 1 around here]

In the case of an interior solution x, the following equation must be satisfied:

$$x(1 - x) - C_X = 0. \tag{1}$$

Obviously, interior equilibria exist only if $0 < C_X < \frac{1}{4}$ and we restrict our further considerations to this range of marginal costs. Indeed, in this case there are two interior equilibria, x^{**} and x^*, where $C_X < x^{**} < \frac{1}{2} < x^* < 1$ (see Figure 2).

[Insert Figure 2 around here]

We now turn to the stability properties of these equilibria.

2.1 Stability Analysis

Consider an adjustment process that starts at the situation where the original network's size $x(0)$ is not necessarily an equilibrium. In this case, in the next time period $t = 1$, the consumers revise their expectations by turning to a new size $x(1)$ which is determined on the basis of the original size $x(0)$ according to the following formula:

$$x(1) = 1 - \alpha(x(0)).$$

Our previous argument implies that if $x(0) \leq C_X$, then $x(1) = 0$. Otherwise the following equality holds

$$\{1 - x(1)\}\, x(0) - C_X = 0.$$

In the next stage $t = 2$, a similar adjustment is made when $x(2) = 1 - \alpha(x(1))$ is chosen. Again, if $x(1) \leq C_X$, then $x(2) = 0$. Otherwise, the following equality holds

$$\{1 - x(2)\}\, x(1) - C_X = 0.$$

By continuing this process, we generate the sequence $\{x(t)\}_{t\,=\,1}^{\infty}$, which satisfies $x(t+1) = 1 - \alpha(x(t))$. That is, $x(t+1) = 0$ if $x(t) \leq C_X$, and

$$\{1 - x(t+1)\}x(t) - C_X = 0 \tag{2}$$

if $C_X < x(t) \leq 1$. We shall call a fulfilled expectations equilibrium *stable* if the adjustment process which starts sufficiently close to the equilibrium would converge to it. More specifically,

Definition 2.1.1: A fulfilled expectations equilibrium $\widehat{x} \in [0, 1]$ is *stable* if there is a positive ε such that for any $x(0) \in (\widehat{x} - \varepsilon, \widehat{x} + \varepsilon) \bigcap [0, 1]$, the adjustment process converges to $\widehat{x}$.

We can assert the following:

Proposition 2.1.2: The fulfilled expectations equilibria $x = x^*$ and $x = 0$ are stable, whereas $x = x^{**}$ is not.

This proposition follows from the following:

Lemma 2.1.3: (i) If the initial size $x(0)$ satisfies $1 > x(0) > x^*$, then the adjustment process converges to x^*.

(ii) If the initial size $x(0)$ satisfies $x^* > x(0) > x^{**}$, then the adjustment process converges to x^*.

(iii) If the initial size $x(0)$ satisfies $0 < x(0) < x^{**}$, then the adjustment process converges to 0.

Proof: (i) Let $x(0) > x^*$. Then, by (2) for $t = 0$, $x(1) < x(0)$. Since (1) holds for x^* it also implies that $x(1) > x^*$. Thus, $x(0) > x(1) > x^*$. By repeating this argument we show that $\{x(t)\}_{t=1}^{\infty}$ is a decreasing monotone sequence to the right of x^*. Since (1) has no root to the right of x^*, it follows that $\lim_{t\to\infty} x(t) = x^*$.
(ii) Let $x^* > x(0) > x^{**}$. Then, by (2) for $t = 0$, $x(1) > x(0)$. Since (1) holds for x^*, it also implies that $x(1) < x^*$. Thus, $x(0) < x(1) < x^*$. By repeating this argument, we show that $\{x(t)\}_{t=1}^{\infty}$ is an increasing monotone sequence in the interval (x^{**}, x^*). Since (1) has no root in this open interval, it follows that $\lim_{t\to\infty} x(t) = x^*$ as well.
(iii) If $0 < x(0) \leq C_X$, then $x(1) = 0$ and therefore, $x(t) = 0$ for all $t \geq 1$. Let, therefore, $C_X < x(0) < x^{**}$. Then, by (2) for $t = 0$, $x(1) < x(0)$. If $x(1) \leq C_X$, then $x(2) = 0$ and all further $x(t)$ will be equal to 0. If $x(1) > C_X$, by repeating this argument we conclude that there is T such that $x(T) = 0$. Otherwise, there would exist a $\bar{y} < x^{**}$ satisfying (1), a contradiction. □

Remark 2.1.4: (a) Since the fulfilled expectations equilibrium x^{**} is unstable, the network X will have to die out unless consumers are somehow convinced that the size of X exceeds the critical mass of x^{**}. Even at this primitive level, therefore, there is an ample room for public policy to sustain the provision of network services.

(b) Since the fulfilled expectations equilibrium x^* is stable, the network size x^* will be eventually established if consumers are convinced that the size of X exceeds the critical mass of x^{**}, but the market will not be fully covered as $x^* < 1$.

Remark 2.1.5: By virtue of the marginal cost pricing in the absence of fixed cost, producer's surplus is zero. Thus, the total welfare consists solely of consumers' surplus, which is defined by

$$CS(x) = \int_{\alpha \in S(x)} (\alpha x - C_X)\, d\alpha = \{1 - \alpha(x)\} \left[\{1 + \alpha(x)\} \frac{x}{2} - C_X \right].$$

Invoking the equilibrium condition $\alpha(x) = 1 - x$, we may obtain $CS(x) = \frac{x^3}{2}$ at $x \in \{0, x^*, x^{**}\}$, so that we obtain the following unambiguous verdict: $CS(0) < CS(x^{**}) < CS(x^*)$.

Remark 2.1.6: It is easy to verify that, if $x(0) > x^{**}$, then the rate of convergence of $x(t)$ to x^* tends to $\frac{x^*}{1-x^*} > 1$. Indeed, denote $a(t) = x(t) - x^*$. Then (2) can be rewritten as

$$(1 - x^*)a(t) - x^* a(t+1) - a(t+1)a(t) = 0,$$

or

$$\frac{a(t+1)}{a(t)} = \frac{1 - x^*}{x^*} - \frac{a(t+1)}{x^*}.$$

Thus, as $a(t)$ approaches 0, the distance between $x(t)$ and x^* shrinks at the rate converging to $\frac{x^*}{1-x^*}$ for large t.

2.2 Monopoly Pricing

The existing network may deviate from the marginal cost pricing and charge a per unit price $P > C_X$. Let us focus on the stable interior fulfilled expectations equilibrium size of the network. We obtain the following expression for the network's profits $\pi(P)$:

$$\pi(P) = \begin{cases} 0 & if \quad P > \frac{1}{4} \\ x(P)(P - C_X) & if \quad \frac{1}{4} \geq P \geq C_X, \end{cases}$$

where

$$x(P) = \frac{1}{2} + \sqrt{\frac{1}{4} - P},$$

which is the larger solution of the equation $x(1 - x) = P$. The graph of $\pi(P)$ is depicted in Figure 3.

[Insert Figure 3 around here]

To find its maximum, we equate the derivative of $\pi(P)$ to zero:

$$\sqrt{\frac{1}{4} - P} - P + C_X + \frac{1}{2} - 2P = 0.$$

This equation has two roots:

$$P_1 = \frac{1 + 3C_X + \sqrt{1 - 3C_X}}{9} \text{ and } P_2 = \frac{1 + 3C_X - \sqrt{1 - 3C_X}}{9}.$$

Since $0 < C_X < \frac{1}{4}$, it is easy to verify that

$$P_2 < C_X < P_1 < \frac{1}{4}.$$

Thus, the monopolistic price $P^m\,(C_X)$ is given by the following expression:

$$P^m(C_X) = \frac{1 + 3C_X + \sqrt{1 - 3C_X}}{9}.$$

Observe that:

Remark 2.2.1: The monopoly price is an increasing function of C_X. Indeed,

$$\frac{dP^m(C_X)}{dC_X} = \frac{1}{6}(2 - \frac{1}{\sqrt{1 - 3C_X}}).$$

It is easy to see that this expression is positive whenever $C_X < \frac{1}{4}$.

Thus, the higher the marginal cost, the higher is the monopolistic price. Moreover, raising the market price above marginal cost level would have a negative impact on the total welfare. Indeed, for any positive price $P \in (C_X, \frac{1}{4})$ the welfare function $W(P, C_X)$ is defined by

$$\begin{aligned} W(P, C_X) &= (P - C_X)x(P) + \int_{1-x(P)}^{1} \{\alpha x(P) - P\}\, d\alpha \\ &= (P - C_X)\, x(P) + \frac{\{x(P)\}^3}{2} \end{aligned} \tag{3}$$

Since the first term has zero first derivative and $x(P)$ is a decreasing function of P, the last expression declines in P. To be more explicit, we may easily check that

$$\frac{\partial W(P, C_x)}{\partial P} = (P - C_X)x'(P) + \frac{1}{2}\{3Px'(P) - x(P)\}, \tag{4}$$

which is negative in view of $x'(P) < 0$. Note that the first term in the RHS of (4) represents the usual increase in deadweight loss, whereas the second term thereof represents the additional welfare loss due to network externalities. Therefore,

Remark 2.2.2: For every C_X, the welfare function $W(\cdot, C_X)$ is decreasing on the interval $(C_X, \frac{1}{4})$. In particular, the total welfare under marginal cost pricing is higher than in the case where the firm charges the monopolistic price.

2.3 Fixed Cost and Government Policies

Let us now consider a situation where the firm X has to pay a fixed cost $F > 0$ in order to enter the industry. In this case, even the monopolistic profits would not necessarily be sufficient to cover its fixed cost. Then it is natural to ask whether any public assistance can help the monopolistic service provider to overcome this difficulty in such a way as to be justified in terms of social welfare.

Before analyzing the types of government assistance which may cope with this problem and make the monopolistic firm viable, let us prove the following useful result.

Lemma 2.3.1: Let a level of per unit subsidy $s < C_X$ be given. Then the total welfare under the monopolistic price without subsidy, $W(P^m(C_X), C_X)$, and that with subsidy, $W(P^m(C_X - s), C_X - s)$, satisfy

$$W(P^m(C_X - s), C_X - s) - W(P^m(C_X), C_X) > sx(P^m(C_X - s)),$$

which means that the introduction of a subsidy causes a raise in the total welfare by the amount which exceeds the budget spent on subsidies.

Proof: By rearranging (3) we have

$$W(P^m(C_X - s), C_X - s) = \{P^m(C_X - s) - (C_X - s)\}\, x(P^m(C_X - s)) + \frac{\{x(P^m(C_X - s))\}^3}{2},$$

whereas

$$W(P^m(C_X), C_X) = \{P^m(C_X) - C_X\}\, x(P^m(C_X)) + \frac{\{x(P^m(C_X))\}^3}{2},$$

where use is made of the equilibrium condition $x(P)\{1 - x(P)\} = P$.
Thus

$$W(P^m(C_X - s), C_X - s) - W(P^m(C_X), C_X) =$$
$$= sx(P^m(C_X - s)) + W(P^m(C_X - s), C_X) - W(P^m(C_X), C_X).$$

However, by Remark 2.2.1, the monopolistic price is an increasing function of C_X, while by Remark 2.2.2, the total welfare is a decreasing function of the price. Thus, it follows that

$$W(P^m(C_X - s), C_X - s) - W(P^m(C_X), C_X) > sx(P^m(C_X - s)).$$

□

Consider now two types of government intervention which are meant to make the monopolistic firm viable in the presence of fixed cost: *budget-neutral subsidy without regulation* and *budget-neutral subsidy with regulation.*

In the first type, the government offers a per unit subsidy s and preserves the budget neutrality by charging consumers and producer the total lump sum payment of $sx(P^m(C_X - s))$ in such a way as to guarantee the firm positive profits if it enters. Since by Lemma 2.3.1, any subsidy is welfare improving, the welfare function is decreasing in price, and the monopolistic price is increasing in marginal cost, the subsidy s should be such that it reduces the marginal cost to zero, i.e., $s = C_X$.

However, under regulation, the best the government can do is to offer the subsidy and, in addition, to force the firm to charge its marginal cost only. Once again, the subsidy should be equal to marginal cost C_X.

It is easy to check that $P^m(0) = \frac{2}{9}$, $x(P^m(0)) = \frac{2}{3}$ and $\pi(P^m(0)) = \frac{4}{27}$. Depending on the level of fixed cost, our verdict on the viability of the monopoly firm goes as follows:

Proposition 2.3.2: (i) $F \geq \frac{1}{2}$. Then even the budget-neutral subsidy with regulation would not work as the total consumer surplus even at zero price, viz. $W(0,0)$, is not sufficient to cover the fixed cost.

(ii) $\frac{1}{2} > F \geq \frac{4}{27}$. Then a budget-neutral subsidy without regulation would not work as even at zero marginal cost the monopoly profit $\pi(P^m(0))$ would not cover the fixed cost. The only way to force the firm to enter is via budget-neutral subsidy with regulation. In this case the firm will charge the zero price and will be compensated for the fixed cost F.

(iii) $\frac{4}{27} > F \geq \{P^m(C_X) - C_X\}\, x(P^m(C_X))$. Then a budget-neutral subsidy without regulation would work as the firm would make a profit $\pi(P^m(0))$ at zero marginal cost.

(iv) $F < \{P^m(C_X) - C_X\}\, x(P^m(C_X))$. Then the firm enters without any intervention.

This completes our preliminary analysis of the single network case. Although it is concerned with an extreme case of no competition, this analysis proves useful in our analysis of competitive networks. To this analysis we now turn.

3 Two Network Case

Back, then, to the case of no fixed cost, and let us consider the case where there are two networks, viz. the incumbent network X and a new entrant network Y. Given previous experience and established reputation of the network X, we assume that, given the size of incumbent network x, all consumers have the same utility from purchasing a unit of service from the network X. At the same time, due to unknown quality of the service provided by a new entrant network, consumers are assumed to have different marginal utility from purchasing a unit of service from the entrant network. Specifically, if the sizes of the two networks are x and y, respectively, the gross benefit of every consumer from purchasing a unit of service are $\bar{\alpha}x$ for network X and βy for network Y, where $0 < \bar{\alpha} < 1$ and β is uniformly distributed over the closed interval $[0, 1]$. The constant marginal costs for two networks are C_X and C_Y, respectively. We assume that $0 < C_X, C_Y < 1$. Note that we are not ruling out the possibility of cost advantage for either firm. The cost advantage may be in favour of the incumbent firm due to the advantageous effect of learning by doing, whereas the cost advantage may be the other way round as the new entrant may be equipped with better and more advanced facilities, and the net effect may go either way. It is also worth pointing out that consumers' evaluation of the two networks may work to the benefit of either competitor.

To modify the notion of fulfilled expectations equilibrium from the case of single network to the case of two networks, we must take into account that the consumers may be partitioned into three groups: those who purchase a unit of service from the network X, those who purchase a unit of service from the network Y, and those who stay out of the market altogether.

3.1 Marginal Cost Pricing

Consider first the case where both firms charge consumers their respective marginal cost only. As in the single network case, the consumers are assumed to form their expectations regarding the networks' size, x and y, where x and y are nonnegative numbers, the sum of which not exceeding 1. We assume that consumers enter the market only if their net benefit from a unit of service is greater than 0. It is also assumed that, if a consumer is indifferent between the two networks, she would purchase a unit of service from the incumbent network. Thus, we have three sets, $S_X(x,y)$, viz. the customers of X, $S_Y(x,y)$, viz. the customers of Y and $S_Z(x,y)$, viz. those who stay out of the market:

$$\begin{aligned} S_X(x,y) &= \{\beta \in [0,1] | \bar{\alpha}x - C_X > 0 \text{ and } \bar{\alpha}x - C_X \geq \beta y - C_Y\} \\ S_Y(x,y) &= \{\beta \in [0,1] | \beta y - C_Y > \max\ [0, \bar{\alpha}x - C_X]\} \\ S_Z(x,y) &= \{\beta \in [0,1] | \max\ [\beta y - C_Y, \bar{\alpha}x - C_X] \leq 0\}. \end{aligned}$$

We have the following definition of a fulfilled expectations equilibrium:

Definition 3.1.1: A pair of two nonnegative numbers x and y satisfying $x + y \leq 1$ is *a fulfilled expectations equilibrium* if the mass of $S_X(x,y)$ is equal to x and the mass of $S_Y(x,y)$ is equal to y.

Let us make two important observations concerning a fulfilled expectations equilibrium. Let (x,y) be a FEE.

Observation 3.1.2: If $x > 0$, no consumer stays out of the market, and the market is fully covered. Indeed, the inequality $\bar{\alpha}x - C_X > 0$ implies that the set $S_Z(x,y)$ is empty. Thus, $x > 0$ cannot but imply $x + y = 1$.

Observation 3.1.3: A size of the entrant network y is always less than 1. Indeed, consumers whose marginal utility β does not exceed C_Y will never purchase a unit of entrant's service.

These two observations imply that there could be only four types of fulfilled expectations equilibria: two corner solutions, viz.,

$(0,0)$: *dead market,*
$(1,0)$: *complete coverage incumbent monopoly*

and two interior solutions, viz.,

$(0,y)$ with $y < 1$: *incomplete coverage entrant monopoly,*
(x,y) with $x, y > 0$ and $x + y = 1$: *complete coverage duopoly.*

The following proposition easily summarizes the analysis of the first two types of equilibria:

Proposition 3.1.4: (0,0), viz. the dead market, is always a FEE.

(1,0), viz. the complete coverage incumbent monopoly, is a FEE if and only if $\bar{\alpha} > C_X$. That is, the incumbent monopoly is sustainable in equilibrium only if consumers' evaluation of the network X is sufficiently favourable.

Before analyzing the interior solutions, note that only consumers with higher marginal utility values of β may consider purchasing a unit of service from the entrant network:

Observation 3.1.5: Let (x, y) be a FEE with $0 < y < 1$. Then $S_Y(x, y) = (1 - y, 1]$.

Proposition 3.1.6: $(0, y)$, viz. an incomplete coverage entrant monopoly, is a FEE if and only if there exists a marginal consumer β who is indifferent whether to purchase a unit of service from the network Y, or to stay out of the market. In other words, $y = 1 - \beta$ should satisfy

$$y(1 - y) - C_Y = 0.$$

As we have shown in the case of a single network, an incomplete coverage entrant monopoly is an equilibrium only if $C_Y \leq \frac{1}{4}$. In particular, $C_Y < \frac{1}{4}$ yields two distinct sizes of the entrant's monopoly:

$$y^* = \frac{1}{2} + \sqrt{\frac{1}{4} - C_Y} \text{ and } y^{**} = \frac{1}{2} - \sqrt{\frac{1}{4} - C_Y}.$$

Now we turn to the duopoly case.

Proposition 3.1.7: (x, y) with $x, y > 0$ and $x + y = 1$, viz. a complete coverage duopoly, is a FEE if and only if there exists a marginal consumer β' who is indifferent between the two networks. Moreover, this consumer is guaranteed a positive net benefit by purchasing a unit of service from either of the two networks. That is, the size of the incumbent network $x = \beta'$ satisfies

$$x(1 - x) - C_Y = \bar{\alpha}x - C_X > 0.$$

In terms of the entrant network, the condition for the complete coverage duopoly amounts to

$$-y^2 + (1 + \bar{\alpha})y - \bar{\alpha} - d = 0, \tag{5}$$

where $d := C_Y - C_X$ is the cost differential of two networks. Let us analyze the equation (5) (See Figure 4).

[Insert Figure 4 around here]

It is immediate to observe that (5) has at least one root within the interval $(0,1)$ only if

$$-\bar{\alpha} < d \leq \frac{(1-\bar{\alpha})^2}{4}. \tag{6}$$

If the RHS inequality is strict, (5) has two roots, which are given by:

$$y'' = \frac{1+\bar{\alpha}-\sqrt{(1-\bar{\alpha})^2-4d}}{2} \text{ and } y' = \frac{1+\bar{\alpha}+\sqrt{(1-\bar{\alpha})^2-4d}}{2}.$$

If $\frac{(1-\bar{\alpha})^2}{4} > d > 0$, then both y' and y'' belong to the open interval $(\bar{\alpha}, 1)$, whereas $y'' \in (0, \bar{\alpha})$ and $y' \in (1, +\infty)$ if $0 > d > -\bar{\alpha}$. Finally, we have to guarantee that the participation constraints are also satisfied. In particular, for y' to be a feasible solution we should have $\bar{\alpha}(1-y') > C_X$, or $y' < 1 - \frac{C_X}{\bar{\alpha}}$. In other words, the value of the parabola $-y^2 + (1+\bar{\alpha})y - \bar{\alpha}$ at the point $1 - \frac{C_X}{\bar{\alpha}}$ should be lower than at y'. That is,

$$-\frac{C_X}{\bar{\alpha}}(\bar{\alpha} - 1 + \frac{C_X}{\bar{\alpha}}) < d,$$

or

$$C_Y > \frac{C_X}{\bar{\alpha}} - (\frac{C_X}{\bar{\alpha}})^2. \tag{7}$$

In addition, it should be guaranteed that the point $1 - \frac{C_X}{\bar{\alpha}}$ is to the right of $\frac{1+\bar{\alpha}}{2}$, i.e.,

$$1 - \frac{C_X}{\bar{\alpha}} > \frac{1+\bar{\alpha}}{2}. \tag{8}$$

Thus, the root y' satisfies (5) together with the participation constraints if and only if (6)-(8) hold. For any given value of $\bar{\alpha} \in (0,1)$ we denote by $F(\bar{\alpha})$ the set of all possible values of C_X and C_Y satisfying (6)-(8). This set is depicted by the shaded area in Figure 5. Note, in particular, that a complete coverage duopoly never exists for $(C_X, C_Y) \notin F(\bar{\alpha})$.

[Insert Figure 5 around here]

An important implication of the participation constraints is that $d > 0$ must be the case. Indeed, it follows from (7) that

$$d = C_Y - C_X > \frac{C_X}{\bar{\alpha}}\{(1-\bar{\alpha}) - \frac{C_X}{\bar{\alpha}}\} > \frac{1-\bar{\alpha}}{2\bar{\alpha}} C_X > 0,$$

where the penultimate inequality is due to (8).

3.2 Stability of Equilibria

In this subsection, we analyze stability properties of all four types of FEE described above. As before, we consider an adjustment process that starts at the situation where the expected size of the incumbent network $x(0)$ and that of the entrant network $y(0)$ do not necessarily form a fulfilled expectations equilibrium. In this case, in the next time period $t = 1$, the consumers revise their expectations by turning to a new size $x(1)$ for the incumbent network and $y(1)$ for the entrant network. However, we have to distinguish between two cases: one is where the incumbent network is driven out from the market, and the other is where the incumbent network has a positive market share.

To begin with, consider a FEE (x, y), where $x = 0$. There are two types of such equilibria, $(0, 0)$, viz. the dead market, and $(0, y)$ with $0 < y < 1$, viz. an incomplete coverage entrant monopoly. The adjustment process determines a revised size of the incumbent network $x(1)$ on the basis of the original size $x(0)$ and that of the entrant network $y(1)$ on the basis of the original size $y(0)$ as follows. If $x(0)$ is small enough, say $\bar{\alpha}x(0) < C_X$, then $x(1) = 0$. It then follows that $x(t) = 0$ for all $2 \leq t < +\infty$. As to the size of the entrant network, we have

$$y(1) = 1 - \beta(y(0)),$$

where

$$\beta(y) = \begin{cases} 1 & if \quad y \leq C_Y \\ \frac{C_Y}{y} & if \quad y > C_Y. \end{cases}$$

Again, if $y(0) \leq C_Y$, then $y(1) = 0$. Otherwise, the following equality holds

$$(1 - y(1))y(0) - C_Y = 0.$$

By continuing this process, we can generate the sequence $\{(x(t), y(t))\}_{t=1}^{\infty}$. We shall call a fulfilled expectations equilibrium *stable* if the adjustment process which starts sufficiently close to the equilibrium would converge to it. More specifically:

Definition 3.2.1: A fulfilled expectations equilibrium $(0, y)$, where $0 < y < 1$, is *stable* if there exists $\varepsilon > 0$ such that, for any $x(0)$ with $0 < x(0) < \varepsilon$ and any positive $y(0)$ with $|y(0) - y| < \varepsilon$, the adjustment process converges to $(0, y)$.

The analysis of the previous section allows us to establish the following:

Proposition 3.2.2: (i) The fulfilled expectations "dead market" equilibrium $(0, 0)$ is stable.

(ii) An incomplete coverage entrant monopoly $(0, y^*)$ where $y^* = \frac{1}{2} + \sqrt{\frac{1}{4} - C_Y}$ is stable, whereas an equilibrium $(0, y^{**})$ where $y^{**} = \frac{1}{2} - \sqrt{\frac{1}{4} - C_Y}$ is not.

Consider now an equilibrium in which the incumbent network enjoys a positive market share. There are two types of such equilibria, $(1, 0)$, viz. a complete coverage incumbent monopoly, and (x, y) with $0 < x, y < 1$ and $x + y = 1$, viz. a complete coverage duopoly.

Take first a complete coverage incumbent monopoly. Let $x(0)$ and $y(0)$ be such that $1 - \varepsilon < x(0) < 1$ and $0 < y(0) < \varepsilon$ for a small $\varepsilon > 0$. Assume that $\bar{\alpha} > C_X$. Assume that $\bar{\alpha} > C_X$ in view of Proposition 3.1.4. Since $y(0) - C_Y < 0 < \bar{\alpha}x(0) - C_X$ for sufficiently small $\varepsilon > 0$, we must have $y(1) = 0$ and $x(t) = 1$ for t large enough. Thus, we obtain the following:

Proposition 3.2.3: A complete coverage incumbent monopoly is stable.

We now turn to a complete coverage duopoly. The adjustment process determines a revised size $y(t+1)$ of the entrant network based on its size in the previous stage $y(t)$ by

$$\bar{\alpha}\{1 - y(t)\} - y(t)\{1 - y(t+1)\} + d = 0. \tag{9}$$

However, if $y(t) \leq \frac{\bar{\alpha}+d}{1+\bar{\alpha}}$, the size of the entrant network is too small to sustain its presence and $y(t+1) = 0$. On the other hand, if $1 - y(t) \leq \frac{C_X}{\bar{\alpha}}$, then the network X is displaced from the market and $y(t+1)$ is determined according to the incomplete entrant monopoly case:

$$\{1 - y(t+1)\}y(t) = C_Y.$$

That is, for every $y(t), t = 0, 1, 2, \ldots$, the value of $y(t+1)$ is determined by

$$y(t+1) = \begin{cases} 0 & if \quad y(t) \leq \frac{\bar{\alpha}+d}{1+\bar{\alpha}} \\ 1 - \frac{\bar{\alpha}\{1-y(t)\}+d}{y(t)} & if \quad 1 - \frac{C_X}{\bar{\alpha}} > y(t) \geq \frac{\bar{\alpha}+d}{1+\bar{\alpha}} \\ 1 - \frac{C_Y}{y(t)} & if \quad y(t) \geq 1 - \frac{C_X}{\bar{\alpha}}. \end{cases}$$

As before, a fulfilled expectations equilibrium is *stable* if the adjustment process, which starts sufficiently close to the equilibrium, converges to it.

Proposition 3.2.4: A complete coverage duopoly FEE $(1 - y', y')$ is stable, and the rate of convergence is $\frac{y'}{\bar{\alpha}+1-y'} > 1$. A complete coverage duopoly FEE $(1 - y'', y'')$ is not stable.

Proof: Let y be a FEE, where either $y = y'$ or $y = y''$. Define $b(t) = y(t) - y$. Then (9) can be rewritten as

$$-\bar{\alpha}b(t) - (1-y)b(t) + yb(t+1) + b(t+1)b(t) = 0,$$

or

$$\frac{b(t+1)}{b(t)} = \frac{\bar{\alpha}+1-y}{y} - \frac{b(t+1)}{y}. \tag{10}$$

Let $y = y''$. Assume, in negation, that $b(t)$ converges to zero. Since $y'' < \frac{1+\bar{\alpha}}{2}$, it follows that the ratio $b(t+1)/b(t)$ is greater than 1 for large t, a contradiction, which shows the instability of y''.

Let $y = y'$. Suppose that $y(t) > y'$ but the difference is sufficiently small for (9) to have a solution. Then by (9), $y(t+1) < y(t)$. (5) also implies that $y(t+1) > y'$. Thus, we obtain a decreasing monotone sequence $\{y(t)\}_{t=1}^{\infty}$ to the right of y', that obviously converges to y'. The case where $y(t) < y'$, but is sufficiently close to y', is examined in a similar manner. Thus, y' is indeed stable. The rate of convergence of the adjustment process follows from (10). □

We can neatly summarize the verdicts of Proposition 3.2.2, Proposition 3.2.3 and Proposition 3.2.4 in Figure 6. Since $d > 0$ by virtue of the participation constraints, we can check that

$$y'' - y^{**} > \sqrt{\frac{1}{4} - C_Y} + \frac{1}{2}(2\bar{\alpha} - 1),$$

the RHS of which being positive if $\bar{\alpha} > \frac{1}{2}$. This is the case described in Figure 6. The ranking between y' and y^* is ambiguous, however, even when $\bar{\alpha} > \frac{1}{2}$; there are reasonable combinations of parameters $\bar{\alpha}$, C_X and C_Y which yield $y' > y^*$ for one case, and $y' < y^*$ for another case.[4] Figure 6 describes the case where $y^* > y'$.

[Insert Figure 6 around here]

3.3 Welfare Performance of Stable FEEs

By virtue of the marginal cost pricing in the absence of fixed cost, the total welfare consists solely of consumers' surplus, which is defined by

$$CS(x,y) = \int_{\beta \in S_X(x,y)} (\alpha\overline{x} - C_X)\, d\beta + \int_{\beta \in S_Y(x,y)} (\beta y - C_Y)\, d\beta.$$

[4]Suppose that $\alpha = \frac{3}{4}$, $C_Y = \frac{1}{8}$ and $d = \frac{1}{128}$. Then we obtain $y' = \frac{1}{8}(7 + \frac{\sqrt{2}}{2}) > y^* = \frac{1}{2}(1 + \frac{\sqrt{2}}{2})$. Suppose next that $\overline{\alpha} = \frac{5}{8}$, $C_Y = \frac{1}{8}$ and $d = \frac{35}{1024}$. Then we obtain $y^* = \frac{1}{2}(1 + \frac{\sqrt{2}}{2}) > y' = \frac{27}{32}$.

It is easy to check that the total welfare at each one of the stable fulfilled expectations equilibria, viz. $(x, y) \in \{(0,0), (1,0), (0, y^*), (1-y', y')\}$, may be computed as follows:

$CS(0,0) = 0$;
$CS(1,0) = \bar{\alpha} - C_X$;
$CS(0, y^*) = \frac{(y^*)^3}{2}$; and
$CS(1-y', y') = \{\bar{\alpha}(1-y') - C_X\} + \frac{(y')^3}{2}$.

Thanks to the participation constraints, it is easy to verify that $CS(1,0) > 0$ and that the first term of $CS(1-y', y')$ is positive. Thus, if the combination of parameters are such that $y' > y^*$, then $CS(1-y', y') > CS(0, y^*)$.

3.4 Price Competition

In this subsection we examine the price competition between two networks and show the existence of the Bertrand equilibrium. Let $F(\bar{\alpha})$ be as defined in Section 3.1. For each pair $(C_X, C_Y) \in F(\bar{\alpha})$, define

$$z_1(C_Y, \bar{\alpha}) = \max\{z_1 : (z_1, C_Y) \in F(\bar{\alpha})\}$$

and

$$z_2(C_X, \bar{\alpha}) = \max\{z_2 : (C_X, z_2) \in F(\bar{\alpha})\}.$$

It is clear that $z_1(C_Y, \bar{\alpha}) \geq C_X$ and $z_2(C_X, \bar{\alpha}) \geq C_Y$. Moreover, every point within the rectangle R with vertices at $(C_X, C_Y), (C_X, z_2(C_X, \bar{\alpha})), (z_1(C_Y, \bar{\alpha}), C_Y)$ and $(z_1(C_Y, \bar{\alpha}), z_2(C_X, \bar{\alpha})1)$ belongs to $F(\bar{\alpha})$.

Let us define the duopoly game played in prices belonging to this rectangle R. For each $\bar{\alpha} \in (0,1)$ and each $(C_X, C_Y) \in F(\bar{\alpha})$, consider the game $\Gamma(\bar{\alpha}; C_X, C_Y)$ between two players, viz. X (incumbent) and Y (entrant), their strategy sets being the closed intervals $A_X = [C_X, z_1(C_Y, \bar{\alpha})]$ and $A_Y = [C_Y, z_2(C_X, \bar{\alpha})]$, respectively. Given a pair of strategies $(P_X, P_Y) \in A_X \times A_Y$, their payoff functions are defined by

$$\pi_X(P_X, P_Y) = x(P_X, P_Y)(P_X - C_X)$$

and

$$\pi_Y(P_X, P_Y) = y(P_X, P_Y)(P_Y - C_Y),$$

respectively, where $x\left(P_X, P_Y\right) + y\left(P_X, P_Y\right) = 1$ and $y\left(P_X, P_Y\right)$ is determined by

$$-y^2 + (1+\bar{\alpha})\,y - \bar{\alpha} - (P_Y - P_X) = 0,$$

which yields

$$y\left(P_X, P_Y\right) = \frac{1 + \bar{\alpha} + \sqrt{(1-\bar{\alpha})^2 - 4\left(P_Y - P_X\right)}}{2}.$$

The pure strategy Nash equilibrium of the game $\Gamma\left(\bar{\alpha}; C_X, C_Y\right)$ is called the *Bertrand equilibrium*, whose existence is guaranteed by the following:

Proposition 3.4.1: For each $\bar{\alpha} \in (0,1)$ and each $(C_X, C_Y) \in F\left(\bar{\alpha}\right)$, the game $\Gamma\left(\bar{\alpha}; C_X, C_Y\right)$ has a Bertrand equilibrium.

Proof: The payoff functions of both players are clearly continuous. Note moreover that the functions $x(\cdot, P_Y)$ and $y(P_X, \cdot)$ are decreasing and concave in their own argument. This immeadiately implies that the profit functions $\pi_X = s(P_X - C_X)$ and $\pi_Y = y(P_Y - C_Y)$ are also concave in their own variables. By virtue of Debreu's (1952) existence theorem, there exists a pure strategy Nash equilibrium of the game $\Gamma\left(\bar{\alpha}; C_X, C_Y\right)$. □

3.5 Welfare Performance Compared

We now provide a welfare comparison among three "alive" and stable fulfilled expectations equilibria: a *complete coverage incumbent monopoly*, an *incomplete coverage entrant monopoly* and a *complete coverage duopoly*.

Complete coverage incumbent monopoly. Regardless of the monopoly price P, the total welfare is $W_{im} = (P - C_X)\,x\left(P\right) + \bar{\alpha}x\left(P\right) - P$, where $x\left(P\right) = 1$, so that we obtain $W_{im} = \bar{\alpha} - C_X$.

Incomplete coverage entrant monopoly. In the absence of any subsidy or regulation, the entrant charges the monopolistic market price

$$P^m(C_Y) = \arg \max_{C_Y \leq P \leq \frac{1}{4}} (P - C_Y)y,$$

where y is the larger root of the equation $y(1-y) = P$. The total welfare is given by

$$\begin{aligned} W_{em}(P^m(C_Y), C_Y) &= \left\{P^m(C_Y) - C_Y\right\} y_{em} + \int_{1-y_{em}}^{1} \left\{\beta y_{em} - P^m(C_Y)\right\} d\beta \\ &= \left\{P^m(C_Y) - C_Y\right\} y_{em} + \frac{y_{em}^3}{2}, \end{aligned}$$

where y_{em} is the larger root of the equation:

$$y(1-y) = P^m(C_Y).$$

However, as we have shown above, the best possible outcome, from the welfare point of view, is when the entrant is given a per unit subsidy (obviously accounted for in our welfare calculus) at the rate of C_Y. In this case, every consumer purchases a unit of service at the zero price, the total consumer surplus is $\frac{1}{2}$, and the subsidy budget is C_Y. That is,

$$W_{em}(0,0) = \frac{1}{2} - C_Y.$$

Complete coverage duopoly. Again, in the absence of subsidy and regulation, the incumbent and the entrant charge the Bertrand equilibrium prices P_X and P_Y, respectively. The total welfare is given by

$$\begin{aligned} W_{du}(P_X, P_Y, C_X, C_Y) &= (P_X - C_X)\{1 - y(P_X, P_Y)\} \\ &+ \int_0^{1-y(P_X,P_Y)} [\bar{\alpha}\{1 - y(P_X, P_Y)\} - P_X]\, d\beta \\ &+ (P_Y - C_Y)\, y(P_X, P_Y) \\ &+ \int_{1-y(P_X,P_Y)}^{1} \{\beta y(P_X, P_Y) - P_Y\}\, d\beta, \end{aligned}$$

where $y(P_X, P_Y)$ is the larger root of the equation

$$\bar{\alpha}(1-y) - P_X = y(1-y) - P_Y.$$

The first and third terms in the above expression represent the profits of networks X and Y, respectively, whereas the second and forth terms represent the consumers' surplus of customers of X and Y, respectively.

Let us show first that if the firms charge their marginal costs, the total welfare will increase.

Lemma 3.5.1: $W_{du}(P_X, P_Y, C_X, C_Y) < W_{du}(C_X, C_Y, C_X, C_Y)$.

Proof: For every $y \in [0,1]$ consider the following function $H(y)$:

$$H(y) = \{\bar{\alpha}(1-y) - C_X\}(1-y) + \int_{1-y}^{1} (\beta y - C_Y) d\beta,$$

which, in the marginal cost pricing case, simply represents an arbitrary assignment of all consumers with $\beta > 1 - y$ to the entrant network and all consumers with $\beta \leq 1 - y$ to the incumbent network. It is important to note that

$$\begin{aligned} W_{du}(P_X, P_Y, C_X, C_Y) &= H(y(P_X, P_Y)) \\ W_{du}(C_X, C_Y, C_X, C_Y) &= H(y(C_X, C_Y)), \end{aligned}$$

where $y(C_X, C_Y)$ is the larger root of the equation

$$\bar{\alpha}(1 - y) - C_X = y(1 - y) - C_Y.$$

The function $H(y)$ can be rewritten as

$$H(y) = \bar{\alpha} - C_X - (2\bar{\alpha} + d)y + (1 + \bar{\alpha})y^2 - \frac{y^3}{2},$$

from which we obtain

$$H'(y) = -(2\bar{\alpha} + d) + 2(1 + \bar{\alpha})y - \frac{3}{2}y^2,$$

Since $H'(0) < 0$, $H''(y) = 2(1 + \bar{\alpha}) - 3y$, $H'(1) = \frac{1}{2} - d > 0$ by virtue of the participation constraint (6) and $H''(y) > 0$ for all $y \in \left(0, \frac{2}{3}(1 + \bar{\alpha})\right)$, it follows that the equation $H'(y) = 0$ has one root, $\bar{y}$, on the open interval $(0, 1)$. See Figure 7. Note that

$$H'(\frac{1 + \bar{\alpha}}{2}) = \frac{5}{8}(1 - \bar{\alpha})^2 + \frac{\bar{\alpha}}{2} - d \geq \frac{3}{8}(1 - \bar{\alpha})^2 + \frac{\bar{\alpha}}{2} > 0,$$

where we made use of the participation constraint (6). It follows that $0 < \bar{y} < \frac{1+\bar{\alpha}}{2}$. See Figure 7. Thus, $H(y)$ is increasing on the closed interval $\left[\frac{1+\bar{\alpha}}{2}, 1\right]$. Since both $y(C_X, C_Y)$ and $y(P_X, P_Y)$ are greater than $\frac{1+\bar{\alpha}}{2}$, it remains to show that $y(C_X, C_Y) > y(P_X, P_Y)$, or $P_Y - P_X > C_Y - C_X$. See Figure 8.

[Insert Figure 7 and Figure 8 around here]

However, the equilibrium conditions imply that

$$\begin{aligned} -(P_X - C_X)y_I(P_X, P_Y) + \{1 - y(P_X, P_Y)\} &= 0 \\ -(P_Y - C_Y)y_I(P_X, P_Y) + y(P_X, P_Y) &= 0, \end{aligned}$$

where $y_I(P_X, P_Y)$ is the derivative of $y(P_X, P_Y)$ with respect to the first variable. Thus,

$$\frac{P_X - C_X}{1 - y(P_X, P_Y)} = \frac{P_Y - C_Y}{y(P_X, P_Y)},$$

and since $y(P_X, P_Y) > \frac{1}{2}$, it follows that $P_Y - C_Y > P_X - C_X$ or $P_Y - P_X > C_Y - C_X$. □

Finally, the welfare maximum in duopoly is achieved when the government gives a per unit subsidy C_X to the incumbent network and a per unit subsidy C_Y to the entrant network.

Proposition 3.5.2: A maximum of the total welfare $W_{du}(C_X, C_Y, C_X, C_Y)$ over the set $F(\bar{\alpha})$ is obtained when $C_X = C_Y = 0$.

Proof: Let $(C_X, C_Y) \in F(\bar{\alpha})$. Then $W_{du}(C_X, C_Y, C_X, C_Y) = W_{du}(0, d, 0, d)$. Indeed, giving the same subsidy C_X to both firms would not change y and, therefore, a value of the total welfare. Consider now an additional per unit subsidy s to the entrant. Before the subsidy, we have:

$$W_{du}(0, d, 0, d) = \bar{\alpha} - (2\bar{\alpha} + d)y(0, d) + (1 + \bar{\alpha})\{y(0, d)\}^2 - \frac{\{y(0, d)\}^3}{2},$$

and after the subsidy, we have:

$$W_{du}(0, d - s, 0, d - s) = \bar{\alpha} - (2\bar{\alpha} + d - s)y(0, d - s) +$$
$$(1 + \bar{\alpha})\{y(0, d - s)\}^2 - \frac{\{y(0, d - s)\}^3}{2} + sy(0, d - s).$$

But $y(0, d - s) > y(0, d)$ implies that $W_{du}(C_X, C_Y, C_X, C_Y) \leq W_{du}(0, 0, 0, 0)$. □

To conclude our verdict on the welfare performance of competition between the incumbent and a new entrant networks, we show that the desirable market equilibrium and, hence, the directions of the government policy crucially depend on the structure of industry costs and the value of consumers' gross benefit from purchasing a unit of service from the incumbent network:

Proposition 3.5.3: (i) If $\bar{\alpha} - C_X \geq \frac{1}{2} - C_Y$ or $\bar{\alpha} \geq \frac{1}{2} - d$, then the reputation of the incumbent network is sufficiently strong and the complete coverage incumbent monopoly would remain the most desirable outcome from the welfare point of view;

(ii) If $\bar{\alpha} - C_X < \frac{1}{2} - C_Y$ or $\bar{\alpha} < \frac{1}{2} - d$, then the most desirable outcome from the welfare point of view would be the support of the new entrant network that would eventually displace the incumbent network.

4 Concluding Remarks

Let us conclude this paper, which was devoted to the simple analysis of competitive economies with network externalities, with three final observations on the further directions to be explored in the future.

In the first place, in order for our model to be relevant for telecommunications industry, where natural monopoly segments and potentially competitive segments exist side by side, the asymmetry between the incumbent network and the new entrant network should be specified more in detail. Recollect that the model used in this paper assumed two potential sources of asymmetry between the incumbent network and the new entrant network; they may differ in demand structures, the incumbent network having the established reputation among consumers, which the new entrant network does not have; they may also differ in their marginal cost structures, the incumbent network having an advantage due to learning by doing, whereas the new entrant network having an access to a newer and better technology to sharpen their competitive edge against the incumbent network. Another source of asymmetry, which is not accommodated in our analysis so far, is that the incumbent network may consist of the local bottleneck facilities together with the long-distance network, whereas the new entrant network may consist only of the long-distance network, the local bottleneck facilities being still too costly to duplicate. If the new entry takes place in the presence of bottleneck monopoly in the local telecommunications services, only the incumbent network can provide integrated telecommunications services and the new entrant network must depend unilaterally on the bottleneck facilities held by the rival network for the provision of integrated network services to the consumers. Needless to emphasize, this structural asymmetry poses many theoretical as well as policy-oriented issues, which are worthwhile to explore in the future. See, among others, Laffont, Rey and Tirole (1998a; 1998b), and Laffont and Tirole (2000) for some of the relevant work in this arena.

In the second place, our model is presumably too simple on the demand side as well. In particular, there is no attempt to accommodate such dynamic factors as inertia formation and switching cost which are known to be relevant in the context of network externalities. It is the task of our future research to accommodate these dynamic factors and see what effects, if any, do they exert on the verdicts we obtained in this paper.

In the third place, it is assumed throughout this paper that the two competing networks are disconnected in the sense that the users of X do not benefit from the size of customers of Y, and vise versa. In telecommunications industry, however, this may not be the case, and this fact may affect the validity of some of our main results. Take, for example, Proposition 3.4.3 which asserts that either the network X alone, or the network Y alone would serve the customers best. This may well be due to the assumed disconnectedness of the two networks, and the story may be substantially different if the two networks are interconnected. It would be worthwhile, to say the least, to examine how our main verdicts would be affected in the presence of interconnection between the competing networks.

Figure 1: Fulfilled Expectations Equilibria

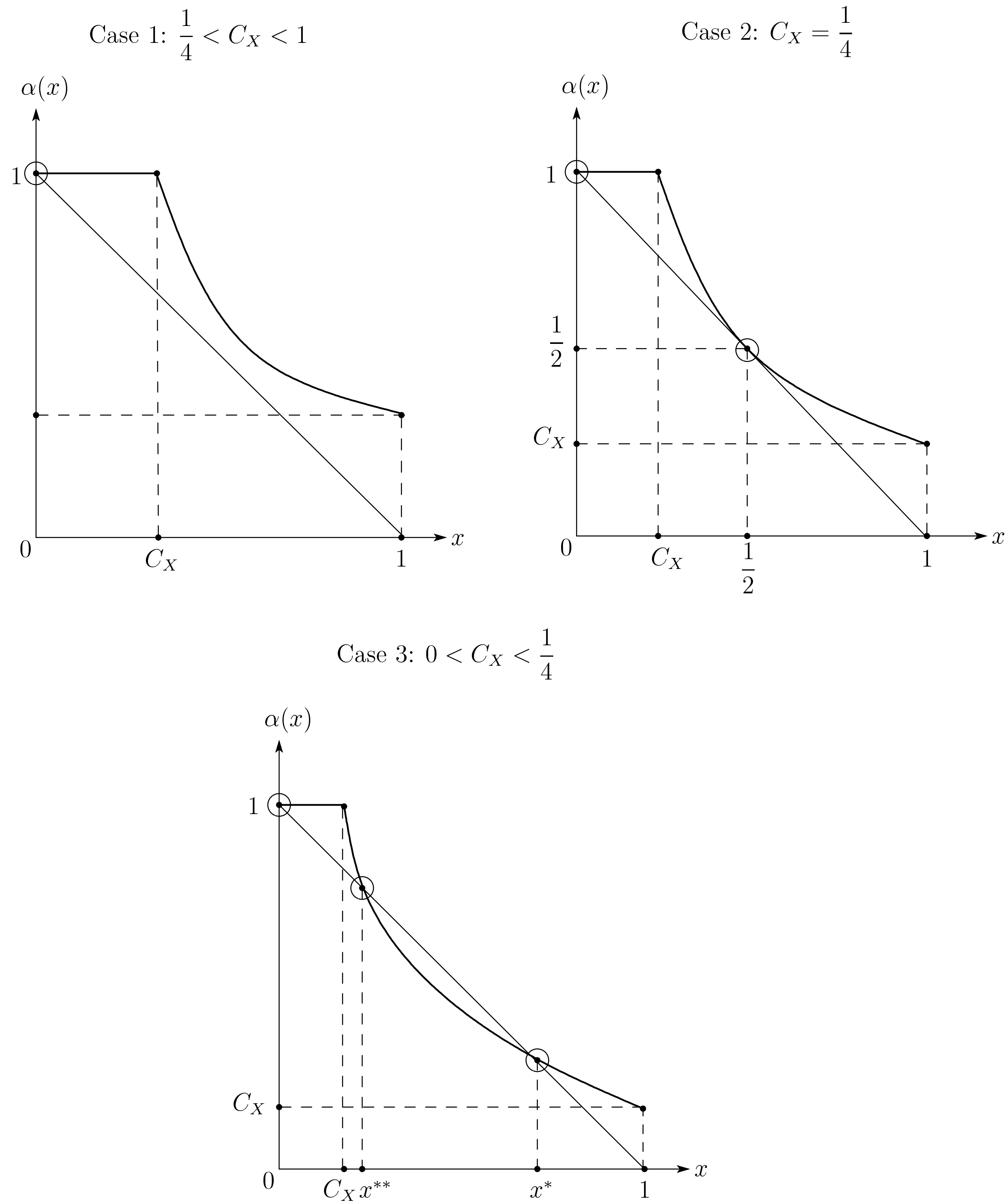


Figure 2: Interior Fulfilled Expectations Equilibria

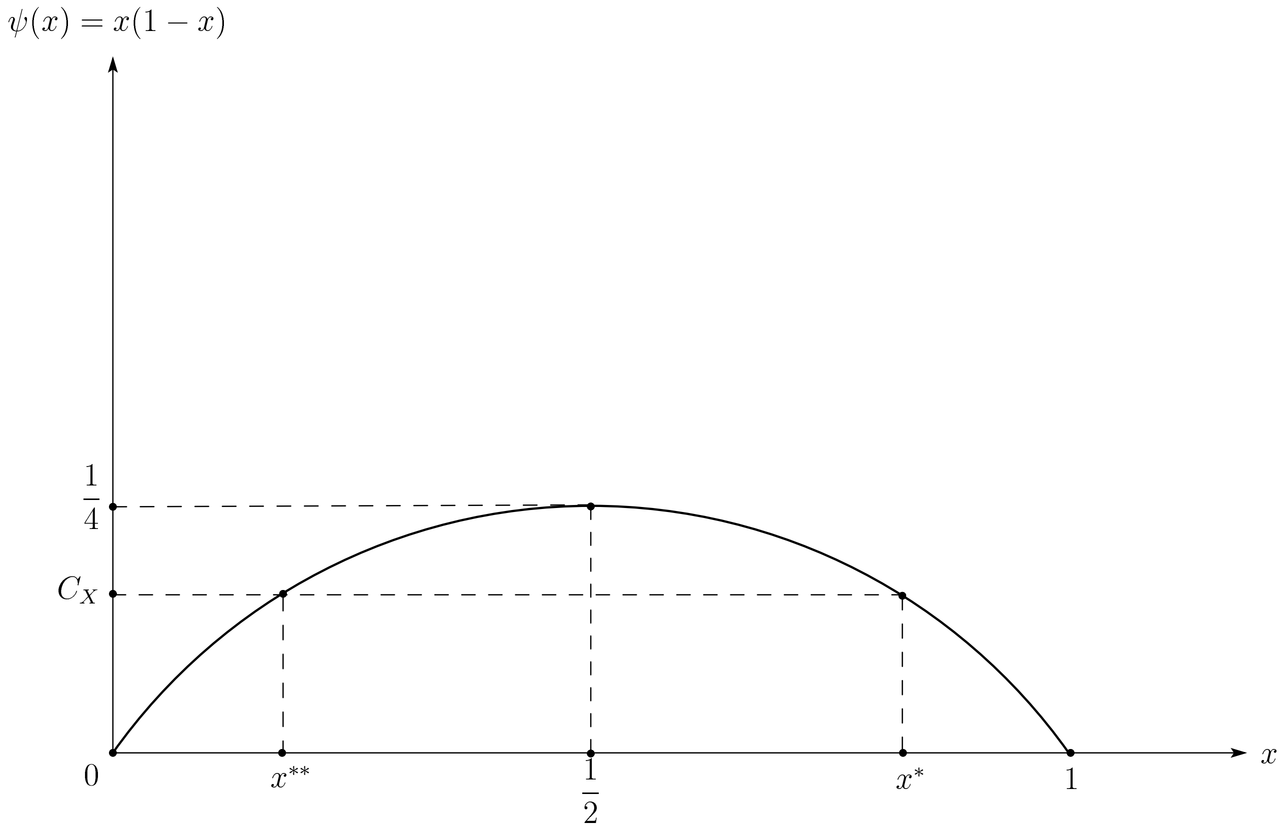


Figure 3: Monopoly Profit Profile

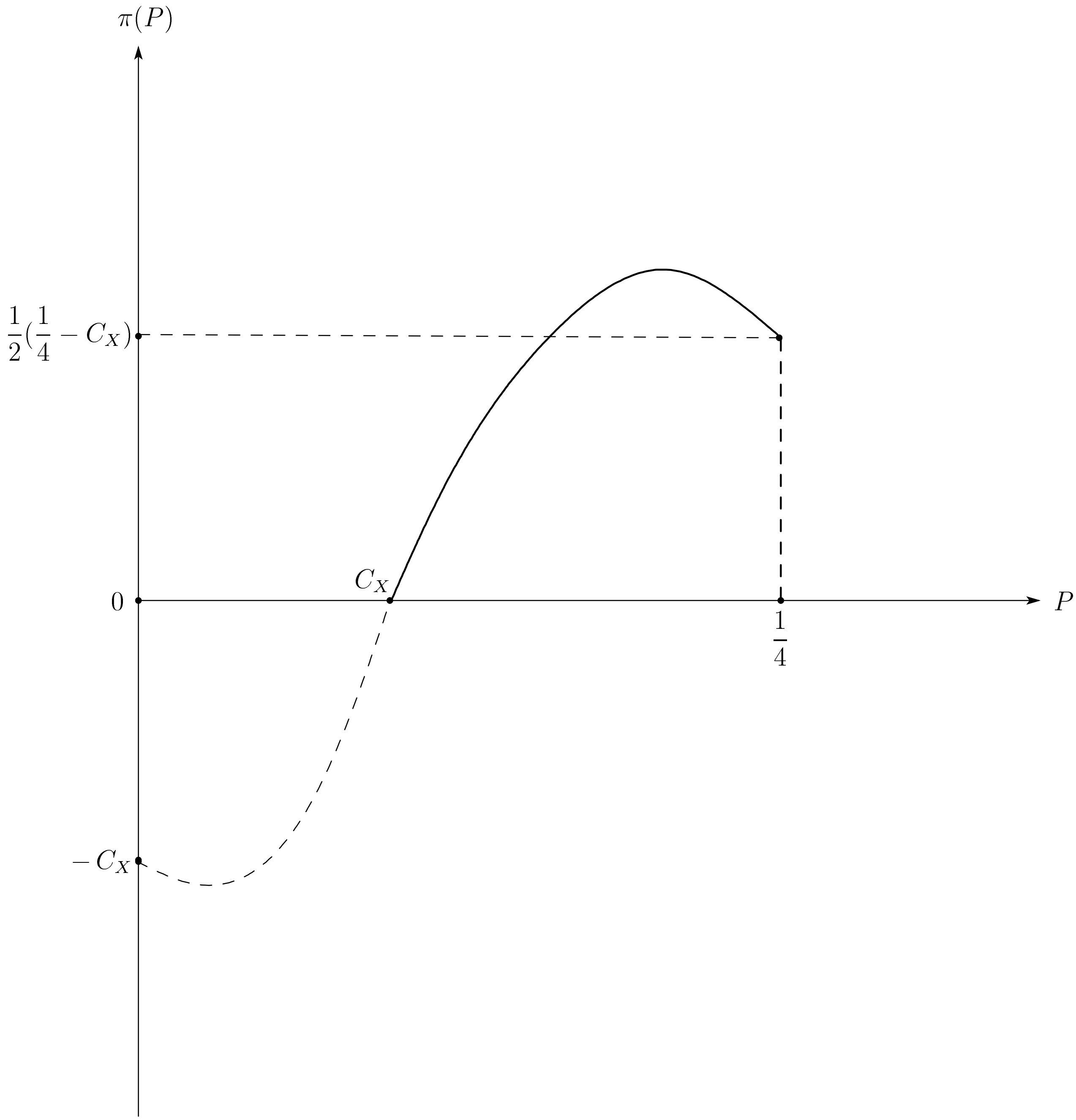


Figure 4: Complete Coverage Duopoly

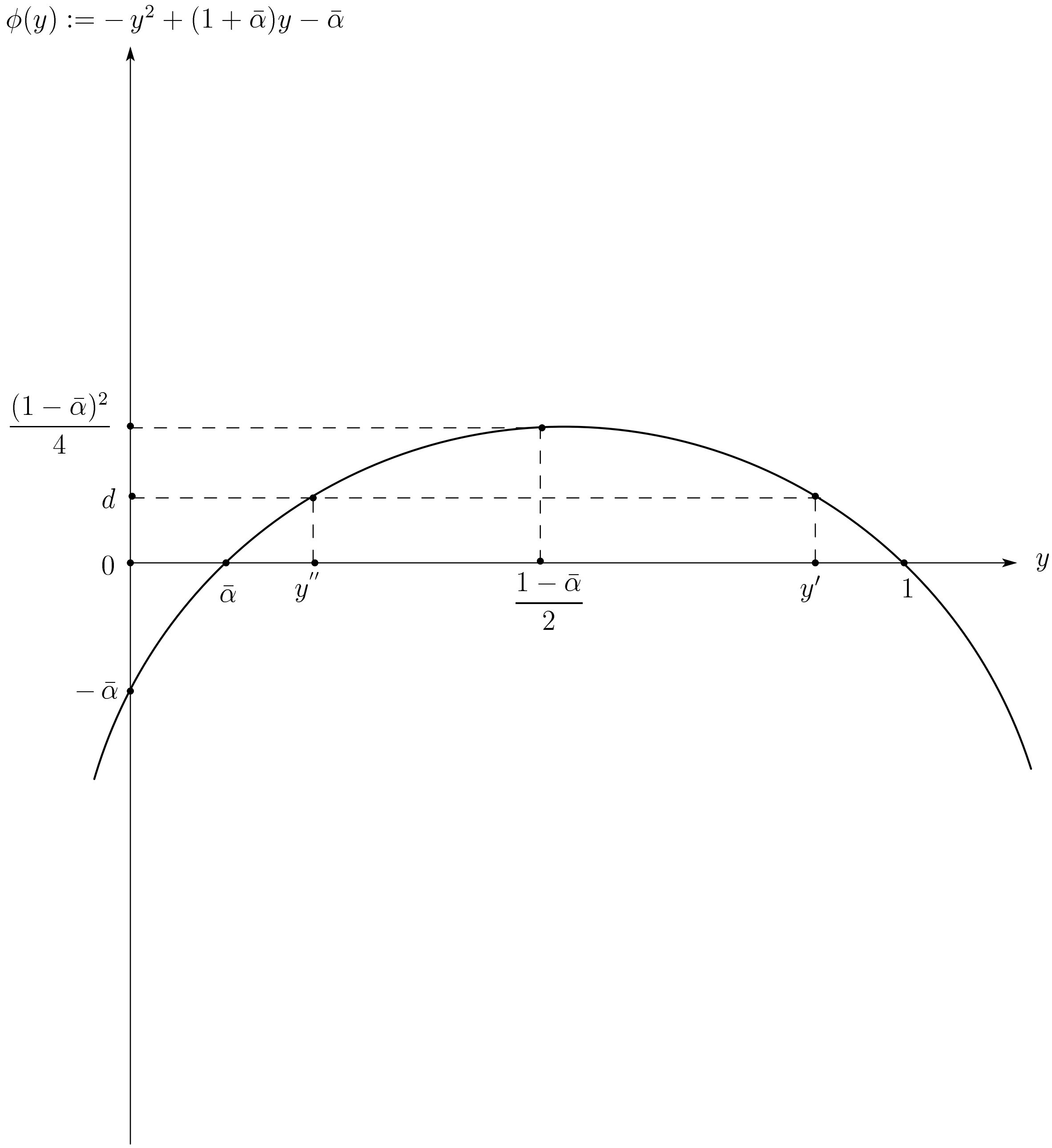


Figure 5: Participation Constraints

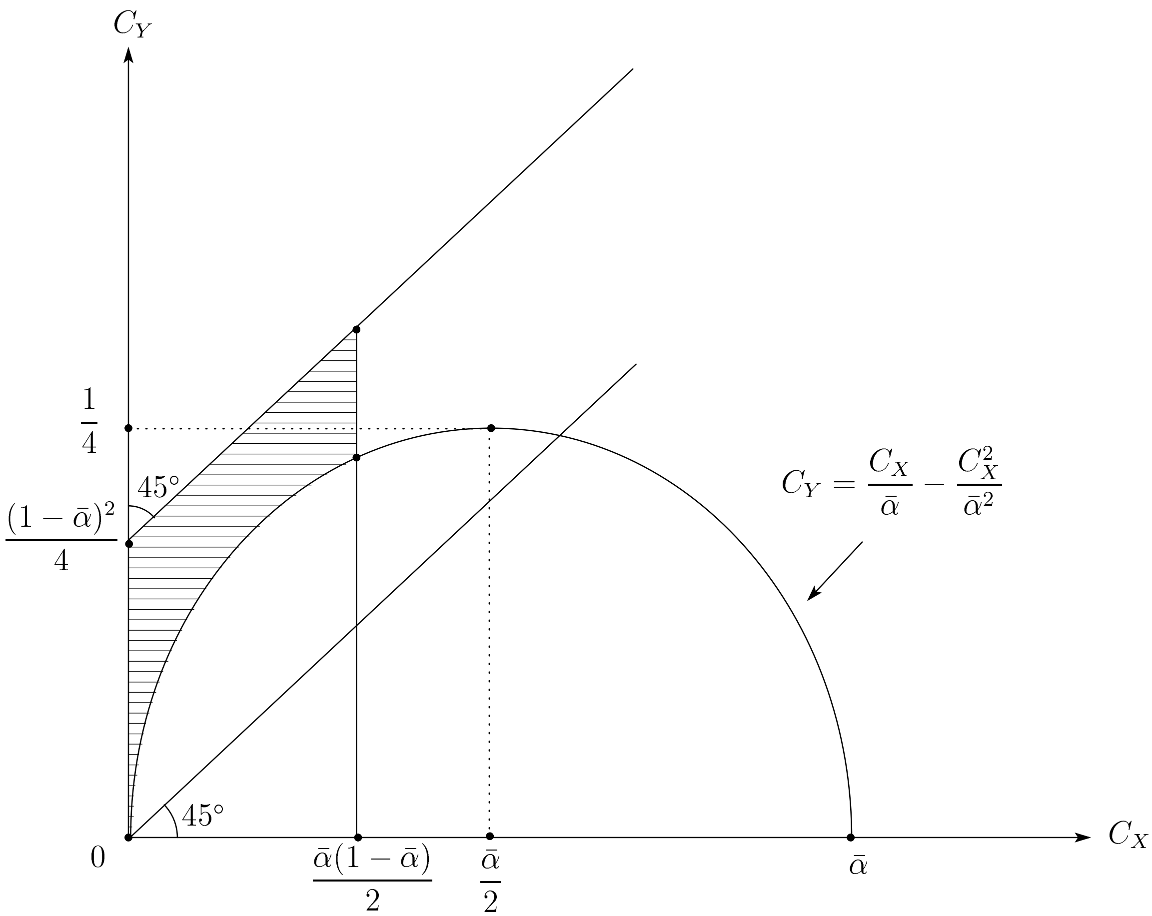


Figure 6: Fulfilled Expectations Equilibria

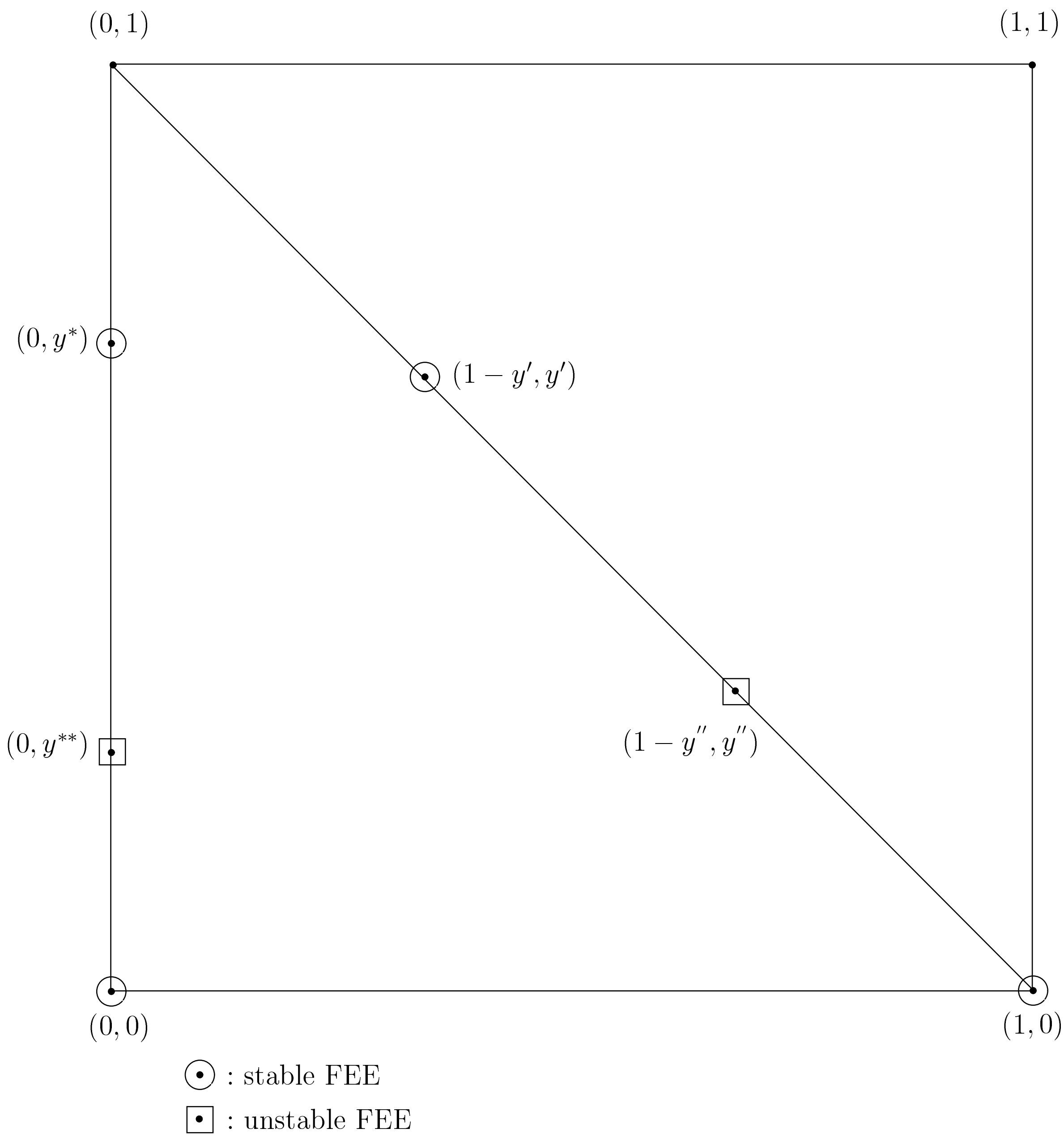


Figure 7: Complete Coverage Duopoly

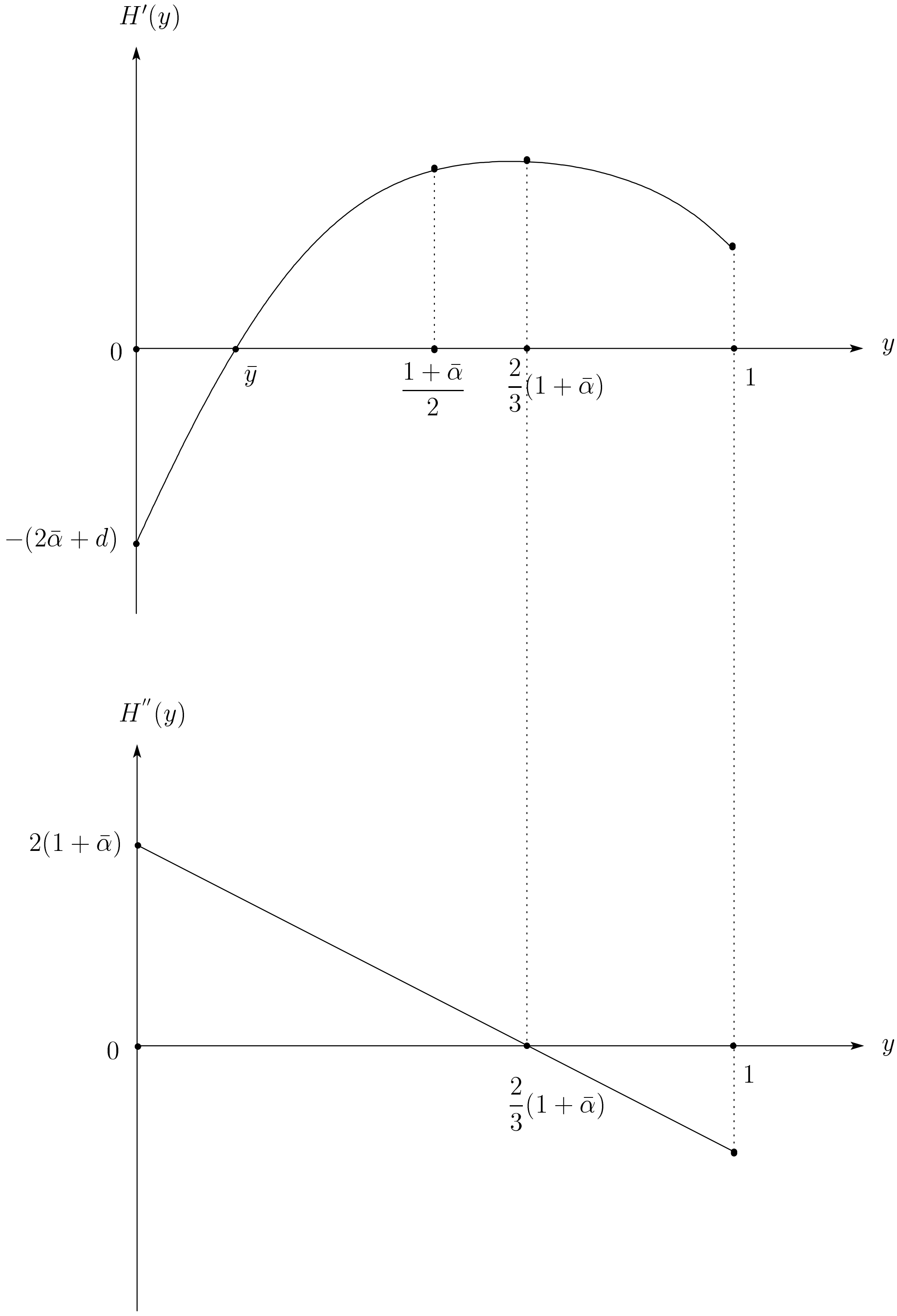
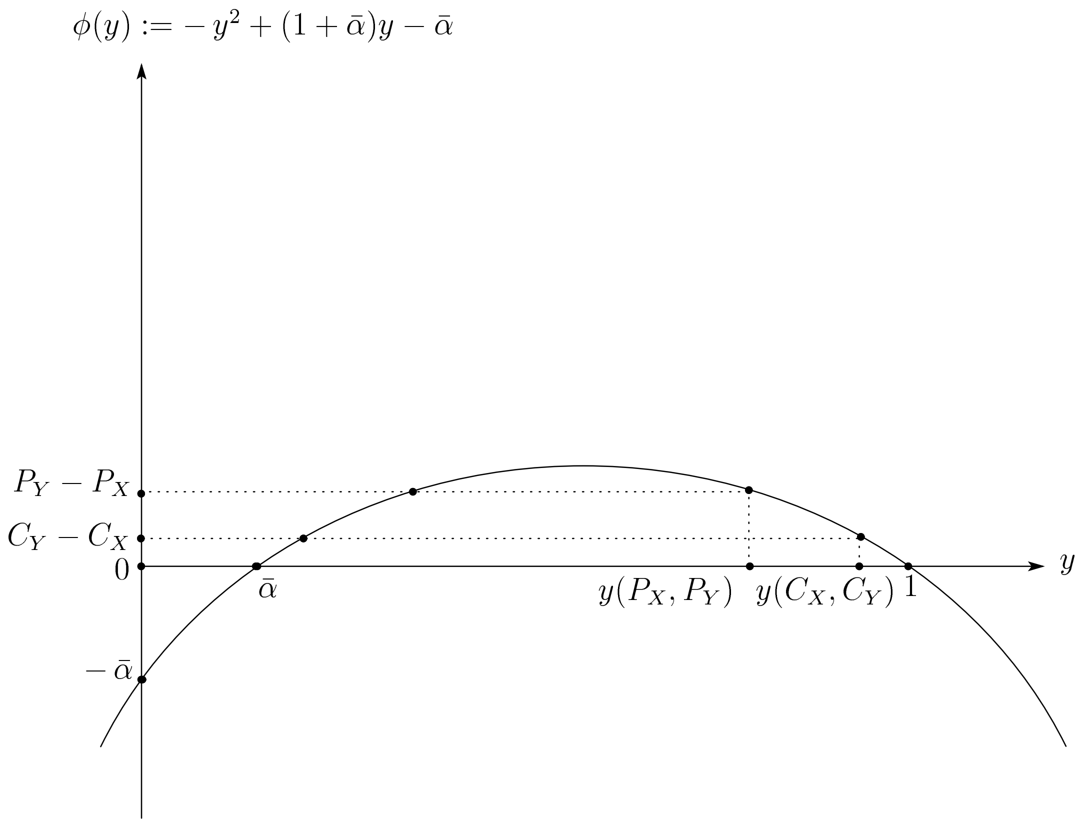


Figure 8: Welfare Performance of Complete Coverage Duopoly



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