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# Long Term Care: the State, the Market and the Family

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# Long Term Care: the State, the Market and the Family*

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**Abstract**

In this paper we study the optimal design of a long term care policy in a setting that includes three types of care to dependent parents: public nursing, private nursing and assistance in time by children. Private nursing can be financed either by financial aid from children or by private insurance. The social planner can use a number of instruments: public nursing, subsidy to aiding children, subsidy to private insurance premiums, which are all financed by a flat tax on earnings. The only source of heterogeneity is children's productivity. Parents can influence their children by leaving them gifts before they know whether or not they will need long term care, yet knowing the productivity of the children. We show that the quality of public nursing homes and the level of tax-transfer depend on their effect on gifts, the distribution of wages and the various inequalities in consumption. We also consider the possibility of private insurance.

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## 1 Introduction

The ongoing demographic ageing process represents a major challenge for the way our economies are organized both from a social, as well as from an

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economic point of view. Ageing can be felt across a large array of domains touching all age groups, ranging from the very young to the oldest old. One often cited example is the provision of long-term care insurance to the oldest old, be it under the form of a private or a public system. Only a handful of countries or regions have set up such long-term care insurance systems which, incidentally, are also sometimes called dependency insurance. The relative scarcity of such systems, and the difficulties of organizing them, is linked to some conceptual problems intrinsically due to the issue at hand. First, a definition of who is a person in need of long-term care cannot always be stated objectively. However, this is not a sufficient reason to justify the lack of long-term care insurance programs around the world, since disability insurance systems are plagued by the same kind of problem but do exist. The second, and probably more fundamental reason, is that a lot of long-term care is not provided through a formal market mechanism, but rather through informal family arrangements. In this respect, the problem is similar to the child-care market, where family care is competing with market-provided care in private or public arrangements. From a social point of view, this duality of providers is an interesting one, as these two types of providers seem to function on a very different basis. While institutional care is essentially a provision of a contribution-based service by a public or private (for-profit or non-profit) provider, family care is at least partly motivated by some degree of altruism, which in turn implies that the caregiving family member also derives utility from this activity. Further, while institutional care usually implies some degree of public subsidization, and hence inter-family redistribution, this does not always hold true for family-care arrangements.

Yet the analogy between these two forms of care is limited. In contrast to the child-care market, the costs involved are much larger in long-term care insurance, as costs of medical and non-medical care are much more expensive at the end of the life-cycle than at the beginning because of the vastly different physical conditions of the people in question. Hence, the choice between family or institutional care has important budgetary implications that a government or a social planner cannot ignore. Another problem raised by cares such as child care or long term care provided out of altruism is that, depending on the opportunity cost of time, they can be provided directly by children in units of time, or obtained from the market through financial aid from children. One sees from this quick overview that the analysis of long term care is very complex, and that all aspects cannot be dealt with at the same time.

In this paper we study a society consisting of a number of pairs of parent-child. Parents are not altruistic, while children have a specific type of altruism: in that they are ready to help their parents if these lose their autonomy.

In the absence of government policy dependent parents can be helped in two ways: either children give them some financial aid or they provide them with assistance in time. Children have different productivities, and parents have a uniform endowment (wealth, pension). Market productivity varies, but productivity in terms of helping dependent parents is the same for all. As a consequence, children of dependent parents are divided into two groups. The low market productivity group helps their parents with time, and the other one provides financial assistance. Before knowing their own health status parents can give part of their endowment to their children in case of they lose so that in case of bad health they get better assistance. Alternatively parents can purchase a private long term care.

We then introduce public policy consisting of three instruments: a uniform payroll tax, a subsidy for dependent parents receiving assistance (in kind or in cash) from their children, and institutionalized nursing assistance. Parents who receive this latter benefit don't receive any help from their children. As it appears, children with middle level wages tend to have their dependent parents going to these nursing homes or purchasing private insurance.

We are ultimately interested in the optimal policy chosen by a utilitarian government. But before doing that, we analyze the comparative statics of our model. In particular, we study the effect of policy variables and exogenous variables on the segmentation of our society into three groups.

Quite clearly such a model does not include all the aspects of long term care and it does rest on a number of assumptions. Some are pretty realistic; others are made to keep the analysis within reasonable limits. The only heterogeneity comes from differences in market productivity. The other characteristics such as altruism, initial endowment, productivity in assistance to dependent parents are equal for all.[1] The instruments are a payroll tax, a lump-sum subsidy to aiding children, an *ad valorem* subsidy to private insurance, and public nursing home. These restrictive policies are adopted for the sake of simplicity. As it will appear the choice of private insurance and public nursing is dichotonous. It will be influenced by the relative efficiency of the two schemes.[2]

In an earlier paper [Pestieau and Sato, 2004], we only consider a tax-transfer policy. The present paper thus extends this work in two different directions. We allow for the possibility of public nursing homes and also for

[1] In Jousten *et al.* (2005), the optimal long term care policy is analyed when the only source of heterogeneity is children's altruism.

[2] In a recent paper Finkelstein and McGarry (2004) underline two sources of heterogeneity in long term care insurance that are not observable: risk types and insurance preferences. They show that this double asymmetric information has negative efficiency consequences on the insurance market. We don't consider this issue here.

the existence of private insurance. As it will be shown public nursing home and private insurance cater to parents with children of middle productivity. Low wages children prefer to help their parents with time; high wages children prefer to assist them with financial transfers.

To avoid confusion, it is important to distinguish among the types of resources dependent parents can count on and among the types of provider of long term care. Assistance in time implies that the dependent parent stays home and is taken care of by his child. Assistance in cash or private insurance benefits allow the dependent parents to stay home and get some nursing service or to go to a private nursing home. Finally, the case of public nursing home is self-explanatory.

Among the scant evidence on upward intergenerational transfers from middle age children to their elderly parents, there is the study by Sloan *et al.* (2002) who use data from HRS. They show that a child with a high wage tends to transfer money rather than time and conversely for a child with a low wage. Zissimopoulos (2001), on the basis of the same data shows that, as children's wage increases, they tend to substitute time for money. Ioannides and Kan (2000) using data from the PSID reach the same conclusion. Children's transfers (both money and time) are determined by their parents' needs and their own resources. High income children and children living far away tend to make transfers in money and not in time.[3]

The rest of the paper is organized as follows. The next section presents the basic model and some comparative statics results along with the *laissez-faire* solution with private insurance. Section 3 introduces the public policy tools. Section 4 is devoted to the design of optimal tax transfer and nursing homes policy. Section 5 is devoted to the choice between public nursing and subsidizing a private long term care insurance. This choice is shown to depend on the relative efficiency of the two schemes, but also on the parent's wealth. A final section concludes.

# 2 The *laissez-faire*

## 2.1 The basic model

We consider a family consisting of a parent and his altruistic child. All families are *ex ante* identical except for the market productivity of children denoted $w$ with density $f(w)$, distribution $F(w)$ and support $(w_{-}, w_{+})$. We assume that the parent chooses to leave a gift $G$ to his child before knowing

[3] See also Prouteau and Wolff (2003) for a study on French data reaching the same conclusion.

whether or not he needs long term care. When this is known, the child decides to help his dependent parent. Each parent faces a probability $\pi$ of losing his autonomy which corresponds to a loss $D$. He has an initial endowment $I$ and consumes $d^D$ if dependent and $d^N$ if autonomous.

His expected utility can be written as:

$$V = \pi \left[ v\left(d^D\right) - D + H \right] + (1-\pi)\, v\left(d^N\right) = v(d) - \pi (D - H)$$

where $d^D = d^N = I - G$, $I$ being his initial endowment. $D$ is the utility loss implied by dependence and $H$ is the help he gets from his child expressed in utility terms as well. Turning to the children, even though they are concerned by the consumption of their parents, dependent or not, they only help them in case of dependency and this help is restricted to health. Denoting their utility by $u(\cdot)$ and their consumption by $c^j\,(j = D, N)$, we have

$$U^D = u\left(c^D\right) + \beta \left(v(d) + H - D\right)$$

and

$$U^N = u\left(c^N\right) + \beta v(d)$$

where $c^D = (1-h)\, w + G - s$, $c^N = w + G$ and $\beta \leqslant 1$ is a factor of altruism. Market labor supply is $(1-h)$ with $h$ being the aid in time provided to dependent parents and $s$, is the amount of financial aid that allows children to purchase market services on behalf of their dependent parents. As we show $h$ and $s$ are mutually exclusive.

It is now time to define $H$. We assume that each child has one unit of time endowment. He can devote part of it to labor market in which case he earns $w$ and he can devote another part of it to his parent. If he provides $h$ to his parent given a constant productivity $\omega_0$, this amount to a help of $\omega_0 h$. This child will also earn $(1-h)\, w$ as market earnings. Instead a child may want to help his dependent parent through financial aid, $s$, which is used to purchase market nursing services.

By assuming perfect substituability between these two forms of assistance, namely by positing:

$$H(\omega_0 h, s) = H(\omega_0 h + s)$$

with $H' > 0$ and $H" < 0$, we know that children with $w \leqslant \omega_0$ will have $1 \geqslant h \geqslant 0$, $s = 0$ and those with $w > \omega_0$, $h = 0$ and $s \geqslant 0$.[4] For $I - G$

[4] In other words if $H(\omega_0 h, s)$ would allow for some complementarity between the two arguments, children could very well provide at the same time assistance in time and in cash.

not too high, we expect interior solutions, namely, either $h > 0$ or $s > 0$. Formally, for $w \leqslant \omega_0$, $h^*$ is the solution of

$$u'\left(c^D\right) w = \beta H'\left(\omega_0 h\right) \omega_0$$

with strict inequality in the case where $h^* = 1$, and for $w > \omega_0$, $s^*$ is the solution of

$$u'\left(c^D\right) = \beta H'\left(s\right).$$

The profile of $\omega_0 h + s$ is represented on Figure 1 below.

Given the expected behavior of his child, each parent can decide to leave him a certain fraction of his endowment. To be precise, the parent aims at maximizing $V\left(I - G\right) + \pi H\left(m^*\right)$ where $m^* = \omega_0 h^*$ for $w \leqslant \omega_0$ and $m^* = s^*$ for $w > \omega_0$. When making this choice, he does not know yet whether or not he will need long term care but he knows his child's productivity. We suppose that the parent takes into account the effect of $G$ on $m^*$, the care provided by the child. There is no parental altruism. With $\pi = 0$, there would not be such a gift. The reason for such an early gift is insurance; it is also the only way to obtain care.

Assuming an interior solution, the first-order condition for $G$ is given by:

$$V'\left(I - G^*\right) = \pi H'\left(m^*\right) \frac{\partial m^*}{\partial G}.$$

The above is difficult to interpret without specifying the functions as we do later, but it can be predicted that $G^*$, the optimal amount of gift, will depend on $w$ as well as $I$.

Up to now we distinguished two regimes depending on $w \gtreqless \omega_0$. We will denote 1 the regime where children provide $\omega_0 h$ and 2 the regime where they provide $s$. We now consider the possibility of a third and intermediate regime defined by $w \in (\hat{w}_1, \hat{w}_2)$ with $\hat{w}_1 < \omega_0 < \hat{w}_2$. To obtain this regime, we introduce the possibility of a private insurance with compensation $a$ and premium $p$, $p\left(a\right) = \pi a \theta$ where $\theta > 1$ reflects the fact that such an insurance cannot be actuarially fair for all sorts of reasons of informational and technological nature.

The parent instead of expecting assistance from his child at the cost $G$ can thus buy that insurance. If the parent opts for private insurance, he chooses $a^*$ defined by

$$v'\left(I - \pi a^* \theta\right) \theta = H'\left(a^*\right).$$

Parents with children of productivity $w < \omega_0$ have to compare the utility $V_1$ they get with aid $h$ but cost $G_1$, and the utility $V_3$ they get with coverage $a$ and cost $p\left(a\right)$. In other words, they choose private insurance if

$$v\left(I - G_1^*\right) + \pi H\left(\omega_0 h\left(w, G_1^*\right)\right) - v\left(I - \pi a^* \theta\right) + \pi H\left(a^*\right) < 0$$

It is not clear that this inequality can be verified. One can expect, e.g., that for an inefficient insurance market (large $\theta$), no parent will ever buy private insurance. For the time being, we assume that there exists a value of $w$ $(< \omega_0)$ for which the above inequality becomes an equality; we denote such a value $\hat{w}_1$.

Similarly we define the treshold value $\hat{w}_2$ as that for which

$$v\left(I - G_2^*\right) + \pi H\left(s\left(w + G_2^*\right)\right) = v\left(I - \pi a^* \theta\right) + \pi H\left(a^*\right).$$

We assume that private insurance and filial assistance are mutually exclusive. Thus, for $w$ sufficient high children will prefer to help their parents than to let them rely on just private insurance whose coverage depends on both $\theta$ and $I$.

To have a better grasp at this problem, we now turn to a simple illustration, using logarithmic utility functions.

## 2.2 The log-linear example

As parents move first and children second, we start by looking at the problem of each child. If his parent is healthy, the child does not help him and benefit from the transfer $G$, if any. $G$ cannot be contingent upon the parent's health status because it is given before hand. If his parent loses his autonomy, the child helps him with $h$ or $s$ depending on his productivity. We suppose that the child choose either $h$ or $s$ but it is the parent's decision whether he seeks for help from his child or purchase insurance.

### 2.2.1 Child's problem

With the specification of the log function, each child solves the following problem:

$$Max \quad \ln\left(w\left(1-h\right) + G - s\right) + \beta \ln\left(\omega_0 h + s\right) - \beta D + \beta \ln\left(I - G\right)$$

From the FOC, we obtain:

$$\begin{aligned} \text{For } w < \omega_0\text{:} \quad & h^* = \frac{\beta}{1+\beta}\frac{w+G_1}{w} && \text{if } G_1 \leqslant \frac{w}{\beta} \\ & h^* = 1 && \text{if } G_1 > \frac{w}{\beta} \\ \text{For } w > \omega_0\text{:} \quad & s^* = \frac{\beta}{1+\beta}\left(w+G\right). \end{aligned}$$

### 2.2.2 Parents' problem without private insurance

$$Max \quad \ln(I-G)+\pi\ln(\omega_0 h+s)-\pi D.$$

Here, the FOC yields a supply function $G_i^*$ that can be summarized by:

$$G_1^* = Max \quad \left[0, Min\left(\frac{\pi I - w}{1+\pi}, \frac{w}{\beta}\right)\right]$$

$$G_2^* = Max \quad \left[0, \frac{\pi I - w}{1+\pi}\right].$$

Note that $G_i^* > 0$ implies $I > w$, so that the parent is wealthier than his child. This may be due to lifetime saving. The general case incorporates the situation where some parents are poor and thus $G = 0$. We consider this explicitly later.

We can now represent the values of $m\,(= h\omega_0$ or $s)$ and $G$ along the $w$-axis on Figure 1. On this axis $\underline{w} \equiv \dfrac{\pi\beta I}{1+\pi+\beta}$ and $\bar{w} \equiv \pi I$. Below $\underline{w}$, the dependent parent could count on a 100% assistance from his child and does not have to leave a lot. Above $\underline{w}$, children are so wealthy that a gift has no effect on the level of $s$. As we show in the appendix the profile we adopt here only holds for particular values of both $I$ and $\omega_0$.

Figure 1: Child's assistance

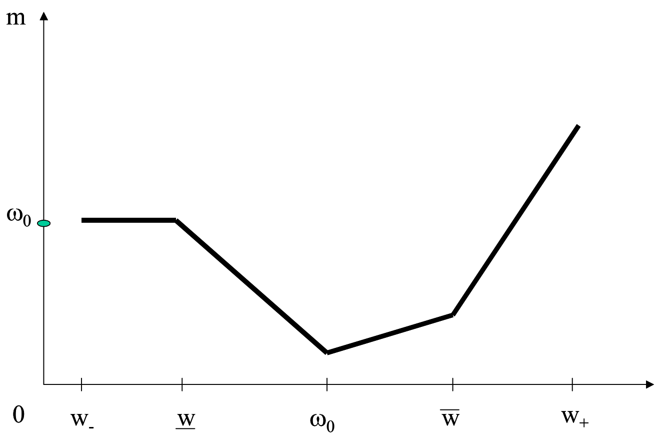
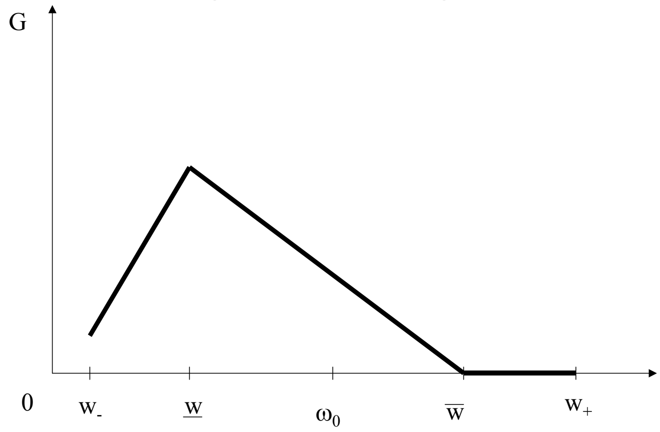


Figure 2: Parent's gift




Substituting these values of $G_i^*$ in the utility function of the parents we have the following expression that depend on the value of $w$.

$$w < \underline{w} : V_1 = \ln\left(I - \frac{w}{\beta}\right) + \pi \ln \omega_0 - \pi D$$

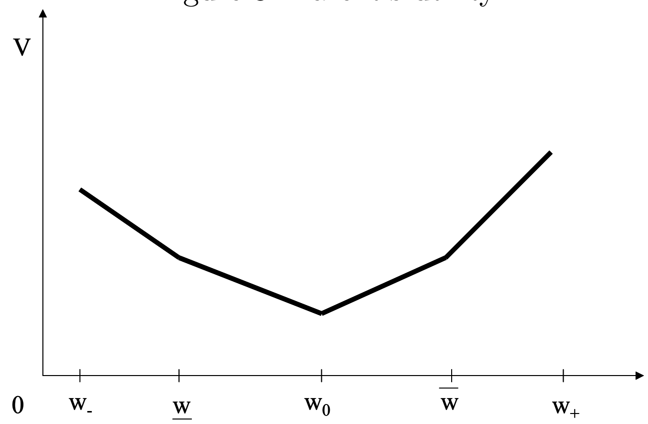
$$\underline{w} < w < \omega_0 : V_1 = \ln(1+\pi)\ln(w+I) - \pi \ln w - (1-\pi)\ln(1+\pi) + \pi \ln \frac{\beta\pi\omega_0}{1+\beta} - \pi D$$

$$\omega_0 < w < \bar{w} : V_2 = (1+\pi)\ln(w+I) - (1+\pi)\ln(1+\pi) + \pi \ln \frac{\beta\pi}{1+\beta} - \pi D$$

$$w > \bar{w} : V_2 = \pi \ln w + \ln I + \pi \ln \frac{\beta}{1+\beta} - \pi D$$

The profile of $V_i$ is given on Figure 3.

Figure 3: Parent's utility



### 2.2.3 Parent's problem with private insurance

With the log-utilities, $a^*$ is simply equal to $\dfrac{I}{\theta(1+\pi)}$ and the utility of the parents:

$$V_3 = (1+\pi)\ln I - (1+\pi)\ln(1+\pi) - \pi\ln\theta - \pi D.$$

To obtain the values of $\hat{w}_1$ and $\hat{w}_2$ (assumed to exist), one respectively solves the following equations:

$$V_3 = V_1(\hat{w}_1) \quad \text{and} \quad V_3 = V_2(\hat{w}_2).$$

Explicitly, this gives

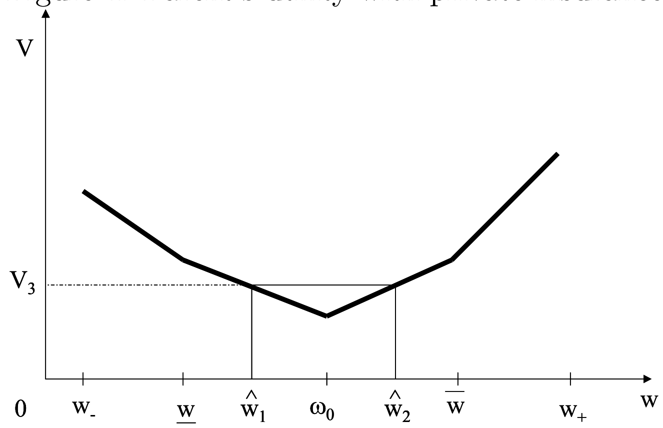
$$V_3 - V_1(\hat{w}_1) = (1+\pi)\left[\ln I - \ln(\hat{w}_1 + I)\right] + \pi\ln\frac{\hat{w}_1}{\omega_0} - \pi\ln\theta - \pi\ln\frac{\beta\pi}{1+\beta} = 0$$

and

$$V_3 - V_2(\hat{w}_2) = (1+\pi)\left[\ln I - \ln(\hat{w}_2 + I)\right] - \pi\ln\theta - \pi\ln\frac{\beta\pi}{1+\beta} = 0.$$

On Figure 4 we represent the value of $V$ along the $w$-axis that is divided in three regimes: assistance in time, private insurance, assistance in cash. It is clear that for high values of $\theta$ (namely for very inefficient markets), the horizontal line $V_3$ could be below the minimum of $V_1$ and $V_2$.

Figure 4: Parent's utility with private insurance



# 3 Public policy

We now introduce three policy instruments: an income tax $t$ levied on children's earnings, a flat subsidy $\sigma$ for children assisting their dependent parents and a public nursing home of quality $g$. In Section 5 we will introduce an additional instrument: an *ad valorem* subsidy on private insurance The income tax is paid by all children; the uniform subsidy and the public nursing home are by assumption mutually exclusive. Parents who end up in a nursing home are not aided by their children.

As we will see in the absence of public nursing home and, for the time being, of private insurance the parents's utility has the $U$-shape of Figure 3. Thus it is pretty intuitive that if providing this facility is not too costly it can be attractive for families with children having productivity around $\omega_0$. For families with very productive children, the quality of the public nursing home may be insufficient. For families with low productivity, the rational choice might be to rely on personal assistance given that children are so more productive at helping their dependent parents than working for an employer (at least in comparative terms).

For clarity sake, let us explicit the sequence of decision.

- Stage 1. The social planner chooses $\tau$, $\sigma$ and $g$.
- Stage 2. Each parent chooses whether or not he leaves some $G$ and how much. If he anticipates that given $(\tau, \sigma, g)$ and in case of bad health he is better off in a nursing home or with a private insurance, he does not leave anything. Otherwise, his child will help him through $h$ or $s$, and he will *ex ante* leave him part of his wealth.
- Stage 3. The child helps his unhealthy parent by comparing the alternatives: assistance in time or in cash.

Note that a child cannot force his parent to go into a nursing home when he receives the gift $G$.

We now look at each child's choice and thus at the parent's choice before turning to the determination of the optimal public policy.

## 3.1 Child's choice

A child with productivity $w$ with a dependent parent chooses $s$ or $h$ to maximize:

$$u\left(\omega\left(1-h\right)+G+\sigma-s\right)+\beta\left[H\left(s+\omega_{0}h\right)+v\left(I-G\right)-D\right]$$

where $\omega = w(1-t)$. As above we have to distinguish between two regimes to determine the optimal choice. For $\omega \leqslant \omega_0$, $s = 0$ and $h^*$ is chosen so that

$$\omega_0 \beta H'(\omega_0 h^*) \geqslant \omega u'(\omega(1-h) + G + \sigma),$$

whereas $h = 0$ and $s^*$ is positive for $\omega > \omega_0$ so that

$$\beta H'(s^*) = u(\omega + G + \sigma - s^*).$$

This yields the following supply functions:

$$h = h(\omega, G_1 + \sigma) \text{ and } s = s(\omega + G_2 + \sigma).$$

Note that the subsidy and the gift have the same effect, but the subsidy is flat whereas the gift varies with $w$. We can also introduce the children's indirect utility functions (without the altruistic component):

$$u_1^D = u(\omega(1 - h(\omega_1 G_1 \sigma)) + G_1 + \sigma) = \underset{+\ \ +\ \ +}{u_1^D(\omega, G_1, \sigma)}$$

and

$$u_2^D = u(\omega + G_2 + \sigma - s(\omega_1 G_2, \sigma)) = \underset{+\ \ +\ \ +}{u_2^D(\omega, G_2, \sigma)}.$$

where the signs of the partial derivatives are given under each argument for well-behaved utility functions. These are used later to characterize the government optimal policy.

With a log-utility:

$$\begin{aligned} h^* &= \frac{\beta}{1+\beta}\left(1 + \frac{G+\sigma}{\omega}\right) && \text{if } G + \sigma \leqslant \omega/\beta \\ h^* &= 1 && \text{if } G + \sigma > \omega/\beta \\ s^* &= \frac{\beta}{1+\beta}(\omega + G + \sigma). \end{aligned}$$

We can also write the consumption of the child with a dependent parent:

$$c_i^D = \frac{1}{1+\beta}(\omega + G + \sigma).$$

It is the same for the two regimes. The consumption of the child when parent is healthy is trivial as it involves no choice:

$$c_i^N = \omega + G_i.$$

## 3.2 Parent's choice without nursing home

Given the above supply function $h^*(\omega, \sigma + G)$ and $s^*(\omega + \sigma + G)$ the parent of a child with productivity $w$ maximizes

$$V_1 = v(I - G) + \pi [H(\omega_0 h^*) - D]$$

or

$$V_2 = v(I - G) + \pi [H(s^*) - D].$$

This yields two supply functions $G_1^*$ and $G_2^*$ depending on whether $\omega \lesseqgtr \omega_0$ and also two indirect utility functions:

$$V_1^* = \underset{- \ \ +}{V_1^*(\omega, \sigma)}$$

and

$$V_2^* = \underset{+ \ \ +}{V_2^*(\omega, \sigma)}.$$

In the log-utility case, these optimal gifts can be written as:

$$G_1^* = Max\left[0, Min\left(\frac{\pi I - \omega - \sigma}{1 + \pi}, \frac{\omega}{\beta} - \sigma\right)\right]$$

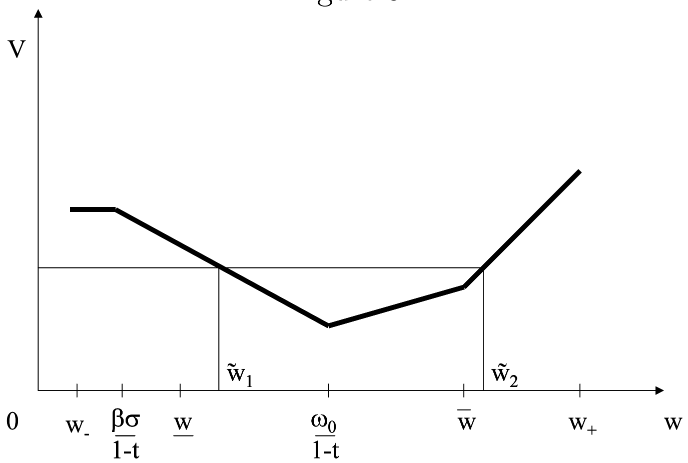
and

$$G_2^* = Max\left[0, \frac{\pi I - \omega - \sigma}{1 + \pi}\right].$$

The values of $m\,(= \omega_0 h$ or $s)$, $G$ and $V$ can be represented as above along the $w$-axis. There is only one difference which comes from the presence of a lump-sum transfer. There is a value of $w = \dfrac{\beta\sigma}{1 - t}$ below which the child devote all his time to his dependent parent and the latter does not find useful to make any gift because even with $G = 0$, $h = 1$.

In any case in this paper we assume that the range of $(w_-, w_+)$ is such that the utility of the parent is first declining and than increasing as on Figure 5.

Figure 5




When $G^* > 0$, $c < d$. For $\underline{w} < w < \bar{w}$, $G^* = G_1^* = G_2^* = \dfrac{\pi I - w - G}{1+\pi}$ yielding

$$c_1^D = c_2^D = \frac{\pi}{1+\beta}\frac{w+\sigma+I}{1+\pi} < d = I - G^* = \frac{w+\sigma+I}{1+\pi}.$$

We also have $d > c^N = w + G = \dfrac{\pi\,(I+w)-\sigma}{1+\pi}$. This result is useful to understand the equity implications of the intergenerational transfers.

## 3.3 Parent's choice with public nursing home

Let us now introduce the possibility for the parent to go to the nursing home. This decision is taken in stage 1 and implies that he does not leave any gift, but also, that in case of bad health, he does not benefit from any filial assistance.

We denote by 4 the regime where children don't help their dependent parents and it is bounded by two levels of wage: $\tilde{w}_1$ and $\tilde{w}_2$.

The first one $\tilde{w}_1$ is determined by the equality between $V_1(\underset{-}{\omega},\underset{+}{\sigma})$ and $V_4 = V_4\,(g) = v\,(I) + \pi\,[H\,(g) - D]$. Similarly, $\tilde{w}_2$ is determined by the equality between $V_2(\underset{+}{\omega},\underset{+}{\sigma})$ and $V_4\,(g)$. There are values of $g$ that are so low that the parent would never choose to go to the public nursing home. This appears clearly on Figure 5.

From the equalities $V_1 = V_4\,(g)$ and $V_2 = V_4\,(g)$, we can write:

$$\tilde{w}_1 = \frac{1}{1-t}\psi_1(\underset{+}{\sigma},\underset{+}{g}) \quad \text{and} \quad \tilde{w}_2 = \frac{1}{1-t}\psi_2(\underset{-}{\sigma},\underset{+}{g}) > \tilde{w}_1.$$

We denote by $n_4$ the fraction of parents opting for the public nursing home.

$$n_4 = F(\tilde{w}_2) - F(\tilde{w}_1) = n_4(\underset{?}{t}, \underset{-}{\sigma}, \underset{+}{g}).$$

The effect of $t$ in $n_4$ is ambiguous. Indeed

$$(1-t)^2 \frac{\partial n_4}{\partial t} = F'(\tilde{w}_2)\psi_2 - F'(\tilde{w}_1)\psi_1.$$

With a uniform density, given that $\psi_2 > \psi_1$, the number of dependent parents going to nursing homes increases with the tax rate, namely $\dfrac{\partial n_4}{\partial t} > 0$.

In this subsection, we have ignored the possibility of resorting to private insurance. Implicitly we were assuming that from the viewpoint of the parents public nursing was preferred over private insurance. Formally,

$$V_4(g) = v(I) + \pi(H(g) - D) \geqslant V_3 = v(I - \pi a^* \theta) - \pi(H(a^*) - D)$$

where $a^*$ is optimally chosen. This is equivalent to say that the parent will choose public nursing if

$$g \geqslant \hat{g},$$

where $\hat{g}$ is defined by $V_4(g) = V_3$. Note the difference between the two ways of financing long term care: $g$ is paid by the young generation whereas $a$ is paid by the parent himself.

## 3.4 The revenue constraint

The government collects a proportional payroll tax on children's earnings and use it to finance both subsidy and nursing homes. The labor supply of workers with productivity higher than $\tilde{w}_1$ is 1; that of workers with productivity below $\tilde{w}_1$ is $(1-h^*)$ or 1 depending on whether or not their parents are dependent. Let us introduce the parameter $q$ that reflects the cost of providing nursing home services. We expect that $q > 1$, which implies some inefficiency. The revenue constraint can be written as

$$\varphi(t, \sigma, g) = t\bar{y} - \pi(1-n_4)\sigma - \pi n_4 q g = 0$$

where

$$\bar{y} = (1-\pi)\bar{w} + \pi \int_{w_-}^{\tilde{w}_1} w(1-h^*)\, dF(w) + \pi \int_{\tilde{w}_1}^{w_+} w dF(w).$$

In this expression $\bar{y}$ and $\bar{w}$ are respectively average income and average wage; $h^*$, $\tilde{w}_1$, $\tilde{w}_2$ and $n_4$ are functions of policy tools.

For further use we can derive $\varphi(t,\sigma,g)$ with respect to its three arguments:

$$\varphi_t = -\pi[\int_{w_-}^{\tilde{w}_1} w\underset{+}{\frac{\partial h^*}{\partial t}}d\ F(w) + \tilde{w}_1 h^* \underset{+}{\frac{d\tilde{w}_1}{dt}}] - \pi(gq-\sigma)\underset{?}{\frac{\partial n_4}{\partial t}}$$

$$\varphi_\sigma = -\pi[\int_{w_-}^{\tilde{w}} w\underset{+}{\frac{\partial h^*}{\partial \sigma}}d\ F(w) + \tilde{w}_1 h^* \underset{+}{\frac{d\tilde{w}_1}{d\sigma}}] - \pi[(1-n_3) + (gq-\sigma)\underset{-}{\frac{\partial n_4}{\partial \sigma}}]$$

$$\varphi_g = -\pi\ \tilde{w}_1\ h^* \underset{-}{\frac{d\tilde{w}_1}{dg}} - \pi[n_4\theta + (gq-\sigma)\underset{+}{\frac{\partial n_4}{\partial g}}].$$

The signs below the derivative hold for general utility functions. Only those pertaining to $h^*$ rest on the logarithmic case.

# 4 Optimal policy

## 4.1 Unconstrained first-best

As a benchmark we first consider the resource allocation that a social planner would implement if he had perfect information and full control of the economy. The objective that we find appropriate is the sum of individual utilities after having removed the altruistic component from the children's utility. In other words we consider a social welfare function:

$$\begin{aligned} SW &= \int_{w_-}^{w_+} \{\pi\left[u\left(c^D\right) + v\left(d^D\right) - D + H\left(h\omega_0 + s + g\right)\right] \\ &\quad + (1-\pi)\left[u\left(c^N\right) + v\left(d^N\right)\right]\} d\ F(w). \end{aligned}$$

This view is not properly utilitarist. Yet, if we were adding individual utilities this would amount to weight the welfare of the elderly people by $(1+\beta)$ and not by 1.[5]

The first-best implies the equality of marginal utilities of consumption: $u'\left(c^D\right) = v'\left(d^D\right) = u'\left(c^N\right) = v'\left(d^N\right)$. It also implies that the best long term care technology is used; this involves using the contribution of children with $w < \omega_0$. Finally, we should have $u'\left(c^D\right) = H'\left(h\omega_0 + s + g\right)$, knowing that these three arguments are mutually exclusive.

[5] See on this Hammond (1997), Cremer and Pestieau (2001).

## 4.2 Second-best optimality

We now turn to a second-best setting with imperfect information and restricted policy tools; namely linear taxation, lump-sum, but conditional subsidy and public nursing homes. We keep in mind that public nursing homes and private insurance are mutually exclusive (The former dominates the latter if $g \geqslant \hat{g}$). In other words, we can have a partition of the interval $(w_-, w_+)$ either in the three subintervals $(w_-, \hat{w}_1)$, $(\hat{w}_1, \hat{w}_2)$, $(\hat{w}_2, w_+)$ if public nursing prevails or the three subintervals $(w_-, \tilde{w}_1)$, $(\tilde{w}_1, \tilde{w}_2)$, $(\tilde{w}_2, w_+)$ if private insurance happens to be more attractive $(g < \hat{g})$. To focus on the choice of public policy, we assume away private insurance for the time being. In section 5 we discuss the choice between the two programs introducing the possibility of subsidizing private insurance.

We write the problem of the government with the following Lagrangean expression.

$$\begin{aligned}\mathcal{L}_1 &= \int_{w_-}^{\hat{w}_1} \left(\bar{u}_1 + V_1\right) dF(w) + \int_{\hat{w}_1}^{\hat{w}_2} \left(\bar{u}_4 + \tilde{V}_4\right) dF(w) + \int_{\tilde{w}_2}^{w_-} \left(\bar{u}_2 + \check{V}_2\right) dF(w) \\ &\quad -\mu\left[(1-n_4)\pi\sigma + n_4\pi q g - t\bar{y}\right].\end{aligned}$$

where the $\bar{u}_i$ denotes the child's indirect utility net of the altruistic component.

$$\bar{u}_1(w) = \pi u\left(w(1-t)\left(1-h^*\left(w(1-t), G_1^*+\sigma\right) + G^* + \sigma\right) + (1-\pi)u\left(w(1-t)+G_1^*\right)\right)$$

$$\bar{u}_2(w) = \pi u\left(w(1-t) + G_2^* + \sigma - s\left(w(1-t), G_2^*+\sigma\right)\right) + (1-\pi)u\left(w(1-t)+G_2^*\right)$$

$$\bar{u}_4 = u\left(w(1-t)\right).$$

We now derive the FOC:

$$\begin{aligned}\frac{\partial \mathcal{L}_1}{\partial t} &= \left(\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+}\right)\left[\pi(1-\beta)H'(m)\frac{\partial m^*}{\partial t} + \left(\bar{u}'(c) - v'(d)\right)\frac{\partial G^*}{\partial t}\right] \\ dF(w) &- \int_{w_-}^{w_+}\left[\pi u'\left(c^D\right)y^D + (1-\pi)u'\left(c^N\right)y^N\right]dF(\omega) + \tilde{\Delta}_1\frac{d\tilde{w}_1}{dt} + \tilde{\Delta}_2\frac{d\tilde{w}_2}{dt} \\ &+\mu\left[\bar{y} + t\frac{\partial \bar{y}}{\partial t} - \pi(qg-\sigma)\frac{\partial n_4}{\partial t}\right] = 0. \qquad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_1}{\partial \sigma} &= \left(\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+}\right)\left[\pi(1-\beta)H'(m)\frac{\partial m^*}{\partial \sigma} + \pi u'\left(c^D\right) + \left(\bar{u}'(c) - v'(d)\right)\frac{\partial G}{\partial \sigma}\right]dF(w) \\ &+\tilde{\Delta}_1\frac{d\tilde{w}_1}{d\sigma} + \tilde{\Delta}_2\frac{\partial \tilde{w}_2}{\partial \sigma} - \mu\left[t\frac{\partial \bar{y}}{\partial \sigma} - \pi(qg-\sigma)\frac{\partial n_4}{\partial \sigma} - (1-n_4)\pi\right] = 0. \qquad (2)\end{aligned}$$

$$\frac{\partial \mathcal{L}_1}{\partial g} = n_4 \ \pi \ H'(g) + \tilde{\Delta}_1 \frac{d\tilde{w}_1}{dg} + \tilde{\Delta}_2 \frac{\partial \tilde{w}_2}{\partial g} + \mu t \frac{\partial \bar{y}}{\partial g}$$
$$-\mu \left[ \pi q n_4 + \pi (qg - \sigma) \frac{\partial n_4}{\partial g} \right] = 0. \tag{3}$$

where $\tilde{\Delta}_1 = \bar{u}_1(\tilde{w}_1) - u((1-t)\tilde{w}_1)$ and $\tilde{\Delta}_2 = u((1-t)\tilde{w}_2) - \bar{u}_1(\tilde{w}_2)$ denote the difference of utility for the child with productivity $\tilde{w}_i$ between helping his parent or not. As the choice is made by the parent, we cannot sign these two differences.

To interpret equations (1) and (2) we combine them as follows: $\dfrac{\partial \mathcal{L}_1^c}{\partial t} = \dfrac{\partial \mathcal{L}_1}{\partial t} + \dfrac{\bar{y}}{\pi(1-n_4)} \dfrac{\partial \mathcal{L}_1}{\partial \sigma}$ where the superscript $c$ stands for compensated.

$$\frac{\partial \mathcal{L}_1^c}{\partial t} = \left( \int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+} \right) \left( \overset{[1]}{(1-\beta)\pi H' \tfrac{\partial m^c}{\partial t}} + \overset{[2]}{[\bar{u}' - \upsilon'] \tfrac{\partial G^c}{\partial t}} \right) dF(w)$$
$$- \overset{[3]}{\sum_{D,N} \pi_j cov\left(u'\left(c^j\right), y^j\right)} + \overset{[4]}{(1-\pi)\bar{y}^N E\left[u'\left(c^D\right) - u'\left(c^N\right)\right]}$$
$$\overbrace{- \bar{y}_4 \left[ \int_{\tilde{w}_1}^{\tilde{w}_2} u'\left(c^D\right) dF(w) - \left( \int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+} \right) \frac{n_4}{1-n_4} u'\left(c^D\right) dF(w) \right]}^{[6]}$$
$$\overbrace{+ \tilde{\Delta}_1 \frac{d\tilde{w}_1^c}{dt} + \tilde{\Delta}_2 \frac{d\tilde{w}_2^c}{dt}}^{[7]}$$
$$+\mu \left[ \overset{[5]}{t \frac{\partial \bar{y}^c}{\partial t}} - \overset{[8]}{\pi (gq - \sigma) \frac{\partial n_4^c}{\partial t}} \right] = 0 \tag{4}$$

We now interpret the tax transfer formula (4) and the formula for $g$ given by (3).

But first we consider the case where $g$ and not only $a$ are not available.

### 4.2.1 The case with $g = 0$ and $a = 0$[6]

Then $n_4 = 0$,and using the operator $E$for the support $(w_-, w_+)$, one can rewrite (4)as:

$$t = \frac{(1-\beta)\pi EH'(m)\frac{\partial m^c}{\partial t} + E[\bar{u}'(c) - v'(d)]\frac{\partial G^c}{\partial t} - \sum_{j=D,N} \pi^j cov\left(u'\left(c^j\right), y^j\right) + (1-\pi)\bar{y}^N E\left[u'\left(c^D\right) - u'\left(c^N\right)\right]}{-\mu \frac{\partial \bar{y}^c}{\partial t}}. \tag{5}$$

To interpret formula (5) we consider each of its components (defined in (4)).

The first term [1] in the numerator reflects the paternalistic action of the social planner. If $\beta = 1$, namely if the social planner and children have the same view on the parents' utility, this term vanishes. For $\beta < 1$, both $t$ and $\sigma$ are desirable if $\frac{\partial m^c}{\partial t} > 0$. In other words, if the tax-transfer policy encourages assistance and if the social planner puts more weight on the parents than the children, this policy should be encouraged. Yet one cannot excludes $\frac{\partial m^c}{\partial t} < 0$ in which case, a paternalistic government will choose a lower tax transfer than if it were not paternalistic. Using the logarithmic example, we see that $\frac{\partial m^c}{\partial t}$ is positive for $w < Max\left[\frac{\omega_0}{1-t}, \frac{\bar{y}}{\pi}\right]$. Roughly speaking, if the majority of children have a low productivity, namely a $w$ below either $\frac{\omega_0}{1-t}$ and $\frac{\bar{y}}{\pi}$, one can expect the tax-transfer policy to stimulate $m$. Then, the first term in the numerator of (5) is positive if $\beta < 1$.

The second term [2] reflects the effect of the tax-transfer on gifts, that expectedly narrows the difference between the marginal utilities of children and parents. If the tax-transfer package induces additional gift, then it will be higher for that reason. With the log utility, we know that $u'(c) > v'(d)$. When parents can afford leaving some gift ($G > 0$) to their children they have a higher consumption than them. $\frac{\partial G^c}{\partial t}$ is positive for $w > \bar{y}/\pi$. It is thus negative if the majority of children have a productivity below that threshold. The second term is negative.

The third term [3] is made of covariances. It expresses the traditional equity consideration. The covariances are negative and they increase (in absolute value) with the concavity of $u(c)$ and the inequality of $w$. As it appears, there is a covariance for each state of nature. This is the traditional equity term that one finds in the literature on linear income tax.

The fourth term [4] in the numerator depends on the gap between children's consumption levels in the two state of nature. To the extent that

[6] This is the case discussed in Pestieau and Sato (2004).

$c^D < c^N$, this term is positive and hence pushes for relatively higher tax-transfers.

It is interesting to observe that we have here a number of sources of inequality: wage inequality, inequality between children with and without dependent parents, inequality among parents leaving different gifts. For the first two, we have some redistribution. Not for the last one.

Finally we come to the denominator [5]. It represents the traditional efficiency term. If the tax transfer policy has a low incidence on the tax base, it will be relatively high. Using the log linear illustration, it clearly appears that $\dfrac{\partial \bar{y}^c}{\partial t}$ is negative. This is because both the tax on earnings and the subsidy tend to foster $h$ and thus to discourage market labor supply.

To sum up, assuming that the majority of children have a productivity below the average, the only term pushing for a low level of tax transfer is the second one. The tax then depresses gifts which contribute to the redistribution between parents and children. All the other terms push for a positive level of tax-transfer.

### 4.2.2 The case with $g > 0$ and $a = 0$

We now have to enlarge our tax formula. Using the notation of equation (4), it becomes:

$$t = \frac{[1] \;+\; [2] \;-\; [3] \;+\; [4] \;-\; [6] \;+\; [7]}{-[5] \;+\; [8]}. \qquad (6)$$

We have to discuss the contribution of three additional terms [6] , [7] and [8] to the optimal tax transfer. Term [6] gives the difference in children's marginal utility when they help their dependent parent or when they have them in a nursing home (in this case $c^D = c^N$). If this difference is positive (which is likely if the majority of children have an income below $\tilde{w}_1$), term [6] has a positive effect on $t$.

Term [7] gives the effect that the tax-transfer has on the bounds $\tilde{w}_1$ and $\tilde{w}_2$ each being weighted by the change in utility the child incurs going from one regime to another. We know that $\dfrac{d\tilde{w}_1^c}{dt} > 0$; the sign of $\dfrac{d\tilde{w}_2^c}{dt}$ is ambiguous. Suppose that it is positive as well. If $\tilde{\Delta}_1$ is positive (the child is better off in regime 4 than in regime 1) and $\tilde{\Delta}_2$ is also positive (the child is better off in regime 2 than in regime 4), then this term will have an ambiguous positive influence on $t$. In general, its signs is uncertain.

The final term [8] is the revenue cost of changing $n_4$. Increasing $n_4$ implies more spending on public nursing homes, but less in subsidies to children. Not surprisingly, if $gq > \sigma$, which is expected (particularly with a large $q$), and if

$\frac{\partial n_4^c}{\partial t} > 0$, which is reasonable, this term is positive and it pushes for a lower tax.

Equation (3) gives the formula for the optimal determination of $g$. The marginal benefit ($g$ for $n_4$ parents plus the effect on the two bounds) is equal to the marginal cost (the cost of providing $g$ to the $n_4$ parents plus the effect of shifting from subsidy to nursing homes). The role of the efficiency parameter $q$ is clearly important.

As above assume that both $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$ are positive. We know that $\frac{d\tilde{w}_1}{dg} < 0$ and $\frac{d\tilde{w}_2}{dg} > 0$. We can reasonably posit that $qg > \sigma$ and $\frac{\partial n_4}{\partial g} > 0$. In words, public nursing costs more than subsidies on filial assistance and increasing the quantity but also the quality of public nursing induces parents to choose that option. Finally, we can also think that $g$ depresses the assistance in time and thus increases aggregate earnings. With these assumptions the cost terms (in square brackets) are straightforward. The benefits include a negative term: the lower bound $\tilde{w}_1$ decreases and this causes a utility loss for the children having that productivity. But this is offset by a gain in tax revenue.

# 5 Private insurance subsidy

## 5.1 Private insurance versus public nursing home

Now we re-introduce the private insurance. Quite clearly, in this paper, private insurance and public nursing home are two ways for the parents to opt out of family solidarity. The difference is that public nursing home is free whereas private insurance costs them a premium equal to $\pi a\theta$. In subsection 4.2.2 we assumed that the optimal value of $g$ were higher or equal to $\hat{g}$. If this were not the case, $g = 0$ and we have formula (5) with a change: the bounds $\tilde{w}_1$ and $\tilde{w}_2$ are to be replaced by the bounds $\hat{w}_1$ and $\hat{w}_2$.

In the same way, as both $h$ and $s$ are subsidized, it is natural to introduce a subsidy of rate $\tau$ on insurance premium. The premium for a coverage $a$ is now:

$$(1-\tau)\,\theta\pi a$$

and with the logarithmic utility:

$$a^* = \frac{1}{1+\pi}\frac{I}{(1-\tau)\,\theta}.$$

This yields the following expected utility for the parent:

$$V_3 = (1+\pi)\ln I - \pi\ \ln(1-\tau) - \pi\ln\theta - (1+\pi)\ln(1+\pi) - \pi D.$$

We can now compute $\hat{g}$, the value of $g$ that makes the parent indifferent between private insurance and public nursing. To be specific, $\hat{g}$ can be obtained by solving $V_3 = V_4(\hat{g})$ $(= \ln\ I + \pi \ln\ g - \pi D)$ and then,

$$\hat{g} = \frac{I}{(1-\tau)\,\theta}\left(\frac{1}{1+\pi}\right)^{\frac{1+\pi}{\pi}}.$$

It is interesting to introduce the subsidy rate $\bar{\tau}$ that implies equal public spending on the two ways of providing care, namely:

$$\pi n_4 q \hat{g} = n_3 \frac{\pi}{1+\pi} I \frac{\bar{\tau}}{1+\bar{\tau}}$$

or

$$\bar{\tau} = \frac{q}{\theta}\left(\frac{1}{1+\pi}\right)^{\frac{1}{\pi}}$$

where $n_3 = n_4$.

Let us now consider the values of public nursing, $g^*$, that was found above. We want to see if resorting to an insurance subsidy can be socially preferable. Let us define the subsidy rate $\tau^*$ such that

$$g^* = \hat{g} = \frac{I}{(1-\tau^*)\,\theta}\left(\frac{1}{1+\pi}\right)^{\frac{1+\pi}{\pi}}.$$

It is clear that if $\tau^* \leqslant \bar{\tau}$, private insurance subsidy dominates public nursing. The proof goes as follows. Start with optimal level of $g$. Then we can obtain $\tau^*$ to yield $V_3 = V_4$ as above. If this $\tau^*$ is less than $\bar{\tau}$, the government can spend less by switching from $g^*$ to $\tau^*$ given $t$ and $\sigma$. This implies that either $t$ can be lowered or $\sigma$ can be increased, enhancing social welfare.

## 5.2 Optimal subsidy

Up to this point we have not derived the optimal insurance subsidy rate. Knowing that $\tau$ and $g$ are mutually exclusive, let us consider the Lagrangean expression for $a > 0$ and $g = 0$.

$$\begin{aligned}\mathcal{L}_2 \quad = \quad & \int_{w_-}^{\hat{\omega}_1} (u_1 + V_1)\, dF(w) + \int_{w_+}^{\hat{w}_2} (\bar{u}_3 + V_2)\, dF(w) \\ & + \int_{\hat{w}_2}^{w_+} (\bar{u}_2 + V_2)\, dF(w) - \mu\left[(1-n_3)\,\pi\sigma + n_3 I\theta\pi a^* - t\bar{y}\right].\end{aligned}$$

After some simplifications we obtain

$$\frac{\partial \mathcal{L}_2}{\partial \tau} = n_3 \pi H' \frac{\partial a^*}{\partial \tau} + \mu t \frac{\partial \bar{y}}{\partial \tau} + \hat{\Delta}_1 \frac{d\hat{w}_1}{d\tau} + \hat{\Delta}_2 \frac{d\hat{w}_2}{d\tau}$$
$$-\mu\pi \left[ n_3 \left( \theta \left( a^* + \tau \frac{\partial a^*}{\partial \tau} \right) \right) + (\tau\theta a^* - \sigma) \frac{\partial n_3}{\partial \tau} \right] = 0. \tag{7}$$

This expression can be compared with the expression for an optimal $g$, namely equation (3). $\hat{\Delta}_i$ is defined similarly as $\tilde{\Delta}_i$ replacing $\tilde{w}_i$ by $\hat{w}_i$.

We can make the same assumptions as in the interpretation of (3): $\hat{\Delta}_1, \hat{\Delta}_2$ positive, $\frac{d\hat{w}_1}{d\tau} < 0$, $\frac{d\hat{w}_2}{d\tau} > 0$, $\frac{\partial n_3}{\partial \tau}$ and $\frac{\partial \bar{y}}{\partial \tau} > 0$. As to $\tau\theta a^* - \sigma$ we take it positive. With these assumptions, we have an equality between the benefits arising from a better health ($H'$), more tax revenue and more (less) utility for children with income $\hat{w}_2$ ($\hat{w}_1$) and the cost of the insurance subsidy increased by the difference between the two subsidies ($\tau\theta a^* - \sigma$).

As discussed in 5.1 there is no obvious way to assert whether $g^*$ or $a^*$ is to be chosen.

### 5.3 Parents with different endowments

Up to now we have assumed that parents had some resources $I$, the same for all, that were high enough for some of them to *ex ante* "buy" the assistance of their children in case of dependency.

Let us now consider the case where there are two levels of parental endowment: a high level $\bar{I}$ as before and a low level $\underline{I}$. This latter level could be so low that parents cannot afford to make any gift and to buy any decent private insurance in the absence of subsidy.

In this new setting we show that a plausible outcome is public care for the poor parents with middle income children and subsidized private insurance for the rich parents with middle income children. The level of both subsidy and pubic care is adjusted to maximize social welfare. Two self-selection constraints have to be taken care of. First, one does not want the rich parent to benefit from public care. Second, given that the subsidy cannot be individualized, public care to the poor is preferred over an heavily subsidized private insurance, even if the latter would be more efficient with observability of parent's income. In Appendix 2 we develop this argument.

## 6 Conclusion

The purpose of this paper was to design an optimal tax transfer policy for long term care. The setting is relatively simple. Each elderly person has an altruistic child who will help him in case of loss of autonomy. Help can be of two types: time for low productivity children, cash for high productivity children. To foster help from their children parents can *ex ante* make a gift to their children. The government can subsidize children's assistance. But it can also directly provide the services of nursing homes. Parents of middle productivity children tend to rely on nursing homes, but in that case they don't give anything to their children. Private insurance appears to be a substitute for public nursing homes, but not for children's assistance.

The case of public nursing homes is quite strong, particularly when private long term care insurance is inefficient. The case of subsidy for either type of assistance is not clear. For redistributive reason, a scheme of tax-subsidy is desirable as it narrows down some differences in consumption. At the same time, it can have undesirable effects on some type of assistance and on the level of *inter vivos* gifts. To clear this ambiguity one has to know more about the distribution of $w$, the level of $I$ and the concavity of the utility function.

Two questions can be raised in conclusion. Is it realistic? Is it not too simplistic? The two questions are naturally related. As mentioned in the introduction, the problem of long term care is very complex. It is also relatively new. There is no much evidence on the socio-economic characteristics of people suffering from loss of autonomy and of those of their close relatives. If productivity was the only distinctive factor, the pattern discussed in this paper could be quite natural. There are however other characteristics. For example, altruism is not uniform across families. Some elderly don't even have children to care about them. Introducing differential altruism along with differential productivity could complicate the model. It is for example clear that in that case public nursing homes would cater not only to parents of middle productivity children with altruism, but also to all parents with non altruistic children. In this case we are faced with a moral hazard problem if altruism cannot be observed (see Jousten *et al.* (2003)). Another difficulty that we have assumed away is that loss of autonomy may not be observable. This leads to another moral hazard problem as it is tempting for healthy parents to mimick unhealthy parents. Again this would add an additional constraint to the design of an optimal tax-transfer scheme.

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# Appendix 1

In the main text we have chosen values of $I$ and $\omega_0$ that cover the different regimes we were interested in. It is important to see that this is not necessarily the case.

With the log-linear utility function, we have the following values for the optimal gift:

$$G_1^* = Max\left(0, Min\left(\frac{\pi I - \omega - \sigma}{1+\pi}, \frac{\omega}{\beta} - \sigma\right)\right) \tag{A.1}$$

$$G_2^* = Max\left(0, \frac{\pi I - \omega - \sigma}{1+\pi}\right) \tag{A.2}$$

and we define two thresholds values of $\omega$, $\underline{\omega}$ and $\bar{\omega}$.

$$\underline{\omega} = \frac{\beta\pi}{1+\pi+\beta}(I+\sigma)$$

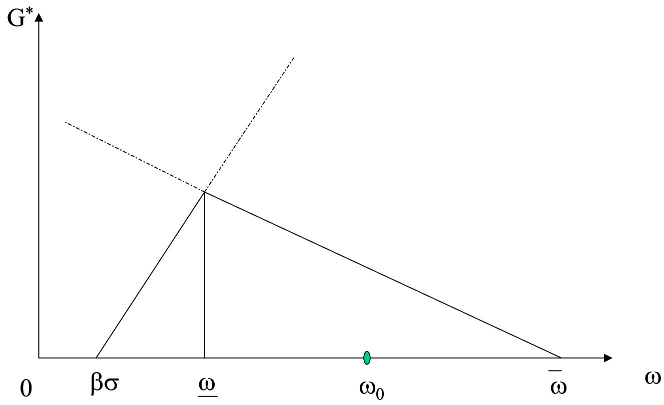
and

$$\bar{\omega} = \pi I - \sigma.$$

$\underline{\omega}$ is the value of $\omega$ at which the two terms in the $Min$ expression of $(A.1)$ coincide and $\bar{\omega}$ is the value of $\omega$ above which $G^* = 0$.

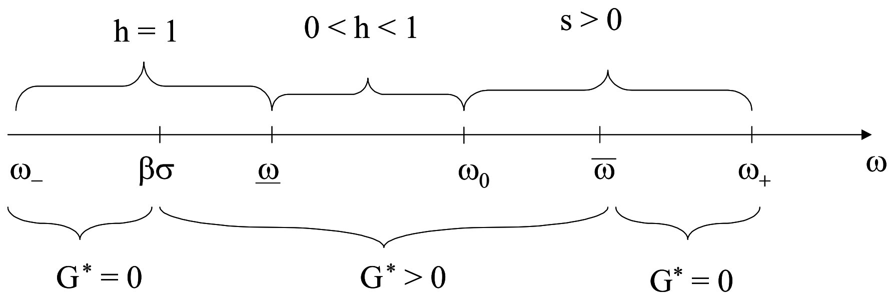
Graphically we can represent $G^*$ on Figure A.1 with $\omega_0$ between $\underline{\omega}$ and $\bar{\omega}$.

Figure A.1

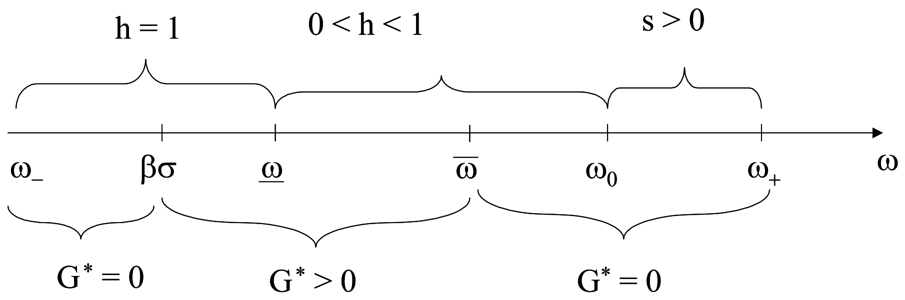



In fact we can distinguish 3 cases depending on the respective values of $I$ and $\omega_0$.

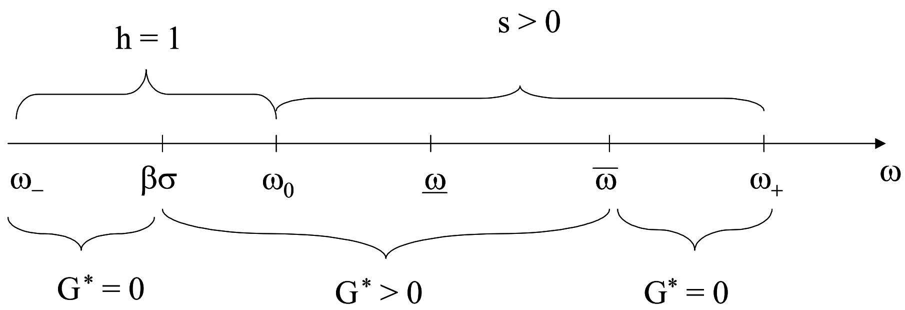
1. $\omega_0 < \bar{\omega}$ or $I > \dfrac{\omega_0 + \sigma}{\pi}$




2. $\omega_0 > \bar{\omega}$ or $I > \dfrac{\omega_0 + \sigma}{\pi}$

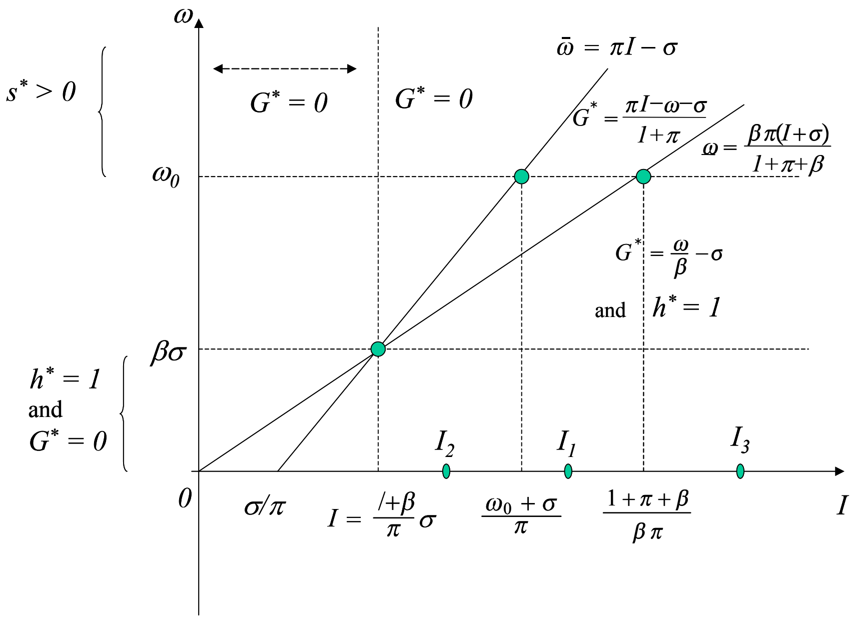



3. $\underline{\omega} > \omega_0$ or $I > \dfrac{1 + \pi + \beta}{\beta\pi}\omega_0 - \sigma$




Note that we assume that $\beta\sigma < \omega_0$ or $\sigma < \dfrac{\omega_0}{\beta}$. We can represent these three cases in the plane $(\omega, I)$ on Figure A.2.

Figure A.2




In this paper we assume that for a given value of $\omega_0$, $I$ is in the interval $\left(\dfrac{\omega_0+\sigma}{\pi}, \dfrac{1+\pi+\beta}{\beta\pi}\omega_0 - \sigma\right)$ which corresponds to case 1 or to a value of $I$ given by $I_1$ on Figure A.2.

For a value $I_2$, we have case 2 and for a value $I_3$ we have case 3.

# Appendix 2

## Two levels of income. Public care versus private insurance

In this appendix we use the logarithmic utility and denote by $p$ and $r$ the poor and the rich parents. We define $\hat{g}^j$ such that

$$V_3^j(\tau) = V_4^j\left(\hat{g}^j\right)$$

or

$$(1+\pi)\ln I^j + \pi \ln \frac{1}{1-\tau} + \pi \ln \frac{1}{\theta} + (1+\pi)\ln \frac{1}{1+\pi} = \ln I^j + \pi \ln \hat{g}^j,$$

where $j = p, r$.

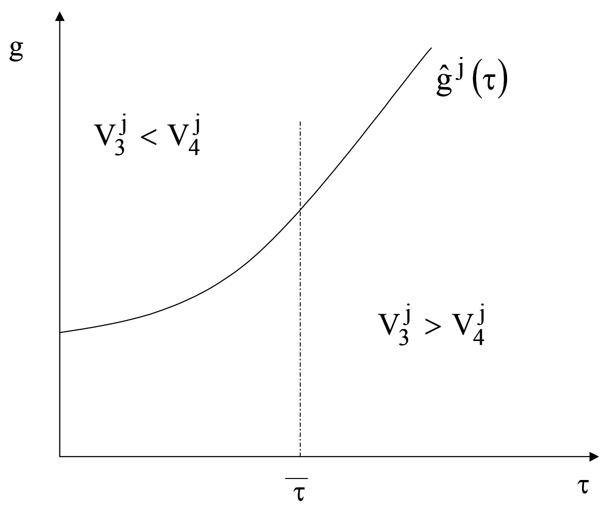
From this equality we derive the function $\hat{g}^j(\tau)$:

$$\hat{g}^j = I^j \left[\frac{1}{1-\tau}\frac{1}{\theta}\right]\left(\frac{1}{1-\pi}\right)^{\dfrac{1+\pi}{\pi}}.$$

This function is represented on Figure A.3 in the plane $(g, \tau)$.

Figure A.3

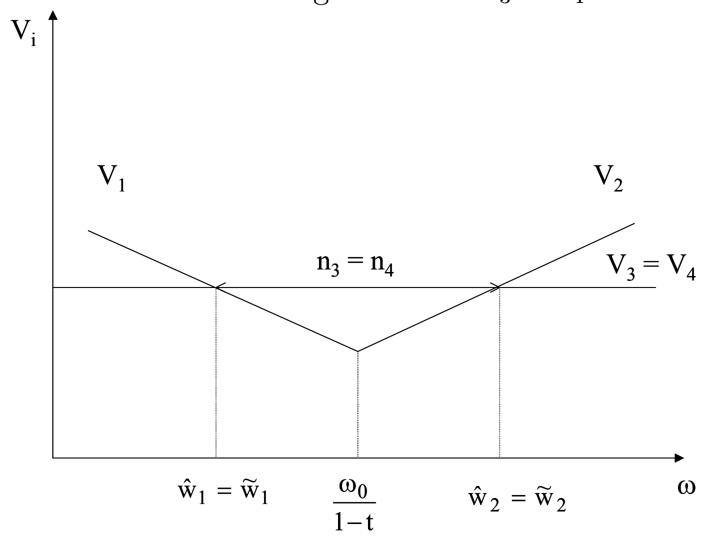
Public care versus private insurance



It is important to observe that when $V_3 = V_4$, $\hat{w}_i = \tilde{w}_i$ and $n_3 = n_4$ as it appears on Figure A.4.

Figure A.4

Alternative regimes when $V_3 = V_4$




Both Figures A.3 and A.4 are drawn for a given value of $I$. One clearly sees that for a higher value of $I$ curve $\hat{g}^i(\tau)$ in Figure A.3 and the the $V$-shape curve $V_1 - V_3\,(= V_4) - V_2$ has to move upward.

When $V_3 = V_4$, total spending by the government with $\tau$ or $g$ is generally not equivalent. Indeed, one can define the value of $\tau$ for which such equivalence would hold. Denoting that value by $\bar{\tau}$, one has:

$$E_3 = \pi(1 - n_3)\sigma + n_3 \pi I \frac{\bar{\tau}}{1-\bar{\tau}}(1+\pi)^{-1} = E_4 \equiv \pi(1-n_4)\sigma + n_4 \hat{g}.$$

After substitution we obtain:

$$\bar{\tau} = \frac{q}{\theta}\left[\frac{1}{1+\pi}\right]^{\frac{1}{\pi}}.$$

On Figure A.3 the dotted line represents the value of $\bar{\tau}$: to the right of $\bar{\tau}$ public spending is higher with a subsidy on private insurance and to the left of $\bar{\tau}$, this is other way around.

The two functions defining $\bar{\tau}$ and $\hat{g}^j$ partition the $(g, \tau)$ plane in four areas. Note that $\bar{\tau}$ is independent of $I$. This partitioning can be useful to compare the desirability of $g^*$ and $\tau^*$, the optimal values of those two parameters obtained through separate optimization.

If we have just one type of parent, we denote the value of those two parameters $g_4^*$ and $\tau_3^*$. Take $g_4^*$ and consider the value of $\tau$ that would make $V_3 = V_4$. If that value of $\tau$ is inferior to $\bar{\tau}$, it is clear that $g_4^*$ is dominated by $\tau_3^*$.

We now introduce explicitly our two levels of $I$. Note that if $I^p$ is low enough it is possible that parents cannot afford leaving any gift $G$ to their children. This implies a simple way to express both $V_1^p$ and $V_2^p$.

With two levels of parental resources, we have Figure A.5 with two curves $\hat{g}^r$ and $\hat{g}^p$.

Figure A.5

Public care versus private insurance with $I^r$ and $I^p$

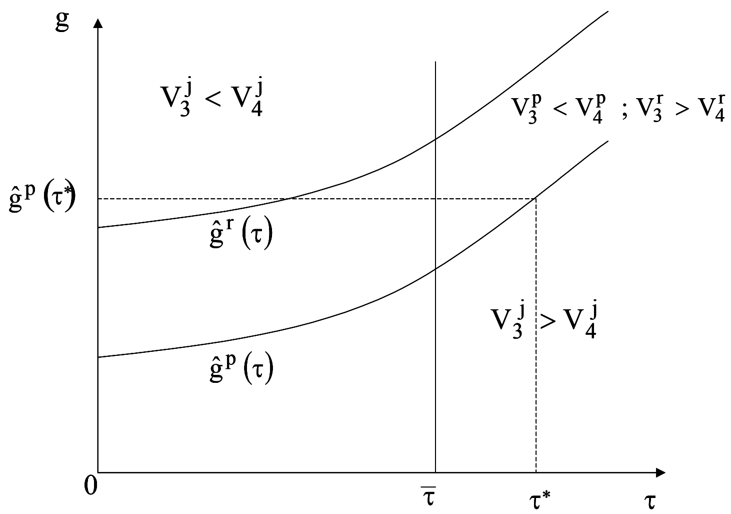



Figure A.5 can be used to show when it might be desirable to have a subsidy $\tau^*$ for the rich parents and public nursing $g^*$ for the poor parents. Suppose that the optimal subsidy $\tau^*$ is to the right of $\bar{\tau}$ and that initially the government provides only the insurance subsidy that is optimized at $\tau^*$. We can choose the value of $g$ given by $\hat{g}^p(\tau^*)$. If we give this amount of public care to the $n_3^p (= n_4^p)$ poor parents, nothing changes except that there are some available resources, which increase social welfare. It is important to make sure that the rich parents are not going to be tempted to use public nursing as well. For that, it suffices that they are better off with $\tau^*$ than with $\hat{g}^p(\tau^*)$ given that $\hat{g}^p(\tau^*) < \hat{g}^r(\tau^*)$.