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Choice, Opportunities, and Procedures: Collected Papers of Kotaro Suzumura

Part V Competition, Cooperation and Economic Welfare

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Chapter 17
Entry Barriers and Economic Welfare*

1 Introduction

Recent studies in the theoretical industrial organization literature have uncovered several instances which cast serious doubt on the reasonableness of "a widespread belief that increasing competition will increase welfare (Stiglitz [14, p.184])".[1] It has been shown that there are cases, which are not altogether unreasonable, where social welfare will be increased by strengthening, rather than weakening, the protection of incumbent firms from the threat of potential entry. This is in sharp contrast with the traditional belief.[2] What is not known, however, is how robust these "pathologies" in fact are. We intend to settle this problem by proving two "excess entry theorems" in the quasi-Cournot (parametric conjectural variations) homogeneous oligopoly model (Seade [11; 12]).

In the first place, we presuppose the existence of a strong ("first-best") government that could costlessly enforce the marginal-cost pricing principle by firms in an oligopolistic market and regulate entry in pursuit of "first-best" social welfare optimization. It is shown that the number of firms at the free-entry equilibrium exceeds the "first-best" welfare optimizing number of firms. The result has already been noted by von Weizsäcker [17; 18] and others in terms of numerical examples. Our Theorem 1 asserts that this phenomenon always holds true for the family of quasi-Cournot models at hand. An implication of this result is that the existence of entry barriers which protect incumbent

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[1] As Baumol [1, p.2] has observed, "... the standard analysis [of industrial organization] leaves us with the impression that there is a rough continuum, in terms of desirability of industry performance, ranging from unregulated pure monopoly as the pessimal arrangement to perfect competition as the ideal, with relative efficiency in resource allocation increasing monotonically as the number of firms expands." Baumol's contestable market theory casts serious doubt on the validity of this impression, so does our excess entry theorem from a somewhat different angle.

[2] Weizsäcker's analysis [17; 18] of the entry in a Cournot market is a good case in point. See, also, Dixit and Stiglitz [2], Kiyono [6], Spence [13], Suzumura [15] and Tandon [16].

firms from potential competitors is not necessarily welfare decreasing in sharp contrast with the traditional belief.

A problem remains with Theorem 1, however, in that an omnipotent "first-best" government does not exist in reality, and the "first-best" ideal is unrealizable in the actual economy.[3] Therefore, even if the intervention by a "first-best" government with a view to restricting the number of firms within an industry may be welfare-improving, it does not justify the intervention by a "second-best" government that lacks the leverage of optimal price regulation. What is needed is an evaluation of the social gains from alternative feasible government actions. This is precisely what we intend to perform in the second part of this chapter. In other words, we analyse an explicit "second-best" social welfare optimum in an oligopolistic economy and show that the "second-best" number of firms will fall short of the number of firms at the free entry equilibrium if the marginal revenue of each firm decreases with an increase of the output of other firms taken together. By the "second-best" we here mean that oligopolistic (marginal-cost-equals-marginal-revenue) pricing is taken for granted by an entry regulating government pursuing social welfare optimization. The implications of our analyses and several qualifications on our results will be discussed in the final section.

2 The Model

2.1. We will be concerned with a model of an oligopolistic industry producing a homogenous good. All firms are assumed to be technologically as well as behaviourally identical. The number of firms in the industry will be denoted by n. The output of the i-th firm is z_i $(i = 1, 2, \ldots, n)$ so that $Q := \Sigma_{i=1}^{n} z_i$ represents the industry output. The cost function of each and every firm and the inverse market demand function are denoted by $C(z_i)$ $(i = 1, 2, \ldots, n)$ and $p = f(Q)$, respectively. It is assumed that C and f are continuously differentiable as often as is required by the following analysis. The profit of the i-th firm will then be given by

$$\pi_i(z_i; Q_i) = z_i f(z_i + Q_i) - C(z_i), \tag{1}$$

where $Q_i := \Sigma_{j \neq i} z_j = Q - z_i$ $(i = 1, 2, \ldots, n)$. Let μ denote the coefficient of conjectural variations, viz., $\mu := \partial Q / \partial z_i$ $(i = 1, 2, \ldots, n)$. The profit maximizing output z_i of the i-th firm corresponding to the output of all other firms taken together, Q_i, will then satisfy

$$f(z_i + Q_i) + \mu z_i f'(z_i + Q_i) - C'(z_i) = 0 \tag{2}$$

and

$$2\mu f'(z_i + Q_i) + \mu^2 z_i f''(z_i + Q_i) - C''(z_i) < 0. \tag{3}$$

Throughout this chapter, we assume the following:

[3] See von Weizsäcker [17, p.400] and Schmalensee [10] on this point.

Assumption A1. The quasi-Cournot conjecture prevails, viz., μ is a positive constant which is less than n.[4]

Assumption A2. The inverse demand function satisfies $-M \leq f'(Q) < 0$ for some $M > 0$ and for all $Q > 0$.

Assumption A3. The cost function satisfies (i) $C'(z) > 0$ for all $z > 0$, and (ii) either $C(0) > 0$ or $\lim_{z \to 0^+} C''(z) := C''(0) < 0$.

Assumption A4. The marginal cost decreases, if ever it does, at a slower rate than the perceived demand curve, viz.,

$$\mu f'(z_i + Q_i) < C''(z_i) \tag{4}$$

for all $z_i > 0$ and $Q_i > 0$ $(i = 1, 2, \ldots, n)$.

Assumption A5. The marginal revenue of any firm is a decreasing function of the aggregate output of the other firms, viz.,

$$f'(z_i + Q_i) + \mu z_i f''(z_i + Q_i) < 0 \tag{5}$$

for all $z_i > 0$ and $Q_i > 0$ $(i = 1, 2, \ldots, n)$.

Assumption A6. For any n and μ satisfying Assumption A1, a symmetric quasi-Cournot equilibrium uniquely exists, and is defined as triplet $\{z(n,\mu), Q(n,\mu), p(n,\mu)\}$ of the firm output $z(n,\mu)$, the industry output $Q(n,\mu)$ and the price $p(n,\mu)$ satisfying

$$f(nz(n,\mu)) + \mu z(n,\mu) f'(nz(n,\mu)) - C'(z(n,\mu)) = 0 \tag{6}$$

$$Q(n,\mu) := nz(n,\mu), \qquad p(n,\mu) := f(Q(n,\mu)). \tag{7}$$

2.2. To simplify our notation, let us introduce the following symbols:

$$m := \frac{n}{\mu}, \qquad K(n,\mu) := 1 - \frac{C''(z(n,\mu))}{\mu f'(Q(n,\mu))}, \qquad E(Q) := \frac{Qf''(Q)}{f'(Q)}. \tag{8}$$

[4] The gist of this assumption is that each firm can predict with confidence the effects of its action on the other firms taken together. Note that the Cournot conjecture, to the effect that each firm supposes that none of the other firms will deviate from their current course of action if the given firm deviates, is a special case of A1, where $\mu = 1$, whereas if all firms are fully aware of their interactions and collude as if they formed a cartel, we have another special case of A1, where $\mu = n$. It follows that $m := n/\mu$ may be construed to be the "effective" number of Cournot oligopolists. In general, $0 < \mu \leq 1$ (*resp.* $1 < \mu \leq n$) may be construed to correspond to the situation of "struggle" (*resp.* "collusion") among firms. We owe these observation to Seade [12].

In what follows, m will be referred to as the "effective" number of firms.[5] In these terms, the inequality (3) may be reduced to

$$E(Q(n,\mu)) + mK(n,\mu) + m > 0 \tag{9}$$

at the symmetric quasi-Cournot equilibrium.

Note that Assumptions A1, A2, A4 and (8) together entail

$$K(n,\mu) > 0 \tag{10}$$

whereas Assumptions A1, A2, (5) and (8) together entail

$$E(Q(n,\mu)) + m > 0. \tag{11}$$

Comparing (9) with (10) and (11), we may assert that Assumptions A1, A2, A4 and A5 ensure that the second-order condition (9) for profit maximization is satisfied.

Note also that Assumptions A4 and A5 are sufficient for an adjustment process in a quasi-Cournot market

$$\dot{z}_i = \alpha_i\{z_i(Q_i) - z_i\}, \qquad \alpha_i > 0 \tag{12}$$

to be dynamically stable, where $z_i(Q_i)$ is a solution to (2) for a given Q_i, and $\dot{z}_i$ denotes the time derivative of z_i.[6]

2.3. For each fixed number of firms n, the *socially optimal firm output* $z_*(n)$ may be defined as the unique maximizer of the market surplus function, viz.,

$$z_*(n) := \arg\max_{z>0}\left\{\int_0^{nz} f(x)dx - nC(z)\right\}. \tag{13}$$

As is well-known, $z_*(n)$ defined by (13) satisfies the *marginal cost principle*:

$$f(nz_*(n)) = C'(z_*(n)). \tag{14}$$

2.4. Finally, let us consider the entry-exit dynamics of the quasi-Cournot market. Suppose that each firm earns positive (*resp.* negative) profit at the symmetric quasi-Counot equilibrium. Then there exists an incentive for a new firm (*resp.* an incumbent firm) to

[5] See footnote 4.

[6] See Hahn [4] and Seade [11].

enter into (*resp.* to exit from) this industry. Treating the number of firms as a continuous variable, we formulate this entry-exit dynamics by a differential equation:[7]

$$\dot{n} = \beta\{f(nz(n,\mu))z(n,\mu) - C(z(n,\mu))\}, \qquad \beta > 0, \tag{15}$$

where the expression within the curly brackets denotes profit, β is an adjustment coefficient and $\dot{n}$ denotes the time derivative of n.

Let $n_e(\mu)$ denote the stationary point of (15) corresponding to $\mu > 0$, viz.,

$$f(n_e(\mu)z(n_e(\mu),\mu))z(n_e(\mu),\mu) = C(z(n_e(\mu),\mu)). \tag{16}$$

We relegate the proof of the uniqueness and stability of $n_e(\mu)$ to the Appendix at the end of the chapter. In what follows, the symmetric quasi-Cournot equilibrium $\{z(n_e(\mu),\mu), Q(n_e(\mu),\mu), p(n_e(\mu),\mu)\}$ will be referred to as the *free-entry quasi-Cournot equilibrium*, whereas $n_e(\mu)$ will be called the *equilibrium number of firms*.

3 The First-Best Excess Entry Theorem

3.1. Let us now set about analysing the "first-best" welfare optimum. Our first order of business is to examine the property of the socially optimal firm output $z_*(n)$ vis-à-vis that of the equilibrium firm output $z(n,\mu)$.

Lemma 1. *For each* $n > 0$ *and* $\mu > 0$, (a) $z_*(n) > z(n,\mu)$ *holds true, and* (b) $z(n,\mu)$ *is a decreasing function of* $\mu > 0$.

Proof. See Appendix. ‖

Several straightforward implications of Lemma 1 are worth mentioning at this stage. First, if we define $Q_*(n) := nz_*(n)$ and $p_*(n) := f(Q_*(n))$, we may conclude from Assumption A2 and Lemma 1(a) that $Q_*(n) > Q(n,\mu)$ and $p_*(n) < p(n,\mu)$ hold true for all n and μ satisfying $0 < \mu < n$. Therefore, for any number of firms $n > 0$ and conjectural coefficient $\mu > 0$ such that the "effective" number of firms m exceeds one, the equilibrium industry output (*resp.* the equilibrium price) is less than (*resp.* greater than) the socially optimal industry output (*resp.* socially optimal price). Second, if we let μ converge to 0 fixing n, $z(n,\mu)$ increases by virtue of Lemma 1(b), which is bounded from above by $z_*(n)$ thanks to Lemma 1(a). Therefore, $z(n,0) := \lim_{\mu\to 0} z(n,\mu)$ exists. On the other hand, if we let μ converge to 0 in (6), taking the boundedness of $\{z(n,\mu)\}$ into consideration, we obtain by continuity that

[7] Treating the number of firms as a continuous variable is a common practice in the analysis of firm entry, which is followed by Dixit and Stiglitz [2], Okuguchi [7], Ruffin [9], Seade [12], von Weizsäcker [17; 18], among many others. See Seade [12, p.482] for an attempt to defend this common practice. See, also the concluding remark (4).

$$f(nz(n,0)) = c'(z(n,0)) \tag{17}$$

holds true. Comparing (14) and (17) and noting the uniqueness of $z_*(n)$, we may conclude that

$$\lim_{\mu\to 0} z(n,\mu) := z(n,0) = z_*(n) \tag{18}$$

holds true for every $n > 0$.[8]

3.2. How about the effect of a change is n on the level of firm output? First, differentiating (14) with respect to n, we obtain

$$z_*'(n) := \frac{d}{dn} z_*(n) = -\frac{z_*(n)}{n - \dfrac{C''(z_*(n))}{f'(nz_*(n))}} \tag{19}$$

which is negative for all n satisfying $n > \mu$. Therefore, *the socially optimal level of firm output decreases in response to the increase in the number of firms in the industry.* Secondly, differentiating (6) with respect to n, we obtain

$$z_n(n,\mu) := \frac{\partial}{\partial n} z(n,\mu) = -\frac{z(n,\mu)}{n} \frac{E(Q(n,\mu)) + m}{E(Q(n,\mu)) + m + K(n,\mu)}, \tag{20}$$

which is negative by virtue of (10) and (11). Therefore, *the equilibrium level of firm output decreases in response to the increase in the number of firms in the industry.*

3.3. How will the equilibrium number of firms $n_e(\mu)$ respond to a change in $\mu > 0$? We may easily verify the following:

Lemma 2. $n_e'(\mu) > 0$ *for all* $\mu > 0$.

Proof. See Appendix. ‖

Roughly speaking, the message of this lemma may be taken as follows: *The more collusive the interfirm relationship becomes the more firms will there be in the industry at the free entry quasi-Cournot equilibrium.* An intuitive reason for this result is that, as

[8]Letting μ converge to 0 corresponds to a situation where each and every firm becomes less and less aware of the effect of its own output change on the industry output. Therefore, (18) may be construed as implying that *the equilibrium firm output converges to the socially optimal firm output when the "subjective size" of a firm becomes infinitesimal.*

the industry becomes more collusive, the profit of each firm increases, thereby enticing prospective firms to enter into this industry.

3.4. So much for preliminaries. Let us now examine the consequence of the intervention by a price and entry regulating government in pursuit of "first-best" welfare optimization. Defining the *first-best market surplus function* by

$$W_f(n) := \int_0^{nz_*(n)} f(x)dx - nC(z_*(n)), \tag{21}$$

we define the *first-best number of firms* n_f by

$$n_f := \arg\max_{n>0} W_f(n). \tag{22}$$

In order to characterize n_f, we differentiate $W_f(n)$ to obtain

$$W_f'(n) = f(nz_*(n))z_*(n) - C(z_*(n)), \tag{23}$$

where use is made of (14) in deleting the terms involving $z_*'(n)$.

Differentiating (23) and making use of (14) and (19), we obtain

$$W_f''(n) = -\frac{\{z_*(n)\}^2 C''(z_*(n))}{n - \dfrac{C''(z_*(n))}{f'(nz_*(n))}}. \tag{24}$$

It follows that $W_f'(n) = 0$ holds true if and only if

$$f(nz_*(n)) = \frac{C(z_*(n))}{z_*(n)}, \tag{25}$$

viz., *price equals average cost*, whereas $W_f''(n) < 0$ holds true under Assumptions A1, A2 and A3 if and only if $C''(z_*(n)) > 0$. Therefore, the first-best number of firms n_f is characterized by (25) if the marginal cost is increasing.

We are now at the stage of putting forward the first main result of this chapter.

Theorem 1 (*First-Best Excess Entry Theorem*). *Assume that Assumptions A1-A6 hold true. Assume further that* (i) *the marginal cost is increasing; and* (ii) *the nominal as well as the "effective" number of firms exceeds one at the free entry quasi-Cournot equilibrium. Then the equilibrium number of firms $n_e(\mu)$ exceeds the first-best number of firms n_f.*

Proof. See Appendix. ‖

Among the conditions A1-A6, which lie behind Theorem 1, A5 may seem to be rather stringent. However, we may replace it by a weaker

Assumption A5*

$$E(Q(n,\mu)) + mK(n,\mu) + m > 0,$$

$$E(Q(n,\mu)) + K(n,\mu) + m > 0.$$

Note that the first part of Assumption A5* is nothing other than the second-order condition for profit maximization (9), whereas the second part is a necessary and sufficient condition, due to Seade [11; 12], for the dynamic stability of the process (12). To the extent that Assumption A5* may replace A5, which may easily be verified to be the case, Theorem 1 can be generalized.

4 The Second-Best Excess Entry Theorem

4.1. However desirable the "first-best" ideal may seem to be, an actual government may be unable to enforce the marginal cost principle required by the "first-best" welfare optimization. Let us suppose, instead, that oligopolistic pricing has to be taken for granted by an actual government and try to see if the main message of our first-best excess entry theorem survives under this change in the leverage of a government.

Let us define the *second-best market surplus function* by

$$W_s(n,\mu) := \int_0^{nz(n,\mu)} f(x)dx - nC(z(n,\mu)), \tag{26}$$

in terms of which the *second-best number of firms* $n_s(\mu)$ is defined by

$$n_s(\mu) := \arg\max_{n>0} W_s(n,\mu). \tag{27}$$

4.2. In what follows, we show that $n_e(\mu) > n_s(\mu)$ necessarily holds true. Assume, to the contrary, that $n_s(\mu) \geq n_e(\mu)$ happens to be the case. Differentiating (26) with respect to n, we obtain

$$\frac{\partial}{\partial n} W_s(n,\mu) = \mu\{z(n,\mu)\}^2 f'(nz(n,\mu)) \frac{E(Q(n,\mu)) + m}{E(Q(n,\mu)) + m + K(n,\mu)} + \pi(n,\mu), \tag{28}$$

where

$$\pi(n,\mu) := f(nz(n,\mu))z(n,\mu) - C(z(n,\mu)) \tag{29}$$

and use is made of (6) and (20). We then note that

$$\frac{\partial}{\partial n}\pi(n,\mu) = f'(nz(n,\mu))\{z(n,\mu)\}^2 \frac{mK(n,\mu) + E(Q(n,\mu)) + m}{E(Q(n,\mu)) + m + K(n,\mu)} < 0 \tag{30}$$

holds true, where use is made of (6), (10), (11) and (20). Since $\pi(n_e(\mu),\mu) = 0$ holds true by the very definition of $n_e(\mu)$, (30) implies that $\pi(n,\mu) < 0$ for all $n > n_e(\mu)$. It then follows that

$$\frac{\partial}{\partial n}W_s(n,\mu) < 0 \text{ for all } n \geq n_e(\mu), \tag{31}$$

where use is made of Assumption A2, (10) and (11). It follows from $n_s(\mu) \geq n_e(\mu)$ and (31) that $(\partial/\partial n)W_s(n_s(\mu),\mu) < 0$ holds true, in contradiction with the definition (27) of $n_s(\mu)$. By *reductio ad absurdum*, we may assert the following result.

Theorem 2 (*Second-Best Excess Entry Theorem*). *Assume that Assumptions A1-A6 hold true. Then the equilibrium number of firms $n_e(\mu)$ exceeds the second-best number of firms $n_s(\mu)$.*

Unlike Theorem 1, the validity of Theorem 2 hinges squarely on Assumption A5. Indeed, if we replace Assumption A5 by A5*, the assertion of Theorem 2 will become untenable. Note also that each incumbent firm's profit is positive at the second-best optimum if Assumption A5 is satisfied, viz., $\pi(n_s(\mu),\mu) > 0$, which may be seen from (28) and $(\partial/\partial n)W_s(n_s(\mu),\mu) = 0$. Therefore, the financial viability of each firm at the second-best optimum is guaranteed.

4.3. Note, in passing, that (31) yields $(\partial/\partial n)W_s(n_e(\mu),\mu) < 0$, viz., the "second-best" surplus $W_s(n,\mu)$ is a decreasing function of n at the equilibrium number of firms $n_e(\mu)$. Therefore, we have the following:[9]

Theorem 3. *Assume that Assumptions A1-A6 hold true. Then a small restriction in the number of firms at the free-entry quasi-Cournot equilibrium raises welfare unambiguously.*

The thrust of this result lies in the following fact: even if we cannot be sure where $n_s(\mu)$ is located exactly (e.g. because of uncertainty on the precise nature of the functions involved), we do know that "exit" is welfare-improving at the margin.

[9]Thanks are due to an anonymous referee who suggested this proposition to us.

5 Concluding Remarks

In this chapter, two excess entry theorems are presented. Presupposing the existence of a strong ("first-best") government, the first theorem asserts that there are an "excessive" number of firms at the free-entry quasi-Cournot equilibrium vis-à-vis the "first-best" number of firms. The second theorem asserts that the main message of the first theorem essentially survives even if we presuppose a "second-best" government, which lacks the leverage of optimal price regulation, instead of an omnipotent "first-best" government. In concluding this chapter, several remarks seem to be in order.

Remark 1. Our excess theorems are proved on the basis of a standard quasi-Cournot oligopoly model satisfying the stability conditions of Hahn [4] and/or Seade [11; 12]. As Seade himself observed, several counter-intuitive results on entry into a Cournot market hold true only when the Cournot equilibrium is unstable. But the same charge cannot be raised against our excess entry theorems, which go counter to the widespread belief of the welfare-improving effects of increasing competitiveness.

Remark 2. Somewhat surprisingly, there are not many attempts in the literature to examine the second-best performance of the quasi-Cournot market. Harris [5] is a possible exception. Note, however, that Harris is concerned with the direct governmental control of the production decisions by private firms, subject only to the constraint that all firms are to be assured of non-negative profit, in pursuit of the maximization of the market surplus function. In contrast, our second-best notion presupposes that the government is deprived of any direct control over the behaviour of firms, leaving them to follow their own private incentives. Care should be taken with this contrast in comparing our results with those of Harris [5].

Remark 3. How does the first-best number of firms n_f compare with the second-best number of firms $n_s(\mu)$? We show in the Appendix that no definite ranking is to be expected in general between n_f and $n_s(\mu)$. In contrast, the second-best number of firms in the sense of Harris [5] either coincides with or is less than the first-best number of firms, the difference in the latter case being exactly one.

Remark 4. Throughout this chapter, we have followed a convention of treating the number of firms as a continuous variable. As a matter of fact, the continuous variables $n_e(\mu)$, n_f and $n_s(\mu)$ are to be regarded as continuous proxies to the discrete variables $N_e(\mu)$, N_f and $N_s(\mu)$, which are defined by $N_e(\mu) = [n_e(\mu)]$, $N_f = [n_f]$ (resp. $[n_f] + 1$) if $W_f([n_f]) \geq W_f([n_f] + 1)$ (resp. $W_f([n_f]) < W_f([n_f] + 1)$), and $N_s(\mu) = [n_s(\mu)]$ (resp. $[n_s(\mu)] + 1$) if $W_s([n_s(\mu)]) \geq W_s([n_s(\mu)] + 1)$ (resp. $W_s([n_s(\mu)]) < W_s([n_s(\mu)] + 1)$), where $[n]$ denotes the greatest integer that does not exceed n. Therefore, a qualification should be made to our excess entry theorems to the following effect: Although $n_e(\mu) > \max\ \{n_f, n_s(\mu)\}$ holds true quite strenuously, $N_e(\mu)$ may still fall short of N_f and/or $N_s(\mu)$ by the margin of at most one, reflecting the above-noted integer problems.

Remark 5. A final remark on the background of our interest in the problem at hand might not be out of place. Throughout the post-war period, a guiding principle of

Japanese industrial policy has been the regulation of so-called "excessive competition". To the extent that the meaning of this key concept has not been made precise, the debates on industrial organization and industrial policy have been rather cloudy to say the least. It is our hope that the analyses in this chapter will help crystallize a possible meaning of this strategic concept, thereby contributing to a more fruitful communication in the future.[10]

Appendix

1. *Uniqueness and Stability of* $n_e(\mu)$

The equilibrium number of firms $n_e(\mu)$ is defined by $\pi(n_e(\mu), \mu) = 0$, where $\pi(n, \mu)$ is given by (29). In view of (30), the uniqueness of $n_e(\mu)$ is clear.

Let $n_t := n(t, n_0)$ be the solution of the differential equation (15), where n_0 denotes the initial value of n. Define the distance between the solution path $\{n_t := n(t, n_0) \mid 0 \leq t < +\infty\}$ and the equilibrium number of firms $n_e(\mu)$ by $V_t := \frac{1}{2}\{n_t - n_e(\mu)\}^2$. It is clear that

$$\dot{V}_t = \{n_t - n_e(\mu)\}\dot{n}_t = \beta\{n_t - n_e(\mu)\}\pi(n_t, \mu). \tag{1*}$$

By virtue of (30), $\pi(n_t, \mu) > 0, = 0$, or < 0 according as $n_t < n_e(\mu)$, $= n_e(\mu)$, or $> n_e(\mu)$. It then follows that $\dot{V}_t < 0$ as far as $n_t \neq n_e(\mu)$, whereas $\dot{V}_t = 0$ obtains if and only if $n_t = n_e(\mu)$. Therefore, $\lim_{t\to\infty} n(t, n_0) = n_e(\mu)$ is guaranteed. ‖

2. *Proof of Lemma* 1

(a) For any n, we define two functions $g_n(z) := f(nz) - C'(z)$ and $h_n(z) := f(nz) - C'(z) + \mu z f'(nz)$ of z. By virtue of Assumptions A1 and A2, $\mu z f'(nz) < 0$ for all $z > 0$, so that we have $g_n(z) > h_n(z)$ for all $z > 0$. By definition of $z_*(n)$ and $z(n, \mu)$, we obtain $g_n(z_*(n)) = h_n(z(n, \mu)) = 0$. Furthermore, Assumptions A2 and A4 entail $g'_n(z) = (n - \mu)f'(nz) + \mu f'(nz) - C''(z) < 0$, which is sufficient to ensure the uniqueness of $z_*(n)$ and the inequality $z_*(n) > z(n, \mu)$.

(b) Differentiating (6) with respect to μ, we may easily verify that

$$z_\mu(n, \mu) := \frac{\partial}{\partial \mu} z(n, \mu) = -\frac{z(n, \mu)}{\mu\{E(Q(n, \mu)) + m + K(n, \mu)\}} \tag{2*}$$

holds true, which is negative by virtue of (10) and (11). ‖

3. *Proof of Lemma* 2

[10] The interested readers are referred to Okuno and Suzumura [8] for our analysis of industrial policy.

Differentiating (16) with respect to μ, and taking (6) for $n = n_e(\mu)$ into consideration, we obtain

$$n_e'(\mu) = -\frac{z_\mu(n_e(\mu),\ \mu)}{z(n_e(\mu),\ \mu) + \{n_e(\mu) - \mu\} z_n(n_e(\mu),\ \mu)}. \tag{3*}$$

Substituting (2*) and (20) for $n = n_e(\mu)$ into (3*) and simplifying, we obtain

$$n_e'(\mu) = -\frac{z_\mu(n_e(\mu),\ \mu) m_e \{E(Q_e) + m_e + K_e\}}{z(n_e(\mu),\ \mu)\{m_e K_e + E(Q_e) + m_e\}}, \tag{4*}$$

where $E(Q_e) := E(Q(n_e(\mu), \mu)), m_e := n_e(\mu)/\mu$ and $K_e := K(n_e(\mu), \mu)$. By virtue of (10) and (11) for $n = n_e(\mu)$, (2*), (20) and (4*) ensure that $n_e'(\mu) > 0$ holds true for all $\mu > 0$. ‖

4. *Proof of Theorem* 1

To begin with, we prove that $n_e(0) := \lim_{\mu \to 0} n_e(\mu)$ exists and it satisfies $n_e(0) = n_f$. Note that

$$z(n_e(\mu), \mu) < z_*(n_e(\mu)) \leq z_*(1) \tag{5*}$$

holds true by virtue of Lemma 1(a), $z_*'(n) < 0$ and the Assumption (ii). Note also that $(d/d\mu) z(n_e(\mu), \mu) = z_n(n_e(\mu), \mu) n_e'(\mu) + z_\mu(n_e(\mu), \mu) < 0$ holds true, where use is made of Lemma 2. Therefore, when μ decreases toward 0, $z(n_e(\mu), \mu)$ increases. Since $\{z(n_e(\mu), \mu)\}$ is bounded from above by $z_*(1)$, $\lim_{\mu \to 0} z(n_e(\mu), \mu) = z(\lim_{\mu \to 0} n_e(\mu), 0)$ exists. Let $n_e(0) := \lim_{\mu \to 0} n_e(\mu)$. Consider (6) for $n = n_e(\mu)$ and let μ converge to 0 to obtain

$$f(n_e(0) z(n_e(0), 0)) = C'(z(n_e(0), 0)), \tag{6*}$$

where use is made of Assumptions A2 and A5*. In view of (18), (6*) may be rewritten as

$$f(n_e(0) z_*(n_e(0))) = C'(z_*(n_e(0))). \tag{7*}$$

Next, we let μ converge to 0 in (16) and take (18) into consideration to obtain

$$f(n_e(0) z_*(n_e(0))) z_*(n_e(0)) = C(z_*(n_e(0))). \tag{8*}$$

Coupled with (7*), (8*) yields $n_e(0) = n_f$, as desired.

Suppose now that $n_f \geq n_e(\mu)$ were true for some $\mu > 0$. By virtue of Lemma 2, we then obtain $n_f \geq n_e(\mu) > n_e(0) = n_f$, which is a contradiction. Therefore, $n_f < n_e(\mu)$ must be the case for all $\mu > 0$ satisfying $n_e(\mu) > \max\{1, \mu\}$, as was to be shown. ‖

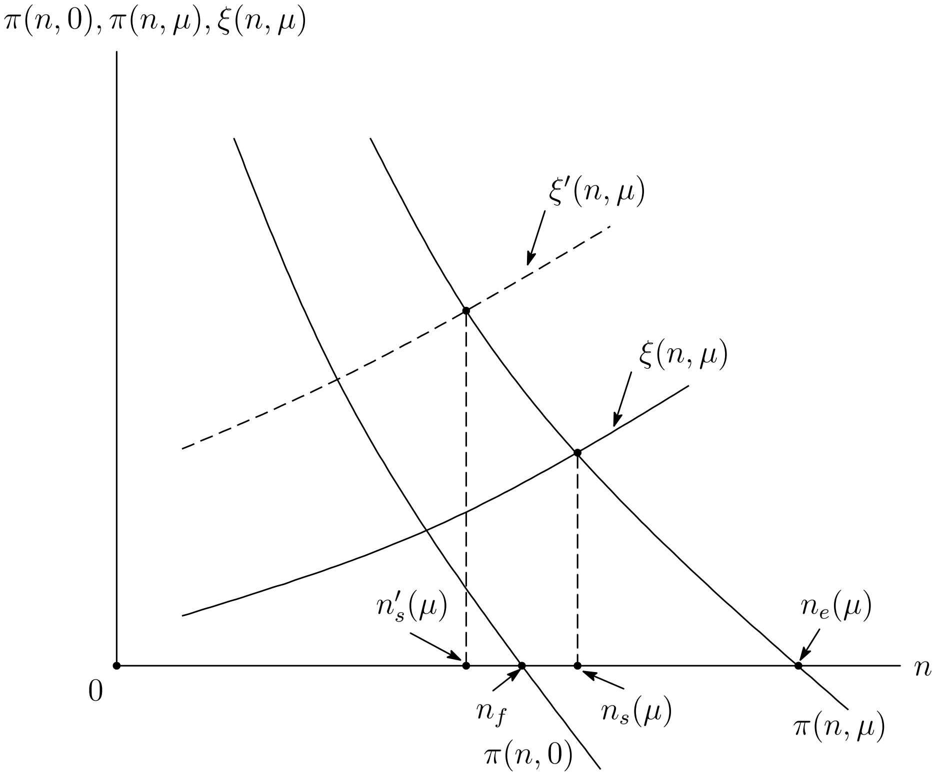

Figure 1
Comparison of $n, n_s(\mu)$ and $n_e(\mu)$

5. *Comparison between* n_f and $n_s(\mu)$

Note that n_f, $n_s(\mu)$ and $n_e(\mu)$ are characterized by $\pi(n_f, 0) = 0$, $\pi(n_s(\mu), \mu) = \xi(n, \mu)$ and $\pi(n_e(\mu), \mu) = 0$, respectively, where $\pi(n, \mu)$ is defined by (29) and $\xi(n, \mu)$ is defined by

$$\xi(n,\mu) := -\mu\{z(n,\mu)\}^2 f'(nz(n,\mu)) \frac{E(Q(n,\,\mu)) + m}{E(Q(n,\,\mu)) + m + K(n,\,\mu)}, \tag{9*}$$

which is positive under our assumption. Note also that (30) guarantees that $(\partial/\partial n)\pi(n,\mu) < 0$, whereas Lemma 1(b) guarantees that

$$\frac{\partial}{\partial \mu}\pi(n,\mu) = (n-\mu)z(n,\mu)f'(Q(n,\mu))z_\mu(n,\mu) > 0. \tag{10*}$$

We now draw the graphs of $\pi(n, 0), \pi(n, \mu)$ and $\xi(n, \mu)$ for fixed $\mu > 0$. As may easily be observed, $n_e(\mu) > n_f$ and $n_e(\mu) > n_s(\mu)$ hold true generally, whereas $n_s(\mu)$ may exceed or fall short of n_f depending on the relative position of $\xi(n, \mu)$ vis-à-vis $\pi(n, \mu)$. $\|$

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Chapter 18
Oligopolistic Competition and Economic Welfare: A General Equilibrium Analysis of Entry Regulation and Tax-Subsidy Schemes*

1 Introduction

The purpose of this chapter is to make two contributions to the general equilibrium analysis of an oligopolistic economy with free entry.

First, we establish a general equilibrium extension of the excess entry theorem due to Mankiw and Whinston [13] and Suzumura and Kiyono [19], which establishes in a partial equilibrium framework that *a marginal decrease in the number of oligopolistic firms from the free-entry equilibrium level improves economic welfare*. As is generally recognized, the advantage of using a partial equilibrium analysis lies in its simplicity, which enables us to crystallize a new theoretical insight. Since the new insight rendered by the excess entry theorem is somewhat paradoxical, it is reassuring that the theorem is essentially kept intact even in the presence of general equilibrium interactions. Indeed, the partial equilibrium verdict on the welfare effect of entry regulation in a free-entry oligopolistic economy is preserved in a general equilibrium setting if the oligopolistic sector uses the same factor more intensively than the competitive sector in the average as well as marginal sense.

Note, however, that the excess entry theorem may be criticized in that a democratic government may lack sufficient leverage enabling it to impose direct entry regulation. Therefore it is interesting to seek other types of government policy instruments than direct entry regulation. With this purpose in mind, the second task of this chapter is to explore tax-subsidy schemes which guarantee an unambiguous Pareto improvement. Recall that the introduction of a tax-subsidy scheme into a perfectly competitive economy necessarily harms at least one economic agent, unless it is of the lump-sum variety. In

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contrast, the introduction of a tax-subsidy scheme can be welfare-improving if the market economy is imperfectly competitive. However, knowing that a tax-subsidy scheme *can be* welfare-improving is quite different from knowing when and precisely what kind of tax-subsidy scheme *is warranted* to be welfare-improving. It is interesting to identify several self-financing tax-subsidy schemes that guarantee Pareto improvement for sure, and this is precisely what we intend to accomplish.

At this juncture, some remarks on the related literature might be in order. In a partial equilibrium framework, Katz and Rosen [9] as well as Seade [17] analysed the issue of tax-shifting in oligopolistic competition, whereas the general equilibrium analysis of the incidence of a corporate profit tax in an imperfectly competitive economy was pioneered by Anderson and Ballentine [1], and analysed further by Atkinson and Stiglitz [2, Chapter 7]. Their analyses differ from ours in several essential respects. First, unlike ours, Anderson and Ballentine analysed the "short-run" incidence and welfare effect of corporate profit tax, where by "short-run" is meant that they fixed the number of Cournot oligopolists. Second, Atkinson and Stiglitz analysed the model of monopolistic competition à la Dixit and Stiglitz [5], which subsumes our model of homogenous product oligopoly as a special case. Note, however, that they focused on a simplified case ($\gamma = 0$ in their notation) where the price-cost margin remains the same regardless of any change in exogenous environment because the residual demand of each and every firm has constant and identical elasticity. On the other hand, the price-cost margin is endogenous in our model, which enables us to analyse the welfare effects through prices as well as through other routes. Despite the formal generality, therefore, their model does not necessarily cover the area which our analysis focuses on. Besides, our focus is on the "short-run" as well "long-run" welfare implications of various policies in contrast with their exclusive concern with the "long-run" incidence of corporate taxation.[1]

2 The Model

We consider a closed economy which consists of two sectors producing two goods, X and Y, using two factors of production, capital and labour. The endowment of these factors is fixed exogenously, and factors are freely mobile between two sectors. Good X is produced under increasing returns to scale due to the existence of fixed costs, which makes industry X oligopolistic. For simplicity, we assume that all firms in industry X are identical and behave as Cournot-Nash quantity competitors, so that we can work with the symmetric Cournot-Nash equilibrium. Good Y is competitively produced under constant returns to scale. We also simplify our model by supposing a single representative consumer, whose

[1] In what follows, we shall focus on the welfare effects of a change in the number of firms. In analysing these effects, a crucial factor is the price-cost margin which reflects the monopoly power of the firms. Using the Atkinson-Stiglitz framework with $\gamma = 1$, however, one cannot meaningfully analyse these effects as the price-cost margin remains the same even if the number of firms changes. On the other hand, the Atkinson-Stiglitz framework is quite suitable for analysing the welfare effects of a change in the number of products. See also, Besley and Suzumura [3], Myles [14], Robinson [15, Chapter 5], and Stern [18], among others, for different aspects of tax and welfare analyses in an oligopolistic economy.

welfare is the central focus of our analysis. The consumer's income is taken as *numeraire*. Moreover, we confine our attention to the "long-run" equilibrium where entry and exit are free in the oligopolistic sector as well as in the perfectly competitive sector. This completes our informal description of the model.

2.1 Representative Consumer

Let $V(p_X, p_Y)$ be the indirect utility function of the representative consumer, where p_X (resp. p_Y) denotes the price of X (resp. Y) and the consumer's income is taken as numeraire. We assume quasi-linearity of her preference for simplicity, which enables us to write the inverse demand function for X as:

$$p_X = p_Y \phi(X), \quad \phi'(X) < 0.^2 \tag{2.1}$$

Moreover, for expositional convenience, the price elasticity of demand for X, denoted by ϵ, is assumed to be constant, i.e.,

$$\epsilon = -\frac{\phi(X)}{X\phi'(X)} = \text{const.}^3 \tag{2.2}$$

2.2. Industry *Y*

Industry Y consists of perfectly competitive firms producing Y under constant returns to scale. Let $g(w, r)$ stand for the unit cost function of the representative firm, where r and w are, respectively, the rental rate of capital and the wage rate. Needless to say, g is homogeneous of degree one. The price of Y being p_Y, we should have

$$p_Y = g(w, r) \tag{2.3}$$

at equilibrium. Industry Y is assumed to be an untaxed sector.

2.3. Industry *X*

Industry X consists of identical and oligopolistically competitive firms. Let the *before tax-subsidy* cost function of each firm be

$$C^*(q; w, r) = m^*(w, r)q + F^*(w, r), \tag{2.4}$$

where q is each firm's output of good X. Clearly, $m^*(w, r)$ and $F^*(w, r)$ denote, respectively, the marginal cost and fixed cost functions which are homogeneous of degree

[2] Quasi-linearity is a strong assumption which makes our analysis close to partial equilibrium analysis. However, this setup still allows us to discuss the crucial general equilibrium adjustment in factor markets in a straightforward way. In fact, weakening quasi-linearity to homotheticity will not alter any essential results of this chapter as we showed in Konishi, Okuno-Fujiwara and Suzumura [11]. This paper is available to any interested reader on request.

[3] The assumption of constant elasticity is made only for simplifying our presentation. Our results are valid even without this assumption as is clear from Konishi, Okuno-Fujiwara and Suzumura [11].

one. Since we are concerned with the "long-run" analysis of the oligopolistic economy where firms enter into and exit from the oligopolistic sector, there is no sunk cost in our economy.

In this chapter, we examine the welfare effects of various *infinitesimal* tax-subsidy schemes applied to the oligopolistic industry, which are represented in terms of the following parameters:

s = production subsidy per unit output,
s_w = rate of subsidy on the wage expenditure component of marginal cost,
s_r = rate of subsidy on the capital expenditure component of marginal cost,
t = lump-sum subsidy,
t_w = rate of subsidy on the wage expenditure component of fixed cost,
t_r = rate of subsidy on the capital expenditure component of fixed cost.

Note that each of s, s_w, s_r, t, t_w and t_r can be negative. If this is in fact the case, we are referring to a tax rather than to a subsidy. By the use of vector notation, let $\boldsymbol{S} = (s, s_w, s_r, t, t_w, t_r)$ be the overall tax-subsidy scheme, which is partitioned into the tax-subsidy scheme on the marginal cost part $\boldsymbol{s} = (s, s_w, s_r)$ and that on the fixed cost part $\boldsymbol{t} = (t, t_w, t_r)$.

Under the given tax-subsidy scheme, we can redefine the *after tax-subsidy* cost function for X industry firms:

$$C(q; w, r, \boldsymbol{S}) = m(w, r, \boldsymbol{s})q + F(w, r, \boldsymbol{t}),$$

where $m(w, r, \boldsymbol{s}) = m^*(w - s_w, r - s_r) - s$, and $F(w, r, \boldsymbol{t}) = F^*(w - t_w, r - t_r) - t$.

Throughout this chapter, we assume that the X industry is in Cournot-Nash competition in quantities. It follows that each firm solves the problem:

$$\max_{q>0} \{p_Y \phi(X_{-i} + q)q - m(w, r, \boldsymbol{s})q - F(w, r, \boldsymbol{t})\}, \tag{2.5}$$

taking the total output of other firms X_{-i}, the price p_Y of good Y, and that of production factors (w, r), and the tax-subsidy scheme $\boldsymbol{S}$ as given. The first-order condition for profit maximization becomes:

$$p_Y \phi(X) \left\{ \frac{\phi'(X)}{\phi(X)} q + 1 \right\} = m(w, r, \boldsymbol{s}).$$

Under the assumption of identical firms, $X = nq$ holds at the symmetric Cournot-Nash equilibrium if there are n firms operative in industry X.

Using (2.2) and (2.3), we can derive the following equilibrium condition in industry X:

$$\phi(nq) \left(1 - \frac{1}{n\epsilon} \right) = \frac{m(w, r, \boldsymbol{s})}{g(w, r)}, \tag{2.6}$$

where $n > 1/\epsilon$ should be satisfied for the internal equilibrium solution to exist.

We also assume that entry and exit are free in industry X and focus on a "long-run" equilibrium of the economy. Thus, the output level of each firm and the number of firms in industry X are important determinants of the allocative efficiency of the economy.

The equilibrium number of firms is determined by the break-even condition. By virtue of (2.3), this condition is reduced to:

$$\phi(nq)q = \frac{m(w,r,\boldsymbol{s})}{g(w,r)}q + \frac{F(w,r,\boldsymbol{t})}{g(w,r)}. \tag{2.7}$$

The integer problem on the equilibrium number of firms is assumed away following the customary practice in the literature (e.g. see Seade [16], and Suzumura and Kiyono [19]).

2.4. Consumer's Income

Recollect that the income of the representative consumer is taken as numeraire. Let K (resp. L) be the fixed supply of capital (resp. labour). Profit earned in industry X, if any, is distributed to the consumer. The resources, which are required to substantiate a tax-subsidy scheme, are collected from the representative consumer in a lump-sum fashion. When tax is collected from the oligopolistic sector, its revenue is distributed to the consumer in the same manner. Let T be the lump-sum subsidy (or tax if it is negative):

$$T = n\{(s + s_w m_w + s_r m_r)q + (t + t_w F_w + t_r F_r)\}.$$

In the above expression, partial derivatives of the cost functions coincide with the levels of factor utilization in the marginal and fixed cost parts by virtue of Shephard's lemma.[4]

Finally, the normalization of the consumer's income implies:

$$wL + rK + n\{g(w,r)\phi(nq)q - m(w,r,\boldsymbol{s})q - F(w,r,\boldsymbol{t})\} - T = 1. \tag{2.8}$$

Note that the third term on the LHS is equal to zero at equilibrium.

2.5. Factor Market Equilibrium

Capital and labour are allocated between industries through the adjustment of rental and wage rates. Both factor markets are assumed to be perfectly competitive. By the use of Shephard's lemma, total factor use in each industry can be written as:

$$K_X = m_r(w,r,\boldsymbol{s})X + nF_r(w,r,\boldsymbol{t}), \qquad L_X = m_w(w,r,\boldsymbol{s})X + nF_w(w,r,\boldsymbol{t}),$$

$$K_Y = g_r(w,r)Y, \qquad L_Y = g_w(w,r)Y,$$

and the market-clearing conditions become:

[4]Throughout this chapter, a subscript to a function signifies partial differentiation with respect to the specified variable. For example, if the relevant function is given by $f(x,y)$, we denote $f_x = \partial f/\partial x$, $f_{xy} = \partial^2 f/\partial x \partial y$, and so on.

$$K_X + K_Y = K, \tag{2.9}$$

$$L_X + L_Y = L. \tag{2.10}$$

The above six equations, (2.3) and (2.6)-(2.10), complete the general equilibrium system of our economy. The market-clearing condition for good Y is omitted because of Walras's law. There are six unknowns, s, q, n, p_Y, Y, w and r.

3 Welfare Criterion

In order to analyze this system, we first define the welfare criterion, which is the basis for evaluating the entry regulation policy and the tax-subsidy schemes to be examined later.

The welfare of the representative consumer is written as:

$$V(p_X, p_Y) = V(g(w,r)\phi(nq), g(w,r)). \tag{3.1}$$

Total differentiation of (3.1) yields:

$$\frac{1}{\lambda}\mathrm{d}V = -g(w,r)X\phi'(X)\mathrm{d}X - \{X\phi(X)+Y\}(g_w\mathrm{d}w + g_r\mathrm{d}r), \tag{3.2}$$

where use is made of Roy's Identity and λ represents the marginal utility of income ($\lambda > 0$).

In addition, total differentiation of (2.8) tells us that changes of variables are restricted by the following relation because of the normalization of the consumer's income:

$$\begin{aligned} &\{L - n(m_w q + F_w) + \phi(X)XXg_w\}\mathrm{d}w + \\ &\quad \{K - n(m_r q + F_r) + \phi(X)XXg_r\}\mathrm{d}r \\ &\quad\quad + g(w,r)\phi'(X)X(\mathrm{d}X - \mathrm{d}q) = 0, \end{aligned} \tag{3.3}$$

where (2.6) and (2.7) are applied. Note that the terms relating to the tax-subsidy parameters do not appear here because we are concerned only with the infinitesimal tax-subsidy schemes. Substituting (2.9) and (2.10), we can convert (3.3) into:

$$\{Y + \phi(X)X\}(g_w\mathrm{d}w + g_r\mathrm{d}r) + g(w,r)\phi'(X)X(\mathrm{d}X - \mathrm{d}q) = 0. \tag{3.4}$$

Thus, using the restriction (3.4), we obtain the following welfare criterion in our economy:

$$\frac{1}{\lambda}dV = -g(w,r)X\phi'(X)\mathrm{d}q. \tag{3.5}$$

The following useful theorem is now established:

Theorem 1 (Welfare criterion). *The necessary and sufficient condition for a change in the number of oligopolistic firms and/or the introduction of an infinitesimal tax-subsidy scheme to be welfare-improving is that it induces an increase in the output of each oligopolistic firm.*

The assertion of this theorem is intuitively clear. In a free-entry oligopolistic economy, average cost, which equals product price, exceeds marginal cost, which equals marginal revenue, so that there remain unexploited increasing returns. Hence, it is socially beneficial to expand the scale of production of each firm in the oligopolistic industry.

4 Perturbation of the General Equilibrium System

In this section we analyse our general equilibrium system using the so-called *hat-calculus* and derive the relations which must hold at equilibrium among the output levels of each oligopolistic firm, the number of firms in the oligopolistic sector, and the relative factor price.[5] This is a customary procedure in the literature of tax incidence pioneered by Harberger [6].[6] To begin with, we examine the effect of a change in the number of firms, and later we investigate the effect of the introduction of a tax-subsidy scheme.

Consider (2.6), which is the Cournot- Nash equilibrium condition in industry X, and assume $\boldsymbol{S} = 0$. The RHS of (2.6) shows the relative marginal cost of industry X to that of industry Y. It is well known that the relative factor intensity between the two industries plays a central role in determining the relation between ω, the wage-rental ratio w/r, and the value of the RHS of (2.6); if the marginal cost part of industry X is more capital intensive than industry Y, an increase in ω decreases the value of the RHS, and vice versa.

The LHS of (2.6), in turn, represents the marginal revenue of the oligopolistic sector in terms of the good Y. Clearly, it depends on the equilibrium number of firms as well as the equilibrium output level of each oligopolistic firm. Using the hat-calculus, we obtain the following equation of change:

$$-\frac{1}{\epsilon}\hat{q} - \frac{(n-1)\epsilon - 1}{\epsilon(n\epsilon - 1)}\hat{n} = \hat{m} - \hat{g}.$$

Let us now introduce a crucial assumption on the strategic behavior of oligopolists: output levels are *strategic substitutes*. The property of strategic substitutes, first formulated by Bulow *et al.* [4], corresponds to the downward-sloping reaction curve for each oligopolistic firm, or equivalently, the negative partial derivative of the marginal revenue with respect to the output chosen by other firms. It is well known that the strategic substitutes property is a natural requirement when oligopolistic competition is in terms of quantities.

Note that the LHS of (2.6) is decreasing in q because of the second-order condition of profit maximization. Moreover, in our formulation, an increase in the number of firms will

[5] For any variable x, we denote $\hat{x} = \mathrm{d}x/x$.

[6] See also Atkinson and Stiglitz [2] and Kotlikoff and Summers [12] for useful surveys.

induce, *ceteris paribus*, an increase in other firms' output. The assumption of strategic substitutes, then, implies that the LHS of (2.6) is decreasing in n. Thus, the assumption of strategic substitutes and the existence of an internal solution imply that the number of firms, n, must satisfy $n > 1 + 1/\epsilon$ at the symmetric free-entry equilibrium.

The implied relation $(\hat{q}, \hat{n}, \hat{\omega})$ in (2.6) is now reduced to:

$$-\frac{1}{\epsilon}\hat{q} - \alpha\hat{n} + A\theta^{\mathrm{M}}\hat{\omega} = 0, \tag{4.1}$$

where $A = (wr/c)m_w L_Y > 0$, $\alpha = \{(n-1)\epsilon - 1\}/\epsilon(n\epsilon - 1) > 0$, and $\theta^{\mathrm{M}} = (m_r/m_w) - (K_Y/L_Y)$.[7] θ^{M} denotes the difference in *marginal* capital intensity between two industries. If θ^{M} is positive (resp. negative), industry X's marginal cost is more capital (resp. labour) intensive than industry Y. Hence, an increase in the wage-rental ratio expands (resp. contracts) the output of each oligopolist if industry X is marginally more capital (resp. labour) intensive than industry Y. On the other hand, other things being equal, an increase in the number of firms in industry X reduces the output of each oligopolistic firm, regardless of the sign of θ^{M}.

Next, we apply the same procedure as above to the break-even condition (2.7) of industry X. The RHS thereof being the total cost of an oligopolistic firm in terms of the good Y, the value of the RHS determines the overall factor intensity. The implied relation of $(\hat{q}, \hat{n}, \hat{\omega})$ becomes:

$$-(1/\epsilon)\hat{X} + \hat{q} = (mq/C)\hat{q} + (wr/C)L_Y L_X\{(K_Y/L_Y) - (K_X/L_X)\}\hat{\omega}.$$

By the use of (2.6), $mw/C = 1 - 1/\epsilon$, so that the above relation is reduced to:

$$-\frac{n-1}{\epsilon}\hat{q} - \frac{1}{\epsilon}\hat{n} + B\theta^{\mathrm{A}}\hat{\omega} = 0, \tag{4.2}$$

where $B = (wr/Cg)L_Y L_X > 0$ and $\theta^{\mathrm{A}} = K_X/L_X - K_Y/L_Y$. θ^{A} denotes the difference in *average* capital intensity between two industries.[8] If it is positive, industry X is overall more capital intensive than industry Y, and vice versa. Note that the average capital intensity is equal to the marginal intensity in industry Y because the industry is under constant returns to scale. By (4.2), if industry X is overall more (resp. less) capital intensive than industry Y, other things being equal, an increase in the output of each oligopolistic firm decreases the number of firms in industry X, and a rise in the wage-rental ratio induces new entry into (resp. exit from) industry X.

Finally, we must derive the factor market equilibrium relation of $(\hat{q}, \hat{n}, \hat{\omega})$, which is obtained by totally differentiating (2.9) and (2.10), and then eliminating $\mathrm{d}Y$. Since the calculation procedure is rather complicated, we shall state only the final result in the main text, relegating a detailed derivation to appendix A:

$$\lambda^{\mathrm{M}}\hat{q} + \lambda^{\mathrm{A}}\hat{n} + \varDelta\hat{\omega} = 0, \tag{4.3}$$

[7] Note that, in the derivation of (4.1), the homogeneity property of the cost function is used.

[8] The distinction between marginal and average factor intensity is due originally to Jones [8].

where λ^{M} and λ^{A} are positively related to the difference in marginal and average factor intensities, θ^{M} and θ^{A}, respectively. $\varDelta$ represents what is usually called the *factor substitution term* and is always positive.

When the number of firms is fixed, interpretation of (4.3) is familiar in the theory of international trade and/or that of tax incidence. If in need, the reader is referred to the pioneering paper on the two-sector general equilibrium model by Jones [7].

Consider an increase in the wage-rental ratio. It induces firms in both industries to choose more capital-intensive technology, which brings about excess demand in the capital market and excess supply in the labour market. For both factor markets to clear, the output of the capital-intensive industry must decrease and that of the labour-intensive industry increase; this is the renowned Rybczynski theorem. In our context, if industry X is marginally more capital intensive, the output of each oligopolistic firm must decrease, and vice versa.

What is rather unfamiliar in (4.3) is the effect of an increase in the number of firms on the output of each oligopolistic firm under a constant wage-rental ratio. This depends not only on the difference in *marginal* factor intensities, but also on that in *average* factor intensities between industries. Let us say that there exists a *factor intensity twist* in industry X when θ^{M} and θ^{A} are of opposite signs.[9] Figure 1(a) depicts the box diagram describing factor utilizations when there is no factor intensity twist at the given wage-rental ratio. Note that, in this figure, industry X is assumed to be overall more capital intensive than industry Y.

In Figure 1(a), E denotes the initial equilibrium. O^X and O^Y denote the origin of industry X and industry Y, respectively. Factor utilization of industry X, represented by the vector O^XE, can be decomposed into the fixed cost part and the marginal cost part:

$$(K_X, L_X) = (m_r, m_w)X + (F_r, F_w)n.$$

In this figure, O^XF corresponds to the fixed cost part and FE to the marginal cost part. The slopes of both O^XE and FE are steeper than that of O^YE, reflecting the assumption of no factor intensity twist.

Now consider an increase in the number of firms in industry X. New equilibrium will occur at E', where factor utilization in the fixed cost part is expanded to O^XF'. However, if the production level of each oligopolistic firm were to remain the same as before, industry X would utilize factors of O^XG. Hence, an increase in the number of firms leads to a fall in the output of oligopolists in industry X.

Figure 1(b) depicts the case with a factor intensity twist; the slope of O^YE is between those of O^XE and FE. An increase in the number of firms changes the equilibrium from E to E' and the individual output level of the oligopolists increases, because $F'G < F'E'$.

[9] The crucial role played by the absence of the factor intensity twist was recognized by Atkinson and Stiglitz [2, Chapter 7] as well, although the name is not theirs.

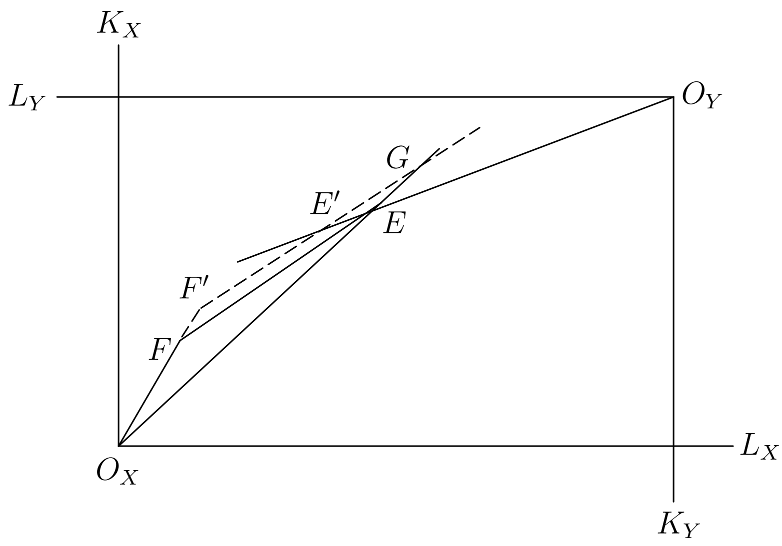

Figure 1(a)

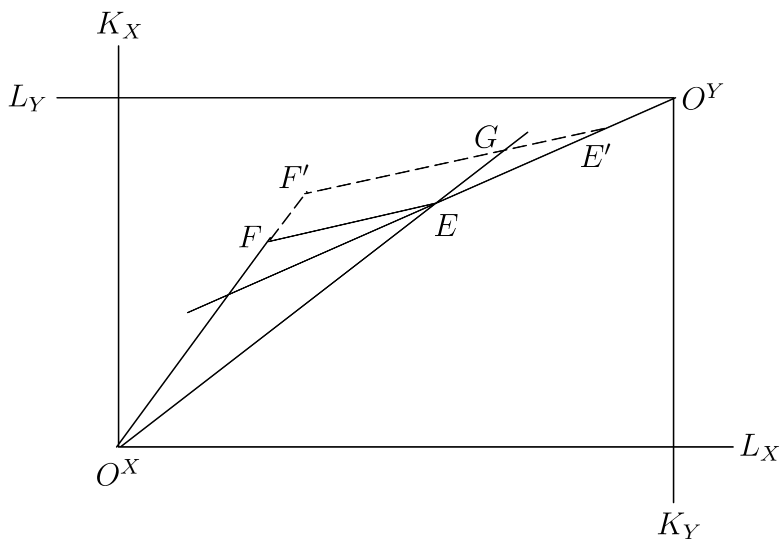

Figure 1(b)

5 Excess Entry Theorem

We are now ready to solve the equations of change, (4.1)-(4.3), and investigate the welfare effects of entry regulation.

Suzumura and Kiyono [19], using a partial equilibrium framework, examined the welfare effects of entry regulation in a free-entry (quasi-) Cournot oligopoly with fixed cost. They showed that a reduction in the number of firms leads to welfare improvement when the only available policy tool is the control of the number of firms. This result critically hinges on the assumption of strategic substitutability. Under this assumption, a reduction in the number of firms gives rise to an increase in the equilibrium output of each oligopolistic firm, which leads to a fall in the average cost of oligopolists due to the existence of unexploited increasing returns to scale.

However, a partial equilibrium result may not hold in a general equilibrium setting in general. One of the major aims of this chapter is to show under what conditions entry regulation assures welfare improvement when we pay due attention to the general equilibrium interactions. In view of Theorem 1, we need only to know a sufficient condition for the output of each oligopolistic firm to increase when the number of firms is reduced marginally from the free-entry equilibrium level. Under entry regulations, n is fixed and the relation (4.2) no longer holds. Solving (4.1) and (4.3), we obtain:

$$\hat{q} = -\frac{1}{\Omega}(\alpha\Delta + A\lambda^{\mathrm{A}}\theta^{\mathrm{M}})\hat{n}, \tag{5.1}$$

where $\Omega = (1/\epsilon)\Delta + A\lambda^{\mathrm{M}}\theta^{\mathrm{M}} > 0$. Thus, barring a factor intensity twist, a marginal reduction in the number of firms from the free-entry equilibrium level increases individual output in the oligopolistic sector. Invoking Theorem 1, we obtain:

Theorem 2 (Excess entry theorem in general equilibrium). *Suppose that strategic substitutability and no factor intensity twist hold. Then a marginal reduction in the number of oligopolistic firms from the free-entry equilibrium level unambiguously improves economic welfare.*

Diagrammatical exposition of Theorem 2 is given in Figure 2, which depicts the case of $\theta^{\mathrm{M}}, \lambda^{\mathrm{M}} > 0$.

The downward-sloping schedule FF and the upward-sloping schedule PP are the implied relation of (q, ω) in the factor market (4.3) and that in the product market (4.1) with the assumption that $\hat{n} = 0$, respectively. The initial free-entry equilibrium is shown by E in the figure. When the number of firms in industry X is reduced marginally, the PP schedule moves to the right by the assumed strategic substitutability. The shift of the FF schedule depends on whether or not the factor intensity twist exists. If there is no factor intensity twist and $\theta^{\mathrm{A}}, \lambda^{\mathrm{A}} > 0$, the FF schedule also moves to the right. It follows that an increase in q is always assured.

However, when there is a twist and $\theta^{\mathrm{A}}, \lambda^{\mathrm{A}} < 0$, the response via factor markets to a reduction in the number of oligopolistic firms counteracts the expansion of the individual oligopolist's output. This is because a reduction in the number of firms induces a change in the wage-rental ratio which worsens the marginal cost condition in industry X relative to that in industry Y. Thus, without the factor intensity twist, equilibrium moves to E' and an increase in q is entailed, but equilibrium may move to E'' and q may decrease with the factor intensity twist. The case of $\theta^{\mathrm{M}}, \lambda^{\mathrm{M}} < 0$ can be examined similarly; we have only to notice that the relative position of the FF and PP schedules are reversed.

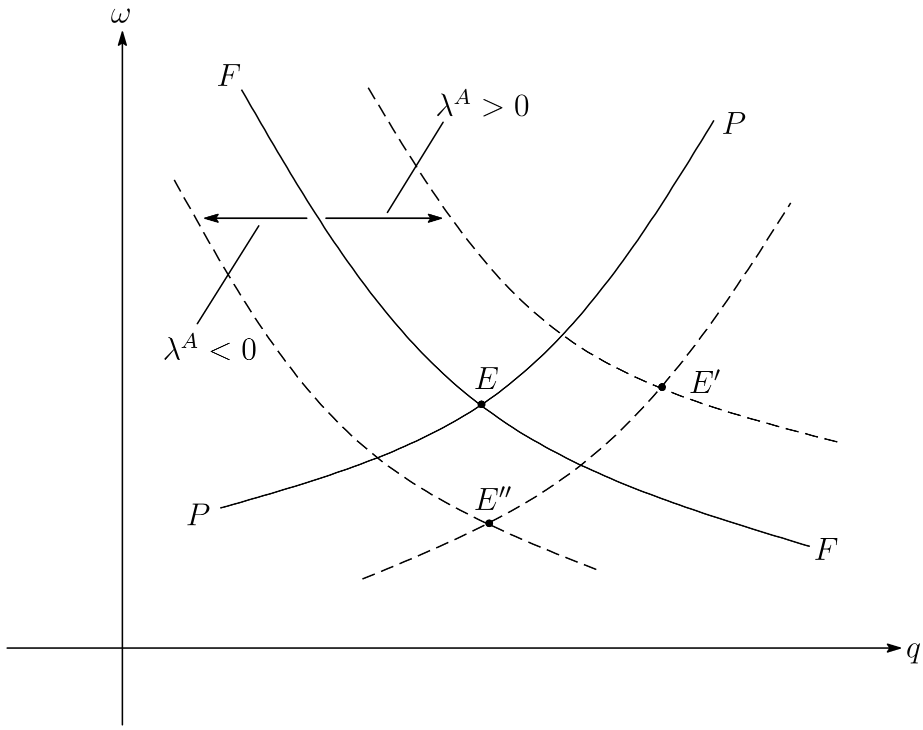

Figure 2

In concluding this section, note that the relevance of Theorem 2 to the effect that restricting competitiveness of the oligopolistic sector directly would contribute to improve welfare may well be suspect, since it may be doubted how the government can directly regulate the number of firms in the oligopolistic sector. Assuming that the government leverage against private incentives is indeed too weak to implement direct entry regulation, can we instead design tax-subsidy schemes which can induce welfare improvement indirectly? We now turn to this problem.

6 Welfare-Improving Tax-Subsidy Schemes

To analyse the effect of a tax-subsidy scheme in our model, the equations of change (4.1), (4.2) and (4.3) must be modified to include the tax-subsidy parameters. In order to avoid unnecessary complications, we will modify the equations of change with a familiar theoretical apparatus, the formal derivation of which being relegated to appendix A.

The Cournot-Nash equilibrium condition (4.1) in industry X will be modified by the introduction of tax-subsidy parameters as:

$$-\frac{1}{\epsilon}\hat{q}-\alpha\hat{n}+A\theta^{\mathrm{M}}\hat{\omega}=\frac{1}{m}m_S\cdot\mathrm{d}\boldsymbol{S}, \tag{6.1}$$

where $m_S = -(1, m_w, m_r; 0, 0, 0)$ and $\mathrm{d}\boldsymbol{S} = (\mathrm{d}s, \mathrm{d}s_w, \mathrm{d}s_r, \mathrm{d}t, \mathrm{d}t_w, \mathrm{d}t_r)$. It is clear from (6.1) that subsidies (resp. taxes) on the marginal cost increase (resp. decrease) the equilibrium output of each oligopolistic firm when the number of firms and factor prices are fixed.

Similarly, (4.2) can be modified by introducing tax-subsidy parameters as:

$$-\frac{n-1}{n\epsilon}\hat{q}-\frac{1}{\epsilon}\hat{n}+B\theta^{\mathrm{A}}\hat{\omega}=\frac{1}{C}C_S\cdot \mathrm{d}\boldsymbol{S}, \tag{6.2}$$

where $C_S = -(q, qc_w, qc_r; 1, F_w, F_r)$. It follows from (6.2) that a cost reduction (resp. increase) in the oligopolistic sector induced by government subsidies (resp. taxes) brings about an increase (resp. decrease) in the number of firms when the individual output level and factor prices are fixed.

The effects of the introduction of taxes and subsidies into factor markets are familiar in the theory of tax incidence. For example, the number of firms and the output level of each oligopolistic firm being fixed, a subsidy on the labour use in industry X, which brings about the factor substitution effect to cause excess demand in the labour market and excess supply in the capital market, increases the wage-rental ratio, and so forth. Formally, the modified equation of change in the factor market equilibrium becomes:

$$\lambda^{\mathrm{M}}\hat{q}+\lambda^{\mathrm{A}}\hat{n}+\Delta\hat{\omega}=-\beta\cdot \mathrm{d}\boldsymbol{S}, \tag{6.3}$$

where $\beta = (0, (1/w)\Delta^{\mathrm{M}}, -(1/r)\Delta^{\mathrm{M}}; 0, (1/w)\Delta^{\mathrm{F}}, -(1/r)\Delta^{\mathrm{F}})$. Δ^M and Δ^F represent what we may call the substitution terms in the marginal cost and the average cost part, respectively. They are proved to be always positive.[10] Clearly, the production subsidy and the lump-sum subsidy do not directly affect the factor market equilibrium. Since we are concerned only with the introduction of infinitesimal tax-subsidy schemes, the signs of λ^{M} and λ^{A} correspond to those of θ^{M} and θ^{A}, respectively.[11] Enumeration of the modified equations of change being now complete, we are ready to examine the welfare implications of several tax-subsidy schemes.

Consider the simultaneous equation system for $(\hat{q}, \hat{n}, \hat{\omega})$ defined by (6.1), (6.2) and (6.3):

$$\begin{bmatrix} -(1/\epsilon) & -\alpha & A\theta^{\mathrm{M}} \\ -(n-1)/n\epsilon & -(1/\epsilon) & B\theta^{\mathrm{A}} \\ -\lambda^{\mathrm{M}} & -\lambda^{\mathrm{A}} & -\Delta \end{bmatrix} \begin{bmatrix} \hat{q} \\ \hat{n} \\ \hat{\omega} \end{bmatrix} = \begin{bmatrix} (1/m)m_S \\ (1/C)C_S\cdot \mathrm{d}\boldsymbol{S} \\ -\beta\cdot \mathrm{d}\boldsymbol{S} \end{bmatrix} \tag{6.4}$$

and let H denote the determinant of the coefficient matrix in (6.4), i.e.

$$H=-\frac{1}{\epsilon}\left(\frac{1}{\epsilon}\Delta+B\theta^{\mathrm{A}}\lambda^{\mathrm{A}}\right)+\frac{n-1}{n\epsilon}(\alpha\Delta+A\theta^{\mathrm{M}}\lambda^{\mathrm{A}})-\lambda^{\mathrm{M}}\left(-\alpha B\theta^{\mathrm{A}}+\frac{1}{\epsilon}A\theta^{\mathrm{M}}\right).$$

[10] A formal derivation of (5.3) is contained in Appendix A.

[11] For the introduction of non-infinitesimal tax-subsidy schemes, see Atkinson and Stiglitz [2, Chapter 5] and the papers cited there.

We prove in appendix B that the general equilibrium system is locally stable if $H < 0$. The rest of the analysis proceeds under this assumption of local stability.

Solving (6.4) for $\hat{q}$, we obtain:

$$\hat{q} = \frac{\mathit{\Delta} + \epsilon B \theta^{\mathrm{A}} \lambda^{\mathrm{A}}}{\epsilon c H} \cdot m_S \cdot \mathrm{d}\boldsymbol{S} - \frac{\alpha \mathit{\Delta} + A \theta^{\mathrm{M}} \lambda^{\mathrm{A}}}{CH} \cdot C_S \cdot \mathrm{d}\boldsymbol{S}$$
$$- \frac{A\theta^{\mathrm{M}} - \alpha \epsilon B \theta^{\mathrm{A}}}{\epsilon H} \cdot \beta \cdot \mathrm{d}\boldsymbol{S}. \qquad (6.5)$$

From the above equation we can see how tax-subsidy schemes affect the equilibrium output level of each oligopolistic firm. The effect in question can be decomposed into three general equilibrium effects, each corresponding to a term in (6.5). We call them the *marginal cost effect*, the *total cost effect*, and the *factor substitution effect*, respectively, in accordance with the order of appearance in (6.2).

The marginal cost effect refers to the coefficient of $m_S \cdot \mathrm{d}\boldsymbol{S}$ which is always negative. Therefore, other things being equal, a tax-subsidy scheme which brings the marginal cost down induces an expansion in the individual output of the oligopolistic firm. It is intuitively clear within a partial equilibrium framework that a reduction in the marginal cost increases the output of each oligopolist if firms are symmetric. We have thus verified that, even when the general equilibrium interactions via factor market are taken into account, such a partial equilibrium result remains true as long as the general equilibrium system is locally stable.

The total cost effect refers to the coefficient of $C_S \cdot \mathrm{d}\boldsymbol{S}$. Suppose that the total cost is increased by the introduction of tax-subsidy. The effect on the output of each oligopolistic firm is two-fold. On the one hand, the output level of each oligopolistic firm will rise due to the exit of some firms from the oligopolistic industry (the excess entry theorem). On the other hand, the output level is also affected by an induced change in the factor price ratio; if industry X is more capital (resp. labour) intensive in the average sense, the wage-rental ratio goes up (resp. down). Thus without the factor intensity twist, an increase in individual output of the oligopolistic firm is enhanced, while it is offset or may even be upset when there is a factor intensity twist.

Finally, the factor substitution effect refers to the coefficient of $\beta \cdot \mathrm{d}\boldsymbol{S}$. A change in the relative factor price through substitution between factors affects the output level of each oligopolistic firm by changing the marginal cost condition and the number of firms. For example, consider the effect of the introduction of a subsidy on the wage expenditure component of marginal cost. Since it motivates the oligopolistic firms to substitute capital for labour, the wage-rental ratio must rise so as to adjust factor markets into equilibrium. When industry X firms are marginally more capital intensive but totally more labour intensive than industry Y, i.e. the factor intensity twist prevails, a rise in the wage-rental ratio necessarily expands the output of each oligopolistic firm as some firms are forced out of the industry. Without the factor intensity twist, however, it is ambiguous whether the output of each oligopolistic firm increases or not.

The following two observations might be in order here. First, those schemes that reduce total cost, i.e. $C_S \cdot \mathrm{d}\boldsymbol{S} < 0$, may not lead to a welfare improvement, because they always induce new entry into industry X. Second, the total cost effect and the factor substitution effect may conflict with each other in yielding an overall effect, for the former gives rise to an unambiguous change in q only *without* the factor intensity twist, while the latter only *with* it.

We say a tax-subsidy scheme is *self-financing* if $C_S \cdot \mathrm{d}\boldsymbol{S} = 0$ holds. Note also that $\beta \cdot \mathrm{d}\boldsymbol{S} = 0$ holds if and only if the tax-subsidy is of the lump-sum category. It then follows from the previous remark that a tax-subsidy scheme induces an unambiguous welfare improvement only if it is either self-financing or of the lump-sum variety.

Invoking Theorem 1, we may now assert the following:

Theorem 3 (Welfare-improving tax-subsidy schemes). *Suppose that strategic substitutability, quasi-linear preferences, and local stability hold simultaneously. Then, any tax-subsidy scheme which belongs to the following four fundamental categories is always welfare-improving:*

(1) $m_S \cdot \boldsymbol{S} < 0, C_S \cdot \mathrm{d}\boldsymbol{S} = 0, \beta \cdot \mathrm{d}\boldsymbol{S} = 0$.
(2a) $m_S \cdot \boldsymbol{S} < 0, C_S \cdot \mathrm{d}\boldsymbol{S} = 0, \beta \cdot \mathrm{d}\boldsymbol{S} > 0$ *in the case of* $\theta^{\mathrm{M}} > 0$ *and* $\theta^{\mathrm{A}} < 0$.
(2b) $m_S \cdot \boldsymbol{S} < 0, C_S \cdot \mathrm{d}\boldsymbol{S} = 0, \beta \cdot \mathrm{d}\boldsymbol{S} < 0$ *in the case of* $\theta^{\mathrm{M}} < 0$ *and* $\theta^{\mathrm{A}} > 0$.
(3) $m_S \cdot \boldsymbol{S} = 0, C_S \cdot \mathrm{d}\boldsymbol{S} > 0, \beta \cdot \mathrm{d}\boldsymbol{S} = 0$ *in the case of* $\theta^{\mathrm{M}}\theta^{\mathrm{A}} > 0$.

A typical scheme that belongs to category (1) is an infinitesimal production subsidy accompanied by a self-financing lump-sum tax. Note that this scheme improves welfare independent of factor intensity conditions.[12]

A typical scheme that belongs to category (3) is a lump-sum tax, which improves welfare when factor intensity twist does not occur. This result is parallel to Theorem 2. What is added to the oligopolists' profit under entry regulation is transformed into tax revenue for the government.

Welfare-improving tax-subsidy schemes that belong to categories (2a) and (2b) are more complex. They guarantee welfare improvement when the factor intensity twist exists. To illustrate, consider the case where industry X is marginally more capital intensive but totally more labour intensive [category (2a)]. Accompanied by a self-financing lump-sum tax, the introduction of an infinitesimal labour subsidy, applied to both the fixed cost and the marginal cost at a uniform rate, is welfare-improving; if $\mathrm{d}\boldsymbol{S} = (0, \mathrm{d}s_w, 0; \mathrm{d}t, \mathrm{d}s_w, 0)$ with $\mathrm{d}s_w > 0$ and $\mathrm{d}t = -(qm_w + F_w)\mathrm{d}s_w$, it belongs to the category (2a).

Even when a self-financing *lump-sum* tax is not available to the government, it is possible to construct a welfare-improving tax-subsidy scheme if the government can distinguish between factors used in the marginal cost part and the fixed cost part of the oligopolistic production. In the case of (2a), for example, the introduction of a production subsidy accompanied by a self-financing tax on the wage expenditure component of the fixed costs warrants welfare improvement. An infinitesimal subsidy on the capital expenditure component of the marginal cost accompanied by the same self-financing tax

[12] Although our analysis here is confined to the class of infinitesimal schemes, a combination of a non-infinitesimal production subsidy and a self-financing lump-sum tax appears to attain the first-best resource allocation. See Konishi [10].

as above, also satisfies (2a). The reader is invited to examine the types of tax-subsidy schemes which satisfy condition (2b).

7 Concluding Remarks

It goes without saying that the first-best policy in an oligopolistic economy such as ours is to simultaneously enforce the marginal cost pricing and control the number of firms at the optimal level. However, this first-best policy is likely to be beyond the reach of any actual government. Still there might remain room for a second-best policy if the government, which is unable to control prices, can nevertheless control the number of firms to improve economic welfare.

In the first part of this chapter we re-examined and generalized the excess entry theorem by allowing general equilibrium interactions in the factor markets. The assumption of strategic substitutability and of quasi-linear preferences, upon which our generalization rests, are rather standard. Our main finding is that the validity of the theorem hinges crucially on whether or not the factor intensity twist exists. If there is a factor intensity twist, entry regulation in the oligopolistic sector may decrease welfare. We also noted that such a direct entry regulation may still be beyond the reach of the government. In the second part of this chapter we analyzed conditions for tax-subsidy schemes to be welfare-improving.[13] Our analysis delineates several types of tax-subsidy schemes which are unambiguously welfare-improving.

We should emphasize, however, that our model is based on several drastically simplifying assumptions such as Cournot-Nash quantity competition, a single consumer (neglecting all the distributional issues), and the specific form of increasing returns to scale via the existence of fixed costs. It should also be noted that identifying capital (and wage) expenditures in marginal and fixed costs separately, which can be easily defined in theory, may be far from obvious in reality. The robustness of our results should be carefully examined before extracting any serious policy implications.

Appendix A: Derivation of (4.3) and/or (6.3)

We use the following notation:

Cost Shares

θ^{M}_{XL} (θ^{M}_{XK}) $= wm_w/m$ (rm_r/m): marginal cost share of labour (capital) in industry X,

θ^{F}_{XL} (θ^{F}_{XK}) $= wF_w/F$ (rF_r/F): fixed cost share of labour (capital) in industry X,

θ^{A}_{XL} (θ^{A}_{XK}) $= w(m_w q + F_w)/F$ ($r(m_r q + F_r)/F$): total cost share of labour (capital) in industry X,

θ_{YL} (θ_{YK}) $= wg_w/g$ (rg_r/g): cost share of labour (capital) in industry Y.

[13]The second-best tax-subsidy scheme, rather than welfare-improving infinitesimal tax-subsidy schemes, is analysed in Konishi [10].

Factor Shares

$\lambda^{\mathrm{M}}_{XL}$ $(\lambda^{\mathrm{M}}_{XK}) = nqm_w/L$ (nqm_r/K): share of total labour (capital) used as variable input in industry X,
$\lambda^{\mathrm{F}}_{XL}$ $(\lambda^{\mathrm{F}}_{XK}) = nF_w/L$ (nF_r/K): share of total labour (capital) used as fixed input in industry X,
$\lambda^{\mathrm{A}}_{XL}$ $(\lambda^{\mathrm{A}}_{XK}) = L_X/L$ (K_X/K): share of total labour (capital) used in industry X,
λ_{YL} $(\lambda_{YK}) = L_Y/L$ (K_Y/K): share of total labour (capital) used in industry Y,

Elasticities of Substitution

$\sigma^{\mathrm{M}}_X = -(\hat{m}_w - \hat{m}_r)/\hat{\omega}$: elasticity of substitution between variable inputs in industry X.
$\sigma^{\mathrm{F}}_X = -(\hat{F}_w - \hat{F}_r)/\hat{\omega}$: elasticity of substitution between fixed inputs in industry X.
$\sigma_Y = -(\hat{g}_w - \hat{g}_r)/\hat{\omega}$: elasticity of substitution in industry Y.

Total differentiation of (2.9) and (2.10) yields:

$$\lambda^{\mathrm{M}}_{XK}\hat{q} + \lambda^{\mathrm{A}}_{XK}\hat{n} + \lambda_{YK}\hat{Y} - (\lambda^{\mathrm{M}}_{XK}\theta^{\mathrm{M}}_{XL}\sigma^{\mathrm{M}}_X + \lambda^{\mathrm{F}}_{XK}\theta^{\mathrm{F}}_{XL}\sigma^{\mathrm{F}}_X + \lambda_{YK}\theta_{YL}\sigma_Y)\hat{\omega}$$
$$= \lambda^{\mathrm{M}}_{XK}\theta^{\mathrm{M}}_{XL}\sigma^{\mathrm{M}}_X\{(1/w)\mathrm{d}s_w - (1/r)\mathrm{d}s_r\}$$
$$+\lambda^{\mathrm{F}}_{XK}\theta^{\mathrm{F}}_{XL}\sigma^{\mathrm{F}}_X\{(1/w)\mathrm{d}t_w - (1/r)\mathrm{d}t_r\} \tag{A.1}$$

and

$$\lambda^{\mathrm{M}}_{XL}\hat{q} + \lambda^{\mathrm{A}}_{XL}\hat{n} + \lambda_{YL}\hat{Y} - (\lambda^{\mathrm{M}}_{XL}\theta^{\mathrm{M}}_{XK}\sigma^{\mathrm{M}}_X + \lambda^{\mathrm{F}}_{XL}\theta^{\mathrm{F}}_{XK}\sigma^{\mathrm{F}}_X + \lambda_{YL}\theta_{YK}\sigma_Y)\hat{\omega}$$
$$= -\lambda^{\mathrm{M}}_{XL}\theta^{\mathrm{M}}_{XK}\sigma^{\mathrm{M}}_X\{(1/w)\mathrm{d}s_w - (1/r)\mathrm{d}s_r\}$$
$$-\lambda^{\mathrm{F}}_{XL}\theta^{\mathrm{F}}_{XK}\sigma^{\mathrm{F}}_X\{(1/w)\mathrm{d}t_w - (1/r)\mathrm{d}t_r\} \tag{A.2}$$

respectively. Eliminating $\hat{Y}$ from the above two equations, we obtain:

$$\lambda^{\mathrm{M}}\hat{q} + \lambda^{\mathrm{A}}\hat{n} + \varDelta\hat{\omega} = -\,\beta\cdot\mathrm{d}\boldsymbol{S}, \tag{6.3}$$

where

$$\lambda^{\mathrm{M}} = \lambda^{\mathrm{M}}_{XK}\lambda_{YL} - \lambda^{\mathrm{M}}_{XL}\lambda_{YK} = (1/K)\lambda^{\mathrm{M}}_{XL}\lambda_{YL}\{(m_r/m_w) - (K_Y/L_Y)\},$$

$$\lambda^{\mathrm{A}} = \lambda^{\mathrm{A}}_{XK}\lambda_{YL} - \lambda^{\mathrm{A}}_{XL}\lambda_{YK} = (1/K)\lambda^{\mathrm{A}}_{XL}\lambda_{YL}\{(K_X/L_X) - (K_Y/L_Y)\},$$

$$\varDelta = \varDelta^{\mathrm{M}} + \varDelta^{\mathrm{F}} + \lambda_{YL}\lambda_{YK}\sigma_Y,$$

$$\varDelta^{\mathrm{M}} = \lambda^{\mathrm{M}}_{XL}\lambda_{YK}\theta^{\mathrm{M}}_{XL} + \lambda^{\mathrm{M}}_{XK}\lambda_{YL}\theta^{\mathrm{M}}_{XK} \text{ and } \varDelta^{\mathrm{F}} = \lambda^{\mathrm{F}}_{XL}\lambda_{YK}\theta^{\mathrm{F}}_{XL} + \lambda^{\mathrm{F}}_{XK}\lambda_{YL}\theta^{\mathrm{F}}_{XK}.$$

Taking $\mathrm{d}\boldsymbol{S} = 0$ yields (4.3) and this completes the derivation of (4.3) and (6.3).

Appendix B: Local Stability of the System

To simplify the stability analysis, we assume that the wage and rental rates are adjusted instantly to equate demand and supply for labour and capital, respectively. Similarly, the market for Y is assumed to be cleared immediately. Thus, (2.3), (2.9) and (2.10) always hold along any adjustment path. We further assume that $\boldsymbol{S} = 0$.

We denote by ω^*, q^* and n^* the equilibrium values of the wage-rental ratio, the individual output of each oligopolistic firm, and the number of firms, respectively. Assumptions on the adjustment process in the market for factors and for good Y warrants that

$$\frac{\omega - \omega^*}{\omega^*} = -\frac{1}{\varDelta}\left\{\lambda^{\mathrm{M}}\frac{q - q^*}{q^*} + \lambda^{\mathrm{A}}\frac{n - n^*}{n^*}\right\} \tag{B.1}$$

and

$$p_Y = g(w, r) \tag{B.2}$$

hold in the neighbourhood of equilibrium.

We next define the dynamic adjustment process by:

$$\dot{q} = \kappa\left\{\phi(nq)\left(1 - \frac{1}{n\epsilon}\right) - \frac{m(w, r, 0)}{g(w, r)}\right\}, \tag{B.3}$$

$$\dot{n} = \eta\left\{\phi(nq)q - \frac{m(w, r, 0)}{g(w, r)}q - \frac{F(w, r, 0)}{g(w, r)}\right\}, \tag{B.4}$$

where $\dot{q}$ and $\dot{n}$ denote the time derivative of q and n, and $\kappa > 0$ and $\eta > 0$ are adjustment coefficients.

Linearly approximating (B.3) and (B.4) around the free-entry equilibrium and using (B.1), we obtain:

$$\dot{q} = \kappa^*\left\{\left(-\frac{1}{\epsilon} - \frac{A}{\varDelta}\theta^{\mathrm{M}}\lambda^{\mathrm{M}}\right)\frac{q - q^*}{q^*} + \left(-\,\alpha - \frac{A}{\varDelta}\theta^{\mathrm{M}}\lambda^{\mathrm{A}}\right)\frac{n - n^*}{n^*}\right\}, \tag{B.5}$$

$$\dot{n} = \eta^*\left\{\left(-\frac{n^* - 1}{n^*\epsilon} - \frac{B}{\varDelta}\theta^{\mathrm{A}}\lambda^{\mathrm{M}}\right)\frac{q - q^*}{q^*} + \left(-\frac{1}{\epsilon} - \frac{B}{\varDelta}\theta^{\mathrm{A}}\lambda^{\mathrm{A}}\right)\frac{n - n^*}{n^*}\right\}, \tag{B.6}$$

where $\kappa^* = \kappa\phi(n^*q^*)\{1 - (1/n^*\epsilon)\} > 0$ and $\eta^* = \eta\phi(n^*q^*)q^* > 0$. Observe from these adjustment equations that the equilibrium is locally stable if

$$-\frac{1}{\epsilon} - \frac{A}{\varDelta}\theta^{\mathrm{M}}\lambda^{\mathrm{M}} < 0$$

and

$$-\frac{H}{\varDelta} = \left(-\frac{1}{\epsilon} - \frac{A}{\varDelta}\theta^{\mathrm{M}}\lambda^{\mathrm{M}}\right)\left(-\frac{1}{\epsilon} - \frac{B}{\varDelta}\theta^{\mathrm{A}}\lambda^{\mathrm{A}}\right) - \left(-\,\alpha - \frac{A}{\varDelta}\theta^{\mathrm{M}}\lambda^{\mathrm{A}}\right)$$

$$\times\left(-\frac{n^*-1}{n^*\epsilon}-\frac{B}{\Delta}\theta^{\mathrm{A}}\lambda^{\mathrm{M}}\right)>0.$$

The first inequality is always satisfied. Hence, $H<0$ is sufficient for the local stability of equilibrium, as was to be verified.[14]

[14]Note that the asterisks are omitted in the main text.

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Chapter 19
Cooperative and Noncooperative R&D in an Oligopoly with Spillovers*

The purpose of this chapter is to examine the effects of cooperative R&D, wherein member firms commit themselves to a joint profit-maximizing level of R&D in a "pre-competitive stage" but remain fierce competitors in the product market. As a standard of reference, this chapter will also examine the characteristic features of noncooperative R&D, socially first-best R&D, and socially second-best R&D. Since the incentives for a firm to undertake R&D on its own depend on there being a sufficient degree of appropriability of the research outcomes (hence, a limited diffusion of knowledge) it is of particular interest and relevance to see how cooperative R&D fares vis-à-vis noncooperative R&D in the presence of R&D spillovers.

This analysis is conducted in terms of a two-stage model of oligopolistic competition. In the first stage, firms decide on their cost-reducing R&D either cooperatively or noncooperatively, whereas in the second stage they engage in quantity competition in the product market. Two-stage models of this type have been studied extensively by Timothy Besley and Suzumura [2], James Brander and Barbara Spencer [3], Spencer and Brander [17], and Masahiro Okuno-Fujiwara and Suzumura [13] in the absence of R&D coordination and R&D spillovers.[1]

Claude d'Aspremont and Alexis Jacquemin [6; 7] presented an interesting analysis of cooperative and noncooperative R&D in terms of a two-stage model of a *duopoly* with R&D spillovers. They make the interesting point that cooperative R&D agreements between otherwise competing firms may increase the R&D expenditure level relative to the fully noncooperative case provided that the R&D spillovers are sufficiently large, although the cooperative R&D may still fall short of the socially first-best level. For at least two reasons, however, the robustness of their results is questionable. The present chapter is meant to provide a careful examination of their findings with the purpose of

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[1]In contrast, Partha Dasgupta and Joseph Stiglitz [5], Richard Levin and Peter Reiss [12], and Michael Spence [16] analyzed one-stage models without strategic commitment to R&D, but with or without R&D spillovers.

locating them in a more appropriate perspective.[2] In so doing, it will develop a systematic method for analyzing the properties of a two-stage model of oligopolistic competition.

In the first place, d'Aspremont and Jacquemin [6] presented their results as surprising on the basis that cooperation should reduce excessive duplication of R&D efforts in the presence of large spillovers.[3] Note, however, that the R&D incentive of a single firm hinges squarely on the extent of appropriability of the R&D benefits, so that the presence of large R&D spillovers may drastically reduce the incentives for cost reduction, with the result that the R&D commitment made voluntarily by a firm tends to be socially too small. From this viewpoint, an enforceable agreement on cooperative R&D efforts seems to facilitate more commitments. The result of the net effect of the R&D cooperation hinges on the relative strength of these competing effects. It will be shown that the latter effect dominates the former not only in the duopoly example with linear inverse demand function and linear marginal cost function assumed by d'Aspremont and Jacquemin [6], but also in a much wider class of oligopolistic industries, thereby supporting the robustness of their results.

In the second place, d'Aspremont and Jacquemin [6] invoked the first-best welfare (market surplus) function as their welfare criterion, but the relevance of this convention may be disputed in that the enforcement of the first-best arrangement may require considerable leverage on the government vis-à-vis private firms, something which may be hard to secure in reality. What is needed is an evaluation of the social gains from cooperative R&D within the alternative feasible arrangements. It will be shown that my results can be extended and made more relevant by invoking a second-best welfare (market surplus) function as an alterative welfare criterion.

1 The Model

Consider an industry with n firms ($2 \leq n < +\infty$) producing a homogeneous product. Let $p = f(Q)$ be the inverse demand function, where p and Q denote, respectively, the price and the aggregate output of this product. Let x_i and q_i denote, respectively, the amount of R&D and the output of firm i. The cost of production and that of R&D are assumed to be $c(x_i; \mathbf{x}_{-i})q_i$ and x_i, respectively, where $\mathbf{x}_{-i} := [x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$, assuming for the sake of notational simplicity that the amount of R&D is measured by its cost. Diminishing returns on R&D can be incorporated without affecting any of the results in this chapter.

Throughout the analysis, I will assume the following:

Assumption 1: *The inverse demand function $f(Q)$ is twice continuously differentiable with $f'(Q) < 0$ for all $Q \geq 0$ satisfying $f(Q) > 0$.*

[2] Several salient features of cooperative research activities are analyzed by Michael Katz [11] and Jacquemin [10], among many others. Needless to say, there are several relevant features thereof that I had to leave out of this chapter, but in the concluding remarks, several directions in which the present analysis should be further developed will be pointed out.

[3] Also, by participating in the cooperative R&D association, firms can conserve on the fixed costs of R&D equipment and administration.

Assumption 2: *The average variable cost function $c(x_i; \mathbf{x}_{-i})$ is twice continuously differentiable and satisfies $c(x_i; \mathbf{x}_{-i}) > 0, (\partial/\partial x_i)c(x_i; \mathbf{x}_{-i}) < 0$ and $(\partial/\partial x_j)c(x_i; \mathbf{x}_{-i}) \leq 0$ for any $\mathbf{x} = [x_1, x_2, \ldots, x_n] \geq \mathbf{0}$ $(i \neq j; i, j = 1, 2, \ldots, n)$. Furthermore, for any symmetric vector $\mathbf{x} = [x_1, x_2, \ldots, x_n] \geq \mathbf{0}$, $(\partial/\partial x_i)c(x_i; \mathbf{x}_{-i}) < (\partial/\partial x_j)c(x_i; \mathbf{x}_{-i})$ $(i \neq j; i.j = 1, 2, \ldots, n)$ holds.*

No particular account is needed for Assumption 1. According to Assumption 2, a firm's R&D is cost-reducing and can benefit other firms without payment. However, the cost-reducing effect of own R&D outweighs the benefits accruing freely from other firms when all firms are spending the same amount on R&D.

In the model, firms are engaging in two-stage competition. In the first stage, firms make an irrevocable commitment to a level of R&D in full anticipation of the equilibrium that will be established in the second stage, where firms compete in the product market. The second-stage strategic variable is assumed to be the level of output.

Following d'Aspremont and Jacqemin [6], I will examine two types of equilibrium. The first equilibrium concept is noncooperative throughout the two stages, so that the equilibrium of the second stage is a Cournot-Nash equilibrium, and that of the entire game is a subgame-perfect equilibrium. The second equilibrium concept is a mixed cooperative and noncooperative equilibrium; firms are supposed to coordinate their R&D in the first stage so as to maximize joint profits with the understanding that they engage in noncooperative competition in the second stage.

To characterize the second-stage equilibrium, let a profile of the amounts of R&D, $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, be parametrically given. Then the profits of firm i in the second-stage game corresponding to an output profile $\mathbf{q} = [q_1, q_2, \ldots, q_n]$ are defined by

$$(1) \quad \pi_i(\mathbf{q}; x_i, \mathbf{x}_{-i}) := \{f(Q) - c(x_i; \mathbf{x}_{-i})\}q_i - x_i \quad (i = 1, 2, \ldots, n)$$

where $Q := \Sigma_{j=1}^n q_j$. In what follows, $\mathbf{q}^{\mathrm{N}}(\mathbf{x}) = [q_1^{\mathrm{N}}(\mathbf{x}), q_2^{\mathrm{N}}(\mathbf{x}), \ldots, q_n^{\mathrm{N}}(\mathbf{x})]$ denotes the Cournot-Nash equilibrium of the second-stage game corresponding to the specified first-stage R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$. Assuming the interior optimum and second-order conditions, $\mathbf{q}^{\mathrm{N}}(\mathbf{x})$ can be characterized by $(\partial/\partial q_i)\pi_i(\mathbf{q}^{\mathrm{N}}(\mathbf{x}); x_i, \mathbf{x}_{-i}) = 0$ $(i = 1, 2, \ldots, n)$:

$$(2) \quad f'(Q^{\mathrm{N}}(\mathbf{x}))q_i^{\mathrm{N}}(\mathbf{x}) + f(Q^{\mathrm{N}}(\mathbf{x})) - c(x_i; \mathbf{x}_{-i}) = 0 \quad (i = 1, 2, \ldots, n)$$

where $Q^{\mathrm{N}}(\mathbf{x}) := \Sigma_{j=1}^n q_j^{\mathrm{N}}(\mathbf{x})$. Throughout this chapter, I focus on the symmetric equilibrium, so that $q_i^{\mathrm{N}}(\mathbf{x}) = q_j^{\mathrm{N}}(\mathbf{x})$ $(i, j = 1, 2, \ldots, n)$ holds if $x_i = x_j$ $(i, j = 1, 2, \ldots, n)$.

To lend substance to the oligopolistic interactions in the model, I now introduce a strategic assumption. Let $\alpha_i(\mathbf{q}; \mathbf{x})$ and $\beta_{ij}(\mathbf{q}; \mathbf{x})$ be defined for any R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ by

$$\alpha_i(\mathbf{q}; \mathbf{x}) := \frac{\partial^2}{\partial q_i^2}\pi_i(\mathbf{q}; x_i, \mathbf{x}_{-i}) \quad (i = 1, 2, \ldots, n)$$

and

$$\beta_{ij}(\mathbf{q};\mathbf{x}) := \frac{\partial^2}{\partial q_i \partial q_j}\pi_i(\mathbf{q}; x_i, \mathbf{x}_{-i}) \quad (i \neq j;\ i,j = 1,2,\ldots,n)$$

respectively. I can then state the following second-stage strategic assumption in a compact form:

Assumption 3: *Firms' strategic variables in the second-stage game are strategic substitutes; that is,* $\beta_{ij}(\mathbf{q};\mathbf{x}) < 0$ $(i \neq j;\ i,j = 1,2,\ldots,n)$ *holds for any specified R&D profile* $\mathbf{x} = [x_1, x_2, \ldots, x_n] \geq 0$.[4,5]

Note that it is quite natural to invoke Assumption 3 in the present context of homogeneous-product Cournot oligopoly, as it corresponds to the assumption of downward-sloping reaction curves (see Bulow et al. [4]; Dixit [8]). Note also that Assumptions 1-3 are satisfied by d'Aspremont and Jacquemin's [6] example.

Turning to the first stage of the game, define the first-stage payoff function of firm i by

$$(3) \quad \Pi_i(\mathbf{x}) := \pi_i(\mathbf{q}^{\mathrm{N}}(\mathbf{x}); x_i, \mathbf{x}_{-i}) \quad (i = 1,2,\ldots,n).$$

Then, the Nash equilibrium of the first-stage game, denoted by $\mathbf{x}^{\mathrm{N}} = [x_1^{\mathrm{N}}, x_2^{\mathrm{N}}, \ldots, x_n^{\mathrm{N}}]$, is characterized under the assumption of interior optimum and second-order conditions by $(\partial/\partial x_i)\Pi_i(\mathbf{x}^{\mathrm{N}}) = 0$ $(i = 1,2,\ldots,n)$; that is,

$$(4) \quad \sum_{j=1}^{n} \frac{\partial}{\partial q_j}\pi_i(\mathbf{q}^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}); x_i^{\mathrm{N}}, \mathbf{x}_{-i}^{\mathrm{N}}) \cdot \frac{\partial}{\partial x_i} q_j^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}) + \frac{\partial}{\partial x_i}\pi_i(\mathbf{q}^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}); x_i^{\mathrm{N}}, \mathbf{x}_{-i}^{\mathrm{N}}) = 0 \quad (i = 1,2,\ldots,n).$$

Note that $\{\mathbf{x}^{\mathrm{N}}, \mathbf{q}^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\}$ is nothing but the subgame-perfect equilibrium of the two-stage game, which is the central focus of subsequent analysis. By definition, $\mathbf{q}^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})$ is the second-stage Cournot-Nash equilibrium, given $\mathbf{x}^{\mathrm{N}}$. Throughout this chapter, I focus on the symmetric $\mathbf{x}^{\mathrm{N}}: x_i^{\mathrm{N}} = x_j^{\mathrm{N}}$ $(i,j = 1,2,\ldots,n)$.[6]

[4] The concept of strategic substitutes and complements was first introduced by Jeremy Bulow et al. [4] and is invoked quite widely in the recent literature on oligopoly theory. See, among others, Avinash Dixit [8], Besley and Suzumura [2], and Okuno-Fujiwara and Suzumura [13].

[5] As Jesus Seade [14; 15] has aptly observed, perverse results often hold in oligopoly theory when the Cournot-Nash equilibrium is unstable. This being the case, it is reassuring that the myopic adjustment process à la Cournot that each firm increases (decreases) its output marginally if marginal profitability is positive (negative) can be shown to be locally stable under Assumptions 1 and 3.

[6] To analyze in detail the structure of the first-stage game, define $\lambda_i(\mathbf{x}) := (\partial^2/\partial x_i^2)\Pi_i(\mathbf{x})$ and $\mu_{ij}(x) := (\partial^2/\partial x_i \partial x_j)\Pi_i(\mathbf{x})$ $(i \neq j;\ i,j = 1,2,\ldots,n)$. As in the case of the second-stage game, the first-stage strategic variables are *strategic substitutes (resp. strategic complements)* if and only if $\mu_{ij}(\mathbf{x}) <$ (resp. $>$) 0 holds $(i \neq j;\ i,j = 1,2,\ldots,n)$. Depending on the extent of R&D spillovers, both

Although Assumptions 1-3 generally will be maintained throughout this chapter, there will be several occasions when Assumption 3 must be replaced by the following assumption for the sake of obtaining unambiguous verdicts:

Assumption 3*: *The inverse demand function* $f(Q)$ *is concave; that is,* $f''(Q) \leq 0$ *holds for all* $Q \geq 0$ *such that* $f(Q) > 0$.

Note that Assumption 3* is the condition that guarantees the concavity of profit function $\pi_i(\mathbf{q}; x_i, \mathbf{x}_{-i})$ with respect to q_i, and it is clearly satisfied by the linear inverse demand function used by d'Aspremont and Jacquemin [6] as well as by many others. In the presence of Assumption 1, Assumption 3* implies Assumption 3, so that the set of Assumptions 1, 2, and 3* is collectively stronger than the set consisting of Assumptions 1, 2 and 3. Finally, note that $\alpha_i(\mathbf{q}; \mathbf{x})$ and $\beta_{ij}(\mathbf{q}; \mathbf{x})$ become independent of the firm indexes i and j ($i \neq j;\ i, j = 1, 2, \ldots, n$) if they are evaluated at the symmetric $\mathbf{q}$ and $\mathbf{x}$. In such a case, it is possible to denote them simply as $\alpha(\mathbf{q}; \mathbf{x})$ and $\beta(\mathbf{q}; \mathbf{x})$.

This completes the description of the two-stage model of oligopolistic competition. I must now set about analyzing the positive as well as the normative properties of the game.

2 R&D Spillovers and Output Response

To begin, I analyze how the individual firm's output and the industry aggregate output at the second-stage Nash equilibrium react to a change in the R&D expenditure by a firm in the first-stage game. With this purpose in mind, define $\omega(\mathbf{x}) := (\partial/\partial x_i) q_i^{\mathrm{N}}(\mathbf{x})$ and $\theta(\mathbf{x}) := (\partial/\partial x_i) q_j^{\mathrm{N}}(\mathbf{x})$ ($i \neq j;\ i, j = 1, 2, \ldots, n$) for any symmetric R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$. Clearly, $\omega(\mathbf{x})$ (resp. $\theta(\mathbf{x})$) denotes the effect of a marginal change in the first-stage R&D expenditure by firm i on the second-stage equilibrium output of firm i (resp. firm j), where $i \neq j$. By virtue of the symmetry of $\mathbf{x}$, $\omega(\mathbf{x})$ and $\theta(\mathbf{x})$ are independent of the firm indexes i and j ($i \neq j;\ i, j = 1, 2, \ldots, n$). Clearly, the effect of a change in x_i on the equilibrium aggregate output $Q^N(\mathbf{x})$ is given by $(\partial/\partial x_i) Q^{\mathrm{N}}(\mathbf{x}) = \omega(\mathbf{x}) + (n-1)\theta(\mathbf{x})$ ($i = 1, 2, \ldots, n$).

In Appendix A, I show that $\omega(\mathbf{x})$ and $\theta(\mathbf{x})$ can be calculated as

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cases are a priori possible. It is easy to verify that the first-stage Nash equilibrium $\mathbf{x}^{\mathrm{N}}$ is locally stable with respect to the myopic adjustment process $\dot{x}_i = \tau(\partial/\partial x_i)\Pi_i(\mathbf{x})$ ($i = 1, 2, \ldots, n$), where $\dot{x}_i$ denotes the time derivative of x_i and $\tau > 0$ is the adjustment coefficient, if the inequality $\lambda + (k-1)\mu < 0$ ($k = 0, 1, \ldots, n$) holds, where $\lambda := \lambda_i(\mathbf{x}^{\mathrm{N}})$ and $\mu := \mu_{ij}(\mathbf{x}^{\mathrm{N}})$. In the case of a duopoly where $n = 2$, this inequality boils down to $\lambda - \mu < 0, \lambda < 0$, and $\lambda + \mu < 0$. Note that the condition $\lambda < 0$ is nothing other than the second-order condition for the first-stage payoff maximization, whereas the other two conditions reduce to $-1 < \mu/\lambda < 1$; that is,

$$-1 < (\partial^2/\partial x_i \partial x_j)\Pi_i(\mathbf{x}^{\mathrm{N}})/(\partial^2/\partial^2 x_i)\Pi_i(\mathbf{x}^{\mathrm{N}}) < 1 \quad (i \neq j;\ i, j = 1, 2)$$

which is precisely the condition analyzed by Irene Henriques [9] in the context of the d'Aspremont and Jacquemin [6] example.

$$(5)\quad \omega(\mathbf{x}) = \frac{1}{\Delta(\mathbf{x})}\Bigg\{[\alpha(\mathbf{x}) - \beta(\mathbf{x})]\frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i}) \\ +(n-1)\beta(\mathbf{x})\left[\frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i}) - \frac{\partial}{\partial x_j}c(x_i;\mathbf{x}_{-i})\right]\Bigg\}$$

and

$$(6)\quad \theta(\mathbf{x}) = \frac{1}{\Delta(\mathbf{x})}\Bigg\{\alpha(\mathbf{x})\left[\frac{\partial}{\partial x_j}c(x_i;\mathbf{x}_{-i}) - \frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i})\right] \\ + [\alpha(\mathbf{x}) - \beta(\mathbf{x})]\frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i})\Bigg\}$$

where $\alpha(\mathbf{x}) := \alpha(\mathbf{q}^{\mathrm{N}}(\mathbf{x}), \mathbf{x}), \beta(\mathbf{x}) := \beta(\mathbf{q}^{\mathrm{N}}(\mathbf{x}), \mathbf{x})$ for notational brevity and where $\Delta(\mathbf{x}) := [\alpha(\mathbf{x}) - \beta(\mathbf{x})][\alpha(\mathbf{x}) + (n-1)\beta(\mathbf{x})]$ and $i \neq j$. One can then obtain

$$(7)\quad \psi(\mathbf{x}) := \omega(\mathbf{x}) + (n-1)\theta(\mathbf{x}) \\ = \frac{\alpha(\mathbf{x}) - \beta(\mathbf{x})}{\Delta(\mathbf{x})}\cdot\left[\frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i}) + (n-1)\frac{\partial}{\partial x_j}c(x_i;\mathbf{x}_{-i})\right].$$

It is easy to verify that

$$(8)\quad \alpha(\mathbf{x}) = 2f'(Q^{\mathrm{N}}(\mathbf{x})) + f''(Q^{\mathrm{N}}(\mathbf{x}))q_i^{\mathrm{N}}(\mathbf{x})$$

$$(9)\quad \beta(\mathbf{x}) = f'(Q^{\mathrm{N}}(\mathbf{x})) + f''(Q^{\mathrm{N}}(\mathbf{x}))q_i^{\mathrm{N}}(\mathbf{x})$$

so that $\alpha(\mathbf{x}) - \beta(\mathbf{x}) = f'(Q^{\mathrm{N}}(\mathbf{x})) < 0$ by virtue of Assumption 1. It then follows from Assumption 3 that $\alpha(\mathbf{x}) < 0$, which in turn guarantees that $\Delta(\mathbf{x}) > 0$. Therefore, $\omega(\mathbf{x}) > 0$ follows from (5), Assumption 2 and Assumption 3. In other words, *an increase in cost-reducing R&D by a firm unambiguously increases the equilibrium output of that firm.* In contrast, because of the R&D spillovers, an increase in cost-reducing R&D by firm i exerts two conflicting effects on the equilibrium output of firm j $(i \neq j)$. On the one hand, it tends to increase j's output by bringing j's cost down through spillovers of cost-reducing benefits. On the other hand, it tends to decrease j's output by strengthening i's competitive edge against j. I will say that *the R&D spillovers are sufficiently large if the former effect outweighs the latter so that $\theta(\mathbf{x}) > 0$ holds.*[7] Finally, it follows from (7) that *the industry aggregate output at the Cournot-Nash equilibrium always increases when one of the firms increases its cost-reducing R&D irrespective of whether the R&D spillovers are large or small.*

[7]This definition is in concordance with the one adopted by d'Aspremont and Jacquemin [6]. Indeed, in their duopoly example where $0 > (\partial/\partial x_j)c(x_i;\mathbf{x}_{-i}) = -\epsilon > -1 = (\partial/\partial x_i)c(x_i;\mathbf{x}_{-i})(i \neq j; i,j = 1,2), f(Q) = a - bQ, a > 0$, and $b > 0$, one may compute that $\theta(\mathbf{x}) = (2\epsilon - 1)/3$, which is positive if and only if $\epsilon > \frac{1}{2}$ (i.e., if and only if the R&D spillovers are "sufficiently large" in the sense d'Aspremont and Jacquemin [6] used the term).

3 Mixed Cooperative and Noncooperative Game

Let us turn now to the second two-stage game. In this game, firms coordinate their R&D in the first stage in order to maximize their joint profits, whereas they compete in the second-stage quantity game.

The cooperative equilibrium $\mathbf{x}^{\mathrm{C}} = [x_1^{\mathrm{C}}, x_2^{\mathrm{C}}, \ldots, x_n^{\mathrm{C}}]$ of the first-stage game can be characterized by $(\partial/\partial x_i)\Sigma_{j=1}^{n}\Pi_j(\mathbf{x}^{\mathrm{C}}) = 0$ $(i = 1, 2, \ldots, n)$ under the assumption of interior optimum and second-order conditions. It is easy, although tedious, to reduce this condition to the following:

$$(10) \quad q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\Big\{(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}))\psi(\mathbf{x}^{\mathrm{C}}) - \left[\frac{\partial}{\partial x_i}c(x_i^{\mathrm{C}}; \mathbf{x}_{-i}^{\mathrm{C}}) + (n-1)\frac{\partial}{\partial x_j}c(x_i^{\mathrm{C}}; \mathbf{x}_{-i}^{\mathrm{C}})\right]\Big\} - 1 = 0 \quad (i \neq j;\ i, j = 1, 2, \ldots, n).$$

Then the equilibrium of the whole game is given by $\{\mathbf{x}^{\mathrm{C}}, \mathbf{q}^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\}$, which is also assumed to be symmetric.

4 First-Best Welfare Analysis

The welfare performance of $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ can be gauged and compared in terms of several alternative criteria. To begin, I invoke the *first-best welfare (market surplus) function* $W^{\mathrm{F}}(x)$ à la d'Aspremont and Jacquemin [6]. For any R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ and output profile $\mathbf{q} = [q_1, q_2, \ldots, q_n]$, let the market surplus function $W(\mathbf{x}, \mathbf{q})$ be defined by

$$(11) \quad W(\mathbf{x}, \mathbf{q}) := \int_0^Q f(Z)dZ - \sum_{j=1}^{n} [c(x_j; \mathbf{x}_{-j})q_j + x_j]$$

where $Q \equiv \Sigma_{j=1}^{n} q_j$, which is nothing other than the sum of the consumer's surplus and the producer's surplus.

Take any symmetric R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, and let

$$\mathbf{q}^{\mathrm{F}}(\mathbf{x}) = [q_1^{\mathrm{F}}(\mathbf{x}), q_2^{\mathrm{F}}(\mathbf{x}), \ldots, q_n^{\mathrm{F}}(\mathbf{x})]$$

be the *socially first-best output profile* corresponding to $\mathbf{x}$, which is defined by

$$(12) \quad \mathbf{q}^{\mathrm{F}}(\mathbf{x}) := \arg\max_{\mathbf{q} > \mathbf{0}} W(\mathbf{x}, \mathbf{q}).$$

As is easily verified, $\mathbf{q}^{\mathrm{F}}(\mathbf{x})$ is characterized by the familiar *marginal cost principle*:

$$(13)\quad f(Q^{\mathrm{F}}(\mathbf{x})) = c(x_i; \mathbf{x}_{-i}) \quad (i = 1, 2, \ldots, n)$$

where $Q^{\mathrm{F}}(\mathbf{x}) := \Sigma_{j=1}^{n} q_j^{\mathrm{F}}(\mathbf{x})$. Comparing $\mathbf{q}^{\mathrm{F}}(\mathbf{x})$ with $\mathbf{q}^{\mathrm{N}}(\mathbf{x})$, one can obtain the following lemma.

Lemma 1: *For any symmetric R&D profile* $\mathbf{x} = [x_1, x_2, \ldots, x_n], q_i^{\mathrm{F}}(\mathbf{x}) > q_i^{\mathrm{N}}(\mathbf{x})$ *holds* $(i = 1, 2, \ldots, n)$.

(See Appendix B for the proof.)

In terms of $\mathbf{q}^{\mathrm{F}}(\mathbf{x})$, the first-best welfare (market surplus) function $W^{\mathrm{F}}(\mathbf{x})$ is defined by

$$(14)\quad W^{\mathrm{F}}(\mathbf{x}) := W(\mathbf{x}, \mathbf{q}^{\mathrm{F}}(\mathbf{x})).$$

One straightforward way to apply the first-best welfare function $W^{\mathrm{F}}(\mathbf{x})$ is to compare $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ directly with the socially first-best R&D, $\mathbf{x}^{\mathrm{F}}$, which is defined by

$$(15)\quad \mathbf{x}^{\mathrm{F}} := \arg\max_{\mathbf{x} > \mathbf{0}} W^{\mathrm{F}}(\mathbf{x}).$$

Unless the model is further specialized in detail so that one can actually compute $\mathbf{x}^{\mathrm{N}}$, $\mathbf{x}^{\mathrm{C}}$, and $\mathbf{x}^{\mathrm{F}}$ as in the d'Aspremont and Jacquemin [6] duopoly example, however, such a direct application of $W^{\mathrm{F}}(\mathbf{x})$ is hard to come by.

An alternative way to proceed is to evaluate the partial derivative $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x})$ at $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ for any $i = 1, 2, \ldots, n$. If it so happens that $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) <$ (resp. $>$) 0, then a marginal decrease (resp. increase) in x_i at $\mathbf{x}^{\mathrm{N}}$ increases the value of $W^{\mathrm{F}}(\mathbf{x})$ marginally, so that the cost-reducing R&D at $\mathbf{x}^{\mathrm{N}}$ is socially excessive (insufficient) at the margin in terms of the first-best welfare function $W^{\mathrm{F}}(\mathbf{x})$. Similarly, one can gauge the marginal social excessiveness/insufficiency of the cost-reducing R&D at $\mathbf{x}^{\mathrm{C}}$ by evaluating the sign of $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x})$ at $\mathbf{x}^{\mathrm{C}}$.

It is easy to verify that the crucial derivative to be evaluated, $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x})$, consists of two terms, which may be called the *commitment effect*,

$$(16)\quad \gamma^{\mathrm{F}}(\mathbf{x}) := -\, q_i^{\mathrm{F}}(\mathbf{x})\frac{\partial}{\partial x_i}c(x_i; \mathbf{x}_{-i}) - 1$$

and the *spillover effect*,

(17) $\sigma^{\mathrm{F}}(\mathbf{x}) := -\sum_{j\neq i} q_j^{\mathrm{F}}(\mathbf{x})\dfrac{\partial}{\partial x_i}c(x_j;\mathbf{x}_{-j}).$

The spillover effect $\sigma^{\mathrm{F}}(\mathbf{x})$ is easy to interpret. A marginal increase in x_i reduces the marginal cost $c(x_j;\mathbf{x}_{-j})$ of firm j $(i \neq j)$ through the spillover of cost-reducing benefits, which will increase the first-best social welfare in proportion to j's output $q_j^F(\mathbf{x})$. Summing up these effects over all $j \neq i$, one immediately obtains $\sigma^{\mathrm{F}}(\mathbf{x})$.

To motivate the commitment effect, consider the problem of social-welfare maximization without strategic commitment and R&D spillovers. One is then working with the following alternative optimization problem:

$$(18) \quad \max_{(\mathbf{q},\mathbf{x})>\mathbf{0}} \left\{ \int_0^Q f(Z)dZ - \sum_{j=1}^{n} [c(x_j;\mathbf{x}_{-j})q_j + x_j] \right\}$$

where $Q := \Sigma_{j=1}^n q_j$. The first-order conditions for this maximization problem are

$$(19) \quad f\left(\sum_{j=1}^{n} q_j^*\right) = c(x_i^*;\mathbf{x}_{-i}^*) \quad (i = 1, 2, \ldots, n)$$

$$(20) \quad -q_i^*\frac{\partial}{\partial x_i}c(x_i^*;\mathbf{x}_{-i}^*) - 1 = 0 \quad (i = 1, 2, \ldots, n)$$

where $\mathbf{q}^* = [q_1^*, q_2^*, \ldots, q_n^*]$ and $\mathbf{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]$ represent the solution to (18). Comparing (16) and (20), one can maintain that $\gamma^{\mathrm{F}}(\mathbf{x})$ captures the portion of the effect of R&D that can be nonzero only in the presence of strategic commitment to R&D (hence the use of the term "commitment effect").

It is shown in Appendix C that both $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) > 0$ and $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}}) > 0$ hold in the presence of sufficiently large R&D spillovers.

Theorem 1: *Suppose that Assumptions* 1-3 *hold. Then,*

(i) the noncooperative equilibrium R&D level is socially insufficient at the margin in terms of the first-best welfare criterion if R&D spillovers are sufficiently large; and
(ii) the cooperative equilibrium R&D level is socially insufficient at the margin in terms of the first-best welfare criterion irrespective of whether R&D spillovers are large or small.

According to Theorem 1, both x_i^{N} and x_i^{C} are socially insufficient at the margin in terms of the first-best welfare criterion. How, then, does x_i^{N} fare vis-à-vis x_i^{C}? To settle this, note that $\mathbf{x}^{\mathrm{C}}$ is the maximizer of $\Sigma_{j=1}^n \Pi_j(\mathbf{x})$ and compute $(\partial/\partial x_i)\Sigma_{j=1}^n \Pi_j(\mathbf{x}^{\mathrm{N}})$. If

Assumption 3^* is used instead of Assumption 3, one can verify that $(\partial/\partial x_i)\Sigma_{j=1}^n \Pi_j(\mathbf{x}^{\mathrm{N}}) > 0$ holds in the presence of sufficiently large R&D spillovers, leading to the following theorem.

Theorem 2: *Suppose that Assumptions 1, 2, and* 3^* *hold. Then, a marginal increase in R&D level at the noncooperative equilibrium increases firms' joint profits marginally in the presence of sufficiently large R&D spillovers.*

(See Appendix D for the proof.)

To highlight the role played by large R&D spillovers in the model, I will briefly examine the other extreme case of no R&D spillover. By definition, then, $(\partial/\partial x_j)c(x_i;\mathbf{x}_{-i}) = 0$ $(i \neq j;\ i,j = 1,2,\ldots,n)$ and $\theta(\mathbf{x}) < 0$, so that the sign of $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}})$ is ambiguous in general. However, I prove in Appendix E that $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) < 0$ holds if Assumption 3^* and $n \geq 3$ are satisfied. On the other hand, I have already shown that $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}}) > 0$ holds regardless of the extent of R&D spillovers. It is shown in Appendix E that $(\partial/\partial x_i)\Sigma_{j=1}^n \Pi_j(\mathbf{x}^{\mathrm{N}}) < 0$ holds in the case of no R&D spillover.

Theorem 3: *Suppose that Assumptions 1-3 hold and there is no R&D spillover. Then, the following three statements hold:*

(i) If assumption 3^* *is satisfied in place of Assumption 3 the noncooperative equilibrium R&D level is socially excessive at the margin in terms of the first-best welfare criterion when* $n \geq 3$.
(ii) The cooperative equilibrium R&D level is socially insufficient at the margin in terms of the first-best welfare criterion.
(iii) A marginal decrease in R&D level at the noncooperative equilibrium marginally increases firms' joint profits.

5 Second-Best Welfare Analysis

Despite its obvious intuitive appeal and the preceding utilization by d'Aspremont and Jacqumin [6] and many others, the relevance of the first-best market surplus function $W^{\mathrm{F}}(\mathbf{x})$ as the welfare criterion must be called into question. Indeed, the assumed enforceability of the marginal-cost principle, which underlies the use of $W^{\mathrm{F}}(\mathbf{x})$ as a welfare criterion, is likely to run into problems of implementation for any government in a democracy. For this reason, it makes sense to invoke the *second-best welfare (market surplus) function* instead, which is defined by

(21) $$W^{\mathrm{S}}(\mathbf{x}) := W(\mathbf{x}, \mathbf{q}^{\mathrm{N}}(\mathbf{x})).$$

Unlike $W^{\mathrm{F}}(\mathbf{x})$, $W^{\mathrm{S}}(\mathbf{x})$ presupposes that the oligopolistic competition in the second-stage quantity game lies beyond the regulatory power of the nonomnipotent government. In other words, $W^{\mathrm{S}}(\mathbf{x})$ is one way to evaluate the second-best performance of the oligopolistic industry.

If $W^{\mathrm{F}}(\mathbf{x})$ is replaced by $W^{\mathrm{S}}(\mathbf{x})$, the crucial derivative $(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x})$ should be decomposed into three, rather than two, terms. In addition to the commitment effect,

$$(22) \quad \gamma^{\mathrm{S}}(\mathbf{x}) := -\, q_i^{\mathrm{N}}(\mathbf{x})\frac{\partial}{\partial x_i}c(x_i;\mathbf{x}_{-i}) - 1$$

and the spillover effect,

$$(23) \quad \sigma^{\mathrm{S}}(\mathbf{x}) := -\sum_{j\neq i} q_j^{\mathrm{N}}(\mathbf{x})\frac{\partial}{\partial x_i}c(x_j;\mathbf{x}_{-j})$$

there is an additional term, which may be called the *distortion effect*:

$$(24) \quad \delta^{\mathrm{S}}(\mathbf{x}) := \sum_{j=1}^{n}[f(Q^{\mathrm{N}}(\mathbf{x})) - c(x_j;\mathbf{x}_{-j})]\frac{\partial}{\partial x_i}q_j^{\mathrm{N}}(\mathbf{x}).$$

Clearly, the distortion effect is nothing other than the sum of marginal distortions generated by a marginal change in x_i.

If $\gamma^{\mathrm{S}}(\mathbf{x})$, $\sigma^{\mathrm{S}}(\mathbf{x})$, and $\delta^{\mathrm{S}}(\mathbf{x})$ are evaluated at $\mathbf{x}^{\mathrm{N}}$, one obtains

$$(25) \quad \gamma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = -\,(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}))\cdot q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\theta(\mathbf{x}^{\mathrm{N}})$$

$$(26) \quad \sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = -\,(n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\cdot\frac{\partial}{\partial x_j}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}}) \quad (i\neq j)$$

$$(27) \quad \delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = [f(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})) - c(x_i^{N};\mathbf{x}_{-i}^{\mathrm{N}})]\cdot\psi(\mathbf{x}^{\mathrm{N}}),$$

all of which are positive under Assumptions 1-3 in the presence of sufficiently large R&D spillovers. Therefore, $(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = \gamma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) + \sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) + \delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) > 0$.

On the other hand, evaluating $\gamma^{\mathrm{S}}(\mathbf{x})$, $\sigma^{\mathrm{S}}(\mathbf{x})$, and $\delta^{\mathrm{S}}(\mathbf{x})$ at $\mathbf{x}^{\mathrm{C}}$, one obtains

$$(28) \quad \gamma^{S}(\mathbf{x}^{\mathrm{C}}) = -\,(n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\cdot f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}))\psi(\mathbf{x}^{\mathrm{C}}) + (n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\cdot\frac{\partial}{\partial x_j}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}})$$

$$(29) \quad \sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) = -(n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}) \cdot \frac{\partial}{\partial x_j} c(x_i^{\mathrm{C}}; \mathbf{x}_{-i}^{\mathrm{C}}) \quad (i \neq j)$$

$$(30) \quad \delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) = [f(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})) - c(x_i^{\mathrm{C}}; \mathbf{x}_{-i}^{\mathrm{C}})] \cdot \psi(\mathbf{x}^{\mathrm{C}}).$$

Since $\gamma^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}})$ cancels the second term of $\sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}})$, one obtains

$$(31) \quad \frac{\partial}{\partial x_i} W^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) = \{-(n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})) + [f(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})) - c(x_i^{\mathrm{C}}; \mathbf{x}_{-i}^{\mathrm{C}})]\} \cdot \psi(\mathbf{x}^{\mathrm{C}})$$

which is unambiguously positive under Assumptions 1-3, irrespective of whether R&D spillovers are large or small.

I have thus established the following theorem, which shows in effect that, when the R&D spillovers are sufficiently large, the social insufficiency of the amount of cooperative as well as noncooperative equilibrium R&D at the margin remains intact even if the first-best welfare criterion is replaced by the more realistic second-best welfare criterion.

Theorem 4: *Suppose that Assumptions 1-3 hold. Then,*

(i) the noncooperative equilibrium R&D level is socially insufficient at the margin in terms of the second-best welfare criterion if the R&D spillovers are sufficiently large; and

(ii) the cooperative equilibrium R&D level is socially insufficient at the margin in terms of the second-best welfare criterion irrespective of whether R&D spillovers are large or small.

By contrast, to crystallize the role played by the assumption of sufficiently large spillovers, examine the opposite polar case of no spillover. In the case of noncooperative equilibrium, $\gamma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) < 0$, $\sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = 0$, and $\delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) > 0$ from (25), (26), and (27) in the absence of R&D spillover, so that $(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}})$ consists of two terms with opposite signs. Furthermore,

$$(32) \quad \frac{\partial}{\partial x_i} W^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) = \delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}}) \cdot \left\{ 1 - (n-1) \left[\frac{-\dfrac{\partial}{\partial x_i} q_j^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}{\dfrac{\partial}{\partial x_i} Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})} \right] \right\}$$

holds, where use is made of (2) for $\mathbf{x} = \mathbf{x}^{\mathrm{N}}$. As Okuno-Fujiwara and Suzumura [13] have observed, the expression within the square brackets in (32) is independent of the number of firms n, so that (32) implies that the crucial term $(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x}^{\mathrm{N}})$ becomes negative as n increases.[8]

On the other hand, in the case of cooperative equilibrium, $\gamma^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) > 0$, $\sigma^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) = 0$, and $\delta^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}}) > 0$ from (28), (29), and (30) in the absence of R&D spillover, so that $(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x}^{\mathrm{C}})$ is unambiguously positive.

In summary, one can assert the following:

Theorem 5: *Suppose that Assumptions 1-3 hold and there exists no R&D spillover. Then,*

(i) if there is a sufficiently large number of firms in the industry, then the noncooperative equilibrium R&D level is socially excessive at the margin in terms of the second-best welfare criterion; and
(ii) the cooperative equilibrium R&D level is socially insufficient at the margin in terms of the second-best welfare criterion.

6 Concluding Remarks

The main conclusions of my analysis are succinctly summarized in Table 1 (large spillover case) and Table 2 (no spillover case), where a "+," "0," or "−" in any cell signifies that the partial derivative in the corresponding row is, respectively, "positive," "zero," or "negative" when it is evaluated at the R&D profile in the corresponding column. When the validity of a particular sign requires more than just the standard Assumptions 1-3, that fact is indicated in the table footnote. These marginal conclusions on the performance of $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ can be converted into global conclusions; that is, the ranking among x_i^{N}, x_i^{C}, x_i^{F}, and x_i^{S}, on the one hand, and the ranking among $Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})$, $Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})$, $Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{S}})$, and $Q^{\mathrm{F}}(\mathbf{x}^{\mathrm{F}})$, on the other, can be obtained if the relevant welfare or joint profit function is guaranteed to be single-peaked.[9]

Comparison between Table 1 and Table 2 yields several policy-relevant conclusions. (a) From comparison of the first and second rows in each table, it appears that the qualitative conclusions remain the same even if one uses the second-best welfare function instead of the first-best function. Note, however, that the policy relevance of these conclusions is substantially increased by the use of the second-best welfare criterion. (b) Comparing the first and second columns in Table 1, one sees that, in the presence of large

[8] How large should the number of firms be for the validity of this assertion? It was shown by Okuno-Fujiwara and Suzumura [13] that the critical number of firms remains small for a wide class of models. Indeed, if the inverse demand function is concave (i.e., under Assumption 3*), it can be shown that the watershed number of firms is exactly two, so that my conclusion applies to *all* oligopoly models with concave inverse demand functions. If the inverse demand function is constantly elastic, the value of the watershed number of firms changes as the value of elasticity changes. However, for a wide range of the values of elasticity, the value of the watershed number of firms remains consistently less than 3.

[9] By the single-peaked nature of a function $f(\mathbf{x})$, where $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, I mean here that $f(x_i; \mathbf{x}_{-i})$ has a unique local (hence global) maximum with respect to x_i for any specified value of $\mathbf{x}_{-i} = [x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$ $(i = 1, 2, \ldots, n)$. Note that, in the duopoly example of d'Aspremont and Jacquemin [6], the first-best welfare function as well as the joint profit function is assured to be concave (hence single-peaked).

spillovers, not only the noncooperative equilibrium R&D level but also the cooperative equilibrium R&D level is socially insufficient at the margin, so that the technology policy which facilitates further investment in R&D is marginally welfare-improving irrespective of whether firms cooperate or not. (c) Comparing the first and second columns in Table 2, one sees that, in the absence of spillovers, while the cooperative equilibrium R&D level remains socially too small at the margin, the noncooperative equilibrium R&D level turns out to be socially excessive at the margin. Therefore, the marginally welfare-improving technology policy should facilitate (resp. restrict) investment in R&D if firms cooperate (resp. do not cooperate). (d) From a comparison of the third row in Table 1 with the corresponding row in Table 2, it appears that an increase in R&D at the noncooperative equilibrium marginally increases (resp. decreases) joint profits if spillover effects are large (resp. small).

Table 1—Large Spillover Case

	R&D profile	
Derivative	$\mathbf{x}^{\mathrm{N}}$	$\mathbf{x}^{\mathrm{C}}$
$(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x})$	$+$	$+$
$(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x})$	$+$	$+$
$(\partial/\partial x_i)\Sigma_{j=1}^{n}\Pi_j(\mathbf{x})$	$+^{a}$	0

Notes: The vectors $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ denote, respectively, the noncooperative equilibrium R&D profile and the cooperative equilibrium R&D profile. The symbol "+," "0," or "−" denotes, respectively, that the partial derivative in the corresponding row is "positive," "zero" or "negative" at the R&D profile in the corresponding column.

[a]This requires Assumption 3*.

Table 2—No-Spillover Case

	R&D profile	
Derivative	$\mathbf{x}^{\mathrm{N}}$	$\mathbf{x}^{\mathrm{C}}$
$(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x})$	$-^{a}$	$+$
$(\partial/\partial x_i)W^{\mathrm{S}}(\mathbf{x})$	$-^{b}$	$+$
$(\partial/\partial x_i)\Sigma_{j=1}^{n}\Pi_j(\mathbf{x})$	$-$	0

Notes: The vectors $\mathbf{x}^{\mathrm{N}}$ and $\mathbf{x}^{\mathrm{C}}$ denote, respectively, the noncooperative equilibrium R&D profile and the cooperative equilibrium R&D profile. The symbol "+," "0," or "−" denotes, respectively, that the partial derivative in the corresponding row is "positive," "zero," or "negative" at the R&D profile in the corresponding column.

[a]This requires Assumption 3* and $n \geq 3$.

[b]This requires that the number of firms is sufficiently large.

Several concluding remarks are in order concerning directions for future analysis of cooperative R&D. First, as Jeffrey Bernstein and Ishaq Nadiri [1] and Richard Levin and Peter Reiss [12] have emphasized, R&D undertaken by firms outside the industry (e.g., by material suppliers and equipment suppliers) may exert an influence on a firm's marginal cost. Such interindustry spillover effects should be taken into consideration along with the intraindustry spillover effects in order to obtain a well-balanced evaluation of the effects of R&D spillovers.

Second, one of the alleged functions of cooperative R&D is precisely to generate synergic effects by pooling various complementary resources, such as research information and experience, teams of researchers, and technological know-how. From this viewpoint, my formulation of R&D spillovers in terms of the average variable cost function, which remains the same irrespective of whether firms cooperate or not, may be seriously inadequate. For fuller analysis, one presumably should endogenize the spillover function by making the cost-reducing technology dependent on the extent to which firms pool their complementary R&D resources.

Third, the potential benefits of cooperative R&D are often related to an acceleration in the speed of invention and innovation by risk-spreading and risk-pooling. In discussing cost-reducing R&D, therefore, one should introduce an element of uncertainty into the analysis.

Fourth, my concentration on the second-stage quantity game and the assumption of symmetric equilibria are likely to restrict the generality of the conclusions. In particular, it may well be worthwhile to select a subgroup of cooperating firms and let this subgroup conduct R&D exclusively—subject to the cost-sharing agreements among all member firms—since the group as a whole can thereby conserve on the fixed cost of equipment installation and avoid any unnecessary duplication of R&D efforts.

The policy implications of my results should be interpreted carefully in the light of these qualifying observations. Nevertheless, I hope that the results, partial though they are, and the method of analysis that has been developed en route, will contribute to a better understanding of the complex issue of cooperative R&D.

Appendix A: Derivation of the Formulas for $\omega(\mathbf{x})$ and $\theta(\mathbf{x})$

To derive formulas (5) and (6), differentiate the first-order condition $(\partial/\partial q_i)\Pi_i(\mathbf{q}^{\mathrm{N}}(\mathbf{x}); x_i, \mathbf{x}_{-i}) = 0$ characterizing $\mathbf{q}^{\mathrm{N}}(\mathbf{x})$ with respect to x_i and x_h ($h \neq i$) and use symmetry to obtain the following simultaneous equations for $\omega(\mathbf{x})$ and $\theta(\mathbf{x})$:

$$\text{(A1)} \quad \alpha(\mathbf{x})\omega(\mathbf{x}) + (n-1)\beta(\mathbf{x})\theta(\mathbf{x}) = \frac{\partial}{\partial x_i} c(x_i, \mathbf{x}_{-i})$$

$$\text{(A2)} \quad \beta(\mathbf{x})\omega(\mathbf{x}) + [\alpha(\mathbf{x}) + (n-2)\beta(\mathbf{x})]\theta(\mathbf{x}) = \frac{\partial}{\partial x_j} c(x_i, \mathbf{x}_{-i})$$

where $i \neq j$. Solving (A1) and (A2) for $\omega(\mathbf{x})$ and rearranging terms appropriately, one obtains (5) in the main text; solving for $\theta(\mathbf{x})$ and rearranging, one obtains (6).

Appendix B: Proof of Lemma 1

Note that (2) and (13) yield $f(nq_i^{\mathrm{F}}(\mathbf{x})) - f(nq_i^{\mathrm{N}}(\mathbf{x})) = f'(nq_i^{\mathrm{N}}(\mathbf{x}))q_i^{\mathrm{N}}(\mathbf{x})$ for any symmetric R&D profile $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, so that there exists a positive number $\zeta(\mathbf{x}), 0 < \zeta(\mathbf{x}) < 1$, such that

$$\text{(B1)} \quad q_i^{\mathrm{F}}(\mathbf{x}) - q_i^{\mathrm{N}}(\mathbf{x}) = \frac{q_i^{\mathrm{N}}(\mathbf{x})f'(nq_i^{\mathrm{N}}(\mathbf{x}))}{nf'(n\{\zeta(\mathbf{x})q_i^{\mathrm{F}}(\mathbf{x}) + [1-\zeta(\mathbf{x})]q_i^{\mathrm{N}}(\mathbf{x})\})}$$

holds by virtue of the mean-value theorem. Lemma 1 is an immediate consequence of (B1) and Assumption 1.

Appendix C: Proof of Theorem 1

Taking the first-order condition (2) characterizing $q_i^{\mathrm{N}}(\mathbf{x})$ and the symmetry of $\mathbf{x}^{\mathrm{N}}$ into consideration, one can rewrite the first-order condition (4) characterizing $\mathbf{x}^{\mathrm{N}}$ as follows:

$$\text{(C1)} \quad -q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\frac{\partial}{\partial x_i}c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}}) - 1 = -(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}))q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\theta(\mathbf{x}^{\mathrm{N}}) \quad (i = 1, 2, \ldots, n).$$

Comparing (16) at $\mathbf{x} = \mathbf{x}^{\mathrm{N}}$ with (C1), one may obtain

$$\text{(C2)} \quad \gamma^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) = -(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}))q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\theta(\mathbf{x}^{\mathrm{N}}) + [q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}) - q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}})]\frac{\partial}{\partial x_i}c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}})$$

which is positive in the presence of sufficiently large R&D spillovers, where use is made of Lemma 1 and Assumptions 1 and 2. On the other hand, at $\mathbf{x} = \mathbf{x}^{\mathrm{N}}$, (17) can be reduced to

$$\text{(C3)} \quad \sigma^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) = -(n-1)q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) \cdot \frac{\partial}{\partial x_j}c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}}) \quad (i \neq j)$$

which is nonnegative by virtue of Assumption 2. It then follows that $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) = \gamma^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) + \sigma^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) > 0$.

Turning to the cooperative R&D equilibrium $\mathbf{x}^{\mathrm{C}}$, one may invoke (10) to obtain

$$\text{(C4)}\quad \gamma^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}}) = -(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}))\cdot q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\psi(\mathbf{x}^{\mathrm{C}}) + [q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}) - q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}})]\cdot\frac{\partial}{\partial x_i}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}})$$
$$+(n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\cdot\frac{\partial}{\partial x_j}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}})\quad (i\neq j).$$

Coupling (C4) with $\sigma^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}})$, which can be reduced to

$$\text{(C5)}\quad \sigma^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}}) = -(n-1)q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}})\cdot\frac{\partial}{\partial x_j}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}})\quad (i\neq j)$$

one obtains

$$\text{(C6)}\quad \frac{\partial}{\partial x_i}W^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}}) = -(n-1)f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}))q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}})\psi(\mathbf{x}^{\mathrm{C}})$$
$$+[q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{C}}) - q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{C}})]\cdot\left[\frac{\partial}{\partial x_i}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}}) + (n-1)\frac{\partial}{\partial x_j}c(x_i^{\mathrm{C}};\mathbf{x}_{-i}^{\mathrm{C}})\right]$$

where $i\neq j$, which is positive by virtue of $\psi(\mathbf{x}^{\mathrm{C}}) > 0$, Lemma 1, Assumption 1 and Assumption 2, irrespective of whether R&D spillovers are large or small.

Appendix D: Proof of Theorem 2

Invoking (C1), one can compute that

$$\text{(D1)}\quad \frac{\partial}{\partial x_i}\sum_{j=1}^{n}\Pi_j(\mathbf{x}^{\mathrm{N}}) = (n-1)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\cdot\left[f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}))\psi(\mathbf{x}^{\mathrm{N}}) - \frac{\partial}{\partial x_j}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}})\right]$$

holds, where $i\neq j$. Having recourse to (5) and (6), one is assured that

$$\text{(D2)}\quad \operatorname{sgn}\frac{\partial}{\partial x_i}\sum_{j=1}^{n}\Pi_j(\mathbf{x}^{\mathrm{N}})$$
$$=\operatorname{sgn}\left\{[-(n-3)\alpha(\mathbf{x}^{\mathrm{N}}) + 2(n-1)\beta(\mathbf{x}^{\mathrm{N}})]\cdot\frac{\partial}{\partial x_j}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}}) - \alpha(\mathbf{x}^{\mathrm{N}})\frac{\partial}{\partial x_i}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}})\right\}$$

so that $(\partial/\partial x_i)\Sigma_{j=1}^{n}\Pi_j(\mathbf{x}^{\mathrm{N}}) >$ (resp. $<$) 0 holds if and only if

$$\text{(D3)} \quad \Omega_i(\mathbf{x}^{\mathrm{N}}) := \alpha(\mathbf{x}^{\mathrm{N}}) + [(n-3)\alpha(\mathbf{x}^{\mathrm{N}}) - 2(n-1)\beta(\mathbf{x}^{\mathrm{N}})] \cdot \frac{\dfrac{\partial}{\partial x_j} c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}})}{\dfrac{\partial}{\partial x_i} c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}})}$$

$$> \text{(resp. } < \text{) } 0$$

holds, where $i \neq j$. Under the assumption of sufficiently large R&D spillovers,

$$\text{(D4)} \quad \frac{\dfrac{\partial}{\partial x_j} c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}})}{\dfrac{\partial}{\partial x_i} c(x_i^{\mathrm{N}}; \mathbf{x}_{-i}^{\mathrm{N}})} > \frac{\beta(\mathbf{x}^{\mathrm{N}})}{\alpha(\mathbf{x}^{\mathrm{N}})}$$

obtains, where $i \neq j$. Note that an inequality $(n-3)\alpha(\mathbf{x}) - 2(n-1)\beta(\mathbf{x}) > 0$ is valid for any symmetric R&D profile $\mathbf{x} = [x_1, x_2, \dots, x_n]$. To verify this fact, define $\kappa^{\mathrm{N}}(\mathbf{x}) := Q^{\mathrm{N}}(\mathbf{x}) f''(Q^{\mathrm{N}}(\mathbf{x}))/f'(Q^{\mathbf{N}}(\mathbf{x}))$, which makes it possible to rewrite (8) and (9) as

$$\text{(D5)} \quad \alpha(\mathbf{x}) = f'(Q^{\mathrm{N}}(\mathbf{x})) \left[2 + \frac{\kappa^{\mathrm{N}}(\mathbf{x})}{n} \right]$$

$$\text{(D6)} \quad \beta(\mathbf{x}) = f'(Q^{\mathrm{N}}(\mathbf{x})) \left[1 + \frac{\kappa^{\mathrm{N}}(\mathbf{x})}{n} \right].$$

It follows that

$$\text{(D7)} \quad (n-3)\alpha(\mathbf{x}) - 2(n-1)\beta(\mathbf{x}) = \frac{\alpha(\mathbf{x})}{2n + \kappa^{\mathrm{N}}(\mathbf{x})} \cdot \{ -[\kappa^{\mathrm{N}}(\mathbf{x}) + 4]n - \kappa^{\mathrm{N}}(\mathbf{x}) \} > 0$$

whose sign is due to $\alpha(\mathbf{x}) < 0$ and $\kappa^{\mathrm{N}}(\mathbf{x}) \geq 0$, which follows from Assumption 3^*.

Putting (D4), (D5), and (D7) together and having recourse to Assumption 3^*, one obtains

$$\text{(D8)} \quad \Omega_i(\mathbf{x}^{\mathrm{N}}) > \alpha(\mathbf{x}^{\mathrm{N}}) + \frac{\beta(\mathbf{x}^{\mathrm{N}})}{\alpha(\mathbf{x}^{\mathrm{N}})} \cdot [(n-3)\alpha(\mathbf{x}^{\mathrm{N}}) - 2(n-1)\beta(\mathbf{x}^{\mathrm{N}})].$$

Using (D5) and (D6), one can verify that

(D9) $$\alpha(\mathbf{x}^{\mathrm{N}}) + (n-3)\beta(\mathbf{x}^{\mathrm{N}}) - 2(n-1)\frac{[\beta(\mathbf{x}^{\mathrm{N}})]^2}{\alpha(\mathbf{x}^{\mathrm{N}})}$$

$$= -\frac{f'(Q^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}))\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}{2n + \kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})} \cdot \{\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}) + n + 1\} \geq 0.$$

One is thus led to conclude that $(\partial/\partial x_i)\Sigma_{j=1}^{n}\Pi_j(\mathbf{x}^{\mathrm{N}}) > 0$.

Appendix E: Proof of Theorem 3

In the case of no R&D spillover, $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) = \gamma^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}})$, whose sign is indeterminate in general. However, if Assumption 3* is also adopted, one may obtain

(E1) $$\frac{\partial}{\partial x_i}W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) = \frac{\partial}{\partial x_i}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}})\cdot\left[\frac{n+\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}{n+1+\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}\left(\frac{n-1}{n}\right)q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}) + q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}}) - q_i^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}})\right]$$

$$\leq \frac{\partial}{\partial x_i}c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}})q_i^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})\cdot\left[\frac{n+\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}{n+1+\kappa^{\mathrm{N}}(\mathbf{x}^{\mathrm{N}})}\left(\frac{n-1}{n}\right) - \frac{1}{n}\right]$$

where use is made of (B1), (C2), and (D5). Since the expression within the brackets is positive when $n \geq 3$, it follows from (E1) that $(\partial/\partial x_i)W^{\mathrm{F}}(\mathbf{x}^{\mathrm{N}}) < 0$ when $n \geq 3$. This proves part (i) of Theorem 3. Part (ii) of Theorem 3 needs no further proof, whereas part (iii) follows from (D2) by setting $(\partial/\partial x_j)c(x_i^{\mathrm{N}};\mathbf{x}_{-i}^{\mathrm{N}}) = 0$ $(i \neq j)$.

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Chapter 20
Symmetric Cournot Oligopoly and Economic Welfare: A Synthesis*

1 Introduction

Contrary to "a widespread belief that increasing competition will increase welfare (Stiglitz [18, p.184])," recent studies have revealed that competition may sometimes be socially "excessive". In particular, Mankiw and Winston [11] and Suzumura and Kiyono [21] have shown that socially excessive firm entry may occur in unregulated oligopolitic markets.[1] This happens because entry is occasionally more desirable to entrants than to the society, as new entry creates an incentive for incumbent firms to reduce their outputs. This result was established in a partial equilibrium framework for symmetric Cournot oligopoly.[2]

The purpose of the present chapter is to add a new dimension to this literature by looking into strategic aspects of cost-reducing R&D investment that may create incentives towards socially excessive investment. We consider an oligopolistic competition played in three stages. In the first stage, firms simultaneously decide whether or not to enter the market. In the second stage, firms make an irrevocable commitment to R&D investment, which affects production cost in the third stage where firms compete in quantities. Since R&D investment is a fixed commitment, firms' investment decisions are affected by strategic considerations.

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[1]See, also, Perry [13] and von Weizsäcker [22; 23].

[2]Konishi, Okuno-Fujiwara and Suzumura [12] have generalized this result with general equilibrium interactions, whereas Lahiri and Ono [10] have shown that this paradoxical result essentially survives with heterogeneous firms by proving that eliminating minor firms increases social welfare through the improvement of average production efficiency which overwhelms the undesirable effect of a change in market structure.

In the first half of the chapter, we analyze the second and the third stage game, with a number of firms fixed. Brander and Spencer [2] analyzed this game in a Cournot *duopoly* setting and showed that the level of investment is higher at the strategic equilibrium than that at the non-strategic equilibrium. They also showed that investment is sometimes socially excessive as it exceeds the level that maximizes second-best social welfare.[3] In this chapter, we identify the causes of this excessive investment and generalize their results in several respects. First, we shall focus on the excessive investment *at the margin* and decompose the welfare effect of an additional investment into the *commitment effect* and the *distortion effect*. Second, by invoking the concept of *strategic substitutes* due to Bulow et al. [4], we shall provide a clear interpretation of the excessive investment result.[4] Third, we shall establish an increase in the number of firms is likely to cause a socially excessive investment.

In the latter half of this chapter, we shall consider the fully-fledged three stage game. Under a set of rather weak assumptions, we shall show that the excessive entry *à la* Mankiw-Whinston and Suzumura-Kiyono is extended even with the existence of strategic investment.

The structure of this chapter is as follows. In Section 2, our model is formulated. Section 3 considers the second and the third stage games with a fixed number of firms, and decomposes the welfare effect of a change in R&D investment into the commitment effect and the distortion effect. In this section, we shall show that, under fairly mild conditions, the strategic R&D investment is socially excessive at the margin if the actual number of firms exceeds a certain critical number. Section 4 extends our analysis to the full three stage model and a marginal reduction of the number of firms from the free entry level is shown to improve social welfare under a slightly more restrictive set of assumptions. Proofs are gathered in Section 5. Section 6 concludes the chapter.

2 Distortion Effect and Commitment Effect: The Homogeneous Product Case

2.1 Consider an industry where operating firms produce a homogeneous product. Firms engage in three-stage competition. There are infinite number of potential entrants. In the first stage, firms decide whether or not to enter the market in a predetermined sequential order. In the second stage each firm makes a strategic commitment to cost-reducing R&D, whereas firms compete in terms of quantities in the third stage.

In this chapter, we will utilize three different equilibrium concepts. Given any arbitrary number of firms and R&D investment profile, the *third stage Cournot-Nash equi-*

[3] Note, however, they assumed the Cournot competition with product differentiation, while in this chapter we assume the Cournot competition with homogeneous products. See also d'Aspremont and Jacquemin [5] and Suzumura [20] which analyzed the role of R&D spillovers and cooperative research associations in the framework of two-stage oligopoly models.

[4] Brander and Spencer [2, p.277] assumed, in effect, that products are strategic substitutes. See, also, Besley and Suzumura [1], Eaton and Grossman [7] and Fundenberg and Tiroie [8] for other contexts where this assumption plays an essential role.

librium is defined. Given an arbitrary number of firms, the *second stage subgame perfect equilibrium* is defined when the relevant game is defined by the second and the third stages of the entire game. Finally, the *first stage free entry equilibrium* is defined as a subgame perfect equilibrium of the entire game. The focus of our analysis is the welfare performances of the second stage symmetric subgame perfect equilibrium and that of the first stage free entry equilibrium.

2.2 The inverse demand function for the product is $p = f(Q)$, where p is the price and Q is the industry output. The cost-reducing R&D and the output level of firm i is denoted by x_i and q_i, respectively, and the variable cost function of firm i is represented by $c(x_i)q_i$, where the function $c(\cdot)$ is assumed to be identical for all firms.

For each specified number of firms $n \geq 2$ and each specified profile of R&D commitments $\mathbf{x} = (x_1, x_2, \ldots, x_n) > 0$, the third stage payoff function of firm i is given by

$$\pi^i(\mathbf{q}; \mathbf{x}; n) := \{f(Q) - c(x_i)\}q_i - x_i, \tag{2.1}$$

where $\mathbf{q} = (q_1, q_2, \ldots, q_n)$ and $Q = \sum_{j=1}^{n} q_j$. For notational simplicity, we assume that the R&D level x_i is measured by the expenditure for equipment installations. Let $\mathbf{q}^N(\mathbf{x}; n)$ denote the *third stage Cournot-Nash equilibrium* corresponding to the specified $(\mathbf{x}; n)$.

We assume throughout that $\mathbf{q}^N(\mathbf{x}; n)$ is unique, symmetric and positive if the R&D profile $\mathbf{x}$ is symmetric and positive.[5] We also assume:

A(1): *$f(Q)$ is twice continuously differentiable and satisfies $f'(Q) < 0$ for all $Q \geq 0$ such that $f(Q) > 0$. Furthermore, there exists a constant $\delta_0 > -\infty$ such that*

$$\delta(Q) := \frac{Qf''(Q)}{f'(Q)} \geq \delta_0 \textit{ for all } Q \geq 0 \textit{ with } f(Q) > 0.^6 \tag{2.2}$$

A(2): *$c(x)$ is twice continuously differentiable and satisfies $c(x) > 0$, $c'(x) < 0$ and $c''(x) > 0$ for all $x \geq 0$.*

For any output profile $\mathbf{q} = (q_1, q_2, \ldots, q_n)$, R&D profile $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and the number of firms n, we define

$$\alpha_i(\mathbf{q}; \mathbf{x}; n) := \frac{\partial^2}{\partial q_i^2}\pi^i(\mathbf{q}; \mathbf{x}; n)$$

$$\beta_{ij}(\mathbf{q}; \mathbf{x}; n) := \frac{\partial^2}{\partial q_i \partial q_j}\pi^i(\mathbf{q}; \mathbf{x}; n) \quad (i \neq j;\ i, j = 1, 2, \ldots, n).$$

[5] An n-vector $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ is *symmetric* if $y_i = y_j$ for all $i, j = 1, 2, \ldots, n$, whereas $\mathbf{y}$ is *positive* if $y_i > 0$ for all $i = 1, 2, \ldots, n$.

[6] The elasticity δ of the slope of inverse demand function plays a crucial role in many contexts of oligopolistic interaction. See Besley and Suzumura [1], Seade [15; 16], Suzumura [19], and Suzumura and Kiyono [21], among others.

Note that $\beta_{ij}(\mathbf{q};\mathbf{x};n)$ is the crucial term that determines whether the second stage strategies are *strategic substitutes* ($\beta_{ij}(\mathbf{q};\mathbf{x};n) < 0$) or *strategic complements* ($\beta_{ij}(\mathbf{q};\mathbf{x};n) > 0$).[7] It will be assumed that:

A(3): *The second stage strategies are strategic substitutes so that* $\beta_{ij}(\mathbf{q};\mathbf{x};n) < 0$ *holds for any* $(\mathbf{q};\mathbf{x};n)$ $(i \neq j;\ i,j = 1,2,\ldots,n)$.

Remark 1. A(1) admits the following class of inverse demand functions with constant elasticity δ of $f'(Q)$:

$$f(Q) = \begin{cases} a - bQ^{\gamma} & \text{if } \gamma = \delta + 1 \neq 0 \\ a - b \cdot \log Q & \text{if } \delta = -1, \end{cases} \tag{2.3}$$

where a is a non-negative constant and b is a positive (resp. negative) constant if $\gamma < 0$ (resp. $\gamma > 0$). Note that (2.3) includes a linear demand ($\gamma = 1$) as well as constantly elastic demand ($a = 0$), so that it still accommodates a wide class of "normal" inverse demand functions.

Remark 2. The assumption of strategic substitutability is quite natural to require in our present context, since it is equivalent to assuming the downward sloping reaction functions in the third stage quantity game.

Remark 3. It is easy to verify that

$$\alpha_i(\mathbf{q}^N(\mathbf{x};n);\mathbf{x};n) = 2f'(Q^N(\mathbf{x};n)) + q_i^N(\mathbf{x};n) \cdot f''(Q^N(\mathbf{x};n)) \tag{2.4}$$

$$\beta_{ij}(\mathbf{q}^N(\mathbf{x};n);\mathbf{x};n) = f'(Q^N(\mathbf{x};n)) + q_i^N(\mathbf{x};n) \cdot f''(Q^N(\mathbf{x};n)) \tag{2.5}$$

hold, where $Q^N(\mathbf{x};n) = \sum_{j=1}^{n} q_j^N(\mathbf{x};n)$. If $\mathbf{x}$ is symmetric, α_i and β_{ij} are identical for all i and j. In this case, invoking (2.2), we can rewrite (2.4) and (2.5) into

$$\alpha(\mathbf{x};n) = n^{-1} \cdot f'(Q^N(\mathbf{x};n)) \cdot \{2n + \delta(Q^N(\mathbf{x};n))\} \tag{2.6}$$

$$\beta(\mathbf{x};n) = n^{-1} \cdot f'(Q^N(\mathbf{x};n)) \cdot \{n + \delta(Q^N(\mathbf{x};n))\} \tag{2.7}$$

respectively, where $\alpha(\mathbf{x};n) := \alpha_i(\mathbf{q}^N(\mathbf{x};n);\mathbf{x};n)$ and $\beta(\mathbf{x};n) := \beta_{ij}(\mathbf{q}^N(\mathbf{x};n);\mathbf{x};n)$ for notational simplicity. Therefore, **A**(3) implies that:

$$n + \delta(Q^N(\mathbf{x};n)) > 0 \tag{2.8}$$

[7] For the concept of strategic substitutes and complements, see Bulow, Geanakoplos and Klemperer [4]. See, also, Eaton and Grossman [7] and Fudenberg and Tirole [8].

for any $\mathbf{x}$ and $n \geq 2$, where use is made of $\mathbf{A}(1)$. Note that (2.8) is satisfied for any $n \geq 2$ if and only if

$$(2.8^*) \qquad 2 + \delta(Q^N(\mathbf{x}; n)) > 0$$

holds. Note also that $\mathbf{A}(1)$ and (2.8) guarantee that $\alpha(\mathbf{x}; n) < 0$ holds for any $(\mathbf{x}; n)$.

2.3 Under the assumption of an interior optimum, the third stage Cournot-Nash equilibrium $\mathbf{q}^N(\mathbf{x}; n)$ is characterized by

$$(2.9) \qquad f(Q^N(\mathbf{x}; n)) + q_i^N(\mathbf{x}; n) \cdot f'(Q^N(\mathbf{x}; n)) = c(x_i) \ (i = 1, 2, \ldots, n).$$

The first aim of our analysis is to ascertain how the Cournot-Nash output $q_i(\mathbf{x}; n)$ behaves in response to a change in x_i, x_j $(i \neq j)$ and n. Defining $\omega(\mathbf{x}; n) := (\partial/\partial x_i) q_i^N(\mathbf{x}; n)$ and $\theta(\mathbf{x}; n) := (\partial/\partial x_j) q_i^N(\mathbf{x}; n)$ $(i \neq j)$, straightforward computations assert the following:

Lemma 1. *For each symmetric* $\mathbf{x}$ *and* n,

$$(2.10) \qquad (\partial/\partial n) q_i^N(\mathbf{x}; n) = -\frac{q_i^N(\mathbf{x}; n) \cdot \beta(\mathbf{x}; n)}{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)} < 0$$

$$(2.11) \qquad \omega(\mathbf{x}; n) = \frac{c'(x_i)}{\Delta(\mathbf{x}; n)} \cdot \{\alpha(\mathbf{x}; n) + (n-2)\beta(\mathbf{x}; n)\} > 0$$

$$(2.12) \qquad \theta(\mathbf{x}; n) = -\frac{c'(x_i)}{\Delta(\mathbf{x}; n)} \cdot \beta(\mathbf{x}; n) < 0$$

hold, where

$$(2.13) \qquad \Delta(\mathbf{x}; n) := \{\alpha(\mathbf{x}; n) - \beta(\mathbf{x}; n)\} \cdot \{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)\} > 0.$$ [8]

2.4 We now turn to the second stage game. For each specified n, the first stage pay-off function of firm i is given by

$$(2.14) \qquad \Pi^i(\mathbf{x}; n) = \pi^i(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n).$$

If we denote the Nash equilibrium of the second stage game by $\mathbf{x}^N(n)$, it is clear that $\{\mathbf{x}^N(n), \mathbf{q}^N(\mathbf{x}^N(n); n)\}$ is nothing other than the *second stage subgame perfect equilibrium*

[8] Since $\beta(\mathbf{x}; n) < 0$ and $\alpha(\mathbf{x}; n) < 0$ hold under $\mathbf{A}(1)$ and $\mathbf{A}(3)$, it follows that

$$(1^*) \qquad \alpha(\mathbf{x}; n) + (k-1)\beta(\mathbf{x}; n) < 0 \ (k = 0, 1, \ldots, n)$$

holds. Note that (1^*) is a sufficient condition for the local stability of the myopic adjustment process

$$(2^*) \qquad \dot{q}_i = \sigma \cdot \frac{\partial}{\partial q_i} \pi^i(\mathbf{q}; \mathbf{x}; n) \ (i = 1, 2, \ldots, n)$$

where $\dot{q}_i$ denotes the time derivative of q_i, and $\sigma > 0$ stands for the adjustment coefficient.

among n firms. We assume throughout that $\mathbf{x}^N(n)$ is unique, symmetric and positive for each n.

Assuming an interior optimum, $\mathbf{x}^N(n)$ is characterised by

$$(2.15)\quad \{f(Q^N(\mathbf{x}^N(n);n)) - c(x_i^N(n))\}\cdot(\partial/\partial x_i)q_i^N(\mathbf{x}^N(n);n) + q_i^N(\mathbf{x}^N(n);n) \cdot\{f(Q^N(\mathbf{x}^N(n);n))\cdot(\partial/\partial x_i)Q^N(\mathbf{x}^N(n);n) - c'(x_i^N(n))\} - 1 = 0 \ (i = 1, 2, \ldots, n).$$

Invoking (2.9) for $\mathbf{x} = \mathbf{x}^N(n)$, (2.15) reduces into

$$(2.16)\quad -c'(x_i^N(n))\cdot q_i^N(\mathbf{x}^N(n);n) - 1 = \{f(Q^N(\mathbf{x}^N(n);n)) - c(x_i^N(n))\}\cdot\sum_{j\neq i}(\partial/\partial x_i)q_j^N(\mathbf{x}^N(n);n) \ (i = 1, 2, \ldots, n),$$

which proves to be crucially important in what follows.

2.5 Consider now the profits $\Pi^i(\mathbf{x}^N(n);n)$ earned by firm i at the second stage subgame perfect equilibrium among n firms. According to the classical entry/exit dynamics, the number of firms n will increase (resp. decrease) whenever $\Pi^i(\mathbf{x}^N(n);n) > 0$ (resp. < 0), viz.,

$$(2.17)\quad \dot{n} > 0 \text{ (resp. } < 0) \Leftrightarrow \Pi^i(\mathbf{x}^N(n);n) > 0 \text{ (resp. } < 0),$$

where $\dot{n}$ denotes the time derivative of n.

Let the *equilibrium number of firms* n_e be defined as the stationary point of the dynamic process specified by (2.17):

$$(2.18)\quad \Pi^i(\mathbf{x}^N(n_e);n_e) = 0 \ (i = 1, 2, \ldots, n_e).$$

Then $\{n_e, \mathbf{x}^N(n_e), \mathbf{q}^N(\mathbf{x}^N(n_e);n_e)\}$ constitutes the *first stage free entry equilibrium*.

2.6 To gauge the welfare performance of the industry, we define the *net market surplus function* by

$$(2.19)\quad W(\mathbf{q};\mathbf{x};n) := \int_0^Q f(R)dR - \sum_{j=1}^n\{c(x_j)q_j + x_j\},$$

where $Q = \sum_{j=1}^n q_j$.

If the government can control this industry in its entirety from the viewpoint of social welfare maximization, the best that can be done is to impose the socially first best R&D, $x^F(n)$, and the socially first best output, $q^F(n)$, on each incumbent firm and to choose the *first best number of firms*, n_f. These are defined by

$$(2.20)\quad f(nq^F(n)) - c(x^F(n)) = 0$$

$$- c'(x^F(n)) \cdot q^F(n) - 1 = 0 \tag{2.21}$$

$$n_f := \arg\max_{n \geq 1} W(\mathbf{q}^F(n), \mathbf{x}^F(n); n). \tag{2.22}$$

Realistically speaking, however, such a first best policy is hard to implement, since firms are thereby imposed to produce in deficit. If the government cannot control firms' competitive strategies, however, the best that can still be done may be to choose the *second best number of firms*:

$$n_s := \arg\max_{n \geq 1} W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n). \tag{2.23}$$

That is, let n_s firms freely compete to establish the second-stage subgame perfect equilibrium $\{\mathbf{x}^N(n_s); \mathbf{q}^N(\mathbf{x}^N(n_s); n_s)\}$.

In the short-run, however, the government may not be able to control the number of firms. It may be forced to control the R&D level of each incumbent firms to the second best level, $x^S(n)$, defined by

$$x^S(n) := \arg\max_{x > 0} W(\mathbf{q}^N(\mathbf{x}; n); \mathbf{x}; n). \tag{2.24}$$

Despite its obvious relevance and appeal, such second best policies may still be difficult to implement. Because of uncertainty on the precise nature of the functions involved, it may be prohibitively hard to identify where exactly $x^S(n)$ is located. What is required is a policy prescription which does not presuppose the availability of detailed knowledge on the nature of demand and cost functions involved. This is precisely what we look for in the next sections.

3 Commitment Effect, Distortion Effect and the Number of Firms

3.1 In this section, we assume the number of firms, n, is uncontrollable but R&D investment is under the government's control. Let $W^N(\mathbf{x}; n)$ be the net market surplus with outputs evaluated at the third stage Cournot-Nash equilibrium:

$$W^N(\mathbf{x}; n) := \int_0^{Q^N(\mathbf{x};n)} f(Q)dQ - \sum_{j=1}^{n} \{c(x_j)q_j^N(\mathbf{x}; n) + x_j\}. \tag{3.1}$$

Suppose $(\partial/\partial x_i)W^N(\mathbf{x}(n); n) <$ (resp. $>$) 0. Then a marginal *decrease* (resp. a marginal *increase*) of firms i's investment at the second stage subgame perfect equilibrium *increases* social welfare, so that the investment at the subgame perfect equilibrium is *socially excessive* (resp. *socially insufficient*) *at the margin*.

To understand what determines the crucial term $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n)$, it is useful to decompose it into the *commitment effect* $C_i(\mathbf{x}^N(n);n)$ and the *distortion effect* $D_i(\mathbf{x}^N(n);n)$. To be concrete, the commitment effect is defined by[9]

$$C_i(\mathbf{x}^N(n);n) := -c'(x_i^N(n)) \cdot q_i^N(\mathbf{x}^N(n);n) - 1, \tag{3.2}$$

which, in view of (2.16), can be reduced into

$$C_i(\mathbf{x}^N(n);n) := \mu_i(\mathbf{x}^N(n);n) \cdot \sum_{j\neq i}(\partial/\partial x_i)q_j^N(\mathbf{x}^N(n);n), \tag{3.3}$$

whereas the distortion effect is defined by

$$D_i(\mathbf{x}^N(n);n) := \sum_{j=1}^{n} \mu_j(\mathbf{x}^N(n);n) \cdot (\partial/\partial x_i)q_j^N(\mathbf{x}^N(n);n), \tag{3.4}$$

where $\mu_j(\mathbf{x}^N(n);n) := f(Q^N(\mathbf{x}^N(n);n)) - c(x_j^N(n))$ denotes the *marginal distortion* of firm j, which is independent of firm index j at the symmetric equilibrium. By simply adding $C_i(\mathbf{x}^N(n);n)$ and $D_i(\mathbf{x}^N(n);n)$, we obtain the crucial term $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n)$.

In view of symmetry of $\mathbf{x}$ and (2.11)-(2.13),

$$C_i(\mathbf{x}^N(n);n) = (n-1)\cdot\mu(\mathbf{x}^N(n);n)\cdot\theta(\mathbf{x}^N(n);n) < 0, \tag{3.5}$$

$$D_i(\mathbf{x}^N(n);n) = \mu(\mathbf{x}^N(n);n)\cdot\{\omega(\mathbf{x}^N(n);n) + (n-1)\cdot\theta(\mathbf{x}^N(n);n)\} > 0, \tag{3.6}$$

where $\mu(\mathbf{x}^N(n);n) := \mu_j(\mathbf{x}^N(n);n) > 0$. Therefore $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n)$ consists of two components with opposite signs.

3.2 It may be useful to illustrate our decomposition of the marginal welfare effect with the help of Figure 1. At the original symmetric subgame perfect equilibrium, each firm produces $q_i^* := q_i^N(\mathbf{x}^N(n);n)$ with the marginal cost $c^* := c(\mathbf{x}_i^N(n);n)$, and the industry output is $Q^* := nq_i^*$. If firm i unilaterally increases its investment by a small amount $\epsilon > 0$, its marginal cost is reduced to $c^{**} := c^* - \epsilon \cdot \{-c'(x_i^N(n))\}$. Products being strategic substitutes, this increase in firm i's aggressiveness reduces other firms' output, so that firm i's residual demand curve shifts up. As a result, industry output increases to Q^{**}, and output of firm i increases to q_i^{**}.

[9] In the *absence* of strategic commitment, the problem of social welfare maximization takes the form of maximizing $\int_0^Q f(R)dR - \sum_{j=1}^{n}\{c(x_j)q_j + x_j\}$ with respect to $\{(q_i;x_i)\}_{i=1}^n$. The first-order conditions are then $f(Q) - c(x_i) = 0$ and $-c'(x_i)q_i - 1 = 0$ $(i = 1,2,\ldots,n)$. Note that the latter condition suggests that $C_i(\mathbf{x}^N(n);n)$ becomes non-zero *only by the presence of strategic commitment*, which motivates our terminology.

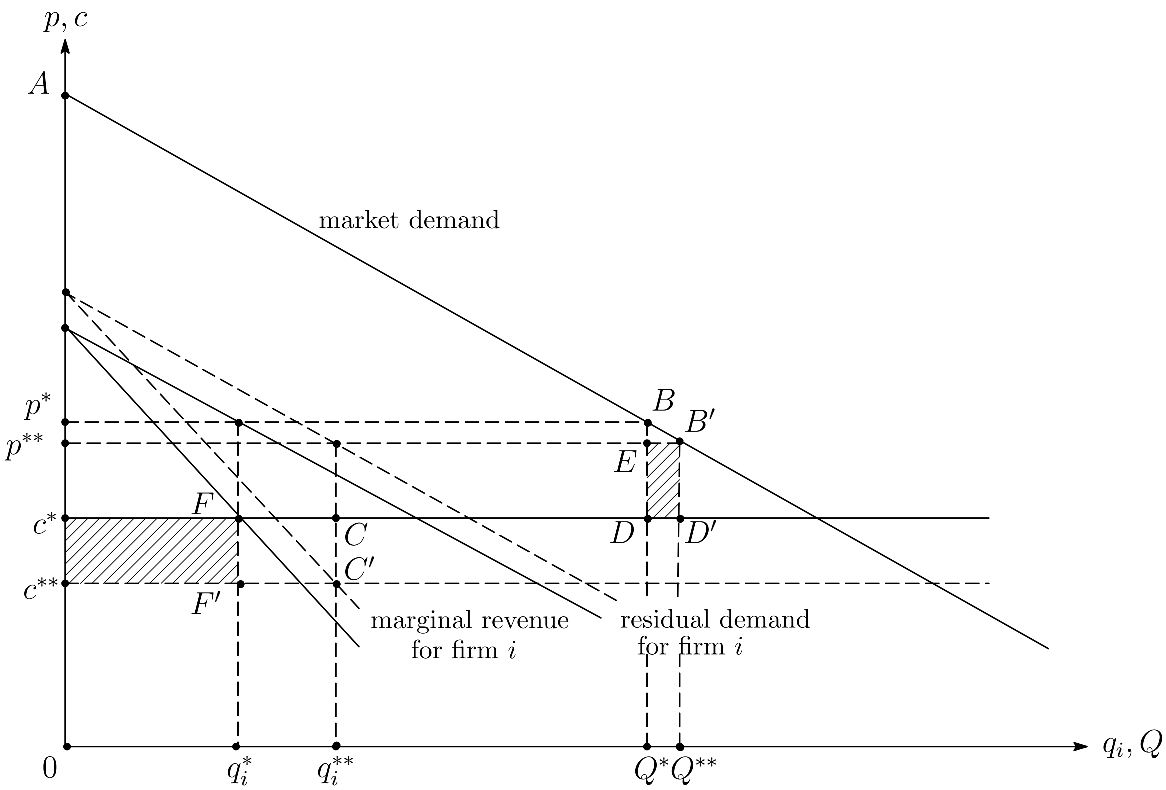

Figure 1. Distortion Effect and Commitment Effect

The net welfare gain from this change consists of

$$\begin{aligned} \text{The change in consumers' surplus} &= \text{area } Ap^{**}B' - \text{area } Ap^*B \\ &= \text{area } Bp^*p^{**}B', \end{aligned}$$

and

$$\begin{aligned} \text{The change in profits} = \; &(\text{area } B'p^{**}c^*D' + \text{ area } Cc^*c{*}{*}C' - \epsilon) \\ &- \text{area } Bp^*c^*D, \end{aligned}$$

which, after neglecting terms of the second order infinitesimal, boils down to (area $B'EDD'$) + (area $Fc^*c^{**}F' - \epsilon$). It is clear that the first term is nothing other than our distortion effect, whereas the second term corresponds precisely to our commitment effect defined by (3.2).

Thus, the distortion effect, which is nothing but the familiar sum of marginal distortions, represents the welfare loss caused by the exercise of firms' monopolistic power on consumers. Clearly, an increase in investment that increases the industry's total supply will generate a positive distortion effect. On the other hand, the commitment effect measures the extent to which a firm can extract additional profits by capturing other firms' market share by taking advantage of a better third stage game structure *via* an increase in investment in x_i. The total effect on economic welfare depends on the relative strength of these conflicting effects.

3.3 In the rest of this section, we shall elucidate that the commitment effect is likely to dominate the distortion effect, so that the term $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n)$ is likely to become negative, if the number of firms is sufficiently large. In view of (3.5), (3.6), (2.6), (2.7), (2.11) and (2.12), and noting A(1) and (2.13), the condition for

$$(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n) = C_i(\mathbf{x}^N(n);n) + D_i(\mathbf{x}^N(n);n) < 0$$

can be reduced into

$$1 - (n-1) \cdot \left\{ -\frac{(\partial/\partial x_i)q_j^N(\mathbf{x}^N(n);n)}{(\partial/\partial x_i)Q^N(\mathbf{x}^N(n);n)} \right\} < 0, \tag{3.7}$$

which can be further reduced into

$$n^2 - 2n + (n-1)\delta(Q^N(\mathbf{x}^N(n);n)) > 0. \tag{3.8}$$

By virtue of $\mathbf{A}(1)$, (3.8) holds whenever $\lambda(n) := n^2 - 2n + (n-1)\delta_0 > 0$ is satisfied. Let $N(\delta_0) > 0$ be the largest root of the quadratic equation $\lambda(n) = 0$. Then $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n) < 0$ holds if $n > N(\delta_0)$. Thus:

Theorem 1 *Under* $\mathbf{A}(1)$, $\mathbf{A}(2)$ *and* $\mathbf{A}(3)$, *there exists a positive number* $N(\delta_0)$ *such that* $(\partial/\partial x_i)W^N(\mathbf{x}^N(n);n) < 0$ *holds, viz., the strategic cost-reducing investment is socially excessive at the margin if* $n > N(\delta_0)$.

An important question still remains. How large is the critical number $N(\delta_0)$ which appears in Theorem 1? In the case of concave inverse demand functions, it is easy to see that $N(\delta_0) = 2$. In the case of constantly elastic inverse demand functions, $N(\delta_0)$ will increase as the elasticity η of the inverse demand function increases, but for all values of η satisfying $0 < \eta < 1$, we have $1 < N(\delta_0) < 2 + \sqrt{2}$. Thus, $N(\delta_0)$ remains fairly small for these important classes of situations.

3.4 It may be useful to graphically illustrate why the number of firms, n, plays an important role in deciding social excessiveness of investment. Define the third stage *reaction function* of firm i by

$$r_i(Q_{-i};x_i^0) := \arg\max_{q_i > 0} \{f(q_i + Q_{-i}) - c(x_i^0)\}q_i, \tag{3.9}$$

where $Q_{-i} := \sum_{j \neq i} q_j$, and an investment profile $\mathbf{x}^0 := (x_1^0, x_2^0, \ldots, x_n^0)$ is fixed. Then the *cumulative reaction function* $R_i(Q;x_i^0)$ is defined by

$$q_i = R_i(Q;x_i^0) \quad \textit{if and only if} \quad q_i = r_i(Q - q_i;x_i^0). \tag{3.10}$$

By construction, the industry output in the third stage Cournot-Nash equilibrium $Q_N(\mathbf{x}^0; n)$ is the fixed point of the mapping

$$\sum_{j=1}^{n} R_j(Q; x_j^0), \text{ viz., } Q^N(\mathbf{x}^0; n) = \sum_{j=1}^{n} R_j(Q^N(\mathbf{x}^0; n); x_j^0).$$

Figure 2 describes the original third stage equilibrium E^0 as a point where the curve $\sum_{j=1}^{n} R_j(Q; x_j^0)$ cuts the 45° line.

Suppose now that firm i increases its investment marginally. Then the aggregate cumulative reaction curve will shift up to $\sum_{j=1}^{n} R_j(Q; x_j^1)$, where $R_j(Q; x_j^1) = R_j(Q; x_j^0)$ for all $j \neq i$, so that the industry output increases by $Q^N(\mathbf{x}^1; n) - Q^N(\mathbf{x}^0; n)$, whereas the output of firm j $(j \neq i)$ decreases by $q_j^N(\mathbf{x}^0; n) - q_j^N(\mathbf{x}^1; n)$, where $x_j^1 = x_j^0$ for all $j \neq i$. The ratio between the two, $[Q^N(\mathbf{x}^1; n) - Q^N(\mathbf{x}^0; n)]/[q_j^N(\mathbf{x}^0; n) - q_j^N(\mathbf{x}^1; n)]$, which closely approximates $-(\partial/\partial x_i)q_j^N(\mathbf{x}^0; n)/(\partial/\partial x_i)Q^N(\mathbf{x}^0; n)$ in (3.7) if an increase of firm i's investment is small enough, is provided by the slope of the cumulative reaction curve.

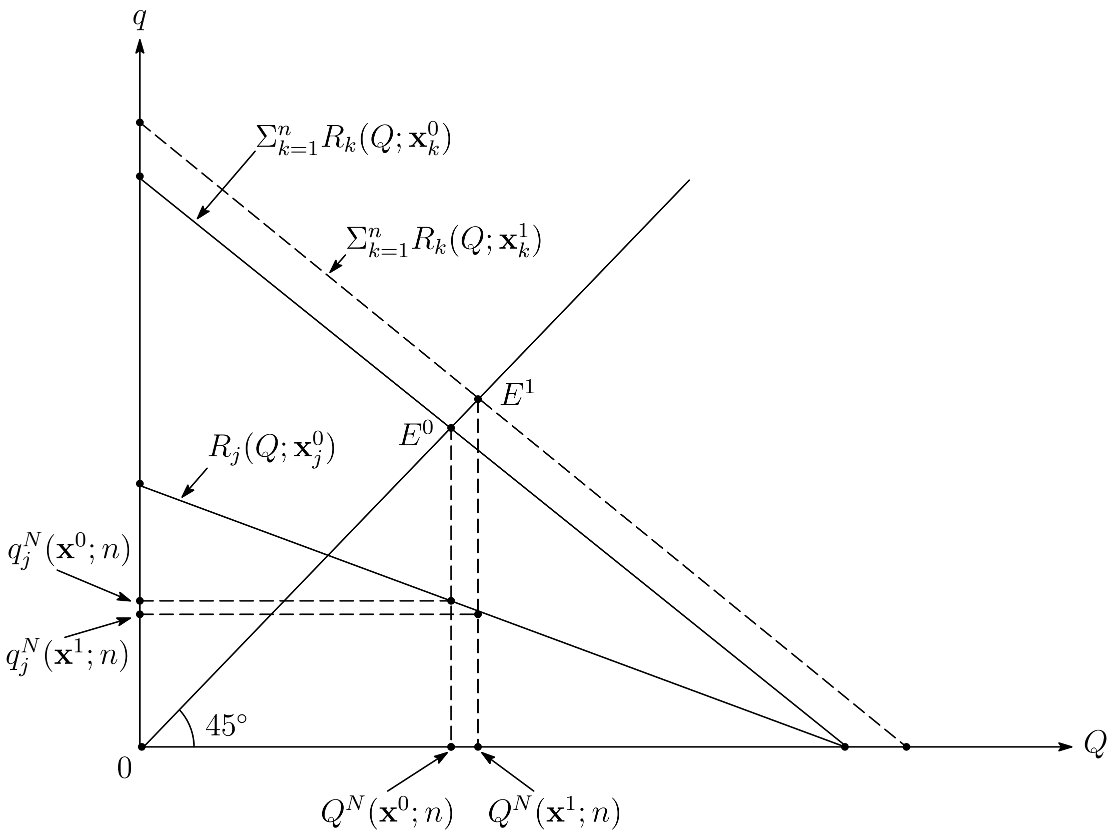

Figure 2. Cumulative Reaction Curves

Figure 2 describes a situation where the inverse demand function is linear, so that the reaction curve is also linear *whose slope is independent of the number of firms n.* In this case, as n becomes large, $(\partial/\partial x_i)W^N(\mathbf{x}^0; n)$ clearly becomes negative, and the equilibrium investment becomes socially excessive at the margin.

3.5 Before closing this section, a final remark is in order. Since our welfare criterion need not be concave in general, a marginally welfare-improving investment may in fact be a "wrong" move from the global viewpoint. However, it is possible to compare level of the second-best investment directly with that of the second stage subgame perfect equilibrium if our model is parametrizable, viz., the inverse demand function as well as the cost function is constantly elastic. Quite consistent with our analysis so far, it can be shown that there exists a critical number of firms as a function of the elasticity η of the inverse demand function, say $n^*(\eta)$, such that *the subgame perfect equilibrium level of investment exceeds the second-best level if* $n > n^*(\eta)$. The critical number is given by $n^*(\eta) := [(\eta+3) + \sqrt{\{(\eta+3)^2 - 4(\eta+1)\}}]/2 < \eta + 3$, which remains fairly small for a wide range of η.

4 Excess Entry in the Long Run

4.1 If the industry is left unregulated for a long time, the first stage free entry equilibrium with n_e firms, viz., $\{n_e, \mathbf{x}^N(n_e), \mathbf{q}^N(\mathbf{x}^N(n_e); n_e)\}$ will be attained. What, then, will be the welfare-improving policy that the government can enforce?

If the government can enforce the marginal cost pricing, it is easy to verify that the welfare-maximizing policy is to restrict the number of firms to either zero or one and impose the marginal cost pricing on the operating firm. Namely, we have the following:

Theorem 2 (First-best excess entry). *Assume that* $\mathbf{A}(2)$ *holds. Then a small reduction in the number of firms n unambiguously improves first-best social welfare in the sense that*

$$(4.1) \qquad (d/dn)W(\mathbf{q}^F(n); \mathbf{x}^F(n); n) < 0$$

holds as long as $n \geq 2$. Indeed, the first-best number of firms n_f is either 0 *or* 1.

4.2 Since enforcing the marginal cost principle is almost impossible for the actual government, we should examine how the second-best welfare function $W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n)$ will be affected when the number of firms changes by a small amount.

Differentiating $W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n)$ totally with respect to n, we obtain

$$(4.2) \qquad \begin{aligned} &(d/dn)W(\mathbf{q}^N(\mathbf{x}^N(n); n); \mathbf{x}^N(n); n) \\ &= \Pi^i(\mathbf{x}^N(n); n) + n \cdot \{f(Q^N(\mathbf{x}^N(n); n)) - c(x_i^N(n))\} \cdot (\partial/\partial n)q_i^N(\mathbf{x}^N(n); n) \\ &\quad + n \cdot \{C(\mathbf{x}^N(n); n) + D(\mathbf{x}^N(n); n)\} \cdot x_i^N(n). \end{aligned}$$

Note that the first term in the RHS of (4.2) is zero when it is evaluated at $n = n_e$ by virtue of the definition (2.18) of n_e, whereas the second term evaluated at $n = n_e$, viz.,

$$\mu(\mathbf{x}^N(n_e); n_e) \cdot (\partial/\partial n) q_i^N(\mathbf{x}^N(n_e); n_e) \tag{4.3}$$

is always negative by virtue of $\mathbf{A}(1)$, (2.9) for $\mathbf{x} = \mathbf{x}^N(n_e)$ and Lemma 1. Note also that (4.3) is the crucial term which leads to the excess entry theorem of Mankiw and Whinston [11] and Suzumura and Kiyono [21] in the context of no strategic commitment.

The third term in the RHS of (4.2) is specific to the oligopoly models with strategic commitment. Its first component evaluated at $n = n_e$, viz. $C(\mathbf{x}^N(n_e); n_e)$, is what we called the commitment effect in Section 3. Its second component evaluated at $n = n_e$, viz. $D(\mathbf{x}^N(n_e); n_e)$, is the distortion effect.

As was shown in Section 3, $C(\mathbf{x}^N(n_e); n_e) < 0$ and $D(\mathbf{x}^N(n_e); n_e) > 0$, so that the presence of strategic commitment seems to introduce some ambiguity in signing (4.2). If we replace $\mathbf{A}(1)$ by the following slightly stronger assumption $\mathbf{A}(1^*)$, however, we can establish an unambiguous result.

$\mathbf{A}(1^*)$: *$f(Q)$ is twice continuously differentiable with $f'(Q) < 0$ for all $Q \geq 0$ such that $f(Q) > 0$. Furthermore, the elasticity of $f'(Q)$ is constant, say, $\delta(Q) = \delta$.*[10]

With this, we can establish:

Theorem 3 *(Second-best excess entry at the margin). Assure that $\mathbf{A}(1^*)$, $\mathbf{A}(2)$ and $\mathbf{A}(3)$ hold. Then a small reduction in the number of firms at the first stage free-entry equilibrium unambiguously improves the second-best social welfare in the sense that*

$$(d/dn) W^N(\mathbf{q}^N(\mathbf{x}^N(n_e); n_e); \mathbf{x}^N(n_e); n_e) < 0 \tag{4.4}$$

holds as long as $n_e \geq 1 - \delta$.

Thanks to Theorem 3, under the assumed conditions, the exit of an incumbent firm at the first stage free-entry equilibrium is welfare-improving at the margin in the second-best sense even if we do not know where exactly n_f and n_s are located. Note that the crucial inequality $n_e \geq 1 - \delta$ is obviously satisfied if the inverse demand function is concave, so that $\delta \geq 0$ holds.

5 Proofs

(a) Proof of Lemma 1

Differentiating (2.9) with respect to n and rearranging terms using $\alpha(\mathbf{x}; n)$ and $\beta(\mathbf{x}; n)$, we obtain

$$\{\alpha(\mathbf{x}; n) + (n-1)\beta(\mathbf{x}; n)\} \cdot (\partial/\partial n) q_i^N(\mathbf{x}; n) = -\, q_i^N(\mathbf{x}; n) \cdot \beta(\mathbf{x}; n), \tag{5.1}$$

[10] See Remark 1 following the statement of $\mathbf{A}(1)$.

which yields (2.10). The negative sign of $(\partial/\partial n)q_i^N(\mathbf{x};n)$ is due to $\mathbf{A}(1)$ and $\mathbf{A}(3)$.

To prove (2.11) and (2.12), we differentiate (2.9) with respect to x_i and x_j $(i \neq j)$, respectively, and rearrange terms using $\omega(\mathbf{x};n)$ and $\theta(\mathbf{x};n)$ to obtain

$$\alpha(\mathbf{x};n)\cdot\omega(\mathbf{x};n) + (n-1)\cdot\beta(\mathbf{x};n)\cdot\theta(\mathbf{x};n) = c'(x_i) \tag{5.2}$$

$$\beta(\mathbf{x};n)\cdot\omega(\mathbf{x};n) + \{\alpha(\mathbf{x};n) + (n-2)\beta(\mathbf{x};n)\}\cdot\theta(\mathbf{x};n) = 0. \tag{5.3}$$

Solving (5.2) and (5.3) for $\omega(\mathbf{x};n)$ and $\theta(\mathbf{x};n)$, we obtain (2.11) and (2.12). The signs of $\omega(\mathbf{x};n)$, $\theta(\mathbf{x};n)$ and $\Delta(\mathbf{x};n)$ are determined by $\mathbf{A}(3)$, (2.6) and (2.7). ■

(b) Proof of Theorem 1

The sketch of the proof is given in the main text and hence it is omitted. ■

(c) Proof of Theorem 2

Differentiating $W(\mathbf{q}^F(n);\mathbf{x}^F(n);n)$ totally with respect to n, we obtain

$$\begin{aligned}(d/dn)W(\mathbf{q}^F(n);\mathbf{x}^F(n);n) &= \{f(nq^F(n)) - c(x^F(n))\}\cdot q^F(n) - x^F(n)\\ &\quad + nq^{F'}(n)\cdot\{f(nq^F(n)) - c(x^F(n))\}\\ &\quad + nx^{F'}(n)\cdot\{-c'(x^F(n))\cdot q^F(n) - 1\}.\end{aligned} \tag{5.4}$$

Invoking (2.20) and (2.21), we are then led to conclude that

$$(d/dn)W(\mathbf{q}^F(n);\mathbf{x}^F(n);n) = -x^F(n), \tag{5.5}$$

which is always negative, as was to be established. ■

(d) Proof of Theorem 3

Step 1. By virtue of $\mathbf{A}(1)$, (2.9) for $\mathbf{x} = \mathbf{x}^N(n_e)$, (3.5) and (3.6), it can easily be verified that the sign of (4.2) coincides with that of

$$\Lambda(n) := (\partial/\partial n)q_i^N(\mathbf{x}^N(n);n) + x_i^{N'}(n)\cdot\{\omega(\mathbf{x}^N(n);n) + 2(n-1)\cdot\theta(\mathbf{x}^N(n);n)\} \tag{5.6}$$

at $n = n_e$. Invoking Lemma 1, $\Lambda(n)$ can be further reduced into

$$\begin{aligned}\Lambda(n) = &\frac{1}{\alpha^N(n) + (n-1)\beta^N(n)}\\ &\cdot\left\{-q_i^N(\mathbf{x}^N(n);n)\cdot\beta^N(n) + \frac{\alpha^N(n) - n\cdot\beta^N(n)}{\alpha^N(n) - \beta^N(n)}\cdot c'(x_i^N(n))\cdot x^{N'}(n)\right\},\end{aligned} \tag{5.7}$$

where $\alpha^N(n) := \alpha(\mathbf{x}^N(n); n)$ and $\beta^N(n) := \beta(\mathbf{x}^N(n); n)$ for short. By virtue of (2.6) and (2.7) for $\mathbf{x} = \mathbf{x}^N(n)$ and $\mathbf{A}(1^*)$, it follows that

$$\text{sgn}\,\Lambda(n) = \text{sgn}\,[A + B \cdot x_i^{N'}(n)], \tag{5.8}$$

where

$$A = q_i^N(\mathbf{x}^N(n); n) \cdot f'(Q^N(\mathbf{x}^N(n); n)) \cdot (n + \delta) < 0 \tag{5.9}$$

and

$$B = c'(x_i^N(n)) \cdot \{n(n-2) + \delta(n-1)\}. \tag{5.10}$$

Note that $A > 0$ follows by virtue of (2.8), but the term $\{n(n-2) + \delta(n-1)\}$ is positive only when $n > N(\delta)$, as was shown in (3.8).

Step 2. We examine some properties of the second stage payoff function $\Pi^i(\mathbf{x}; n)$ with the purpose of evaluating $x_i^{N'}(n)$ which appears in (5.8). To begin with, simple yet complicated computation using (2.2), (2.4), (2.5), $\mathbf{A}(1^*)$ and Lemma 1 establishes that

$$\begin{aligned} \Pi_i^i(\mathbf{x}; n) &:= (\partial/\partial x_i)\Pi^i(\mathbf{x}; n) \\ &= -\, c'(x_i) \cdot q_i^N(\mathbf{x}; n) \cdot \xi(n) - 1 \end{aligned} \tag{5.11}$$

holds, where

$$\xi(n) := 1 + \frac{n-1}{n} \cdot \frac{n+\delta}{1+n+\delta} > 0 \tag{5.12}$$

in view of (2.8).

Differentiating (5.11) partially with respect to x_i and x_j $(i \neq j)$, respectively, we obtain

$$\begin{aligned} \Pi_{ii}^i &:= (\partial^2/\partial x_i^2)\Pi^i(\mathbf{x}; n) \\ &= -\, \xi(n) \cdot \{c''(x_i) \cdot q_i^N(\mathbf{x}; n) + c'(x_i) \cdot \omega(\mathbf{x}; n)\} < 0 \end{aligned} \tag{5.13}$$

$$\begin{aligned} \Pi_{ij}^i(\mathbf{x}; n) &:= (\partial^2/\partial x_i \partial x_j)\Pi^i(\mathbf{x}; n) \\ &= -\, \xi(n) \cdot c'(x_i) \cdot \theta(\mathbf{x}; n) \\ &= \xi(n) \cdot c'(x_i) \cdot \frac{c'(x_i) \cdot (n+\delta)}{f'(Q^N(\mathbf{x}; n)) \cdot n \cdot (n+1+\delta)} < 0 \quad (i \neq j) \end{aligned} \tag{5.14}$$

where the last equality of (5.14) is obtained in view of (2.6) and (2.7). Note that the second order condition for profit maximization at the second stage game requires that $\Pi_{ii}^i(\mathbf{x}^N(n); n) < 0$ holds, while $\mathbf{A}(2)$, Lemma 1 and (5.14) ensure that $\Pi_{ij}^i(\mathbf{x}; n) < 0$

$(i \neq j)$ holds for any $(\mathbf{x}; n)$. Therefore, the second stage strategies are warranted to be strategic substitutes if the third stage strategies are.[11]

Differentiating (5.11) partially with respect to n and nothing that

$$\xi'(n) = \frac{2n^2 + 2\delta n + \delta(\delta + 1)}{n^2(1 + n + \delta)^2} \tag{5.15}$$

follows from (5.12), we can finally obtain

$$(\partial/\partial n)\Pi^i_i(\mathbf{x}^N(n); n) = c'(x^N_i(n)) \cdot q^N_i(\mathbf{x}^N(n); n) \cdot \frac{(1-n)\{2(n+\delta)^2 + \delta\}}{n^2(1+n+\delta)^2}. \tag{5.16}$$

Step 3. By definition, $\mathbf{x}^N(n)$ is characterized by

$$\Pi^i_i(\mathbf{x}^N(n); n) = 0 \quad (i = 1, 2, \ldots, n). \tag{5.17}$$

Differentiating (5.17) totally and invoking symmetry, we obtain

$$x^{N\prime}_i(n) = -\frac{(\partial/\partial n)\Pi^i_i(\mathrm{x}^N(n); n)}{\Pi^i_{ii}(\mathrm{x}^N(n); n) + (n-1)\Pi^i_{ij}(\mathrm{x}^N(n); n)} \tag{5.18}$$

In view of (5.18), (5.8) is reduced into

$$\mathrm{sgn}\ \Lambda(n) = \mathrm{sgn}\left[\frac{A\Pi_{ii}}{\Pi_{ii} + (n-1)\Pi_{ij}} + \frac{(n-1)A \cdot \Pi_{ij} - B \cdot \Pi_{in}}{\Pi_{ii} + (n-1)\Pi_{ij}}\right] \tag{5.19}$$

where $\Pi_{ii} := \Pi^i_{ii}(\mathbf{x}^N(n); n), \Pi_{ij} := \Pi^i_{ij}(\mathbf{x}^N(n); n)$ and $\Pi_{in} := (\partial/\partial n)\Pi^i_i(\mathbf{x}^N(n); n)$.

It follows that (5.9), (5.13) and (5.14) assure that first term of the right hand side of (5.19) is unambiguously negative. Thus, for the sign of $\Lambda(n)$ to be negative, a sufficient condition is

$$\Gamma(n) := (n-1)A \cdot \Pi_{ij} - B \cdot \Pi_{in} < 0. \tag{5.20}$$

Invoking (5.9),(5.13),(5.14) and (5.16), a straightforward calculation yields

$$\Gamma(n) = q_i(n) \cdot \{c'(n)\}^2 \cdot (n-1) \cdot \Omega(n) \tag{5.21}$$

where

$$\Omega(n) := \left\{\frac{(n+\delta)^2}{n \cdot (n+1+\delta)} \cdot \zeta(n) + \frac{\{n(n-2) + \delta(n-1)\} \cdot \{2(n+\delta)^2 + \delta\}}{n^2 \cdot (1+n+\delta)^2}\right\}, \tag{5.22}$$

$q_i(n) := q^N_i(\mathrm{x}^N(n); n)$ and $c'(n) := c'(x^N_i(n))$.

[11]It is the latter half of $\mathbf{A}(1^*)$ that is responsible for this nice property. In general, this property does not necessarily hold. See Besley and Suzumura [1] and Suzumura [19].

In view of (5.21), if $n > 1$, the sufficient condition for $\Lambda(n)$ to be negative boils down to the condition that $\Omega(n)$ to be negative. In view of (5.12), a straightforward computation yields that

$$\Omega(n) = \phi_\delta(n)/[n^2 \cdot (n+1+\delta)^2], \tag{5.23}$$

where

$$\phi_\delta(n) := -4n^3 - 8\delta n^2 - \delta(5\delta - 2)n - \delta^2(\delta+1). \tag{5.24}$$

Step 4. The proof of Theorem 3 is complete if we can show that $\phi_\delta(n) < 0$ as long as $n \geq 1-\delta$. Since $\phi_\delta(n) < 0$ holds for all $n > 0$ if $\delta \geq 0$, we have only to examine the case where $\delta < 0$. With this goal in mind, let $n^*(\delta)$ stand for the largest real root of the cubic equation $\phi_\delta(n) = 0$. The coefficient of the highest order term of this cubic equation being negative, we have $1 - \delta > n^*(\delta)$ if all of $\phi_\delta(n)$, $\phi'_\delta(n)$ and $\phi''_\delta(n)$ are negative at $n = 1-\delta$. This is indeed the case, as we have

$$\phi_\delta(1-\delta) = 2(\delta - 2) < 0,$$

$$\phi'_\delta(1-\delta) = -\delta^2 + 6\delta - 12 < 0,$$

and

$$\phi''_\delta(1-\delta) = -8(3-\delta) < 0$$

for $\delta < 0$. if $n \geq 1-\delta$, we have $n > n^*(\delta)$, so that we obtain $\phi_\delta(n) < 0$, as was to be verified. ■

6 Concluding Remarks

In this chapter, we have examined the welfare performance of oligopoly with strategic commitments, which culminated into the excess entry results. The second best excess entry theorem at the margin, which is the main result of this chapter, is based on three explicit assumptions. The first assumption is on the admissible class of inverse demand functions. Despite its restrictive nature, we should note that a wide class of demand functions satisfies this assumption, as it does accommodate all linear inverse demand functions as well as all constantly elastic inverse demand functions. The second assumption is on the nature of cost reduction technology, which seems to be on the safe ground. The third assumption is on the nature of strategic interrelatedness of competitive measures. Within a model of quantity competition, the assumed strategic substitutability seems to be widely recognized as a normal case. Despite its rather paradoxical implications, therefore, our welfare verdicts cannot be flatly discarded as pathological. The fact that our results hold even in the presence of strategic commitments seems to enhance its relevance rather substantially.

It goes without saying that there are other implicit assumptions on which our results hinge. To cite just a few, quantity competition rather than price competition, exclusive focus on the symmetric equilibria, no uncertainty in cost-reducing R&D, and no product differentiation and no R&D spillovers can be referred to. It is almost certain, and in some cases demonstrably certain, that the mileage of our excess entry results are severely limited by these implicit assumptions. Nevertheless, the fact remains that the arena where our results do have their bites is in no sense negligible. Presumably, we are in need for more careful analyses of the role of competition as an efficient allocator of resources. The purpose of this chapter will be served if it succeeds in bringing this simple point home.

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