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Abstract

The paper examines the formation of free trade agreements (FTAs) as a network formation game. We consider a general n-country model in which (possibly asymmetric) countries trade differentiated industrial commodities as well as a numeraire good. We show that if all countries are symmetric, a complete global free trade network is pairwise stable and it is the unique stable network if industrial commodities are not highly substitutable. We also compare FTAs and customs unions (CUs) as to which of these two regimes facilitate global trade liberalization, emphasizing the fact that unlike in the case of a CU, each country signing an FTA can have a new FTA with an outside country without consent of other member countries.

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1 Introduction

The network of preferential trade agreements (PTAs) covers most countries in a complex way. The tendency towards "regionalism," a movement to form regional trade agreements, has been steadily growing especially since 1980s (Bhagwati, 1993). Since the Treaty of Rome established the European Economic Community (EEC) in 1957, European Union (EU) has been growing with the accession of new members. The North American Free Trade Agreement (NAFTA) has started negotiations with Latin American countries to form the Free Trade Area of the Americas. Japan has recently signed a free trade agreement (FTA) with Singapore, and now seeks the possibilities separately with South Korea and Mexico. The website of the World Trade Organization (WTO) on regionalism provides us with an excellent introduction to this topic.

> The vast majority of WTO members are party to one or more regional trade agreements. The surge in RTAs has continued unabated since the early 1990s. Some 250 RTAs have been notified to the GATT/WTO up to December 2002, of which 130 were notified after January 1995. Over 170 RTAs are currently in force; an additional 70 are estimated to be operational although not yet notified. By the end of 2005, if RTAs reportedly planned or already under negotiation are concluded, the total number of RTAs in force might well approach 300.
>
> (http://www.wto.org/english/tratop_e/region_e/region_e.htm#intro, September 4, 2003)

> One of the most frequently asked questions is whether these regional groups help or hinder the multilateral trading system of the WTO. A committee is keeping an eye on developments.
>
> (http://www.wto.org/english/thewto_e/whatis_e/tif_e/bey3_e.htm, September 4, 2003)

Whether PTAs serve as "building blocs" or "stumbling blocs" is a central question in this topic (Bhagwati, 1993). Of course, multilateral trade liberalization efforts and PTA formation interact with each other.[1] However, putting this feature aside for a while, another

[1] Levy (1997) and Krishna (1998) show in their political economy models that PTA formation hinders multilateral trade liberalization. Freund (2000b) demonstrates that countries have more incentive to form PTAs as multilateral trade negotiations lower tariffs imposed by every country. See also Bagwell and Staiger (1997a,b), Bond, Syropoulos, and Winters (2001), and Ethier (1998).

important question remains. Will successive PTA formation alone effectively achieve global free trade, or will the process stop prematurely so that the world is divided into several, mutually exclusive trading blocs? If PTA formation continues until global free trade network is achieved, we may conclude that PTAs are "building blocs." But otherwise, PTAs can be "stumbling blocs."[2]

Ohyama (1972) and Kemp and Wan (1976) demonstrate a positive result for this "dynamic" path problem. The so-called Kemp-Wan theorem states that member countries can appropriately adjust external tariffs and make internal transfers so that a newly formed customs union (CU) is Pareto-improving, not only to members themselves but also to all countries in the world.[3] Continuous application of this Kemp-Wan process implies that the CU expansion continues until all countries in the world are covered.[4] Although the theorem looks promising, it should be taken as an existence theorem (of a Pareto-improving CU expansion). In reality, it is extraordinarily difficult to adjust external tariffs such that each nonmember country's welfare is not reduced by the CU formation. Indeed, as Viner (1950) taught us, adverse trade-diversion effects often prevents PTAs from being Pareto improving.[5] It is far from obvious that in reality, countries always have incentives to form PTAs so that we will eventually observe the complete global free trade network (global free trade). Indeed, Yi (1996) shows that even if countries are symmetric, the world would be divided into two CUs of asymmetric size when the number of countries is a realistic number.

CUs are not the only form of PTAs. A PTA can take a form of an FTA, such as NAFTA, in which member countries choose their individual external tariffs without consent of other member countries unlike in the case of CU where all member countries adopt the same external tariff schedule.[6] An important consequence of this difference, which seems to be overlooked more or less in the literature, is that under an FTA, each member country (or a

[2] Bhagwati and Panagariya (1996) raise this "PTA time-path" question. The complete global free trade network may still be different from global free trade attained through multilateral trade negotiations, as Freund (2000a) demonstrates in a model where firms incur distribution network costs, for example. The global free trade network may be more complex and inefficient ("spaghetti bowl" phenomenon) than global free trade attained through multilateral trade negotiations, as Bhagwati and Panagariya (1996) claim.

[3] See Panagariya and Krishna (2002) for an FTA version of the Kemp-Wan theorem.

[4] Baldwin (1993) demonstrates that as a regional trading bloc expands, outside countries have more incentive to join the bloc. Haveman (1996) and Goto and Hamada (1999) conduct interesting simulations that investigate welfare consequences of CU expansion.

[5] Krugman (1991b) claims that if a "natural" trading bloc, within which a large share of trade takes place even in the absence of a PTA, is formed, the gains from trade creation are likely to outweigh the losses from trade diversion. Deardorff and Stern (1994) show the possibility that the situation where the world is divided into trading blocs is almost as beneficial as global free trade.

[6] Richardson (1993), Yi (2000) and Bond, Riezman, and Syropoulos (2002) show that each member country's optimal tariffs against nonmember countries decline as an FTA expands. See Krugman (1991a) and Bond and Syropoulos (1996) in the case of CUs.

subset of member countries) can sign another FTA with outside countries without consent of other member countries. Whereas in the case of CUs, such as European Union, all member countries should be involved when an outside country tries to form a PTA with a member country of a CU. Thus, FTAs are more flexible than CUs: A hub-and-spoke system, for example, is allowed only under FTAs.[7]

The main goal of this paper is to investigate whether or not the worldwide movement toward FTAs continues until the complete global free trade network is attained.[8] An FTA may be added to the current free trade network in the world in various ways, such as expansion of an FTA, and a hub-and-spoke system where a hub country is involved in multiple FTAs at the same time. An appropriate way to deal with such various forms of FTAs is to model the problem as a network formation game developed by Jackson and Wolinsky (1996).[9] Given any FTA configuration in the world, we examine whether or not a pair of countries have incentives to sign an FTA, and whether or not a country has an incentive to cut an existing FTA. A network that is immune to such deviations is called *(pairwise) stable* (Jackson and Wolinsky, 1996). Then we ask if the complete global free trade network is stable, and if it is, we further ask if it is a unique one. If the complete global free trade network is a unique stable network, the world is likely to attain global free trade, building many bilateral FTAs.[10]

In order to find stable free trade networks, we need to analyze each country's incentive to sign or abandon an FTA. As Krugman (1991b) and Grossman and Helpman (1995) suggest, the asymmetry of countries is an important factor when we assess countries' incentives for

[7]Kowalczyk and Wonnacott (1992) discuss the hub-and-spoke system in the argument about NAFTA. Mukunoki and Tachi (2001) investigate dynamic formation of bilateral FTAs in a three-country model. See also Puga and Venables (1997) and Wonnacott (1996)

[8]Driven by the same motivation, Freund (2000c) builds a model such that each country calls out the number of countries with which it wants to have FTAs, and shows that global free trade is effectively attained as a unique Nash equilibrium. However, she seems to assume implicitly that a bilateral FTA between two countries is made effective as long as one of the countries benefits from an agreement, even if the other strictly prefers not to sign the agreement. This "open membership" rule (see also Yi, 1996) does not seem to be appealing for discussions of FTAs. If FTAs require consent from both sides, then we will run into the multiplicity problem of Nash equilibria (see footnote 19).

[9]Coalition formation games such as Yi (1996, 2000) is not rich enough to capture this complex feature of FTAs.

[10]To derive a definite prediction regarding the time-path to global free trade, we may need to build a dynamic network formation model with farsightedness. Mukunoki and Tachi (2001) show in a dynamic, symmetric, three-country model that under certain parameter values, only one bilateral FTA may be signed in equilibrium so that global free trade is not attained. As Kennan and Riezman (1990) suggest, countries in a bilateral FTA may in some cases prefer the current situation to global free trade. Then, each member country may not sign a new bilateral free trade agreement with an outside country since it would induce an FTA between spoke countries, effectively attaining global free trade, in the future. However, extending Mukunoki and Tachi's (2001) analysis to the case of many countries is not an easy task.

FTAs. Viner (1950), on the other hand, suggests that substitutability of commodities traded internationally is also an important factor. The model of this paper is general enough to allow us to observe how these factors play a role in countries' decisions to sign an FTA with other countries. We consider the model in which the world consists of n countries that trade a numeraire good and a continuum of differentiated industrial commodities. Consumers in all countries share a common quasi-linear utility function, in which substitutability of industrial commodities is parameterized. Countries may be different in the market size (population size) and the size of the industrial good industry (measure of firms). Each of the differentiated industrial commodities is produced by one firm that belongs to one of n countries. Each country has a tariff schedule for imported industrial commodities, and an FTA between countries i and j simply means that countries i and j simultaneously eliminate tariffs on commodities imported from each other.

When the utility function is quasi-linear, social welfare, which is merely a representative consumer's utility, can be decomposed into two parts: the consumer's gross utility and the (industrial) trade surplus that is defined as the value of export minus import payments in the industrial good sector. An FTA with another country is likely to raise the gross utility, although the second-best effect (Lipsey and Lancaster, 1956) may sometimes outweigh the benefits from tariff reduction.[11] On the other hand, the impact on the (industrial) trade surplus is generally ambiguous, and is often crucial in determining whether or not an FTA is welfare improving.

The effect on a country's trade surplus of signing an FTA with another country can be further decomposed into two: one on the trade surplus between these two countries (the direct surplus effect) and the other on the trade surplus with third countries (the third country effect). The latter effect is always positive, since the country's exports to third countries are not affected by the FTA, while its imports from them decrease because their commodities become relatively more expensive after the FTA. Thus, the third country effect always serves to encourage countries to sign FTAs at the costs of third countries: all other countries including existing FTA partners are hurt by these new FTAs. In contrast, the sign of the direct surplus effect depends on the two countries' characteristics such as the market and industry size, and the characteristics of their current partners. Let us consider, for example, an FTA between a highly-industrialized small country and a less-industrialized large country. The FTA increases trade flows from the former to the latter disproportionately, dramatically increasing the trade surplus of the small highly-industrialized country and decreasing that of

[11]If tariffs have been imposed on a large portion of commodities, it may not be welfare-improving to get rid of tariffs for a small portion of commodities since it enlarges distortions between these commodities and the ones with high tariffs.

the large less-industrialized country. The direct surplus effect for the large less-industrialized country is likely to be negative, and it may outweigh the third country effect. Consequently, the large less-industrialized country is likely to oppose to sign the FTA.[12] If two countries are similar in their characteristics, however, the direct surplus effects would be small, and the countries are likely to benefit from signing an FTA due to the third country effect.

The main results of this paper are as follows. When all countries are symmetric in the market size and the industry size, we show that the global free trade network, a network in which any pair of countries has an FTA, is pairwise stable (Proposition 1). If commodities are highly substitutable among themselves, however, there may also be other pairwise stable networks. It is because the difference in the number of FTA partners can create a large differential in the impacts on the direct surplus, even though all countries are symmetric in the market size and industry size. We show that if commodities are *not* highly substitutable among themselves, the global free trade network is the unique pairwise stable network (Proposition 2). If countries are asymmetric, on the other hand, the global free trade network may not be attained. In a special case where all industrial commodities are independent from each other, a pair of countries sign an FTA if and only if their industrialization levels are close to each other (Proposition 3). This proposition implies that developed countries and less developed countries, for example, respectively form mutually exclusive trading blocs. We also compare FTAs and CUs as to which of these two regimes facilitate global trade liberalization. The direct surplus effect is smaller if the existing partnership of the country takes the form of a CU rather than a regional FTA since the new partner opens its market also to the country's existing CU partners. Moreover, the third country effect is smaller under CUs since existing CU partners also open their own markets to the new partner. The third country effect against non-member countries of the CU remain, whereas the effect against CU partners are offset. Indeed, we find that if all countries are symmetric, and if industrial commodities are not highly substitutable among themselves, a pair of countries have less incentive to form a new FTA if either of them is a member of a CU as opposed to an FTA (Proposition 4). If countries are asymmetric, on the other hand, the CU formation averages out member countries' industrialization levels, which may help further PTA formation. We illustrate this possibility in the case of mutually independent industrial commodities.

[12]It is interesting to notice that countries in our model have a view that Krugman (1991b) calls GATT-think: '(1) Exports are good, (2) Imports are bad, (3) and other things being equal, an equal increase in imports and exports is good.' Our model gives an economic reasoning to this "enlightened mercantilism." See Furusawa and Konishi (2003) for details.

An independent work by Goyal and Joshi (2001) also investigates the FTA formation as a network formation game, and obtains the result that the complete global free trade network is pairwise stable. The main part of their analysis assumes that all countries are symmetric with respect to the (Cournot-oligopolistic) market size and the number of domestic firms. Our model is richer in many aspects, enabling us to obtain insights on incentives to sign FTAs, especially in the case of asymmetric countries. We also discuss the difference between FTAs and CUs as we have briefly described above.

Our paper is organized as follows. In Section 2, we present the model and derive the market equilibrium in the presence of tariffs. Then, we propose a convenient way to decompose a country's social welfare, which plays an important role in the subsequent analysis. We also derive the optimal tariff. In Section 3, we analyze a country's incentive to sign an FTA, and then derive our main results about stable networks both in the symmetric country and asymmetric country cases. Section 4 compares FTAs and CUs as to which of these enhances the PTA formation. Section 5 concludes the paper.

2 The Model

2.1 Overview

Let N be the set of n countries ($n \geq 2$), each of which is populated by a continuum of identical consumers who consume a numeraire good and a continuum of horizontally differentiated commodities that are indexed by $\omega \in [0, 1]$. A differentiated commodity can be considered as a variety of an industrial good. Each industrial commodity ω is produced by one firm which is also indexed by the same ω. We assume that there is no entry of firms into this industry. Each firm is owned equally by all domestic consumers who receive equal shares of all firms' profits. The numeraire good is produced competitively, on the other hand. Each consumer is endowed with l units of labor, which is used for production of the industrial and numeraire goods. Each unit of labor produces one unit of the numeraire good, so that the wage rate equals 1. We also assume that industrial commodities are produced with a linear technology, and normalize the unit labor requirement to be equal to 0 for each industrial commodity, without loss of generality. Alternatively, we can interpret the model such that each consumer is endowed with l units of the numeraire good, which can be transformed by a linear technology into industrial commodities.

In country i ($i = 1, 2, \cdots, n$), measure μ^i of consumers and measure s^i of firms that produce industrial commodities are located. Thus, country i produces s^i industrial commodities, which are consumed in every country in the world. We assume that the markets are segmented so that firms can perfectly price discriminate among different countries. We normalize the size of total population so that $\sum_{k=1}^{n} \mu^k = 1$ as well as $\sum_{k=1}^{n} s^k = 1$. The ratio $\theta^i \equiv s^i/\mu^i$ measures country i's industrialization level. The higher the ratio, the higher the country's industrialization level. This ratio plays an important role later in our analysis. Country i imposes a specific tariff at a rate of t_j^i on the imports of the industrial commodities that are produced in country j. For simplicity, we assume that every commodity produced in country j faces the same tariff rate t_j^i.[13] We assume that there is no commodity tax, so that $t_i^i = 0$. We also assume that the countries do not impose tariffs on the numeraire good which may be traded internationally to balance the trade. Tariff revenue is redistributed equally to domestic consumers.

2.2 Consumer Demands

A representative consumer's utility is given by the following quasi-linear utility function:

$$U(q, q_0) = \alpha \int_0^1 q(\omega)d\omega - \frac{\beta}{2}\int_0^1 q(\omega)^2 d\omega - \frac{\delta}{2}\left[\int_0^1 q(\omega)d\omega\right]^2 + q_0, \tag{1}$$

where $q : [0,1] \rightarrow \Re_+$ is an integrable consumption function, and q_0 denotes the consumption level of the numeraire good.[14] The second last term represents the substitutability among differentiated commodities, which may become clearer if we notice $\left[\int_0^1 q(\omega)d\omega\right]^2 = \int_0^1 \int_0^1 q(\omega)q(\omega')d\omega' d\omega$. Letting y denote the consumer's income, the budget constraint can be written as

$$y = \int_0^1 \tilde{p}(\omega)q(\omega)d\omega + q_0, \tag{2}$$

where $\tilde{p} : [0,1] \rightarrow \Re_+$ denotes the consumer price function. The first order condition for the consumer's maximization problem gives us the inverse demand function for each good ω:

$$\tilde{p}(\omega) = \alpha - \beta q(\omega) - \delta \int_0^1 q(\omega')d\omega'.$$

Integrating over $[0,1]$, we obtain

$$\int_0^1 q(\omega)d\omega = \frac{1}{\beta+\delta}\left(\alpha - \tilde{P}\right),$$

[13] In our simple model, country i's optimal tariff rates, for example, are the same across all commodities imported from country j.

[14] This utility function is a continuous-goods version of the ones of Shubik (1984) and Yi (1996, 2000) who analyze the case where there are only finitely many differentiated commodities. Our continuum of commodity setup is based on Ottaviano, Tabuchi, and Thisse (2002). Neary (forthcoming) uses a similar functional form but without quasi-linearity in order to capture general equilibrium effects. We keep quasi-linearity in this paper since other aspects of our model are highly complex.

where $\tilde{P} = \int_0^1 \tilde{p}(\omega)d\omega$. Substituting this equation back into the first order condition, we have

$$q(\omega) = \frac{\alpha}{\beta} - \frac{1}{\beta}\tilde{p}(\omega) - \frac{\delta}{\beta(\beta+\delta)}\left(\alpha - \tilde{P}\right).$$

2.3 Equilibrium in Country i

Letting $p^i(\omega)$ and $\tilde{P}^i$ denote the producer price for commodity ω sold in country i, and the average consumer price in country i, respectively, a representative consumer's demands in country i for a commodity ω produced in country j can be written as

$$q^i(\omega) = \frac{\alpha}{\beta} - \frac{1}{\beta}(p^i(\omega) + t^i_j) - \frac{\delta}{\beta(\beta+\delta)}(\alpha - \tilde{P}^i). \tag{3}$$

The firm ω in country j chooses $\{p^i(\omega)\}_{i=1}^n$ in order to maximize its profits $\pi(\omega) = \sum_{i=1}^n \mu^i p^i(\omega) q^i(\omega)$. The first order condition for this maximization gives us

$$p^i(\omega) = \frac{1}{2}\left[\frac{\alpha\beta}{\beta+\delta} - t^i_j + \frac{\delta}{\beta+\delta}\tilde{P}^i\right], \tag{4}$$

for any i. Notice that $p^i(\omega)$ does not vary with ω. Prices charged by firms depend only on the import country's tariff policies. For simplicity, we henceforth suppress the argument ω.

It follows from (4) that country i's average consumer price is

$$\begin{aligned}\tilde{P}^i &= \sum_{j=1}^n s^j(p^i + t^i_j) \\ &= \frac{1}{2}\left[\frac{\alpha\beta}{\beta+\delta} + \frac{\delta}{\beta+\delta}\tilde{P}^i\right] + \frac{1}{2}\bar{t}^i,\end{aligned}$$

where $\bar{t}^i \equiv \sum_{j=1}^n s^j t^i_j$. Thus, country i's average consumer price $\tilde{P}^i$ is given by

$$\tilde{P}^i = \frac{\alpha\beta}{2\beta+\delta} + \frac{\beta+\delta}{2\beta+\delta}\bar{t}^i. \tag{5}$$

Substituting (5) into (4) yields the equilibrium producer price that each firm in country j charges for the market of country i, as a function of country i's tariff vector $\mathbf{t}^i = (t^i_1, ..., t^i_n)$:

$$p^i_j(\mathbf{t}^i) = \frac{\alpha\beta}{2\beta+\delta} - \frac{1}{2}t^i_j + \frac{\delta}{2(2\beta+\delta)}\bar{t}^i.$$

Then it follows from (3) that a representative consumer's demand in country i for a commodity produced in country j is

$$q^i_j(\mathbf{t}^i) = \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t^i_j + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i. \tag{6}$$

Notice that $p^i_j(\mathbf{t}^i) = \beta q^i_j(\mathbf{t}^i)$ holds for any tariff vector $\mathbf{t}^i$.

2.4 Social Welfare

Under the world tariff vector $\mathbf{t} = (\mathbf{t}^1, ..., \mathbf{t}^n)$, each firm in country i earns the profits:

$$\pi_i(\mathbf{t}) = \sum_{k=1}^{n} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k) = \sum_{k=1}^{n} \mu^k \beta q_i^k(\mathbf{t}^k)^2. \tag{7}$$

Country i's *per capita* tariff revenue is

$$T^i(\mathbf{t}^i) = \sum_{k=1}^{n} t_k^i s^k q_k^i(\mathbf{t}^i). \tag{8}$$

A representative consumer's income in country i is the sum of labor income, redistributed tariff revenue, and the profit shares of the firms in country i:

$$y = l + T^i(\mathbf{t}^i) + \frac{s^i \pi_i(\mathbf{t})}{\mu^i}. \tag{9}$$

Then it follows from (2) that

$$\begin{aligned} q_0^i(\mathbf{t}) &= l + T^i(\mathbf{t}^i) + \frac{s^i \pi_i(\mathbf{t})}{\mu^i} - \sum_{k=1}^{n} s^k [p_k^i(\mathbf{t}^i) + t_k^i] q_k^i(\mathbf{t}_k^i) \\ &= l + \sum_{k=1}^{n} s^k t_k^i q_k^i(\mathbf{t}^i) + \frac{s^i}{\mu^i} \sum_{k=1}^{n} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k) - \sum_{k=1}^{n} s^k [p_k^i(\mathbf{t}^i) + t_k^i] q_k^i(\mathbf{t}^i) \\ &= l - \sum_{k \neq i} s^k p_k^i(\mathbf{t}^i) q_k^i(\mathbf{t}^i) + \frac{s^i}{\mu^i} \sum_{k \neq i} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k), \end{aligned}$$

where $q^i(\omega) = q_k^i(\mathbf{t}^i)$ if ω is produced in country k.

Substituting this equilibrium demand into (1) and letting $q^i(\mathbf{t}^i)$ represent country i's equilibrium consumption function under the tariff $\mathbf{t}^i$, i.e., $q^i(\mathbf{t}^i) = (q_j^i(\mathbf{t}^i))_{j \in N}$, we obtain a representative consumer's utility in country i as a function of the world tariff vector, which can be considered as country i's *per capita* social welfare:

$$W^i(\mathbf{t}) \equiv U(q^i(\mathbf{t}^i), q_0^i(\mathbf{t})) = V^i(\mathbf{t}^i) + X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i), \tag{10}$$

where

$$V^i(\mathbf{t}^i) = U(q^i(\mathbf{t}^i), l), \tag{11}$$

$$M^i(\mathbf{t}^i) = \sum_{k \neq i} s^k p_k^i(\mathbf{t}^i) q_k^i(\mathbf{t}^i) = \sum_{k \neq i} \beta s^k q_k^i(\mathbf{t}^i)^2, \tag{12}$$

$$X^i(\mathbf{t}^{-i}) = \frac{s^i}{\mu^i} \sum_{k \neq i} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k) = \frac{s^i}{\mu^i} \sum_{k \neq i} \beta \mu^k q_i^k(\mathbf{t}^k)^2, \tag{13}$$

with $\mathbf{t}^{-i} = (\mathbf{t}^1, \cdots, \mathbf{t}^{i-1}, \mathbf{t}^{i+1}, \cdots, \mathbf{t}^n)$. The functions $V^i(\mathbf{t}^i)$, $M^i(\mathbf{t}^i)$, and $X^i(\mathbf{t}^{-i})$ represent a consumer's gross utility, import payments, and the export value of industrial commodities, respectively. Country i's social welfare consists of a consumer's gross utility $V^i(\mathbf{t}^i)$ and the per-capita industrial trade surplus $X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)$.[15] Country i's tariffs affect social welfare through the effects on $V^i(\mathbf{t}^i)$ and $M^i(\mathbf{t}^i)$. Other countries' tariffs also affect country i's social welfare through the effect on $X^i(\mathbf{t}^{-i})$.

Now, we examine the effects of tariff changes on the three components of social welfare: $V^i(\mathbf{t}^i)$, $X^i(\mathbf{t}^{-i})$, and $M^i(\mathbf{t}^i)$. We notice from (11)-(13) that an increase in a tariff rate affects these components only through the changes in consumption of industrial commodities. We see from (6) that the consumption of an industrial commodity depends on the tariff rate imposed on that commodity and the average tariff rate, i.e., $q^i_k(\mathbf{t}^i) \equiv \tilde{q}^i_k(t^i_k, \bar{t}^i)$. Thus, we can write, for example, $V^i(\mathbf{t}^i) = \tilde{V}^i(\tilde{q}^i_1(t^i_1, \bar{t}^i), \cdots \tilde{q}^i_n(t^i_n, \bar{t}^i))$. An increase in t^i_j does not only affect q^i_j directly, but also affects q^i_k indirectly, for all $k = 1, 2, \cdots, n$. These changes in consumption affect $V^i(\mathbf{t}^i)$ and $M^i(\mathbf{t}^i)$, in turn. As for the effect on $V^i(\mathbf{t}^i)$, for example, we have

$$\frac{\partial V^i}{\partial t^i_j} = \sum_{k=1}^{n} \frac{\partial \tilde{V}^i}{\partial \tilde{q}^i_k} \left(\frac{\partial \tilde{q}^i_k}{\partial t^i_j} + \frac{\partial \tilde{q}^i_k}{\partial \bar{t}^i} \frac{\partial \bar{t}^i}{\partial t^i_j} \right).$$

An increase in another country's tariff rate on country i's commodity affects the export profits $X^i(\mathbf{t}^{-i})$ in a similar fashion. We can easily obtain the following lemma that shows the effects of raising a tariff rate on the three components of social welfare. The proof is straightforward and hence omitted.

Lemma 1 *The first order effects of raising t^i_j on V^i and M^i and the effect of raising t^j_i on X^i are:*

$$\begin{aligned}
\frac{\partial V^i}{\partial t^i_j} &= s^j \left[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)} \sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i) + \frac{1}{2} q^i_j(\mathbf{t}^i) \right], \\
\frac{\partial X^i}{\partial t^j_i} &= \frac{\mu^j s^i}{\mu^i} \beta \left[q^j_i(\mathbf{t}^j) \left(-\frac{1}{\beta} + \frac{\delta}{\beta(2\beta+\delta)} s^i \right) \right], \\
\frac{\partial M^i}{\partial t^i_j} &= s^j \beta \left[q^i_j(\mathbf{t}^i) \left(-\frac{1}{\beta} + \frac{\delta}{\beta(2\beta+\delta)} s^j \right) + \sum_{k \neq i,j} q^i_k(\mathbf{t}^i) \frac{\delta}{\beta(2\beta+\delta)} s^k \right].
\end{aligned}$$

[15] Furusawa and Konishi (2003) show that when consumers have quasi-linear utility functions and all countries share the same constant-returns-to-scale production technology for each commodity they produce, social welfare of a country can be represented by the sum of consumers' gross utilities and trade surplus of non-numeraire goods. It should be emphasized that this welfare decomposition would not support mercantilism, since an increase in imports, for example, is not necessarily bad as it raises consumers' gross utilities as well as it lowers trade surplus.

It may appear that an increase in a tariff rate of country i, say t^i_j, necessarily decreases the domestic consumer's gross utility V^i. Each consumer in country i reduces the consumption of country j's commodities as a consequence, which is detrimental. However, each agent consumes other commodities more than before, which tends to increase the consumer's gross utility. The latter indirect effect may outweigh the former so that an increase in a tariff rate may increase the domestic consumer's gross utility, if the industrial commodities are highly substitutable among themselves. Similarly, an increase in a tariff rate may not always decrease the import payments. If the industrial commodities are highly substitutable, the resulting decrease in q^i_j may be outweighed by increases in q^i_k for $k \neq i, j$. However, it is easy to see from Lemma 1 that an increase in another country's tariff unambiguously decreases the domestic profits obtained from the export to that country.

Now, let us derive the optimal tariffs. Country i's optimal tariff maximizes $V^i(\mathbf{t}^i) - M^i(\mathbf{t}^i)$ since $X^i(\mathbf{t}^{-i})$ does not depend on $\mathbf{t}^i$. Let $C_i = \{j \in N | t^i_j = 0\}$ represent the set of countries that produce commodities on which country i does not impose tariffs. (Notice that C_i includes country i itself since $t^i_i = 0$.) We consider here the situation in which country i has signed FTAs, rather than CUs, with all other countries in C_i. Therefore, country i chooses its external tariffs without any coordination with other countries in C_i. As the following lemma shows, a country's optimal tariff rates only depend on its own characteristics due to the separability of a consumer's utility function.[16] The proof of lemma 2 is straightforward and hence omitted.

Lemma 2 *Country i's optimal tariff rate τ^i is a function of s^i, $s^{C_i}(\equiv \sum_{k \in C_i} s^k)$, and parameters α, β and δ:*

$$\tau^i(s^i, s^{C_i}; \alpha, \beta, \delta) = \frac{4\alpha\beta(\beta + \delta s^i)}{3(2\beta + \delta)^2 - \delta(1 - s^{C_i})[4(2\beta + \delta) - \delta(1 - 2s^i)]} > 0,$$

which is increasing in s^i, and decreasing in s^{C_i}. Thus, a country of size s^i has the highest optimal tariff rate when $s^{C_i} = s^i$, or $C_i = \{i\}$, and a more industrialized country, whose s^i tends to be high, has a higher optimal tariff rate for a given s^{C_i}.

[16] Somewhat surprisingly, they do not depend on country i's own market size μ^i. It is because the market size is only related to the scale effect.

3 Free Trade Agreements

3.1 Incentives to sign an FTA

We consider an FTA between countries i and j, such that they eliminate all tariffs imposed on commodities imported from each other, while they keep all other tariffs at their original levels. For an FTA to be signed, both countries i and j must benefit from the agreement. That is, we must have both (i) $W^i(t^i_j, \mathbf{t}^i_{-j}; t^j_i, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}}) \leq W^i(0, \mathbf{t}^i_{-j}; 0, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}})$ and (ii) $W^j(t^j_i, \mathbf{t}^j_{-i}; t^i_j, \mathbf{t}^i_{-j}; \mathbf{t}^{-\{i,j\}}) \leq W^j(0, \mathbf{t}^j_{-i}; 0, \mathbf{t}^i_{-j}; \mathbf{t}^{-\{i,j\}})$. The condition (i), for example, can be written as

$$\Delta V^i(\mathbf{t}^i) + \left[\Delta X^i(\mathbf{t}^{-i}) - \Delta M^i(\mathbf{t}^i)\right] \geq 0, \tag{14}$$

where $\Delta V^i(\mathbf{t}^i) \equiv V^i(0, \mathbf{t}^i_{-j}) - V^i(\mathbf{t}^i)$, $\Delta X^i(\mathbf{t}^{-i}) \equiv X^i(0, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}}) - X^i(\mathbf{t}^j; \mathbf{t}^{-\{i,j\}})$, and $\Delta M^i(\mathbf{t}^i) \equiv M^i(0, \mathbf{t}^i_{-j}) - M^i(\mathbf{t}^i)$. As we will see shortly, a tariff reduction is likely to increase a consumer's gross utility, unless the industrial commodities are highly substitutable. Since the FTA increases the country's export profits and is also likely to increase the import payments, on the other hand, the FTA has an ambiguous impact on country i's industrial trade surplus. We assume for simplicity that each country i imposes the same external tariff rate, which is denoted by t^i, on all commodities imported from countries outside of C_i.

First, let us investigate the sign of $\Delta V^i(\mathbf{t}^i)$. The next lemma shows that an FTA increases a consumer's gross utility either if the substitutability among the industrial commodities is low or if the original tariff rate is small.

Lemma 3 *A bilateral FTA increases a consumer's gross utility, i.e., $\Delta V^i(\mathbf{t}^i) > 0$, if either one of the following conditions is satisfied:*

$$(i)\ 4\beta(\beta+\delta) - \delta^2(1 - 2s^{C_i} - s^j) \geq 0,$$

$$(ii)\ t^i < \frac{8\alpha\beta^2}{\delta^2(1 - 2s^{C_i} - s^j) - 4\beta(\beta+\delta)}.$$

In particular, when country i levies the tariff rate t^i that is not greater than the optimal tariff rate $\tau^i(s^i, s^{C_i}; \alpha, \beta, \delta)$ (for any s^i, s^j and s^{C_i}), it is sufficient that $\delta \leq 10\beta$ for $\Delta V^i(\mathbf{t}^i)$ to be positive.

Remark 1 *Note that condition (i) is satisfied if $1 - 2s^{C_i} - s^j \leq 0$, or equivalently $s^{C_i} + \frac{1}{2}s^j \geq \frac{1}{2}$. This corresponds to the second best effect: In an economy with distortions, partial removal of tax distortions may reduce efficiency (see Dixit, 1975, and Hatta, 1977). When a tariff* *on a commodity is eliminated, distortions between this commodity and untaxed commodities shrink, whereas distortions with taxed commodities expand. Thus, if there are more untaxed commodities than taxed commodities, the second best theory tells us that a bilateral FTA between i and j is likely to raise a consumer's gross utility. The condition $s^{C_i} + \frac{1}{2}s^j \geq \frac{1}{2}$ matches exactly to this observation. Turning to condition (ii), the right-hand side is positive only when $s^{C_i} + \frac{1}{2}s^j < \frac{1}{2}$. Thus, condition (ii) is meaningful only when (i) is not satisfied, and it shows that the detrimental second best effect is negligible if the rate of tariffs, which cause distortions in the first place, is small.*

Next, we turn to investigating the effect of an FTA between countries i and j on the industrial trade surplus. Let M_k^i and X_k^i be country i's (per capita) import payments to country k and country i's (per capita) export profits from country k, respectively:

$$\begin{aligned} M_k^i(\mathbf{t}^i) &= \beta s^k q_k^i(\mathbf{t}^i)^2, \\ X_k^i(\mathbf{t}^k) &= \frac{s^i}{\mu^i}\beta\mu^k q_i^k(\mathbf{t}^k)^2 \left(= \frac{\mu^k}{\mu^i}M_i^k(\mathbf{t}^k)\right). \end{aligned}$$

Then, we can rewrite country i's industrial trade surplus as

$$X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i) = \sum_{k\neq i}\left[X_k^i(\mathbf{t}^k) - M_k^i(\mathbf{t}^i)\right].$$

An FTA between i and j only involves changes in $\mathbf{t}^i$ and $\mathbf{t}^j$ so that it does not affect $X_k^i(\mathbf{t}^k)$ for any $k \neq i, j$. Consequently, a change in country i's industrial trade surplus can be written as

$$\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] = \underbrace{\Delta\left[X_j^i(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right]}_{\text{direct surplus effect}} + \underbrace{\left(-\sum_{k\neq i,j}\Delta M_k^i(\mathbf{t}^i)\right)}_{\text{third country effect}}.$$

The third country effect, represented by the terms in the parentheses, is always positive since the reduction of t_j^i makes commodities imported from country j relatively less expensive, and hence country i's imports from third countries decrease, i.e., $\Delta M_k^i(\mathbf{t}^i) < 0$. Although the third country effect works positively for countries i and j, it implies that the FTA between them hurts all other countries.

Having shown that the third country effect is positive, let us now investigate the direct surplus effect, which can be rewritten as follows from the definitions of $M_i^j(\mathbf{t}^j)$ and $M_j^i(\mathbf{t}^i)$:

$$\Delta\left[X_j^i(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right] = \mu^j\beta\Delta\left[\theta^i q_i^j(\mathbf{t}^j)^2 - \theta^j q_j^i(\mathbf{t}^i)^2\right],$$

where $\theta^i = s^i/\mu^i$ as defined above. The higher θ^i and the lower θ^j, the larger an increase in country i's industrial trade surplus. That is, other things being equal, the relatively more industrialized country is more enthusiastic than the less industrialized country in signing a bilateral FTA.[17] The more industrialized country derives a large benefit from the opening of the partner's relatively large market. In addition, opening its own market to the partner's firms does not significantly increase import payments since the resulting penetration by the partner's firms is relatively small. Another important factor that affects the incentives to form an FTA is the difference in the original tariff rates. It is easy to see that if $t^i < t^j$, for example, then $\Delta q_j^i(\mathbf{t}^i) < \Delta q_i^j(\mathbf{t}^j)$. Country i's export to country j increases more than its import from country j, and hence the FTA between i and j tends to be more beneficial to country i.

3.2 Stable Free Trade Networks

An FTA that involves more than two countries can be considered as a collection of bilateral FTAs between member countries, so it is convenient to describe FTAs in terms of graph theory terminology. An FTA between countries i and j is described by a *link*, which is an unordered pair of two countries. An *FTA graph* is a nondirected graph, (N, Γ), consisting of the set of countries N and a (free trade) *network* Γ that is a collection of links. The set of country i's FTA *partners* in a network Γ is $C_i(\Gamma) = \{i\} \cup \{k \in N : (i,k) \in \Gamma\}$, which includes i, as we have already described. We continue to write it as C_i without confusion, as long as a network Γ is fixed.

If tariff rates are exogenously determined, or if they are determined uniquely for each free trade network Γ (such as in the case where all countries set their individual optimal tariffs given the prevailing network Γ), then country i's payoff (social welfare) can be written uniquely by $u_i(\Gamma)$. The set of countries N and their payoff functions define a *network formation game*.[18]

Network formation games are first studied by Jackson and Wolinsky (1996). A *pairwise stable network* is a network Γ^* such that (i) for any $(i,j) \in \Gamma^*$, $u_i(\Gamma^*) \geq u_i(\Gamma^* \backslash (i,j))$ and $u_j(\Gamma^*) \geq u_j(\Gamma^* \backslash (i,j))$, i.e., for any pair of linked countries, neither country has an incentive

[17]Indeed, if one country's direct surplus effect is positive, the partner's direct surplus effect must be negative since the sum of two countries' direct surplus effects is always zero ($\Delta X_j^i(\mathbf{t}^j) = \Delta M_i^j(\mathbf{t}^j)$ for any $i, j \in N$ with $i \neq j$).

[18]In the case where countries are (*ex ante*) symmetric, our network formation game belongs to the class of local spillover games studied by Goyal and Joshi (2003). A local spillover game is a game in which a player's incentive to form a link with her potential partner depends only on how many links she has and how many links her potential partner has.

to cut the link, and (ii) for any $(i,j) \notin \Gamma^*$ with $i \neq j$, if $u_i(\Gamma^*) < u_i(\Gamma^* \cup (i,j))$ then $u_j(\Gamma^*) > u_j(\Gamma^* \cup (i,j))$, i.e., for any pair of non-linked countries, at least one of them has no incentive to form a link with the other.[19]

We are particularly interested in the situation where the entire world is involved in a large FTA, i.e., the world attains global free trade. This situation can also be described by graph theory terminology. A *complete* graph is a graph (N, Γ^{comp}) that contains all possible links, i.e., for any $i, j \in N$ with $i \neq j$, $(i,j) \in \Gamma^{comp}$. We call Γ^{comp} a *complete network*. The global free trade is a complete graph of the free trade network formation game.

3.3 Symmetric Countries

We say that countries i and j are *symmetric* if $s^i = s^j$ and $\mu^i = \mu^j$. This subsection considers the case in which the world consists of n symmetric countries so that $s^i = \mu^i = 1/n$ for any $i \in N$. In this case, country i's direct surplus effect can be simplified as

$$\begin{aligned} \Delta\left[X_j^i(\mathbf{t}^{-i}) - M_j^i(\mathbf{t}^i)\right] &= \mu^j \beta \Delta\left(\frac{s^i}{\mu^i} q_i^j(\mathbf{t}^j)^2 - \frac{s^j}{\mu^j} q_j^i(\mathbf{t}^i)^2\right) \\ &= \frac{\beta}{n}\left[\Delta(q_i^j(\mathbf{t}^j)^2) - \Delta(q_j^i(\mathbf{t}^i)^2)\right]. \end{aligned}$$

The current network structure affects the impact of the FTA between i and j on country i's industrial trade surplus through its effects on commodity demands. Especially important are the size of C_i and the size of C_j.

Let us say that countries i and j are *completely symmetric* if they are symmetric and $|C_i| = |C_j|$. If the original tariffs are the same between completely symmetric countries i and j, i.e., $t^i = t^j = t$, then $\bar{t}^i = \bar{t}^j$ and $q_i^j(\mathbf{t}^j) = q_j^i(\mathbf{t}^i)$, and hence we have $\Delta q_i^j(\mathbf{t}^j) = \Delta q_j^i(\mathbf{t}^i)$ and $\Delta X_j^i(\mathbf{t}^{-i}) = \Delta M_j^i(\mathbf{t}^i)$. Thus, the direct surplus effect disappears if countries i and j are completely symmetric and their original tariffs are the same. An increase in country i's export to country j and an increase in country i's import from county j are completely canceled out. On the other hand, the third country effect is nonnegative (if $n \geq 3$). Thus, we have $\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] \geq 0$ if countries i and j are completely symmetric.

Therefore, completely symmetric countries always have incentives to sign an FTA as long as one of the conditions in Lemma 3 is satisfied. One important case is that all pairs but

[19] Readers may be tempted to formulate a strategic form game such that each player (country) announces the names of players with whom she wants to be linked, and a link is formed if and only if both sides of the link announce each other's names. In such a game, however, there would be too many Nash equilibria, always including the one without any link. It is because a player has no incentive to announce the name of the player who does not announce her name. See Dutta and Mutuswami (1997) for the coalition-proof Nash equilibrium, a refinement of the Nash equilibrium in such games.

(i,j) have already formed free trade links. Since most tariffs are already eliminated, an FTA between i and j reduces distortions (condition (i) of Lemma 3), and hence enhances a consumer's gross utility in these countries ($\Delta V^i > 0$). Thus, the two countries can improve social welfare by signing an FTA, which leads to our first proposition.[20]

Proposition 1 *Suppose that there are n symmetric countries in the world, and that their external tariff rates are the same if they are imposed. Then, global free trade (a complete network Γ^{comp}) is a stable network.*

Proof. If all pairs but (i,j) have already formed free trade links, i.e., the free trade network is $\Gamma^{comp}\backslash(i,j)$, then countries i and j are completely symmetric. As a result, we know from the above observation that each country's industrial trade surplus does not decrease by signing an FTA. Moreover, since $s^{C_i} = 1 - \frac{1}{n}$ and $s^j = 1/n$, and similarly for country j, when the free trade network is $\Gamma^{comp}\backslash(i,j)$, we have $s^{C_i} + \frac{1}{2}s^j = 1 - \frac{1}{2n} > 1/2$ for all $n \geq 2$. Then, it follows from Lemma 3 and Remark 1 that consumers' gross utilities in countries i and j strictly increases. Therefore, we have

$$\begin{aligned} u_i(\Gamma^{comp}) &> u_i(\Gamma^{comp}\backslash(i,j)), \\ u_j(\Gamma^{comp}) &> u_j(\Gamma^{comp}\backslash(i,j)), \end{aligned}$$

implying that Γ^{comp} is a stable network.

Q.E.D.

Remark 2 *Note that this proposition holds even if we assume that each country adjusts its tariff rate optimally according to the current free trade network. Starting at the complete graph, if a pair of countries cut their FTA, they would impose the same tariff rates $\tau^i(1/n,(n-1)/n;\alpha,\beta,\delta)$ by symmetry. Thus, the assumptions of Proposition 1 are satisfied even if tariff rates are endogenously determined.*

A natural question now is whether or not the complete graph is a unique stable network. Unfortunately, it is not the case in general. If $q_i^j(\mathbf{t}^j)$ is significantly smaller than $q_j^i(\mathbf{t}^i)$ and hence $\Delta q_i^j(\mathbf{t}^j)$ is significantly smaller than $\Delta q_j^i(\mathbf{t}^i)$, the direct surplus effect for country i is

[20] Bagwell and Staiger (1999) argue that reciprocal trade liberalization between two countries is beneficial to both countries since it leaves each country's terms of trade unchanged so that it eliminates negative terms-of-trade externalities. An FTA between two completely symmetric countries fits their argument in that it leaves the bilateral (industrial commodity) terms of trade unaffected. In addition, each country's bilateral terms of trade against a third country improves as $q_k^i(\mathbf{t}^i)$ and hence $p_k^i(\mathbf{t}^i)$ decline for $k \neq i,j$.

negative and it may outweigh the third country effect. This situation arises when country j has many FTAs with other countries, while country i has a small number of FTAs as the following numerical example illustrates. As a benchmark, we set each country's tariff rate at $\tau(n) \equiv \tau^i(1/n, 1/n; \alpha, \beta, \delta)$, the optimal level without FTAs.

Example 1 *Suppose that countries are symmetric,* $n = 12$ *and* $\delta = 12\beta$*. Suppose further that* $t^i = \tau(n)$ *for any* $i \in N$*. In this case, graph* $\Gamma^{-1} = \{(j,k) : j,k \neq 1\}$ *(country 1 does not have any FTA, while all other countries have FTAs with one another) is pairwise stable. The reason why* Γ^{-1} *is stable is that the isolated country* 1 *does not have an incentive to have a bilateral FTA with any other country, although each of other countries has an incentive to sign a bilateral FTA with country* 1*.*

If δ is large, as in the above example, consumer demands are price sensitive. In the absence of an FTA, therefore, country 1 does not import much of industrial commodities, and most of industrial commodities consumed are domestically produced. However, once country 1 signs an FTA with country 2, say, much of (about a half of) the consumption of domestic commodities is substituted by those produced in country 2 so that country 1 experiences a dramatic increase in its import payments. In contrast, country 2 has already opened its market to all but country 1 before the FTA. Therefore, the FTA with country 1 does not increase its import much even if δ is large. Therefore, the direct surplus effect of country 1 is negative and is large in its magnitude, which outweighs the third country effect and the effect on $\Delta V^i(\mathbf{t}^i)$ in Example 1. Although it is hoped that (preferential) trade liberalization continues under GATT Article XXIV, it is quite possible that the process of FTA formation stops prematurely even if all countries are symmetric, as this example shows.

In the following, we seek a condition under which every pair of countries has incentives to form an FTA regardless of their existing FTAs. In such a case, it is obvious that the complete graph (global free trade) becomes the unique stable network.

Lemma 4 *Suppose that countries* i *and* j *are symmetric and that the external tariffs are the same, i.e.,* $t^i = t^j = t$*. Then, regardless of free trade networks of these two countries with the rest of the world,* $\Delta(X^i - M^i) > 0$ *holds if the following condition is satisfied.*

$$t \leq \frac{2\alpha\beta}{4\beta + (1 + \frac{1}{n})\delta}$$

If the optimal tariff rate, derived in Lemma 2, is smaller than $\frac{2\alpha\beta}{4\beta+(1+\frac{1}{n})\delta}$, we can conclude that a bilateral FTA between i and j increases the industrial trade surplus for these countries regardless of the network structure, under the mild condition that their external tariffs are not greater than their individual optimal tariffs. The next proposition states that it is indeed the case if the industrial commodities are not highly substitutable.

Proposition 2 *Suppose that there are n symmetric countries in the world, and that their external tariff rates are the same at t that is not greater than the optimal tariff rate without any FTA, i.e., $t \leq \tau(n)$. Suppose further that $\delta \leq 6\beta$. Then, under any network Γ, any pair of countries i and j without a bilateral FTA have incentives to form a free trade link. As a result, global free trade (a complete network Γ^{comp}) is the unique stable network.*

The significance of this proposition is that it applies regardless of these two countries' existing FTAs. Consider the case where there exist several FTAs, possibly different in size. If the industrial commodities are not highly substitutable among themselves, any pair of countries from different FTAs has incentives to form a new FTA. As far as trading blocs take the form of FTAs, as opposed to CUs, they are likely to be "building blocs" rather than "stumbling blocs" towards global free trade if countries are symmetric. Moreover, Proposition 2 suggests that if countries myopically make decisions as to whether or not they have FTAs with other countries, the world free trade network will eventually reach the complete network such that global free trade is effectively attained.

3.4 Asymmetric Countries

Now, let us turn to a more realistic case in which countries are asymmetric. Unfortunately, it is difficult to obtain any analytical result in a general setting. Thus, in this subsection, we assume $\delta = 0$ in order to simplify the analysis. This simplification is definitely restrictive, since it makes demands for industrial commodities completely independent of one another, and hence the third country effects vanish and the existing FTA network becomes irrelevant for each country's incentive to sign a new FTA. Despite of these restrictions, it is a convenient assumption for illustrating how the asymmetry of countries affects the FTA network formation. It follows from Lemma 2 that the optimal tariff rate is constant for any country i, irrespective of s^i, μ^i, and C_i. Therefore, we can naturally assume that each country imposes the optimal tariff rate, $t = \frac{\alpha}{3}$ as its external tariffs.

In this special case of no substitution among industrial commodities, we can easily calculate social welfare of each country. Since commodity demands are independent of one another when $\delta = 0$, the main part of a consumer's gross utility can be written as a simple sum of utilities derived from the consumption of all individual commodities. Let $p(t)$ and

$q(t)$ denote the equilibrium producer price and quantity of the industrial commodity that is faced with the tariff rate t, and let $v(t)$ denote a consumer's utility derived from the consumption of that commodity. Then, we can write

$$
\begin{aligned}
V^i(\mathbf{t}^i) &= \sum_{k \in C_i} s^k v(0) + \sum_{h \notin C_i} s^h v(\alpha/3) + l, \\
X^i(\mathbf{t}^{-i}) &= \frac{s^i}{\mu^i}\left[\sum_{k \in C_i \setminus \{i\}} \mu^k p(0)q(0) + \sum_{h \notin C_i} \mu^h p(\alpha/3)q(\alpha/3)\right], \\
M^i(\mathbf{t}^i) &= \sum_{k \in C_i \setminus \{i\}} s^k p(0)q(0) + \sum_{h \notin C_i} s^h p(\alpha/3)q(\alpha/3).
\end{aligned}
$$

As Figure 1 shows, we have $p(t) = (\alpha - t)/2$ and $q(t) = (\alpha - t)/2\beta$, and hence $v(\alpha/3) = 5\alpha^2/(18\beta)$, $p(\alpha/3)q(\alpha/3) = \alpha^2/(9\beta)$, $v(0) = 3\alpha^2/(8\beta)$, and $p(0)q(0) = \alpha^2/(4\beta)$. Thus, social welfare of country i can be written as

$$
\begin{aligned}
& W^i(\mathbf{t}) \\
&= V^i(\mathbf{t}^i) + X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i) \\
&= \frac{\alpha^2}{\beta}\left\{\frac{3}{8}s^{C_i} + \frac{5}{18}(1 - s^{C_i}) + \frac{s^i}{\mu^i}\left[\frac{1}{4}(\mu^{C_i} - \mu^i) + \frac{1}{9}(1 - \mu^{C_i})\right] - \left[\frac{1}{4}(s^{C_i} - s^i) + \frac{1}{9}(1 - s^{C_i})\right]\right\}.
\end{aligned}
$$

If countries i and j sign an FTA, then C_i expands to include j. Thus, the impact on country i's welfare is

$$
\begin{aligned}
\Delta W^i &= \frac{\alpha^2}{\beta}\left[s^j\left(\frac{3}{8} - \frac{5}{18}\right) + \frac{s^i}{\mu^i} \cdot \mu^j \left(\frac{1}{4} - \frac{1}{9}\right) - s^j\left(\frac{1}{4} - \frac{1}{9}\right)\right] \\
&= \frac{\mu^j \alpha^2}{\beta}\left[\frac{7}{72}\theta^j + \frac{5}{36}\left(\theta^i - \theta^j\right)\right].
\end{aligned}
$$

The first term in the brackets corresponds to ΔV^i, which is always positive (see Lemma 3 and Remark 1), and the latter corresponds to the direct surplus effect, $\Delta X^i_j - \Delta M^i_j$. As we have discussed in Section 3.1, the direct surplus effect depends on the two countries' industrialization levels. By rearranging the above formula, we obtain

$$
\Delta W^i = \frac{\mu^j \alpha^2}{72\beta}\left(10\theta^i - 3\theta^j\right).
$$

Country i has an incentive to sign an FTA with country j if and only if θ^j does not exceed $10/3$ times θ^i. This implies that two countries sign an FTA if their industrialization levels are not very different. The following proposition is an immediate consequence of this observation.

Proposition 3 *Suppose that $\delta = 0$ and that countries impose the optimal tariff rate $t = \alpha/3$ as their external tariffs. Then, countries i and j form a link if $\theta^j \leq \frac{10}{3}\theta^i$ and $\theta^i \leq \frac{10}{3}\theta^j$. The* *stable network is a collection of all links, each of which connects such a pair of countries, and is generically unique.*

This proposition gives us an interesting prediction. Suppose that there are two groups of countries: one is a group of developed countries with similar and high industrialization levels, and the other is a group of less developed countries with similar and low industrialization levels. Suppose also that the industrialization level of each developed country is far greater (more than 10/3 times) than the one of any less developed country. Then, the FTA formation process leads to a stable network in which all countries within each group are linked with each other, while there is no link across the two groups. The FTA formation process may end with two (stumbling) trading blocs if industrialization levels of two groups are very different from each other.

4 Free Trade Agreements vs. Customs Unions

This section investigates the difference in member countries' incentives to sign a new FTA emphasizing the fact that a CU requires that all members be involved when a member country wants to have a free trade link with an outside country. Our main goal of the paper is to assess how far the process of PTAs continues and whether or not global free trade is effectively attained as a complete world-wide web of PTAs. The analysis in this section may possibly tell us which form of PTAs, CU or FTA, should be encouraged for the purpose of facilitating more PTAs in the world. In order to focus on the issue, we assume that external tariff rates are fixed and the same in both cases.

First, we examine incentives for country i having engaged in free trade with other countries in C_i to have a new free trade link with country $j \notin C_i$, and compare the incentives between two cases: the case where C_i forms a CU and the case where C_i is a *regional FTA* such that every pair of countries in C_i has a bilateral FTA. Let us begin with investigating the impact on a consumer's gross utility V^i. As we have seen in Section 3.1, the impact on V^i is ambiguous in both cases. However, these effects are exactly the same between the two cases, since V^i only depends on $\mathbf{t}^i$ and changes in $\mathbf{t}^i$ are the same between the two cases. Thus, the difference in changes of the industrial trade surplus between these two cases will determine whether or not country i's incentive to have an FTA with country j is higher in the case where C_i is a CU rather than a regional FTA. Here, we decompose the third country effect into the member country and nonmember country effects:

$$\Delta X^i - \Delta M^i = \underbrace{\left(\Delta X^i_j - \Delta M^i_j\right)}_{\text{direct surplus effect}} + \underbrace{\sum_{k \in C_i \backslash \{i\}} \left(\Delta X^i_k - \Delta M^i_k\right)}_{\text{member country effect}} + \underbrace{\sum_{h \notin C_i \cup \{j\}} \left(\Delta X^i_h - \Delta M^i_h\right)}_{\text{nonmember country effect}},$$

where country k is a representative partner of i, i.e., $k \in C_i \backslash \{i\}$, and country h is a representative outsider of i, i.e., $h \notin C_i \cup \{j\}$. Table 1 depicts the signs of the effects, and compares these two cases item by item.

	FTA		CU
ΔV^i	?	=	?
ΔM^i_j	+	=	+
ΔX^i_j	+	>	+
ΔM^i_k	−	=	−
ΔX^i_k	0	>	−
ΔM^i_h	−	=	−
ΔX^i_h	0	=	0

Table 1. FTA vs. CU

We start with $M^i_j = M^i_j(\mathbf{t}^i)$, country i's import payments to country j. Since country i's import is solely determined by $\mathbf{t}^i$ and country i's post-FTA tariff vectors are the same between the two cases, the effects are exactly the same. This effect is positive since country i lowers its tariff rate for commodities imported from country j. In contrast, the effects on $X^i_j = X^i_j(\mathbf{t}^j)$ are different especially when $|C_i \backslash \{i\}|$ is large. It is because country j eliminates tariffs against all countries in C_i in the case of CU while it eliminates tariffs only for commodities imported from country i in the case of FTA. Since industrial commodities are substitutable among themselves, it is obvious that an increase in X^i_j is smaller in the case of CU. Consequently, the direct surplus effect is lower in the case of CU than in the case of FTA.

Next, we investigate the effects on country i's industrial trade surplus with a member country $k \in C_i \backslash \{i\}$. As before, the effects on $M^i_k = M^i_k(\mathbf{t}^i)$ are the same in both cases. However, the effects on $X^i_k = X^i_k(\mathbf{t}^k)$ are different again. In the case of FTA, $\mathbf{t}^k$ is unaffected and hence X^i_k does not change. In the case of CU, on the other hand, country k also eliminates tariffs against country j, and country i's export to country k is reduced due to the substitution effect. Country i's industrial trade surplus with a member country k is again lower in the case of CU. Finally, it is easy to see that the third country effect with nonmembers is the same in both cases. Import payments from country h decrease by the same amount due to the tariff reduction for commodities imported from country j, and country i's export to country h stays the same in both cases since $\mathbf{t}^h$ is not affected.

We have shown that the impacts of a new FTA on a consumer's gross utility are the same between the two cases, but the effect on the industrial trade surplus is unambiguously lower in the case of CU. We record this result as a lemma.

Lemma 5 *Country i has less incentive to have an FTA with country $j \notin C_i$ when C_i forms a CU rather than a regional FTA, unless the industrial commodities are independent of one another, i.e., $\delta = 0$, in which case the incentives are the same.*

Whether or not country i's incentive to have a free trade link with country j is lower when C_j forms a CU rather than a regional FTA is generally ambiguous, however. The difference between these two cases in our terminology is that country i adds only one link with country j in the case of a regional FTA, whereas in the case of a CU country i adds $|C_j|$ links simultaneously with all individual countries in C_j. The latter case is effectively equivalent to the case where country i has an FTA with an integrated economy that consists of all countries in C_j. Whether country i prefers having an FTA with country j alone or with the whole C_j depends on the relative characteristics of j and C_j.

However, we can make a strong statement in the symmetric country case with a low substitution parameter δ. Proposition 2 indicates that if all countries are symmetric, and δ is not very high, country i has an incentive to have an FTA with any country in any FTA configuration, in particular with country j alone or with all countries comprising C_j. Therefore, country i wants to have an FTA with country j whether C_j forms a CU or an FTA. Combining this observation together with Lemma 5, we find that two countries are less likely to have an FTA if either of them is a member of a CU in such a situation. Indeed, global free trade is the unique stable network if all PTAs take a form of FTA (Proposition 2), whereas several CUs of asymmetric size may co-exist in a stable network if all FTAs take a form of CU.[21]

[21] Employing a coalition bargaining game (see Bloch (1996) and Ray and Vohra (1999)), Yi (1996) shows that in equilibrium, two CUs of different size are formed when the world consists of a reasonable number of symmetric countries. We can conduct the same exercise in our model and obtain qualitatively the same result. Suppose that countries are symmetric, $n = 50$ and $\delta = 5\beta$. Suppose further that each country sets its external tariff rate at $\tau(n)$, the optimal level without any PTA. (Proposition 2 suggests that in this case, a unique stable network in the FTA regime is the complete graph Γ^{comp}.) Then, two CUs $\{43, 7\}$ are formed in equilibrium. The first proposer calls 43 countries to form a coalition, which is accepted by all countries involved. Then a country outside of this coalition calls all other remaining countries to form another coalition, which is also accepted by those countries.

Proposition 4 *An FTA is less likely to be signed if a country involved is a member of a CU rather than a regional FTA, if all countries are symmetric, imposing the same external tariff rate, and* $\delta \leq 6\beta$*, i.e., the industrial commodities are not highly substitutable among themselves.*

If countries are not symmetric, CUs can facilitate global trade liberalization more than FTAs. Consider again the asymmetric country case with $\delta = 0$. Let us order n asymmetric countries according to their industrialization levels such that $\theta^1 \geq \theta^2 \geq \cdots \geq \theta^n$. We know from Proposition 3 that if $\theta^1 > \frac{10}{3}\theta^n$, countries 1 and n will not sign an FTA, and the process of bilateral FTA formation will never reach global free trade. However, if all PTAs take a form of CU, the process of CU formation may reach global free trade. Let us consider a CU by $C(k) \equiv \{1, 2, ..., k\}$, formed by k countries with highest industrialization levels. The industrialization level of the entire $C(k)$, i.e., $\theta^{C(k)} \equiv \sum_{h \in C(k)} s^h / \sum_{h \in C(k)} \mu^h$, is the "average" industrialization level of all individual members of $C(k)$, so that $\theta^1 \geq \theta^{C(k)} \geq \theta^k$. Now, it follows from Proposition 3 that $C(k)$ and $k+1$ sign an FTA, or in other words, CU by $C(k)$ expands to include $k+1$, if $\theta^{C(k)} \leq \frac{10}{3}\theta^{k+1}$. Notice that this inequality can hold even if $\theta^1 > \frac{10}{3}\theta^{k+1}$. The CU formation averages out member countries' industrialization levels, and hence encourages a less industrialized country to join the group. In particular, if $\theta^{C(k)} \leq \frac{10}{3}\theta^{k+1}$ for any $k = 1, \cdots, n-1$, CUs serve as "building blocs" and the process of CU formation will reach global free trade.[22]

5 Concluding Remarks

We have introduced a general analytical framework that is suitable for the investigation of PTAs and shown how countries' incentives vary with the country size, industrialization level, substitutability among industrial commodities, etc. We have found that if all countries are symmetric, a complete global free trade network is pairwise stable and it is the unique stable network if industrial commodities are not highly substitutable among themselves. We have also compared FTAs and CUs as to which of these two regimes facilitates PTA formation. We have shown that in the symmetric country case where industrial commodities are not highly substitutable, countries are likely to have less incentives to have a new FTA if one of the countries is a member of a CU rather than an FTA. If countries are asymmetric, however, the CU formation averages out member countries' industrialization levels, which may help

[22] We should note that history of CU expansion may matter. It is possible for the CU expansion to stop prematurely if two unions, one by developed countries and the other by less developed countries, are formed, and the difference in the industrialization levels of these two unions is quite large.

further CU formation.

We must note that Propositions 2 and 4 are obtained under the assumption that external tariffs are fixed when countries form FTAs. Since Lemma 2 implies that a country's optimal tariffs decrease as the country have more free trade links, countries would lower their external tariffs as they form more free trade links. Indeed, countries' incentives to have an FTA are affected by this tariff adjustment. We have assumed that external tariff rates are exogenously fixed, since it is necessary to simplify the model for analyzing various forms of complicated FTA networks. Moreover, it is far from obvious that in reality, countries optimally adjust external tariffs every time when they sign a new FTA. Of course, it is interesting to know how these tariff adjustments affect countries' incentives to form PTAs. We leave this extension for future research.[23]

[23] Our primitive numerical analysis in the case of n symmetric countries shows that even if the optimal tariffs are constantly adjusted in the process of free trade network formation, a variant of Proposition 2 holds: Social welfare of each side of a new bilateral free trade agreement improves under any prevailing free trade network, if δ is not very high and n is reasonably large.

Appendix

Proof of Lemma 3. Using (6), we have

$$\begin{aligned}\sum_{k=1}^{n} s^k q_k^i(\mathbf{t}^i) &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}\sum_{k=1}^{n} s^k t_k^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i \\ &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta+\delta}\bar{t}^i.\end{aligned}$$

By substituting this result and (6) into $\partial V^i/\partial t_j^i$ in Lemma 1, we obtain

$$\frac{\partial V^i}{\partial t_j^i} = s^j\left[-\frac{\alpha\beta}{(2\beta+\delta)^2} + \frac{\delta^2}{4\beta(2\beta+\delta)^2}\bar{t}^i - \frac{1}{4\beta}t_j^i\right].$$

Let $\mathbf{t}(\gamma)$ denote the bilateral tariff reform schedule between countries i and j. This schedule satisfies $t_j^i(\gamma) = (1-\gamma)t^i$ and $t_i^j(\gamma) = (1-\gamma)t^j$, where $\gamma \in [0,1]$ and hence $t_j^i(0) = t^i$ and $t_j^i(1) = 0$, for example. All other tariff rates are kept unchanged, i.e., $t_k^i(\gamma) = t^i$ and $t_k^j(\gamma) = t^j$ for any k where $k \neq i, j$. Notice that $\bar{t}^i$ also changes in the course of tariff reform such that $\bar{t}^i(\gamma) = \sum_{k\notin C_i\cup\{j\}} s^k t^i + s^j(1-\gamma)t^i = (1 - s^{C_i} - \gamma s^j)t^i$, and similarly for $\bar{t}^j(\gamma)$. By substituting $\bar{t}^i(\gamma)$ and $t_j^i(\gamma)$ for $\bar{t}^i$ and t_j^i, respectively, and using $dt_j^i/d\gamma = -t^i$, we obtain

$$\frac{dV^i(\mathbf{t}^i(\gamma))}{d\gamma} = s^j t^i\left[\frac{\alpha\beta}{(2\beta+\delta)^2} - \frac{\delta^2}{4\beta(2\beta+\delta)^2}(1 - s^{C_i} - \gamma s^j)t^i + \frac{1}{4\beta}(1-\gamma)t^i\right].$$

By integrating over γ, the welfare change of country i due to the FTA with j becomes

$$\begin{aligned}\Delta V^i &\equiv V^i(0, \mathbf{t}_{-j}^i) - V^i(\mathbf{t}^i) \\ &= s^j t^i\left[\frac{\alpha\beta}{(2\beta+\delta)^2} - \frac{\delta^2}{4\beta(2\beta+\delta)^2}(1 - s^{C_i} - \frac{s^j}{2})t^i + \frac{1}{8\beta}t^i\right] \\ &= \frac{s^j t^i}{8\beta(2\beta+\delta)^2}\left\{8\alpha\beta^2 + \left[4\beta(\beta+\delta) - (1 - 2s^{C_i} - s^j)\delta^2\right]t^i\right\}. \qquad (15)\end{aligned}$$

The sufficient condition (i) immediately follows.

Let us suppose now that the condition (i) does not hold so that the terms in the square brackets of the last equation in the above are negative. Then the condition (ii) also follows immediately.

In order to derive the sufficient condition $\delta \leq 10\beta$ for $\Delta V^i(\mathbf{t}^i)$ to be positive, we need to show that $\tau(s^i, s^{C_i}; \alpha, \beta, \delta) \leq \frac{8\alpha\beta^2}{\delta^2(1-2s^{C_i}-s^j)-4\beta(\beta+\delta)}$ for any s^i, s^{C_i}, and s^j, when (i) is violated; i.e.,

$$\frac{4\alpha\beta(\beta+\delta s^i)}{3(2\beta+\delta)^2 - \delta(1-s^{C_i})[4(2\beta+\delta) - \delta(1-2s^i)]} \leq \frac{8\alpha\beta^2}{\delta^2(1-2s^{C_i}-s^j) - 4\beta(\beta+\delta)}.$$

Note that the left-hand side is decreasing in s^{C_i} and the right-hand side is increasing in s^{C_i} and s^j. Given that $s^{C_i} \geq s^i$, therefore, a sufficient condition for the above inequality is

$$\frac{\beta + \delta s^i}{3(2\beta+\delta)^2 - \delta(1-s^i)[4(2\beta+\delta) - \delta(1-2s^i)]} \leq \frac{2\beta}{\delta^2(1-2s^i) - 4\beta(\beta+\delta)}.$$

Letting $\hat{\beta} = \frac{\beta}{\beta+\delta}$ and $\hat{\delta} = \frac{\delta}{\beta+\delta}$, we rewrite the above inequality as

$$\frac{\hat{\beta} + (1-\hat{\beta})s^i}{3(1+\hat{\beta})^2 - (1-\hat{\beta})(1-s^i)[4(1+\hat{\beta}) - (1-\hat{\beta})(1-2s^i)]} \leq \frac{2\hat{\beta}}{(1-\hat{\beta})^2(1-2s^i) - 4\hat{\beta}}.$$

Then what we need to show is

$$6\hat{\beta}(1+\hat{\beta})^2 - 2\hat{\beta}(1-\hat{\beta})(1-s^i)[4(1+\hat{\beta}) - (1-\hat{\beta})(1-2s^i)] \\ - \left(\hat{\beta} + (1-\hat{\beta})s^i\right)\left[(1-\hat{\beta})^2(1-2s^i) - 4\hat{\beta}\right] \geq 0.$$

Now, we rewrite the left-hand side as

$$\begin{aligned}
&\hat{\beta}(1+\hat{\beta})^2 - 2\hat{\beta}(1-\hat{\beta})(1-s^i)[4(1+\hat{\beta}) - (1-\hat{\beta})(1-2s^i)] \\
&\quad - \left(\hat{\beta} + (1-\hat{\beta})s^i\right)\left[(1-\hat{\beta})^2(1-2s^i) - 4\hat{\beta}\right] \\
&= \hat{\beta}\left[6(1+\hat{\beta})^2 - 8(1-\hat{\beta})(1+\hat{\beta}) + (1-\hat{\beta})^2(1-2s^i) + 4\hat{\beta}\right] \\
&\quad +(1-\hat{\beta})s^i\left[8\hat{\beta}(1+\hat{\beta}) - 2\hat{\beta}(1-\hat{\beta})(1-2s^i) - (1-\hat{\beta})^2(1-2s^i) + 4\hat{\beta}\right] \\
&= \hat{\beta}\left[6(1+\hat{\beta})^2 - 8(1-\hat{\beta})(1+\hat{\beta}) + (1-\hat{\beta})^2 + 4\hat{\beta}\right] \\
&\quad +(1-\hat{\beta})s^i\left[8\hat{\beta}(1+\hat{\beta}) - 2\hat{\beta}(1-\hat{\beta}) - (1-\hat{\beta})^2 + 4\hat{\beta} - 2\hat{\beta}(1-\hat{\beta}) + 2s^i(1-\hat{\beta}^2)\right] \\
&= \hat{\beta}(15\hat{\beta}-1)(\hat{\beta}+1) + (1-\hat{\beta})s^i\left[(11\hat{\beta}-1)(\hat{\beta}+1) + 2s^i(1-\hat{\beta}^2)\right].
\end{aligned}$$

Since $\hat{\beta}$ takes a value in an interval $[0,1]$, it suffices that $\hat{\beta} \geq \frac{1}{11}$ for the above formula to take a positive value. This condition is equivalent to $\delta \leq 10\beta$.

Q.E.D.

Proof of Lemma 4. Recall from the proof of Lemma 3 the definition of the bilateral tariff reform schedule between countries i and j, denoted by $\mathbf{t}(\gamma)$ where $t^i_j(\gamma) = (1-\gamma)t$ and $\bar{t}^i(\gamma) = (1 - s^{C_i} - s^j\gamma)t = \left(1 - \frac{|C_i|}{n} - \frac{\gamma}{n}\right)t$, and similarly for j, while $\mathbf{t}^k(\gamma) = \mathbf{t}^k$ for any $k \neq i, j$, and any $\gamma \in [0,1]$. Then, it follows from (6) that

$$\begin{aligned}
q^i_j(\mathbf{t}^i(\gamma)) &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}(1-\gamma)t^i + \frac{\delta}{2\beta(2\beta+\delta)}\left(1 - \frac{|C_i|}{n} - \frac{\gamma}{n}\right)t^i, \\
q^i_k(\mathbf{t}^i(\gamma)) &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t^i + \frac{\delta}{2\beta(2\beta+\delta)}\left(1 - \frac{|C_i|}{n} - \frac{\gamma}{n}\right)t^i.
\end{aligned}$$

Consequently, we have

$$\begin{aligned}\frac{dq_j^i}{d\gamma} &= \frac{1}{2\beta}t - \frac{\delta}{2\beta(2\beta+\delta)n}t,\\ \frac{dq_k^i}{d\gamma} &= -\frac{\delta}{2\beta(2\beta+\delta)n}t.\end{aligned}$$

Now, we can rewrite a change in country i's industrial trade surplus.

$$\begin{aligned}&\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right]\\ &= \int_0^1 \left[\left(\frac{dX_j^i(\mathbf{t}^j(\gamma))}{d\gamma} - \frac{dM_j^i(\mathbf{t}^i(\gamma))}{d\gamma}\right) - \sum_{k\neq i,j}\frac{dM_k^i(\mathbf{t}^i(\gamma))}{d\gamma}\right]d\gamma\\ &= \frac{\beta}{n}\int_0^1\left[\left(2q_i^j(\mathbf{t}^j)\frac{dq_i^j}{d\gamma} - 2q_j^i(\mathbf{t}^i)\frac{dq_j^i}{d\gamma}\right) - \sum_{k\neq i,j}2q_k^i(\mathbf{t}^i)\frac{dq_k^i}{d\gamma}\right]d\gamma\\ &= \frac{2\beta t}{n}\int_0^1\left\{(|C_i|-|C_j|)\frac{\delta}{2\beta(2\beta+\delta)n}\left(\frac{1}{2\beta} - \frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right.\\ &\quad\left. + \left(\frac{\delta}{2\beta(2\beta+\delta)n}\right)\sum_{k\neq i,j}\left[\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t_k^i + \frac{\delta}{2\beta(2\beta+\delta)}\left(1-\frac{|C_i|}{n}-\frac{\gamma}{n}\right)t\right]\right\}d\gamma\\ &= \frac{\delta t}{n^2(2\beta+\delta)}\int_0^1\left\{(|C_i|-|C_j|)\left(\frac{1}{2\beta}-\frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right.\\ &\quad\left. +(n-2)\frac{\alpha}{2\beta+\delta} - [n-2+(|C_i|-1)]\frac{1}{2\beta}t + (n-2)\frac{\delta}{2\beta(2\beta+\delta)}\left(\frac{n-|C_i|-\gamma}{n}\right)t\right\}d\gamma.\end{aligned}$$

The value of this formula is minimized when $|C_i| = 1$ and $|C_j| = n-1$. Thus,

$$\begin{aligned}&\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right]\\ &\geq \frac{\delta t}{n^2(2\beta+\delta)}\int_0^1\left[-(n-2)\left(\frac{1}{2\beta}-\frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right.\\ &\quad\left. +(n-2)\frac{\alpha}{2\beta+\delta} - (n-2)\frac{1}{2\beta}t + (n-2)\frac{\delta}{2\beta(2\beta+\delta)n}(n-1-\gamma)t\right]d\gamma\\ &= \frac{(n-2)\delta t}{2n^2\beta(2\beta+\delta)^2}\int_0^1\left\{t\left[-2(2\beta+\delta)+\frac{n-\gamma}{n}\delta\right]+2\alpha\beta\right\}d\gamma.\end{aligned}$$

The contents in the curly brackets is nonnegative for any γ if and only if

$$\begin{aligned}t &\leq \frac{2\alpha\beta}{2(2\beta+\delta) - \frac{n-1}{n}\delta}\\ &= \frac{2\alpha\beta}{4\beta + \left(1+\frac{1}{n}\right)\delta}.\end{aligned}$$

Indeed, they are strictly positive under this condition, except at $\gamma = 1$ in which case they becomes zero. Therefore, $\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] > 0$ if this condition is satisfied.

Q.E.D.

Proof of Proposition 2. If $n = 2$, then $s^i = s^j = \frac{1}{2}$, and hence the industrial trade surplus does not change. Then, it follows from Lemma 3 that if $\delta \leq 10\beta$, countries i and j have incentives to sign an FTA.

If $n \geq 3$, on the other hand, the upper bound for t in Lemma 4 becomes important. As Lemma 2 shows, the optimal tariff is highest when a country does not have any FTA, i.e., $s^{C_i} = s^i$. Thus, in the case of n (≥ 3) symmetric countries, the optimal tariff rate under any cooperation structures is bounded above by $\tau(n)$. Now, let us examine how $\tau(n)$ varies with n. To see this, we define the function τ^* by $\tau^*(s^i) = \tau(1/s^i)$, keeping in mind that $n = 1/s^i$ for any i when all countries are completely symmetric. Then, $\tau(n) \leq \tau(3)$ for any $n \geq 3$ if and only if $\tau^*(s^i) \leq \tau^*(1/3)$ for any $s^i \leq 1/3$.

Since $\tau^*(s^i) = \tau^i(s^i, s^i; \alpha, \beta, \delta)$ for a given set of α, β, and δ, we have $d\tau^*/ds^i = \partial\tau^i/\partial s^i + \partial\tau^i/\partial s^{C_i}$. Then, it follows from the optimal tariff formula in Lemma 2 that

$$\begin{aligned}
&\left\{3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i)\right\}^2 \frac{d\tau^*}{ds^i} \\
&= 4\alpha\beta\delta\{3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i)\} \\
&\quad -4\alpha\beta\{(\beta+\delta) - \delta(1-s^i)\}\{4(2\beta+\delta)\delta - \delta^2(1-2s^i) - 2\delta^2(1-s^i)\} \\
&= 4\alpha\beta\delta[4\beta^2 + 3\beta\delta - 4s^i\beta\delta - 2(s^i)^2\delta^2],
\end{aligned}$$

or

$$\frac{d\tau^*}{ds^i} = 4\alpha\beta\delta\frac{A(s^i)}{B(s^i)^2},$$

where

$$\begin{aligned}
A(s^i) &= 4\beta^2 + 3\beta\delta - 4s^i\beta\delta - 2(s^i)^2\delta^2, \\
B(s^i) &= 3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i).
\end{aligned}$$

Now, we show that if $\delta \leq 9\beta$, then $A(s^i) > 0$ and $B(s^i) > 0$ for any $s^i \in [0, 1/3]$, which implies that τ^* is increasing on $[0, 1/3]$. First, it is easy to see that $A(0) > 0$ and $dA/ds^i = -4\beta\delta - 4s^i\delta^2 < 0$. We can also see $A(\frac{1}{3}) = \frac{1}{9}(36\beta^2 + 15\beta\delta - 2\delta^2) > 0$ if $\delta \leq (15 + \sqrt{513})/4$, which in turn is satisfied if $\delta \leq 9\beta$. Thus, $A(s^i) > 0$ on $[0, 1/3]$ if $\delta \leq 9\beta$. Turning to $B(s^i)$, it is also easy to see that $B(0) = 4\beta(3\beta+\delta) > 0$ and

$$\begin{aligned}
\frac{dB}{ds^i} &= 4(2\beta+\delta)\delta - \delta^2(1-2s^i) - 2\delta^2(1-s^i) \\
&= 8\beta\delta + \delta^2(1+4s^i) > 0.
\end{aligned}$$

Thus, we have $B(s^i) > 0$ for any $s^i \in [0, 1/3]$.

We have found that $d\tau^*/ds^i$ is positive for any $s^i \in (0, 1/3]$ so that τ^* takes the largest value at $s^i = 1/3$. Thus, if $\delta \leq 9\beta$, then $\tau^*(s^i) \leq \tau^*(1/3)$ for any $s^i \leq 1/3$ and hence $\tau(n) \leq \tau(3)$ for any $n \geq 3$.

Next, we calculate the optimal tariff rate when $n = 3$.

$$\begin{aligned} \tau(3) &= \frac{4\alpha\beta\left(\beta + \frac{1}{3}\delta\right)}{3(2\beta+\delta)^2 - \left(4(2\beta+\delta)\delta - \frac{1}{3}\delta^2\right)\frac{2}{3}} \\ &= \frac{12\alpha\beta(3\beta+\delta)}{108\beta^2 + 60\beta\delta + 5\delta^2}. \end{aligned}$$

Since $2\alpha\beta/[4\beta + (1+\frac{1}{n})\delta]$ is increasing in n, we need only show $\tau(3) \leq 2\alpha\beta/[4\beta + (1+\frac{1}{3})\delta]$ to prove that the condition in Lemma 4 is satisfied. Now, we have

$$\frac{2\alpha\beta}{4\beta + \frac{4}{3}\delta} - \tau(3) = \frac{9\alpha\beta}{2(3\beta+\delta)(108\beta^2 + 60\beta\delta + 5\delta^2)}\left(6\beta - \delta\right)\left(2\beta + \delta\right).$$

This value is nonnegative if and only if $\delta \leq 6\beta$. Thus, as long as $\delta \leq 6\beta$ (as we have seen before, $\delta \leq 9\beta$ guarantees that $\tau(n)$ is decreasing in n for $n \geq 3$), the condition in Lemma 4 is satisfied when $n \geq 3$. Moreover, if $\delta \leq 6\beta$, then $\delta \leq 10\beta$ is also satisfied, so that Lemma 3 implies that a consumer's gross utility increases by the FTA. Consequently, countries i and j have incentives to have an FTA if $\delta \leq 6\beta$.

Q.E.D.

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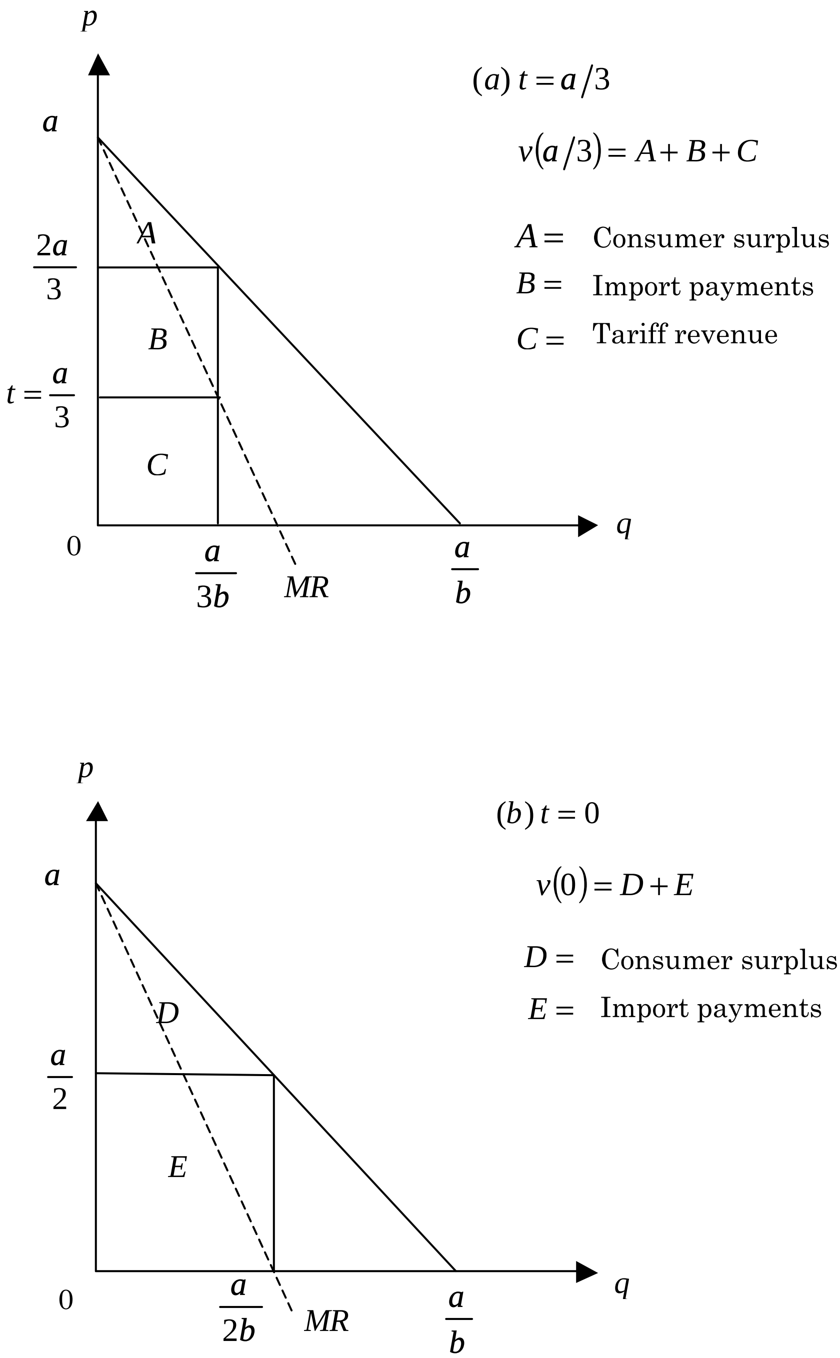

Figure 1. The Total Surplus in a Commodity Market $(d = 0)$

[note: MR =Marginal revenue curve]