

**COE-RES Discussion Paper Series
Center of Excellence Project
The Normative Evaluation and Social Choice of
Contemporary Economic Systems**

**Graduate School of Economics and Institute of Economic Research
Hitotsubashi University**

COE/RES Discussion Paper Series, No.249

March 2008

A Factor-Proportions Theory of Firm Heterogeneity

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A Factor-Proportions Theory of Endogenous Firm Heterogeneity*

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This Draft: August 2008

Abstract

In the model where the choice of technology by firms endogenously determines productivity differences, we investigate the link between factor endowment and the productivity both in the firm and industry levels. We find among others that firms in capital-abundant countries tend to adopt new advanced technologies more in their production processes, but opening to international trade will equalize factor prices across countries and as a result productivity differences across countries disappear, if their states of technologies are the same. If the states of technologies are different, on the other hand, the country with a higher state of technology has a lower wage-rental ratio than the other. We argue that the capital-abundant country tends to have a higher state of technology, so if that is the case the wage-rental ratio is lower in the capital-abundant country in free trade.

JEL Classification Numbers: F12, O30

Keywords: technology adoption, firm heterogeneity, international trade

*We are grateful to Yoichi Sugita and Robert Staiger for helpful comments. We are also helpful for participants of Asia Pacific Trade Seminars 2007, Far Eastern Meeting of the Econometric Society, Hitotsubashi Conference, Midwest International Group Meeting, and seminar participants at Federal Reserve Bank of Dallas, National University of Singapore, Ritsumeikan University, Tohoku University, and Yale University for fruitful discussions and comments.

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1 Introduction

Productivity varies across firms even within the same industry. Some firms are more efficient than others as they hire more-productive workers or they adopt more-advanced production technology. Bartel and Lichtenberg (1987) show the evidences that educated workers have comparative advantage in implementing new technology. Doms et al. (1997) also find that the plants that adopt new automation technology tend to have more skilled workers. Even if new advanced production technology is in the public domain, some firms choose not to adopt it perhaps due to the lack of human capital that is essential to the adoption of new technology.

This linkage between the difference in the intrinsic productivity of firms, due to the difference in the amount of human capital, and the adoption rate of new technology explains the coexistence of capital-intensive advanced technology and labor-intensive old technology within the same industry. Anecdotal evidences suggest that even the same firm often uses different technologies depending on the plant location. Firms tend to adopt capital-intensive advanced technology in capital-abundant developed countries while they tend to use labor-intensive old technology in labor-abundant less-developed countries. Firms in less-developed countries use less-advanced technology on average, not just because of its lack of capacity to absorb new technology but perhaps because that less-developed countries tend to be labor-abundant so firms that operate there deliberately adopt old technology to save capital.¹

In this paper, we formalize the idea that the endowment of production factors affects firm-level productivity heterogeneity through firms' technology adoption. To this end, we extend the framework of monopolistic competition and heterogeneous firms developed by Melitz (2003), by introducing firms' appropriate technology choices à la Zeira (1998, 2006). Melitz (2003) introduces productivity difference across firms within the same industry and builds a general equilibrium model in which less productive firms sell only domestically while productive firms sell both domestically and internationally. He also investigates how trade liberalization affects firms with different productivity levels differently. In his model, different productivity levels are assigned to *ex ante* symmetric firms exogenously, and productivity of a firm remains unchanged once it is assigned. In reality, however, efficiency gaps between productive firms and unproductive ones tend to expand with technological

¹Examining a sample of 230 U.S. cities over the period from 1980 to 2000, Beaudry et al. (2006) find that firms located in cities endowed with relatively abundant and cheap skilled labor on average adopted personal computers more aggressively than cities with relatively expensive skilled labor.

progress.

In our model, firms are allowed to choose either labor-intensive old technology or capital-intensive new technology; unlike Zeira (1998, 2006), however, firms are different in their intrinsic productivity levels, which leads to the dispersion of technology adoption within the same industry. Labor-intensive old technology is standardized so that all firms can utilize it in the same manner. Capital-intensive new technology, on the other hand, requires cognitive skills that vary across firms. Thus, the difference in productivity arises once they adopt new production technologies. Our model specification is rich enough that efficiency gap between inherently-productive firms and inherently-unproductive firms expands with technological progress; inherently-productive firms become more efficient as the technology advances while inherently-unproductive firms stop adopting new technologies once the technological frontier reaches certain levels.

In the model where firms' technological choice endogenously determines productivity differences, we investigate the link between the factor endowment and productivity both in the firm and industry levels. The wage-rental ratio in free trade will be higher in the country with a higher frontier of new technology, which we argue may well depend on the factor endowment. We find among others that firms in capital-abundant countries tend to adopt new advanced technologies more in their production processes, but opening to international trade will equalize factor prices across countries and as a result productivity differences across countries disappear, if their technological frontiers are the same. If the technological frontiers are different, on the other hand, the country with a higher technological frontier has a lower wage-rental ratio than the other. We argue that the capital-abundant country tends to have a higher state of technology, so if that is the case the wage-rental ratio is lower in the capital-abundant country in free trade.

Bernard et al. (2007) also investigate the link between the factor endowment and the industry-wide productivity distribution across firms, assuming as Melitz (2003) that the productivity of each firm is exogenously given. Ederington and McCalman (2008, forthcoming) consider the models in which ex ante identical firms become heterogeneous in their productivities as a result of their diversified timing of the date of technology adoption. In their model, firms are faced with the tradeoff between adopting the new technology at an earlier date so that they obtain cost advantage over their rivals and delaying the adoption until the adoption cost becomes sufficiently small. In equilibrium, firms are indifferent as to when to adopt the new technology, so they are dispersed in their adoption timing of the

new technology. Unlike theirs, our model emphasizes the effects of factor endowments on technology adoption.

2 The Model

This section lays out the basic one-country model that contains two sectors, two factors, and a continuum of potentially heterogeneous firms; section 4 extends the model to a two-country free-trade model. One sector (sector Z) competitively produces a homogenous, numeraire good from labor and capital. The other sector (sector Y) produces a continuum of differentiated varieties from a continuum of inputs, which in turn are produced from labor and capital.

2.1 Preferences

We consider an economy with many identical consumers who own as a whole L units of labor and K units of capital. A representative consumer's preferences are characterized by the utility function

$$U = \left[\int_{i \in \Omega} y(i)^\alpha di \right]^{\gamma/\alpha} z^{1-\gamma}, \quad \gamma \in (0, 1),$$

where $y(i)$ denotes the consumption of variety i of good Y , Ω the set of available varieties, and z the consumption of the homogeneous good. The varieties of differentiated goods are substitutable from one another with the elasticity of substitution $\sigma = 1/(1 - \alpha) > 1$. These preferences yield the iso-elastic demand function for each variety i such that

$$y(i) = \frac{\gamma E p_y(i)^{-\sigma}}{P^{1-\sigma}},$$

where E represents the total expenditure, $p_y(i)$ the price of variety i , and P the aggregate price index in sector Y such that

$$P = \left[\int_{i \in \Omega} p_y(i)^{1-\sigma} di \right]^{1/(1-\sigma)}. \quad (1)$$

2.2 Production Technology

The numeraire good Z is produced from labor and capital with a constant-returns-to-scale production technology characterized by the unit cost function $c_z(w, r)$, where w and r are the wage rate and rental rate, respectively. Perfect competition ensures that

$$c_z(w, r) = 1. \quad (2)$$

For analytical concreteness, we specify the functional form of $c_z(w, r)$ such that $c_z(w, r) = w^{1-m_z} r^{m_z}$ where $m_z \in (0, 1)$.

Production of a variety of good Y requires a continuum of inputs, which can be either intermediate goods or tasks. The set of inputs is given by the interval $[0, 1]$. The following Cobb-Douglas function characterizes the process of production for each variety i from the inputs $\{x_i(j)\}_{j \in [0, 1]}$:

$$y(i) = \exp \left[\int_0^1 \ln x_i(j) dj \right]. \quad (3)$$

The inputs are produced from labor and capital. There potentially exist two types of technologies for each input production. The first one is a standardized technology that is labor intensive. We assume for simplicity that the production of each unit of input j requires $l(j)$ units of labor without any input of capital. This production technology is well known to the economy and always available to any firm.

In contrast, the second type is a new technology that replaces labor with capital, i.e., it is capital intensive. The new technology is skill complement in the sense that firms with higher productivity levels use it more efficiently than others. Specifically, we assume that the input production with the new technology requires capital only such that the unit requirement of capital equals $k(j)/\varphi$ for input j where $\varphi > 0$ denotes the firm-specific intrinsic productivity level. Moreover, the new technology is not in general available for all the inputs; it is available only up to a certain input on the technological frontier that expansion can be considered as technological progress of the industry (or the country) as a whole.

We order inputs in the interval $[0, 1]$ so that the one with a higher index corresponds to a larger requirement of capital that is needed to replace one unit of labor when firms switch from the old technology to the new one. Namely, defining the following function $\lambda(\varphi, j)$ by

$$\lambda(\varphi, j) = \frac{k(j)}{\varphi l(j)},$$

we order j such that $\lambda(\varphi, j)$ is non-decreasing in j , which implies that the new technology is more efficient for firms with higher productivity levels in the sense that they would save the same amount of labor with less capital than those with lower productivity levels.

We define the technological frontier as a critical input $\theta \in [0, 1]$ such that new technology is known for any input $j \in [0, \theta]$ while it is yet to be discovered for any remaining input $j \in (\theta, 1]$. Firms can choose whether or not to adopt the new technology for input $j \in [0, \theta]$

whereas they have to use the old technology to produce the remaining set of inputs. An increase in θ means that the new technology becomes available for more inputs with higher λ . This specification captures the real-world observation that newly discovered technologies are skill complement and require more capital to replace labor than those discovered earlier. These specifications are consistent with empirical findings by Autor et al. (1998), Doms et al. (1997), and Bresnahan et al. (2002), for example.

Firms adopt the new technology as long as it reduces the production costs. That is, they choose the new technology rather than the old one for the production of input j if and only if $rk(j)/\varphi \leq wl(j)$, or equivalently $\lambda(\varphi, j) \leq w/r$. Of course, they cannot adopt the new technology for input $j > \theta$. This constraint is binding for productive firms with φ higher than $rk(\theta)/[wl(\theta)]$. At the other extreme, unproductive firms with φ lower than $rk(0)/[wl(0)]$ will not adopt the new technology at all. But otherwise, firms adopt the new technologies for all inputs up to m that satisfies $\lambda(\varphi, m) = w/r$. Letting λ_φ^{-1} denote the inverse function of λ with respect to the second argument for a given φ , therefore, we write a firm's adoption level of the new technology—the extent to which the firm with φ adopts the new technology—as

$$m(\varphi, w, r, \theta) = \begin{cases} 0 & \text{if } \varphi \leq rk(0)/[wl(0)], \\ \lambda_\varphi^{-1}(w/r) & \text{if } rk(0)/[wl(0)] < \varphi < rk(\theta)/[wl(\theta)], \\ \theta & \text{if } \varphi \geq rk(\theta)/[wl(\theta)]. \end{cases} \quad (4)$$

The unit variable costs for firms with productivity level φ are given by

$$c(\varphi, w, r, \theta) = \exp \left[\int_0^{m(\varphi, w, r, \theta)} \ln rk(j) dj - m(\varphi, w, r, \theta) \ln \varphi + \int_{m(\varphi, w, r, \theta)}^1 \ln wl(j) dj \right]. \quad (5)$$

Figure 1 illustrates the technology choice by three representative firms with different productivities which are faced with a given wage-rental ratio $(w/r)'$ and technological frontier θ' . The upper panel shows the graphs of $\lambda(\varphi, \cdot)$ with three levels of φ . The function λ is non-decreasing in j by construction. For the firm with the highest productivity φ_1 , $\lambda(\varphi_1, j) < (w/r)'$ for the entire range of inputs. The firm adopts the new technologies for every input $j \leq \theta$, and hence, its unit variable cost is monotonically decreasing in θ as the graph of $c(\varphi_1, w, r, \cdot)$ shows in the lower panel of the figure. The firm with the intermediate productivity φ_2 adopts the new technologies up to $\lambda_\varphi^{-1}((w/r)')$ if they are available. But it uses the old technologies for all inputs beyond this range as the new technologies are too costly for the firm; the unit variable cost $c(\varphi_2, w, r, \theta)$ is flat for $\theta \geq \lambda_\varphi^{-1}((w/r)')$. Finally, the firm with the lowest productivity considers the new technologies to be too costly to adopt

so that it sticks to the old technologies for all inputs even though the new technologies are available. The unit variable cost $c(\varphi_3, w, r, \theta)$ is constant in θ .

As the lower panel of Figure 1 illustrates, the higher is the technological frontier θ , the larger is the disparity in the unit variable costs across firms with different intrinsic productivities. In this sense, firms become more heterogeneous in their productivities as the technological frontier expands (i.e., θ increases). Firms with lower intrinsic productivities drop out of the technology adoption race and cease to adopt the new technologies for more inputs while those with higher intrinsic productivities keep up with the pace of technological advances and continue to adopt the new technologies.

Moreover, all firms but those with the highest (and possibly lowest) intrinsic productivities adopt the new technologies for more inputs when the wage-rental ratio becomes higher; they have more incentive to replace labor with capital when faced with a higher wage-rental ratio. Firms with the highest intrinsic productivities adopt the new technologies for all inputs up to the technological frontier and their adoption level is unaffected by a small change in the wage-rental ratio.

With the optimal choice of production technology, firms operate under monopolistic competition. As is standard in the monopolistic competition model á la Dixit and Stiglitz (1977), firm i with productivity level φ sets the price at $p(c(\varphi, w, r, \theta)) = c(\varphi, w, r, \theta)/\alpha$ and produces

$$y(\varphi, w, r, \theta) = \frac{\gamma E}{P} \left[\frac{p(c(\varphi, w, r, \theta))}{P} \right]^{-\sigma}$$

units of variety i .² The production of differentiated varieties also requires fixed costs. We assume that all firms have a common fixed cost of purchasing f units of the numeraire good Z . Then, the profit, net of the fixed cost, is expressed as

$$\pi(\varphi, w, r, \theta) = \frac{\gamma E}{\sigma} \left[\frac{p(c(\varphi, w, r, \theta))}{P} \right]^{1-\sigma} - f.$$

The profit $\pi(\varphi, w, r, \theta)$ is increasing in φ if φ is high enough to adopt the new technology for at least some input: i.e., $\varphi > rk(0)/[wl(0)]$. Otherwise, the profits are independent of firms' intrinsic productivities as they use the old technologies for all inputs.

Finally, it is useful to note that the ratio of any two firms' outputs and that of revenues $R(\varphi, w, r, \theta)$ depend only on the ratio of their unit variable costs: i.e.,

$$\frac{y(\varphi_1, w, r, \theta)}{y(\varphi_2, w, r, \theta)} = \left[\frac{c(\varphi_1, w, r, \theta)}{c(\varphi_2, w, r, \theta)} \right]^{-\sigma}, \quad \frac{R(\varphi_1, w, r, \theta)}{R(\varphi_2, w, r, \theta)} = \left[\frac{c(\varphi_1, w, r, \theta)}{c(\varphi_2, w, r, \theta)} \right]^{1-\sigma}. \quad (6)$$

²With the abuse of notation, we redefine y as a function of φ, w, r , and θ .

2.3 Entry and Exit

We model firms' entry to the good Y sector similarly to Melitz (2003). In order to obtain a blueprint of a variety, potential firms are required to invest f_e units of good Z , and independently draw their intrinsic productivity levels φ from the common probability distribution on $(0, \infty)$ which is characterized by the cumulative distribution function H with the continuous density function h . Firms enter sector Y until their expected profits become equal to the entry cost f_e . As the unit variable cost $c(\varphi, w, r, \theta)$ is decreasing and hence $\pi(c(\varphi, w, r, \theta))$ is increasing in φ , firms with φ smaller than a critical value, which we call φ^* , would earn negative profits and hence immediately exit from the market. This critical productivity φ^* is given by

$$\frac{\gamma E}{\sigma} \left[\frac{p(c(\varphi^*, w, r, \theta))}{P} \right]^{1-\sigma} = f. \quad (7)$$

We focus on the case in which the cutoff productivity φ^* exceeds $rk(0)/[wl(0)]$ so that all operating firms adopt the new technologies at least for some inputs. Letting N_e be the mass of the entrants and N the mass of surviving firms, we have, therefore, the relationship between N_e and N as

$$[1 - H(\varphi^*)]N_e = N. \quad (8)$$

Now, we can express key variables in terms of φ^* , w , r , and θ . First, we define the conditional distribution of φ for operating firms as

$$\mu_{\varphi^*}(\varphi) = \begin{cases} \frac{h(\varphi)}{1 - H(\varphi^*)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{otherwise.} \end{cases}$$

The price index P given by (1) can be written as:

$$P = \left[\int_0^\infty p(c(\varphi, w, r, \theta))^{1-\sigma} N d\mu_{\varphi^*}(\varphi) \right]^{1/(1-\sigma)} = N^{1/(1-\sigma)} p(\tilde{c}(\varphi^*, w, r, \theta)), \quad (9)$$

where $\tilde{c}(\varphi^*, w, r, \theta)$ denotes the unit variable cost for the average firms:

$$\tilde{c}(\varphi^*, w, r, \theta) = \left[\int_0^\infty c(\varphi, w, r, \theta)^{1-\sigma} d\mu_{\varphi^*}(\varphi) \right]^{1/(1-\sigma)}. \quad (10)$$

The price $p(\tilde{c}(\varphi^*, w, r, \theta))$ is the average price in the sense that the price index P can be replicated by N *homogenous* firms that set the price $p(\tilde{c}(\varphi^*, w, r, \theta))$ as shown in (9). Substituting (9) into (7), we can rewrite the zero-profit condition as

$$\frac{\gamma E}{\sigma N} \left[\frac{c(\varphi^*, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} = f. \quad (11)$$

Note that the gross profit for the critical firm expressed on the left-hand side equals $R(\varphi^*, w, r, \theta)/\sigma$ where $R(\varphi^*, w, r, \theta)$ is the revenue for the average firms, $\gamma E/N$, multiplied by $[c(\varphi^*, w, r, \theta)/\tilde{c}(\varphi^*, w, r, \theta)]^{1-\sigma}$ as (6) indicates.

The free-entry condition ensures that the *ex ante* expected profit equal the entry cost f_e . Noting that the profit of the average firms, net of the fixed cost, is $\tilde{\pi} = (\gamma E)/(\sigma N) - f$, the free entry condition can be expressed as

$$[1 - H(\varphi^*)] \left[\frac{\gamma E}{\sigma N} - f \right] = f_e. \quad (12)$$

Using the zero-profit condition (11), the profit earned by the average firm can be written as

$$\tilde{\pi}(\varphi^*, w, r, \theta) = f\phi(\varphi^*, w, r, \theta), \quad (13)$$

where $\phi(\varphi^*, w, r, \theta) \equiv [\tilde{c}(\varphi^*, w, r, \theta)/c(\varphi^*, w, r, \theta)]^{1-\sigma} - 1$. Substituting this expression into the free-entry condition (12), we obtain

$$\Phi(\varphi^*, w, r, \theta) \equiv [1 - H(\varphi^*)] \phi(\varphi^*, w, r, \theta) = f_e/f. \quad (14)$$

Equation (14) restates the free-entry condition in terms of φ^* , w , r , and θ . It should be noted that unlike Bernard et al. (2007), the free-entry condition (14) depends on factor prices. This is because in our model, firms are heterogeneous in their intrinsic productivities and hence they are different in their factor intensities.

2.4 Factor Market

Having obtained the free-entry condition (14), we turn our attention to factor-market clearing conditions to derive the equilibrium. Capital-market clearing requires $K = K_{Yp} + K_{Yf} + K_{Yfe} + K_Z$ where K_{Yp} denotes capital demands as variable inputs in the production of good Y , K_{Yf} and K_{Yfe} capital demands to cover the fixed cost and the entry investment, respectively, and K_Z capital used in sector Z .

Applying Shephard's lemma to (5), we obtain the capital requirements for the unit production of a variety as $\partial c/\partial r = c(\varphi, w, r, \theta)m(\varphi, w, r, \theta)/r$. Then, defining the average level of technological adoption by

$$\tilde{m}(\varphi^*, w, r, \theta) = \int_0^\infty m(\varphi, w, r, \theta) \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi), \quad (15)$$

where the relative revenues are used as the weights (see (6)), we can rewrite the total amount of capital used as variable inputs in the production of varieties of good Y as

$$K_{Yp} = \int_0^\infty \frac{c(\varphi, w, r, \theta)m(\varphi, w, r, \theta)}{r} y(\varphi, w, r, \theta) N d\mu_{\varphi^*}(\varphi) = \frac{\alpha \gamma E}{r} \tilde{m}(\varphi^*, w, r, \theta).$$

Due to the mark-up pricing where price equals $1/\alpha$ times the unit costs, the fraction α of the total revenue covers the variable costs while the remaining $1 - \alpha$ of it is the gross profits for every firm. Noting that the production function given by (3) is of the Cobb-Douglas type, this implies that $\alpha\gamma E\tilde{m}(\varphi^*, w, r, \theta)$ represents the total spending on capital as variable inputs by firms in sector Y .

Total capital demands in sector Y to cover the fixed costs of production and the investment cost upon entry can be written, with the use of (8), as $K_{Yf} + K_{Yfe} = N[f + f_e/(1 - H(\varphi^*))]m_z/r$. The capital demands in sector Z is simply $m_z(1 - \gamma)E/r$. Consequently, the capital-market clearing condition is expressed as

$$\alpha\gamma E\tilde{m}(\varphi^*, w, r, \theta) + N \left[f + \frac{f_e}{1 - H(\varphi^*)} \right] m_z + (1 - \gamma)Em_z = rK.$$

Using (12), we can rewrite this equation as

$$\{\alpha\gamma\tilde{m}(\varphi^*, w, r, \theta) + [(1 - \alpha)\gamma + 1 - \gamma]m_z\}E = rK. \quad (16)$$

Note that the total gross profits in sector Y equal $(1 - \alpha)\gamma E$, which in turn equal the sum of total fixed costs and total investment costs.

Likewise, the labor market clearing condition is given by

$$\{\alpha\gamma[1 - \tilde{m}(\varphi^*, w, r, \theta)] + [(1 - \alpha)\gamma + 1 - \gamma](1 - m_z)\}E = wL. \quad (17)$$

Eliminating E from these two factor-market clearing conditions, (16) and (17), we obtain a rather familiar form of the factor-market clearing condition:

$$\frac{w}{r} = \frac{1 - \hat{m}(\varphi^*, w, r, \theta)}{\hat{m}(\varphi^*, w, r, \theta)} \frac{K}{L}, \quad (18)$$

where \hat{m} is the average capital share of the economy such that $\hat{m}(\varphi^*, w, r, \theta) \equiv \alpha\gamma\tilde{m}(\varphi^*, w, r, \theta) + [(1 - \alpha)\gamma + 1 - \gamma]m_z$.

Since the extent to which the average firms adopt the new technology, \tilde{m} (which critically affect \hat{m}), is a summary index of technology adoption in the economy, it is important to understand how \tilde{m} depends on the cutoff productivity level, φ^* , and factor prices, w and r . If the cutoff productivity level exceeds a certain value, which will be discussed further in the next section, all firms adopt the new technologies for all inputs. As a result, the average productivity level \tilde{m} equals θ and becomes insensitive to φ^* , w , and r . Otherwise, there will be the two types of firms in equilibrium: firms that partially adopt the new technologies and firms that fully adopt them. As expected, given that not all firms adopt

the new technologies up to the technological frontier θ , the average technology adoption \tilde{m} is increasing in φ^* and w and decreasing in r .

Lemma 1. *Unless all firms in sector Y adopt the new technologies for all inputs up to θ , we have*

$$\frac{\partial \tilde{m}}{\partial \varphi^*} > 0, \quad \frac{\partial \tilde{m}}{\partial w} > 0, \quad \frac{\partial \tilde{m}}{\partial r} < 0, \quad \frac{\partial \tilde{m}}{\partial \theta} > 0. \quad (19)$$

Proof. See the Appendix. □

An increase in the cutoff productivity level φ^* forces firms with the lowest productivity levels to exit from the market so that the average level of technology adoption rises. The responses of the average technology adoption to changes in the factor prices involve somewhat more, although they also seem intuitive. Obviously, if the wage rate decreases or the rental rate increases, firms have less incentive to adopt the new technologies, which results in a smaller \tilde{m} . In addition to this substitution effect, however, changes in the factor prices affect the average adoption level through changing firms' weights in the average technology adoption. Recall that we use the ratios of each firm's revenue to the average revenue as the weights when we calculate the average technological adoption level \tilde{m} (see (6) and (15)). A decrease in w or an increase in r will lower the revenues for productive firms relative to the average since productive firms use more capital-intensive technology to produce their varieties. As a result, the weights attached to firms that adopt the new technologies above the average fall while those attached to firms that adopt the new technologies below the average rise, which lowers the average technology adoption level. The impact of the expansion of the technological frontier also involves subtle effects. But most importantly, most productive firms adopt the up-to-date technologies as a result, which raises the average technology adoption level.

3 Closed-Economy Equilibrium

As we have shown in the last section, endogenous variables yet to be determined are reduced to φ^* , w , and r , which must satisfy the three equations: the zero profit condition in sector Z (equation (2)), the free-entry condition (equation (14)), and the factor-market clearing condition (equation (18)). Now, with a slight abuse of notation, we let $w(r)$ denote the wage rate that satisfies (2) for a given r . Then, the system is reduced to two unknown variables φ^* and r with two equations, (14) and (18), where $w(r)$ is substituted for w .

Before we investigate the case of interest in which some firms adopt new technologies only partially, we examine the extreme case where φ^* is greater than $rk(\theta)/[wl(\theta)]$ so that all firms fully adopt the new technologies (see (4)). When all firms fully adopt the new technologies, \hat{m} becomes constant at $\hat{m}_\theta \equiv \alpha\gamma\theta + (1 - \alpha\gamma)m_z$. Then the factor-market clearing condition (18) becomes

$$\frac{w}{r} = \frac{1 - \hat{m}_\theta K}{\hat{m}_\theta L}, \quad (20)$$

so the wage-rental ratio in equilibrium is linear in the capital-labor endowment ratio. Substituting (20), we can rewrite $rk(\theta)/[wl(\theta)]$, the threshold value of φ^* , as

$$\varphi_\theta \equiv \frac{\hat{m}_\theta L k(\theta)}{1 - \hat{m}_\theta K l(\theta)}.$$

When all firms fully adopt the new technology, the free-entry condition (14) is reduced to

$$[1 - H(\varphi^*)] \left[\int_0^\infty \left(\frac{\varphi}{\varphi^*} \right)^{\theta(\sigma-1)} d\mu_{\varphi^*}(\varphi) - 1 \right] = f_e/f, \quad (21)$$

which implies that the cutoff productivity level φ^* is solely determined by the free-entry condition, independent of the factor prices (and factor endowments). We define $\bar{\varphi}$ as φ^* that satisfies (21). If $\bar{\varphi} \geq \varphi_\theta$, then in equilibrium all firms adopt the new technology for all inputs up to θ . Equilibrium factor prices are readily derived from (2) and (20) as

$$w_\theta = \left[\frac{1 - \hat{m}_\theta K}{\hat{m}_\theta L} \right]^{m_z} \quad \text{and} \quad r_\theta = \left[\frac{\hat{m}_\theta L}{1 - \hat{m}_\theta K} \right]^{1-m_z},$$

respectively. In this type of equilibrium, all firms in sector Y employ the same production technologies (with different productivities) regardless of the diversification in their intrinsic productivities. This case is in essence the same as the one considered by Bernard et al. (2007).

Now, we turn to the case of our main interest in which firms are diversified in their technology adoption. To derive an equilibrium, we examine properties of the free-entry condition (14) and the factor-market clearing condition (18). Noting that these two conditions can be expressed by two endogenous variables, φ^* and r , with the use of (2), it is rather straightforward to establish the following lemma.

Lemma 2. *Consider the case in which firms are diversified in their technology adoption. Then, the free-entry condition is represented by a down-sloping schedule in the (r, φ^*) -space, whereas the factor-market clearing condition is illustrated by an upward-sloping schedule.*

Proof. See the Appendix. □

When the entry to the market becomes more difficult (a higher φ^*), the average profit $\tilde{\pi}$ (which equals the *ex ante* expected profit conditioning on the successful entry) must rise in order to make firms willing to incur the initial investment f_e . This is enabled by a reduction of the rental rate, which raises the profits earned by inframarginal firms through both reducing the production costs directly (as sector Y is capital intensive relative to sector Z) and encouraging the technology adoption. Thus, the free-entry (FE) schedule is downward-sloping in the (r, φ^*) -space.

As for the factor-market clearing (FM) schedule, a higher rental rate lowers the demands for capital by discouraging the technology adoption and raising the production costs. In order to compensate this demand fall, firms must become more productive on average, demanding more capital, with a rise in φ^* .

As Figure 2 shows, the equilibrium is uniquely determined as the FE schedule is downward-sloping while the FM schedule is upward-sloping. Thus, we have shown the following proposition.

Proposition 1. *There exists the environment such that in equilibrium, only the most productive firms fully adopt the new technologies, while firms with lower productivity levels adopt the new technologies only partially.*

Once φ^* and r are determined, the characteristics of the average operating firms, such as the average unit variable cost \tilde{c} (equation (10)), the average profit $\tilde{\pi}$ (equation (13)), and the average technology adoption \tilde{m} (equation (15)), are identified, which means that the firms' distribution including the mass of operating firms is also determined. Notice that a change in the size of the economy (i.e., a proportional change in K and L) alters neither the FE schedule nor the FM schedule. This means that the profile of the average firm is independent of the size of the economy; an increase in the country size simply makes the distribution of firms thick (i.e., a rise in N).

Now, let us examine further properties of equilibrium in which firms are heterogeneous in their adoption of the new technologies.

First, we consider the impact of an increase in the capital-labor endowment ratio K/L . It is immediate to see that only the factor-market clearing (FM) schedule is affected. That is, the FM schedule shifts to the left as expected while the FE schedule is left unchanged. As a result, the rental rate r declines (while the wage rate w increases) and the cutoff

productivity level φ^* rises as shown in Figure 2. Firms with the lowest productivities exit from the market since they become less competitive as the costs of labor, which they use intensively, increase. Operating firms adopt the new technologies for more inputs than before as a response to an increase in the wage-rental ratio. Together with the observation that least productive firms exit from the market, we find that an increase in the capital-labor endowment ratio raises the average productivity in sector Y .

Proposition 2. *An increase in the capital-labor ratio induces the least productive firms to exit from the market and the operating firms to adopt the new technologies more than before. Consequently, the average productivity of the industry rises.*

Proof. See the Appendix. □

Next, we consider the effect of a decrease in the fixed cost f (or equivalently an increase in f_e). A decrease in f leaves the factor-market clearing (FM) schedule unchanged, while it shifts the free-entry (FE) schedule downward since all operating firms become more profitable for any given factor prices, and therefore the cutoff productivity level must decline. As a result, both the cutoff productivity level φ^* and the rental rate r fall. Lowering the fixed cost invites less productive firms to enter the market. Since the tail of firm distribution is stretched, however, the average technology adoption decreases. Consequently, the rental rate declines.³ These results are recorded as follows.

Proposition 3. *A decrease in the fixed cost f raises profitability of operating firms and encourages less productive firms to enter the market. The average technology adoption level falls as a result, leading to a lower rental rate.*

Proof. See the Appendix. □

An expansion of the technological frontier (i.e., an increase in θ) brings about relatively more ambiguous results. A rise in θ would increase the average profits for the operating firms, which induces the free-entry (FE) schedule to shift up to lower the expected profits upon entry to counter the effect. On the other hand, an increase in θ shifts the factor-market clearing (FM) schedule to the right. An increase in θ directly increases most-productive firms' demands for capital. Moreover, it makes most productive firms more competitive than others, which also increases the aggregate demand for capital. As a consequence, r increases

³Although each operating firm adopts the new technologies more, the effect of an increase in the mass of low productivity firms outweighs, lowering the average technology adoption.

for any given φ^* . Since the FE schedule shifts up while the FM schedule shifts to the right, r increases unambiguously, but whether or not φ^* increases is not clear. The resulting increase in the rental rate dampens the favorable effect of the technological progress on the expected profits of the operating firms. Therefore, it is not clear whether or not the cutoff productivity rises as a consequence of the expansion of the technological frontier.

Proposition 4. *An expansion of the technological frontier induces the most productive firms to adopt the new technologies for more inputs. Since the wage-rental ratio decreases as a result, however, other firms retreat in their adoption of the new technologies.*

Proof. See the Appendix. □

4 Two-Country Model

This section extends the basic model to the setting in which two countries, Home and Foreign, differ in general in their factor proportions and technology frontiers trade each other. Without loss of generality, we assume that Home is capital abundant while Foreign is labor abundant, i.e., $K_H/L_H > K_F/L_F$ where H and F signify Home and Foreign, respectively. The two countries engage in free trade in both good Y and good Z , so intra-industry trade of good Y occurs as well as inter-industry trade.

In sector Y , N_T firms operate in the world as a whole, where T signifies variables that are sum of corresponding Home's and Foreign's variables in free trade equilibrium. The fraction n_H of them are located in Home, so the mass of firms in Home is $N_H = n_H N_T$ while that in Foreign is $N_F = (1 - n_H) N_T$. We define the world-wide average of the unit cost in sector Y by

$$\tilde{c}_T = [n_H \tilde{c}(\varphi_H^*, w_H, r_H, \theta_H)^{1-\sigma} + (1 - n_H) \tilde{c}(\varphi_F^*, w_F, r_F, \theta_F)^{1-\sigma}]^{1/(1-\sigma)},$$

where we suppress the arguments of \tilde{c}_T to simplify the exposition. Then, the price index, which is common to both countries, is given by

$$P_T = N_T^{1/(1-\sigma)} p(\tilde{c}_T).$$

To derive the free-trade equilibrium, we first investigate the zero-profit conditions for sector Y in both countries and derive the relationship between (r_H, φ_H^*) and (r_F, φ_F^*) given that the wage rates are determined by $w_H = w(r_H)$ and $w_F = w(r_F)$. Then, we derive

the free-entry (FE) conditions and factor-market clearing (FM) conditions to identify an equilibrium.

It follows from (7) that the zero-profit conditions for the least productive cutoff firms in Home and Foreign can be written respectively as

$$\begin{aligned}\frac{\gamma E_T}{\sigma N_T} \left[\frac{c(\varphi_H^*, w(r_H), r_H, \theta_H)}{\tilde{c}_T} \right]^{1-\sigma} &= f, \\ \frac{\gamma E_T}{\sigma N_T} \left[\frac{c(\varphi_F^*, w(r_F), r_F, \theta_F)}{\tilde{c}_T} \right]^{1-\sigma} &= f.\end{aligned}\tag{22}$$

Consequently, we have

$$c(\varphi_H^*, w(r_H), r_H, \theta_H) = c(\varphi_F^*, w(r_F), r_F, \theta_F).\tag{23}$$

The condition (23) gives us the relationship between (r_H, φ_H^*) and (r_F, φ_F^*) . Now, it follows from (A.7) in the Appendix and the definition of η that the total derivative of $c(\varphi, w(r), r, \theta)$ with respect to r is given by

$$\frac{dc(\varphi, w(r), r, \theta)}{dr} = \frac{c(\varphi, w(r), r, \theta)}{r} \frac{m - m_z}{1 - m_z},\tag{24}$$

where we use $dc(\varphi, w(r), r, \theta)/dr$ to denote the total derivative in order to make a clear distinction from the partial derivative of c with respect to r . The total derivative (24) is positive as we focus on the model environment such that sector Y is capital intensive relative to sector Z . Together with the fact that c is decreasing in φ , therefore, we obtain that (r_H, φ_H^*) and (r_F, φ_F^*) must lie on a common upward-sloping zero-profit condition (ZPC) schedule.

Turning to the free-entry condition, we first write the condition in Home under which the expected profits upon entry equals the entry cost:

$$[1 - H(\varphi_H^*)] \left\{ \frac{\gamma E_T}{\sigma N_T} \left[\frac{\tilde{c}(\varphi_H^*, w(r_H), r_H, \theta_H)}{\tilde{c}_T} \right]^{1-\sigma} - f \right\} = f_e,$$

where the expression in the curly brackets is the average profits for the operating firms located in Home. Together with (22), we obtain

$$\Phi(\varphi_H^*, w(r_H), r_H, \theta_H) = f_e/f,\tag{25}$$

where Φ is defined in (14). Note that the functional form of (25) is the same as that in autarky (equation (14)). Thus, the location of the free-entry (FE) schedule will not change

by opening to trade. Note also that once r_H is determined, so is $\varphi_H^* = \varphi_H^*(r_H)$. Similarly, we obtain the free-entry condition in Foreign as

$$\Phi(\varphi_F^*, w(r_F), r_F, \theta_F) = f_e/f. \quad (26)$$

It follows immediately from (23) and (26) that φ_F^* and r_F can also be expressed as a function of r_H .

As depicted in Figure 3, the FE schedule for Home (FE_H) and that for Foreign (FE_F) coincide with each other if $\theta_H = \theta_F$. But FE_H is located to the right of FE_F if $\theta_H > \theta_F$, as shown in Figure 4. This is because Φ is increasing in θ as the proof of Proposition 4 in the Appendix shows while Φ is decreasing in r as (A.9) in the proof of Lemma 2 shows.

Next, we derive the world-wide factor-market clearing condition. We define the average technological adoption level of Home's firms as

$$\tilde{m}_H = \int_0^\infty m(\varphi, w(r_H), r_H, \theta_H) \left[\frac{c(\varphi, w(r_H), r_H, \theta_H)}{\tilde{c}_T} \right]^{1-\sigma} d\mu_H^*(\varphi),$$

and similarly for Foreign's as \tilde{m}_F . The aggregate demands for capital by all firms in sector Y to produce varieties equal $\alpha\gamma E_T(n_H\tilde{m}_H + n_F\tilde{m}_F)$, while those as fixed costs equal $(1-\alpha)\gamma E_T m_z$. Since the demands for capital in sector Z are $(1-\gamma)E_T m_z$, the world-capital-market-clearing condition can be written as

$$\hat{m}_T E_T = r_H K_H + r_F K_F,$$

where $\hat{m}_T = \alpha\gamma[n_H\tilde{m}_H + n_F\tilde{m}_F] + [(1-\alpha)\gamma + 1 - \gamma]m_z$. The world-labor-market-clearing condition, on the other hand, can be written as

$$(1 - \hat{m}_T)E_T = w(r_H)L_H + w(r_F)L_F.$$

Combining these two conditions, we obtain the world-factor-market clearing condition as

$$\frac{1 - \hat{m}_T}{\hat{m}_T} = \frac{w(r_H)L_H + w(r_F)L_F}{r_H K_H + r_F K_F}. \quad (27)$$

The last condition we need in order to determine free-trade equilibrium is Home's factor-market clearing condition. Let ξ denote the share of Home's production in the world-wide production of good Z . Then, the capital-market clearing condition for Home can be written as

$$E_T \left\{ \alpha\gamma n_H \tilde{m}_H + (1-\alpha)\gamma \left[\frac{n_H \tilde{c}(\varphi_H^*, w(r_H), r_H, \theta_H)^{1-\sigma}}{\tilde{c}_T^{1-\sigma}} \right] m_z + (1-\gamma)\xi m_z \right\} = r_H K_H.$$

The labor-market clearing condition for Home can be similarly written as

$$E_T \left\{ \alpha \gamma n_H (1 - \tilde{m}_H) + (1 - \alpha) \gamma \left[\frac{n_H \tilde{c}(\varphi_H^*, w(r_H), r_H, \theta_H)^{1-\sigma}}{\tilde{c}_T^{1-\sigma}} \right] (1 - m_z) + (1 - \gamma) \xi (1 - m_z) \right\} = w(r_H) L_H.$$

Canceling ξ from these equations, we obtain the Home's factor-market clearing condition:

$$\alpha \gamma E_T n_H (\tilde{m}_H - m_z) = r_H K_H - m_z [w(r_H) L_H + r_H K_H], \quad (28)$$

where $E_T = w(r_H) L_H + w(r_F) L_F + r_H K_H + r_F K_F$.

Five equations, (23), (25), (26), (27), and (28), determine five unknown variables, r_H , r_F , φ_H^* , φ_F^* , and n_H , which give us the free-trade equilibrium.

In the remainder of this section, we examine the properties of free-trade equilibrium and compare them with those in autarky. We first investigate the case in which $\theta_H = \theta_F$ and then the case in which $\theta_H > \theta_F$. We infer from Proposition 2 that in autarky, the capital abundant country, Home, has a higher wage-rental ratio than Foreign and hence more firms in sector Y use the new technologies, especially for the frontier input θ_H , if the size of Home is similar to that of Foreign. Higher rewards to new technologies for inputs beyond θ_H than Foreign give firms in Home more incentive to expand the technological frontier. Rigorous modeling of the endogenous expansion of the technological frontier is beyond the scope of this paper. Here we simply assume that $\theta_H > \theta_F$ and explore the consequences of this assumption.

When $\theta_H = \theta_F$, we find from (25) and (26) that the free-entry schedules of the two countries, FE_H and FE_F , coincide as Figure 3 shows. Figure 3 depicts autarky equilibria H and F for Home and Foreign, respectively. We confirm as a corollary to Proposition 2 that in autarky, the capital-abundant Home has a lower r and higher φ^* . Free trade induces the rental rates and cutoff productivity levels to coincide between Home and Foreign, i.e., $r_H = r_F$ and $\varphi_H^* = \varphi_F^*$, as we see from (23), (25), and (26). Then rental rate and n_H are determined by (27) and (28). In free trade, the rental rates (and hence the wage rates) and cutoff productivity levels are same between Home and Foreign, as shown by T in Figure 3. We have shown the following proposition.

Proposition 5. *Suppose that the technological frontiers are the same between Home and Foreign. In autarky, capital-abundant Home has a lower rental rate and higher cutoff productivity level than Foreign. Opening to trade will equalize the factor prices and cutoff pro-*

ductivity levels, respectively, between Home and Foreign. As a result, firms with the same intrinsic productivity adopt the same production technology regardless of their locations.

Next, we examine the case in which $\theta_H > \theta_F$. It follows from (25) that once r_H is determined, so is φ_H^* . Then (23) and (26) determine r_F and φ_F^* as functions of r_H . Finally, r_H together with n_H are solved from the factor-market clearing conditions (27) and (28). Figure 4 illustrates the autarkic equilibria H for Home and F for Foreign; it also shows the free-trade equilibria H_T , which is the intersection between FE_H and FM_T , for Home and F_T , which is the intersection between ZPC and FF_F , for Foreign as the above argument suggests.

Let us consider the autarkic equilibrium first. As the figure shows, if θ_H is not much larger than θ_F , Home's rental rate is smaller than Foreign's as we correctly anticipate from the last result when $\theta_H = \theta_F$. If θ_H is sufficiently larger than θ_F , however, this effect may outweigh the effect of factor endowment, so Home's rental rate is greater than Foreign's. In both cases, the cutoff productivity level is higher in Home than in Foreign. Opening to trade will not equalize factor prices in this case. Since the ZPC schedule is upward-sloping and the FE_H schedule is located above the FE_F schedule reflecting the difference in the technological frontier, H_T must be located to the northeast of F_T . Thus, in free trade, both rental rate and cutoff productivity level are higher in Home than in Foreign.

Proposition 6. *Suppose that the technological frontier is higher in capital-abundant Home than Foreign. Then, in autarky, the cutoff productivity level is higher in Home than in Foreign. The rental rate is lower in Home than in Foreign if the difference in the technological frontiers is small, but it is larger in Home if the difference is large. Free trade will not equalize factor prices. The rental rate is larger and hence the wage rate is lower in Home whose technological frontier is higher than Foreign. The cutoff productivity level is also higher in Home in the free-trade equilibrium.*

Remark 1. *If the capital-abundant country has a higher technological frontier as we argue above, the wage-rental ratio is lower in the capital-abundant country than in the labor-abundant country, contrary to the ranking in autarky when the difference in technological frontier between the countries is small.*

5 Concluding Remarks

We have set up a general equilibrium trade model in which firm heterogeneity in their productivities arise as a result of the choice of technology. The choice of technology depends on the factor prices, which in turn depend on factor endowments in autarky. Thus, the production technology is influenced by factor endowments. We have found among others that firms in the capital-abundant country tends to adopt new advanced technologies more in their production processes than the labor-abundant country, but opening to international trade will equalize factor prices between them and as a result productivity differences across countries disappear, if their states of technologies, which we have measured by the technological frontiers, are the same. If the states of technologies are different, on the other hand, the country with a higher state of technology has a lower wage-rental ratio than the other. We argue that the capital-abundant country tends to have a higher state of technology, so if that is the case the wage-rental ratio is lower in the capital-abundant country in free trade.

We have assumed that there is no trade cost. It is easy to see that in the presence of trade costs, unproductive firms only sell domestically whereas productive firms export their products as well as selling them domestically. This is true for both countries. Thus, even the labor-abundant country exports products that are most capital-intensive of all production process carried out in the country.

A Appendix

A.1 Proof of Lemma 1

The partial derivative of $\tilde{m}(\varphi^*, w, r, \theta)$ with respect to φ^* is given by

$$\begin{aligned} \frac{\partial \tilde{m}}{\partial \varphi^*} &= \frac{\int_{\varphi^*}^{\infty} [c(\varphi^*)^{1-\sigma} h(\varphi^*) m(\varphi) c(\varphi)^{1-\sigma} - m(\varphi^*) c(\varphi^*)^{1-\sigma} h(\varphi^*) c(\varphi)^{1-\sigma}] dh(\varphi)}{\left[\int_{\varphi^*}^{\infty} c(\varphi)^{1-\sigma} dh(\varphi) \right]^2} \\ &= \frac{c(\varphi^*)^{1-\sigma} h(\varphi^*) \int_{\varphi^*}^{\infty} [m(\varphi) - m(\varphi^*)] c(\varphi)^{1-\sigma} dh(\varphi)}{\left[\int_{\varphi^*}^{\infty} c(\varphi)^{1-\sigma} dh(\varphi) \right]^2} \\ &= \frac{h(\varphi^*) [\tilde{m}(\varphi^*) - m(\varphi^*)]}{1 - H(\varphi^*)} \left[\frac{c(\varphi^*)}{\tilde{c}(\varphi^*)} \right]^{1-\sigma}, \end{aligned} \quad (\text{A.1})$$

where we have suppressed arguments $w, r,$ and θ from all the relevant functions to simplify the exposition. Since $m(\varphi, w, r, \theta)$ is non-decreasing in φ , it follows that $m(\varphi, w, r, \theta) \geq m(\varphi^*, w, r, \theta) \geq 0$ on the support of h , and hence $\tilde{m}(\varphi^*) > m(\varphi^*)$ in the case where firms are heterogeneous in their technology adoption. Therefore, we obtain $\partial \tilde{m} / \partial \varphi^* > 0$.

The partial derivative of $\tilde{m}(\varphi^*, w, r, \theta)$ with respect to r is given by

$$\begin{aligned} \frac{\partial \tilde{m}}{\partial r}(\varphi^*, w, r, \theta) &= \int_0^{\infty} \frac{\partial m}{\partial r}(\varphi, w, r, \theta) \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \\ &\quad + \frac{1-\sigma}{r} \left\{ \int_0^{\infty} m(\varphi, w, r, \theta)^2 \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - \tilde{m}(\varphi^*, w, r, \theta)^2 \right\}. \end{aligned} \quad (\text{A.2})$$

It is straightforward to see that the first term on the right-hand side is non-positive since

$$\frac{\partial m}{\partial r}(\varphi, w, r, \theta) = \begin{cases} -\frac{\lambda(\varphi, m)}{r[(\partial \lambda / \partial j)(\varphi, m)]} < 0 & \text{if } rk(0)/[wl(0)] < \varphi < rk(\theta)/[wl(\theta)], \\ 0 & \text{if } \varphi \geq rk(\theta)/[wl(\theta)]. \end{cases} \quad (\text{A.3})$$

As for the second term, the expression in the curly brackets is positive as long as firms are heterogeneous in their technology adoption. This is because it equals the weighted average variance of $m(\varphi, w, r, \theta)$, which we call $Var(m)$, with the relative revenues as the weights:

$$Var(m) = \int_0^{\infty} [m(\varphi, w, r, \theta) - \tilde{m}(\varphi, w, r, \theta)]^2 \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) > 0.$$

Thus, we have shown $\partial m / \partial r < 0$.

The proof of $\partial \tilde{m} / \partial w > 0$ is similar to that of $\partial \tilde{m} / \partial r < 0$, and hence omitted.

To derive the sign of $\partial \tilde{m} / \partial \theta$, we define $\iota(j)$ by $\iota(j) = k(j)/l(j)$ and rewrite the average adoption level \tilde{m} as

$$\begin{aligned} \tilde{m}(\varphi^*, w, r, \theta) &= \int_0^{r\iota(\theta)/w} m(\varphi, w, r, \theta) \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \\ &\quad + \int_{r\iota(\theta)/w}^{\infty} \theta \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi). \end{aligned}$$

Now, we take the derivative of $c(\varphi, w, r, \theta)$ with respect to θ to obtain

$$\frac{\partial c}{\partial \theta}(\varphi, w, r, \theta) = c(\varphi, w, r, \theta) \ln \frac{r\iota(\theta)}{\varphi w}.$$

Moreover, we have

$$\frac{\partial \tilde{c}}{\partial \theta}(\varphi^*, w, r, \theta) = \tilde{c}(\varphi^*, w, r, \theta) \int_{r\iota(\theta)/w}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \ln \frac{r\iota(\theta)}{\varphi w} d\mu_{\varphi^*}(\varphi). \quad (\text{A.4})$$

Consequently, we obtain the derivative of $[c(\varphi, w, r, \theta)/\tilde{c}(\varphi^*, w, r, \theta)]^{1-\sigma}$ with respect to θ as

$$(1 - \sigma) \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \left[\ln \frac{r\iota(\theta)}{\varphi w} - A \right],$$

where

$$A \equiv \int_{r\iota(\theta)/w}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \ln \frac{r\iota(\theta)}{\varphi w} d\mu_{\varphi^*}(\varphi).$$

Now, noting that $c(\varphi, w, r, \theta)$ and $m(\varphi, w, r, \theta)$ are constant with respect to θ for any $\varphi < r\iota(\theta)/w$ and letting $\bar{\varphi}_\theta \equiv r\iota(\theta)/w$, we have

$$\begin{aligned} \frac{\partial \tilde{m}}{\partial \theta}(\varphi^*, w, r, \theta) &= \frac{r\iota'(\theta)}{w} [m(\bar{\varphi}_\theta, w, r, \theta) - \theta] \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \\ &+ \int_0^{\bar{\varphi}_\theta} m(\varphi, w, r, \theta) \frac{1 - \sigma}{\tilde{c}(\varphi^*, w, r, \theta)} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \frac{\partial \tilde{c}}{\partial \theta}(\varphi, w, r, \theta) d\mu_{\varphi^*}(\varphi) \\ &+ \int_{\bar{\varphi}_\theta}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \\ &+ \theta(1 - \sigma) \int_{\bar{\varphi}_\theta}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \left[\ln \frac{r\iota(\theta)}{\varphi w} - A \right] d\mu_{\varphi^*}(\varphi). \end{aligned} \quad (\text{A.5})$$

The first term on the right-hand side equals zero as $m(\bar{\varphi}_\theta, w, r, \theta) = \theta$. Since $r\iota(\theta)/(\varphi w) < 1$ for $\varphi > \bar{\varphi}_\theta$, we find $\partial \tilde{c}/\partial \theta < 0$ (see (A.4)) so that the second term is positive. The third term is obviously positive. As for the last term, we have

$$\begin{aligned} &\int_{\bar{\varphi}_\theta}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} \left[\ln \frac{r\iota(\theta)}{\varphi w} - A \right] d\mu_{\varphi^*}(\varphi) \\ &= A \left\{ 1 - \int_{\bar{\varphi}_\theta}^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \right\}. \end{aligned} \quad (\text{A.6})$$

It follows again from $r\iota(\theta)/(\varphi w) < 1$ for $\varphi > \bar{\varphi}_\theta$ that $A < 0$. Moreover, since

$$\int_0^{\infty} \left[\frac{c(\varphi, w, r, \theta)}{\tilde{c}(\varphi^*, w, r, \theta)} \right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) = 1,$$

and the integrand is positive, the expression inside the curly brackets is positive. Thus, the expression in (A.6) is negative, which makes the last term positive. Since all terms on the right-hand side of (A.5) are positive, we have shown $\partial \tilde{m}/\partial \theta > 0$.

A.2 Proof of Lemma 2

Let $w(r)$ define the wage rate that satisfies (2) for a given r . For later purposes, we define the elasticity of w with respect to r by $\eta \equiv -w'(r)r/w = m_z/(1 - m_z) > 0$. Now the total derivative of $c(\varphi, w(r), r, \theta)$ with respect to r is given by

$$\frac{dc(\varphi, w(r), r, \theta)}{dr} = \frac{c(\varphi, w(r), r, \theta)}{r} [(1 + \eta)m(\varphi, w(r), r, \theta) - \eta], \quad (\text{A.7})$$

where we use $dc(\varphi, w(r), r, \theta)/dr$ to denote the total derivative to make a clear distinction from the partial derivative of c with respect to r , and hence

$$\frac{d\tilde{c}(\varphi^*, w(r), r, \theta)}{dr} = \frac{\tilde{c}(\varphi^*, w(r), r, \theta)}{r} [(1 + \eta)\tilde{m}(\varphi^*, w(r), r, \theta) - \eta]. \quad (\text{A.8})$$

We first show that the free-entry (FE) schedule is downward-sloping. It follows directly from (14) that

$$\left. \frac{\partial \varphi^*}{\partial r} \right|_{FE} = - \frac{\partial \Phi(\varphi^*, w(r), r, \theta) / \partial r}{\partial \Phi(\varphi^*, w(r), r, \theta) / \partial \varphi^*}.$$

It follows from (A.7) and (A.8) that

$$\begin{aligned} \frac{d\phi(\varphi^*, w(r), r, \theta)}{dr} &= (1 - \sigma) \left[\frac{\tilde{c}(\varphi^*, w(r), r, \theta)}{c(\varphi^*, w(r), r, \theta)} \right]^{-\sigma} \\ &\quad \times \frac{\frac{d\tilde{c}(\varphi^*, w(r), r, \theta)}{dr} c(\varphi^*, w(r), r, \theta) - \tilde{c}(\varphi^*, w(r), r, \theta) \frac{dc(\varphi^*, w(r), r, \theta)}{dr}}{c(\varphi^*, w(r), r, \theta)^2} \\ &= \frac{(1 - \sigma)(1 + \eta)}{r} \left[\frac{\tilde{c}(\varphi^*, w(r), r, \theta)}{c(\varphi^*, w(r), r, \theta)} \right]^{-\sigma} [\tilde{m}(\varphi^*, w(r), r, \theta) - m(\varphi^*, w(r), r, \theta)] \\ &< 0, \end{aligned}$$

and hence

$$\frac{d\Phi(\varphi^*, w(r), r, \theta)}{dr} = [1 - H(\varphi^*)] \frac{d\phi(\varphi^*, w(r), r, \theta)}{dr} < 0. \quad (\text{A.9})$$

Similarly, we have

$$\frac{d\Phi(\varphi^*, w(r), r, \theta)}{d\varphi^*} = [1 - H(\varphi^*)] \frac{(1 - \sigma)}{\varphi^*} \left[\frac{\tilde{c}(\varphi^*, w(r), r, \theta)}{c(\varphi^*, w(r), r, \theta)} \right]^{1 - \sigma} m(\varphi^*, w(r), r, \theta) < 0. \quad (\text{A.10})$$

Thus, the slope of the FE schedule is

$$\frac{d\varphi^*}{dr} = - \frac{(1 + \eta)\varphi^*}{r} \left[\frac{\tilde{m}(\varphi^*, w(r), r, \theta)}{m(\varphi^*, w(r), r, \theta)} - 1 \right] < 0.$$

Next, we show that the factor-market clearing (FM) schedule is upward-sloping. Defining $\kappa \equiv K/L$ and

$$g(\varphi^*, r, \theta) \equiv \hat{m}(\varphi^*, w(r), r, \theta)w(r) - [1 - \hat{m}(\varphi^*, w(r), r, \theta)]r\kappa, \quad (\text{A.11})$$

we see that (18) is equivalent to $g(\varphi^*, r, \theta) = 0$. We have

$$\begin{aligned} \frac{\partial g}{\partial r}(\varphi^*, r, \theta) &= \frac{d\hat{m}(\varphi^*, w(r), r, \theta)}{dr} [w(r) + r\kappa] + \hat{m}(\varphi^*, w(r), r, \theta)w'(r) \\ &\quad - [1 - \hat{m}(\varphi^*, w(r), r, \theta)]\kappa, \end{aligned} \quad (\text{A.12})$$

$$\frac{\partial g}{\partial \varphi^*}(\varphi^*, r, \theta) = \frac{\partial \hat{m}}{\partial \varphi^*}(\varphi^*, w(r), r, \theta)[w(r) + r\kappa]. \quad (\text{A.13})$$

Now, it follows directly from the definition of \hat{m} that

$$\frac{d\hat{m}(\varphi^*, w(r), r, \theta)}{dr} = \alpha\gamma \left[\frac{\partial \tilde{m}}{\partial w}(\varphi^*, w(r), r, \theta)w'(r) + \frac{\partial \hat{m}}{\partial r}(\varphi^*, w(r), r, \theta) \right],$$

which is negative as $\partial \tilde{m}/\partial w > 0$ and $\partial \tilde{m}/\partial r < 0$ (Lemma 1), together with $w'(r) < 0$. Moreover, we have

$$\frac{\partial \hat{m}}{\partial \varphi^*}(\varphi^*, w(r), r, \theta) = \alpha\gamma \frac{\partial \tilde{m}}{\partial \varphi^*}(\varphi^*, w(r), r, \theta),$$

which is positive as $\partial \tilde{m}/\partial \varphi^* > 0$ (Lemma 1). Therefore, we have $\partial g/\partial r < 0$ and $\partial g/\partial \varphi^* > 0$, and hence

$$\left. \frac{\partial \varphi^*}{\partial r} \right|_{FM} = -\frac{\partial g/\partial r}{\partial g/\partial \varphi^*} > 0.$$

A.3 Proof of Proposition 3

Using the free-entry condition (14) and the factor-market clearing condition (18), we define the vector-valued function G as

$$\begin{aligned} G(\varphi^*, r, \theta, \kappa, f) &\equiv \begin{pmatrix} G_1(\varphi^*, r, \theta, f) \\ G_2(\varphi^*, r, \theta, \kappa) \end{pmatrix} \\ &\equiv \begin{pmatrix} \Phi(\varphi^*, w(r), r, \theta) - (f_e/f) \\ g(\varphi^*, r, \theta, \kappa) \end{pmatrix}, \end{aligned} \quad (\text{A.14})$$

$$(\text{A.15})$$

where g is defined by (A.11). In equilibrium, we have $G(\varphi^*, r, \theta, \kappa, f) = 0$, where 0 is the two-dimensional vector with 0 in each element. The Jacobian derivative of G is given by

$$J \equiv \begin{pmatrix} \frac{\partial G_1}{\partial \varphi^*} & \frac{\partial G_1}{\partial r} \\ \frac{\partial G_2}{\partial \varphi^*} & \frac{\partial G_2}{\partial r} \end{pmatrix}.$$

It follows from (A.9), (A.10), (A.12), and (A.13) that $\partial G_1/\partial \varphi^* < 0$, $\partial G_1/\partial r < 0$, $\partial G_2/\partial \varphi^* > 0$, $\partial G_2/\partial r < 0$. Thus, the Jacobian determinant is positive, i.e., $|J| > 0$.

Applying the implicit function theorem with the use of Cramer's rule, we then have

$$\begin{aligned} \frac{\partial \varphi^*}{\partial \kappa} &= \frac{1}{|J|} \left| \begin{array}{cc} -\frac{\partial G_1}{\partial \kappa} & \frac{\partial G_1}{\partial r} \\ -\frac{\partial G_2}{\partial \kappa} & \frac{\partial G_2}{\partial r} \end{array} \right| = -\frac{1}{|J|} [1 - \hat{m}(\varphi^*, w(r), r, \theta)]r \frac{\partial G_1}{\partial r} > 0, \\ \frac{\partial r}{\partial \kappa} &= \frac{1}{|J|} \left| \begin{array}{cc} \frac{\partial G_1}{\partial \varphi^*} & -\frac{\partial G_1}{\partial \kappa} \\ \frac{\partial G_2}{\partial \varphi^*} & -\frac{\partial G_2}{\partial \kappa} \end{array} \right| = \frac{1}{|J|} [1 - \hat{m}(\varphi^*, w(r), r, \theta)]r \frac{\partial G_1}{\partial \varphi^*} < 0. \end{aligned}$$

A.4 Proof of Proposition 3

Similarly to the proof of Proposition 2, we obtain

$$\begin{aligned}\frac{\partial \varphi^*}{\partial f} &= \frac{1}{|J|} \begin{vmatrix} -\frac{\partial G_1}{\partial f} & \frac{\partial G_1}{\partial r} \\ -\frac{\partial G_2}{\partial f} & \frac{\partial G_2}{\partial r} \end{vmatrix} = -\frac{1}{|J|} \frac{f_e}{f^2} \frac{\partial G_2}{\partial r} > 0, \\ \frac{\partial r}{\partial f} &= \frac{1}{|J|} \begin{vmatrix} \frac{\partial G_1}{\partial \varphi^*} & -\frac{\partial G_1}{\partial f} \\ \frac{\partial G_2}{\partial \varphi^*} & -\frac{\partial G_2}{\partial f} \end{vmatrix} = \frac{1}{|J|} \frac{f_e}{f^2} \frac{\partial G_2}{\partial \varphi^*} > 0.\end{aligned}$$

A.5 Proof of Proposition 4

To use the implicit function theorem, we first obtain

$$\begin{aligned}\frac{\partial G_1}{\partial \theta} &= \frac{\partial \Phi}{\partial \theta}(\varphi^*, w(r), r, \theta) \\ &= [1 - H(\varphi^*)] \frac{\partial \phi}{\partial \theta}(\varphi^*, w(r), r, \theta).\end{aligned}$$

Since $\partial \tilde{c}/\partial \theta < 0$ and $c(\varphi^*, w(r), r, \theta)$ is constant with respect to θ as the least productive operating firms do not adopt the new technology for the frontier input θ , we have $\partial \phi/\partial \theta > 0$. Therefore, we have $\partial G_1/\partial \theta > 0$. As for the sign of $\partial G_2/\partial \theta$, we have

$$\begin{aligned}\frac{\partial G_2}{\partial \theta} &= \frac{\partial g}{\partial \theta} \\ &= \frac{\partial \hat{m}}{\partial \theta}(\varphi^*, w(r), r, \theta)[w(r) + r\kappa] \\ &= \alpha\gamma \frac{\partial \tilde{m}}{\partial \theta}(\varphi^*, w(r), r, \theta)[w(r) + r\kappa],\end{aligned}$$

which is positive as Lemma 1 shows.

Now, we have

$$\frac{\partial \varphi^*}{\partial \theta} = \frac{1}{|J|} \begin{vmatrix} -\frac{\partial G_1}{\partial \theta} & \frac{\partial G_1}{\partial r} \\ -\frac{\partial G_2}{\partial \theta} & \frac{\partial G_2}{\partial r} \end{vmatrix} = \frac{1}{|J|} \left(\frac{\partial G_1}{\partial r} \frac{\partial G_2}{\partial \theta} - \frac{\partial G_1}{\partial \theta} \frac{\partial G_2}{\partial r} \right).$$

Since $\partial G_1/\partial r < 0$, $\partial G_2/\partial \theta > 0$, $\partial G_1/\partial \theta > 0$, and $\partial G_2/\partial r < 0$, the sign of $\partial \varphi^*/\partial \theta$ is ambiguous. On the other hand, we have

$$\frac{\partial r}{\partial \theta} = \frac{1}{|J|} \begin{vmatrix} \frac{\partial G_1}{\partial \varphi^*} & -\frac{\partial G_1}{\partial \theta} \\ \frac{\partial G_2}{\partial \varphi^*} & -\frac{\partial G_2}{\partial \theta} \end{vmatrix} = \frac{1}{|J|} \left(\frac{\partial G_1}{\partial \theta} \frac{\partial G_2}{\partial \varphi^*} - \frac{\partial G_1}{\partial \varphi^*} \frac{\partial G_2}{\partial \theta} \right).$$

Since $\partial G_1/\partial \theta > 0$, $\partial G_2/\partial \varphi^* > 0$, $\partial G_1/\partial \varphi^* < 0$, and $\partial G_2/\partial \theta > 0$, we obtain $\partial r/\partial \theta > 0$.

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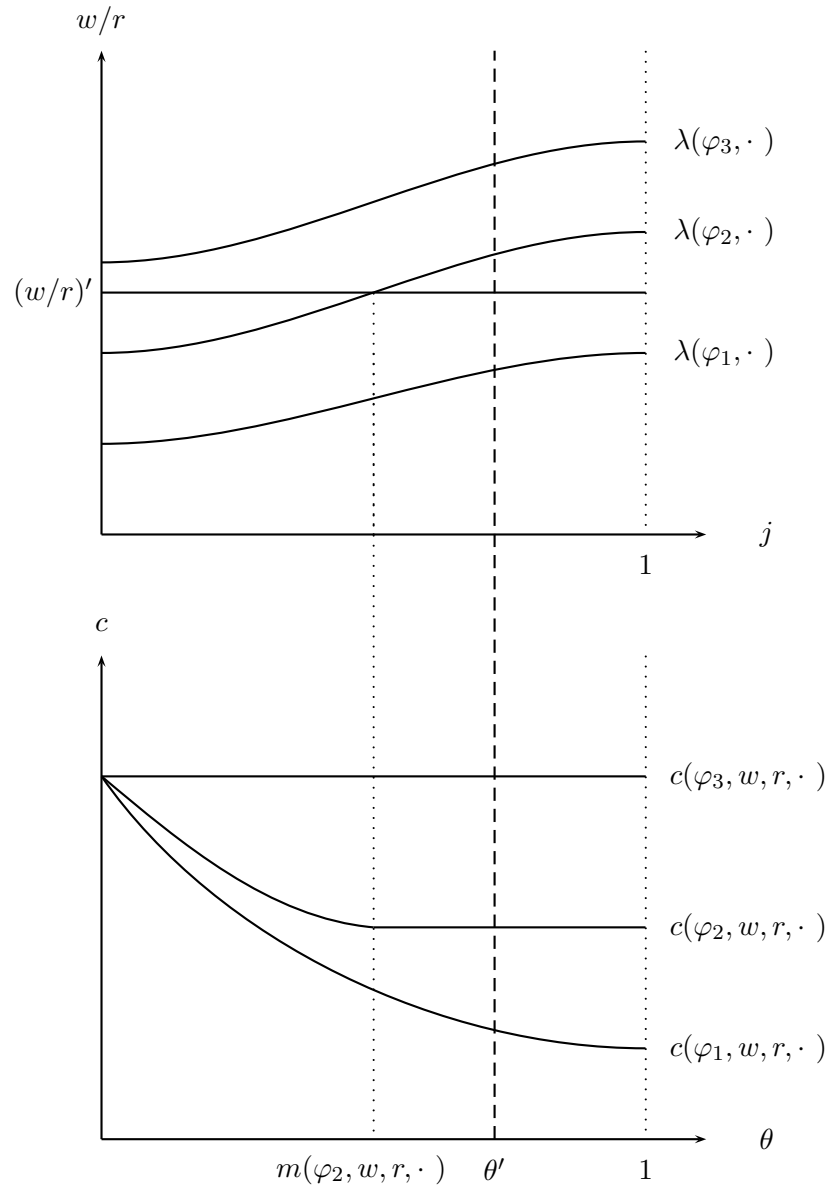


Figure 1: **Technology Adoption and the Unit Variable Costs**

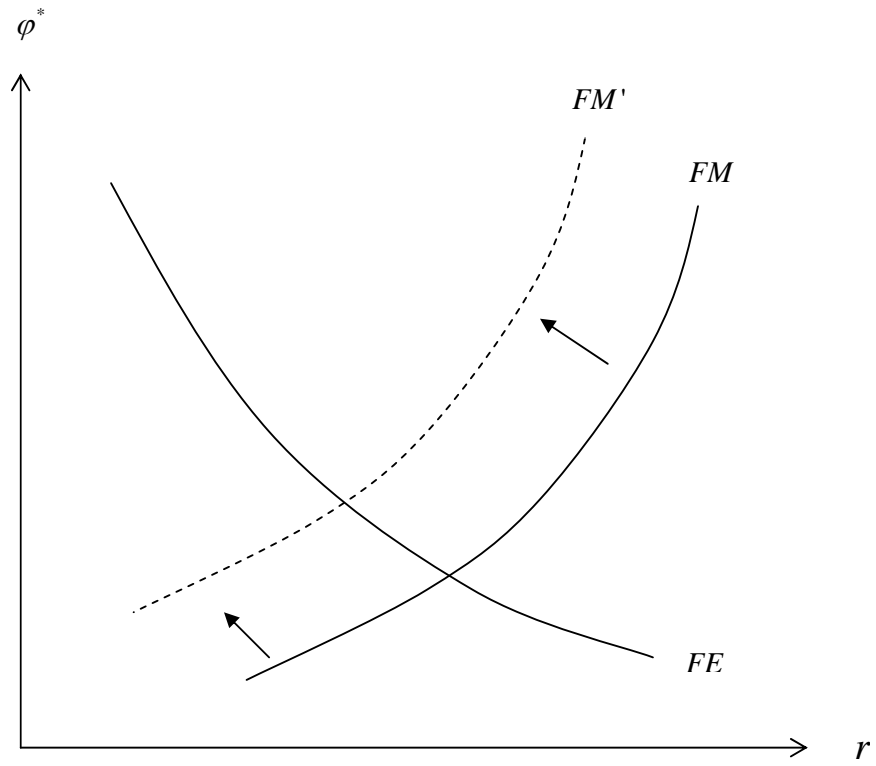


Figure 2: The Equilibrium Cutoff Productivity Level and Rental Rate

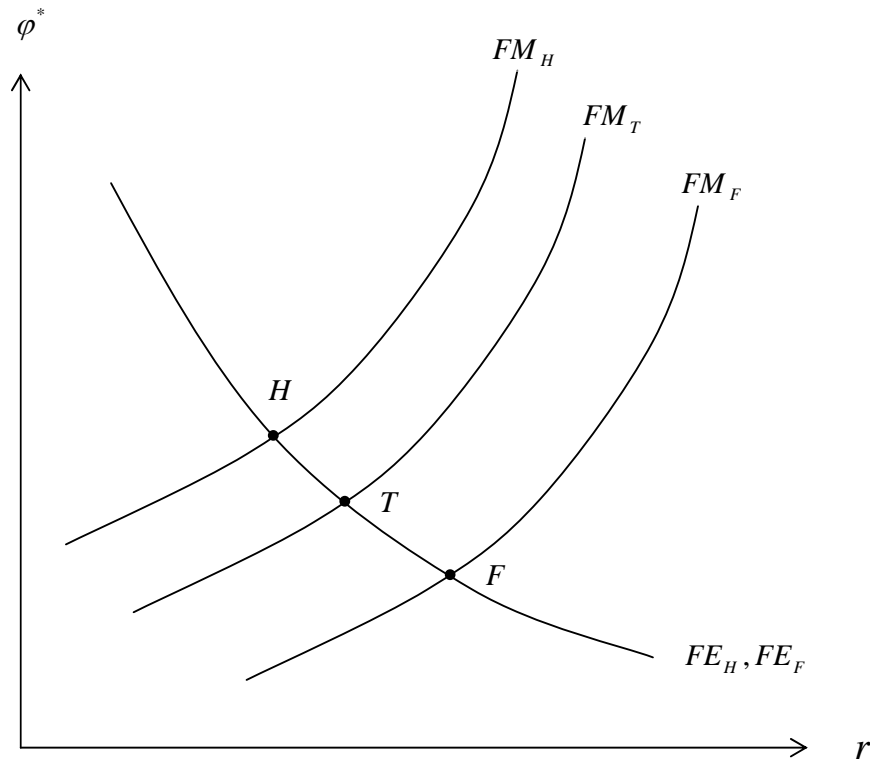


Figure 3: Trade Equilibrium ($\theta_H = \theta_F$)

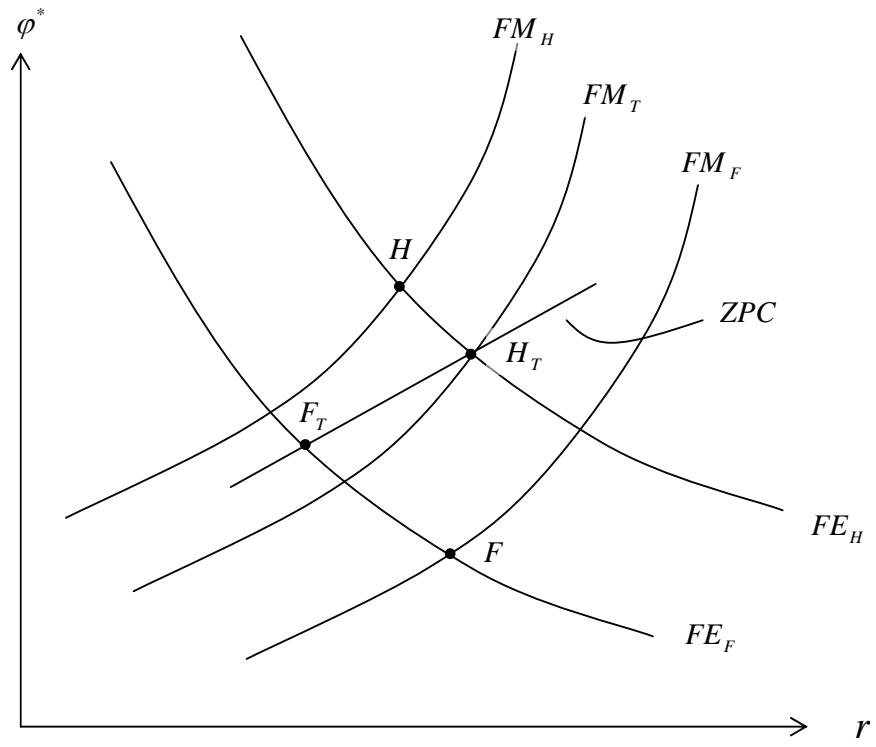


Figure 4: Trade Equilibrium ($\theta_H > \theta_F$)