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Endogenous Present-Bias and Policy Implementation*

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Abstract

This paper presents a detailed analysis of the present-bias of governments in a multi-party political system and the resulting inefficiency in policy implementation. We show that under a two-party political system the party in office tends to be present-biased and time-inconsistent. This may lead to inefficient procrastination of socially beneficial projects that impose upfront social/economic costs but yield long-term social/economic benefits, such as trade liberalization. However, procrastination is often not indefinite. There exist equilibria in which the project is carried out, and in many cases completed in finite time. The procrastination problem tends to get more serious as the cost of the project gets higher. When the cost is low, there is no procrastination problem. When the cost is high, the project can be procrastinated indefinitely, though there may exist equilibria in which the project is implemented gradually. When the cost is intermediate, there is an array of equilibria, all characterized by some form of procrastination. Thus, the model can explain why sometimes even unilateral trade liberalization is carried out gradually or procrastinated.

JEL Classification: C70, D11, D60, D72, D78, D91

Keywords: endogenous present-bias, hyperbolic discounting, time-inconsistency, procrastination, multi-party political system, policy implementation

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1 Introduction

People often procrastinate doing things that generate lasting benefits but require the payment of an immediate cost, to the detriment of their long-term interests. Quitting bad habits, such as smoking and drinking, is one prominent example. Other examples include house-cleaning, studying for an examination, and writing a referee report. A recent literature (e.g., Akerlof, 1991 and O'Donoghue and Rabin, 1999) explains this phenomenon by focusing on the existence of present-biased preferences, which induce time-inconsistent behavior. As a present-biased individual considers trade-offs between two future periods, stronger relative weight is given to the earlier period as it approaches. This creates time-inconsistent behavior because an individual's relative preference for payoff at an earlier period over a later period strengthens as the earlier period approaches. Procrastination may therefore ensue as the present self cannot commit the future selves to future actions. A present-biased, time-inconsistent individual may procrastinate completing a task forever, even though it is in her best long-term interest to complete the task immediately.

Similarly, it is often observed that politicians procrastinate implementing socially beneficial policies that require incurring immediate costs, but generate long-lasting benefits. For example, politicians are reluctant to raise income taxes even though it may benefit citizens in the long-run by helping to reduce the government deficit and hence lower the long-term interest rate. The delay of trade liberalization, despite its long-term benefits to the country as a whole, can be explained by the fact that the costs of resource reallocation (such as unemployment of workers) are incurred immediately while social benefits (of lower prices of goods for domestic consumers and users) are spread far into the future. Another prominent example of government procrastination is that of pension reform. As Feldstein (2005) states, "Many economists and policy analysts acknowledge the long-run advantages of shifting from a pay-as-you go [tax-financed] system to a mixed system [that combines pay-as-you-go benefits with investment-based personal retirement account] but believe that the transition involves unacceptable costs. This is often summarized by saying that the transition generation would have to pay 'double' — to finance the social security benefits of current retirees and to save for its own retirement." This might explain why many countries delay pension reform.[1]

In this paper, we provide a theory to explain government procrastination based on a model of endogenous present-bias, which is a consequence of a two-party political system. In our model, a party has the same intertemporal preferences as a typical citizen, which is characterized by geometric discounting, if the party believes it will be in office in every future period. Its discount factor between any two consecutive periods is constant, and its intertemporal utility function does not give rise to time-inconsistency. Nevertheless, under a two-party political system, the government of each term becomes present-biased and time-inconsistent. Present-bias arises if a government's probability of getting elected in the future is less than one, and it puts more weight on social welfare when it is in office than when it is not. As a result, each government in a two-party system has incentives to procrastinate carrying out projects that carry upfront social/economic costs but yield long-term social/economic benefits, and that should be undertaken immediately if social welfare is to be maximized. Specifically, we consider a divisible project which is socially beneficial, and demonstrate that, depending on the cost of the project relative to the discount factor, the present-biased governments may (i) carry out the project immediately exactly in accordance with citizens' interests, (ii) procrastinate somewhat, but still manage to complete the whole project in some period in finite time, (iii) undertake the project in stages, with the process continuing for a long time, or (iv) completely fail to undertake the socially beneficial project.

Indefinite procrastination of socially beneficial projects can sometimes be explained by a model of myopic government who cares more about current constituents and discounts heavily future unborn generations. That is, the government discounts future more heavily than the typical citizen but they both remain time-consistent. The government has incentives to procrastinate a socially beneficial project indefinitely if and only if the government discounts future sufficiently heavier than the citizens. Since the government remains time-consistent, the project is either completed immediately or procrastinated indefinitely depending on the

[1]Other examples include the repair of obsolete highways and construction of dykes for storm defense.

government's discount factor. On the contrary, ours is not a model of myopia. Instead, it is a model of endogenous time-inconsistency of the political parties. A present-biased government may wish a future government (but not itself) to carry out the project; such time-inconsistency never occurs for myopic governments. The outcome of the model is also different from that of myopic government in that there exist equilibria in which, despite certain degree of procrastination, a socially beneficial project is carried out and completed in finite time. Thus, our analysis reveals the distinction between two sources of procrastination by governments. The first arises from the government being more impatient than the citizens, i.e. a myopic government. The second arises from endogenous present-bias as political parties face uncertainty about the prospect of being elected and put less weight on social welfare when they are out of office. In this paper, we focus on the second source, which is the more interesting one.

We shall assume that a policy can be partially carried out by a government. For example, a government can choose to partially liberalize the trade regime by cutting only some tariffs, or lowering those tariffs somewhat but not all the way to free-trade level. In the case of balancing the budget, a government can choose to reduce the deficit somewhat but not all the way to a balanced budget. According to our analysis, the possibility of partially carrying out the project allows the present-biased government to bypass the fate of indefinite procrastination of the project when the implementation cost is high. Seen in this light, this paper identifies a new source of gradualism in the literature on dynamic contribution to a public good, namely the endogenous present-bias arising from the two-party political system.[2]

Our paper is related to the work of Alesina and Tabellini (1990) and Amador (2003). In their studies of government debt, they argue that the government saves too little, or accumulates too much debt, due to the political uncertainty caused by the two-party system. Amador (2003) observes that the time-inconsistency with which the government is faced is

[2]Compte and Jehiel (2004), for example, obtain endogenous gradualism in a contribution game by assuming that raising a player's contribution in the negotiation phase increases the other player's outside option value. Each player gradually makes contributions to prevent their respective partner from terminating the game.

equivalent to the problem faced by a present-biased consumer. In contrast, our paper is a general analysis of the mechanism through which a government comes to have present-biased preferences in the two-party political system and how these affect the inefficiency of its policy implementation in a dynamic setting. For example, we have introduced a lot of asymmetries in our model. The probability that a party is elected in any period depends on whether it is a dominant party or non-dominant party and whether it is an incumbent or non-incumbent.[3] The two parties can also differ in how much they value social welfare when they are not in office. Moreover, instead of applying the model to a specific policy issue, we focus on the analysis of how different policy implementation equilibria arise under different circumstances in a more general setup, which can be further refined to apply to various specific policy issues.

In section 2, we lay down the basic assumptions and setup of the model. In section 3, we show how a two-party political system gives rise to present-bias of the party in office. We consider a socially beneficial project with immediate cost outlay and future flows of benefits. Given that two parties compete for office in each period, the party currently in office plays a game with all future governments (including its future selves) in choosing the fraction of the project to be implemented today. In section 4, we compute the subgame perfect equilibria corresponding to different implementation costs. In section 5, we summarize the results and conclude.

2 The Basic Setup of the Model

There are two political parties, A and B, which seek power in the government. One of them is in office in period $t \in \{0, 1, 2, \cdots\}$. Let each period be a term. The party in office makes

[3]In our model, the election outcome is characterized by a Markov process, such that the current ruling party will be re-elected with an arbitrarily fixed probability between 0 and 1. Moreover, that probability can be different for the two parties. Alesina and Tabellini (1990) and Amador (2003), however, assume that every party has an equal probability of being elected in every election. That is a special case of ours, when the probability of being re-elected equals one half for both parties. Although Alesina and Tabellini (1990) mention in a footnote of their paper that the analysis can be extended to a similar framework to ours, they have not explored how the likelihood of being re-elected affects the government present-bias as much as we do in this paper.

policy decisions in accordance with its own preferences; therefore the objective function of the current government is the same as that of the party in office. Both parties discount future with a common discount factor $\delta \in (0,1)$, which is also that same as the discount factor of the citizens.

The selection of the party in office in each election is characterized by a Markov process, such that the incumbent party will lose in election with a constant probability. This probability, however, is different between the parties; we let $q^i \in (0,1)$ denote the probability that party i loses in the next election when it is currently in office. We assume for simplicity that the probability of a party being elected is independent of how the policy is implemented by the party or its rival. This is clearly a limitation. But this assumption allows us to focus on the issue of interest and to present our main findings transparently. Moreover, it enables us to conduct a simple analysis as to how party dominance affects the policy implementation outcome when such dominance is exogenously given.[4] Even if the party's performance in carrying out the policy can have some effect on its probability of being elected in the future, as long as the effect is limited, the results in this paper are still valid.

The policy that we consider is about undertaking a project that involves an immediate implementation cost of c but generates a constant benefit flow of 1 in the current period and every period thereafter. We assume that the project is divisible in the sense that a government can choose to carry out only a fraction of the project in its term so that a fraction a_t of the project undertaken in period t poses an immediate cost $a_t c$ to society while generating benefit flows of a_t in each period thereafter. We assume that $1/(1-\delta) > c$, so the project is worth carrying out from the citizens' point of view.

The flow of social welfare enjoyed by citizens in period t is given by

$$u_t = \sum_{k=0}^{t} a_k - a_t c.$$

The first term on the right-hand side shows the benefit that society enjoys in period t from the fraction of the project that has been completed, whereas the second term represents the

[4]It is often the case that party dominance is quite exogenous for historical reasons. For example, the Liberal Democratic Party in post-war Japan has been dominating for a long time. So has the People's Action Party in post-independence Singapore.

cost that society incurs from the part of the project undertaken in period t. We assume that the party in office in period t puts a (normalized) weight of one on the flow of social welfare in period t, and so its one-shot payoff in period t equals u_t, while the opposition party puts a weight of $\alpha \in [0, 1]$ on the flow of social welfare in the same period.[5] In other words, a party puts more weight on social well-being when it is in power than when it is not in power. This discounting is motivated by the presumption that, while members of the opposition party are part of the citizenry and so they care about social welfare just like the average citizen, the opposition party also treats the success of the ruling party as unfavorable, perhaps because it undermines its political status. On the contrary, the ruling party in period t has a different view about payoff in period t. Members of the incumbent party care about social welfare in that period as they are part of the citizenry; in addition, they have incentives to care more about social welfare in period t because a higher social welfare gives them higher political status.

3 Endogenous Present-Bias

In this section, we show that in a two-party political system, the party in office will possess present-biased preferences. By present-bias, we mean that the discount rate for the next-period payoff is greater in the current period than that in any other future period. In other words, the government of any period puts a disproportionately high weight on the current payoff. We also show that if an incumbent advantage (which is defined shortly) exists, the party in office will possess a payoff function with generalized hyperbolic discounting. By generalized hyperbolic discounting, we mean one such that the discounting of the next period's payoff in period t weakly diminishes as t increases (thus preferences with generalized hyperbolic discounting are present-biased).[6]

[5] The model can easily be accommodated to the case where the parties have different values of α. We assume that they have the same value of α only to simplify the exposition.

[6] The psychological basis for present-bias in individuals' preferences is that the distinction between two consecutive periods is most salient between today ($t = 0$) and tomorrow ($t = 1$), and it becomes less and less so when the two consecutive periods are further and further into the future. Akerlof (1991) gives an excellent discussion about the salience of the present for a present-biased individual.

To be more specific, let

$$U_t^i = \sum_{k=0}^{\infty} \beta_k^i u_{t+k} \tag{1}$$

represent the present discounted payoff function for party i when it is in office in period t. Then, U_t^i exhibits generalized hyperbolic discounting if the ratio of the two consecutive discount functions β_{k+1}^i/β_k^i weakly increases with k.[7] We show that U_t^i exhibits generalized hyperbolic discounting if $q^A + q^B \leq 1$.

Let p_k^i denote the probability that party i currently in office will also be in office k periods later. For concreteness, let us suppose temporarily that party A is in office in the current period. Then the probability that party A will also be in office $k+1$ periods later can be linked with the probability that the party A is in office one period earlier as follows:

$$\begin{aligned} p_{k+1}^A &= (1-q^A)p_k^A + q^B(1-p_k^A) \\ &= q^B + (1-q^A-q^B)p_k^A, \end{aligned}$$

with $p_0^A = 1$. When $q^A + q^B \neq 1$, we can solve this difference equation explicitly to obtain

$$p_k^A = \frac{q^B + q^A(1-q^A-q^B)^k}{q^A+q^B}. \tag{2}$$

When $q^A + q^B = 1$, it is obvious that $p_0^A = 1$ and $p_k^A = q^B$ for $k \geq 1$. We define $q \equiv q^A + q^B \in (0,2)$ and p^A such that $q^A = (1-p^A)q$ and $q^B = p^A q$. Then we can rewrite (2) as

$$p_k^A = p^A + (1-p^A)(1-q)^k, \tag{3}$$

which is valid for any $k \geq 0$ when $q \neq 1$; when $q = 1$, we have $p_0^A = 1$ and $p_k^A = p^A$ for $k \geq 1$. Similarly, we have

$$p_k^B = 1 - p^A + p^A(1-q)^k. \tag{4}$$

[7] The instantaneous discount rate of the "usual" exponential discount function $\beta_e(t) \equiv e^{-rt}$ in continuous time models is given by $-\beta_e'(t)/\beta_e(t) = r$, whereas that of hyperbolic discount function $\beta_h(t) \equiv (1+\alpha t)^{-\gamma/\alpha}$ is given by $-\beta_h'(t)/\beta_h(t) = \gamma/(1+\alpha t)$ that decreases with t (for hyperbolic discounting, see Loewenstein and Prelec, 1992, who call it *generalized* hyperbolic discounting contrary to our terminology). Phelps and Pollak (1968) develop an intertemporal utility function of the form: $U_t = u_t + \beta \sum_{k=1}^{\infty} \delta^k u_{t+k}$ (where $0 < \beta < 1$ and $0 < \delta < 1$) to capture imperfect altruism for future generations. Laibson (1997) introduces this utility function with quasi-hyperbolic discounting to behavioral economics in order to capture important properties of hyperbolic discounting. Note that quasi-hyperbolic discounting is a special case of generalized hyperbolic discounting as β_{k+1}/β_k weakly increases with k ($\beta_1/\beta_0 = \beta\delta$ and $\beta_{k+1}/\beta_k = \delta$ for $k \geq 1$).

As we see from (3) and (4) (together with $0 < q < 2$) that p_k^A and p_k^B approach p^A and $1 - p^A$, respectively. That is, p^A is the steady state probability that party A is in office. Without loss of generality, we assume that $p^A \geq 1/2$, or equivalently $q^A \leq q^B$. That is, we assume that party A is a (weakly) dominant party.

The parameter q measures the incumbent disadvantage in the next election. The probability that party i wins the election generally depends on whether or not it is currently in office. The winning probability is greater when it is currently in office than otherwise if and only if $1 - q^i > q^j$, where $j \neq i$, or equivalently $q < 1$. That is, the incumbent has an advantage in the next election if $0 < q < 1$. As we see from (3), the probability that party A is in office decreases over time from $p_0^A = 1$ to p^A: when $0 < q < 1$, the incumbent A has an advantage in election, but this advantage diminishes over time. The case where party B is currently in office is similar; the probability that party B is in office decreases over time from $p_0^B = 1$ to $1 - p^A$. If $q = 1$, there is no incumbent advantage nor disadvantage. The probability that a party is in office remains constant over time, and this probability is independent of whether or not the party is currently in office: $p_k^A = 1 - p_k^B = p^A$ and $p_k^B = 1 - p_k^A = 1 - p^A$ for any $k \geq 1$. Finally, there exists an incumbent disadvantage when $1 < q < 2$. The probability that party A is in office k periods later, p_k^A or $1 - p_k^B$, fluctuates around p^A, and converges to p^A. Similarly, the probability that party B is in office fluctuates around $1 - p^A$ and converges to $1 - p^A$.

To show that the party in office has present-biased preferences, consider a payoff stream $\{u_{t+k}\}_{k=0}^{\infty}$. Recalling that party i discounts social welfare by the factor α when it is not in office, we write the expected payoff for the party i that is in office in period t as $U_t^i = \sum_{k=0}^{\infty} \beta_k^i u_{t+k}$, where $\beta_k^i = \delta^k[p_k^i + (1 - p_k^i)\alpha]$. For concreteness, let us consider the case where party A is in office in period 0. Then, the discount function for party A can be written as

$$\begin{aligned}
\beta_k^A &= \delta^k[p_k^A + (1 - p_k^A)\alpha] \\
&= \delta^k\{\alpha + (1 - \alpha)[p^A + (1 - p^A)(1 - q)^k]\}. \qquad (5)
\end{aligned}$$

The payoff function for the incumbent party i exhibits generalized hyperbolic discounting if β^i_{k+1}/β^i_k weakly increases with k. It directly follows from (5) that

$$\frac{\beta^A_{k+1}}{\beta^A_k} = \delta \left[\frac{\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^{k+1}\}}{\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^k\}} \right]; \tag{6}$$

we have similar expression for β^B_{k+1}/β^B_k. As Figure 1 indicates, this ratio of discount functions changes with k differently depending on the value of q. First, it can be readily verified from (6) that if $q < 1$, then β^i_{k+1}/β^i_k increases with k and converges to δ as k tends to infinity. Thus, the government's payoff function exhibits generalized hyperbolic discounting in this case. If $q > 1$, on the other hand, β^i_{k+1}/β^i_k fluctuates around δ as k increases, such that it is less than δ when k is even, is greater than δ when k is odd, and converges to δ as k tends to infinity. Moreover, β^i_{k+1}/β^i_k takes on the smallest value when $k = 0$, which implies that the discount rate is greatest in the current period, i.e., the government has a present-biased preferences as in the case where $q < 1$.

Finally, if $q = 1$, it follows from $\beta^A_0 = 1$ and (6) that $\beta^A_1/\beta^A_0 = \delta[\alpha + (1-\alpha)p^A] < \delta$ and $\beta^A_{k+1}/\beta^A_k = \delta$ for $k \geq 1$, and similarly for party B. Therefore, each party's payoff function exhibits quasi-hyperbolic discounting (Laibson, 1997; see also footnote 7). The incumbent party discounts social welfare in the next period more heavily than the discounting brought about by the discount factor δ as it will be out of office with a certain probability. Since the probability of being in office is the same for all future periods whether or not a party is currently in office (i.e., the party never enjoys incumbent advantage nor disadvantage in future elections), discounting between future consecutive periods is stationary.

In a similar multi-party political environment as ours, Amador (2003) shows that if all political parties including the incumbent party have equal probabilities of being elected in the next election, the preferences of the incumbent party is characterized by quasi-hyperbolic discounting. His model therefore corresponds to the case where $q = 1$ and $p^A = 1/2$ in our model.[8]

We record the above findings in the following proposition.

[8]Our argument can easily be generalized to the case of multi-party political system with more than two parties. We demonstrate our argument in the case of two parties to avoid the discussion of issues such as coalition formation to gain a majority, which are not of central interest in our analysis.

Proposition 1 *The two-party political system leads to present-biased preferences of the party in office. The preferences of the party in office are characterized by generalized hyperbolic discounting in the presence of a (weak) incumbent advantage in elections.*

We have shown that the party in office is present-biased, regardless of the degree of the incumbent advantage. Present-bias causes time-inconsistency in the governments' policy decision making even if the parties are symmetric (i.e., $p^A = 1/2$) so that they have exactly the same preferences when in office. To make our points more transparent, we henceforth assume that $q \leq 1$. In this case, each party's payoff function exhibits generalized hyperbolic discounting, which plays an important role especially in the existence of the equilibrium with gradual policy implementation when the cost of the project is relatively high.

Before turning to the issue of policy implementation by present-biased parties, we investigate how the basic parameters affect the degree of present-bias. First, it is readily verified from (6) that β_{k+1}^A/β_k^A increases with α for any k. That is, the less the party discounts social welfare when its rival party is in office, the less present-biased and less hyperbolic are its preferences. In an extreme case where $\alpha = 1$, we have $\beta_{k+1}^i/\beta_k^i = \delta$, i.e., each party i's preferences exhibit geometric discounting, and there is no present-bias. Next, an increase in the degree of party A's dominance will make the payoff function of the dominant party A exhibit less present-biased and less hyperbolic, and make the payoff function of the dominated party B exhibit more present-biased and more hyperbolic. This can be seen from the observation that β_{k+1}^A/β_k^A increases with p^A and β_{k+1}^B/β_k^B decreases with p^A (which can also be readily verified), with q held constant. Indeed, when $0 < q < 1$, the preferences for the dominant party A are more present-biased and hyperbolic than those for the dominated party B, i.e., $\beta_{k+1}^A/\beta_k^A > \beta_{k+1}^B/\beta_k^B$ for any k.

4 Policy Implementation

This section analyzes the policy decision of the governments. It has been shown that an individual with a quasi-hyperbolic payoff function exhibits time-inconsistent behavior, which includes inefficient procrastination of costly actions that generate a future stream of large benefits (see, for example, O'Donoghue and Rabin, 1999). In the current setting, the party in office has a present-biased payoff function. Therefore, it is also faced with a time-inconsistency problem, and we expect that it may procrastinate. Each party may procrastinate also because it prefers the other party to carry out the project and hence to bear the immediate cost of policy implementation, rather than carrying out the project for itself. The policy implementation game is a war of attrition; each party has an incentive to wait hoping the other party to concede in the costly policy implementation.[9]

We find that procrastination sometimes occurs, and the problem gets worse as implementation cost gets higher. However, even when it does happen, procrastination needs not be indefinite. Although the government sometimes procrastinates about implementing socially beneficial projects, there exist equilibria in which the project is undertaken, and may be completed in finite time. Specifically, we show that (i) the entire project is carried out immediately in period 0 if the cost of the project is small; (ii) there may be some finite delay in undertaking the project or the project is carried out gradually over many periods of time if the cost is in the intermediate range; and (iii) if the cost is high, the project may never be carried out, but there may also exist other equilibria in which the project is carried out gradually. The equilibrium with gradual policy implementation when the project cost is high exists, precisely because the party in office possesses hyperbolic discounting.

To see that the party in office has an incentive to procrastinate due to its hyperbolic discounting, we examine how the incumbent party i in period 0 evaluates a nonstochastic implementation schedule represented by $\{a_t\}_{t=0}^{\infty}$, when $a_t \geq 0$ and $\sum_{t=0}^{\infty} a_t \leq 1$. Now, we define the present discounted value of net benefits for the party i currently in office of the project that is carried out t periods later as

$$B_t^i \equiv \sum_{k=0}^{\infty} \beta_{t+k}^i - \beta_t^i c.$$

We rewrite party i's discounted payoff function given in (1) for $t = 0$, making use of the fact

[9] Alesina and Drazen (1991) also investigate a war-of-attrition political game in which socioeconomic groups may attempt to shift the burden of a policy onto others.

that the fraction a_t of the project undertaken in period t yields an expected payoff $a_t B_t^i$:

$$U_0^i = \sum_{t=0}^{\infty} a_t B_t^i. \tag{7}$$

Since (7) is linear with respect to $\{a_t\}_{t=0}^{\infty}$, it is in the best interest of the incumbent party i in period 0 to have the project carried out in a period where the present value of its net benefit is greatest. Let us define t^{i*} by

$$t^{i*} \in \arg\max_{t \in \mathbf{Z}_+} B_t^i,$$

where $\mathbf{Z}_+$ denotes the set of non-negative integers. Then, it is clear that the best sequence of $\{a_t\}_{t=0}^{\infty}$ is that $a_t = 1$ if $t = t^{i*}$ and $a_t = 0$ if $t \neq t^{i*}$.[10]

To find t^{i*}, we compare the present values of the expected payoffs for two consecutive periods t and $t+1$. Party i that is in office in period 0 (weakly) prefers having the project undertaken in period t to having it done in period $t+1$ if and only if

$$\begin{aligned} & B_t^i \geq B_{t+1}^i \\ \Leftrightarrow \quad & \beta_t^i \geq (\beta_t^i - \beta_{t+1}^i)c \\ \Leftrightarrow \quad & \frac{\beta_{t+1}^i}{\beta_t^i} \geq \frac{c-1}{c}. \end{aligned} \tag{8}$$

The second inequality is easy to interpret: the incumbent party in period 0 prefers having the project undertaken in period t to having it done in $t+1$ if and only if the reduction in payoff of postponing the project by one period, β_t^i, is at least as high as the reduction in cost by doing so, $(\beta_t^i - \beta_{t+1}^i)c$. If c is large enough that $\beta_1^i (= \beta_1^i/\beta_0^i) < (c-1)/c$, both parties wish (whenever they are in office) that the project be carried out at some point in the future. Since $p^A > 1/2$, the best timing of policy implementation is (weakly) earlier for the dominant party A than party B, i.e., $t^{A*} \leq t^{B*}$.

If neither party discounts social welfare when it is out of office (i.e., $\alpha = 1$), then $\beta_t^i = \delta^t$ for any $i = A, B$, and inequality (8) holds for any t since it reduces to $1/(1-\delta) \geq c$. Then, the government in period 0, regardless of the party in office, prefers having the project undertaken in period t to having it postponed to the next period, no matter what t is. This

[10]Generically, t^{i*} is uniquely determined.

implies that $t^{i*} = 0$, for any $i = A, B$, and so it is in the incumbent party's best interest to carry out the entire project within its term. Note that, since $\beta_t^i = \delta^t$, its payoff function is exactly the same as that of the citizens. Therefore, in this case, the government's action maximizes the welfare of the citizens. We summarize this finding in the following proposition.

Proposition 2 *Suppose that neither party discounts social welfare when it is out of office, i.e., $\alpha = 1$. Then neither party would procrastinate about implementing a socially beneficial project when it is in office.*

On the other hand, if the parties discount social welfare when it is out of office (i.e., $\alpha < 1$), then postponing the project may be preferable for the current government since, by doing so, the reduction in cost can outweigh the loss in benefit.

As Figure 1 illustrates, we have shown that if $q < 1$, then β_{t+1}^i/β_t^i strictly increases with t and that it converges to δ as t tends to infinity. Since $\delta > (c-1)/c$, there exists a threshold value of t such that (8) holds if and only if t is greater than or equal to the threshold value. As the party in office in period 0 prefers having the project undertaken in period t to having it done in period $t+1$ for all t greater than or equal to the threshold value, while it prefers postponing the project from t to $t+1$ for any t smaller than the threshold value, we infer that this party prefers having the project undertaken in this threshold period, i.e.,

$$t^{i*} = \min\left\{ t \in \mathbf{Z}_+ \,\middle|\, \frac{\beta_{t+1}^i}{\beta_t^i} \geq \frac{c-1}{c} \right\}. \tag{9}$$

If $q = 1$, then β_1^A/β_0^A, for example, equals $\delta[\alpha + (1-\alpha)p^A] < \delta$ and $\beta_{k+1}^A/\beta_k^A = \delta$ for $k \geq 1$. Since $(c-1)/c < \delta$, we see from (8) that $t^{A*} = 0$ if $\delta[\alpha + (1-\alpha)p^A] \geq (c-1)/c$ (as shown in Figure 1) and $t^{A*} = 1$ otherwise (and similarly for B).

4.1 Non-Cooperative Subgame Perfect Equilibrium

Since $\beta_1^i/\beta_0^i = \beta_1^i < \beta_{t+1}^i/\beta_t^i$ for any $t \geq 1$, we see from (9) that $t^{i*} = 0$ if $\beta_1^i \geq (c-1)/c$. If $\beta_1^i \geq (c-1)/c$ for any $i = A, B$, therefore, any government will carry out the project, so the project is completely carried out in period 0. But otherwise, at least one party has an incentive to procrastinate, so the project may not be implemented immediately. This subsection derives non-cooperative subgame perfect equilibria, and shows that despite of the existence of hyperbolic discounting, the project is carried out with a possible delay if the implementation cost is not very large. The next subsection shows that even in the case where the cost is so large that there exists no non-cooperative equilibrium with successful implementation of the project, there exists a "cooperative" equilibrium with a trigger strategy with a possible punishment, in which the project is gradually implemented.

To find the subgame perfect equilibrium in all cases (with different values of c), it is useful to find a possible mixed-strategy equilibrium and examine conditions for its existence. The mixed-strategy equilibrium exists when each party is willing to carry out the project immediately but prefers the other party to carry out the project in some future period. In such situations, each party i may randomize in its policy decision so that the other party j is made indifferent between carrying out the project or procrastinating when party j is in office.

Recall that the payoff for party i when it carries out the project immediately is $B_0^i = \sum_{k=0}^{\infty} \beta_k^i - c$, and define $\tilde{\beta}_k^i = \delta^k[1 - p_k^j + p_k^j \alpha]$ (where $j \neq i$) and $\tilde{B}_0^i = \alpha(1-c) + \sum_{k=1}^{\infty} \tilde{\beta}_k^i$, the latter as the payoff for party i when its rival party j is in office and carries out the project immediately. Let σ^i denote the stationary probability that party i carries out the project (completely) when it is in office. Also let V^i and $\tilde{V}^i$ denote the expected payoff (when the project is yet to be implemented) for party i at the beginning of each period (before the election) when party i was in office in the last period and that when the rival party j was in office in the last period, respectively.

Then, in the mixed-strategy equilibrium, V^A and $\tilde{V}^A$, for example, must simultaneously satisfy

$$V^A = (1 - q^A)[\sigma^A B_0^A + (1 - \sigma^A)\delta V^A] + q^A[\sigma^B \tilde{B}_0^A + (1 - \sigma^B)\delta \tilde{V}^A], \tag{10}$$

$$\tilde{V}^A = q^B[\sigma^A B_0^A + (1 - \sigma^A)\delta V^A] + (1 - q^B)[\sigma^B \tilde{B}_0^A + (1 - \sigma^B)\delta \tilde{V}^A]. \tag{11}$$

In the mixed-strategy equilibrium, party A is indifferent between carrying out the project and procrastinating whenever it is in office, i.e., $B_0^A = \delta V^A$. Substituting this into (11) and solving it for $\tilde{V}^A$, we obtain

$$\tilde{V}^A = \frac{q^B B_0^A + (1-q^B)\sigma^B \tilde{B}_0^A}{1-\delta(1-q^B)(1-\sigma^B)}.$$

Then, we substitute this equality and $B_0^A = \delta V^A$ into (10) to obtain

$$V^A = \frac{[1-q^A-\delta(1-q^A-q^B)(1-\sigma^B)]B_0^A + \sigma^B q^A \tilde{B}_0^A}{1-\delta(1-q^B)(1-\sigma^B)}. \tag{12}$$

We use $B_0^A = \delta V^A$ one more time to get

$$\sigma^B = \frac{1-\delta(2-q)-\delta^2(1-q)}{\delta\left[q^A \frac{\tilde{B}_0^A}{B_0^A} + q^B - 1\right] + \delta^2(1-q)}. \tag{13}$$

Similarly, we have

$$\sigma^A = \frac{1-\delta(2-q)-\delta^2(1-q)}{\delta\left[q^B \frac{\tilde{B}_0^B}{B_0^B} + q^A - 1\right] + \delta^2(1-q)}.$$

In the mixed-strategy equilibrium, σ^B, for example, is chosen so that party A is indifferent between carrying out the project and procrastinating. Thus, in the situations where party A's incentive to procrastinate decreases, σ^B must increase to preserve this indifference. Consequently, if σ^B calculated in (13) is greater than 1, party A will carry out the project whenever it is in office even though party B will also carry out the project whenever B is in office. On the other hand, if $\sigma^B < 0$, party A will procrastinate regardless of the policy-implementation strategy of party B.

To closely examine the properties of the mixed-strategy equilibrium, let us assume for the rest of the section that $q = 1$, i.e., there exists neither incumbent advantage nor disadvantage. In this case, we have $p_k^A = 1 - p_k^B = p^A$ and $p_k^B = 1 - p_k^A = 1 - p^A$. We further assume that $\alpha = 0$ to simplify the exposition. Then, we have

$$B_0^A = 1 - c + \frac{\delta p^A}{1-\delta},$$
$$\tilde{B}_0^A = \alpha(1-c) + \frac{\delta p^A}{1-\delta}.$$

We substitute them and $q = 1$ into (13) to obtain

$$\sigma^B = \frac{\delta p^A - (1-\delta)(c-1)}{\delta(1-p^A)(c-1)}. \tag{14}$$

It is readily verified that σ^B increases if c decreases or p^A increases. It is necessary for σ^B to increase to reduce party A's incentive to carry out the project when either of these pro-implementation forces arises.

Now, we derive the conditions under which $\sigma^B > 1$ and $\sigma^B < 0$, respectively. It follows directly from (14) that $\sigma^B > 1$ is equivalent to

$$1 - c + \frac{\delta p^A}{1-\delta} > \frac{\delta}{1-\delta}(1-p^A)(c-1). \tag{15}$$

In equilibrium, σ^B must lie in $[0, 1]$. When $\sigma^B = 1$, we have from (12) that $V^A = p^A B_0^A + (1 - p^A)\tilde{B}_0^A$. Using this equality, we find that the right-hand side of (15) equals $\delta(V^A - B_0^A)/(1-\delta)$ and hence (15) is equivalent to $B_0^A > \delta V^A$; party A strictly prefers carrying out the project to procrastinating. Now, we rewrite (15) as

$$\frac{c-1}{c} < \delta p^A.$$

This is the condition under which party A carries out the project regardless of its rival party's strategy. On the other hand, it follows from (14) that $\sigma^B < 0$ is equivalent to

$$1 - c + \frac{\delta p^A}{1-\delta} < 0, \tag{16}$$

which clearly states that carrying out the project gives party A a negative payoff. This inequality can be rewritten as

$$\frac{c-1}{c} > \frac{\delta p^A}{1-\delta(1-p^A)}.$$

Under this condition, party A procrastinates regardless of party B's strategy.

We can conduct a similar analysis for party B to obtain

$$\sigma^A = \frac{\delta(1-p^A) - (1-\delta)(c-1)}{\delta p^A(c-1)}.$$

Party B carries out the project regardless of party A's policy decision if

$$\frac{c-1}{c} < \delta(1-p^A),$$

whereas B procrastinates regardless of A's choice if

$$\frac{c-1}{c} > \frac{\delta(1-p^A)}{1-\delta p^A}.$$

Figure 2 illustrates these results regarding the parties' decision in policy implementation. In the intermediate rage of $(c-1)/c$ for party A (where $0 < \sigma^B < 1$), for example, party A's best response can take any of the following three alternatives: (i) party A randomizes when it is made indifferent between carrying out the project and procrastinating when $\sigma^B \in (0,1)$ is properly adjusted, (ii) party A will carry out the project if party B does not, and (iii) party A will procrastinate if B carries out the project.

Figure 2 shows the case where $p^A > 1/2$, i.e., party A is a strictly dominant party. The above observation indicates that there are three subgame perfect equilibria as illustrated in Figure 3. In all equilibria, the project is immediately carried out in period 0 if $(c-1)/c \leq \delta(1-p^A)$, while it is never implemented if $(c-1)/c \geq \delta p^A/[1-\delta(1-p^A)]$. Also common is a possible implementation delay in the intermediate range.

The intermediate range of $(c-1)/c$ is divided into three parts in equilibrium (a). When the implementation cost, or $(c-1)/c$, is smallest in the intermediate range, party A carries out the project regardless of party B's decision, while party B procrastinates if party A carries out the project. In equilibrium, therefore, party A carries out the project while party B procrastinates. When the implementation cost is largest in the intermediate range, party B procrastinates regardless of party A's decision, so party A carries out the project while party B procrastinates in this case too. In the middle range, both parties randomize.

In equilibrium (b), party A carries out the project while party B procrastinates even in the middle range of the intermediate implementation cost. Consequently, only party A carries out the project in the entire intermediate range of the implementation cost. Finally in equilibrium (c), less present-biased party A will procrastinate while more present-biased party B will carry out the project in the middle range of intermediate implementation cost.

If the implementation cost falls in the intermediate range, the project is carried out only stochastically. In the equilibrium where party A implements while party B procrastinates, the probability (conditional on the event that the project has not been carried out) for the project to be carried out equals p^A in each period. This implementation probability increases as party A's dominance becomes more prominent. Party A's dominance, however, acts adversely on the policy implementation in the equilibrium where party A procrastinates while party B implements (in equilibrium (c)). Finally, in the mixed-strategy equilibrium of equilibrium (a), the conditional probability of policy implementation equals

$$p^A\sigma^A + (1-p^A)\sigma^B = \frac{\delta - 2(1-\delta)(c-1)}{\delta(c-1)}$$

in each period. The implementation probability does not depend on party A's dominance; an increase in p^A leads to an increase in σ^B and a decrease in σ^A so that parties A and B are kept indifferent between carrying out the project and procrastinating. This probability of policy implementation, however, increases if δ increases or c decreases.

We summarize these findings in the following proposition.

Proposition 3 *If the cost of the project is small, the project is immediately carried out despite the fact that each party is present-biased. If the cost is high, on the other hand, neither party carries out the project. If the cost is in the intermediate range, some delay in implementation is expected. The delay may arise because one of the two parties procrastinates, or because both parties mix their decision as to whether or not they carry out the project when they are in office.*

As mentioned earlier, a situation depicted in Figure 2 will arise when party A is strictly dominant. If both parties are perfectly symmetrical (i.e., $p^A = 1/2$), the critical implementation costs are the same between the parties: $\delta/2$ and $\delta/(2-\delta)$ as indicated in Figure 4. As p^A increases from $1/2$, the critical implementation costs increase for party A while they decrease for party B (so a situation illustrated in Figure 2 arises). If the parties become even more asymmetric such that $p^A > \bar{p}$, where

$$\bar{p} = \frac{1-\sqrt{1-\delta}}{\delta} \in \left(\frac{1}{2}, 1\right),$$

δp^A exceeds $\delta(1-p^A)/(1-\delta p^A)$. In this case, only equilibrium (b) in Figure 3 will survive.

Party A's dominance may hinder policy implementation if the implementation cost is relatively small. When $(c-1)/c$ is slightly smaller than $\delta/2$, the project is carried out immediately if the two parties are symmetrical. If party A's dominance is sufficiently large, however, the project is carried out only when party A is in office; party A's dominance causes a possible implementation delay in this case. On the contrary, party A's dominance may encourage policy implementation if the implementation cost is relatively large such that $(c-1)/c$ is slightly greater than $\delta/(2-\delta)$. In this case, both parties procrastinate if they are symmetrical, but party A will carry out the project (while party B procrastinates) if party A is sufficiently dominant.

In addition to the equilibrium summarized in Proposition 3, there exists an equilibrium with gradual implementation of the project if the implementation cost falls in the intermediate range where the mixed-strategy equilibrium exists. Indeed, we find that this "gradual implementation equilibrium" has a one-to-one correspondence with the mixed-strategy equilibrium.

Let us consider a stationary strategy profile such that whenever party i is in office, it carries out the fraction a^i of the remainder of the project of the size $\theta \in (0,1]$. Then, party A's expected payoff when A was in office in the last period and that when B was in office in the last period can be written as functions of θ:

$$\begin{aligned} V^A(\theta) &= (1-q^A)[a^A\theta B_0^A + \delta V^A((1-a^A)\theta)] + q^A[a^B\theta\tilde{B}_0^A + \delta\tilde{V}^A((1-a^B)\theta)], \\ \tilde{V}^A(\theta) &= q^B[a^A\theta B_0^A + \delta V^A((1-a^A)\theta)] + (1-q^B)[a^B\theta\tilde{B}_0^A + \delta\tilde{V}^A((1-a^B)\theta)]. \end{aligned}$$

We guess that $V^A(\theta)$ and $\tilde{V}^A(\theta)$ are linear such that $V^A(\theta) = \theta v^A$ and $\tilde{V}^A(\theta) = \theta\tilde{v}^A$. Then, these equations can be rewritten as

$$v^A = (1-q^A)[a^A B_0^A + (1-a^A)\delta v^A] + q^A[a^B\tilde{B}_0^A + (1-a^B)\delta\tilde{v}^A], \tag{17}$$

$$\tilde{v}^A = q^B[a^A B_0^A + (1-a^A)\delta v^A] + (1-q^B)[a^B\tilde{B}_0^A + (1-a^B)\delta\tilde{v}^A]. \tag{18}$$

It is immediate to see that (17) and (18) correspond to (10) and (11), respectively. Again, focusing on the case in which $q = 1$, we know from the analysis of the mixed-strategy equilibrium that if

$$\begin{aligned} a^A &= \frac{\delta(1-p^A) - (1-\delta)(c-1)}{\delta p^A(c-1)}, \\ a^B &= \frac{\delta p^A - (1-\delta)(c-1)}{\delta(1-p^A)(c-1)}, \end{aligned}$$

then both parties are indifferent between carrying out the project and procrastinating, and hence it is party i's best response that it carries out the fraction a^i of the remainder of the project when it is in office. It is also readily confirmed that V^A and $\tilde{V}^A$ are indeed linear functions of θ as we have guessed. We record this finding in the following proposition.

Proposition 4 *If the cost of the project is in the intermediate range where the mixed-strategy equilibrium exists, there also exists equilibrium in which the project is gradually carried out.*

4.2 Cooperative Equilibrium with Gradual Policy Implementation

We have shown that if the implementation cost is large enough, there exists a subgame perfect equilibrium in which neither party carries out the project. In this case, we have $B_0^i < 0$ (see (16) for the case where $q = 1$), for $i = A, B$, so that each party would obtain a negative payoff from carrying out any positive fraction of the project. Nevertheless, each party when it is in office wishes that the project be undertaken sometime in the future since B_t^i is positive if t is large enough. To see this claim, we note that

$$B_t^i = \beta_t^i \left[\sum_{k=0}^{\infty} \frac{\beta_{t+k}^i}{\beta_t^i} - c \right]. \tag{19}$$

As we have seen in Section 3, the behavior of present-biased preferences is very similar to that of geometric discounting far off in the future, i.e., β_{k+1}^i/β_k^i converges to δ as k tends to infinity. Thus, $\beta_{t+k}^i/\beta_t^i = \Pi_{i=0}^{k-1}(\beta_{t+i+1}^i/\beta_{t+i}^i)$ approaches δ^k as t gets larger and larger, and hence the expression in square brackets on the right-hand side of (19) converges to $\sum_{k=0}^{\infty} \delta^k - c$ as t tends to infinity. Since $\sum_{k=0}^{\infty} \delta^k - c > 0$ under the assumption $1/(1-\delta) > c$, we have $B_t^i = \beta_t^i \left[\sum_{k=0}^{\infty}(\beta_{t+k}^i/\beta_t^i) - c\right] > 0$ when t exceeds a certain level. This contrasts sharply with the case of myopia. If the parties are simply myopic (heavily discounting the future with geometric discounting), neither party wishes that the project be undertaken in the future when it would obtain a negative payoff from the immediate policy implementation. Time inconsistency arises precisely because the parties are present-biased when they are in office.

It follows from $B_0^i < 0$ that if the incumbent party expects all future governments to refrain from carrying out the project, it should also stay out of the project. That is, no government wants to be the last to undertake a fraction of the project. The strategy profile in which $a_t = 0$ for any t is a subgame perfect equilibrium as we have seen. This is certainly bad news for the citizens. Although the project is socially beneficial, there is a possibility of indefinite procrastination. Does there exist any subgame perfect equilibrium in which some governments at least carry out part of the project?

No government would want to carry out the project to completion since it would incur a net loss from undertaking the last part of it. Suppose that, contrary to our original assumption, the project is indivisible, then the project will never get done. Thus, we have the following proposition.

Proposition 5 *If the cost of the project is so high that $B_0^i < 0$, for $i = A, B$, and if the project is not divisible, then the socially beneficial project never gets implemented.*

Under circumstances where partial completion of the project is not feasible, there is indefinite procrastination.

Indeed, if the project is to be implemented at all, it must be spread out over time to assure a non-negative payoff for every government. Moreover, the policy implementation process must continue indefinitely, since otherwise the government that completes the project would suffer a loss from the part of the project it undertakes. The following analysis presents such a gradual implementation equilibrium.

We shall show that when $B_0^i < 0$, a symmetric gradual implementation equilibrium exists if $\sum_{k=0}^{\infty} B_k^i > 0$ for any $i = A, B$, i.e., the simple sum of all current and future net benefits is positive for both parties. The situation in which $\sum_{k=0}^{\infty} B_k^i > 0$ arises if c is relatively small in the high-cost range. The following lemma implies that $B_k^i > 0$ for all $k \geq 1$ when c takes the value such that $B_0^i = 0$, and hence $\sum_{t=0}^{\infty} B_k^i > 0$ by continuity if c is sufficiently small even when $B_0^i < 0$ (recall that B_k^i decreases in c).

Lemma 1 *If $\alpha < 1$, then $B_t^i > \beta_t^i B_0^i$ for any $t \geq 1$.*

The proof of Lemma 1 is relegated to the Appendix. Under the usual geometric discounting such that $\beta_t^i = \delta^t$, B_t^i would be equal to $\beta_t^i B_0^i$. Under the present-biased preferences, however, the current government puts a disproportionately high weight on the cost incurred in the current period, and so B_0^i is disproportionately small.

Now, consider the stationary action profile, symmetric between the two parties, such that regardless of a party in office, $a_t = a\,(1-a)^t$ for some constant $a \in (0,1)$. According to this action profile, both parties undertake the fraction a of the remainder of the project whenever they are in office, and this process continues indefinitely. Consequently, the relevant payoff for the party in office in period t as evaluated in that period equals

$$\sum_{k=0}^{\infty}\left[a(1-a)^k B_k^i\right]. \tag{20}$$

Lemma 2 *Suppose $\sum_{k=0}^{\infty} B_k^i > 0$. Then, there exists $\bar{a} \in (0,1)$ such that for any $a \in (0,\bar{a})$, the relevant payoff for the party in office in period t given by (20) is positive.*

Proof: We first notice that $\sum_{k=0}^{\infty}(1-a)^k B_k^i$ converges to $\sum_{k=0}^{\infty} B_k^i > 0$ as $a \to 0$. Thus, there exists an $\bar{a}$ such that for any $a \in (0,\bar{a})$, $\sum_{k=0}^{\infty}(1-a)^k B_k^i > 0$, and hence $\sum_{k=0}^{\infty} a(1-a)^k B_k^i > 0$.

Q.E.D.

Can this gradual implementation scheme with $a \in (0,\bar{a})$ be supported as a subgame perfect equilibrium? The answer is "yes" as the following strategy profile is subgame perfect.

$$a_t = \begin{cases} a\,(1-a)^t & \text{if there has been no deviation from } a_k = a\,(1-a)^k \text{ for all } k \le t-1 \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Hence, we obtain the following proposition.

Proposition 6 *If the cost of the project is sufficiently high that $B_0^i < 0$ for $i = A, B$, but small enough that $\sum_{k=0}^{\infty} B_k^i > 0$ for $i = A, B$, there is a subgame perfect equilibrium in which every government carries out a constant fraction of the remainder of the project so the implementation process goes on indefinitely.*

Proof: We show here that the strategy profile (21) is subgame perfect. Since indefinite procrastination is a subgame perfect equilibrium, we need only show that no government has an incentive to deviate from the prescribed actions when there has been no deviation in the past. If there has been no deviation, the party in office in period t is to choose

$a_t = a(1-a)^t$, obtaining a positive payoff from its action (Lemma 2). If it chooses some other level of a_t, on the other hand, the equilibrium path would switch to the "punitive equilibrium" of indefinite procrastination, making the present value of future payoffs zero. Since the one-shot payoff from choosing a positive a_t for the party in office in period t is negative, the discounted sum of payoffs would be non-positive if it chooses any a_t other than $a(1-a)^t$. Hence, the party in office in period t is better off by conforming to the equilibrium path than choosing any other levels of a_t. Therefore, it will choose $a_t = a(1-a)^t$ if there has been no deviation before period t.

Q.E.D.

This cooperative equilibrium exists because both parties have hyperbolic discounting. There are two reasons why the party in office may procrastinate. First, it may prefer the other party to carry out the project in the near future, rather than carrying it out for itself. Second, the present discounted net benefit from the project may become greater if the project is carried out some time in the future due to the hyperbolic discounting. The first one was a predominant cause of the mixed-strategy (and gradual implementation) equilibrium derived in the last subsection. The second one, on the other hand is the primary cause of this gradual implementation equilibrium. Each party has an incentive to carry out part of the project only when a large portion of the project is sufficiently delayed so that the entire process of policy implementation yields a positive present discounted net benefit.

Summarizing the above results, we note that inefficient procrastination of the government is a result of the discrepancy between the socially optimal timing of implementation of the project and the optimal timing from the point of view of the government. When the implementation cost is small, the project is worth completing for both the citizens and for the current government, and there is no discrepancy between the optimal timing of implementation for the citizens and that for the government. When the implementation cost is intermediate, the project is again worth completing for both the citizens and the current government. However, the citizens and the government disagree on the timing of the policy implementation — the government does not want to implement the policy immediately. When the implementation cost is high, the project is not worth undertaking for any government acting alone, even though it is socially beneficial. In this case, indefinite procrastination is a subgame perfect equilibrium, though the project can also be gradually implemented when c is not too high.

5 Concluding Remarks

This paper presents a detailed analysis of the present-bias of governments in a multi-party political system and the policy implementation inefficiency therefrom when politicians value social welfare more when they are in office than when they are not. We have shown that under such circumstance, in a two-party political system, the party in office tends to be present-biased and time-inconsistent. This may lead to inefficient procrastination of socially beneficial projects that impose upfront social/economic costs but yield long-term social/economic benefits, such as trade liberalization. Procrastination arises because a party's chance of being in office in any future period is less than one, and that it puts less weight on social welfare when it is not in office. We assume that the probability that a party is elected in any period depends on whether it is an incumbent and whether it is a dominant party. We find that there is an array of equilibria, which can be categorized according to the cost-benefit ratio of the policy. The procrastination problem tends to get more serious as the cost-to-benefit ratio gets higher. When the cost is relatively low, there is no procrastination problem. When the cost is somewhat intermediate, there is likely to be some procrastination, such as having a probability of implementation less than one in each period, or certain implementation with a finite delay. When the cost is relatively high, the project can be procrastinated indefinitely, though there may exist equilibria in which the project is implemented gradually. Thus, the model can explain why sometimes even unilateral trade liberalization is carried out gradually or procrastinated.

There are many policies whose cost rises with time while the flows of future benefits remain the same. We can easily modify our model to capture this situation, and it is expected that the results will remain qualitatively the same.

One can easily derive corresponding results when the government is faced with a project that confers immediate benefits and incur future flows of costs. In this setting, a present-biased government is expected to preproperate, i.e. to carry out a project which is not socially beneficial.

A possible extension of this research is to endogenize the probability of a party being elected. Moreover, this model can be easily applied to address specific policy issues, such as trade liberalization, by adding more structure.

Appendix

Proof of Lemma 1:

To prove Lemma 1, it suffices to show that $\beta^i_{t+k}/\beta^i_t > \beta^i_k$, or $\beta^i_{t+k} > \beta^i_t\beta^i_k$, for any $t \geq 1$ and $k \geq 1$, since $B^i_t = \beta^i_t\left[\sum_{k=0}^{\infty}(\beta^i_{t+k}/\beta^i_t) - c\right]$ and $B^i_0 = \sum_{k=0}^{\infty}\beta^i_k - c$. Indeed, we only show that $\beta^A_{t+k} > \beta^A_t\beta^A_k$ since party B's counterpart is obvious. Recall equation (5) and define

$$\begin{aligned} f(\alpha) \equiv\ & \alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^{t+k}\} \\ & -[\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^t\}][\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^k\}]. \end{aligned}$$

It is easy to see that $\beta^A_{t+k} > \beta^A_t\beta^A_k$ if and only if $f(\alpha) > 0$.

Now,

$$\begin{aligned} f(0) &= p^A + (1-p^A)(1-q)^{t+k} - [p^A + (1-p^A)(1-q)^t][p^A + (1-p^A)(1-q)^k] \\ &= p^A(1-p^A)[1-(1-q)^t][1-(1-q)^k] > 0, \end{aligned}$$

since $-1 < 1-q < 1$. In addition, $f(1) = 0$. Moreover, since

$$f''(\alpha) = -2[1-p^A-(1-p^A)(1-q)^t][1-p^A-(1-p^A)(1-q)^k] < 0,$$

the function f is a concave function. Thus, we have shown that $f(\alpha) > 0$ for any $\alpha \in [0,1)$.

Proof of the claim that $\sum_{k=0}^{\infty}(\beta^i_1)^k < \sum_{k=0}^{\infty}\beta^i_k < \sum_{k=0}^{\infty}\delta^k$:

We first note that $(\beta^i_1)^0 = \beta^i_0 = \delta^0 = 1$ when $k = 0$, and that $\beta^i_1 < \delta$ when $k = 1$. For $k \geq 2$, we use the equation

$$\beta^i_k = \beta^i_1\Pi_{l=1}^{k-1}\frac{\beta^i_{l+1}}{\beta^i_l} \tag{22}$$

to show that $(\beta^i_1)^k < \beta^i_k < \delta^k$. It is obvious that we need only show these inequalities in order to prove the claim.

The first inequality is easy to show. Indeed, it follows immediately from (22) and $\beta^i_1 < \beta^i_{l+1}/\beta^i_l$ that $(\beta^i_1)^k < \beta^i_k$. If $q \leq 1$, it is also straightforward to derive the second inequality $\beta^i_k < \delta^k$, since the fact that $\beta^i_0 = \delta^0$, $\beta^i_1 < \delta$, and $\beta^i_{l+1}/\beta^i_l \leq \delta$ for any $l \geq 1$ together with (22) imply that $\beta^i_k < \delta^k$ for any $k \geq 1$.

In the case where $q > 1$, however, the above proof does not apply since $\beta^i_{l+1}/\beta^i_l > \delta$ for any odd i as Figure 1 shows. So, in this case, we use the inequality (for $i = A$ for concreteness)

$$\frac{\beta^i_{l+1}}{\beta^i_l} \cdot \frac{\beta^i_{l+2}}{\beta^i_{l+1}} = \frac{\delta^2[\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^{l+2}\}]}{\alpha + (1-\alpha)\{p^A + (1-p^A)(1-q)^l\}]} < \delta^2,$$

which is valid when l is even, to show that $\beta^i_k < \delta^k$ for any $k \geq 1$:

$$\beta^i_k = \sum_{j=0}^{(k/2)-1} \frac{\beta^i_{2j+1}}{\beta^i_{2j}} \cdot \frac{\beta^i_{2(j+1)}}{\beta^i_{2j+1}} < \delta^{2(k/2)} = \delta^k$$

for an even k, and

$$\beta_k = \left[\sum_{j=0}^{(k-3)/2} \frac{\beta^i_{2j+1}}{\beta^i_{2j}} \cdot \frac{\beta^i_{2(j+1)}}{\beta^i_{2j+1}}\right] \frac{\beta^i_k}{\beta^i_{k-1}} < \delta^{2((k-1)/2)} \cdot \delta = \delta^k$$

for an odd k.

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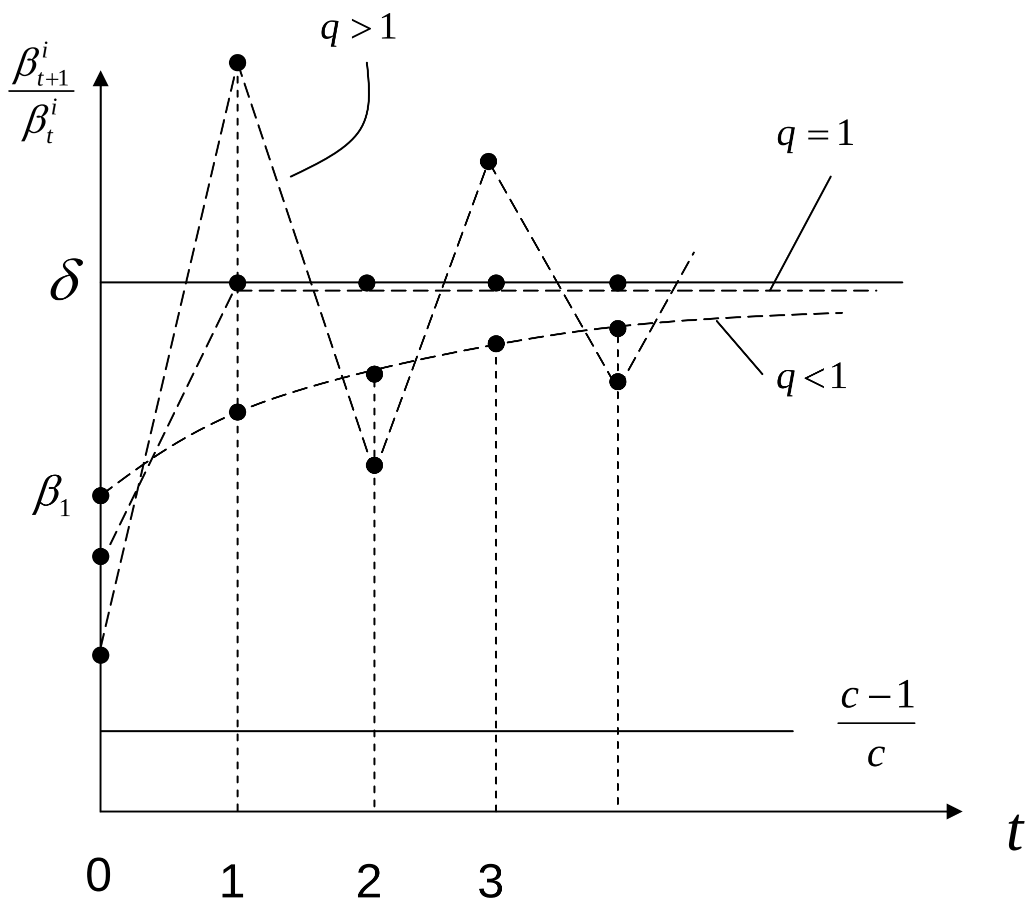

Figure 1. Present-Bias

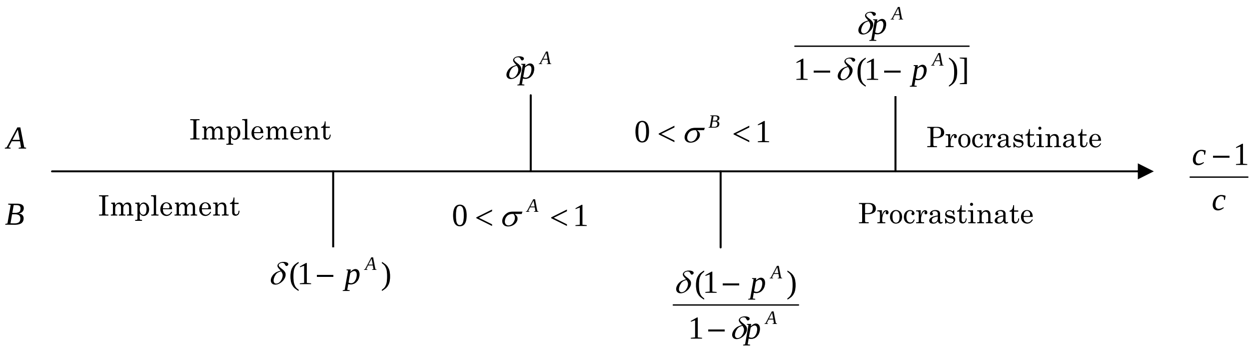

Figure 2. Policy Decision (case where $\alpha = 0$)

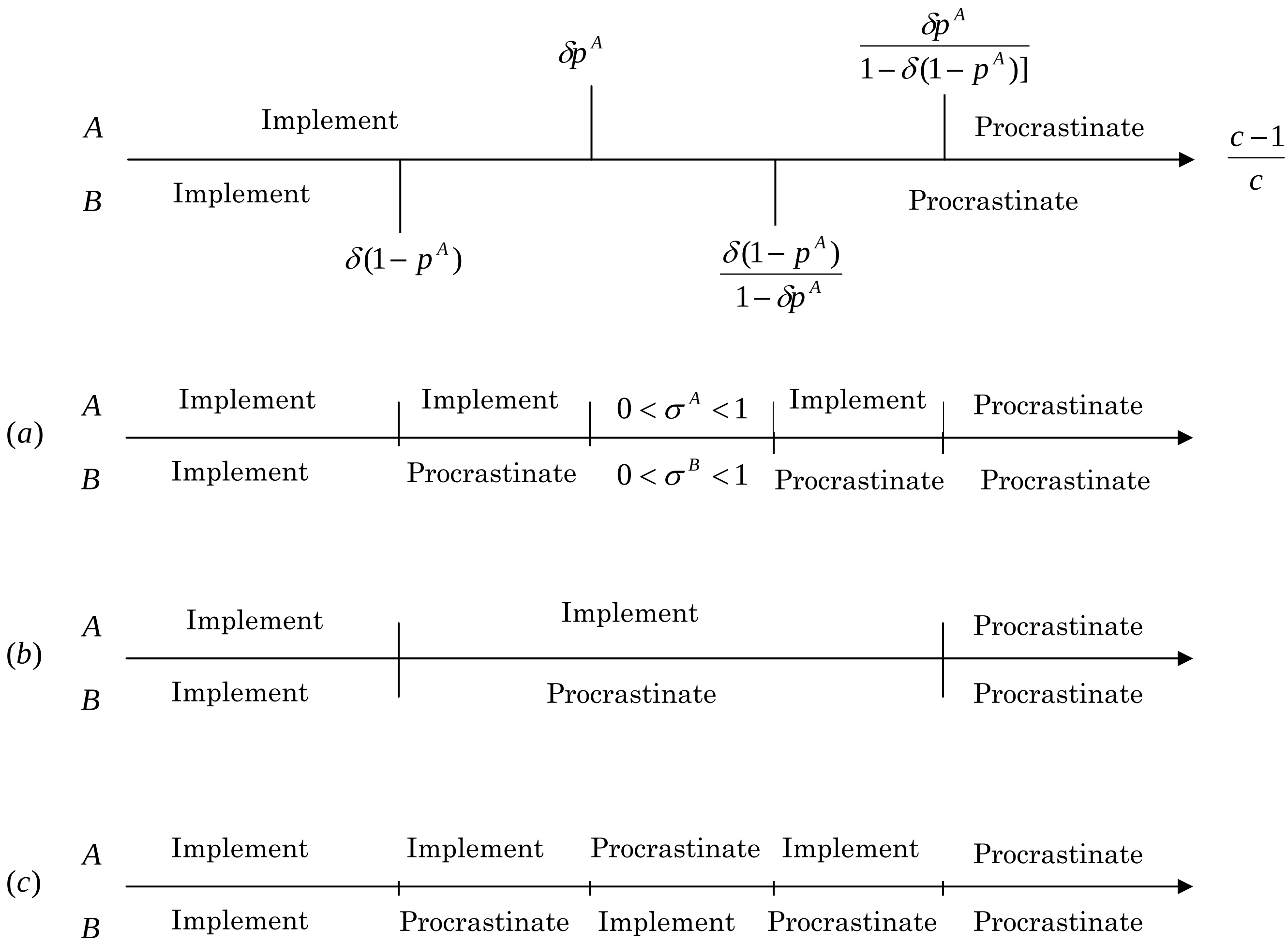

Figure 3. Subgame Perfect Equilibria

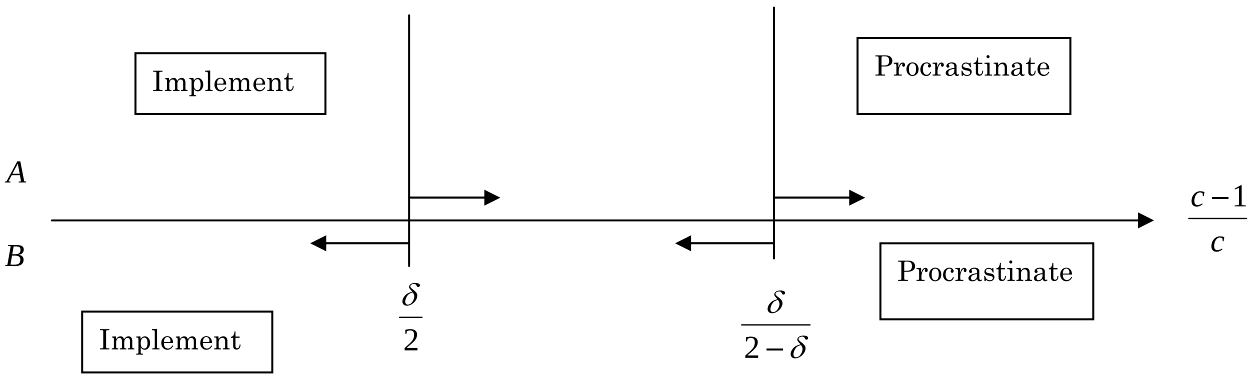

Figure 4. Effect of $p^A \uparrow$ on the critical costs, starting from $p^A = 1/2$