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The Second-Order Dilemma of Public Goods and Capital Accumulation*

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The Second-Order Dilemma of Public Goods and Capital Accumulation[1]

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Abstract

The second-order dilemma of public goods arises from individuals' incentive to free ride on a mechanism to solve the provision problem (first-order dilemma) of public goods. Without relying on social and behavioral arguments, we show by a voluntary participation game that the accumulation of public goods can mitigate the second-order dilemma in the long run. The analysis also shows that population decrease damages the accumulation of public goods.

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1 Introduction

It has been widely argued in the theory of public goods since the seminal work of Olson (1965) that public goods would be undersupplied by voluntary contribution, due to free-riding incentives (see Bergstrom, Blume and Varian (1986) and Andreoni (1988) among others for recent works). To solve the free-riding problem, a large volume of literature investigates suitable mechanisms to achieve an efficient provision of public goods. First, the theory of repeated games considers how long-term relationships facilitate cooperation among selfish individuals. It investigates various behavioral rules with decentralized punishments to attain cooperation. For studies of collective actions in the context of repeated games, see Cremer (1986), Bendor and Mookherjee (1987) and Taylor (1987), etc. Secondly, the theory of mechanism design investigates many kinds of mechanisms to implement an efficient provision of public goods (more generally, a desirable collective choice rule). Groves and Ledyard (1977) propose a mechanism that achieves a Pareto efficient allocation in a public good economy. For a survey on the mechanism design, see Groves and Ledyard (1987). More recently, Moore and Repullo (1988) show that almost any social choice rule can be implemented by subgame perfect equilibria of multi-stage mechanisms in an economic environment with at least one private good. Varian (1994) proposes simple mechanisms which solve a wide variety of externalities problems including the implementation of the Lindahl allocations. For other types of works, it has been studied by cooperative game theory, especially by the core theory, how individuals can reach an efficient solution through voluntary bargaining. For example, see Foley (1970) and Mas-Colell (1980).

Most previous studies on mechanisms to solve the free-riding problem implicitly assume one undesirable property.[2] It has been assumed either that all individuals in question

[2] In the literature cited above, one can broadly regard repeated game strategies and cooperative bargaining games as mechanisms to achieve efficient allocations.

have already participated (or forced to participate) in a mechanism, or that individuals are willing to participate in a mechanism if they become better-off by doing so than in the status-quo. The latter condition is called the individual rationality (sometimes called the participation constraint in the mechanism design literature). It, however, should be remarked that *any* mechanism itself which achieves an efficient provision of public goods is a kind of public goods. Individuals may have an incentive to free ride on the mechanism. The free-riding incentive may lead to the failure of a mechanism, This problem is called *the second-order dilemma* of public goods (Oliver 1980 and Ostrom 1990). There is no external power to force individuals to participate in a mechanism. Participation should be voluntary.

Recently, the voluntary participation problem has been studied by several authors (Palfrey and Rosenthal (1984), Okada (1993), Saijo and Yamato (1999) and Dixit and Olson (2000) among others). By a two-stage game model, they analyze whether or not individuals voluntarily participate in a mechanism which implements an efficient provision of public goods.[3] It has been shown that all individuals do not necessarily participate in the mechanism, and that the likelihood of all individuals' participation becomes lower as population becomes larger. These results partially support a negative view widely-held in the literature that an efficient provision of public goods is not possible due to the second-order dilemma. To solve the second-order dilemma, the recent literature explores extensively social and behavioral factors such as norm, community, social sanctions, trust, fairness, reciprocity and inequity-averse preferences, etc. These factors serve as devices to change individuals' free-riding incentives.

In this paper, we reexamine the voluntary participation game from the viewpoints of public goods accumulation. The motivation of our analysis is two-fold. First, most studies

[3] Palfrey and Rosenthal (1984) consider a one-stage model of voluntary participation in providing binary public goods. Their model has a payoff structure similar to that in the reduced form of the two-stage games studied by others.

on voluntary participation have been done without the possibility of accumulation. On the contrary, many kinds of public goods can be accumulated for generations. Some examples of them are forest, parks, common lands, irrigation systems, public libraries and museums, etc. The relation between the accumulation and individuals' strategic behavior of group formation is subtle, and it is worth investigating. Second, individuals' free-riding incentives naturally change as public goods are accumulated. We will consider whether or not the accumulation can mitigate the second-order dilemma in the long run.

We present a dynamic model of a public good economy with the Cobb-Douglas utility functions. The standard voluntary participation game is played by (non-overlapping) generations. In the first stage, every individual decides independently to participate in a group or not. In the second stage, all participants negotiate for creating a mechanism (say, Groves and Ledyard's mechanism) that implements an efficient provision within the group. It is assumed that if all participants agree to establish it, then the mechanism can be created and implemented effectively with some costs which participants bear.[4] Any non-participant is allowed to free ride on the mechanism. Depending on the outcome in the voluntary participation game, the public goods are accumulated and are inherited by the next generation. We will prove that if the depreciation rate of public goods is low, then the public good stock monotonically increases owing to group formation, and it can exceed the maximum level in the feasible region of groups in the long run. In this sense, we conclude that the accumulation can mitigate the second-order dilemma of public goods. The maximum level of public goods is an increasing function of population. We show that depopulation damages the accumulation of public goods in the long run.

While our analysis presumes the mechanism design approach as a solution of the provision problem of public goods, the analysis can be extended to other types of solutions. For example, suppose that individuals play a repeated voluntary contribution game. If

[4]The implementation problem of mechanisms provides an important research field in its own. However, since we focus on the second-order dilemma, this issue is out of the scope of the paper.

individuals are sufficiently patient, a trigger-type strategy in which individuals stop contributing once any player defects can attain an efficient provision of public goods as a self-binding agreement (Nash equilibrium). A crucial question, however, is whether or not individuals are willing to participate in implementing such a strategy. For every group with positive surplus, there exists a subgame perfect equilibrium of the repeated game in which only participants contribute to public goods. This fact implies that every individual has an incentive to free ride on other individuals' participation.

Fershtman and Nitzan (1991) and Marx and Matthews (2000) consider dynamic contribution games of infinitely-lived individuals. Similar to the folk theorem of repeated games, they describe two polar cases of voluntary contribution. In a linear-quadratic differential game, Fershtman and Nitzan (1991) show the inefficiency of voluntary contribution by a feedback Nash equilibrium in Markovian strategies. Marx and Matthews (2000) show efficiency results implemented by a trigger-type strategy in a long-horizon model. Glomm and Lagunoff (1999) consider a migration choice between a voluntary provision mechanism and an involuntary provision mechanism in a sequential-move game of infinitely-lived individuals. The second-order dilemma of public goods is not at issue in these studies. In the growth theory literature, the accumulation of public capital has been studied extensively since the seminal works of Shell (1967) and Arrow and Kurz (1970). Almost studies in this literature are done in the competitive equilibrium framework, and investment in public capital is chosen by a government authority which maximizes the welfare of a representative infinitely-lived consumer.

The paper is organized as follows. Section 2 presents a dynamic model of the voluntary provision of public goods. Section 3 analyzes the group formation. Section 4 examines the accumulation of public goods. Section 5 discusses implications of the model to the second-order dilemma of public goods and to the depopulation problem. Section 6 concludes the paper.

2 The model

Consider a simple economy with one private good and one public good for generations t (=1, 2,$\cdots$). The public goods are assumed to be nonexcludable and nonrival. Let $n_t \in [n, N]$ be population in generation t. In each generation t, every individual i is initially endowed with wealth ω and contributes an amount a_i $(0 \leq a_i \leq \omega)$ of the private goods to the provision of public goods. The public goods are produced under a constant return to scale technology, and the production y_t of public goods is given by $y_t = \beta \sum_{i=1}^{n_t} a_i$ where β (> 0) is the marginal productivity of private goods.

The total amount K_t of public goods in generation t is given by

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + y_t \\ &= (1-\delta)K_{t-1} + \beta \sum_{i=1}^{n_t} a_i, \end{aligned} \tag{2.1}$$

where K_{t-1} is the stock of public goods in generation $t-1$ and δ $(0 < \delta < 1)$ is the depreciation rate of public goods. In this paper, for clarity of analysis, we employ the Cobb-Douglas utility function of individual i

$$u_i(x_i, K_t) = x_i^{\alpha} K_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $x_i + a_i = \omega$.

Given K_{t-1}, the voluntary contribution game in generation t is defined as follows. Every individual i $(= 1, \cdots, n_t)$ has a pure action $a_i \in [0, \omega]$, and his payoff function is given by

$$u_i(a_1, \cdots, a_n, K_{t-1}) = (\omega - a_i)^{\alpha}((1-\delta)K_{t-1} + \beta \sum_{i=1}^{n_t} a_i)^{1-\alpha}.[5] \tag{2.2}$$

[5] To save symbols, we abuse notation u_i to denote various types of payoffs for individual i whenever no confusion arises.

We can see without much difficulty that $\partial u_i / \partial a_i < 0$ for all contribution profiles $a = (a_1, \cdots, a_n)$ if

$$\frac{1-\alpha}{\alpha} < \frac{(1-\delta)K_{t-1}}{\beta\omega}. \tag{2.3}$$

This fact means that if (2.3) holds, then every individual i is better off by contributing nothing ($a_i = 0$) than by contributing any positive amount ($a_i > 0$) of private goods, regardless of others' contributions. That is, the zero contribution is the dominant action for each individual. Then, the voluntary contribution game has a unique Nash equilibrium $a^* = (0, \cdots, 0)$ in which every individual receives payoff

$$u(0, K_{t-1}) = \omega^\alpha [(1-\delta)K_{t-1}]^{1-\alpha}. \tag{2.4}$$

Generally, the zero-contribution equilibrium is not Pareto-efficient. (2.3) shows that either the marginal productivity (β) or the preference-weight ratio $\frac{1-\alpha}{\alpha}$ of public good to private good is low. Throughout the paper, we assume (2.3).[6]

The voluntary contribution game of generation t has the structure of an n-person prisoners' dilemma under (2.3). The individually rational action (zero contribution) does not lead to a collectively rational outcome (Pareto-efficient allocation). Many types of solutions for this problem (often called the first-order dilemma of public goods) have been explored in the literature. As we have discussed in the introduction, the solutions are broadly considered to design suitable mechanisms to implement efficient allocations. In the next section, we will consider how the mechanism design approach is effective to solve the first-order dilemma of public goods.

[6]When (2.3) does not hold, it can be shown that the voluntary contribution game has a unique Nash equilibrium in which every individual contributes a positive amount $a_i^* = \frac{(1-\alpha)\beta\omega - \alpha(1-\delta)K_{t-1}}{(1+n\alpha-\alpha)\beta}$. Then, stock K_{t-1} increases when the depreciation rate δ is low. This fact suggests that (2.3) would become satisfied as the economy develops.

3 The group formation

We consider a situation where individuals voluntarily form a group to provide public goods. We assume that if a group is formed, its members can establish a suitable mechanism to implement an efficient provision of public goods within the group. It is certainly costly to have such a mechanism for providing public goods collectively. The costs of a mechanism include those of communication, negotiations, monitoring, punishment, staffing, and maintaining the group organization. In what follows, these costs are simply referred to as *group costs*. The participants bear the group costs. The focus of our analysis is on whether or not individuals voluntarily participate in a costly group to enforce an efficient provision of public goods, and (if they do) on how many individuals participate. All non-participants are free from the enforcement by a group. They are allowed to free ride on the contributions by participants.

Before we present a two-stage process of group formation, we first analyze the optimal contribution of a group. Let S be a set of participant. The optimal contributions of group S maximize the sum of participants' payoffs

$$U^S = \sum_{i \in S} u_i(a_1, \cdots, a_{n_t}, K_{t-1})$$
$$\text{subject to} \quad 0 \leq a_i \leq \omega \ (i \in S) \quad \text{and} \quad a_i = 0 \ (i \notin S).$$

Note that the optimal contribution of non-participant is $a_{np} = 0$ under (2.3), independent of group size s. Assuming that all participants contribute an amount x of private goods equally, the total payoff U^S is given by

$$U^S = s(\omega - x)^\alpha((1-\delta)K_{t-1} + \beta s x)^{1-\alpha}.$$

It can be seen without much difficulty that $\partial U^S/\partial x = 0$ implies

$$x = \frac{(1-\alpha)\beta s\omega - \alpha(1-\delta)K_{t-1}}{\beta s}.$$

Therefore, given the group size s and the stock K_{t-1} of public goods, the optimal (symmetric) contribution $a_p(s, K_{t-1})$ of each participant is given by

$$a_p(s, K_{t-1}) = \max\ (0,\ \frac{(1-\alpha)\beta s\omega - \alpha(1-\delta)K_{t-1}}{\beta s}). \tag{3.1}$$

When

$$\frac{1-\alpha}{\alpha} s > \frac{(1-\delta)K_{t-1}}{\beta\omega}, \tag{3.2}$$

the group rational behavior of each participant is to contribute the positive amount $a_p(s, K_{t-1})$ of private goods. The optimal payoff $u_p(s, K_{t-1})$ of participant i is given by

$$u_p(s, K_{t-1}) = \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(\beta s)^\alpha}[\beta s\omega + (1-\delta)K_{t-1}]. \tag{3.3}$$

When the optimal contribution is implemented in the group S, the non-participant's payoff $u_{np}(s, K_{t-1})$ is given by

$$u_{np}(s, K_{t-1}) = \omega^\alpha(1-\alpha)^{1-\alpha}[\beta s\omega + (1-\delta)K_{t-1}]^{1-\alpha}. \tag{3.4}$$

When (3.2) does not hold, the optimal choice of the group is the zero contribution. From (2.3) and (3.2), we can see that the voluntary contribution game is described as an n-person prisoners' dilemma if

$$\frac{1-\alpha}{\alpha}\frac{\beta\omega}{(1-\delta)} < K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{(1-\delta)}n_t.$$

The following properties of payoff functions $u_p(s, K_{t-1})$ and $u_{np}(s, K_{t-1})$ are important to our analysis.

Lemma 3.1. When (3.2) satisfies,

(1) $u_p(s, K_{t-1}) > u(0, K_{t-1})$,

(2) $u_p(s, K_{t-1})$ and $u_{np}(s, K_{t-1})$ are monotonically increasing in s,

(3) $\frac{u_{np}(s-1, K_{t-1})}{u_p(s, K_{t-1})}$ is monotonically increasing in s.

Proof. (1): proved by (3.1) and (3.3). (2): $\log\, u_p(s, K_{t-1})$ is a monotonically increasing function of s since

$$\frac{d}{ds}\log\, u_p(s, K_{t-1}) = \frac{\beta\omega}{\beta s\omega + (1-\delta)K_{t-1}} - \frac{\alpha}{s} > 0$$

if (3.2) holds. Therefore, $u_p(s, K_{t-1})$ is monotonically increasing in s. It follows easily from (3.4) that $u_{np}(s, K_{t-1})$ is monotonically increasing in s. (3): It is sufficient to show that $f(s) \equiv \log\, u_{np}(s-1, K_{t-1}) - \log\, u_p(s, K_{t-1})$ is monotonically increasing in s. This can be proved by

$$\begin{aligned}
\frac{df}{ds} &= \frac{\alpha}{s} + \frac{\beta\omega(1-\alpha)}{\beta(s-1)\omega + (1-\delta)K_{t-1}} - \frac{\beta\omega}{\beta s\omega + (1-\delta)K_{t-1}} \\
&> \frac{\alpha}{s} - \frac{\alpha\beta\omega}{\beta s\omega + (1-\delta)K_{t-1}} > 0.
\end{aligned}$$

Q.E.D.

Every individual is faced with two conflicting incentives about group formation. First of all, he has an incentive to free ride on others' contributions, and thus not to participate in a group. But, if all individuals' actions are governed by this incentive, a group is not formed, and no contribution is made to provide public goods. Participants in a group are better-off than in the non-contribution outcome if (3.2) holds. Therefore, every individual has an incentive to participate in a group, *provided that a sufficient number of other individuals participate.* Due to these different incentives of individuals, the group formation problem in the public goods economy is not simple.

To consider the problem of voluntary participation in a pure form, we present a two-stage game of group formation in each generation t $(= 1, 2, \cdots)$ as follows.

(1) Participation decision stage:

Given the stock K_{t-1} of public goods, all individuals of generation t decide independently whether to participate in a group or not. Let S be the set of all participants, and let s be the number of the participants. If $s = 0, 1$, then no group forms.[7]

(2) Group negotiation stage:

All participants decide independently whether or not they should agree to establish a mechanism to enforce the optimal provision (3.1) of public goods on them. The mechanism is established if and only if all participants agree. We say that a group is formed, if such a unanimous agreement is reached. When a group is formed, every participant is assumed to bear the group cost equally. The group cost per member is denoted by $c(s)(> 0)$ where s is the group size. All non-participants can choose their contributions freely. When a group is not formed, the voluntary contribution game in section 2 is played. When every individual makes his choice in the group negotiation stage, it is assumed that he knows perfectly the outcome in the participation decision stage.

In the process of group formation, individuals decide to participate in a group or not, anticipating rationally the outcome of the group negotiation stage. To consider the participation decision of individuals, we characterize a subgame perfect equilibrium of the two-stage game of group formation. In this paper, we consider only pure strategy equilibria.[8]

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[7] If $s = 1$, then no single participant has an incentive to contribute under (2.3).

[8] Okada (1993) characterizes a (symmetric) mixed strategy equilibrium in the context of an n-person prisoners' dilemma.

By backward induction, we first analyze the group negotiation stage. For a group of size s, we define the *group surplus (per member)*, denoted by $w(s, K_{t-1})$, as

$$w(s, K_{t-1}) = u_p(s, K_{t-1}) - c(s) - u(0, K_{t-1}) \tag{3.5}$$

where $u(0, K_{t-1}) = \omega^\alpha[(1-\delta)K_{t-1}]^{1-\alpha}$ is the payoff for every individual when no contribution is made. $u(0, K_{t-1})$ can be regarded as the opportunity cost of a group. The group surplus $w(s, K_{t-1})$ is the participant's net payoff $u_p(s, K_{t-1}) - c(s)$ minus the opportunity cost of the group.

We assume the following property of the group cost function $c(s)$.

Assumption 3.1. (1) $c(s)$ is differentiable, and $\frac{\partial u_p}{\partial s} > \frac{dc}{ds}$ for all $s \leq n_t$. (2) $w(s, K_{t-1}) > 0$ for some $s \leq n_t$.

The first assumption implies that the net payoff $u_p(s, K_{t-1}) - c(s)$ of a participant is a monotonically increasing function of group size s as $u_p(s, K_{t-1})$ is (see Lemma 3.1). If the second assumption does not hold, the surplus of no group is positive, and thus the group formation problem is vacuous.

Lemma 3.2. There exists a unique solution s^*, denoted by $g_c(K_{t-1})$, of $w(s, K_{t-1}) = 0$. The function g_c satisfies: (1) g_c is a monotonically increasing function of K_{t-1}, (2) $g_0(K_{t-1}) = \frac{\alpha}{1-\alpha}\frac{(1-\delta)K_{t-1}}{\beta\omega}$ when $c(s) \equiv 0$, and (3) if the group cost function $c(s)$ shifts upward, then $g_c(K_{t-1})$ shifts upward, too.

Proof. By Lemma 3.1, $u_p(s, K_{t-1}) - u(0, K_{t-1})$ is a monotonically increasing function of s where $s \geq \frac{\alpha}{1-\alpha}\frac{(1-\delta)K_{t-1}}{\beta\omega}$, and its value is zero at $s = \frac{\alpha}{1-\alpha}\frac{(1-\delta)K_{t-1}}{\beta\omega}$. Then, Assumption 3.1 implies that the group surplus $w(s, K_{t-1})$ is a monotonically increasing function of s where

$w(s,K_{t-1}) < 0$ for $s = \frac{\alpha}{1-\alpha}\frac{(1-\delta)K_{t-1}}{\beta\omega}$ and $w(s,K_{t-1}) > 0$ for some $s \leq n_t$. Accordingly, we can prove the first part of the lemma. By applying the Implicit Function Theorem to $w(s,K_{t-1}) = 0$, we obtain

$$(\frac{\partial u_p}{\partial s} - \frac{dc}{ds})\frac{ds}{dK_{t-1}} = \frac{\partial u(0,K_{t-1})}{\partial K_{t-1}} - \frac{\partial u_p}{\partial K_{t-1}}.$$

By Assumption 3.1, $\frac{\partial u_p}{\partial s} - \frac{dc}{ds} > 0$. By (2.4) and (3.3), we have

$$\begin{aligned}\frac{\partial u(0,K_{t-1})}{\partial K_{t-1}} - \frac{\partial u_p}{\partial K_{t-1}} &= \frac{(1-\alpha)(1-\delta)^{1-\alpha}\omega^{\alpha}}{K_{t-1}^{\alpha}} - \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\delta)}{(\beta s)^{\alpha}} \\ &= (1-\alpha)(1-\delta)((\frac{\omega}{(1-\delta)K_{t-1}})^{\alpha} - (\frac{\alpha}{(1-\alpha)\beta s})^{\alpha}).\end{aligned}$$

It can be shown that the RHS of the last equality is positive if (3.2) holds. Thus, (1) can be proved. (2) can be proved by (3.1). Finally, (3) can be proved by the fact that the group surplus function $w(s,K_{t-1})$ shifts downward as the group cost function $c(s)$ does upward. Q.E.D.

The next lemma characterizes the equilibrium outcome of the group negotiation stage.

Lemma 3.3. In the group negotiation stage, a group with s participants is formed in a Nash equilibrium if and only if $s \geq g_c(K_{t-1})$.

Proof. If all s participants agree to form a group, then each of them receives the net payoff $u_p(s,K_{t-1}) - c(s)$. If any single participant does not agree, the group is not formed and all participants receive payoff $u(0,K_{t-1})$. Therefore, the group of s participants can be formed in a Nash equilibrium if and only if $w(s,K_{t-1}) \geq 0$. It follows from Lemma 3.2 that $w(s,K_{t-1}) \geq 0$ is equivalent to $s \geq g_c(K_{t-1})$. Q.E.D.

The intuition for this result is clear. All participants agree to form their group if and only if the group surplus per member is positive, that is, they all are better off by forming a group than by in the zero-contribution outcome. We remark that there are many other non-strict Nash equilibria leading to the failure of a group.[9] For example, all situations where there are at least two participants who do not agree to form the group, are such equilibria. We exclude from our analysis these non-strict equilibria. Proposition 3.1 motivates the following definition.

Definition 3.1. Given K_{t-1}, the *threshold* of a group is defined to be the minimum integer s that $s > g_c(K_{t-1})$ (that is, $w(s, K_{t-1}) > 0$) and is denoted by $s(K_{t-1})$.[10]

Next, we analyze the participation decision stage. In this stage, all individuals decide independently to participate in a group or not, anticipating rationally what will happen in the group negotiation stage. There always exists a (non-strict) Nash equilibrium in which no individuals participate. We focus on a Nash equilibrium in which a group is formed in the group negotiation stage. We call such a Nash equilibrium a *group equilibrium*. The following theorem characterizes a group equilibrium of the participation decision stage.

Theorem 3.1. A group equilibrium of the participation decision stage is characterized as follows.

(1) There exists a group equilibrium if and only if $s(K_{t-1}) \leq n_t$. The number s^* of participants in a group equilibrium satisfies $s(K_{t-1}) \leq s^* \leq n_t$.

(2) The largest group with n_t participants is a group equilibrium if and only if either

[9]A Nash equilibrium is called *strict* if every player has a unique best response. In a non-strict Nash equilibrium, every individual is indifferent between agreeing and disagreeing to form a group.

[10]When $s = g_c(K_{t-1})$, all participants receive payoffs $u(0, K_{t-1})$ in the group negotiation stage, regardless of their actions. Then, every action profile is trivially a (non-strict) Nash equilibrium. We rule out this degenerate case from our analysis.

$n_t = s(K_{t-1})$ or $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$.

(3) Suppose that $\frac{dc(s)/ds}{c(s)} \geq \frac{\partial u_p(s,K_{t-1})/\partial s}{u_p(s,K_{t-1})}$. Then, the number of participants in a group equilibrium is uniquely determined (except a degenerate case).

Proof. (1): If a group with s ($\leq n_t$) participants forms in a Nash equilibrium, then it must be true from Lemma 3.3 that $s(K_{t-1}) \leq s$. Therefore, $s(K_{t-1}) \leq n_t$. Conversely, suppose that $s(K_{t-1}) \leq n_t$. Consider the two distinct cases, (i) $s(K_{t-1}) = n_t$ and (ii) $s(K_{t-1}) < n_t$. In case (i), we will show that the largest group with n_t participants is a Nash equilibrium. Every participant in the largest group receives payoff $u_p(n_t, K_{t-1}) - c(n_t)$. By Lemma 3.3, if any participant deviates from the group, the remaining $n_t - 1$ participants do not agree to form their group since $n_t - 1 < s(K_{t-1})$, and thus the deviator receives payoff $u(0, K_{t-1})$. Since $u_p(n_t, K_{t-1}) - c(n_t) > u(0, K_{t-1})$ from $s(K_{t-1}) = n_t$, the largest group is a Nash equilibrium. Consider case (ii). In this case, even if any participant deviates from the largest group, the remaining $n_t - 1$ participants agree to form their group since $s(K_{t-1}) \leq n_t - 1$. This means that the deviator can free-ride on the other participants' contributions, and thus receives payoff $u_{np}(n_t - 1, K_{t-1})$. Consider the two subcases: case (iia) $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$, and case (iib) $u_p(n_t, K_{t-1}) - c(n_t) < u_{np}(n_t - 1, K_{t-1})$. In subcase (iia), it is clear that the largest group is a Nash equilibrium. In subcase (iib), consider further the two distinct subcases, (iib-1) $u_p(s, K_{t-1}) - c(s) < u_{np}(s-1, K_{t-1})$ for all s with $s(K_{t-1}) + 1 \leq s \leq n_t$, and (iib-2) $u_{np}(s-1, K_{t-1}) \leq u_p(s, K_{t-1}) - c(s)$ for some s with $s(K_{t-1}) + 1 \leq s \leq n_t$. In subcase (iib-1), only groups with $s(K_{t-1})$ participants are Nash equilibria. In subcase (iib-2), denote by s^* the maximum integer which satisfies $u_{np}(s-1, K_{t-1}) \leq u_p(s, K_{t-1}) - c(s)$. Then, we have $u_{np}(s^*-1, K_{t-1}) \leq u_p(s^*, K_{t-1}) - c(s^*)$ and $u_p(s^*+1, K_{t-1}) - c(s^*+1) < u_{np}(s^*, K_{t-1})$. These two inequalities mean that a group with s^* participants is a Nash equilibrium.

(2): The arguments in cases (i) and (iia) of (1) prove (2).

(3): First, we show that $\frac{u_{np}(s-1,K_{t-1})}{u_p(s,K_{t-1})-c(s)}$ is a monotonically increasing function of s where $s \geq s(K_{t-1})$ if $\frac{dc(s)/ds}{c(s)} \geq \frac{\partial u_p(s,K_{t-1})/\partial s}{u_p(s,K_{t-1})}$. To do this, it suffices to see that $f(s) \equiv \log u_{np}(s-1, K_{t-1}) - \log\,(u_p(s, K_{t-1}) - c(s))$ is a monotonically increasing function. We have

$$\frac{df(s)}{ds} = \frac{\partial u_{np}(s-1, K_{t-1})/\partial s}{u_{np}(s-1, K_{t-1})} - \frac{\partial u_p(s, K_{t-1})/\partial s - dc(s)/ds}{u_p(s, K_{t-1}) - c(s)}.$$

The condition $\frac{dc(s)/ds}{c(s)} \geq \frac{\partial u_p(s,K_{t-1})/\partial s}{u_p(s,K_{t-1})}$ implies

$$\frac{\partial u_p(s, K_{t-1})/\partial s}{u_p(s, K_{t-1})} \geq \frac{\partial u_p(s, K_{t-1})/\partial s - dc(s)/ds}{u_p(s, K_{t-1}) - c(s)}.$$

This gives

$$\frac{df(s)}{ds} \geq \frac{\partial u_{np}(s-1, K_{t-1})/\partial s}{u_{np}(s-1, K_{t-1})} - \frac{\partial u_p(s, K_{t-1})/\partial s}{u_p(s, K_{t-1})}.$$

Lemma 3.1.(3) implies that the RHS of the inequality above is positive. Thus, $f(s)$ is an increasing function of s. The fact that $\frac{u_{np}(s-1,K_{t-1})}{u_p(s,K_{t-1})-c(s)}$ is an increasing function of s yields the following two properties (for notational simplicity, stock variable K_{t-1} is omitted in payoff functions u_p and u_{np}):

$$u_{np}(s-1) \geq u_p(s) - c(s) \;\text{ implies } u_{np}(t-1) > u_p(t) - c(t) \;\text{ for } s < t \tag{3.6}$$

$$u_{np}(s-1) \leq u_p(s) - c(s) \;\text{ implies } u_{np}(t-1) < u_p(t) - c(t) \;\text{ for } t < s \tag{3.7}$$

According to the proof of (1), it suffices to show that the number of participants in a group equilibrium is unique in each of three distinct cases: (iia), (iib-1), and (iib-2) (except a degenerate case). In case (iia), the largest group is a Nash equilibrium since $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$. By (3.7), we have $u_p(t, K_{t-1}) - c(t) > u_{np}(t-1, K_{t-1})$ for all t with $s(K_{t+1}) \leq t < n_t$. This means that any other group is not a Nash equilibrium. In subcase (iib-1), only groups with $s(K_{t-1})$ participants are Nash equilibria. In subcase (iib-2), as we have shown in the proof of (1), there exists some s^* which satisfies

$u_{np}(s^* - 1, K_{t-1}) \le u_p(s^*, K_{t-1}) - c(s^*)$ and $u_p(s^* + 1, K_{t-1}) - c(s^* + 1) < u_{np}(s^*, K_{t-1})$. The first inequality and (3.7) imply that

$$u_{np}(s-1) < u_p(s) - c(s) \quad \text{if} \ \ s(K_{t+1}) + 1 \le s \le s^* - 1, \tag{3.8}$$

and the second inequality and (3.6) imply that

$$u_p(s+1, K_{t-1}) - c(s+1) < u_{np}(s, K_{t-1}) \quad \text{if} \ \ s^* \le s \le n_t. \tag{3.9}$$

We can see from (3.8) and (3.9) that any group with s $(\neq s^* - 1, s^*)$ participants is not a Nash equilibrium. It can be shown that if $u_{np}(s^* - 1, K_{t-1}) < u_p(s^*, K_{t-1}) - c(s^*)$, then only groups with s^* participants are Nash equilibria, and that if $u_{np}(s^* - 1, K_{t-1}) = u_p(s^*, K_{t-1}) - c(s^*)$, then groups with either $s^* - 1$ or s^* are Nash equilibria. Q.E.D.

Theorem 3.1 clarifies the strategic nature of the second-order dilemma of public goods. It shows that if population n_t is not less than the threshold of a group (i.e., $s(K_{t-1}) \le n_t$), then it is possible that some group, although not necessarily the largest one, is formed voluntarily by self-interested individuals. Thus, a positive amount of contributions is possible. One may wonder why self-interested individuals are willing to form a group for collective provision of public goods in a situation that they have an incentive to free-ride on others' contributions. The answer consists of two parts. First, individuals are better-off than in the non-contribution outcome, by forming a group larger than the threshold. Second, if any participant opts out of the equilibrium group, then the remaining participants have an opportunity to respond to the deviation in the two-stage process of group formation. Actually, they decrease their contributions, and even worse, they may dissolve a group when the number of new participants is less than the threshold. This counter-response by other participants damages the deviator. The condition $n_t \ge s(K_{t-1})$

tends to be satisfied if population (n_t), marginal productivity (β) and depreciation rate (δ) are large while the preference-weight ratio ($\frac{\alpha}{1-\alpha}$) of private goods to public goods and stock (K_{t-1}) of public goods are small.

Our analysis shows that the first-order dilemma of and the second-order dilemma of public goods describe strategic situations of different kinds. Formally, the first-order dilemma arises in the voluntary contribution game, which is formulated as an n-person prisoners' dilemma where zero-contribution is the dominant action of every individual. On the other hand, the second-order dilemma arises in the voluntary participation game, in which non-participation is not the dominant action of an individual.

The theorem also shows that all individuals do not participate in a group. Furthermore, it can be shown that as population grows large, the likelihood that the largest group is a Nash equilibrium becomes negligible.

Corollary 3.1. When population n_t becomes large, the class of the Cobb-Douglas utility functions under which the largest group is a Nash equilibrium is negligible.

Proof. In Theorem 3.1.(2), the first condition $n_t = s(K_{t-1})$ is violated as n_t becomes large, noting that the threshold $s(K_{t-1})$ of a group is independent of n_t. The second condition $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$ implies that

$$1 > (\frac{1}{\alpha})^{\alpha} \ \frac{\beta(n-1)\omega + (1-\delta)K_{t-1}}{\beta n\omega + (1-\delta)K_{t-1}} \ (\frac{\beta n\omega}{\beta(n-1)\omega + (1-\delta)K_{t-1}})^{\alpha}.$$

As n_t goes to infinity, the RHS of the inequality above converges to $(\frac{1}{\alpha})^{\alpha}$, which is larger than 1 for any $0 < \alpha < 1$. Therefore, the condition does not hold in the limit that n_t goes to infinity. Q.E.D.

This result shows a negative aspect of population growth in voluntary provision of public goods. Saijo and Yamato (1999) prove a similar result in the case of no stock of public goods.

To conclude this section, the voluntary participation game can not solve the second-order dilemma of public goods in the sense that the provision of public goods is not efficient. Against this conclusion, one may argue that a different rule of a participation game from ours may solve the second-order dilemma yielding an efficient provision of public goods. Probably, one of the simplest rules of this kind seems to be that in the first stage, every individual makes a conditional statement, "I will participate if and only if all others also participate." If such a commitment were credible, every individual would recognize that if he does not participate, then he can not enjoy free-riding benefits since any group is not formed, and thus would participate. However, this type of prior commitments should be made in negotiations among individuals (in the group negotiation stage in our two-stage game). Individuals should make the decision to participate or not before negotiations start.[11] More generally, if anyone could design a game in which all individuals participate in a group to provide public goods, such a game itself is a public good, and every individual has an incentive to free ride. We are again faced with the second-order dilemma of public goods.

From a purely theoretical point of view, the arguments of the second-order dilemma are so strong that without any devises that change individuals' incentives, one can not hope that the dilemma is solved. As such devices, the recent literature explores extensively social and behavioral factors such as norm, community, social sanctions. trust, fairness and inequity-averse preferences, etc. In the next section, we will consider that the accumulation of public goods can also serve as a device to mitigate the second-order dilemma.

[11] See Dixit and Olson (2000, p. 313) for related arguments.

4 The accumulation of public goods

We investigate whether or not the accumulation of public goods can mitigate the second-order dilemma when the group formation game is played by generations. It follows from (2.1) that the magnitude of accumulation in each generation is determined by the total contribution of individuals. Theorem 3.1 shows that the equilibrium group is formed if population n_t exceeds the threshold $s(K_{t-1})$ of a group at K_{t-1}. Recall that $s(K_{t-1})$ is the smallest integer that the group surplus $w(K_{t-1}, s)$ per member is positive (Definition 3.1). Let $s^*(K_{t-1})$ be the number of participants in a group equilibrium at K_{t-1}. It holds from Theorem 3.1 that $s(K_{t-1}) \leq s^*(K_{t-1}) \leq n_t$.

Let $a^*(K_{t-1})$ be the total contribution of an equilibrium group when the public good stock is K_{t-1}. By (3.1), it holds that

$$a^*(K_{t-1}) = \frac{1}{\beta}[(1-\alpha)\beta\omega \cdot s^*(K_{t-1}) - \alpha(1-\delta)K_{t-1}]. \tag{4.1}$$

By (2.1), the dynamics of public good stock K_{t-1} is given by

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + \beta a^*(K_{t-1}) \quad \text{if} \quad s(K_{t-1}) \leq n_t \\ &= (1-\delta)K_{t-1} \qquad\qquad\qquad\quad \text{otherwise.} \end{aligned} \tag{4.2}$$

Substituting (4.1) to (4.2), we obtain

$$K_t = (1-\alpha)((1-\delta)K_{t-1} + \beta s\omega) \quad \text{where} \quad s = s^*(K_{t-1}). \tag{4.3}$$

The dynamic system (4.3) has the following property. Define

$$K^*(s) = \frac{(1-\alpha)\beta\omega}{1-(1-\alpha)(1-\delta)}s. \tag{4.4}$$

Let $s = s^*(K_{t-1})$. When $K_{t-1} < K^*(s)$, the stock K_{t-1} increases to $K_t < K^*(s)$, and when $K_{t-1} > K^*(s)$, K_{t-1} decreases to $K_t > K^*(s)$. In either case, given s, the stock sequence $\{K_t\}$ monotonically converges to $K^*(s)$ as t goes to infinity, provided that the group size were fixed at s over generations. We call $K^*(s)$ the *long-run level of public goods under a group with s participants.*

Finally, we formulate the population change by

$$n_{t+1} = n_t + \Delta n_t. \tag{4.5}$$

When $\Delta n_t > 0$, population increases, and when $\Delta n_t < 0$, population decreases. When $\Delta n_t = 0$, there is no population change.

To analyze the accumulation of the public good stock, we define the *feasible region of groups*, denoted by F, as the set of (K_{t-1}, n_t) at which a group forms in equilibrium. By Theorem 3.1, the feasible region of groups is given by

$$F = \{(K_{t-1}, n_t) \in R_+ \times [n, N] \mid (K_{t-1}, n_t) \text{ satisfies (2.3) and } n_t > g_c(K_{t-1})\}.$$

Let k_c denote the inverse function of g_c (note that g_c is a monotonically increasing function by Lemma 3.2.). Then, $(K_{t-1}, n_t) \in F$ if and only if $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta} < K_{t-1} < k_c(n_t)$. The $k_c(n_t)$ is the maximum level of public goods among those at which a group can be formed. If the stock of public goods exceeds this level, then the zero contribution is the individually rational action and also the optimal action of any group. In this case, the second-order dilemma disappears since there exists no provision problem of public goods. In the following, we will prove that if depreciation rate is sufficiently low, then public goods can be accumulated through group formation, and that it can exceed the maximum level $k_c(n_t)$ in the feasible region of groups in the long run.

By Lemma 3.2, the maximum level of public goods in F is given by

$$k_0(n_t) = \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}n_t \quad \text{when} \quad c(s) \equiv 0. \tag{4.6}$$

Comparing (4.4) and (4.6), we can see that for all s, $K^*(s) = \frac{(1-\alpha)\beta\omega}{1-(1-\alpha)(1-\delta)}s \leq k_0(s) = \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}s$ for all $\delta \geq 0$ (the equality holds if $\delta = 0$). By Lemma 3.2, if the group cost function $c(s)$ moves upward from zero, then g_c moves upward, too. This implies that $k_c(s) < k_0(s)$ for every s. As the group cost $c(s)$ becomes larger, $k_c(s)$ becomes smaller. $K^*(s)$ becomes as close to $k_0(s)$ as one wants, when depreciation rate δ converges to zero. Therefore, when depreciation rate δ is sufficiently low and the group cost $c(s)$ is positive, we can obtain $k_c(s) < K^*(s) < k_0(s)$ for all s. With this in mind, we assume the following.

Assumption 4.1. $k_c(s) < K^*(s)$ for all s.

Geometrically, this assumption means that the graph $K = K^*(s)$ of the long-run levels of public goods locates outside the feasible region F of groups (see Figure 4.1 where the feasible region F is the quadrangle $ABCD$). More importantly, it means that the stock K_{t-1} of public goods increases whenever a group of individuals forms to provide public goods collectively. The contributions of group members outweigh the depreciation of public goods.

We now investigate the accumulation of public goods derived by group formation. To examine purely the relationship between the accumulation and the group formation, we first consider the case of no population change ($\Delta n_t = 0$).

Theorem 4.1. Suppose that $n_t = n_1$ for all $t \geq 1$. If Assumption 4.1 holds, then, starting from any initial point $(K_0, n_1) \in F$, the sequence of the public goods stock K_t ($t = 1, 2, \cdots$) generated by (4.2) monotonically increases, and for some t K_t exceeds the maximum level of the feasible region F of groups.

Proof. First note that the threshold $s(K_{t-1})$ of a group is equal to s $(= 2, \cdots, n_1)$ if $s - 1 \leq g_c(K_{t-1}) < s$, equivalently, $k_c(s-1) \leq K_{t-1} < k_c(s)$ where k_c is the inverse function of g_c. With help of this relation, we divide the set of all K_{t-1} with $(K_{t-1}, n_1) \in F$ into distinct intervals $I_s = [k_c(s-1), k_c(s))$, $s = 2, \cdots, n_1$. We will prove the theorem in two steps.

step 1: We show that for every s, $K_{t-1} \in I_s$ implies $K_{t-1} < K_t$. From $K_{t-1} \in I_s$, we have $K_{t-1} < k_c(s)$. Since k_c is a monotonically increasing function, we have $K_{t-1} < k_c(s) < k_c(m) < K^*(m)$ for all m with $s \leq m \leq n_1$. The last inequality is held by Assumption 4.1. By Theorem 3.1, the equilibrium group size s^* at $K_{t-1} \in I_s$ satisfies $s \leq s^* \leq n_1$. By the property of the dynamic system (4.2), $K_{t-1} < K^*(s^*)$ implies $K_{t-1} < K_t$.

step 2: We show that for some large t, $k_c(n_1) \leq K_t$. Suppose not. Then, $K_t < k_c(n_1)$ for all t. Since $\{K_t\}$ is a monotonically increasing sequence, it must be true that there exists some $m \leq n_0$ such that $K_t \in I_m$ for almost all t. By Theorem 3.1, an equilibrium group size s^* at $K_t \in I_m$ is greater than or equal to m. Since $k_c(m) < K^*(m) < K^*(s^*)$, it follows from the dynamic property of (4.2) that we must have $k_c(m) < K_t$ for any sufficiently large t. This contradicts that $K_t \in I_m$ for almost all t. Q.E.D.

The theorem shows that when the depreciation rate δ is so small that the contributions of group members can overweigh the depreciation of public goods, the public goods stock monotonically increases for generations, and that it eventually exceeds the maximum level of public goods in the feasible region F of groups. In Figure 4.1, the public goods accumulation is shown by a horizontal arrow. Beyond the maximum level, there exists no "public goods problem" since the zero contribution is the optimal action of any group. An intuition for the theorem is the following. In the feasible region of groups, some group, not necessarily the largest one, can be formed for the joint contribution. When the depreciation rate of public goods is low, the group contribution can increase the stock of public goods. That is, the accumulation is possible even in the presence of the second-order dilemma. As public goods are accumulated, individuals' free-riding incentives change and the threshold of a group increases. The public good stock continues to increase as long as a group contribution is beneficial.

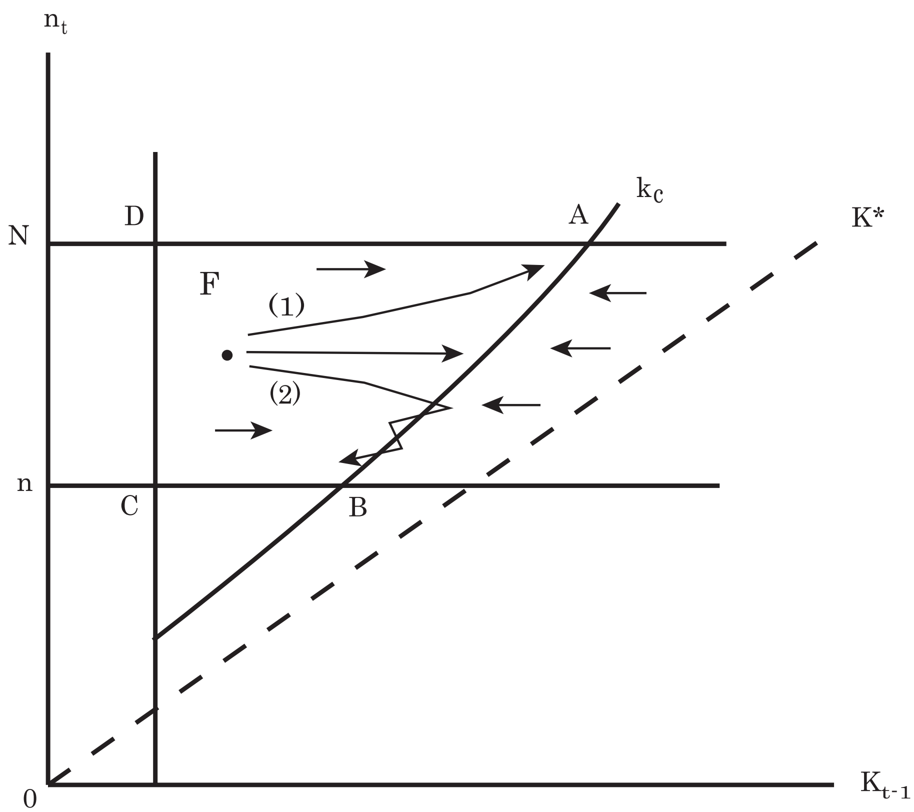

Figure 4.1 Accumulation of public goods

Theorem 4.1 shows the accumulation of public goods in the case of a fixed population. The maximum level $k_c(n_t)$ of public goods in the feasible region F is a monotonically increasing function of population n_t. Therefore, the population growth has a positive effect on the accumulation of public goods in the long run. In the following, we will roughly sketch accumulation patterns of public goods under population change with help of the phase diagram of (K_{t-1}, n_t) (Figure 4.1).

Case (1): population increase

Suppose that population n_t monotonically increases and converges to the upper limit N. In this case, the following two patterns of the accumulation process are possible. The first pattern is roughly like path (1) illustrated in Figure 4.1, on which the public good stock approaches to point A, without a pair (K_{t-1}, n_t) going outside the feasible region F. In the second pattern, a pair (K_{t-1}, n_t) may go outside the feasible region F. If this happens, then (K_{t-1}, n_t) moves to the north-west direction since K_{t-1} decreases due to the failure of group formation while population n_t increases. After some number of generations, (K_{t-1}, n_t) moves back into F, and thereafter the accumulation process starts again. In general, the accumulation process moves outside and inside the feasible region F repeatedly. If we consider a small time interval $\Delta t (> 0)$ instead of one in the dynamic system (4.2), then the system is modified as

$$K(t + \Delta t) - K(t) = \Delta t[-\delta K(t) + \beta a^*(K(t))].$$

This modified system has the same properties as (4.3) in that the stock $K(t)$ monotonically converges to the limit point $K^*(s)$ given in (4.4) as t goes to infinity, provided that the group size were fixed at s. In the modified system with small time intervals Δt, the difference $K(t+\Delta t) - K(t)$ can become arbitrarily small as time interval Δt goes to zero. With this remark, we can say that, in the second pattern, the public good stock-population pair $(K(t), n(t))$ moves to the north-east direction until it reaches the boundary of F and thereafter it moves along the boundary approximately and approaches to point A.

Case (2): population decrease

Suppose that population n_t monotonically decreases and converges to the lower limit n. Like the case of population increase, the accumulation process typically has the two patterns. In the first pattern, the public good stock monotonically decreases and approaches to point B, without a pair (K_{t-1}, n_t) going outside the feasible region F. In the second pattern roughly illustrated as path (2) in Figure 4.1, a pair (K_{t-1}, n_t) may go outside the feasible region F. Thereafter, the pair (K_{t-1}, n_t) can go back into the feasible region F unless population n_t decreases very steeply. By the similar arguments to case (1), the public good stock-population pair $(K(t), n(t))$ moves to the south-east direction until it reaches the boundary of F and thereafter it moves along the boundary approximately and approaches to point B. Comparing accumulation paths in cases (1) and (2), we can see that the population decrease damages severely the accumulation of public goods in the long run.

5 Discussion

5.1 the Second-Order Dilemma

The second-order dilemma of public goods arises from individuals' incentive to free-ride on a mechanism which is designed to implement an efficient provision of public goods. Any mechanism itself to solve the (first-order) dilemma of public goods is a kind of public goods. Due to the second-order dilemma, the effectiveness of the institutional approach to solve the provision problem of public goods has received some doubt in the literature since Parsons (1937, p.89-94), at least on a theoretical level. Parsons raised a logical inconsistency in the arguments of Hobbes on the Leviathan. If individuals are rational egoists, then why would they act in the collective interest by establishing a coercive state? For more recent criticisms to the institutional approach, see Bates (1988) and Dixit and Olson (2000), etc. In his critique to the "new institutionalism," Bates (1984) contends that an institution would provide a collective good and rational individuals would seek to secure its benefits for free. The proposed institution is subject to the very incentive problems it is supposed to resolve. Although not as pessimistic as these critics, the recent works on voluntary participation games (Palfrey and Rosenthal (1984), Okada (1993), Saijo and Yamato (1999) and Dixit and Olson (2000)) may be considered to provide a partial support for the negative view on the institutional approach to solve the provision problem of public goods. It has been shown that if individuals have the opportunity to decide freely whether or not they should participate in a group (or mechanism) to provide public goods, then they all do not participate due to the incentive of free-riding. The roles of norm, community, social sanctions, trust, fairness and reciprocity to solve the second-order dilemma have been discussed extensively in the literature. These social factors serve as devices to change individuals' incentives.

In this paper, we have shown, without relying on social and behavioral arguments, that the accumulation of public goods can mitigate the second-order dilemma in the long run. A positive amount of contributions, even if not efficient, can be made by self-interested individuals if they have the opportunity to organize a group to provide public goods. Thus, the accumulation of public goods is possible even in the presence of the second-order dilemma. The accumulation may change the degree of individuals' incentive to free ride, and it increases the threshold of a group. The accumulation may motivate more individuals to participate in a group. In the long run, the maximum level of public goods in the feasible region of groups can be attained. From studies on several cases which illustrate how people solve the second-order dilemma in real common-pool resource problems, Ostrom (1990) observed that the investment in institutional change occurred in an incremental and sequential process. The accumulation process of public goods may be considered as a part of the incremental and sequential process to solve the second-order dilemma.

5.2 Depopulation

In Section 4, we have examined the effects of population change on the accumulation of public goods. In particular, we have shown that population decrease damages the accumulation of public goods severely (see Figure 4.1). A serious problem of depopulation happens in many industrialized countries. In these countries, population moves from rural to urban areas.

For example, the decline of agricultural labor force causes agricultural abandonment and the decline of common resources in rural areas. MacDonald and Crabtree etc. (2000) review twenty-four case studies in European mountain areas and examine environmental impacts of agricultural abandonment and decline in traditional farming practices caused by rural depopulation. They find that abandonment is widespread and that it has an undesirable effect on biodiversity, landscape and soils including natural hazards such as soil erosion and landslides.

The depopulation in rural areas damages not only the maintenance of common resources but also the participation in various voluntary works. The depopulation started in Japan in the late of 1950. Population in rural areas of Japan decreases from 55 million in 1955 to 44 million in 2000 while the whole population increased from 90 million to 127 million for the same period (FAO, 2005). We discuss a case of Volunteer Fire Corps in Japan as an example of declining voluntary works (FDMA, 2005). Fire defence forces in Japan are composed of permanent fire defence professionals and members of Volunteer Fire Corps. Fire defence professionals are employed by cities, towns and villages as dedicated personnel as fire services. The number of professionals are about 155,000 in 2005. Members of Volunteer Fire Corps have their own occupations as a living, and voluntarily participate in Corps to devote themselves to fire services. The number of Volunteer Fire Corps is about 3,600 and their members are about 928,000 in the whole country. The members' activities are wide, including fire-fighting activities, lifesaving and rescue activity, patrolling and guidance for evacuation in natural disaster, and also the organization of first-aid classes, fire prevention instruction to residents and public relations activities. The members participate in physical training in their free time, mostly on weekends. Although the members are employed as part-time municipal officers, the character of their work is voluntary.[12] The total number of members in Volunteer Corps has decreased drastically since the middle of 1950, and it becomes almost half in 2005 compared to in 1956. Most Corps have now a difficulty to find new members. The main cause of this drastic decrease is depopulation in rural areas and changes of working styles in urban areas.

5.3 Majority Voting

We have employed a unanimous voting rule as a collective choice rule in the group negotiation stage. The unanimous rule gives every participant a veto power in the collective choice. While we think that the unanimous rule should be a fundamental choice rule for institutional change involving punishments, Theorem 3.1 (and thus Theorem 4.1) still holds even if majority voting rules are employed. Suppose that a majority rule is employed in the group negotiation stage. Then, Lemma 3.3 changes in that even a group with negative surplus can be formed in equilibrium since a single participant does not change the voting outcome when a majority of participants agree to establish a non-beneficial mechanism. No individual, however, participates in such a group if he anticipates that the mechanism is enforced in it. Therefore, the number of participants in every group equilibrium must be greater than or equal to the threshold as Theorem 3.1 claims.

[12] A regular-class member receives only 36,000 yen (approximately \$300) annually and is paid 6,900 yen (\$58) each fire-fighting work. A regular-class member who has engaged in Corps for 30 years receives about 640,000 yen (\$5333) at retirement.

5.4 Applications

The group formation game studied in this paper can be applied to a broad class of social dilemma situations. We briefly discuss two applications done in our previous works: environmental pollution (Okada (1993)) and the emergence of a state (Okada, Sakakibara and Suga (1997)).

The environmental pollution problem is a typical example of social dilemma in which cooperative actions are needed in order to prevent pollution. In Okada (1993), we consider how a voluntary organization for environmental regulation is formed among economic agents (factories) pursuing private benefits. Any agent in the organization is enforced to treat wastes before discharging them into the environments (air, lake, ocean, etc.) If the treatment cost is higher than the marginal benefit of environmental improvements, then every agent has the dominant action not to treat wastes. By applying a mixed-strategy equilibrium of the group formation game, we have shown that there is a positive probability that all agents form voluntarily a regulatory organization, and that the probability of an organization (not necessarily the largest one) may be strictly positive even when the number of agents goes to infinity.

The state is another example of social dilemma. According to the contractarian viewpoints, people in a society make a social contract to establish a state in order to escape from "warre of every one against every one." To achieve a common benefit, the state must have enforcement power to control the constituents' behavior. As we have discussed in subsection 5.1, a theoretical issue then is: how is the formation of the state consistent with the free-rider problem? In Okada, Sakakibara and Suga (1997), we investigate how the state as a tax enforcement institution can emerge in a dynamic production economy with non-overlapping generations. In the economy, individuals have linear utility functions and public goods are accumulated without depreciation. To capture an enforcer-enforcee relation in the state, we incorporate an enforcement agent in the group formation game.

The enforcement agent is randomly selected from the set of participants. We have shown that the state can be established if social productivity (measured in terms of public capital stock) is less than the threshold, and that, in case of no population change, social productivity stochastically converges to (and beyond) the threshold as generations go to infinity.

6 Concluding Remarks

We have shown that the accumulation of public goods can mitigate the second-order dilemma of public goods in the long run. We remark that the analysis never denies the importance of social and behavioral factors in human decision to solve the second-order dilemma. Rather, we conclude that the accumulation of public goods is complement to other social and behavioral factors such as community, norm and trust in an incremental process where people solve the second-order dilemma for generations.

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