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Incentive Efficient Risk Sharing in Settlement Mechanism

by

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Abstract

The purpose of this paper is to address a question concerning risk management in continuing, multi-party, contractual, clearing and settlement arrangements through which large-value payments are typically made. We are particularly interested in the issues of incentive compatibility when a third party possesses a private information concerning the riskiness of transfers being made. If a third party possesses private information that would be of value in determining how best to settle a payment, how does the exposure of that party to the settlement risk affect the quality of information that the party chooses to provide? In this paper, we address thes question by analyzing a specific class of parametric environments of a schematic, formal, model of a settlement arrangement or a payment network.

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I Introduction

During the past several decades, there has been progressively increasing awareness of the importance of risk management in continuing, multi-party, contractual, clearing and settlement arrangements through which large-value payments are typically made. Because a very large loss can potentially be incurred if settlement of a payment fails, how such a loss would be shared should be a matter of substantial concern for the participants in an arrangement. Moreover, to the extent that complete contingent-claims markets do not exist for insurance against settlement failures and that there are political pressures for governments or central banks to assume losses from such failures, management of settlement risk is also a public policy issue. The purpose of this paper is to address a question concerning risk management in such settlement arrangements. We are particularly interested in the issues of incentive compatibility when a third party possesses a private information concerning the riskiness of transfers being made. If a third party possesses private information that would be of value in determining how best to settle a payment, how does the exposure of that party to the settlement risk affect the quality of information that the party chooses to provide? In this paper, we address this question by analyzing a specific class of parametric environments of a schematic, formal, model of a settlement arrangement or a payment network.

Let us contemplate that a third party within a payment network has some private information, not possessed by either the payer or the payee, about the level of risk. In such a case, the information is potentially relevant to how the transaction should be conducted and even to how large a transaction ought to be undertaken.

If the privately informed third party were involved solely as the reporter of that information to the concerned parties, then there would be no problem about ensuring the truthfulness of the report. In particular, if compensation were required to induce reporting, that compensation could be made in the form of a flat fee. If the possessor of information functions in the payments process as an agent for one of the principals in the transaction, though, then there will generally be an issue of whether there is incentive for truthful reporting.[1] One might think, for example, that efficiency would generally require a payments network would have to penalize an information provider when a payment would fail without a warning of particularly risky circumstances having been given.

We will show that there is indeed an incentive-compatibility issue for the payments

[1]This idea, that a dual role of privately informed members of a payments network is critical for understanding how the institutional design of a payment arrangement is related to the attainment of economic efficiency, has previously been studied by Rochet and Tirole [4]. In their model, unlike the present one, traders' information can only be revealed through their trades, and not by making explicit report. In many actual payments networks, the limited opportunities for traders to make explicit reports seem to fall between the absence of opportunity modeled by Rochet and Tirole and the completely adequate opportunity modeled here. When traders are required to set prior limits (which will not necessarily ever be binding in equilibrium) on their bilateral exposure to counterparties, for instance, their choices of which limits to set can be regarded as partially informative reports of their private information about those counterparties' riskiness.

network to resolve, but that there is no simple generalization about how to resolve it. The incentive for truthful revelation of information depends on the pattern of risk sharing within the payments network, the differences in risk attitudes among members, and the distribution of rents that is to be achieved by an incentive efficient transaction mechanism in a private-information environment. In fact, for the parametric environment that we study, there are two distinctive cases in which some incentive efficient mechanisms involve a binding incentive-compatibility constraint for truthful revelation that a transfer is likely to fail, while other incentive efficient mechanisms involve a binding constraint for truthful revelation that failure is unlikely. The case in which incentive efficient mechanisms involve a binding incentive-compatibility constraint for truthful revelation that a transfer is likely to fail, that is, a constraint to prevent under-reporting of risks, prevails when costly but safe transfer technology which is alternative to a risky transfer technology is not regarded too costly from the social point of view. On the other hand, the case in which incentive efficient mechanisms involve a binding constraint for truthful revelation that failure is unlikely, that is, a constraint to prevent over-reporting of risks, prevails when an alternative safe transfer technology is too costly to be used from the social point of view. It may be worthwhile to point out that the fact that the nature of incentive constraint depends upon allocations on the Pareto efficient frontier is very distinctive to the problem of settlement mechanisms that seems not to have noticed in the literature.

The paper proceeds as follows. In section II we give a detailed description of a model of transfers of assets involving a risky transfer along with a description of sequence of information and economic activities. A settlement system is formulated as an incentive efficient transaction mechanism. In the following section III, after specifying utility functions of traders, analysis and discussions on incentive efficient mechanisms are presented in an environment with perfectly accurate private information. In section IV we look at incentive efficient mechanisms in an environment with imperfectly accurate private information and discuss the effects of imperfection of information on the issues of incentive compatibility. A concluding remark is given in section V.

II Model of a transaction — a transfer with a third party

We first formulate a simplest kind of model of a transaction that involves a risky asset transfer. The model is simple enough to be analytically tractable.

A transaction is a related set of asset transfers between traders. The assets involved might be either commodities or financial assets. An asset transfer involves two traders, the donor and the recipient, but a transaction can generally involve more than two traders. Thus, a simplest model of a transaction involving a risky transfer will include three traders. A distinction can be drawn thereby between a participant in the broad transaction and a participant (that is, the donor or the recipient) in the specific transfer where the risk occurs.

In order for the participant who is neither the donor nor the recipient of the risky transfer to be essential to making a mutually beneficial transaction, there should be no "double coincidence of wants" between the donor and the receiver. This consideration suggests modeling the three participants as a "Wicksell triangle."

A third party to risky transfer in a Wicksell triangle might be intrinsically necessary in the sense that the donor and recipient of the risky transfer would have no double coincidence of wants, even if the transfer did not involve risk (that is, if the recipient would receive the expected value of the transfer with certainty). Alternatively, the riskiness of the transfer might impair a double coincidence of wants that would exist under certainty between the donor and the recipient, and the third party might be needed solely to restore that double coincidence by serving as a guarantor or insurer of the transfer. [2]

On the basis of these considerations, we will specify the set of traders in a simplest possible way to be $\{1, 2, 3\}$. We will assume that trader 1 is essential to a mutually beneficial transaction but that trader 2 is the donor and trader 3 is the recipient of the risky transfer. In the following subsections we give a mathematical description of our model.

Probability structure

The risky transfer will be formalized in terms of a probability space of events (in Ω) on which a probability measure Pr is defined. There is a distinguished event S, with $0 < \Pr(S) < 1$. Assume that the risky transfer from trader 2 to trader 3 succeeds in S, and that it fails in the complementary event F. When we say that the transfer succeeds, we mean that trader 3 receives the entire quantity of the asset that is transferred. When we say that the transfer fails, we mean that the quantity of the asset that was intended to be transferred disappears irretrievably from the economy.[3]

To model private information, we suppose that an event that is statistically relevant to the outcome of that risky transfer will be privately observed by trader 1, who is not directly involved in the risky transfer but who is an essential participant in a mutually beneficial transaction among the traders. To consider the simplest case of nontrivial private information, suppose that trader 1 observes which element of the partition $\mathcal{P} = \{H, L\}$ of Ω contains the true state of nature. We suppose that H and L satisfy

$$0 \leq \Pr(S|L) < \Pr(S) < \Pr(S|H) \leq 1. \tag{1}$$

Write $\sigma_L = \Pr(S|L)$, $\sigma_S = \Pr(S|H)$, and $\sigma = \Pr(S)$ so that $0 \leq \sigma_L < \sigma < \sigma_S \leq 1$. The elements of $\mathcal{P}$ will be referred to as signals that trader 1 observes.

[2] We find it useful to characterize the differences between the roles of these two types of third parties in discussing a role of the central bank or the government with regards to settlement failures. In such a case, a four-trader model (including both an intrinsic third party and a trader whose only involvement would be to share risk) can be useful. See, e.g., Fujiki, Green, and Yamazaki (1999).

[3] Failure of an actual transfer seldom involves such an irretrievable loss, although there are some contemporary examples and many historical examples of that type of failure.

Endowments and preferences

Assume that each trader i has an endowment consisting solely a type of commodity that only he possesses. We denote that type of commodity also by i. Intuitively, trader i is endowed with one unit of commodity of type i with certainty.

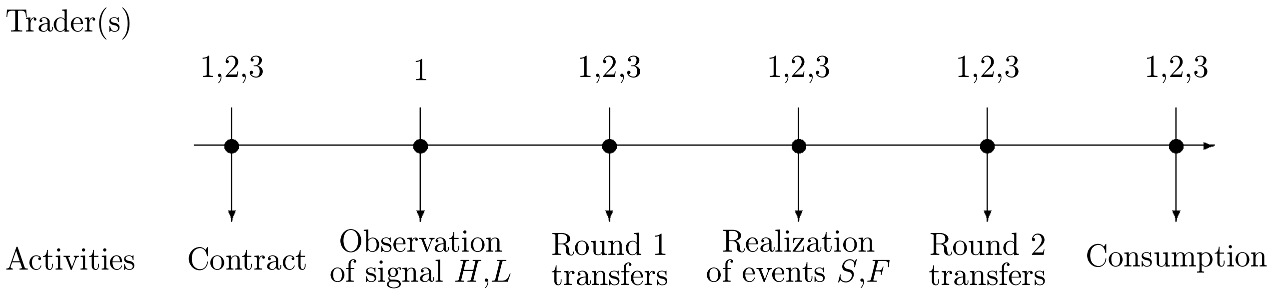
In general, a commodity bundle provides a random amount of each of the three types of commodity in the economy. That is, each trader can acquire commodities by transactions with others, and randomness is introduced by the riskiness of the transactions technology.[4] We use the letter γ to denote such a random commodity bundle.

Each trader's preference between commodity bundles conforms to expected utility. Trader i has a von Neumann-Morgenstern utility function $U^i\colon \mathsf{R}^3_+ \to \mathsf{R} \cup \{-\infty\}$. Trader i's expected utility of consuming a commodity bundle γ is the expectation of the random variable $U^i(\vec{\gamma})$.

Information and a sequence of economic activities

The sequence of economic activities in this economy is as follows. Initially, before any information is observed, traders make an agreement with one another for transfers of goods among them. The agreement among the traders is binding. Trader 1 will observe a signal H or L before the first round of transfers takes place. Thus it is natural for the agreement to specify that trader 1 will report what he observes, and that his report will determine which transaction to make. Note, however, that this contingent agreement is made *ex ante*, before trader 1 has had an opportunity to make any observation.

Figure 1: A sequence of information and economic activiteis



The traders agree on an initial round of transfers among them. With one exception, the

[4]One could introduce the kind of randomness treated in this paper as an endowment shock. However, in order to do so, one needs to distinguish goods by the location of each trader. While it is a standard convention to distinguish commodities by their location, given a very simple environment studied in this paper, it is not only convenient to treat the randomness as one coming from transfer technologies but is also appropriate to think the randomness is due to transfer technologies in this paper.

transfers are safe. That is, everything sent out reaches its intended recipient in its entirety and with certainty. The exception is the transfer of trader 2's endowment to trader 3. Recall that this transfer reaches trader 3 in its entirety in event S, but is completely and irretrievably lost in event F. Whether the transfer will succeed or fail is not known at the time that the traders make their agreement.

The traders also agree *ex ante* on a second round of transfers, to be made after the first transfers have been completed and the result of the risky transfer has become known. Thus the transfer to be made in the second round can be made contractually contingent on which of the events S and F has occurred.[5] All second-round transfers, including the one from trader 2 to trader 3, are nonstochastic. However, second-round transfers are costly. Only a proportion ρ, $(0 < \rho < 1)$, of the goods that a trader sends in the second round are received.[6]

After the agreement has been made, trader 1 privately observes whether signal H or L will occur. Trader 1 reports this observation to the others. It is feasible for trader 1 to make an untruthful report, since the other traders do not directly observe H or L, and they cannot infer with certainty which of those events have been observed from their subsequent observation of S or F.[7]

The two rounds of contractually specified transfers are then made. Traders consume their stocks of goods after these two rounds of transfers have been completed.

Consumption determined by transfers

To simplify the characterization of traders' consumption resulting from settlement, we make two assumptions: that a trader is able to transfer only his own endowment good, and that only a few of the possible flows of those goods are feasible. Specifically, trader 1 can make a transfer to 2, 2 to 3, and 3 to 1.[8] (Traders 1,2, and 3 together will constitute the Wicksell triangle to which reference was made earlier.) Thus, a *round of transfers* is a vector $\phi \in \mathsf{R}^3_+$.

[5] Strictly speaking, this sentence describes a different information structure from the preceding one. If traders can only distinguish between events S and F on the basis of observing the success or failure of a transfer, then they can not make any distinction unless a (non-zero) transfer has been attempted. To assume that they can make a state-contingent transfer in the second round even if no first-round transfer from 2 to 3 has been attempted neglects this limitation of their opportunity for inference. In the case where there is no private information, this ambiguity is harmless because risk-averse traders would not cooperatively choose to make a state-contingent transfer in the second round unless they had exposed themselves to settlement risk in the first round. How the ambiguity is resolved is important in the private-information case, though, and we will discuss this issue further below.

[6] This assumption, sometimes called "iceberg cost," can be viewed as a crude way of reflecting various intuitive considerations including time preference and exposure to business loss due to delayed availability of transferred funds.

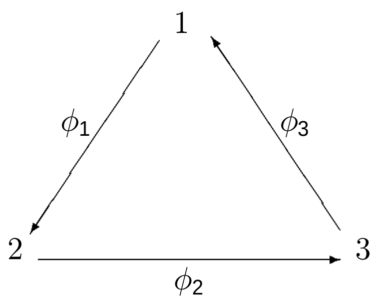
[7] To be exact, this statement applies to the cases below in which either information is not perfectly accurate or the events S and F cannot be verified without a positive risky transfer made from trader 2 to 3 even if information is perfectly accurate.

[8] Strictly speaking, possible flows of goods will be expressed via utility functions of traders.

The coefficients of ϕ are interpreted as follows.

1. ϕ_1 is the amount to be sent from trader 1 to trader 2;
2. ϕ_2 is the amount to be sent from trader 2 to trader 3;
3. ϕ_3 is the amount to be sent from trader 3 to trader 1.

Figure 2: A Round of Transfers: A Wicksell Triangle



As described above, either all, a proportion ρ, or none of the goods sent may be received. A *transaction* is a sequence $\tau = (\tau^1, \tau^S, \tau^F)$ of rounds of transfers. The elements τ^1, τ^S, and τ^F specify the initial round of transfers, the second round of transfers in event S, and the round of transfers in event F, respectively.

A transaction is feasible if no trader is ever required to send a cumulative amount that would exceed his endowment. That is, transaction τ is feasible if

$$\forall i \;\; \tau_i^1 + \max\{\tau_i^S, \tau_i^F\} \leq 1. \tag{2}$$

Let $\mathcal{T}$ denote the set of feasible transactions. [9]

Now we provide an explicit definition of traders' consumptions resulting from a transaction. To begin, let τ^χ be the random net trade that results from transaction τ, which depends on whether event S or F instead occurs. That is, we define for each ω, $\tau^\chi(\omega) = \chi_S(\omega)\tau^S + \chi_F(\omega)\tau^F$ where χ_S and χ_F denote the indicator functions of S and F respectively. Let $z^1 = (1,0,0), z^2 = (0,1,0)$, and $z^3 = (0,0,1)$ in terms of which these consumptions are defined. By adding the positions of a particular trader in all such net trades in which that trader is involved to the trader's endowment, the random consumption of the trader is determined. Specifically the consumption vector $c^i(\tau, \omega)$ that trader i receives as a consequence of transaction τ is as follows.

[9] As noted in the footnote above, the informational constraint that, if $\tau_2^1 = 0$, then $\tau^S = \tau^F$, may or not be added to the definition of feasibility for a transaction.

$$
\begin{aligned}
c^i(\tau,\omega) &= (1-(\tau_i^1+\tau_i^\chi(\omega))z^i + (\tau_{i-1}^1+\rho\tau_{i-1}^\chi(\omega))z^{i-1} \\
& \qquad i = 1,2,3 \pmod 3
\end{aligned} \tag{3}
$$

Incentive-efficient transaction mechanisms

The agreement among traders regarding the structure of the transaction, described above, is an *ex ante* agreement, made before trader 1 has received any information. However, trader 1 will observe H or L before the first round of transfers takes place. Thus it is natural for the agreement to specify that trader 1 will report what he observes, and that his report will determine which transaction to make. That is, the agreement among the traders specifies a *transaction mechanism* rather than a single transaction. Formally, a transaction mechanism is a mapping $\mu: \mathcal{P} \to \mathcal{T}$.[10] Thus, given a mechanism μ, $\mu(H)$ denotes the contractually agreed upon transaction when trader 1 reports the signal to be H.

Transaction mechanism μ will elicit truthful reporting from trader 1 if the following incentive-compatibility condition is satisfied.

$$
\forall A \in \mathcal{P} \quad \forall B \in \mathcal{P} \quad \mathsf{E}[U^1(c^1(\mu(B),\omega))|A] \leq \mathsf{E}[U^1(c^1(\mu(A),\omega))|A]. \tag{4}
$$

Let $\mathcal{M}$ denote the set of incentive-compatible transaction mechanisms. We restrict attention to incentive-compatible mechanisms, as is justified by the revelation principle.[11] If $\mu \in \mathcal{M}$, then the resulting transaction τ and the consumption Γ^i for each trader $i = 1,2,3$ is defined as follows.

$$
\begin{aligned}
\tau(\mu,\omega) &= \sum_{A\in\mathcal{P}} \chi_A(\omega)\mu(A), \\
\Gamma^i(\mu,\omega) &= \sum_{A\in\mathcal{P}} \chi_A(\omega)c^i(\mu(A),\omega).
\end{aligned} \tag{5}
$$

A mechanism $\mu \in \mathcal{M}$ is *feasible* if it is *individually rational*, i.e.

$$
\forall i \quad \mathsf{E}[U^i(\Gamma^i(\mu,\omega))] \geq U^i(z^i),
$$

and the resulting transactions are feasible. Finally, define a feasible mechanism $\mu \in \mathcal{M}$ to be *incentive efficient* if it is Pareto efficient within $\mathcal{M}$, that is, if there exist no other feasible mechanism $\nu \in \mathcal{M}$ such that

[10] Strictly speaking, μ is defined on {'H', 'L'} where 'H' and 'L' are names for the events H and L. They are merely linguistic reports of events and may defer from actual events themselves if not reported truthfully. However, we find it convenient to abuse notation and treat μ to be defined on $\mathcal{P}$.

[11] Myerson (1991) provides an exposition of incentive compatibility and the revelation principle.

$$
\begin{array}{lrcl}
\forall i & \mathsf{E}[U^i(\Gamma^i(\mu,\omega))] & \leq & \mathsf{E}[U^i(\Gamma^i(\nu,\omega))], \quad \text{and} \\
\exists i & \mathsf{E}[U^i(\Gamma^i(\mu,\omega))] & < & \mathsf{E}[U^i(\Gamma^i(\nu,\omega))]
\end{array} \tag{6}
$$

III A parametric environment with accurate private information

For convenience, we assume throughout that success and failure as well as their private signals have equal probability. That is, we assume that $\Pr(S) = \Pr(F) = \Pr(H) = \Pr(L) = 1/2$. We start from the case in which trader 1's signal is perfectly accurate in the sense that $\Pr(S|H) = 1$ and $\Pr(S|L) = 0$.

We work with piecewise-linear utility functions for the traders. Their utilities will be defined in terms of parameters δ and ϵ, which are assumed to satisfy $0 < \epsilon < \delta < 1/4$.[12] Utility functions are defined in terms of the following functions on the nonnegative real numbers.

$$
\begin{array}{rcl}
V(x) & = & \min\{1, x\} + \epsilon \max\{0, x-1\}; \\
W(x) & = & \min\{1/2, x\} + \epsilon \max\{0, x-1/2\}.
\end{array} \tag{7}
$$

Define the agents' utilities, for $c = (c_1, c_2, c_3) \in \mathsf{R}^3_+$, as

$$
\begin{array}{rcl}
U^1(c) & = & c_1 + c_3; \\
U^2(c) & = & V(c_1 + c_2); \\
U^3(c) & = & W(c_2) + \delta c_3.
\end{array} \tag{8}
$$

Example 1 [Trades]: The model most directly resembles settlement by delivery of goods in early international and interregional trade. Trader 1, who is an intermediary, ships the endowment good of trader 2 trader 3 by giving trader 2 a part of his endowment good as a settlement payment, and trader 2 gives trader 1 his endowment good to settle the transaction. A part of the first round transfers may be interpreted as "insurance premium" and that of the second round transfers as insurance payments.

Example 2 [Money payment through an intermediary]: Suppose that trader 2 transfers a good to trader 3, who settles in money through intermediary trader 1. And think of the endowment good of trader 2 as being produced by a technology with constant marginal cost 1. The utility functions of trader 1 and 2 in (8) can be thought of as specifying indirect utility of wealth.

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[12] One may note that only the value of δ and ϵ relative to that of $\Pr(S)$ is important.

Because trader 1's utility function is linear and information is perfectly accurate, the incentive-compatibility constraint reduces to the following two equations. The first and second equations state that trader 1 has incentive to report truthfully in event H and L, respectively.

$$
\begin{aligned}
&(\mu_3^1(H) + \rho\mu_3^S(H)) - (\mu_1^1(H) + \mu_1^S(H)) \geq (\mu_3^1(L) + \rho\mu_3^S(L)) - (\mu_1^1(L) + \mu_1^S(L)) \\
&(\mu_3^1(L) + \rho\mu_3^F(L)) - (\mu_1^1(L) + \mu_1^F(L)) \geq (\mu_3^1(H) + \rho\mu_3^F(H)) - (\mu_1^1(H) + \mu_1^F(H))
\end{aligned} \tag{9}
$$

Subject to both technological and incentive constraints, a transfer mechanism is incentive efficient if and only if it implements a Pareto efficient allocation that is individually rational for each trader. An allocation that maximizes a weighted sum of traders' expected utilities, with all weights strictly positive is Pareto efficient. Therefore, for $\alpha \in \mathsf{R}^3_{++}$, consider

$$
U(\mu, \alpha) = \sum_{i=1}^{3} \alpha_i \mathsf{E} U^i(\Gamma_i(\mu, \omega)). \tag{10}
$$

Although a vector $(\alpha_1, \alpha_2, \alpha_3)$ corresponds to a specific point in the Pareto efficient utility allocation frontier, it is useful to think these vectors as generating different transfer "flows" among traders when analyzing transactions in our framework. This will become apparent below.

Regarding the transfer technology, we will assume that[13]

$$
\rho = 1/2. \tag{11}
$$

A IC constraint can bind in H

Let α satisfy the following conditions.

$$
\begin{aligned}
\delta\alpha_3 < \alpha_1 < \alpha_2 < \alpha_3; \\
\rho\alpha_3 = \alpha_3/2 < \alpha_2.
\end{aligned} \tag{12}
$$

(For example, if $\epsilon = 1/10$ and $\delta = 2/10$, then $\alpha = (2, 3, 4)$ satisfies the inequalities (12).) These values of α_i's correspond to a case in which first round transfers between traders 1 and 2, 2 and 3, and 3 and 1 are warranted by the social welfare maximization of (10), and second round transfer from 2 to 3 is viewed socially too costly to be performed. Consider a transfer, contingent on an announcement of trader 1's information that is assumed for the moment to be truthful, that would maximize the social welfare U for this value of α. It must have the following features, where c_j^i denotes the amount of consumption by trader i of the endowment good of trader j.

- If trader 1 announces event H, then the following transfers should be made in the first round. Trader 3 should send his entire endowment to trader 1, because $\delta\alpha_3 < \alpha_1$.

[13] Again the specific value of ρ is not important as long as $\rho \leq \sigma$.

Trader 2 should send half a unit to trader 3, because $\epsilon\alpha_3 < \alpha_2 < \alpha_3$. After having made that transfer, $c_2^2 = 1/2$. Therefore trader 1 should send half a unit to trader 2, so that $c_1^2 + c_2^2 = 1$, because $\epsilon\alpha_2 < (\delta\alpha_3 <)\alpha_1 < \alpha_2$.

- If trader 1 announces event L, then the following transfers should be made. In this case, also, trader 3 should send his entire endowment to trader 1 in the first round, because $\delta\alpha_3 < \alpha_1$. However, trader 2 should send nothing to trader 3 in either round. Such a transfer would fail in the first round by the assumption that trader 1 announces the truth, and it should not be made in the second round because $\rho\alpha_3 < \alpha_2$. Thus, since the condition that $c_1^2 + c_2^2 \geq 1$ will be satisfied, trader 1 should send nothing to trader 2, because $\epsilon\alpha_2 < \alpha_1$.

- Since trader 1 has perfectly accurate information, transfers in round 1 can be optimized contingent on whether a transfer from trader 2 to trader 3 would actually succeed or fail. Therefore, no further transfer is needed in round 2. Since making a transfer in round 2 would incur an "iceberg cost" that is avoidable in round 1, no transfer should be made in round 2 after the transfers described above have been made in round 1.

We find it convenient to represent the above arguments by diagrams. In Figure 3 a circled letter H or L is the report on a signal that have been observed by trader 1. To each report of a signal corresponds three triangles whose vertices represent a trader numbered from 1 to 3. A number placed along an edge between two vertices represent a transfer between two traders. Each triangle represents a round of transfers. The first round transfer, the second round transfer in the event S, and the second round transfer in the event F are represented in this order from the outer triangle to the inner triangle. Thus, an optimal transaction mechanism that would imply the above transfers and its associated contingent allocation if trader 1 were to report truthfully, is expressed by Figure 3.

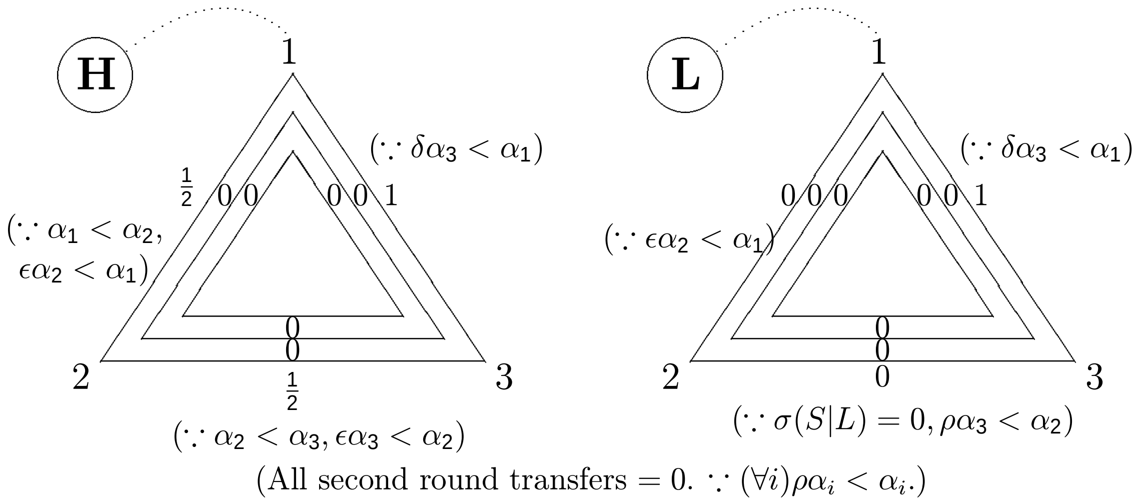
The corresponding transaction mechanism is specified by

$$\tilde{\mu} = \Big(\tilde{\mu}(H), \tilde{\mu}(L)\Big) = \Big(\big((1/2,1/2,1)(0,0,0)(0,0,0)\big),\ \big((0,0,1)(0,0,0)(0,0,0)\big)\Big). \tag{13}$$

This mechanism is individually rational for all traders, but it is not incentive compatible. Specifically, trader 1 always has incentive to announce event L, regardless of his actual observation, in order to avoid having to give up half of his endowment.

In order to make the mechanism incentive compatible, it must be modified so that, if the trader 1 observes event H, then he will consume no more (total of goods 1 and 3) by misrepresenting his observation as L than by truthfully announcing H. This can be accomplished while retaining the same equilibrium allocation as $\tilde{\mu}$ would provide, by defining transaction mechanism μ to be identical to $\tilde{\mu}$ except that $\mu_1^S(L) = 1$. The contingent transfer $\mu_1^S(L)$ will never be made in equilibrium (by the IC constraint) so it does not reduce the value of the planer's objective function relative to truthful reporting subject

Figure 3: Optimal but not Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \alpha_3$, $\rho\alpha_3 < \alpha_2$

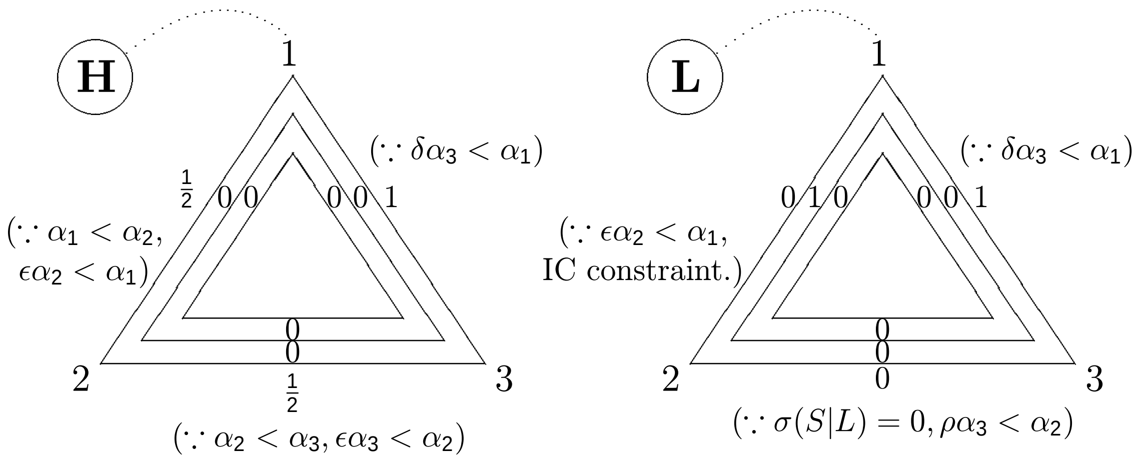


to the transfer mechanism $\tilde{\mu}$. That is, an incentive efficient transaction mechanism that maximizes $U(\mu, \alpha)$ is

$$\mu = \big(\mu(H), \mu(L)\big) = \Big(\big((1/2, 1/2, 1)(0,0,0)(0,0,0)\big), \big((0,0,1)(1,0,0)(0,0,0)\big)\Big) \tag{14}$$

(see Figure 4).

Figure 4: Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \alpha_3$, $\rho\alpha_3 < \alpha_2$



As we pointed out in the introduction, one might have thought that efficiency would generally require a payments network would have to penalize an information provider when a payment would fail without a warning of particularly risky circumstances having been given. Instead, the above argument shows that when the second round transfer from trader 2 to 3 is regarded socially too costly, then the payment network need to penalize the informed trader for causing the network to miss an opportunity of mutually beneficial transactions by misleading uniformed traders to believe first round transfer from 2 to 3 is extremely risky. Since informed trader 1, interpreted to be acting as an intermediary between traders 2 and 3, can also be thought of as a provider of an insurance, it might sound unintuitive, at least at first glance, that an insurer has an incentive to report that the transfer is extremely risky when it is indeed very safe.

This may be a good place to explain basic differences existing between a standard model of optimal insurance and the model of this paper. There seems to be at least two fundamental differences between these two models. The first difference is that "insurance" payments themselves might face a risky transfer. (This particularly is the case when the second round transfer from trader 2 to 3 is too costly, i.e., $\rho\alpha_3 < \alpha_2$, so that virtually the network cannot avail itself a safe second round transfer technology between traders 2 and 3.) The insured are not information holders concerning riskiness, but the insurer is. These two features seem to create unique aspects of the incentive compatibility issues in the present model. An intuitive explanation of why there is an incentive constraint in reporting the signal H might be given as follows. If trader 1 reports signal L truthfully, then everyone knows that the first round transfer from 2 to 3 surely fails. Now, from social point of view, trader 3 should make the first round transfer of his endowment good to 1 regardless of signals observed by 1. A part of this transfer is considered to be a payment of "insurance premiums" of traders 2 and 3. However, if trader 1 announces L truthfully, no transfers are made from 2 to 3 because the first round transfer is sure to fail and the second round transfer should not be made from the social point of view. This means that even the insurance payments to trader 3 must be transferred from 1 to 3 through 2 when 1 announces H whereas no insurance payments are made to 3 in case of L. Thus, trader 1, as an insurer, is put in the position where he is better off to announce L even when he observes the signal H.

In summary, care has to be taken to specify a transfer mechanism that maximizes $U(\mu, \alpha)$ in a way that achieves incentive compatibility, but having to impose the incentive-compatibility constraint need not make any trader worse off than he would be in an environment where all traders could observe $\mathcal{P}$ directly. In this sense, incentive-compatibility is not a binding constraint in the environment where the IC constraint binds in H and the partition $\{S, F\}$ is observed regardless of whether transfer from trader 2 to 3 is actually made. In a sense this result is naturally expected because second round transfers are Arrow-Debreu type contingent contracts. However, it is important to realize that the fact that Arrow-Debreu type contingent contracts can be written among traders, itself will not eliminate the issues of incentive compatibility. Later in this paper, we will establish that incentive-compatibility can be a binding constraint when the information of trader 1 is less than perfectly accurate.

B What if S could not be distinguished from F without a non-zero transfer being made?

The foregoing discussion has assumed that the events S and F could be distinguished even if no transfer were attempted (that is, if a zero transfer were specified). The alternative assumption, that the uniformed traders can only learn about these events through the actual success or failure of a non-zero transfer, may be thought to be more widely applicable and leaves little rooms for questioning the verifiability of the events.

Under this assumption, incentive compatibility in case of IC constraint binding in H, can be achieved by requiring trader 1 to transfer half of his endowment to trader 2 in L, or else by having trader 3 retain half of his endowment rather than transferring it to trader 1 in L. (Because the environment is piecewise linear, combinations of these two modifications do not have to be considered.) Because $\epsilon < \delta$, having trader 3 retain half of his endowment achieves the higher value of U. That is $U(\mu, \alpha)$ achieves a higher value by defining μ so that

- Trader 1 sends half a unit to trader 2, and trader 2 sends half a unit to trader 3, in H;
- Trader 1 sends nothing to trader 2, and trader 2 sends nothing to trader 3, in L;
- Trader 3 sends one unit to trader 1 in H, but only half a unit in L.

That is,

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\big((1/2, 1/2, 1)(0,0,0)(0,0,0)\big), \big((0,0,1/2)(0,0,0)(0,0,0)\big)\Big). \quad (15)$$

There are two other possibilities to achieve incentive compatibility when the events S and F can not be distinguished without a non-zero first round transfer from trader 2 to 3. One possibility is that when no first round transfer is attempted by trader 2, trader 1 in second round could make non-contingent second round transfers. In this case trader 1 need to send half a unit to trader 2 regardless of the events S and F when L is announced. This would obviously give a lower value of U than would having trader 1 send half a unit in the first round because of the iceberg cost in the second round.

The second possibility is to have trader 2 send a small amount, say λ, to trader 3 in round 1 to find out prevailing event and then to have trader 1 send one unit in round 2 in event S. In this case μ would be defined by

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\big((1/2, 1/2, 1)(0,0,0)(0,0,0)\big), \big((0,\lambda,1)(1,0,0)(0,0,0)\big)\Big). \quad (16)$$

For a positive value of λ sufficiently small, for example any values of λ less than $(\alpha_1 - \delta\alpha_3)/2\alpha_2$, μ in (16) will give a greater value of U than the one in (15). And this value of

U becomes larger as λ becomes smaller, hence there is no maximum value for U. At first glance it might seem worrisome that the maximum value of U does not exist. However, it helps to consider what it means to be able to have a value of λ arbitrarily close to zero in the context of transfer of assets. If we are interested in the question of the settlement risk arising from a solvency risk, we may not want to allow the values of λ arbitrarily close to zero. In such a case we might be content with the incentive compatible mechanism μ given in (15). On the other hand, if we are concerned with operational risks arising from the operation of a network, then settlement risks could involve the values of λ arbitrarily close to zero. But, then, to the extent that zero transfers can be made to find out whether network transfers are safe or not, we can be content with the mechanism μ in (14) instead of (16).

Compared to the level of U attained by μ defined in (14), the level of U given by μ in (15) declines by the amount of $(\alpha_1 - \delta\alpha_3)/4$ whereas μ in (16) decreases the level by the amount of $\lambda\alpha_2/2$. Therefore, in any case, when events S and F cannot be distinguished without a positive round 1 transfer made from trader 2 to trader 3, the incentive compatibility conditions really become binding in consumption to be achieved even when information is perfectly accurate. Thus the elicitation of truthful report from the informed trader will impose social costs to network participants. This may not be surprising because the fact that traders cannot distinguish the events without some positive transfer made in round 1 despite a complete risk of non-arrival may be thought of as another way of introducing an inaccuracy of information. By summarizing the arguments of the two subsections, we obtain the following proposition.

Proposition 1 *Under the given parametric specification of the above two subsections, in which a safe second round transfer from trader 2 to trader 3 is too costly from social point of view, (for an optimal and individually rational transaction mechanism) there will be a problem of possible over-reporting by the informed trader concerning the risk involved in a risky transfer from 2 to 3. If the signal observed by the informed trader is perfectly accurate, and if information can be verified by uniformed traders regardless of contents of their contracts, then there will be no social costs of imposing incentive compatibility. However, if there is a problem of verification of signal observed by the informed trader without a positive risky transfer made from trader 2 to 3, then there will be a social cost of imposing incentive compatibility.*

C IC constraint can bind in L

Let us now look at the case in which the use of a safe second round transfer technology from trader 2 to 3 is not too costly from the social point of view. This means that we change α so that making transfer from trader 2 to trader 3 even in the second round is appropriate to maximize the α-weighted sum of utilities. Thus, let α satisfy the following conditions.

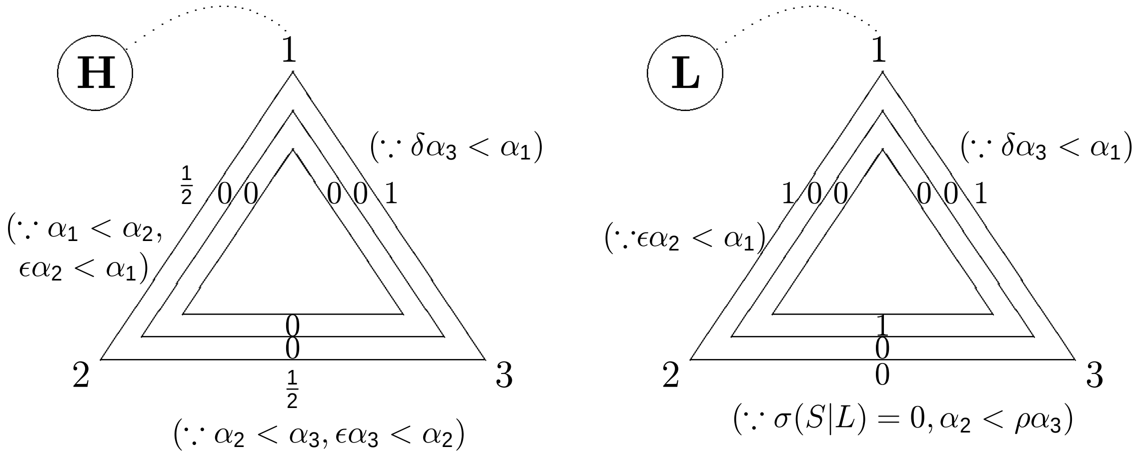
$$\delta\alpha_3 < \alpha_1 < \alpha_2 < \rho\alpha_3 = \alpha_3/2. \tag{17}$$

(For example, if $\epsilon = 1/10$ and $\delta = 2/10$, then $\alpha = (2, 3, 8)$ satisfies the inequalities (17).)

Since trader 1, as an intermediary and an insurer, can transfer "payments" to trader 3 through trader 2 in the event F, one would expect just as in a standard insurance model that trader 1 would have incentive to under-report the risk involved in the transfer from 2 to 3. The following argument shows that this is exactly what happens in this case.

The transaction mechanism $\tilde{\mu}$ that maximizes U for the value of α satisfying (17) has the following characteristics. (See Figure 5.)

Figure 5: Optimal but not Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \rho\alpha_3$



(Second round transfers for $i = 1, 3$ are 0. $\because \rho\alpha_i < \alpha_i$ for $i = 1, 3$.)

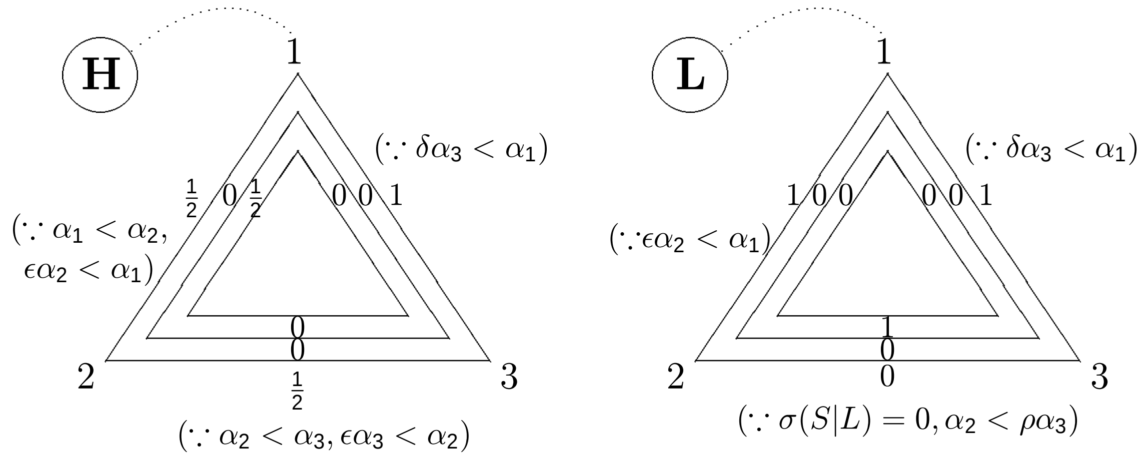
- If trader 1 announces H truthfully, then all the transfers among traders should be the same as in the previous case since the only difference from the previous case is the social evaluation of iceberg costs in terms of α_2 and $\rho\alpha_3$ and there are no second round transfers when H is announced.
- If trader 1 announces L truthfully, then trader 3 sends one unit to trader 1. Trader 2 sends nothing to trader 3 in the first round, but sends one unit to trader 3 in the second round since we now have $\alpha_2 < \rho\alpha_3$. Trader 3 only receives half a unit because of the iceberg cost. Trader 1 sends one unit to trader 2 in the first round.

Thus, the transaction mechanism $\tilde{\mu}$ is specified as follows.

$$\tilde{\mu} = \left(\tilde{\mu}(H), \tilde{\mu}(L)\right) = \left(\left((1/2, 1/2, 1)(0, 0, 0)(0, 0, 0)\right), \left((1, 0, 1)(0, 0, 0)(0, 1, 0)\right)\right). \tag{18}$$

Again, this mechanism is individually rational for all traders but is not incentive compatible. Specifically, trader 1 always has incentive to announce the signal H (i.e., under-report the risk of transfer), regardless of his actual observation, so that he will be required to transfer only half of his endowment, rather than all of it.

Figure 6: Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \rho\alpha_3$



(Second round transfers for $i = 1, 3$ are 0. $\because \rho\alpha_i < \alpha_i$ for $i = 1, 3$.)

In order to make the mechanism incentive compatible, it must be modified so that, if the trader 1 observes the signal L, then he will consume no more by misrepresenting his observation as H than by truthfully announcing L. This can be accomplished while retaining the same equilibrium allocation as $\tilde{\mu}$ would provide, by defining transaction mechanism μ to be identical to $\tilde{\mu}$ except that $\mu_1^F(H) = 1/2$. The contingent transfer $\mu_1^F(H)$ will never be made in equilibrium so again it does not reduce the value of the planer's objective function relative to truthful reporting subject to the transfer mechanism $\tilde{\mu}$. Thus, an incentive efficient transaction mechanism that maximizes $U(\mu, \alpha)$ is (see Figure 6)

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\big((1/2,1/2,1)(0,0,0)(1/2,0,0)\big),\ \big((1,0,1)(0,0,0)(0,1,0)\big)\Big). \quad (19)$$

Hence, the incentive-compatibility is not a binding constraint in the environment where the IC constraint binds in L and the partition $\{S, F\}$ is observed regardless of whether transfer from trader 2 to 3 is actually made.

S and F distinguished only with a non-zero transfer

Let us assume that the uniformed traders can only learn about the events S and F through the actual success or failure of a non-zero transfer made in the first round, and see how the requirement alters an incentive compatible mechanism in this case.

When trader 1 announces his observation of signal to be H, the mechanism μ in (19) requires trader 1 to send one half unit to trader 2 contingent on the event F in the second round in addition to one half unit sent in the first round. Since trader 2 also sends one half unit to trader 3 in the first round, there is no problem of verification of event by the uniformed traders. However, when trader 1 announces his observation of signal to be L, the mechanism μ requires trader 2 to send one unit to trader 3 in the event F, but uniformed traders 2 and 3 will not be able to verify whether F took place or not because trader 2 sends nothing to trader 3 in the first round. Therefore, one possibility is for trader 2 to make non-contingent second round transfers. This means that trader 2 sends one unit to 3 regardless of the events S and F. The level of U will not be changed by this modification of μ because $Pr(S|L) = 0$. The other possibility is for trader 2 to send trader 3 a small amount λ in round 1 for a verification of event. This, however, reduces the level of U by the amount of $(1/2)\lambda\alpha_2$. Thus, the former way of changing μ maximizes U. Therefore, the transaction mechanism μ that is incentive compatible and maximizes $U(\mu, \alpha)$ is given by the following.

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\big((1/2, 1/2, 1)(0, 0, 0)(1/2, 0, 0)\big), \big((1, 0, 1)(0, 1, 0)(0, 1, 0)\big)\Big). \quad (20)$$

Let us summarize the arguments of this subsection as follows.

Proposition 2 *Under the given parametric specification of this subsection, in which a safe second round transfer from trader 2 to trader 3 is not too costly from social point of view, (for an optimal and individually rational transaction mechanism) there will be a problem of possible under-reporting by the informed trader concerning the risk involved in a risky transfer from 2 to 3. If the signal observed by the informed trader is perfectly accurate, then there will be no social costs of imposing incentive compatibility. In this case, even if there is a problem of verification by the uniformed traders of the signal observed by the informed trader without a positive risky transfer made from trader 2 to 3, there will not be a social cost of imposing incentive compatibility because an appropriate incentive for truthful reporting is created in the case of H when a positive first round transfer from trader 2 to 3 is perfectly safe.*

From proposition 1 and proposition 2 one could infer that the preciseness of information must be related to the costs of imposing incentive compatibility of truthful reporting by an information holder. Thus, we now like to turn to a parametric environment in which the information conveyed by H and L is not perfectly accurate.

IV A parametric environment with imperfectly accurate private information

Let us consider the case where the information conveyed by H and L is not perfectly accurate so that

$$0 < \sigma_L = \Pr(S|L) < \sigma = \Pr(S) < \sigma_H = \Pr(S|H) < 1. \quad (21)$$

Since we are interested in the incentive compatibility issues of truthful reporting of information observed by an informed trader, it seems natural to require the information to be sufficiently accurate. We express this by requiring conditional probabilities σ_H and σ_L to satisfy

$$\sigma_H = \Pr(S|H) > 1-\epsilon, \sigma_L = \Pr(S|L) < \epsilon. \tag{22}$$

We assume the parameter values to satisfy

$$0 < \epsilon < \delta < \frac{1-\epsilon}{4}, \quad \delta\alpha_3 < \alpha_1 < \alpha_2 < \alpha_3, \rho = 1/2. \tag{23}$$

In addition we impose the condition

$$\frac{1}{1-\epsilon}\alpha_2 < \alpha_3, \tag{24}$$

which insures

$$\alpha_2 < \sigma_H\alpha_3 \tag{25}$$

and is interpreted to say that a risky transfer from trader 2 to trader 3 is appropriate to maximize the α-weighted sum of utilities when H is truthfully announced.

With less than perfectly accurate information, the incentive-compatibility constraints in (9) need to be revised as in the following two equations.

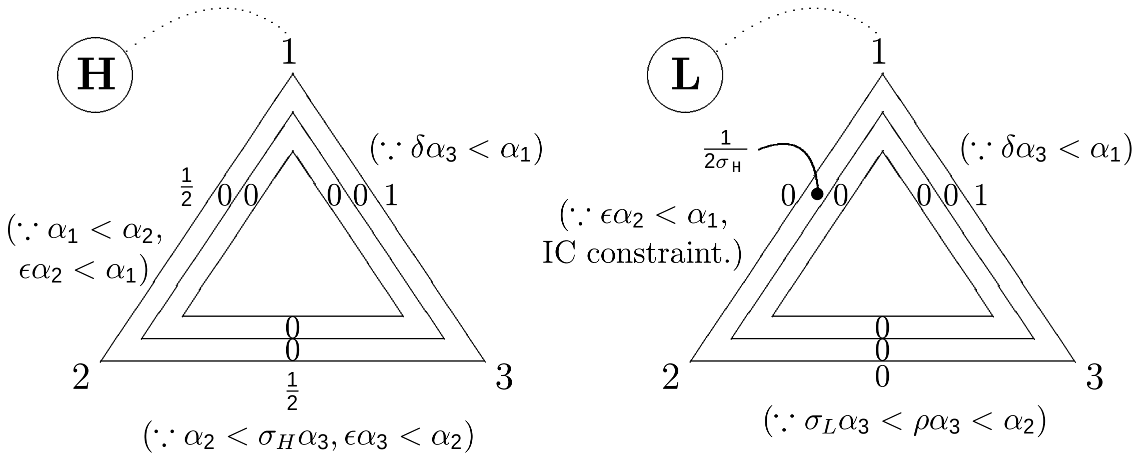
$$\begin{aligned}
&(\mu_3^1(H) - \mu_1^1(H)) + \sigma_H(\rho\mu_3^S(H) - \mu_1^S(H)) + (1-\sigma_H)(\rho\mu_3^F(H) - \mu_1^F(H)) \\
&\quad \geq \; (\mu_3^1(L) - \mu_1^1(L)) + \sigma_H(\rho\mu_3^S(L) - \mu_1^S(L)) + (1-\sigma_H)(\rho\mu_3^F(L) - \mu_1^F(L)), \\
&(\mu_3^1(L) - \mu_1^1(L)) + \sigma_L(\rho\mu_3^S(L) - \mu_1^S(L)) + (1-\sigma_L)(\rho\mu_3^F(L) - \mu_1^F(L)) \\
&\quad \geq \; (\mu_3^1(H) - \mu_1^1(H)) + \sigma_L(\rho\mu_3^S(H) - \mu_1^S(H)) + (1-\sigma_L)(\rho\mu_3^F(H) - \mu_1^F(H)).
\end{aligned} \tag{26}$$

A Costly safe transfer technology

As in the previous section III, we like to analyze two different environments, one with costly second round safe transfer technology between traders 2 and 3 and the other with less costly case from the social point of view. Thus, let α satisfy the condition, $\rho\alpha_3 < \alpha_2$ in this subsection. Then, an incentive compatible transaction mechanism μ that is individually rational and maximizes the value of U must have the following features. (See Figure 7.)

- If trader 1 announces H, then the following transfers should be made in the first round. Trader 3 should send his entire endowment to trader 1 because $\delta\alpha_3 < \alpha_1$. Trader 2 should make a transfer of his endowment to trader 3 even if there is a risk in transfer because this risk is small relative to his contribution to social utility U given the information H and assuming trader 1 announces truth, because $\alpha_2 < \sigma_H\alpha_3$. However, trader 2 should send only up to half a unit to trader 3 because $\epsilon\alpha_3 < \alpha_2$.

Figure 7: Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \sigma_H\alpha_3$, $\sigma_L\alpha_3 < \rho\alpha_3 < \alpha_2$



(Since there is a probability $1 - \sigma_H$ of non-arrival, trader 3's expected utility is $\sigma_H/4$ which is greater than δ even when he receives nothing in L. In other words, trader 3's expected utility is higher than that obtained by consuming his own entire endowment.) Trader 1 should make a transfer of his endowment to trader 2 because $\alpha_1 < \alpha_2$, but no less and no more than half a unit because $\epsilon\alpha_2 < \alpha_1$.

- If trader 1 announces L, then the following transfers should be made. Trader 3 should send his entire endowment to trader 1 in the first round, because $\delta\alpha_3 < \alpha_1$. Despite trader 1's announcement of L, it is not a perfectly accurate information of the event F. There is a possibility of σ_L that a transfer he might make in the first round may succeed (under the assumption that trader 1 announces the truth). However, trader 2 should send nothing to trader 3 in the first round, because his information is accurate enough to have $\sigma_L\alpha_3 < \epsilon\alpha_3 < \alpha_2$. Transfer should not be made in the second round either because $\rho\alpha_3 < \alpha_2$. If trader 1 were to send nothing to trader 2 in either round because $\epsilon\alpha_2 < \alpha_1$, then the mechanism μ would not be incentive compatible. Trader 1 would have incentive to announce L regardless of his actual observation in order to avoid giving up half of his endowment which he is required to do if he announces H. Thus, the mechanism should specify an amount of $\mu_1^S(L)$ in the interval $[1/2\sigma_H, 1]$ since, then, trader 1's expected utility of announcing L instead of announcing the true H would not exceed that of announcing H.

- Although trader 1 does not have perfectly accurate information, no further transfer is needed in round 2 since making a transfer in round 2 would incur an iceberg cost that is avoidable in round 1.

Thus, an incentive compatible transaction mechanism that maximizes $U(\mu, \alpha)$ is specified by the following transaction mechanism.

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\Big((1/2, 1/2, 1)(0,0,0)(0,0,0)\Big), \Big((0,0,1)(1/2\sigma_H, 0, 0)(0,0,0)\Big)\Big) \tag{27}$$

If we do not require incentive compatibility, $U(\tilde{\mu}, \alpha)$ is maximized by defining $\tilde{\mu}$ as follows just as in (13) although information is not perfectly accurate.

$$\tilde{\mu} = \Big(\tilde{\mu}(H), \tilde{\mu}(L)\Big) = \Big(\Big((1/2, 1/2, 1)(0,0,0)(0,0,0)\Big), \Big((0,0,1)(0,0,0)(0,0,0)\Big)\Big). \tag{28}$$

Thus, as anticipated earlier, the incentive compatibility requirement becomes a real constraint as the utility $U(\mu, \alpha)$ is reduced by the amount of $(1/4)(\sigma_L/\sigma_H)(\alpha_1 - \rho\epsilon\alpha_2)$ as compared to the level given by $\tilde{\mu}$ in (28). Let us note that the extent of this reduction becomes larger as the precision of information as reflected by the inverse of the ratio σ_L/σ_H declines. In the case of perfectly accurate information, we have $\sigma_L/\sigma_H = 0$.

If S could not be distinguished from F without a non-zero transfer being made as in the previous section, then incentive compatibility can be achieved in four different ways.

1. Trader 1 makes non-contingent second round transfer of $1/2\sigma_H$ units to trader 2.
2. Require trader 1 to transfer half of his endowment to trader 2 in L. Thus, in this case, $\mu_1^1(L) = 1/2$.
3. Require trader 3 to retain half of his endowment rather than transferring it to trader 1 in L so that $\mu_3^1(L)$ is reduced to $1/2$.
4. Require trader 2 in L to send an amount λ to trader 3 in round 1 so that trader 1 can find out the event S, and then trader 1 to send the amount $1/2\sigma_H$ in round 2 in event S, that is, $\mu_2^1(L) = \lambda$, $\mu_1^S(L) = 1/2\sigma_H$.

Among these alternatives the first one is clearly worse than the second, and the second one in turn is worse than the third because we have $\delta\alpha_3 > \epsilon\alpha_2$. But just as in the case of the previous section, for values of λ sufficiently small, the fourth alternative will give a higher value of $U(\mu, \alpha)$ but otherwise the third alternative maximizes the value of $U(\mu, \alpha)$.

The following proposition summarizes the arguments of this section.

Proposition 3 *Under the given parametric specification of this subsection, in which a safe second round transfer from trader 2 to trader 3 is too costly from the social point of view, (for an optimal and individually rational transaction mechanism) there will be a problem of possible over-reporting by the informed trader concerning the risk involved in a risky transfer from 2 to 3. If the signal observed by the informed trader is imperfectly accurate, then there will be social costs of imposing incentive compatibility which increase as the accuracy of information conveyed by the signals declines.*

B Not too costly safe transfer technology

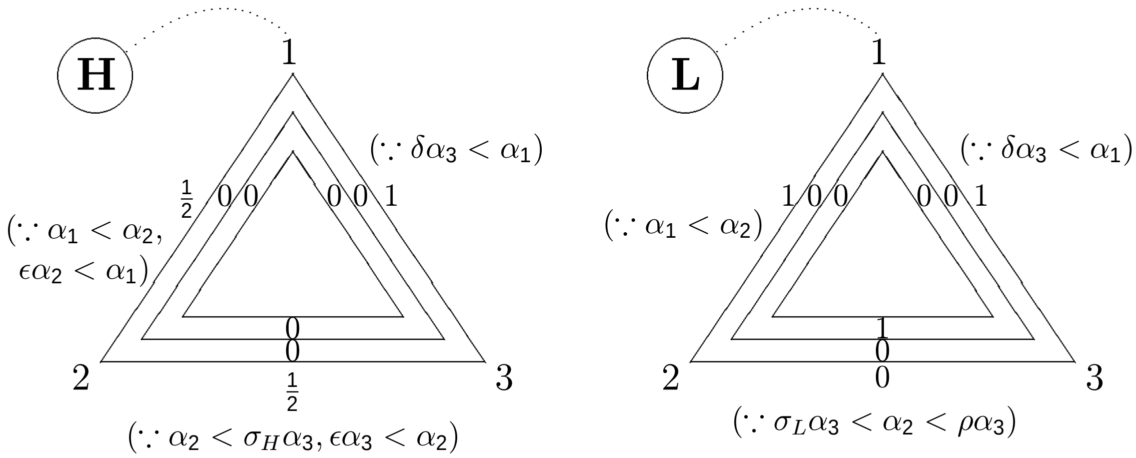
Now, let us assume that the safe second round transfer technology from trader 2 to 3 is not too costly so that transfer from trader 2 to 3 even with iceberg cost is appropriate to maximize the α-weighted sum of utilities. Thus, let α satisfy the condition $\alpha_2 < \rho\alpha_3$. Then, a transaction mechanism $\tilde{\mu}$ that is individually rational and maximizes the value of U have the following characteristics.

- If trader 1 announces H, trader 3 sends entire unit of his endowment to trader 1. Trader 2 sends half a unit of his endowment to trader 3 despite a risk of transfer failure because this risk is small relative to the importance of making such a transfer, i.e. $\sigma_H\alpha_3 > \alpha_2$. Trader 1 sends half a unit to trader 2. When trader 1 announces H, transfers to be made are exactly the same as in the case of the previous subsection.

- If trader 1 announces L, trader 3 sends a full unit to trader 1. Trader 2 sends nothing to trader 3 in round 1 because the risk of non-arrival is sufficiently high that the maximization of U does not warrant such a transfer (i.e, $\sigma_L\alpha_3 < \alpha_2$), but sends one unit in round 2 regardless of the state to prevail, because a second round transfer from trader 2 to trader 3 even with iceberg cost is appropriate, i.e. $\alpha_2 < \rho\alpha_3$, and this cost reduces the actual receipt of transfer by trader 3 by half a unit. Trader 1 sends his entire endowment of one unit to trader 2.

Therefore, the following $\tilde{\mu}$ maximizes the weighted sum U of utilities. (See Figure 8.)

Figure 8: Optimal but not Incentive Efficient Mechanism: $\delta\alpha_3 < \alpha_1 < \alpha_2 < \rho\alpha_3$



$$\tilde{\mu} = \left(\tilde{\mu}(H), \tilde{\mu}(L)\right) = \left(\left((1/2, 1/2, 1)(0,0,0)(0,0,0)\right), \left((1,0,1)(0,1,0)(0,1,0)\right)\right). \tag{29}$$

Again, this mechanism $\tilde{\mu}$ is not incentive compatible. There are four possible ways to make the mechanism incentive compatible.

1. In H, trader 3 withholds half a unit and sends to trader 1 only half a unit so that $\mu_3^1(H) = 1/2$.
2. In H, require trader 1 to make an additional second round transfer of $1/2(1-\sigma_L)$ units in the event F, i.e., $\mu_1^F(H) = 1/2(1-\sigma_L)$.
3. In H, trader 1 sends an additional one half unit to trader 2 so that $\mu_1^1(H) = 1$.
4. In L, trader 1 withholds half a unit and sends only half a unit to trader 2 so that $\mu_1^1(L) = 1/2$.

All the above modifications will reduce the weighted sum of utilities. The third modification will reduce the utilities more than the first modification, because $\delta\alpha_3 > \epsilon\alpha_2$. One can check that it is also worse than the second since $\sigma_H > \sigma_L$. One can also verify that if the information conveyed by the signals H and L is sufficiently accurate and $\sigma_H \simeq 1, \sigma_L \simeq 0$, then the second alternative above maximizes U. But as the quality of information deteriorates, there is a range of values of σ_H and σ_L such that either the first or the third alternative gives the highest value of U depending upon whether $(\alpha_2 + \delta\alpha_3)/2 > \alpha_1$ or not. Therefore, the transaction mechanism μ that is incentive compatible and maximizes $U(\mu, \alpha)$ is given by the following.

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\Big((1/2, 1/2, 1)(0,0,0)(1/2(1-\sigma_L), 0, 0)\Big), \Big((1,0,1)(0,1,0)(0,1,0)\Big)\Big) \tag{30}$$

if $\sigma_H \simeq 1, \sigma_L \simeq 0$. Otherwise,

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\Big((1/2, 1/2, 1/2)(0,0,0)(0,0,0)\Big), \Big((1,0,1)(0,0,0)(0,1,0)\Big)\Big) \tag{31}$$

if $(\alpha_2 + \delta\alpha_3)/2 > \alpha_1$, and

$$\mu = \Big(\mu(H), \mu(L)\Big) = \Big(\Big((1/2, 1/2, 1)(0,0,0)(0,0,0)\Big), \Big((1/2,0,1)(0,0,0)(0,1,0)\Big)\Big) \tag{32}$$

if $(\alpha_2 + \delta\alpha_3)/2 \leq \alpha_1$.

As in the previous subsection let us note again that the cost of imposing incentive compatibility, as measured by a reduction of weighted-sum of utilities $U(\mu, \alpha)$, increases as the precision of information declines because it results in a decline of the denominator of the term $1/2(1-\sigma_L)$. But this cost is bounded because when the quality of information deteriorates beyond some point, then the mechanism expressed by either (31) or (32) gives a higher value that does not depend upon the precision of the information.

By summarizing the arguments above we obtain the following proposition.

Proposition 4 *Under the given parametric specification of this subsection, in which a safe second round transfer from trader 2 to trader 3 is not too costly from the social point of view, (for an optimal and individually rational transaction mechanism) there will be a problem of possible under-reporting by the informed trader concerning the risk involved in a risky transfer from 2 to 3. If the signal observed by the informed trader is not perfectly accurate, then there will be social costs of imposing incentive compatibility. This cost increases as the precision of information conveyed by the signals declines but is bounded.*

V Conclusion

If we regard the crucial issue in the settlement system as the efficient risk sharing among the participants in the presence of private information, the rules in a settlement system must encourage the participants to take optimal degrees of risk in accordance with their attitude towards risk. If some participants have socially useful private information, the rules in a settlement system must be constructed that it does not give participant adverse incentives to mask their information. Policy makers can achieve this objective if they think about the rules in a settlement system as a mechanism design problem. If policy makers ignore those points and introduce new rules into a settlement system, the equilibrium allocation of goods might be distorted.

We have shown that there is indeed an incentive-compatibility issue for the payments network to resolve, but that there is no simple generalization about how to resolve it. In fact, we have noted that there are two distinctive cases in which some incentive efficient mechanisms involve a binding incentive-compatibility constraint for truthful revelation that a transfer is likely to fail, while other incentive efficient mechanisms involve a binding constraint for truthful revelation that failure is unlikely. As a practical matter, then, an implication of using the incentive efficient transaction mechanisms as payment arrangements is that supervisory authorities ought to accord substantial discretion to the governing body of a payments network to establish rules aimed at eliciting accurate information from members.

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