

Discussion Paper #2005-11

Non-governmental Copyright Protection
in a Music Market
by
Yasuhiro Arai

December, 2005

Non-governmental Copyright Protection in a Music Market*

Yasuhiro Arai

Graduate School of Economics, Hitotsubashi University

2-1, Naka, Kunitachi, Tokyo 186-8601, Japan

E-mail: yasuhira68@yahoo.co.jp

December, 2005

Abstract

We consider non-governmental copyright protection in a music market in a game theoretical framework. Music composers voluntarily form a non-governmental association to prevent illegal uses of music and to impose music fees collectively. We prove that the formation of an association increases social welfare when differences in composers' abilities are large enough. We also show that a uniform pricing rule, currently employed by the association in Japan, is desirable from the welfare viewpoint than a non-uniform pricing rule where members can set music fees individually to maximize their private profits.

*I am very grateful to Munemoto Ando, Reiko Aoki, Taiji Furusawa, Hideshi Itoh, Kohei Kawamura, and members in Hitotsubashi Game Theory Workshop for helpful comments and suggestions. I am truly indebted to Akira Okada, my advisor, for his invaluable guidance. Of course, any remaining errors are my own responsibility. I also gratefully acknowledge financial support from the 21st Century COE program "Normative Evaluation and Social Choice of Contemporary Economic System " at Graduate School of Economics, Hitotsubashi University.

1 Introduction

The music market is drastically changing due to the emergence of new computer technology. A growing number of people enjoy music by using internet technology. There are about 62 million people (80% households) who use internet in Japan. This information technology has a great impact on our music life. We can buy and sell music through the internet. For example, a lot of people buy the music on the iTunes Music Store and enjoy the music by using iPod in these days. The number of the accumulation sales of iPod exceeded 20 million units and the total number of downloaded music on the iTunes Music Store exceeded 500 million in July 2005. In the U.S.A., the internet music market yield about \$270 million sales in 2004 and the sales are expected to reach \$1.7 billion in 2009 according to a report released by Jupiter Research (2004).

The emergence of these new markets increases the possibility of illegal uses of music. By new computer technologies, one can make easily digital copies of CD and of DVD without the licence of copyrights. A lot of illegal copies are distributed through the network. Many illegal music files are exchanged by PtoP software such as Napster and Gnutella. According to the Association of Copyright for Computer Software (ACCS), the number of illegal music files which are exchanged by PtoP is about 16.1 million and 92% of these files are exchanged without right holder's license. The protection of copyrights in the music market becomes critical. Non-governmental associations to prevent illegal uses play an important role in many countries.

The purpose of this paper is to present a game theoretical model of a non-governmental association of music composers to prevent illegal uses, and to examine economic effects of it. In particular, our main interest is in how non-governmental protection can increase social welfare. The non-governmental association affects the social welfare at least in two ways. First, composers can save the costs to monitor illegal uses by forming an association. Second, composers joining the association can not set their monopoly prices due to a uniform pricing rule employed by the association. Because of these conflicting affects, it is not clear whether or not the non-governmental association can increase the social welfare. Another issue of the non-governmental association is voluntary participation of composers. Composers can decide to participate in an association or not, without any enforcement of a government authority. High-ability composers may be reluctant

to join the association. It is not clear whether all composers join the association under the uniform pricing rule. The social welfare depends on the number of composers who join the association.

In this paper, we will consider how the social welfare is affected by the formation of an association. Composers participate voluntarily in the association or not. Participants can share the monitoring costs, and can choose a uniform pricing rule to maximize the joint profits. The uniform pricing rule is currently employed by the association named JASRAC in Japan. The association distributes the total profits to its members. By a two-stage game model, we analyze the voluntary participation of composers and the profit distribution in the association.

We obtain the following results. First, the effect on the social welfare depends on two factors: the monitoring cost to protect copyrights and the ability of composers. When the monitoring cost is smaller than low-type composers' monopoly profits, the social welfare increases by the formation of an association. When the monitoring cost is higher than high-type composers' monopoly profits, the social welfare decreases by the formation of an association. In the intermediate case, the social welfare increases if the difference of composers' performances is large enough.

Second, it is desirable from the viewpoint of society to employ a uniform pricing rule rather than a non-uniform pricing rule where every member composer can set his own monopoly price. This result may provide a theoretical support for the current pricing rule employed by JASRAC.

Most works in the literature study copyright protection by governments (see Novos and Waldman (1984), Johnson (1985), Besen and Kirby (1989), and Yoon (2002) for example). As far as we know, the works on non-governmental protection of copyrights are few. Besen, Kirby and Salop (1992) and Snow and Watt (2005) consider the formation of non-governmental music association such as JASRAC. Besen, Kirby and Salop (1992) consider how the social welfare is affected by a relationship between an membership policy and the broadcasters' willingness to pay, and show that an open membership policy maximizes the social welfare. Snow and Watt (2005) analyzes a distribution mechanism in an association and show the association can alleviate the risk-bearing situation of its members when the value of each copyright is a random variable. However, these literature do not consider regulations by the associations. Yooki and Scotchmer (2004) consider the joint development of firms for new technology preventing illegal copies. However, they did not

take voluntary participation problem into account.

The paper is organized as follows. Section 2 gives a brief summary of JASRAC. Section 3 presents the music market model. Section 4 analyzes the group formation game of an association. Section 5 provides welfare analysis. Section 6 provides concluding remarks. All proofs are given in Appendix.

2 JASRAC

The first copyright association called SACEM (Societe des Auteurs, Compositeurs et Editeurs de Musique) was formed in France in 1851. At that time, many pieces of music was used in cafes without license of right holders. An association of composers was formed to monitor illegal uses of music. After that, similar associations were formed in all over the world. In Japan, the association called JASRAC was formed in 1939.

Japanese copyright law was implemented in 1899, and acceded to the Berne Convention. But foreign music had been used freely to promote the music modernization until 1931. In 1932, Wilhelm Plage who was the ambassador of the association in England, Germany, France, Italy and Austria visited Japan to collect music fees. He got conflict with Japanese users, demanded expensive music fees. In order to collect music fees, he invited Japanese composers to join an association. A group of composers worked on the Japanese government to implement a copyright law in 1939, and formed JASRAC to prevent Plage from establishing the association.

JASRAC had managed copyright exclusively until law revision in 2001. Copyright associations must have license given by the Agency for Cultural Affairs to manage copyright. JASRAC is the largest association in Japan which has over 13,000 membership and total sales is about 110 billion yen in 2004. Figure 1 shows the number of composers joining the JASRAC from 1995 to 2005. The figure suggests that almost all composers join the JASRAC in 2005. Figure 2 shows the total sales of JASRAC. The total sales have been growing during the last ten years.

A contract between a composer and JASRAC is as follows. Composers transfer their copyrights to JASRAC. JASRAC uses these copyrights to collect music fees from users. JASRAC distributes the total profit to its members. The composers in the JASRAC can concentrate on creating their

Figure 1 The number of membership in JASRAC

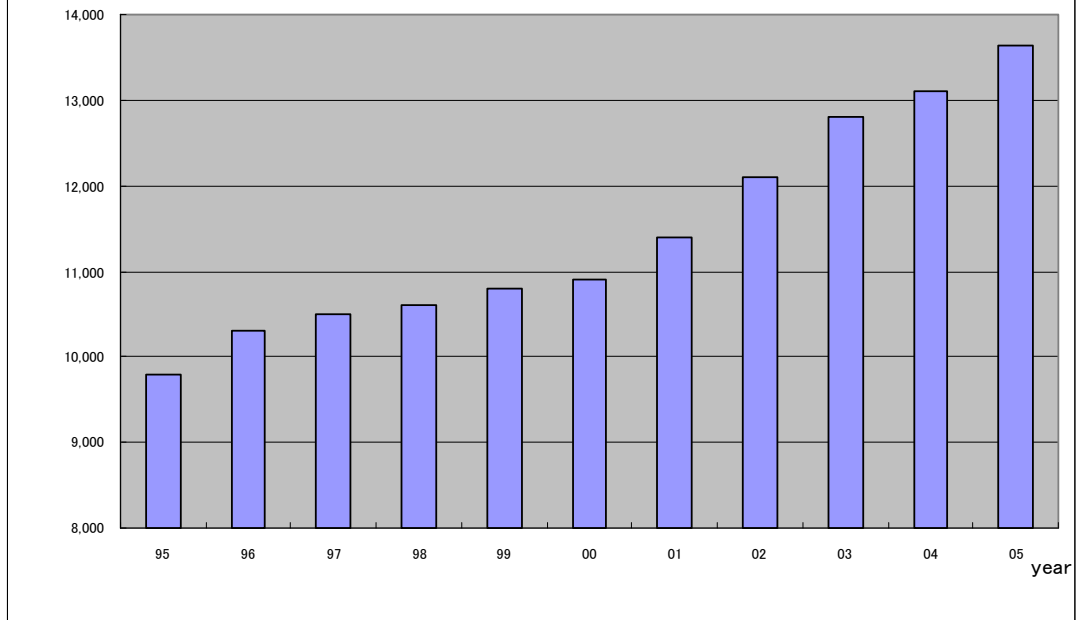
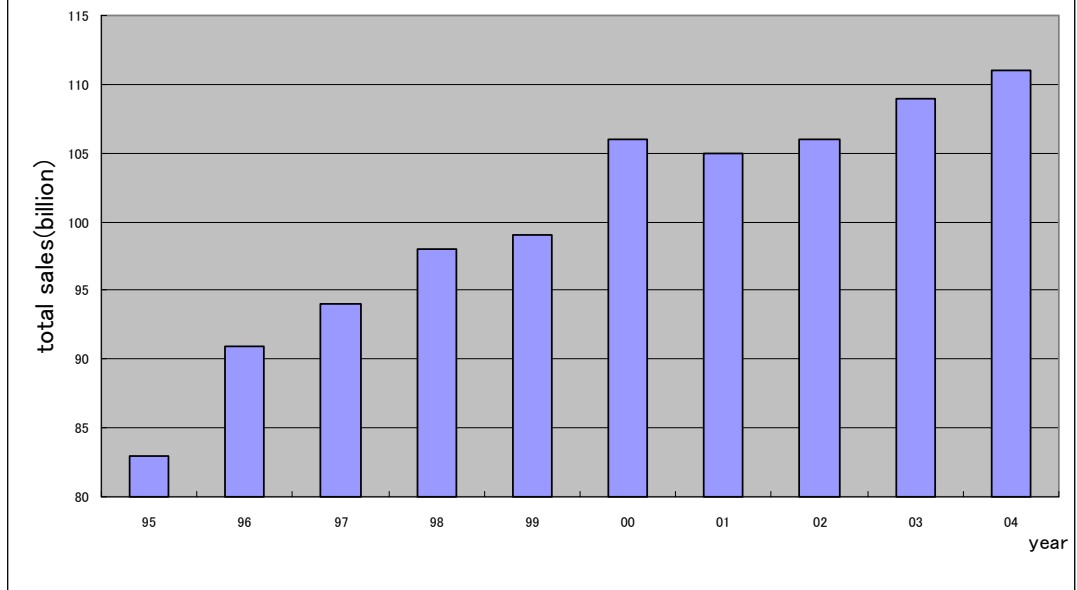


Figure 2 Total sales of JASRAC



songs. They can share monitoring costs with other members. However, the composers can not set the music fee by themselves since they transfer their all copyrights to JASRAC. Some composers do not join the association because of this demerit. JASRAC has a membership rule which requires a high sales performance or a certain experience of concerts. JASRAC restricts the participation of low-performance composers.

3 The Music Market

There are n composers of two types, high(H) and low(L). A type H composer has a high ability to compose music, and a type L composer has a low ability. The number of type i composers is denoted by n_i where $n_H + n_L = n$. We assume that each composer is a monopolist. The prominent character of music goods is that their substitutability is low. Many pieces of music made by different composers are considered to be different goods.

When the illegal use of music is prevented perfectly, the market demand for a type i composer is given by $D_i = 1 - \theta_i p$ where θ_i means performance parameter. We assume that $\theta_L > \theta_H > 0$. The monopoly price p^* of the type i composer is the optimal solution of

$$\max_{p \geq 0} p(1 - \theta p)$$

We can obtain $p^* = 1/2\theta$, and the monopoly profit is $\pi^* = 1/4\theta$. When the illegal use of music is not prevented, the composer's profit is 0.

To prevent the illegal use of music, the composers can organize a private association to protect their copyrights. The primary functions of the association are to monitor whether or not the copyrights of the composers in the association are protected, to set the uniform price of their music and to distribute the total profits to its members. The monitoring cost is denoted by c and it is allocated to the members of the association. We assume that the monitoring cost c is constant and does not depend on the number of the composers who join the association. Suppose that the number of type i composers in the association is given by s_i , $i = H, L$. The number of all composers in the association is denoted by s where $s = s_H + s_L$. Then, the total profit of the

association is given by

$$\pi = s_H p (1 - \theta_H p) + s_L p (1 - \theta_L p) - c.$$

where p is the uniform price of music for the composers in the association. The uniform price p_G that maximizes the total profit is given by

$$p_G = \frac{s_H + s_L}{2(\theta_H s_H + \theta_L s_L)}$$

and the total profit under p_G is given by

$$\pi_G = \frac{(s_H + s_L)^2}{4(\theta_H s_H + \theta_L s_L)} - c.$$

Let θ_G denote the average type of composers in the association, that is $\theta_G = (\theta_H s_H + \theta_L s_L)/(s_H + s_L)$. Then, the optimal price and the total profit of the association are rewritten by

$$p_G = \frac{1}{2\theta_G}, \pi_G = \frac{s}{4\theta_G} - c$$

4 The Group Formation Game

Every composer can voluntarily decide to participate in the association or not. It is not always true that all composers participate. In this section, to analyze how many composers join in the association, we present a game theoretical model of group formation.

The group formation game consists of the following two stages

1. Participation decision stage

All n composers decide independently to join in the association or not.

2. Bargaining stage

All participants negotiate about the uniform price p_G of their music, about how to distribute their total profit π_G and how to allocate monitoring cost c .

We analyze this game by backward induction. To analyze the second stage of bargaining, we apply the Nash bargaining solution. If negotiations break down, the association is not organized and all composers have to decide if they protect their copyrights by themselves. In this case, the payoff of type i composer is given by

$$d_i = \max\left(\frac{1}{4\theta_i} - c, 0\right), i = H, L.$$

This payoff is considered to be the disagreement point in the Nash bargaining solution. If the monitoring cost c is larger than the monopoly profit $1/4\theta_i$, then a type i composer does not monitor the illegal uses, and thus his profit is 0. The Nash bargaining solution is given by the optimal solution $f_i(i = H, L)$ of the maximization problem

$$\begin{aligned} & \max (f_H - d_H)^{s_H} (f_L - d_L)^{s_L}, \\ & \text{s.t. } s_H f_H + s_L f_L = \pi_G \\ & \quad f_H \geq d_H \\ & \quad f_L \geq d_L. \end{aligned}$$

We define $G(s_H, s_L)$ by the total profit π_G minus the sum of the members payoffs at the disagreement point, that is

$$G(s_H, s_L) = \pi_G - s_H d_H - s_L d_L. \quad (1)$$

We call $G(s_H, s_L)$ the bargaining surplus of composers in the association. The bargaining surplus per a member is given by

$$g(s_H, s_L) = \frac{G(s_H, s_L)}{s_H + s_L}.$$

It is not difficult to see that when $G(s_H, s_L) \geq 0$, the Nash bargaining solution is given by

$$f_i(s_H, s_L) = g(s_H, s_L) + d_i, i = H, L.$$

When $G(s_H, s_L) < 0$, bargaining will break down. In the following analysis, we assume :

(A) There exist some natural numbers $s_H, s_L (s_H + s_L \leq n)$ such that $G(s_H, s_L) > 0$.

If this assumption is violated, the association is not formed for any outcome of the participation decision stage.

We next analyze a Nash equilibrium of the participation decision stage. For simplicity of notations we denote a strategy profile for composers by a pair (s_H, s_L) where s_i is the number of type i composers who decide to participate in the association. The first proposition shows under what conditions all composers join the association. In the following two propositions we mean a Nash equilibrium by a strict Nash equilibrium¹.

Proposition 1 (1) *There exists a unique (strict) Nash equilibrium (n_H, n_L) of the participation decision stage where all composers join the association if for all pairs (s_H, s_L) with $G(s_H, s_L) > 0$,*

$$G(t_H, t_L) > 0 \text{ for all } t_H \geq s_H \text{ and all } t_L \geq s_L. \quad (2)$$

(2) *There exists a unique (strict) Nash equilibrium (n_H, n_L) of the participation decision stage where all composers join the association if for all pairs (s_H, s_L) with $G(s_H, s_L) > 0$,*

$$\frac{\partial G}{\partial s_H} > 0 \quad \frac{\partial G}{\partial s_L} > 0. \quad (3)$$

The intuition for the proposition is as follows. If the association's profit is positive for some participation pair (s_H, s_L) with $s_H < n_H$, the association's profit is still positive at $(s_H + 1, s_L)$ under the condition in the proposition. Then, the profit of type H composer who is a new member

¹A strict Nash equilibrium is a Nash equilibrium where every player's payoff strictly decreases if he deviates from the equilibrium.

increases from d_H to $g(s_H + 1, s_L) + d_H$. This means that the pair of (s_H, s_L) is not a Nash equilibrium. The same arguments are applied to the case that $s_L < n_L$.

Proposition 2 (1) *There exists a Nash equilibrium (s_H^*, s_L^*) of the participation decision stage if*

$$G(s_H^*, s_L^*) > 0 > G(s_H^* + 1, s_L^*),$$

$$G(s_H^*, s_L^*) > 0 > G(s_H^*, s_L^* + 1).$$

(2) *There exists a Nash equilibrium (n_H, s_L^*) of the participation decision stage if*

$$G(n_H, s_L^*) > 0 > G(n_H, s_L^* + 1).$$

(3) *There exists a Nash equilibrium (s_H^*, n_L) of the participation decision stage if*

$$G(s_H^*, n_L) > 0 > G(s_H^* + 1, n_L).$$

The second proposition shows other types of equilibria where all composers do not participate in the association. The intuition for Proposition 2 (1) is as follows. The left inequality in the first condition means that the profit of every type H composer who joins the association is larger than that obtained when he does not join. The right inequality means that the profit of any new type H participant decreases. The same arguments can be applied for type L composers if the second conditions holds. The other inequalities in the proposition can be interpreted in the similar manner.

As we have stated in Section 2, a large majority of Japanese composers participate in JASRAC. By this reason in the following analysis, we will focus the Nash equilibrium where all composers join the association. The next proposition provides a sufficient condition for such an equilibrium in terms of performance parameters θ_L, θ_H and monitoring costs c .

Proposition 3 *There exists a unique Nash equilibrium of the participation decision stage where*

all composers join the association if

$$c > \frac{(\theta_L - \theta_H)^2}{4\theta_H^2\theta_L}, \quad (4)$$

$$\theta_L > \theta_H > \frac{\theta_L}{2}. \quad (5)$$

The primary advantage to every composer joining the association is to share monitoring cost with other participants and to prevent illegal use of copyrights. The first condition shows that this merit of saving cost is large enough. However, there exists a disadvantage in joining the association. Since the association sets a uniform price for music use which is determined by the average type of composers in the association, it may differ from the monopoly price of each composer. If the performance parameters of type L composers and type H composers are divergent the disadvantage becomes high. The second condition shows that the difference of composers' performance parameters is not very large.

5 The Welfare Analysis

In this section, we will first consider how the formation of an association with a uniform price affects social welfare. In the last part of this section we will consider an alternative case that the association can set differentiated prices for composers.

To compute the social welfare, we introduce utility functions $U = v/\theta_i - p$ of consumers when they consume the music whose copyright is owned by type i composers with price p . Here, v is assumed to be distributed uniformly over $[0, 1]$. It is not difficult to see that the demand function $D_i = 1 - \theta_i p$ can be derived from these utility functions.

The next theorem shows how the social welfare changes by the formation of an association. Recall that c is the monitoring cost for the association.

Theorem 4 *The following results hold for an association with arbitrary size.*

(1) *In the case of $\frac{1}{4\theta_L} > c$, social welfare is increased by the formation of an association.*

(2) *In the case of $\frac{1}{4\theta_H} > c \geq \frac{1}{4\theta_L}$, social welfare is increased by the formation of an association if the differences between θ_L and θ_H is large enough.*

(3) *In the case of $c \geq \frac{1}{4\theta_H}$, social welfare is decreased by the formation of an association.*

In the case of $1/4\theta_L > c$, both types of composers outside an association can set their monopoly prices and can protect their copyrights because monitoring cost c is smaller than the monopoly profits. The consumers who use the music composed by high performance composers in the association can obtain more surplus because the uniform price set by the association is lower than the monopoly price. On the other hand, the situation is reversed for consumers who use the music composed by low performance composers. The uniform price by the association is higher than the monopoly price of these composers. Producer surplus is increased by the formation of an association due to sharing of monitoring cost. Theorem 1 guarantees that the increases of producer surplus for type L and of consumer surplus derived by the use of type H composers outweigh the decrease of consumer surplus derived by the use of type L composers.

In the second case of $1/4\theta_H > c \geq 1/4\theta_L$, the monitoring cost is larger than the monopoly profit of type L composers. When the association is not formed, type L composers do not protect their copyright, and thus their music is supplied at zero price as if the market is perfectly competitive. Compare to the first case, the decrease of consumer surplus derived by the use of type L composers is larger. Theorem 1 shows that if the differences between θ_L and θ_H is large enough ($\theta_L - \theta_H > s\theta_H/s_H$), the social welfare is increased by the formation of an association.

In the third case of $c \geq 1/4\theta_H$, monitoring cost is so high that neither types of composers protect their copyright without an association. Therefore, all types of music are supplied at zero price when the association is not formed and so the social welfare is maximized. The social welfare is decreased by the formation of an association.

Theorem 1 gives us the following implication to the case of JASRAC. In real situations there are many types of composers varying from professional musicians to amateur musicians. The JASRAC sets the requirement of membership that the music which is composed by any new member had to be published by the third person in the past year. This membership rule by JASRAC essentially prevent low ability composers from joining the association. Although this requirement can be interpreted as that the association protects participating composers' profits, the second result of the theorem shows that such a restriction may harm the increase of social welfare.

It seems to us that the actual music market corresponds to the second case of theorem 1, since there are some composers who can not obtain sufficient sales to pay the monitoring cost.

Theorem 1 (2) show that the social welfare increases by the formation of an association when the differences of composer's performance are sufficiently large. In practice composers can not join the association if they can not clear the requirement from the association. These requirements narrow the defereces of performance down. It is not desirable to set these requirement from the view of social welfare.

So far, we have considered the case that an association sets an uniform music fee for members , which is the actual price rule employed by JASRAC. After the copyright law was revised in 2001, a few member of private associations other than JASRAC are formed newly in Japan. These new associations such as e-License, or Japan Rights Clearance Inc. employ a non-uniform pricing rule that different music fees are set for different composers. In what follows, we examine how the social welfare may affect if JASRAC employs such a non-uniform pricing rule.

We assume that the composers can set the music fee individually in the association, unlike the uniform pricing rule. Then the total profit of the association is given by

$$\pi' = s_H p_H (1 - \theta_H p_H) + s_L p_L (1 - \theta_L p_L) - c$$

where $p_i (i = H, L)$ is the price of music made by type i composers. The total profit is maximized by the monopoly prices

$$p_H = \frac{1}{2\theta_H}, \quad p_L = \frac{1}{2\theta_L} \tag{6}$$

and the total profit is given by

$$\pi'_G = \frac{s_H}{4\theta_H} + \frac{s_L}{4\theta_L} - c. \tag{7}$$

The disagreement point of negotiation is the same as before, that is

$$d_i = \max \left(\frac{1}{4\theta_i} - c, 0 \right), \quad i = H, L.$$

The Nash bargaining solution in the new case is given by the optimal solution $f_i (i = H, L)$ of the

maximization problem

$$\begin{aligned} & \max (f_H - d_H)^{s_H} (f_L - d_L)^{s_L}, \\ & s.t. \quad s_H f_H + s_L f_L = \pi_G^a \\ & \quad \quad f_H \geq d_H \\ & \quad \quad f_L \geq d_L. \end{aligned}$$

We define the bargaining surplus $G^a(s_H, s_L)$ by the total profit π_G^a minus the sum of the members payoffs at the disagreement point, that is

$$G^a(s_H, s_L) = \pi_G^a - s_H d_H - s_L d_L.$$

The bargaining surplus per member is given by

$$g^a(s_H, s_L) = \frac{G^a(s_H, s_L)}{s_H + s_L}.$$

It is not difficult to see that when $G^a(s_H, s_L) \geq 0$, the Nash bargaining solution is given by

$$f'_i(s_H, s_L) = g^a(s_H, s_L) + d_i, i = H, L.$$

The next proposition shows the result of the group formation game presented in section 3 when the association can employ the non-uniform pricing rule.

Proposition 5 *There exists a unique Nash equilibrium of the group formation game. In equilibrium, all composers join the association under the non-uniform pricing rule.*

The intuition of this proposition is clear. If composers in the association can set the music fee by themselves, they can not only set their monopoly prices but also can share the monitoring cost. We can show that $\partial G^a / \partial s_H > 0, \partial G^a / \partial s_L > 0$ for any value of c . Therefore every composer can increase his payoff by joining the association.

We next consider the effect of the non-uniform pricing rule by the association on the social welfare.

Theorem 6 *The social welfare under the non-uniform pricing rule by the association is smaller than that under the uniform pricing rule when all composers join the association.*

It is clear that the producer surplus under the non-uniform pricing rule is larger than that under the uniform pricing rule. The effect of the non-uniform pricing rule on consumer surplus is as follows. Since the uniform equilibrium price is larger than the monopoly price for type L composers, and is smaller than the monopoly price for type H composers, the consumer surplus derived by type L composers increases under the non-uniform pricing rule, and that derived by type H composers decreases. Theorem 2 shows that the decrease of type H consumer surplus is larger than the sum of the increases of producer surplus and of type H consumer surplus. The theorem implies that the uniform pricing rule currently employed by JASRAC is desirable from the viewpoint of society.

6 Conclusion

In this paper, we have presented a game theoretic model of the formation of a private sector to protect copyrights, based on the case of JASRAC. We have examined how the private protection of copyright affects the social welfare. The main conclusions of this paper are as follows.

First, the effect on the social welfare depends on the two factors: monitoring costs to protect copyrights and the ability of composers. When the monitoring cost is smaller than the low type composers' monopoly profits, the social welfare increases by the formation of an association. When the monitoring cost is higher than the high type composer's monopoly profits, the social welfare decreases by the formation of an association. In the intermediate case, the social welfare increases if the deference of composers' performances is large enough.

Second, it is desirable from the viewpoint of society to employ the uniform pricing rule rather than the non-uniform pricing rule where every member composer can set his own monopoly price. This result may provide a theoretical support for the current pricing rule by JASRAC.

Finally, we point out two problems remained for future research. In this paper, we consider only one association for simplicity of analysis. It is interesting to consider interactions among associations. In real economy, governments play important roles to protect copyrights. It is a promising research issue to analyze how government and private associations compliment each other in protecting copyrights. These topics will be next research issues.

References

- [1] ACCS (2004), "2004 PtoP Software Research", <http://www2.accsjp.or.jp/index.html>.
- [2] Besen, S. and S, Kirby (1989), "Private Copying, Appropriability, and Optimal Copying Royalties", *Journal of Law and Economics*, 32, 255-280.
- [3] Johnson, W (1985), "The Economics of Copying", *Journal of Political Economy*, 93, 158-174.
- [4] Jupiter Research (2004), "Consumer Survey Report :Music, 2004", <http://www.jupiterresearch.com/bin/item.pl/home>
- [5] Novos, I. and M, Waldman (1984), "The Effect of Increased Copyright Protections: An Analytic Approach", *Journal of Political Economy*, 92, 236-246.
- [6] Snow, A. and R, Watt (2005), "Risk Sharing and the Distribution of Copyright Collective Income", in *Developments in the Economics of Copyright*, ed. L. Takeyama., WJ. Gordon., and R. Towse.
- [7] Yooki, P. and S, Scotchmer (2004), "Digital Rights Management and The Pricing of Digital Products", Working paper, Cambridge, MA: National Bureau of Economic Research.
- [8] Yoon, K (2002), "The Optimal Level of Copyright Protection", *Information Economics and Policy*, 14, 327-348.

7 Appendix

Proof of Proposition 1

(1) We first prove that the pair (n_H, n_L) is a Nash equilibrium. If any type i ($i = H, L$) composers unilaterally deviate from the association, then his profit decreases from $g(n_H, n_L) + d_i$ to d_i . Notice that $g(n_H, n_L)$ is positive under (2) and Assumption (A). We next prove that all other pairs (s_H, s_L) are not Nash equilibria. Without any loss of generality, we can assume that $s_H < n_H$. When $g(s_H, s_L) > 0$, (2) implies that $g(s_H + 1, s_L) > 0$. Then, the profit of type H composer who is a new member increases from d_H to $g(s_H + 1, s_L) + d_H$. When $g(s_H, s_L) < 0$, every type i composer who deviates from the association can increase his profit from $g(s_H, s_L) + d_i$ to d_i . When $g(s_H, s_L) = 0$, every member is indifferent between joining the association and being outside the association. Therefore (s_H, s_L) is not a strict Nash equilibrium.

(2) It is clear that (3) implies (2) in the proposition. Q.E.D.

Proof of Proposition 2

(1) We prove that (s_H^*, s_L^*) is a Nash equilibrium. The left inequality in the first condition means that the profit of every type H composer who joins the association is larger than that obtained when he does not join. The right inequality in the first condition means that the profit of any new type H participant decreases. The same arguments can be applied for type L composers if the second condition holds.

(2) The left inequality $G(n_H, s_L^*) > 0$ in the condition means that every member increases his profit by joining the association. The right inequality $0 > G(n_H, s_L^* + 1)$ means that any type L composer outside the association does not have an incentive to join. These arguments prove the proposition.

(3) We can prove the proposition by the same manner as in (2). Q.E.D.

Proof of Proposition 3

We consider the following three cases.

- Case of $1/4\theta_L > c$

In this case, the disagreement point is $d_i = 1/4\theta_i - c$. The total surplus of the association is given by

$$G(s_H, s_L) = \frac{(s_H + s_L)^2}{4(\theta_H s_H + \theta_L s_L)} - \frac{s_H}{4\theta_H} - \frac{s_L}{4\theta_L} + c(s_H + s_L - 1).$$

We can write

$$\frac{\partial G}{\partial s_H} = \frac{-s_L^2(\theta_L - \theta_H)^2}{4\theta_H(\theta_H s_H + \theta_L s_L)^2} + c$$

$$\frac{\partial G}{\partial s_L} = \frac{-s_H^2(\theta_L - \theta_H)^2}{4\theta_L(\theta_H s_H + \theta_L s_L)^2} + c$$

and so we can show $\partial G/\partial s_H > 0$ if and only if

$$c > \frac{s_L^2(\theta_L - \theta_H)^2}{4\theta_H(\theta_H s_H + \theta_L s_L)^2}. \quad (8)$$

Similarly, we can obtain $\partial G/\partial s_L > 0$ if and only if

$$c > \frac{s_H^2(\theta_L - \theta_H)^2}{4\theta_L(\theta_H s_H + \theta_L s_L)^2}. \quad (9)$$

It is clear that the right hand side of (8) is decreasing function of s_H . We can obtain the following inequality by substituting $s_H = 0$ into (8),

$$c > \frac{(\theta_L - \theta_H)^2}{4\theta_H\theta_L^2}.$$

Similarly, we can obtain the following inequality by substituting $s_L = 0$ into (9),

$$c > \frac{(\theta_L - \theta_H)^2}{4\theta_H^2\theta_L}.$$

If $c > (\theta_L - \theta_H)^2/4\theta_H^2\theta_L$, we can prove $\partial G/\partial s_L > 0$ ($i = H, L$) since $\theta_L > \theta_H$.

- Case of $1/4\theta_H > c \geq 1/4\theta_L$

In this case, the disagreement point is $d_H = 1/4\theta_H - c, d_L = 0$. The total surplus of the association is given by

$$G(s_H, s_L) = \frac{(s_H + s_L)^2}{4(\theta_H s_H + \theta_L s_L)} - \frac{s_H}{4\theta_H} + c(s_H - 1).$$

From proposition 1 (2), we can obtain

$$\frac{\partial G}{\partial s_H} = \frac{-s_L^2(\theta_L - \theta_H)^2}{4\theta_H(\theta_H s_H + \theta_L s_L)^2} + c,$$

$$\frac{\partial G}{\partial s_L} = \frac{(s_H + s_L)(2s_H\theta_H - s_H\theta_L + s_L\theta_L)}{4(\theta_H s_H + \theta_L s_L)^2}.$$

We can rewrite $\partial G/\partial s_H > 0$ as

$$c > \frac{s_L^2(\theta_L - \theta_H)^2}{4\theta_H(\theta_H s_H + \theta_L s_L)^2}. \quad (10)$$

Similarly, we can obtain $\partial G/\partial s_L > 0$, if

$$\theta_L > \theta_H > \frac{\theta_L}{2}. \quad (11)$$

It is clear that the right-hand side of (10) is a decreasing function of s_H . We can obtain the next inequality by substituting $s_H = 0$ into (10),

$$c > \frac{(\theta_L - \theta_H)^2}{4\theta_H^2\theta_L}.$$

Therefore, if $\theta_L > \theta_H > \frac{\theta_L}{2}$ and $c > (\theta_L - \theta_H)^2 / 4\theta_H^2\theta_L$, we can prove $\partial G/\partial s_L > 0$ ($i = H, L$) in this case.

- Case of $c \geq 1/4\theta_H$

In this case, the disagreement point is $d = 0$. The total surplus of the association is given by

$$G(s_H, s_L) = \frac{(s_H + s_L)^2}{4(\theta_H s_H + \theta_L s_L)} - c.$$

From proposition 1 (2), we can obtain

$$\frac{\partial G}{\partial s_H} = \frac{(s_H + s_L)(2s_L\theta_L - s_L\theta_H + s_H\theta_H)}{4(\theta_H s_H + \theta_L s_L)^2},$$

$$\frac{\partial G}{\partial s_L} = \frac{(s_H + s_L)(2s_H\theta_H - s_H\theta_L + s_L\theta_L)}{4(\theta_H s_H + \theta_L s_L)^2}.$$

All type H composers join in the association since $\partial G/\partial s_H > 0$.

We can show $\partial G/\partial s_L > 0$ if

$$\theta_L > \theta_H > \frac{\theta_L}{2}. \quad (12)$$

Therefore, if $\theta_L > \theta_H > \theta_L/2$, we can prove $\partial G/\partial s_i > 0$ ($i = H, L$) in this case.

From the analysis above, we can obtain the proposition 3. Q.E.D.

Proof of Theorem 1

We compute the social welfare in the following three ranges of monitoring cost c .

- Case of $1/4\theta_L > c$

In this case, the disagreement point is $d_i = 1/4\theta_i - c$. We denote the social welfare by W and divide W into the three parts $W_G, W_H,$ and W_L ($W_G + W_H + W_L = W$). W_G is the sum of producer surplus and consumer surplus by the music supplied from the association. W_i ($i = H, L$) is the sum of producer surplus and consumer surplus by the music supplied by type i composers outside the association. We can compute $W_G, W_H,$ and W_L separately.

$$\begin{aligned} W_G &= \pi_G + s_H \int_{\theta_H p_G}^1 \left(\frac{1}{\theta_H} v - p_G\right) dv + s_L \int_{\theta_L p_G}^1 \left(\frac{1}{\theta_L} v - p_G\right) dv \\ &= \pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2} p_G^2 \theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2} p_G^2 \theta_L \right) \end{aligned}$$

$$\begin{aligned} W_H &= (n_H - s_H) \left(d_H + \int_{\theta_H p_H}^1 \left(\frac{1}{\theta_H} v - p_H\right) dv \right) \\ &= (n_H - s_H) \left(d_H + \frac{1}{8\theta_H} \right) \end{aligned}$$

$$\begin{aligned} W_L &= (n_L - s_L) \left(d_L + \int_{\theta_L p_L}^1 \left(\frac{1}{\theta_L} v - p_L\right) dv \right) \\ &= (n_L - s_L) \left(d_L + \frac{1}{8\theta_L} \right). \end{aligned}$$

We next calculate the social welfare W^0 when the association is not formed. By the same way as for W , we divide W^0 into the two parts W_H^0 and W_L^0 ($W_H^0 + W_L^0 = W^0$). We can compute

$$W_H^0 = n_H(d_H + \frac{1}{8\theta_H})$$

$$W_L^0 = n_L(d_L + \frac{1}{8\theta_L}).$$

The social welfare increases by the formation of an association if

$$\pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right) > s_H(d_H + \frac{1}{8\theta_H}) + s_L(d_L + \frac{1}{8\theta_L}).$$

We can rewrite this equation as

$$\frac{(s_H + s_L)^2}{8(\theta_H s_H + \theta_L s_L)} + c < \frac{s_H}{8\theta_H} + \frac{s_L}{8\theta_L} + c(s_H + s_L).$$

This inequality holds since

$$\frac{(s_H + s_L)^2}{(\theta_H s_H + \theta_L s_L)} < \frac{s_H}{\theta_H} + \frac{s_L}{\theta_L}.$$

- Case of $1/4\theta_H > c \geq 1/4\theta_L$

In this case, the disagreement point is $d_H = 1/4\theta_H - c$, $d_L = 0$. We can calculate W_G , W_H , and W_L as follows.

$$W_G = \pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right)$$

$$W_H = (n_H - s_H)(d_H + \frac{1}{8\theta_H})$$

$$W_L = \frac{1}{2\theta_L}(n_L - s_L).$$

We can obtain W_H^0 and W_L^0 as follows:

$$W_H^0 = n_H(d_H + \frac{1}{8\theta_H})$$

$$W_L^0 = \frac{n_L}{2\theta_L}.$$

We can obtain the following condition which assures the association raises social welfare

$$\pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right) > s_H \left(d_H + \frac{1}{8\theta_H} \right) + \frac{s_L}{2\theta_L}.$$

We can rewrite this equation as

$$\frac{(s_H + s_L)^2}{8(\theta_H s_H + \theta_L s_L)} + c < \frac{s_H}{8\theta_H} + c s_H.$$

Notice that, if this inequality holds when $c = 1/4\theta_L$, then it always holds in this case. The inequality above holds if the next condition holds when $c = 1/4\theta_L$:

$$\frac{(s_H\theta_L - 2s_H\theta_H - s_L\theta_H)s_L}{8\theta_H(\theta_H s_H + \theta_L s_L)} > 0.$$

It is clear that $W > W^0$ holds when the difference between θ_L and θ_H is large enough.

- Case of $c \geq 1/4\theta_H$

In this case, the disagreement point is $d = 0$. We can obtain $W_G, W_H,$ and W_L as follows.

$$W_G = \pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right)$$

$$W_H = \frac{1}{2\theta_H}(n_H - s_H)$$

$$W_L = \frac{1}{2\theta_L}(n_L - s_L).$$

We can obtain W_H^0 and W_L^0 as follows.

$$W_H^0 = \frac{n_H}{2\theta_H}, \quad W_L^0 = \frac{n_L}{2\theta_L}.$$

We can obtain the following condition which assures that the association raises social welfare:

$$\pi_G + s_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + s_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right) > \frac{s_H}{2\theta_H} + \frac{s_L}{2\theta_L}.$$

We can rewrite this equation as follows

$$-\frac{(s_H + s_L)^2}{8(\theta_H s_H + \theta_L s_L)} - c > 0$$

It is clear that this equation does not hold. Therefore, we can prove $W^0 > W$ in this case. Q.E.D.

Proof of Proposition 4

We first remark that there exists some pair (s_H, s_L) such that $G'(s_H, s_L) > 0$ from Assumption (A) in Section 3 since $G'(s_H, s_L) > G(s_H, s_L)$ for all s_H and s_L .

- Case of $1/4\theta_L > c$

In this case, the disagreement point is $d_i = 1/4\theta_i - c$. The total surplus of the association is given by

$$G(s_H, s_L) = c(s_H + s_L - 1).$$

It holds that $\frac{\partial G}{\partial s_i} > 0$, since

$$\frac{\partial G}{\partial s_H} = \frac{\partial G}{\partial s_L} = c > 0.$$

- Case of $1/4\theta_H > c \geq 1/4\theta_L$

In this case, disagreement point is $d_H = 1/4\theta_H - c, d_L = 0$. The total surplus of the association is given by

$$G(s_H, s_L) = \frac{s_L}{4\theta_L} + c(s_H - 1)$$

It is clear that $\partial G/\partial s_i > 0$, since

$$\begin{aligned} \frac{\partial G}{\partial s_H} &= c > 0 \\ \frac{\partial G}{\partial s_L} &= \frac{1}{4\theta_L} > 0 \end{aligned}$$

- Case of $c \geq 1/4\theta_H$

In this case, the disagreement point is $d = 0$. The total surplus of the association is given by

$$G(s_H, s_L) = \frac{s_H}{4\theta_H} + \frac{s_L}{4\theta_L} - c.$$

It holds that $\partial G/\partial s_i > 0$, since

$$\frac{\partial G}{\partial s_H} = \frac{1}{4\theta_H} > 0$$

$$\frac{\partial G}{\partial s_L} = \frac{1}{4\theta_L} > 0.$$

It follows from Proposition 1, that all composers join the association under the non-uniform pricing rule. Q.E.D.

Proof of Theorem 2

By Proposition 4, all composers join the association when they can set the music fees by themselves. Then, the social welfare is given by

$$W^a = \frac{3n_H}{8\theta_H} + \frac{3n_L}{8\theta_L} - c.$$

The social welfare under the uniform pricing rule when all composers join the association is given by

$$W = \pi_G + n_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + n_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right).$$

It holds that $W > W^a$ if

$$\pi_G + n_H \left(\frac{1}{2\theta_H} - p_G + \frac{1}{2}p_G^2\theta_H \right) + n_L \left(\frac{1}{2\theta_L} - p_G + \frac{1}{2}p_G^2\theta_L \right) > \frac{3n_H}{8\theta_H} + \frac{3n_L}{8\theta_L} - c.$$

This inequality can rewrite as

$$\frac{(n_H + n_L)^2}{8(\theta_H n_H + \theta_L n_L)} < \frac{s_H}{8\theta_H} + \frac{s_L}{8\theta_L},$$

which clearly holds. Q.E.D.