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Accumulation of Public Goods and Population Change  
in a Voluntary Participation Game

by

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# Accumulation of Public Goods and Population Change in a Voluntary Participation Game<sup>1</sup>

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## Abstract

We consider how public goods can be provided and accumulated over generations of self-interested individuals in the absence of a government authority. A dynamic voluntary participation game is presented to analyze interrelations among group formation, accumulation and population change. We show that the accumulation of public goods can mitigate the second-order dilemma of public goods and that it necessarily leads all individuals to participate in providing public goods in the long run. We discuss a positive effect of population increase on the accumulation of public goods.

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Key words: public goods, voluntary participation, accumulation, group formation, population change

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# 1 Introduction

This paper considers how public goods can be provided and accumulated over generations of self-interested individuals in the absence of a government authority which compels contributions to provision. Imagine a situation that residents in a community voluntarily construct and maintain a park. Investment today benefits both current and future generations. Furthermore, the stock of public goods (park) affects contributions from future generations. The aim of the paper is to investigate dynamic properties of public goods accumulation when investment requires voluntary contribution of individuals. In this paper, we consider pure public goods which are nonexcludable and nonrival.

It has been widely argued in the theory of public goods since the seminal work of Olson (1965) that public goods would be undersupplied by voluntary contribution, due to the free-riding incentives (see Bergstrom, Blume and Varian (1986) and Andreoni (1988) among others for recent works). There also exists a large volume of literature which attempts to solve the free-riding problem. The literature investigates suitable ways to achieve an efficient provision of public goods. First, the theory of repeated games has investigated how long-term relationships facilitate cooperation among selfish individuals. It studies various behavioral rules such as trigger strategies (Friedman 1971) and tit-for-tat strategies (Axelrod 1984) to attain cooperation. For studies of collective actions in the context of repeated games, see Cremer (1986), Bendor and Mookherjee (1987) and Taylor (1987), etc. In repeated games, cooperation is explained by mutual punishments by individuals against those who deviate from cooperation. The folk theorem of repeated games (e.g., Fudenberg and Maskin 1986) shows that an efficient outcome can be achieved by decentralized punishing behavior under suitable conditions. Secondly, the theory of mechanism design has investigated the design of many mechanisms to implement an efficient provision of public goods (more generally, a desirable collective choice rule). Groves and Ledyard (1977) proposed a mechanism that achieves a Pareto efficient allocation in

a public good economy. For a survey on the mechanism design, see Groves and Ledyard (1987). More recently, Moore and Repullo (1988) show that almost any choice rule can be implemented by subgame perfect equilibria of multi-stage mechanisms in an economic environment with at least one private good. Bagnoli and Lipman (1989) present a voluntary contribution mechanism implementing the core. Varian (1994) proposes simple mechanisms which solve a wide variety of externalities problems including implementation of the Lindahl allocations. For other types of works, it has been studied by cooperative game theory, especially by the core theory, how individuals can reach an efficient solution through voluntary bargaining. For example, see Foley (1970) and Mas-Colell (1980).

Most previous studies on the mechanism to solve the free-riding problem implicitly assume one undesirable property.<sup>2</sup> It has been assumed either that all individuals in question have already participated (or forced to participate) in a mechanism, or that individuals are willing to participate in a mechanism if they become better-off by doing so than in the status-quo, by imposing the individual rationality constraint (sometimes called the participation constraint in the mechanism design literature). It, however, should be remarked that *any* mechanism itself which achieves an efficient outcome is a kind of public goods. Individuals may have incentive to free ride on the mechanism. This problem is called *the second-order dilemma* of public goods (Ostrom 1990 and 1998). There is no external power to force individuals to participate. Individuals can decide freely whether they should participate in the mechanism or not. The previous literature has not considered the incentive problem about participation in mechanisms in a sufficient manner.

Recently, the voluntary participation in a mechanism which implements an efficient provision of public goods has been studied by several authors (Palfrey and Rosenthal (1984), Okada (1993), Saijo and Yamato (1999) and Dixit and Olson (2000) among oth-

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<sup>2</sup>In the literature cited above, one can broadly regard strategies for repeated games and cooperative bargaining games as mechanisms to achieve an efficient allocation.

ers). By a two-stage game model, they analyze whether or not individuals voluntarily participate in the mechanism in the presence of the non-participation incentive.<sup>3</sup> In the first stage, every individual decides independently to participate in a group or not. In the second stage, all participants negotiate for creating a mechanism that implements an efficient provision within the group. Any non-participant is allowed to free ride on the mechanism. It has been shown that all individuals do not participate in a group to provide public goods, and that the likelihood of all individuals' participation becomes lower as population becomes larger. These results partially support a widely-held view on the second-order dilemma of public goods that an efficient provision of public goods is not possible due to the incentive of free-riding on the mechanism itself.

In this paper, we reexamine the voluntary participation game from the viewpoints of public goods accumulation and of population change. The motivation of our analysis is as follows. First, most studies on voluntary participation have been done without the possibility of accumulation. On the contrary, many types of public goods can be accumulated over generations. Some examples of them are forest, parks, common lands, irrigation systems, public libraries and museums, etc. We will show that the accumulation of public goods critically affects individuals' participation in providing public goods. Second, the standard one-shot model shows a negative effect of population on the voluntary provision of public goods. The population growth, however, implies an increase of human and economic resources. The previous studies can not capture well this positive side of population increase. Currently, a serious problem of depopulation happens in many industrialized countries. In these countries, population moves from rural to urban areas. For example, population in rural areas of Japan decreases from 55 million in 1955 to 44 million in 2000 while the whole population increased from 90 million to 127 million for the same period

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<sup>3</sup>Palfrey and Rosenthal (1984) consider a one-stage model of voluntary participation in providing binary public goods. Their model has a payoff structure similar to that in the reduced form of the two-stage games studied by others.

(FAO, 2005). Agricultural labor force in Japan decreases dramatically from 30 million in 1960 to 5 million in 2000. Depopulation in rural and mountain areas causes agricultural abandonment and the decline of communities. It has affected traditional ways that people cooperate and manage their common resources.

We will present a dynamic model of a public good economy with the Cobb-Douglas utility functions to consider how public goods can be accumulated through voluntary provision over generations. The model includes three factors: voluntary participation, accumulation and population change. We first prove that given a stock of public goods, the equilibrium group size in the voluntary participation game is uniquely determined (except a degenerate case). When the accumulation level of public goods is low, all individuals do not participate in a group. However, in contrast to the result of the one-shot model, we show that if the depreciation rate of public goods is low, the accumulation of public goods necessarily leads all individuals to participate in a group in the long run. We illustrate typical patterns of the accumulation of public goods. The accumulation pattern shows that population growth can enhance the accumulation of public goods through voluntary provision in the long run.

As far as we know, there are not many works on the accumulation of public goods by voluntary provision. Fershtman and Nitzan (1991) and Marx and Matthews (2000) consider dynamic contribution games of infinitely-lived individuals. Similar to the folk theorem of repeated games, they describe two polar cases of voluntary contribution. In a linear-quadratic differential game, Fershtman and Nitzan (1991) show the inefficiency of voluntary contribution by a feedback Nash equilibrium in Markovian strategies. Marx and Matthews (2000) show efficiency results implemented by a trigger-type strategy in a long-horizon model. Glomm and Lagunoff (1999) consider a migration choice between a voluntary provision mechanism and an involuntary provision mechanism (proportional

tax rules by a majority vote) in a sequential-move game of infinitely-lived individuals.<sup>4</sup> In their model, the accumulation of public goods has a role to equalize income levels of individuals, which makes them to choose the involuntary provision mechanism in the long run. The participation problem is not at issue in these studies of dynamic games. In the growth theory literature, the accumulation of public capital (infrastructure) provided by a government authority has been studied extensively since the seminal works of Shell (1967) and Arrow and Kurz (1970). Recent endogenous growth models in Barro (1990), Glomm and Ravikumar (1994) and Futagami, Morita and Shibata (1993) among others examine long-term growth generated by cumulated infrastructure. Almost studies in this literature are done in the competitive equilibrium framework where strategic behavior of agents is out of the scope of analysis. Investment in public capital is chosen by a government which maximizes the welfare of a representative infinitely-lived consumer.

The paper is organized as follows. Section 2 presents a dynamic model of the voluntary provision of public goods. Section 3 analyzes the group formation for voluntary provision. The equilibrium group size is characterized. Section 4 examines the accumulation of public goods with population change. Section 5 discusses implications of the model to the second-order dilemma of public goods and to the depopulation problem in rural areas. Section 6 concludes the paper.

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<sup>4</sup>Okada and Sakakibara (1991) and Okada, Sakakibara and Suga (1995) consider the formation of a constitutional state in a dynamic public goods economy with capital accumulation. In these works, a political system of the state determines rules of participation, provision of public goods, and distribution. Okada, Sakakibara and Suga (1995) study a constitutional choice between a centralized system (monarchy) and a decentralized system (democracy) from the viewpoint of social contract.

## 2 The model

Consider a simple economy with one private good and one public good over generations  $t$  ( $=1, 2, \dots$ ). The public goods are assumed to be nonexcludable and nonrival. Let  $n_t$  be population in generation  $t$ . In each generation  $t$ , every individual  $i$  is initially endowed with wealth  $\omega$  and contributes an amount  $a_i$  ( $0 \leq a_i \leq \omega$ ) of the private goods to the provision of public goods. The public goods are produced under a constant return to scale technology, and the production  $y_t$  of public goods is given by  $y_t = \beta \sum_{i=1}^{n_t} a_i$  where  $\beta$  ( $> 0$ ) is the marginal productivity of private goods.

The total amount  $K_t$  of public goods in generation  $t$  is given by

$$\begin{aligned} K_t &= (1 - \delta)K_{t-1} + y_t \\ &= (1 - \delta)K_{t-1} + \beta \sum_{i=1}^{n_t} a_i, \end{aligned} \tag{2.1}$$

where  $K_{t-1}$  is the stock of public goods in generation  $t - 1$  and  $\delta$  ( $0 < \delta < 1$ ) is the depreciation rate of public goods. In this paper, for clarity of analysis, we employ the Cobb-Douglas utility function of individual  $i$

$$u_i(x_i, K_t) = x_i^\alpha K_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where  $x_i + a_i = \omega$ .

Given  $K_{t-1}$ , the voluntary contribution game in generation  $t$  is defined as follows. Every individual  $i$  ( $= 1, \dots, n_t$ ) has a pure action  $a_i \in [0, \omega]$ , and his payoff function is given by

$$u_i(a_1, \dots, a_n, K_{t-1}) = (\omega - a_i)^\alpha \left( (1 - \delta)K_{t-1} + \beta \sum_{i=1}^{n_t} a_i \right)^{1-\alpha}. \tag{2.2}$$

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<sup>5</sup>To save symbols, we abuse notation  $u_i$  to denote various types of payoffs for individual  $i$  whenever no confusion arises.



We can see without much difficulty that  $\partial u_i / \partial a_i < 0$  for any contribution  $a_j$  of all individuals  $j (\neq i)$  if the marginal productivity  $\beta$  of private goods satisfies

$$\beta < \frac{\alpha}{1 - \alpha} \frac{(1 - \delta)K_{t-1}}{\omega}. \quad (2.3)$$

That is, if (2.3) holds, then every individual  $i$  is better off by contributing nothing ( $a_i = 0$ ) than by contributing a positive amount ( $a_i > 0$ ) of private goods, regardless of others' contributions. The zero contribution is the dominant action for each individual.

Under (2.3), the voluntary contribution game of generation  $t$  has a unique Nash equilibrium in which no individuals contribute. Generally, the Nash equilibrium outcome is not Pareto-efficient. In the next section, we will consider individuals' group forming behavior to improve the zero-contribution equilibrium.

### 3 The group formation

We consider a situation that individuals voluntarily form a group to provide public goods. We assume that if a group is formed, its members can establish a suitable mechanism to implement an efficient provision of public goods within the group. It is certainly costly to have such a mechanism for providing public goods collectively. The costs of a mechanism include those of communication, negotiations, monitoring, punishment, staffing, and maintaining the group organization. We will call simply these costs the *group costs*. The participants bear the group costs. As we have stated in the introduction, we are primarily concerned with whether or not individuals voluntarily participate in such a group, and (if they do) with how many individuals participate. All non-participants are free from the enforcement by the group. They are allowed to free ride on the contributions by participants.

Before we present a two-stage process of group formation, we analyze the optimal contribution of a group. Let  $S$  be a set of participant. The optimal contributions of all participants in  $S$  maximize the sum of participants' payoffs

$$U^S = \sum_{i \in S} u_i(a_1, \dots, a_{n_t}, K_{t-1})$$

subject to  $0 \leq a_i \leq \omega$  ( $i \in S$ ) and  $a_i = 0$  ( $i \notin S$ ).

Note that the optimal contribution of non-participant is  $a_{np} = 0$  under (2.3), independent of group size  $s$ . Assuming that all participants contribute an amount  $x$  of private goods equally, the total payoff  $U$  is given by

$$U^S = s(\omega - x)^\alpha ((1 - \delta)K_{t-1} + \beta s x)^{1-\alpha}.$$

It can be seen without much difficulty that  $\partial U^S / \partial x = 0$  implies

$$x = \frac{(1 - \alpha)\beta s \omega - \alpha(1 - \delta)K_{t-1}}{\beta s}.$$

Therefore, given the group size  $s$  and the stock  $K_{t-1}$  of public goods, the optimal (symmetric) contribution  $a_p(s, K_{t-1})$  of each participant is given by

$$a_p(s, K_{t-1}) = \max \left( 0, \frac{(1 - \alpha)\beta s \omega - \alpha(1 - \delta)K_{t-1}}{\beta s} \right). \quad (3.1)$$

When  $a_p(s, K_{t-1}) > 0$ , that is,

$$s > \frac{\alpha(1 - \delta)K_{t-1}}{(1 - \alpha)\beta \omega}, \quad (3.2)$$

the group rational behavior of participants is to contribute the positive amount  $a_p(s, K_{t-1})$

of private goods. The optimal payoff  $u_p(s, K_{t-1})$  of participant  $i$  is given by

$$u_p(s, K_{t-1}) = \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(\beta s)^\alpha} [\beta s \omega + (1-\delta)K_{t-1}]. \quad (3.3)$$

We remark that the participant's payoff  $u_p(s, K_{t-1})$  is an increasing function of group size  $s$  when  $a_p(s, K_{t-1}) > 0$ . When the optimal contribution is implemented in the group  $S$ , the non-participant's payoff  $u_{np}(s, K_{t-1})$  is given by

$$u_{np}(s, K_{t-1}) = \omega^\alpha(1-\alpha)^{1-\alpha} [\beta s \omega + (1-\delta)K_{t-1}]^{1-\alpha}. \quad (3.4)$$

Clearly, the non-participant's payoff  $u_{np}(s, K_{t-1})$  is an increasing function of group size  $s$ .

Every individual is faced with two conflicting incentives about group formation. First of all, he has an incentive to free ride on others' contributions, and thus not to participate in a group. But, if all individuals' actions are governed by this incentive, a group is not formed, and no contribution is made to provide public goods. To avoid this inefficient outcome, every individual has an incentive to participate in a group, *provided that a sufficient number of other individuals participate*. Due to these different incentives of individuals, the group formation problem in the public goods economy is not simple.

To consider the problem of voluntary participation in a pure form, we present a two-stage game of group formation in each generation  $t$  ( $= 1, 2, \dots$ ) as follows.

(1) Participation decision stage:

Given the stock  $K_{t-1}$  of public goods, all individuals of generation  $t$  decide independently whether to participate in a group or not. Let  $S$  be the set of all participants, and let  $s$  be the number of the participants. If  $s = 0, 1$ , then no group forms.<sup>6</sup>

(2) Group negotiation stage:

All participants decide independently whether or not they should agree to establish a

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<sup>6</sup>If  $s = 1$ , then no single participant has an incentive to contribute under (2.3).

mechanism to enforce the optimal provision (3.1) of public goods on them. The mechanism is established if and only if all participants agree. We say that a group is formed, if such a unanimous agreement is reached. When a group is formed, every participant is assumed to bear the group cost equally. The group cost per member is denoted by  $c(s)$  where  $s$  is the group size. All non-participants can choose their contributions freely. When a group is not formed, the voluntary provision game in section 2 is played. When every individual makes his choice in the group negotiation stage, it is assumed that he knows perfectly the outcome in the participation decision stage.

In the process of group formation, individuals decide to participate in a group or not, anticipating rationally the outcome of the group negotiation stage. To consider the participation decision of individuals, we characterize a subgame perfect equilibrium of the two-stage game of group formation. In this paper, we consider only pure strategy equilibria. To simplify the analysis, we assume the zero group cost. This assumption does not affect the result of the paper in any critical manner.<sup>7</sup>

By backward induction, we first analyze the group negotiation stage. For a group of size  $s$ , we define the *group surplus (per member)*, denoted by  $w(s, K_{t-1})$ , as

$$w(s, K_{t-1}) = u_p(s, K_{t-1}) - u(0, K_{t-1}) \quad (3.5)$$

where  $u(0, K_{t-1}) = \omega^\alpha[(1 - \delta)K_{t-1}]^{1-\alpha}$  is the payoff when no individuals contribute.  $u(0, K_{t-1})$  can be regarded as the opportunity cost of a group. The group surplus  $w(s, K_{t-1})$  is the participant's payoff minus the opportunity cost of the group.

The next proposition shows the equilibrium of the group negotiation stage.

**Proposition 3.1.** In the group negotiation stage, a group is formed in a strict Nash

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<sup>7</sup>When  $c(s) > 0$ , the participant's payoff  $u_p(s, K_{t-1})$  should be replaced with  $u_p(s, K_{t-1}) - c(s)$ . If  $u_p(s, K_{t-1}) - c(s)$  is an increasing function of  $s$ , the result of the paper holds true.

equilibrium<sup>8</sup> if and only if the group surplus per member  $w(s, K_{t-1})$  given in (3.5) is positive where  $s(\geq 2)$  is the number of participants.

**Proof.** If all  $s$  participants agree to form a group, then each of them receives payoff  $u_p(s, K_{t-1})$ . If any single participant does not agree, the group is not formed and all participants receive payoff  $u(0, K_{t-1})$ . Therefore, the group of  $s$  participants can be formed in a strict Nash equilibrium if and only if  $w(s, K_{t-1})$  is positive. Q.E.D.

The intuition for this proposition is clear. If the group surplus per member is positive, then all participants are better off by forming a group than by not forming it. In this case, there is a unique strict Nash equilibrium of the group negotiation stage in which all participants agree to form the group. We remark that there are many other non-strict Nash equilibria leading to the failure of a group. For example, all situations where there are at least two members who do not agree to form the group, are such equilibria. We exclude from our analysis these non-strict equilibria. When the group surplus  $w(s, K_{t-1})$  is zero, the zero contribution is the optimal choice of the group. In this case, all action profiles of the group negotiation stage are non-strict Nash equilibria leading to the zero contribution.

Next, we analyze the participation decision stage. All individuals decide to participate in a group or not, anticipating rationally what will happen in the group negotiation stage. As Proposition 3.1 shows, the group surplus plays a critical role in their decision.

**Definition 3.1.** Given  $K_{t-1}$ , the *critical size* of a group is defined to be the minimum integer  $s$  that satisfies  $w(s, K_{t-1}) > 0$ , and is denoted by  $s(K_{t-1})$ .<sup>9</sup>

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<sup>8</sup>A Nash equilibrium is called *strict* if every individual's payoff decreases by deviating from the equilibrium unilaterally. A Nash equilibrium is non-strict if every individual is indifferent between equilibrium and non-equilibrium actions when all other individuals stick to their equilibrium actions.

<sup>9</sup>If there exists no integer  $s$  such that  $w(K_{t-1}, s) > 0$ , we put  $s(K_{t-1}) = +\infty$  for convenience.

Since the participant's payoff  $u_p(s, K_{t-1})$  is an increasing function of  $s$ , the critical size  $s(K_{t-1})$  of a group is determined uniquely for every  $K_{t-1}$ . By (3.1),  $s(K_{t-1})$  is the minimum integer  $s$  that satisfies (3.2). It can be seen that  $s(K_{t-1}) = s$  if and only if

$$\frac{(1-\alpha)\beta\omega}{\alpha} \frac{1}{1-\delta} (s-1) \leq K_{t-1} < \frac{(1-\alpha)\beta\omega}{\alpha} \frac{1}{1-\delta} s. \quad (3.6)$$

It follows from Proposition 3.1 that a group is formed if the number of participants is greater than or equal to the critical size  $s(K_{t-1})$ .

The equilibrium group is characterized by the two conditions.

- No participant wants to opt out:  $u_p(s, K_{t-1}) \geq u_{np}(s-1, K_{t-1})$ ,
- No outsider wants to opt in:  $u_{np}(s, K_{t-1}) \geq u_p(s+1, K_{t-1})$ .

The next proposition characterizes the Nash equilibrium of the participation decision stage.

**Theorem 3.2.** The Nash equilibrium of the participation decision stage satisfies the following properties.

1. An equilibrium group is formed if and only if  $s(K_{t-1}) \leq n_t$ , that is,

$$n_t > \frac{\alpha(1-\delta)K_{t-1}}{(1-\alpha)\beta\omega}. \quad (3.7)$$

2. The equilibrium group size  $s$  is uniquely determined (except a degenerate case), and it satisfies  $s(K_{t-1}) \leq s \leq n_t$ .
3. All individuals participate if and only if either  $n_t = s(K_{t-1})$  or  $u_p(n_t) \geq u_{np}(n_t-1)$ .

**Proof.** First, it follows from (3.2) with  $s = n_t$  that  $s(K_{t-1}) \leq n_t$  implies (3.7). We consider the following three cases.

Case (1).  $s(K_{t-1}) > n_t$ : The group surplus is zero, independent of the number of participants, and thus no group is formed in the group negotiation stage by Proposition 3.1. In this case, any action profile in the participation stage is a (non-strict) Nash equilibrium with the failure of group formation.

Case (2).  $s(K_{t-1}) = n_t$ : By the definition of the critical size  $s(K_{t-1})$ , note that  $w(n_t, K_{t-1}) > 0$  and  $w(s, K_{t-1}) = 0$  for all  $s \leq n_t - 1$ . We show that the group of all individuals is a Nash equilibrium. If all individuals participate in a group, then the group is formed in the negotiation stage by Proposition 3.1. If any one does not participate, then the remaining group of  $n_t - 1$  individuals is not formed. Thus, the payoff of the non-participant decreases from  $u_p(n_t, K_{t-1})$  to  $u(0, K_{t-1})$ . Any other group with size  $s \leq n_t - 1$  is not formed in equilibrium since  $w(s, K_{t-1}) = 0$ . Any action profile in which there exist at least two non-participants is a (non-strict) Nash equilibrium with the failure of group formation.

Case (3).  $s(K_{t-1}) < n_t$ : Consider two subcases, (3a)  $u_p(n_t) > u_{np}(n_t - 1)$ , and (3b)  $u_p(n_t) \leq u_{np}(n_t - 1)$ . In subcase (3a), we show that  $n_t$  is a unique equilibrium group size. It can be easily seen that the group of all individuals is a Nash equilibrium since if any participant opts out, then he is worse off. Since  $f(s) \equiv u_{np}(s - 1) - u_p(s)$  is an increasing function of  $s$ , we have  $u_p(s) > u_{np}(s - 1)$  for every  $s(K_{t-1}) \leq s \leq n_t$ . This implies that any group of size less than  $n_t$  is not a Nash equilibrium since any outsider wants to participate. Consider subcase (3b). If  $u_p(s) < u_{np}(s - 1)$  for all  $s(K_{t-1}) + 1 \leq s \leq n_t$ , it is easy to show that  $s(K_{t-1})$  is a unique equilibrium group size. Next, assume that  $u_p(s) \geq u_{np}(s - 1)$  for some  $s(K_{t-1}) + 1 \leq s \leq n_t$ . Let  $s^*$  be the maximum integer of such an  $s$ . Since  $f(s)$  is an increasing function of  $s$ , we have  $u_p(s) < u_{np}(s - 1)$  for all  $s^* < s \leq n_t$ , and  $u_p(s) > u_{np}(s - 1)$  for all  $s(K_{t-1}) + 1 \leq s < s^*$ . The first inequality implies that any group with more than  $s^*$  participants is not a Nash equilibrium. The second inequality implies that any group less than  $s^* - 1$  participants is not a Nash equilibrium. Since  $u_p(s^* + 1) < u_{np}(s^*)$  and  $u_p(s^*) \geq u_{np}(s^* - 1)$ , a group with  $s^*$  participants is a Nash

equilibrium. Finally, if  $u_p(s^*) > u_{np}(s^* - 1)$ , then a group with  $s^* - 1$  participants is not a Nash equilibrium. If  $u_p(s^*) = u_{np}(s^* - 1)$ , then a group with  $s^* - 1$  participants is also a Nash equilibrium. Q.E.D.

The logic of group formation shown by Theorem 3.2 is intuitive. Since the surplus of an equilibrium group should be positive, the equilibrium group size is greater than or equal to the critical size  $s(K_{t-1})$ . It is clear that the largest group with all individuals is a unique equilibrium when population  $n_t$  is equal to the critical size. There exists the other case that the largest group is an equilibrium. If any participant is not better off by deviating from the group (formally,  $u_p(n_t) \geq u_{np}(n_t - 1)$ ), then the largest group is an equilibrium. One may wonder why this is possible in a situation that zero-contribution is the dominant action of every individual. A key observation is that the remaining participants can adjust and decrease their contribution levels in the two-stage process of group formation, triggered by the deviation of one participant. This counter-response by other participants may damage the deviator. Depending on the payoffs of participants and of non-participants, the equilibrium size may not be the largest one, but may be either the critical size or an intermediate one between the critical size and the population.

The condition (3.7) shows that the group formation for public goods provision depends on population ( $n_t$ ), preference weight of public goods relative to private goods ( $\frac{1-\alpha}{\alpha}$ ), endowment ( $\omega$ ), marginal productivity of private goods ( $\beta$ ), depreciation rate of public goods ( $\delta$ ), and stock of public goods ( $K_{t-1}$ ). The likelihood of group formation increases as the first five parameters increase and as the stock of public goods decreases. In the analysis, we assume that preference, endowment, production technology and depreciation rate of stock are fixed, but population and the stock of public goods change over generations. Our main interest is in examining how population change and capital accumulation affect the provision of public goods. The effects of population ( $n_t$ ) and of capital stock



( $K_{t-1}$ ) on the group formation can be explained as follows. First, given  $K_{t-1}$ , population  $n_t$  should be so large that it exceeds the critical size  $s(K_{t-1})$  of a group. If population is not enough, there exists no group with positive surplus, and thus individuals do not form any group voluntarily. Second, given  $n_t$ , the stock  $K_{t-1}$  of public goods should be less than a certain level. If the stock of public goods is affluent, individuals prefer to consume their resources rather than to invest them to public goods. In such an affluent economy, there is no “public goods problem.”

Two remarks on Theorem 3.2 are in order. First, given a stock of public goods, the theorem complements the result of a voluntary participation game studied in previous literature (Palfrey and Rosenthal (1984), Okada (1993), Saijo and Yamato (1999) and Dixit and Olson (2000) among others) that all individuals do not participate in a group (or a mechanism) to provide public goods, and that the likelihood of all individuals’ participation becomes lower as population becomes larger. In particular, the theorem shows the following effect of a large population on the full participation. Given the stock  $K_{t-1}$  of public goods, the condition  $n_t = s(K_{t-1})$  is easily violated as  $n_t$  becomes large. The other condition  $u_p(n_t) \geq u_{np}(n_t - 1)$  means

$$\left(\frac{1}{\alpha}\right)^\alpha \frac{\beta(n-1)\omega + (1-\delta)K_{t-1}}{\beta n\omega + (1-\delta)K_{t-1}} \left(\frac{\beta n\omega}{\beta(n-1)\omega + (1-\delta)K_{t-1}}\right)^\alpha \leq 1.$$

As  $n_t$  goes to infinity, the LHS of the inequality above converges to  $(\frac{1}{\alpha})^\alpha$ , which is larger than 1 for any  $0 < \alpha < 1$ . Therefore, the condition does not hold in the limit that  $n_t$  goes to infinity. This means that when population  $n_t$  becomes large, the class of the Cobb-Douglas utility functions under which the full participation is an equilibrium is negligible.

Second, one may argue that a different rule of the participation game from ours leads to the result that all individuals participate. Probably, one of the simplest rules of this kind is that in the first stage, every individual makes a conditional statement, “I will

participate if and only if all others also participate.” If such a commitment were credible, every individual would recognize that if he does not participate, then he does not enjoy free-riding benefits since any group is not formed, and thus would participate. However, this type of prior commitments should be made in negotiations among individuals (in the group negotiation stage in our two-stage game). Individuals should make the decision to participate or not before negotiations start.<sup>10</sup> More generally, if anyone could design a game in which all individuals participate in a group to provide public goods, such a game itself is a public good, and every individual has an incentive to free ride. We now come back to the start of our analysis!

In the next section, we will consider how the group formation for public goods changes when the stock of public goods is accumulated and population changes over generations.

## 4 The dynamics of accumulation

We investigate how public goods can be accumulated when the group formation game is played over generations. It follows from (2.1) that the magnitude of accumulation in each generation is determined by the total contribution of the equilibrium group. Theorem 3.2 shows that the equilibrium group is formed if population  $n_t$  exceeds the critical size  $s(K_{t-1})$  of a group at  $K_{t-1}$ . Recall that  $s(K_{t-1})$  is the smallest integer that the group surplus  $w(K_{t-1}, s)$  per member is positive (see (3.5)). Let  $s^*(K_{t-1})$  be the equilibrium size of the group at  $K_{t-1}$ . It holds that  $s(K_{t-1}) \leq s^*(K_{t-1})$ .

Let  $g(K_{t-1})$  be the total contribution of the equilibrium group when the public good

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<sup>10</sup>See Dixit and Olson (2000, p. 313) for related arguments.

stock is  $K_{t-1}$ . By (3.1), it holds that

$$g(K_{t-1}) = \frac{1}{\beta}[(1 - \alpha)\beta\omega \cdot s^*(K_{t-1}) - \alpha(1 - \delta)K_{t-1}]. \quad (4.1)$$

By (2.1), the dynamics of public good stock  $K_{t-1}$  is given by

$$\begin{aligned} K_t &= (1 - \delta)K_{t-1} + \beta g(K_{t-1}) \quad \text{if } s(K_{t-1}) \leq n_t \\ &= (1 - \delta)K_{t-1} \quad \text{otherwise.} \end{aligned} \quad (4.2)$$

We formulate the population change by

$$n_{t+1} = n_t + \Delta n_t. \quad (4.3)$$

When  $\Delta n_t > 0$ , population increases, and when  $\Delta n_t < 0$ , population decreases. When  $\Delta n_t = 0$ , there is no population change.

When a group is formed (i.e.,  $s(K_{t-1}) \leq n_t$ ), the dynamic system (4.2) has the following property. Let

$$K^*(s) = \frac{\beta\omega}{1 - (1 - \alpha)(1 - \delta)}s. \quad (4.4)$$

When  $K_{t-1} < K^*(s)$  for  $s = s^*(K_{t-1})$ , the stock  $K_{t-1}$  increases to  $K_t < K^*(s)$ , and when  $K_{t-1} > K^*(s)$  for  $s = s^*(K_{t-1})$ , the stock  $K_{t-1}$  decreases to  $K_t > K^*(s)$ . In either case, the stock  $K_t$  monotonically converges to  $K^*(s)$  as  $t$  goes to infinity, provided that the group size were fixed at  $s$  over generations.

To analyze the dynamics of the public good stock, we define the *feasible region of cooperation*, denoted by  $F$ , in the  $(K_{t-1}, n_t)$ -plane as

$$F = \{(K_{t-1}, n_t) \in R_+^2 \mid (K_{t-1}, n_t) \text{ satisfies (3.7)}\}.$$

We are now ready to characterize dynamic patterns of the accumulation of public goods. The dynamics critically depends on the feasible region  $F$  of cooperation and on the function  $K^*(s)$  defined by (4.4). To examine purely the relationship between the accumulation and the group formation, we first consider the dynamics of the public goods stock under no population change ( $\Delta n_t = 0$ ).

**Theorem 4.1.** Suppose that  $n_t = n$  for all  $t$ . If

$$0 < \delta < \frac{\alpha^2}{1 - \alpha + \alpha^2}, \quad (4.5)$$

then, starting from any initial state  $(K_0, n) \in F$  satisfying (2.3), the public goods stock  $K_t$  ( $t = 1, 2, \dots$ ) monotonically increases over generations, and reaches the level at which all individuals participate in a group. Thereafter, the largest group continues to form as long as  $(K_t, n)$  belongs to the feasible region  $F$  of cooperation.

**Proof.** With help of (3.6), we divide the set of  $K_{t-1}$  with  $(K_{t-1}, n) \in F$  into  $n$  intervals  $I_s = [K(s-1), K(s))$ ,  $s = 1, \dots, n$ , where  $K(s) = \frac{(1-\alpha)\beta\omega}{\alpha(1-\delta)}s$ . It follows from (3.6) that the critical size of a group at  $K_{t-1} \in I_s$  is  $s$ . It can be seen without much difficulty that (4.5) is equivalent to

$$\frac{(1-\alpha)\beta\omega}{\alpha(1-\delta)} < \frac{\beta\omega}{1 - (1-\alpha)(1-\delta)},$$

which implies  $K(s) < K^*(s)$  for all  $s$  (see (4.4)). Suppose that  $K_{t-1} \in I_s$ . Then,  $K_{t-1} < K(s) \leq K(m) < K^*(m)$  for all  $s \leq m \leq n$ . By Theorem 3.2, the equilibrium group size  $m = s^*(K_{t-1})$  at  $K_{t-1} \in I_s$  satisfies  $s \leq m \leq n$ . The dynamic system (4.2) has the property that if  $K_{t-1} < K^*(m)$  and a group with size  $m$  is formed, then  $K_{t-1} < K_t$ . Therefore, when (4.5) holds, the stock  $K_t$  of public goods monotonically increases whenever an equilibrium group is formed to contribute to the provision of public goods.

We will show that if the stock  $K_{t-1}$  belongs to some interval  $I_m$  with  $m \leq n - 1$ , then  $(K_t, n) \in F$ . In other words, we will show that the sequence  $\{K_t\}$  never jumps the last interval  $I_n$  to go outside the feasible region  $F$ . Since  $K_t - K_{t-1}$  is maximized when the largest group is formed, it is sufficient for us to assume that the largest group is formed at  $K_{t-1}$ .  $s(K_{t-1}) \leq n - 1$  means that

$$K_{t-1} < \frac{(1 - \alpha)\beta\omega}{\alpha(1 - \delta)}(n - 1),$$

which implies

$$(1 - \delta)K_{t-1} \leq \left(\frac{1}{\alpha(1 - \delta)} - 1\right)\beta n\omega$$

(note that  $\frac{1 - \alpha}{\alpha} \frac{n - 1}{n} \leq \frac{1}{\alpha(1 - \delta)} - 1$ ). The last inequality and (4.1) implies that

$$K_t = (1 - \delta)K_{t-1} + (1 - \alpha)\beta n\omega - \alpha(1 - \delta)K_{t-1} \leq \frac{(1 - \alpha)\beta\omega}{\alpha(1 - \delta)}n.$$

That is,  $(K_t, n) \in F$ .

Finally, we will show that there exists some  $t$  such that  $K_{t-1} \in I_n$ , which means that the largest group with size  $n$  necessarily forms at  $K_{t-1}$ . On the contrary, suppose that  $s(K_t) < n$  for all  $t$ . Since  $\{K_t\}$  is an increasing sequence under (4.5) as we have shown, it must hold that there exists some  $m \leq n - 1$  such that  $K_t \in I_m$  for almost all  $t$ . The equilibrium group size at  $K_t \in I_m$  is greater than or equal to  $m$ . Since  $K(m) < K^*(m)$ , it follows from the dynamic property of (4.2) that we must have  $K(m) < K_t$  for any sufficiently large  $t$ . This contradicts that  $K_t \in I_m$  for almost all  $t$ . Q.E.D.

The theorem shows that when the depreciation rate  $\delta$  is so small that the contribution of a group can outweigh the depreciation of the public goods, the public goods stock monotonically increases through voluntary formation of groups over generations, and that it eventually reaches the level at which all individuals participate in a group. When public

goods are accumulated sufficiently, all individuals necessarily participate in a group to provide public goods. This effect of the accumulation on the group formation can not be captured well by the one-shot model without accumulation studied in the literature.

The intuition of this result can be explained with help of the critical size of a group. It follows from (3.6) that the critical size of a group is equal to the number  $n$  of all individuals if and only if the stock  $K_t$  of public goods satisfies

$$\frac{(1-\alpha)}{\alpha} \frac{\beta\omega}{1-\delta} (n-1) \leq K_t < \frac{(1-\alpha)}{\alpha} \frac{\beta\omega}{1-\delta} n. \quad (4.6)$$

When the stock  $K_t$  is accumulated owing to contributions by generations, (4.6) can be satisfied as the economy is repeated over generations. In an economy with a high stock of public goods, individuals can enjoy high welfare without a new amount of public goods. In other words, the opportunity costs of a group are high in the economy with a high stock. In this case, only the group of all individuals can produce a positive surplus, and thus the largest group is formed in equilibrium. The largest group continues to be formed as long as it can produce a positive surplus, namely, (4.6) is satisfied.

When population  $n$  is fixed, there exists an upper limit of the stock of public goods, which is given by  $K(n) = \frac{(1-\alpha)\beta\omega}{\alpha(1-\delta)}n$ . If the stock exceeds this limit, there is no “public goods problem” in the sense that any collective contribution does not increase the welfare of individuals. We call  $K(n)$  the *upper limit function of public goods* for population  $n$ . When the stock of public goods exceeds  $K(n)$ , no group is formed to provide public goods.

So far we have examined a pure effect of the accumulation of public goods on the group formation for voluntary provision, assuming no change of population. Finally, we will analyze how the public goods are accumulated through voluntary groups in a dynamic economy with population change. We first remark that the upper limit function  $K(n) = \frac{(1-\alpha)\beta\omega}{\alpha(1-\delta)}n$  of public goods is increasing in population  $n$ . Thus, when population grows, the upper limit of public goods is also expanded. In this sense, the population

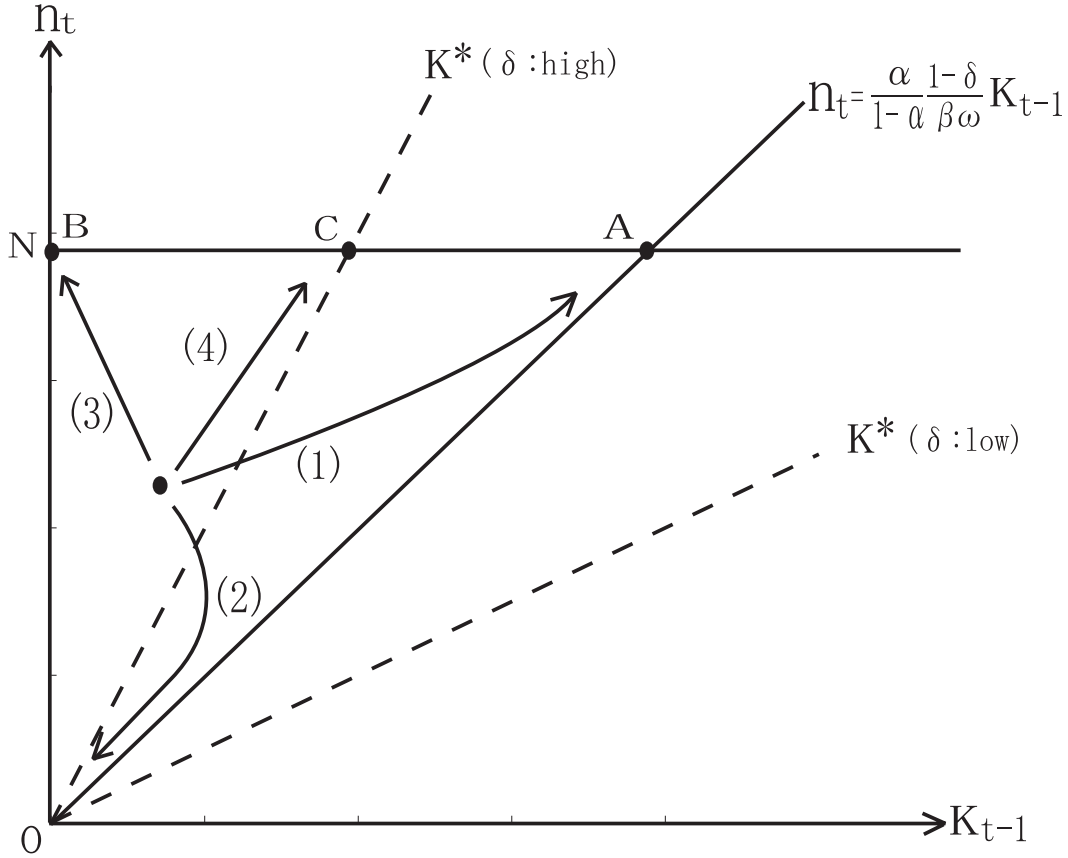


Figure 4.1 Accumulation of public goods

growth has a positive effect on the accumulation of public goods.

In the following, we will illustrate four basic patterns of the public goods accumulation with help of the phase diagram of  $(K_{t-1}, n_t)$  (Figure 4.1).

Case (1): population increase with group formation

Suppose that population  $n_t$  monotonically increases and converges to some capacity  $N$ . The capacity of population is determined by physical, economic and environmental conditions. In this case, the accumulation process is typically like path (1) illustrated in Figure 4.1 and it converges to point  $A$ . More precisely, if the pair  $(K_{t-1}, n_t)$  goes outside the feasible region  $F$  during the process, then  $(K_{t-1}, n_t)$  moves to the north-west direction since  $K_{t-1}$  decreases due to no group formation. After some number of

generations,  $(K_{t-1}, n_t)$  moves in  $F$ , and thereafter the accumulation process starts again. Repeating this process,  $(K_{t-1}, n_t)$  converges to point  $A$  in the long run.

Case (2): population decrease with group formation

Suppose that population  $n_t$  monotonically decreases and converges to zero. In this case, the accumulation process is typically like path (2) illustrated in Figure 4.1 and converges to point  $O$ . The population decrease damages severely the accumulation of public goods.

Case (3): population increase without group formation

Consider the same pattern of population increase as in case (1). Unlike our analysis so far, assume that no group can establish a mechanism to implement the efficient provision of public goods, due to high costs. In this case, no positive contributions can be made over generations, and thus the accumulation process is given by path (3) in Figure 4.1 and converges to point  $B$  with zero stock of public goods.

Case (4): high depreciation rates with group formation

Suppose that the depreciation rate  $\delta$  is so high that (4.5) in Theorem 4.1 does not hold. In this case, the line  $K_{t-1} = K^*(n_t)$  (illustrated as the dotted line in Figure 4.1) is in the feasible region  $F$  of cooperation. In the right area of this line, the stock  $K_t$  decreases. A typical pattern of accumulation is path (4) which converges to an intermediate point  $C$ . Compared to case (1), the accumulation of public goods in the long run is inefficient, due to the high depreciation.

The phase diagram analysis in Figure 4.1 shows that the accumulation of public goods depends on the three factors, (1) population, (2) institutional conditions, and (3) technology to produce and maintain public goods. The institutional conditions in an economy determine whether or not individuals can have the opportunity to form a group, and whether or not the group can establish a suitable mechanism to implement an efficient provision of public goods. In the model, the production technology of public goods is



described by parameter  $\beta$  and the maintenance technology by parameter  $\delta$ . When population increases and the institutional conditions are favorable for group formation, the accumulation of public goods converges to the efficient level in the long run (point A in case (1)). The efficient level increases as production technology ( $\beta$ ) does. However, if either of the two conditions is not satisfied, the level of public goods declines in the long run (cases (2) and (3)). In particular, as Figure 4.1 shows, depopulation (case 2) damages the provision of public goods severely. When the maintenance technology ( $1 - \delta$ ) is low, the group contributions may not be sufficient to increase the stock of public goods. In case (4), the long-run accumulation does not reach the efficient level.

## 5 Discussion

### 5.1 Capital Accumulation Mitigates the Second-Order Dilemma

As we have stated in the introduction, the second-order dilemma of public goods describes individuals' incentive to free-ride on a mechanism which can implement an efficient provision of public goods. The dilemma is derived by the argument that any mechanism itself to solve the (first-order) dilemma of public goods is a kind of public goods. Due to the second-order dilemma, the effectiveness of the institutional approach to solve the provision problem of public goods has received some doubt in the literature since Parsons (1937, p.89-94), at least on a theoretical level. Parsons raised a logical inconsistency in the arguments of Hobbes on the Leviathan. If individuals are rational egoists, then why would they act in the collective interest by establishing a coercive state? For more recent criticisms to the institutional approach, see Bates (1988) and Dixit and Olson (2000), etc. In his critique to the "new institutionalism," Bates (1984) contends that an institution would provide a collective good and rational individuals would seek to secure its benefits

for free. The proposed institution is subject to the very incentive problems it is supposed to resolve. In the literature, it has been argued even that the second-order dilemma is not easier to solve than the first-order dilemma. Although not as pessimistic as these critics, the recent works on voluntary participation games (Palfrey and Rosenthal (1984), Okada (1993), Saijo and Yamato (1999) and Dixit and Olson (2000)) may be considered to provide a partial support for the negative view on the institutional approach to solve the second-order dilemma. It has been shown that if individuals have the opportunity to decide freely whether or not they should participate in a group (or mechanism) to provide public goods, then they all do not participate due to the incentive of free-riding. The roles of trust, norm, community, and social sanctions to solve the second-order dilemma has been discussed extensively in the literature.

In this paper, we have shown, without relying on social and behavioral arguments, that the accumulation of public goods can mitigate the second-order dilemma. A positive amount of contributions, even if not efficient, can be made by self-interested individuals if they have the opportunity to organize a group to provide public goods. Thus, the accumulation of public goods is possible. If the depreciation rate of public goods is not high, the accumulation changes the degree of individuals' incentive to free ride, and it can lead to the full participation of individuals in the long run. From studies on several cases which illustrate how people solve the second-order dilemma in real common-pool resource problems, Ostrom (1990) observed that the investment in institutional change occurred in an incremental and sequential process. The accumulation process of public goods may be considered as a part of the incremental and sequential process to solve the second-order dilemma.

## 5.2 Depopulation Damages the Provision of Public Goods

In Section 4, we have considered the effects of population change on the accumulation of public goods. In particular, we have shown that population decrease damages the accumulation of public goods severely (see Figure 4.1). A serious problem of depopulation happens in many industrialized countries. In these countries, population moves from rural to urban areas.

The decline of agricultural labor force causes agricultural abandonment and the decline of common resources in rural areas. MacDonald and Crabtree etc. (2000) review twenty-four case studies in European mountain areas and examine environmental impacts of agricultural abandonment and decline in traditional farming practices caused by rural depopulation. They find that abandonment is widespread and that it has an undesirable effect on biodiversity, landscape and soils including natural hazards such as soil erosion and landslides.

The depopulation in rural areas damages not only the maintenance of common resources but also the participation of various voluntary works. The depopulation started in Japan in the late of 1950. As stated in the introduction, population in rural areas of Japan decreases from 55 million in 1955 to 44 million in 2000 while the whole population increased from 90 million to 127 million for the same period (FAO, 2005). We discuss a case of Volunteer Fire Corps in Japan as an example of declining voluntary works (FDMA, 2005). Fire defence forces in Japan are composed of permanent fire defence professionals and members of Volunteer Fire Corps. Fire defence professionals are employed by cities, towns and villages as dedicated personnel as fire services. The number of professionals are about 155,000 in 2005. Members of Volunteer Fire Corps have their own occupations as a living, and voluntarily participate in Corps to devote themselves to fire services. The number of Volunteer Fire Corps is about 3,600 and their members are about 928,000 in the whole country. The members' activities are wide, including fire-fighting activities,

lifesaving and rescue activity, patrolling and guidance for evacuation in natural disaster, and also the organization of first-aid classes, fire prevention instruction to residents and public relations activities. The members participate in physical training in their free time, mostly on weekends. Although the members are employed as part-time municipal officers, the character of their work is voluntary. A regular-class member receives only 36,000 yen (approximately \$300) annually and is paid 6,900 yen (\$58) each fire-fighting work. A regular-class member who has engaged in Corps for 30 years receives about 640,000 yen (\$5333) at retirement. Due to depopulation in rural areas and changes of working styles in urban areas, the total number of members in Corps has decreased drastically since the middle of 1950, and it becomes almost half in 2005 compared to in 1956. Most Corps have now difficulty to find new members.

## 6 Conclusion

We have considered how public goods can be provided and accumulated over generations of self-interested individuals in the absence of a government authority. By a dynamic voluntary participation game with accumulation and population change, we have shown that the accumulation of public goods can mitigate the second-order dilemma of public goods and that it can attain an efficient provision of public goods in the long run. Depopulation damages the provision of public goods severely.

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