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IN COINTEGRATED VECTOR  
AUTOREGRESSIONS

by

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# THE GRANGER NON-CAUSALITY TEST IN COINTEGRATED VECTOR AUTOREGRESSIONS

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## ABSTRACT

The usual Wald test for the Granger non-causality in cointegrated vector autoregressive (VAR) processes is known to have the asymptotically non-standard distribution. There have been proposed a few alternative (inefficient) methods which give the asymptotically standard distribution. However, their power of test is relatively low because of their inefficiency. In this paper, we propose a procedure that always gives the asymptotically standard distribution with higher power of test than the alternative ones, by suitably combining the usual Wald test and the alternative ones, based upon the certain rank condition of the submatrix of the cointegration matrix. We also propose a simple modifications to the test statistics in order to obtain reasonable empirical size of test in finite samples. The relevant Monte Carlo experiment reveals that our proposed procedure works favorably in finite samples in comparison with conventional tests.

# 1 INTRODUCTION

The Granger non-causality test provides a particular summary of the forecasting relation between two subsets of the variables. In vector autoregressive (VAR) process, it is based on the least squares prediction. The Wald test for the Granger non-causality is known to follow usual manner in stationary systems.

However, such a test is typically more complicated in cointegrated systems. See, for example, Sims, Stock, and Watson (1990), Park and Phillips (1989), Toda and Phillips (1993). Among their conclusions are that the usual Wald test statistic for Granger non-causality in levels VAR's regression (hereafter, the Std-VAR approach) may have a non-standard asymptotic distribution and possibly depends on nuisance parameters. The test based upon the non-standard distribution is very difficult, if not impossible, to use in practice. In terms of Johansen's (1988) vector error correction (VEC) models format, Toda and Phillips (1994) proposed sequential testing procedures for three special cases, that is, causalities from one variable to a set of variables, from a set of variables to one variable, and from one variable to one variable, respectively. However, their method seems difficult to be extended to a test for a causality between sets of variables. They have not dealt with the testing procedure in levels VAR's format. The present paper is concerned with a testing procedure for such a general case in levels VAR's format.

There have been proposed a few testing procedures that give an asymptotically standard distribution. See, for example, the fully modified VAR (FM-VAR) approach by Phillips (1995) and the lag augmented VAR (LA-VAR) approach by Toda and Yamamoto (1995). The FM-VAR approach removes the asymptotic bias of the OLS estimator and the asymptotic distribution of the Wald statistic is shown to be bounded from above by a chi-square distribution. The LA-VAR approach is the OLS estimation of a VAR model with an artificially augmented lag, and the Wald statistic, which is based upon it, is known to be always asymptotically chi-square distributed.

However, they have respective drawbacks. In appropriate small sample experiments, Yamada and Toda (1998) showed that the test based upon the FM-VAR approach results in severe size distortion of the test, and thus it appears very difficult to use it in practice with reasonable reliability, and we will not be concerned with it in the paper. They also showed that the one based upon the LA-VAR approach exhibits a relatively weak power of the test because of its inefficiency due to an artificially augmented lag in the

regression, while its empirical size of the test is acceptable. We will be concerned with the improvement of the power of the LA-VAR approach in the paper.

The present paper proposes sequential testing procedures for the Granger non-causality in levels VAR's regression. It is a suitable combination of the LA-VAR approach and the Std-VAR approach. Theorem 1 of Toda and Phillips (1993) gives a certain rank condition on a sub-matrix of a cointegrating matrix for the Wald statistic in the levels VAR's to be asymptotically chi-square distributed. If the condition is satisfied, we should use the Std-VAR approach rather than the LA-VAR approach, since the former is obviously asymptotically more efficient. Dolado and Lutkepohl (1996) showed in small sample experiments with a simple 2-variate VAR model that the empirical power of the test of the Std-VAR approach can be significantly greater than that of the LA-VAR approach in some cases, when the rank condition is satisfied. On the other hand, if the rank condition is not satisfied, we should use the LA-VAR approach, since the Std-VAR approach cannot be used in this case because its Wald statistic has a non-standard asymptotic distribution and cannot be properly tested. Obviously, the above testing procedure should be more powerful than the test solely based upon the LA-VAR approach, since it adopts the Std-VAR approach whenever it is applicable.

We note that, when the rank condition is not satisfied, it corresponds to a situation where the relevant matrix in the Wald test statistic is degenerate. The LA-VAR approach is an easy way to circumvent the degeneracy. Needless to say, a generalized inverse (GI) procedure is a usual practice for inverting a degenerate matrix. See, for example, Lutkepohl and Burda (1977). in stationary VAR systems. Thus, when the rank condition is not satisfied, the GI-VAR approach can be used as an alternative to the LA-VAR approach. In order to use the GI-VAR procedure in practice we need to know the exact rank of a degenerate matrix. When it is applicable, the GI-VAR approach may be more powerful than the LA-VAR approach because it uses the rank information fully, while the LA-VAR approach only needs to know whether the relevant matrix is degenerate or not.

In the above procedures a test for a rank of a submatrix of a cointegrating matrix plays a key role. Here, we adopt a newly proposed test for it by Kurozumi (2003). Kurozumi's testing procedure is convenient in the sense its test statistic is also asymptotically chi-square distributed. In a companion paper, Yamamoto and Kurozumi (2003), Kurozumi's testing procedure also plays a key role in detecting the rank of a possibly degenerate

matrix in testing for the long-run Granger non-causality. We may note that, while the Wald test in the final stage in the sequential procedure is based upon levels VAR's format, we also need to estimate a VEC model in order to obtain the cointegrating matrix of the system concerned.

Further, we propose small sample modifications in test statistics of the three approaches in order to reduce their empirical size distortions. While the empirical size of the Wald test based upon the LA-VAR approach is generally acceptable as remarked above, the size distortions of the test based upon the Std-VAR approach and the GI-VAR approach can be sizable in some cases in finite samples. Kurozumi and Yamamoto (2000) proposed the bias correction method in order to reduce the size distortion in the LA-VAR approach. In this paper we propose further modifications in the estimated variance covariance matrix of the estimator in addition to the bias correction method. We may note that these modifications are designed to affect only small sample properties of test statistics, but not their asymptotic properties.

The remainder of the paper is organized as follows. Section 2 details a new sequential procedure for the Granger non-causality test in cointegrated VAR systems. Section 3 explains the small sample modifications in proposed test statistics that reduce empirical size distortions. Section 4 provides some Monte Carlo evidence about the finite sample behavior of our testing procedures relative to conventional testing procedures. Section 5 contains an empirical illustration of testing causality among long-term interest rates of four countries. Section 6 gives some concluding remarks.

A summary word on notation. We denote the rank of a matrix  $D$  by  $rank(D)$ . We use  $vec(D)$  to stack the *rows* of  $D$  into a column vector.  $\xrightarrow{p}$ , and  $\xrightarrow{d}$  signify convergence in probability, convergence in distribution, respectively.  $I_k$  denotes the identity matrix of rank  $k$ .

## 2 SEQUENTIAL GRANGER NON-CAUSALITY TEST

Consider  $m$ -vector process  $\{x = [x_i]\}$  generated by vector autoregressive (VAR) model of order  $p$ ,

$$A(L)x_t = \mu + \Theta D_t + \varepsilon_t, \tag{1}$$

where  $x_t = [x_{it}]$ ,  $\mu$  is the constant vector,  $A(L) = I_m - A_1L - \dots - A_pL^p$ ,  $L$  is the lag operator,  $\Theta$  is the  $m \times g$  coefficient matrix, and  $\{\varepsilon_t\}$  is a Gaussian white noise process with mean zero and nonsingular covariance matrix  $\Sigma_{\varepsilon\varepsilon}$ . The deterministic terms  $D_t$  can contain a linear time, seasonal dummies, intervention dummies, or other regressors that we consider fixed and non-stochastic. Following Johansen (1988,1991), we assume the following:

Assumption : *System (1) satisfies*

- (i)  $|A(z)| = 0$  has its all roots outside the unit circle or equal to 1.
- (ii)  $\Pi = \alpha\beta'$ , where  $\Pi = -A(1)$ ,  $\alpha$  and  $\beta$  are  $m \times r$  matrices of rank  $r$ ,  $0 < r < m$ , and  $\text{rank}\{\Pi\} = r$ . Without loss of generality, it will be assumed that  $\beta$  is orthonormal.
- (iii)  $\text{rank}\{\alpha'_\perp \Gamma \beta_\perp\} = m - r$ , where  $\alpha_\perp$  and  $\beta_\perp$  are  $m \times (m - r)$  matrices such that  $\alpha'_\perp \alpha = 0$ ,  $\beta'_\perp \beta = 0$ , and  $\Gamma = -(\partial A(z)/\partial z)_{z=1} - \Pi$ .

Under the above assumption, each component of  $\{x_t\}$  is I(1), and components of  $\{x_t\}$  are cointegrated with  $r$  cointegrating vectors  $\beta$ . Reparameterizing (1), we get Johansen's (1991) vector error correction (VEC) form of the process,

$$\Delta x_t = \mu + \alpha\beta'x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \Theta D_t + \varepsilon_t, \quad (2)$$

where  $\Gamma_j = -\sum_{i=j+1}^p A_i$  for  $j = 1, \dots, p-1$ .

Without loss of generality, we consider the case where the last  $p_2$  ( $p_2 \geq 1$ ) variables  $R_R^*x$  do not cause the first  $p_1$  ( $p_1 \geq 1$ ) variables  $R_Lx$ , where  $R_R^*$  and  $R_L$  are the choice matrices such that  $R_R^* = [0, I_{p_2}]$ , and  $R_L = [I_{p_1}, 0]$ . Let  $A = [A_1, A_2, \dots, A_p]$  in (1). The null hypothesis  $H_0^G$  that  $R_R^*x$  do not Granger cause  $R_Lx$  is given by

$$R_L A_i R_R^{*'} = 0 \quad (i = 1, 2, \dots, p)$$

or equivalently

$$R \text{vec}(A) = 0, \quad (3)$$

where  $R = R_L \otimes R_R$  and  $R_R = I_p \otimes R_R^*$ .

The least squares estimator of  $A$  in (1) is given by

$$\hat{A} = Y'X(X'X)^{-1}M, \quad (4)$$

where  $X' = [x_1^*, x_2^*, \dots, x_T^*]$ ,  $x_t^* = [x'_{t-1}, x'_{t-2}, \dots, x'_{t-p}, 1, D_t']'$ ,  $Y' = [x_1, x_2, \dots, x_T]$ ,  $M$  is the  $(mp + g + 1) \times mp$  choice matrix such that  $M = [I_{mp}, 0]'$ . We have the following result for  $\hat{A}$ .

*Theorem 1 : Let Assumption holds and let  $\hat{A}$  be the least squares estimator defined in (4). Then, we have*

$$\sqrt{T}vec(\hat{A} - A) \xrightarrow{d} N(0, \Sigma), \quad (5)$$

where  $\Sigma = \Sigma_{\varepsilon\varepsilon} \otimes Q\Sigma_{\xi\xi}^{-1}Q' = \Sigma_{\varepsilon\varepsilon} \otimes K'^{-1}G_\xi\Sigma_{\xi\xi}^{-1}G_\xi'K^{-1}$ ,  $\Sigma_{\xi\xi} = E(\xi_t\xi_t')$ ,  $\xi_t = [(\beta'x_{t-1})', \Delta x'_{t-1}, \dots, \Delta x'_{t-p+1}]'$ ,

$$K^{-1} = \begin{bmatrix} I_m & & & & & \\ I_m & -I_m & & & & \\ & & \cdot & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & 0 & & \cdot & \cdot & \\ & & & & I_m & -I_m \end{bmatrix}, \text{ and } G_\xi = \begin{bmatrix} \beta & 0 \\ 0 & I_{(p-1)m} \end{bmatrix}.$$

*Proof :* See Theorem 2.3 of Phillips (1998).

This theorem implies that, if  $R\Sigma R'$  is of full rank, a conventional Wald test statistic,  $W_0$ , for  $H_0^G$  in levels VAR's format, here termed as the Std-VAR approach, has a chi-square distribution with degrees of freedom equal to the number of restrictions as  $T$  grows:

$$W_0 = T\{Rvec(\hat{A})\}'(R\hat{\Sigma}R')^{-1}\{Rvec(\hat{A})\} \xrightarrow{d} \chi_{pp_1p_2}^2. \quad (6)$$

However,  $R\Sigma R'$  can be degenerate, as is well known. When  $R\Sigma R'$  is degenerate, the Wald statistic has an asymptotically non-standard distribution, and cannot be easily tested. See, for example, Sims, Stock, and Watson (1990) and Toda and Phillips (1993). Thus, the Std-VAR approach has not been used in practice in possibly cointegrated systems. For the rank of  $R\Sigma R'$ , we have the following.

*Proposition 1 : The rank of  $R\Sigma R'$  is given by*

$$rank(R\Sigma R') = p_1\{rank(R_R^*\beta) + (p - 1)p_2\} \quad (7)$$

*Proof :* See Appendix.

From the above proposition, we notice that  $R\Sigma R'$  is of full rank, if and only if  $rank(R\Sigma R') = pp_1p_2$ . The latter condition is an alternative expression of the rank condition,  $rank(R_R^*\beta) =$



$p_2$ , in Theorem 1 of Toda and Phillips (1993). See also a similar result in Yamamoto and Kurozumi (2003, Proposition 3).

When  $R\Sigma R'$  is degenerate, we propose to use the GI-VAR approach or the LA-VAR approach. The Wald statistics of these approaches are denoted as  $WG_0$  and  $WL_0$ , respectively, and their asymptotic distributions are given as follows:

$$WG_0 = T\{Rvec(\hat{A})\}'(R\hat{\Sigma}R')^{-}\{Rvec(\hat{A})\} \xrightarrow{d} \chi_s^2, \quad (8)$$

where  $(R\Sigma R')^{-}$  is the generalized inverse of  $R\Sigma R'$  and  $s$  is the rank of  $R\Sigma R'$ . See, for example, Rao and Mitra (1971, Th. 9.2.2).

$$WL_0 = T\{Rvec(\hat{A})\}'(R\hat{\Sigma}R')^{-1}\{Rvec(\hat{A})\} \xrightarrow{d} \chi_{pp_1p_2}^2, \quad (9)$$

where “ $\hat{\cdot}$ ” indicates LA-VAR estimates. The LA-VAR estimates are obtained by fitting a levels VAR model with an artificially augmented lag. It is known that  $WL_0$  gives an asymptotically chi-square distribution in possibly cointegrated systems. See Toda and Yamamoto (1995) for detail.

*Remark 1:* When  $p = 1$  and  $rank(R_R^*\beta) = 0$ , we have that  $rank(R\Sigma R') = 0$ . In this case we can check the Granger causality immediately without resorting to the statistical tests. In this case, we have

$$\begin{aligned} R_L A R_R &= R_L I_m R_R^{*'} + R_L \alpha \beta' R_R^{*'} \\ &= R_L I_m R_R^{*'} + 0. \end{aligned}$$

The second equality holds because of the fact that  $rank(R_R^*\beta) = 0$  means that  $R_R^*\beta = 0$ . Therefore, in this case,  $R_R^*x_t$  does not Granger cause  $R_L x_t$  when  $R_L R_R^{*'} = 0$ , and  $R_R^*x_t$  Granger causes  $R_L x_t$  otherwise.

The testing procedure proposed in this paper crucially depends upon how we detect the rank of  $R\Sigma R'$ , or more specifically that of  $R_R^*\beta$ . Here, we resort to a newly proposed testing procedure by Kurozumi (2003). He has developed a testing procedures for

$$H_0^s : rank(R_R^*\beta) = f \text{ v.s. } H_1^s : rank(R_R^*\beta) > f, \quad (10)$$

where  $0 \leq f < \min(p_2, r)$ . Then, we have

*Theorem 2:* Suppose that there is no trend but  $d \neq 0$  in the model (2). Let  $\hat{\mu}_1 \geq \hat{\mu}_2 \geq \dots \geq \hat{\mu}_{p_2}$  be the ordered characteristic roots of

$$\left| \hat{\beta}_1 \hat{\Psi} \hat{\beta}_1' - \hat{\mu} \hat{\Phi} \right| = 0, \quad (11)$$

where  $\hat{\beta}_1 = R_R^* \hat{\beta}$ ,  $\hat{\beta}_{\perp,1} = R_R^* \hat{\beta}_{\perp}$ .  $\hat{\Psi} = \hat{\alpha}' \hat{\Sigma}_{\varepsilon\varepsilon}^{-1} \hat{\alpha}$ ,  $\bar{\hat{\beta}}_{\perp} = \hat{\beta}_{\perp} (\hat{\beta}'_{\perp} \hat{\beta}_{\perp})^{-1}$ ,  $S_{11}^+ = T^{-1} \sum_{t=1}^T R_{1t} R'_{1t}$ ,  $R_{1t}$  being the regression residual of  $x_{t-1}^+$  on  $\Delta x_{t-1}, \dots, \Delta x_{t-p+1}$ ,  $x_{t-1}^+ = [x'_{t-1}, 1]'$ , “ $\hat{\cdot}$ ” indicates the sample estimate of the corresponding parameter,  $L$  and  $\Upsilon_T$  are  $(m-r+1) \times (m-r)$  and  $(m-r) \times (m-r+1)$  matrices defined by

$$L = \begin{bmatrix} I_{m-r} \\ 0 \end{bmatrix}, \quad \Upsilon_T = \begin{bmatrix} T^{-1/2} \bar{\hat{\beta}} & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{and}$$

$$\hat{\Phi} = \hat{\beta}_1 (\hat{\beta}'_1 \hat{\beta}_1)^{-1} \hat{\beta}'_1 + \hat{\beta}_{\perp,1} (\hat{\beta}'_{\perp,1} \hat{\beta}_{\perp,1})^{-1} L' (\Upsilon'_T S_{11}^+ \Upsilon_T)^{-1} L (\hat{\beta}'_{\perp,1} \hat{\beta}_{\perp,1})^{-1} \hat{\beta}_{\perp,1},$$

Then, under  $H_0^s$ , we have

$$\mathcal{L} = T^2 \sum_{i=f+1}^{p_2} \hat{\mu}_i \xrightarrow{d} \chi_{(p_2-f)(r-f)}. \quad (12)$$

*Proof:* See Theorem 3 in Kurozumi (2003).

The above theorem specifically concerns with the case where the constant term  $\mu$  in (2) is such that  $\mu = \alpha \rho_0$  where  $\rho_0$  is the  $r \times 1$  vector, and the model (2) can be specifically rewritten as

$$\Delta x_t = \alpha \beta^{+'} x_{t-1}^+ + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \Theta D_t + \varepsilon_t, \quad (13)$$

where  $\beta^+ = [\beta', \rho_0]'$ . This specification of  $\mu$  corresponds to an empirical application discussed in section 5. For different specifications of  $\mu$ , the test statistics should be slightly modified. See Kurozumi (2003) for detail.

The rank of  $R_R^* \beta$  is detected sequentially using the above procedure. For example, to decide the rank of  $R_R^* \beta$ , we firstly test the null of  $f = 0$ . If the null hypothesis is accepted, the rank of  $R_R^* \beta$  is found to be zero. Otherwise, we proceed to test the hypothesis of  $f = 1$ . We sequentially continue the process until the null hypothesis is accepted. When the null of  $f = \min(p_2, r) - 1$  is rejected, we consider that  $R_R^* \beta$  is of full rank.

In sum, the sequential testing procedure proposed in this paper consists of the following three steps:

**Step 1 :** Determine the cointegration rank  $r$  by the Johansen procedure (1988, 1991).

**Step 2 :** Given the cointegration rank  $r$ , determine the rank of  $R \Sigma R'$  by testing  $H_0^s$  by the Kurozumi procedure (2003).

**Step 3** : If  $R\Sigma R'$  is found to be of full rank, test  $H_0^G$  with  $W_0$  (Std-VAR approach).

Otherwise, test  $H_0^G$  with  $WG_0$  (GI-VAR approach) or with  $WL_0$  (LA-VAR approach).

The above combination of  $W_0$  and  $WL_0$  circumvents difficulty of  $W_0$  when  $R\Sigma R'$  is degenerate, and inefficiency of  $WL_0$  when  $R\Sigma R'$  is of full rank. A similar effect is expected for the combination of  $W_0$  and  $WG_0$ .

It may be noted that, while the hypothesis of the Granger non-causality is tested in terms of levels VAR's format in the final step, we need to estimate a VEC model in the first step in order to obtain the cointegrating vectors  $\beta$  of the process.

### 3 SMALL SAMPLE MODIFICATIONS OF TEST STATISTICS

It is well known that the Wald type test based upon time series regressions, say  $W_0$  in (6), usually has large size distortion in finite samples. That is, the empirical size can be significantly greater than the nominal size. In order to reduce the size distortion, we propose to apply a few modifications to the test statistics developed in the previous section. We first take up Kurozumi and Yamamoto's method (2000), which eliminates the quasi-asymptotic bias of the least squares estimator up to  $O_p(T^{-1})$  using the jackknife principle. While their method was developed for the LA-VAR approach, it is readily applicable to the Std-VAR approach, and reproduce it as the first modification in the paper.

*Theorem 3* : Suppose a sample size  $T$  is an even integer. Let the bias corrected estimator for  $A$  be

$$\hat{A}_m = 2\hat{A} - \frac{1}{2}(\hat{A}_1 + \hat{A}_2), \quad (14)$$

where  $\hat{A}_1$  and  $\hat{A}_2$  are least squares estimators based on sample of the 1st period ( $t = 1, \dots, T/2$ ) and the 2nd period ( $t = T/2 + 1, \dots, T$ ), respectively. Then,

- (i)  $\hat{A}_m$  has no quasi-asymptotic bias irrespective of the order of integration of  $\{x_t\}$ , and
- (ii) The asymptotic distribution of  $\hat{A}_m$  is normal irrespective of the order of integration of  $\{x_t\}$ .

*Proof*: See Kurozumi and Yamamoto (2000).

Furthermore, following Kurozumi and Yamamoto (2000), it can be shown that, when  $R\Sigma R'$  is of full rank, the modified Wald statistic, here denoted as  $W_a$ , constructed from the bias corrected estimator  $\hat{A}_m$ , has an asymptotic chi-square distribution with degrees of freedom equal to the number of restrictions.

$$W_a = T\{Rvec(\hat{A}_m)\}'(R\hat{\Sigma}_a R')^{-1}\{Rvec(\hat{A}_m)\} \xrightarrow{d} \chi_{pp_1 p_2}^2, \quad (15)$$

where

$$\begin{aligned} \hat{\Sigma}_a &= 4\hat{\Sigma}_{\varepsilon\varepsilon} \otimes M'(X'X)^{-1}M + \frac{1}{4}\{\hat{\Sigma}_{\varepsilon\varepsilon,1} \otimes M'(X'_1 X_1)^{-1}M + \hat{\Sigma}_{\varepsilon\varepsilon,2} \otimes M'(X'_2 X_2)^{-1}M\} \\ &\quad - 2(\hat{\Sigma}_{\varepsilon\varepsilon,1} + \hat{\Sigma}_{\varepsilon\varepsilon,2}) \otimes M'(X'X)^{-1}M, \\ \hat{\Sigma}_{\varepsilon\varepsilon} &= \frac{1}{T} \sum_{t=1}^T \hat{e}_t \hat{e}'_t, \quad \hat{\Sigma}_{\varepsilon\varepsilon,1} = \frac{2}{T} \sum_{t=1}^{T/2} \hat{e}_{1t} \hat{e}'_{1t}, \quad \hat{\Sigma}_{\varepsilon\varepsilon,2} = \frac{2}{T} \sum_{t=1}^{T/2} \hat{e}_{2t} \hat{e}'_{2t}, \end{aligned}$$

$X_1$  and  $X_2$  are regressor matrices for the whole period, the 1st period and the 2nd period, respectively, such that  $X'_1 = [x_1^*, x_2^*, \dots, x_{T/2}^*]$ ,  $X'_2 = [x_{T/2+1}^*, x_{T/2+2}^*, \dots, x_T^*]$ , and  $\hat{e}_t$ ,  $\hat{e}_{1t}$  and  $\hat{e}_{2t}$  are residuals from regressions in each period.

The above statistic  $W_a$  has been known to reduce the size distortion to some degrees, but there still remains a room for improvement, as can be seen in the experiments in the next section. Here, we propose two additional modifications. The basic idea is to slightly inflate the estimate of the variance-covariance matrix  $\Sigma$ , and it in turn slightly reduces the value of Wald statistic. Because the formula we use for the variance-covariance matrix is based upon the asymptotic theory and it presumably underestimates the true (unknown) one in finite samples. The second modified Wald statistic, denoted as  $W_b$ , is  $W_a$  with  $\hat{\Sigma}_a$  being replaced by  $\hat{\Sigma}_b$  where

$$\begin{aligned} \hat{\Sigma}_b &= 4\tilde{\Sigma}_{\varepsilon\varepsilon} \otimes M'(X'X)^{-1}M + \frac{1}{4}\{\tilde{\Sigma}_{\varepsilon\varepsilon,1} \otimes M'(X'_1 X_1)^{-1}M + \tilde{\Sigma}_{\varepsilon\varepsilon,2} \otimes M'(X'_2 X_2)^{-1}M\} \\ &\quad - 2(\tilde{\Sigma}_{\varepsilon\varepsilon,1} + \tilde{\Sigma}_{\varepsilon\varepsilon,2}) \otimes M'(X'X)^{-1}M, \end{aligned} \quad (16)$$

where

$$\tilde{\Sigma}_{\varepsilon\varepsilon} = \frac{1}{T} \sum_{t=1}^T \tilde{e}_t \tilde{e}'_t, \quad \tilde{\Sigma}_{\varepsilon\varepsilon,1} = \frac{2}{T} \sum_{t=1}^{T/2} \tilde{e}_{1t} \tilde{e}'_{1t}, \quad \tilde{\Sigma}_{\varepsilon\varepsilon,2} = \frac{2}{T} \sum_{t=1}^{T/2} \tilde{e}_{2t} \tilde{e}'_{2t},$$

and  $\tilde{e}_t$ ,  $\tilde{e}_{1t}$  and  $\tilde{e}_{2t}$  are residuals of regressions in each period uniformly evaluated with  $\hat{A}_m$  instead of  $\hat{A}$ ,  $\hat{A}_1$ , and  $\hat{A}_2$ , respectively.

Since the formulae  $\hat{\Sigma}_a$  and  $\hat{\Sigma}_b$  in the above two modifications are a little complicated, we propose the third one that is simpler, but still keeps a spirit of slightly inflating  $\hat{\Sigma}$ . It is denoted as  $W_c$ , and is  $W_a$  with  $\hat{\Sigma}_a$  being now replaced by  $\hat{\Sigma}_c$  where

$$\hat{\Sigma}_c = \tilde{\Sigma}_{\varepsilon\varepsilon} \otimes M'(X'X)^{-1}M + \text{vec}(\hat{A}_m - \hat{A})\{\text{vec}(\hat{A}_m - \hat{A})\}'. \quad (17)$$

It should be noted that the modified statistics,  $W_a$ ,  $W_b$ , and  $W_c$  have the same asymptotic distribution as  $W_0$ . In other words, the modifications are of order  $O_p(T^{-1})$  and they are effective only in finite samples.

In the experiments in the next section, we apply these three modifications to  $WL_0$  in the LA-VAR procedure and to  $WG_0$  in the GI-VAR procedure, in addition to  $W_0$  in the Std-VAR procedure, and evaluate effectiveness of these modifications.

## 4 THE EXPERIMENT AND THE RESULTS

In this section, we employ Monte Carlo technique to evaluate our testing procedures and compare them with conventional testing procedures.

### *The Monte Carlo Design*

In this section, we consider the following simple VEC form with  $m = 4$ ,  $p = 2$ , and  $r = 2$ ,

$$\Delta x_t = \alpha\beta'x_{t-1} + \Gamma_1\Delta x_{t-1} + \varepsilon_t, \quad (18)$$

where  $\{\varepsilon_t\}$  is *i.i.d.*  $N(0, I_4)$ .

We are concerned with the test for non-causality from  $x_3$  and  $x_4$  to  $x_1$ . That is, we test the hypothesis  $H_0^G$  in (3) with  $R_L = [1, 0, 0, 0]$  and  $R_R^* = [0, I_2]$ . The following two data generating processes (GDPs) are employed:

Case 1:

$$\alpha = \begin{bmatrix} 0 & 0 \\ 0.3 & -0.3 \\ -0.5 & 0.1 \\ -0.5 & 0.5 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0.4 & -0.8 \\ -0.5 & 0 \\ 1 & 1 \\ 0.5 & 0 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0.3 & -0.5 & 0 & \delta \\ 0.5 & -0.5 & -0.1 & 0.1 \\ -0.1 & 0.1 & -0.2 & 0.1 \\ -0.3 & 0.3 & -0.1 & 0.2 \end{bmatrix}$$

Case 2:

$$\alpha = \begin{bmatrix} 0 & 0 \\ 0.3 & -0.3 \\ -0.5 & 0.1 \\ -0.5 & 0.5 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0.4 & -0.8 \\ -0.5 & 0 \\ 1 & 1 \\ 0.5 & 0.5 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0.3 & -0.5 & 0 & \delta \\ 0.5 & -0.5 & -0.1 & 0.1 \\ -0.1 & -0.1 & -0.2 & 0.1 \\ -0.3 & 0.3 & -0.1 & 0.2 \end{bmatrix}$$

The important point in designing the DGPs is the rank of  $R_R^*\beta$ . In Case 1,  $\text{rank}(R_R^*\beta) = p_2$ , i.e.,  $\text{rank}(R_R^*\beta) = 2$ , whereas in Case 2,  $\text{rank}(R_R^*\beta) < p_2$ , i.e.,  $\text{rank}(R_R^*\beta) = 1$ . The presence of causation is controlled through the parameter  $\delta$  in  $\Gamma_1$ . We set  $\delta = 0$  to examine empirical size and  $\delta = 0.1$ , and  $0.2$  to evaluate empirical power.

Throughout the experiments, 5000 samples of size  $T + 500$  were generated with the last  $T$  observations used for estimation and testing purposes. For each DGP, three sample sizes were considered:  $T = 100, 200$  and  $400$ . All simulations were carried out using the matrix programming language GAUSS.

For each sample, we first estimated a VAR(2) model by the least squares. The cointegrating rank was selected by the trace test in Johansen (1988) at the significance level of 1%. The entries of Table 0 of Osterwald-Lenum (1992) was used as the critical value. The rank of a sub-matrix of cointegration was detected by the testing procedure in Kurozumi (2003) at the significance level of 1%. We may note that the test statistic for the model (18) is slightly different from that for the model (14), and is given in Kurozumi (2003, Th. 1). The Granger non-causality test in the final step of our procedures was set to 5% significance level. The tabulated results of the experiment are presented in Tables from 1s to 2p. Tables 1s and 1p show the results for Case 1, while Tables 2s and 2p for Case 2.

### *Notation for Tables from 1s to 2p*

Now, we explain the notation in Tables 1s to 2p. The column “ $r$ ” indicates a possible cointegration rank to be selected by the trace test. The column “%” next to it shows an empirical distribution of the selected cointegration rank. Note that the row for  $r = 0$  is omitted from the table, since there were virtually no occurrence. The column “ $s$ ” indicates the rank of  $R\Sigma R'$  detected by the Kurozumi (2003) procedure. “full” means that  $R\Sigma R'$  is of full rank, i.e.,  $R\Sigma R' = pp_1p_2$ , and “deg” means that  $R\Sigma R'$  is degenerate, i.e.,  $\text{rank}(R\Sigma R') < pp_1p_2$ . The column “%” next to  $s$  shows an empirical distribution of  $\text{rank}(R\Sigma R')$  for a given  $r$ . When  $r = 4$ , the system is purely stationary and the Wald test for the Granger non-causality follow usual manner, and then there is no need to test  $H_0^s$ . Because of this, there should be no entries in column “ $s$ ” and “%” next to it when  $r = 4$ . The headings “Sequential procedures” and “Exclusive Std-VAR and LA-VAR” stand for our testing procedures and conventional testing procedures that employ exclusively the Wald statistics based on Std-VAR approach and LA-VAR approach, respectively.

The columns  $W_k$ ,  $WL_k$ , and  $WG_k$  ( $k = 0, a, b$ , and  $c$ ) show the results for testing the hypothesis  $H_0^G$ .  $W$ ,  $WL$ , and  $WG$  stand for the Wald test statistics computed from the Std-VAR, the LA-VAR, and the GI-VAR approaches, respectively. The subscripts 0,  $a$ ,  $b$  and  $c$  stand for the conventional and three modifications proposed in the previous section, respectively. Recall that we use  $W_k$ 's when  $R\Sigma R'$  is of full rank, and  $WL_k$ 's and  $WG_k$ 's when it is degenerate. The columns  $CL_k$  ( $k = 0, a, b$ , and  $c$ ) show weighted sums of the corresponding columns  $W_k$ 's and  $WL_k$ 's. The columns  $CG_k$  ( $k = 0, a, b$ , and  $c$ ) are similar weighted sums  $W_k$ 's and  $WG_k$ 's. The columns  $CL_k$ 's and  $CG_k$ 's represent the performance of the proposed procedures for testing the long-run Granger non-causality in this paper. Finally, the row "total" shows over-all performance in each sample size, that stands for the weighted average of rejection percentages of  $r = 1, 2, 3$ , and 4 for each test statistic.

### *The Monte Carlo Results: Case 1*

Table 1s shows the empirical size for Case 1. In this case,  $W_0$  is a theoretically appropriate statistic, since  $R\Sigma R'$  is of full rank in the GDP. The empirical sizes for "W<sub>0</sub> in Exclusive Std-VAR" for  $T=100, 200$  and 400 are 10.6%, 7.0% and 5.9%, respectively. "W<sub>0</sub> in Exclusive Std-VAR" suffers from size distortion in small samples, although it approximates the correct size as the sample size increases. The "total" empirical sizes for  $CL_0$  and  $CG_0$  do not differ substantially from those of "W<sub>0</sub> in Exclusive Std-VAR." This is because  $WL_0$  and  $WG_0$  were not adopted into  $CL_0$  and  $CG_0$ , respectively, except for the case where  $r=1$  and  $T= 100$ . It is a consequence of the result that Kurozumi's test has sufficiently high power to detect the correct rank of  $R\Sigma R'$  in this case.

Concerning the small sample modifications, it appears that these modifications are quite effective in reducing the size distortion. The modifications "b" and "c" seem to work well, while the modification "a" still leaves a little room for improvement in reducing size distortion. Although large size distortion due to incorrect selection of  $r$  and  $s$ , say  $r =3$ , is selected, may be inevitable, the "total" empirical size appears to perform well, since contributions of those incorrect selection of  $r$  and/or  $s$  are relatively small.

Table 1p contains the tabulated size corrected power for Case 1. The results of  $CL_k$ 's,  $CG_k$ 's, and "W<sub>k</sub>'s in Exclusive Std-VAR" do not differ substantially each other as in Table 1s. Further, the rejection rates of the causality test with no modifications, with subscript "0", are not very much different from those with modifications, with subscript

“ $a$ ,  $b$ , and  $c$ ”. On the other hand, the exclusive use of the LA-VAR approach, “ $WL_k$ ’s in Exclusive LA-VAR,” shows relatively low power as expected, because of its inefficiency. Consequently,  $CL_k$ ’s and  $CG_k$ ’s substantially dominate “ $WL_k$ ’s in Exclusive LA-VAR”.

In sum, there are not much difference between two sequential procedures,  $CL_k$ ’s and  $CG_k$ ’s. This is because the theoretically appropriate statistic,  $W_k$ ’s in this case, is correctly selected most of times. Thus,  $CL_k$ ’s and  $CG_k$ ’s show similar performance as “ $W_k$ ’s in the Exclusive Std-VAR” and they are substantially powerful than “ $WL_k$ ’s in the Exclusive LA-VAR”. It is one of the desired properties of the sequential procedures. In terms of size property, the modifications “ $b$ ” and “ $c$ ” are effective in reducing the original size distortion.

### *The Monte Carlo Results: Case 2*

Table 2s contains the tabulated empirical size for Case 2. In this case,  $WL_0$  and  $WG_0$  are appropriate statistics, since  $R\Sigma R'$  is degenerate in the GDP. In particular,  $WL_0$  is the basic one, since unlike  $WG_0$ , it does not require preliminary tests for the cointegration rank and for the rank of a sub-matrix of the cointegration matrix. The empirical sizes for “ $WL_0$  in Exclusive LA-VAR” for  $T=100$ , 200 and 400 are 9.1%, 7.5% and 5.7%, respectively. “ $WL_0$  in Exclusive LA-VAR” suffers from size distortion in small samples, although it also approximates the correct size as the sample size increases. The “total” empirical sizes for sequential procedures  $CL_0$  and  $CG_0$  are slightly higher than those of “ $WL_0$  in Exclusive LA-VAR.” This is because  $W_0$ , an inappropriate statistic in this case, has substantially higher rejection rates than the nominal size even for larger samples. and it is adopted in the sequential procedures more frequently than expected. Namely, Kurozumi procedure appears to be liberal in this case, rejecting  $H_0^s : rank(R_R^*\beta) = 1$  a little too often, especially when  $T=100$ . It requires a sample size of at least 200 to achieve a relatively desired frequencies that the true  $s$  is selected.

In this case the modifications “ $a$ ” and “ $b$ ” appears to be quite effective in reducing original size distortions. But, the modification “ $c$ ”, seems to overcorrect when sample size is small, i.e.,  $T=100$ . Further, it may be noted that, if the incorrect  $r$  and/or  $s$  are selected, the effect of the modifications can be unstable for some cases.

Table 2p contains the tabulated size corrected power for Case 2. The power increases smoothly as  $\delta$  or  $T$  increases. Generally, the power is slightly higher for the conventional statistics than the corresponding modified ones. We may notice a difference in power when



we compare  $CL_k$ 's with  $CG_k$ 's. For any  $T$ ,  $CG_k$  uniformly dominates the corresponding  $CL_k$  ( $k = 0, a, b$ , and  $c$ ). It should be the result of the fact that the GI-VAR approach uses the rank information fully, while the LA-VAR approach only uses the information on whether  $R\Sigma R'$  is degenerate or not.

In sum, the appropriate statistics in this case are  $WL_k$ 's and  $WG_k$ 's, and the size performance of sequential testing procedure  $CG_k$ 's and  $CG_k$ 's are substantially better than the inappropriate statistics, “ $W$ 's in Exclusive Std-VAR”. It is another of the desired properties of the sequential procedures.  $CG_k$ 's are more preferable than  $CL_k$ 's, since the former are generally more powerful than the latter, while they show similar size performances. Among modifications in  $CG_k$ 's, “b” seems to be the best in terms of size performance, while “c” appears to overcorrect the original size distortion.

### *Recommended Procedure*

Throughout above experiments, we have seen that  $CL_b$  and  $CG_b$  exhibit reasonable size, although we also observed that they can be a little conservative, say, when  $T = 100$  in Case 2. The modifications “a” and “c” are inferior to “b”, since they tended to undercorrect or overcorrect the original size distortion, respectively. In terms of empirical power,  $CG_b$  appeared to be more powerful than  $CL_b$  as shown in Case 2. In conclusion, we recommend the use of the procedure  $CG_b$  in practice.

### *Effects of Misspecification*

For each sample, we also examined cases of estimating a VAR(1) model for underfitting and estimating VAR(3) and VAR(4) models for overfitting in order to examine the effects of misspecifications of the lag length of a VAR model on our procedures. These results are not presented here due to limited space<sup>2</sup>, but its summary is briefly discussed below.

In the underfitting case,  $W_k$ 's and  $WG_k$ 's are computed from a VAR(1) estimation. Hence, “ $W_0$  in Exclusive Std-VAR” and  $CG_0$  suffer severe size distortion in both Cases 1 and 2. The modifications cannot correct these size distortions. On the other hand,  $WL_0$  is computed on a VAR(2). Clearly, the tests based on the LA-VAR approach do not suffer size distortion much, especially for Case 1, since it is an accidentally correct method for Case 1.

The overfitting case causes inefficiency. Although some size distortions and losses of power arise from the inefficiency, the results are not very different from those of the

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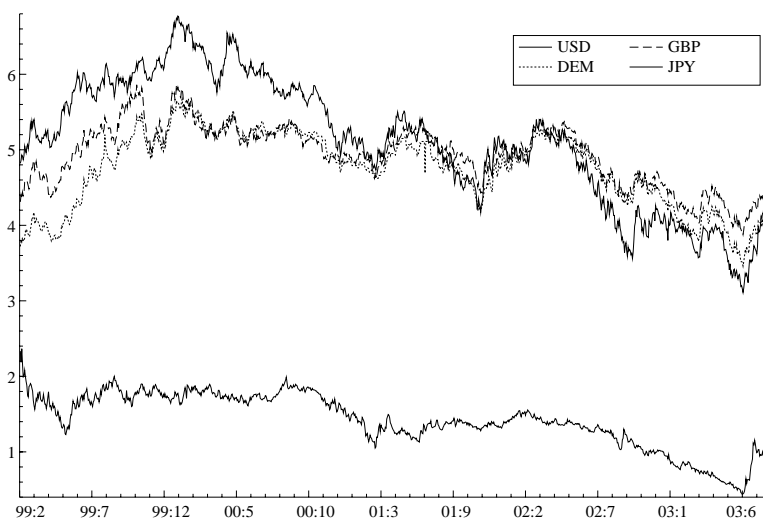
<sup>2</sup>These results will be provided by the author upon request.

exactfitting case.  $CG_b$  performs best in both Case 1 and 2. These results indicate that it is safe to fit a longer model than a shorter one.

## 5 EMPIRICAL ILLUSTRATION

In this section, we examine the Granger non-causality among long-term interest rates among several countries. We consider a system of long-term interest rates of the US dollar (USD), the Great Britain pound (GBP), the Deutschmark (DEM), and the Japanese yen (JPY). Daily time series of 10-year benchmark interest rates for each country are covered from February 2, 1999 to July 31, 2003 with the sample size  $T = 1033^3$ .

Figure 1 Long-Term Interest Rates



Main estimation and test results are given in Table 3. In Table 3, superscripts a, b, and c indicate that statistics are statistically significant at 1%, 5% and 10% level, respectively.

Whether a time series contains a unit root or not is assessed by the Zivot and Andrews (1992) test. It tests for the null hypothesis of a unit root against the alternative hypothesis of trend stationarity with a structural change in the trend. We use the following regression model to test for a unit root.

$$y_t = \eta + \theta t + \gamma DT_t^*(\lambda) + \rho y_{t-1} + \sum_{j=1}^k c_j \Delta y_{t-j} + \varepsilon_t, \quad (19)$$

---

<sup>3</sup>Data come from the web page of NIKKEI MONEY; <http://nk-money.topica.ne.jp>

where  $DT_t^*(\lambda) = t - \lambda T$  if  $t > T\lambda$ , 0 otherwise and  $\lambda$  is a break fraction. We estimate (19) by OLS and calculate  $t$  statistic for testing  $\rho = 1$  for each possible break date  $T_B = [T\lambda]$ . For each break date  $T_B$ ,  $k$  was determined using the BIC criteria. Panel (A) of Table 3 presents the minimum  $t$  statistic which shows that the existence of a unit root is not rejected in each series. The critical values for the test are drawn from Table 3 in Zivot and Andrews (1992). Next, the VEC model (2) is fitted by Johansen's (1991) maximum likelihood method. The optimal lag length is chosen by sequential reduction using the BIC criterion (Panel (B)). A time trend is found to be insignificant by Johansen (1991) likelihood ratio test (Panel (C)). Given this information, the VEC model (13) is estimated. The test for cointegration rank is carried by the well known Johansen (1991) tests. Panel (D) presents the results of the test for cointegration rank, where "Eig" denotes the ordered eigen values, "trace" the trace test statistic, and "l-max" the maximum eigen value test statistic. We conclude that the cointegration rank is one at 5% significance level. The critical values for the tests are drawn from Table 1\* in Osterwald-Lenum (1992). Panels (E) and (F) display the standardized loading vector  $\alpha$  and the standardized cointegrating vector  $\beta$ , respectively, where the last element in  $\beta$  is an estimate of a constant term in the cointegrating vector.

Here, we test the Granger non-causality by  $CG_b$ , since  $CG_b$  was shown to exhibit relatively stable size and be more powerful than the other test based on the LA-VAR approach, i.e.  $CL_b$ , in the previous section. Specifically, we adopt  $W_b$  when Kurozumi's procedure indicates that the relevant variance-covariance matrix is of full rank and adopt  $WG_b$  when it is detected to be degenerate. First, we examine a single variable causality, that is, a causal relation from one variable to one variable. Kurozumi's procedure revealed that USD and DEM are excluded from the cointegrating vector. In other words, the sub-matrices of the cointegrating matrix corresponding to USD and DEM are both degenerate, i.e.  $rank(R_R^*\beta) = 0$  for USD and DEM. This means that, when the causing variable is either USD or DEM, the relevant variance-covariance matrix is degenerate, and  $WG_b$  is called for. Panel (G) gives the test results of the Granger non-causality calculated from the OLS estimation of a levels VAR model (1), and Figure 2 depicts the Granger causality which is statistically significant at 5% significance level. Next, we examine a block causality, that is, a causal relation from a set of variables to a set of variables, For illustrative purpose, we test the Granger non-causality between a group of USD and DEM which are both excluded from the cointegrating vector and a group of GBP and JPY

which are included in the cointegrating vector. We may note that, since the cointegration rank is one, the relevant variance-covariance matrix is necessarily degenerate and  $WG_b$  is called for. It turns out that there is a bidirectional block causality. It appears to be conformable with the results of the single variable causality.

## 6 CONCLUDING REMARKS

In this paper, we proposed two operational procedures to test the hypothesis of the Granger non-causality in cointegrated systems. One based on the GI-VAR approach and the other on the LA-VAR approach. They circumvent the problem of possible degeneracy of the variance-covariance matrix associated with the usual Wald type test statistic. In order to detect degeneracy or the rank of the matrix, the testing procedure by Kurozumi (2003) plays an important role. Further, we proposed two modifications in order to reduce the size distortion of the test, in addition to the one proposed in Kurozumi and Yamamoto (2000).

The finite sample experiments suggested that the test procedure based on GI-VAR approach, denoted here  $CG_k$ 's are preferable because they were shown to be more powerful than  $CL_k$ 's in finite samples, while they exhibited similar size performances. In terms of the modifications proposed in the paper, the one denoted as "b" appeared to perform well. In sum, we recommend  $CG_b$  for testing the Granger non-causality in cointegrated systems.

In empirical applications, we examined the causal relations among long-term interest rates of four countries. We encountered many situations where the degeneracy occurs and the proposed procedure appears to be useful.

We believe that the proposed procedure is practical. Here, the meaning of the term "practical" is twofold. Firstly, the testing procedure consists of test statistics whose asymptotic distributions are all chi-square, except the well known test for the cointegration rank by Johansen (1988, 1991). Thus, we do not need an exotic table of critical values or we do not have to simulate critical values by ourselves. Secondly, the small sample modifications of the test statistics give them reasonable empirical sizes in finite samples, which is essential for a testing procedure to be practical.

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Appendix Proof of Proposition 1

The rank of  $R\Sigma R'$  can be expressed explicitly as follows:

$$\begin{aligned}
 \text{rank}(R\Sigma R') &= \text{rank}(R_L \Sigma_{\varepsilon\varepsilon} R'_L) \times \text{rank}(R_R \Sigma_{XX} R'_R) \\
 &= p_1 \times \text{rank}\{R_R(K'^{-1}G_\xi \Sigma_{\xi\xi}^{-1}G'_\xi K^{-1})R'_R\} \\
 &= p_1 \times \text{rank}(R_R K'^{-1}G_\xi).
 \end{aligned}$$

Because  $\Sigma_{\xi\xi}^{-1}$  is of full rank, the third equality holds. In order to investigate the rank of  $R_R K'^{-1}G_\xi$ , let us write it explicitly as

$$\begin{aligned}
 R_R K'^{-1}G_\xi &= \left[ \begin{array}{c|ccc} R_R^* \beta & R_R^* & 0 & \dots & 0 \\ 0 & -R_R^* & R_R^* & & 0 \\ \vdots & & -R_R^* & \ddots & \\ \vdots & & & \ddots & R_R^* \\ 0 & 0 & & & -R_R^* \end{array} \right] \\
 &= \begin{bmatrix} G_{11}^* & G_{12}^* \\ G_{21}^* & G_{22}^* \end{bmatrix} \text{ (say)}.
 \end{aligned}$$

Then, it is turned out that

$$\begin{aligned}
 \text{rank}(R_R K'^{-1}G_\xi) &= \text{rank}(G_{11}^*) + \text{rank}(G_{22}^*) \\
 &= \text{rank}(R_R^* \beta) + (p-1)p_2.
 \end{aligned}$$

Hence, we have

$$\text{rank}(R\Sigma R') = p_1 \times (\text{rank}(R_R^* \beta) + (p-1)p_2).$$

This completes the proof of Proposition 1.

Table 1s (Empirical size in Case 1,  $p = 2$ )

| $T$ | $r$   | %    | $s$         | %            | Sequential procedures |           |           |           |                  |          |          |          |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |
|-----|-------|------|-------------|--------------|-----------------------|-----------|-----------|-----------|------------------|----------|----------|----------|-----------|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|------|--|--|--|--|--|--|
|     |       |      |             |              | $W_0$                 | $W_a$     | $W_b$     | $W_c$     | $WL_0$           | $WL_a$   | $WL_b$   | $WL_c$   | $WG_0$    | $WG_a$   | $WG_b$   | $WG_c$   | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |      |  |  |  |  |  |  |
| 100 | 1     | 7.0  | full<br>deg | 0.0<br>100.0 | .<br>.                | .<br>.    | .<br>.    | .<br>.    | .<br>11.2        | .<br>6.0 | .<br>5.7 | .<br>5.2 | .<br>10.3 | .<br>5.4 | .<br>5.7 | .<br>5.7 | 11.2   | 6.0    | 5.7    | 5.2    | 10.3   | 5.4    | 5.7    | 5.7    |      |  |  |  |  |  |  |
|     | 2     | 91.1 | full<br>deg | 100.0<br>0.0 | 10.2<br>.             | 6.6<br>.  | 5.4<br>.  | 4.7<br>.  | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | 10.2   | 6.6    | 5.4    | 4.7    | 10.2   | 6.6    | 5.4    | 4.7    |      |  |  |  |  |  |  |
|     | 3     | 1.8  | full<br>deg | 100.0<br>0.0 | 21.3<br>.             | 9.0<br>.  | 11.2<br>. | 11.2<br>. | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 21.3   | 9.0    | 11.2   | 11.2   | 21.3   | 9.0    | 11.2   | 11.2 |  |  |  |  |  |  |
|     | 4     | 0.1  |             |              | 0.0                   | 0.0       | 0.0       | 0.0       | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0  |  |  |  |  |  |  |
|     | total |      |             |              | 10.4                  | 6.6       | 5.5       | 4.8       | 11.1             | 6.0      | 5.7      | 5.1      | 10.3      | 5.4      | 5.7      | 5.7      | 10.4   | 6.6    | 5.5    | 4.8    | 10.4   | 6.5    | 5.5    | 4.9    |      |  |  |  |  |  |  |
| 200 | 1     | 0.0  | full<br>deg | .<br>.       | .<br>.                | .<br>.    | .<br>.    | .<br>.    | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. |      |  |  |  |  |  |  |
|     | 2     | 98.7 | full<br>deg | 100.0<br>0.0 | 7.0<br>.              | 5.7<br>.  | 5.5<br>.  | 4.4<br>.  | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 7.0    | 5.7    | 5.5    | 4.4    | 7.0    | 5.7    | 5.5    | 4.4  |  |  |  |  |  |  |
|     | 3     | 1.2  | full<br>deg | 100.0<br>0.0 | 6.8<br>.              | 1.7<br>.  | 5.1<br>.  | 3.4<br>.  | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 6.8    | 1.7    | 5.1    | 3.4    | 6.8    | 1.7    | 5.1    | 3.4  |  |  |  |  |  |  |
|     | 4     | 0.1  |             |              | 0.0                   | 0.0       | 0.0       | 0.0       | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0  |  |  |  |  |  |  |
|     | total |      |             |              | 7.0                   | 5.6       | 5.5       | 4.4       | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 7.0    | 5.6    | 5.5    | 4.4    | 7.0    | 5.6    | 5.5    | 4.4  |  |  |  |  |  |  |
| 400 | 1     | 0.0  | full<br>deg | .<br>.       | .<br>.                | .<br>.    | .<br>.    | .<br>.    | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. | .<br>. |      |  |  |  |  |  |  |
|     | 2     | 98.6 | full<br>deg | 100.0<br>0.0 | 5.9<br>.              | 5.5<br>.  | 5.5<br>.  | 4.8<br>.  | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 5.9    | 5.5    | 5.5    | 4.8    | 5.9    | 5.5    | 5.5    | 4.8  |  |  |  |  |  |  |
|     | 3     | 1.4  | full<br>deg | 100.0<br>0.0 | 10.3<br>.             | 10.3<br>. | 10.3<br>. | 10.3<br>. | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 10.3   | 10.3   | 10.3   | 10.3   | 10.3   | 10.3   | 10.3   | 10.3 |  |  |  |  |  |  |
|     | 4     | 0.0  |             |              | 0.0                   | 0.0       | 0.0       | 0.0       | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0  |  |  |  |  |  |  |
|     | total |      |             |              | 5.9                   | 5.5       | 5.7       | 4.9       | .<br>.           | .<br>.   | .<br>.   | .<br>.   | .<br>.    | .<br>.   | .<br>.   | .<br>.   | .<br>. | 5.9    | 5.5    | 5.5    | 4.9    | 5.9    | 5.5    | 5.5    | 4.9  |  |  |  |  |  |  |
|     |       |      |             |              | Exclusive Std-VAR     |           |           |           | Exclusive LA-VAR |          |          |          |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |
|     |       |      |             |              | $W_0$                 | $W_a$     | $W_b$     | $W_c$     | $WL_0$           | $WL_a$   | $WL_b$   | $WL_c$   |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |
| 100 | .     |      |             |              | 10.6                  | 6.7       | 5.7       | 4.8       | 9.8              | 4.0      | 4.2      | 3.5      |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |
| 200 | .     |      |             |              | 7.0                   | 5.6       | 5.5       | 4.4       | 7.0              | 3.8      | 4.8      | 4.1      |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |
| 400 | .     |      |             |              | 5.9                   | 5.5       | 5.5       | 4.9       | 6.0              | 4.4      | 5.3      | 4.9      |           |          |          |          |        |        |        |        |        |        |        |        |      |  |  |  |  |  |  |

Note: For explanation of the notation, see subsection *Notation for Tables from 1a to 2b* in section 4.



Table 1p (Empirical power in Case 1,  $p = 2$ )

| $\delta$ | $T$ | $r$   | %     | $s$         | %            | Sequential procedures |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |
|----------|-----|-------|-------|-------------|--------------|-----------------------|-------|-------|-------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|-------|------|------|
|          |     |       |       |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ | $WG_0$ | $WG_a$ | $WG_b$ | $WG_c$ | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |       |       |      |      |
| 22       | 100 | 1     | 10.4  | full<br>deg | 0.0<br>100.0 | .                     | .     | .     | .     | 6.7              | 5.0    | 6.3    | 6.7    | 8.4    | 8.8    | 7.1    | 7.6    | 6.7    | 5.0    | 6.3    | 6.7    | 8.4    | 8.8    | 7.1    | 7.6    |       |       |      |      |
|          |     | 2     | 88.2  | full<br>deg | 100.0<br>0.0 | 11.3                  | 10.4  | 10.0  | 10.4  | .                | .      | .      | .      | .      | .      | .      | .      | .      | 11.3   | 10.4   | 10.0   | 10.4   | 11.3   | 10.4   | 10.0   | 10.4  |       |      |      |
|          |     | 3     | 1.3   | full<br>deg | 100.0<br>0.0 | 1.6                   | 10.9  | 3.1   | 3.1   | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 1.6    | 10.9   | 3.1    | 3.1    | 1.6    | 10.9   | 3.1   | 3.1   |      |      |
|          |     | 4     | 0.1   |             |              | 20.0                  | 20.0  | 20.0  | 20.0  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0  | 20.0  |      |      |
|          |     | total |       |             |              | 11.1                  | 10.4  | 9.9   | 10.3  | 6.7              | 5.0    | 6.3    | 6.7    | 8.4    | 8.8    | 7.1    | 7.6    | 10.7   | 9.9    | 9.5    | 9.9    | 10.8   | 10.3   | 9.6    | 10.0   |       |       |      |      |
|          | 0.1 | 200   | 1     | 0.0         | full<br>deg  | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .     | .     |      |      |
|          |     |       | 2     | 98.8        | full<br>deg  | 100.0<br>0.0          | 22.5  | 21.3  | 21.4  | 22.0             | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 22.5   | 21.3   | 21.4   | 22.0   | 22.5   | 21.3  | 21.4  | 22.0 |      |
|          |     |       | 3     | 1.1         | full<br>deg  | 100.0<br>0.0          | 30.4  | 33.9  | 30.4  | 30.4             | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 30.4   | 33.9   | 30.4   | 30.4   | 30.4  | 33.9  | 30.4 | 30.4 |
|          |     |       | 4     | 0.1         |              |                       | 60.0  | 40.0  | 40.0  | 40.0             | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 60.0   | 40.0   | 40.0   | 40.0   | 60.0   | 40.0  | 40.0  | 40.0 |      |
|          |     |       | total |             |              |                       | 22.6  | 21.5  | 21.6  | 22.1             | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 22.6   | 21.5   | 21.6   | 22.1   | 22.6   | 21.5  | 21.6  | 22.1 |      |
|          | 0.1 | 400   | 1     | 0.0         | full<br>deg  | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .     | .     |      |      |
|          |     |       | 2     | 98.6        | full<br>deg  | 100.0<br>0.0          | 48.8  | 47.4  | 47.7  | 47.8             | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 48.8   | 47.4   | 47.7   | 47.8   | 48.8   | 47.4  | 47.7  | 47.8 |      |
| 3        |     |       | 1.3   | full<br>deg | 100.0<br>0.0 | 44.6                  | 46.2  | 40.0  | 38.5  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 44.6   | 46.2   | 40.0   | 38.5   | 44.6   | 46.2  | 40.0  | 38.5 |      |
| 4        |     |       | 0.1   |             |              | 100.0                 | 100.0 | 100.0 | 100.0 | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  | 100.0 | 100.0 |      |      |
| total    |     |       |       |             |              | 48.8                  | 47.4  | 47.6  | 47.7  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 48.8   | 47.4   | 47.6   | 47.7   | 48.8   | 47.4   | 47.6  | 47.7  |      |      |
|          |     |       |       |             |              | Exclusive Std-VAR     |       |       |       | Exclusive LA-VAR |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |
|          |     |       |       |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |
| 0.1      | 100 | .     |       |             |              | 11.0                  | 10.1  | 9.7   | 10.1  | 8.5              | 7.7    | 7.9    | 8.9    |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |
|          | 200 | .     |       |             |              | 22.8                  | 21.4  | 21.6  | 22.4  | 17.0             | 15.0   | 15.2   | 16.1   |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |
|          | 400 | .     |       |             |              | 48.7                  | 47.3  | 47.1  | 47.8  | 30.0             | 28.3   | 27.9   | 28.4   |        |        |        |        |        |        |        |        |        |        |        |        |       |       |      |      |

Table 1p - contd. (Empirical power in Case 1,  $p = 2$ )

| $\delta$ | $T$   | $r$ | %     | $s$   | %     | Sequential procedures |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |
|----------|-------|-----|-------|-------|-------|-----------------------|-------|-------|-------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
|          |       |     |       |       |       | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ | $WG_0$ | $WG_a$ | $WG_b$ | $WG_c$ | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |      |
| 0.2      | 100   | 1   | 14.7  | full  | 0.0   | .                     | .     | .     | .     | 24.3             | 13.7   | 19.7   | 22.0   | 34.2   | 30.5   | 25.8   | 29.5   | 24.3   | 13.7   | 19.7   | 22.0   | 34.2   | 30.5   | 25.8   | 29.5   |      |
|          |       |     | deg   | 100.0 | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |
|          |       | 2   | 83.9  | full  | 100.0 | 39.2                  | 33.6  | 33.2  | 32.9  | .                | .      | .      | .      | .      | .      | .      | .      | .      | 39.2   | 33.6   | 33.2   | 32.8   | 39.2   | 33.6   | 33.2   | 32.8 |
|          |       |     | deg   | 0.0   | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |
|          |       | 3   | 1.3   | full  | 100.0 | 26.9                  | 40.3  | 25.4  | 17.9  | .                | .      | .      | .      | .      | .      | .      | .      | .      | 26.9   | 40.3   | 25.4   | 17.9   | 26.9   | 40.3   | 25.4   | 17.9 |
|          |       | deg | 0.0   | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
|          | 4     | 0.1 |       |       | 50.0  | 33.3                  | 66.7  | 83.3  | .     | .                | .      | .      | .      | .      | .      | .      | .      | 50.0   | 33.3   | 66.7   | 83.3   | 50.0   | 33.3   | 66.7   | 83.3   |      |
|          |       | deg | 0.0   | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
|          | total |     |       |       | 39.0  | 33.7                  | 33.1  | 32.7  | 24.3  | 13.6             | 19.6   | 22.0   | 34.1   | 30.4   | 25.8   | 29.5   | 36.8   | 30.8   | 31.2   | 31.1   | 38.3   | 33.2   | 32.1   | 32.2   |        |      |
|          | 200   | 1   | 0.0   | full  | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |
|          |       |     | deg   | 100.0 | 78.9  | 75.1                  | 76.1  | 76.8  | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | 78.9   | 75.1   | 76.1   | 76.8   | 78.9   | 75.1   | 76.1   | 76.8 |
|          |       |     | deg   | 0.0   | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |
| 3        |       | 1.0 | full  | 100.0 | 76.5  | 78.4                  | 74.5  | 72.5  | .     | .                | .      | .      | .      | .      | .      | .      | .      | 76.5   | 78.4   | 74.5   | 72.5   | 76.5   | 78.4   | 74.5   | 72.5   |      |
|          |       | deg | 0.0   | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
| 4        | 0.1   |     |       | 83.3  | 83.3  | 83.3                  | 83.3  | .     | .     | .                | .      | .      | .      | .      | .      | .      | 83.3   | 83.3   | 83.3   | 83.3   | 83.3   | 83.3   | 83.3   | 83.3   |        |      |
|          | deg   | 0.0 | .     | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |        |      |
| total    |       |     |       | 78.9  | 75.1  | 76.1                  | 76.8  | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | 78.9   | 75.1   | 76.1   | 76.8   | 78.9   | 75.1   | 76.1   | 76.8   |      |
| 400      | 1     | 0.0 | full  | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
|          |       | deg | 100.0 | 98.9  | 98.5  | 98.5                  | 98.7  | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | 98.9   | 98.5   | 98.5   | 98.7   | 98.9   | 98.5   | 98.5   | 98.7   |      |
|          |       | deg | 0.0   | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
|          | 3     | 1.1 | full  | 100.0 | 98.2  | 96.4                  | 96.4  | 94.6  | .     | .                | .      | .      | .      | .      | .      | .      | .      | 98.2   | 96.4   | 96.4   | 94.6   | 98.2   | 96.4   | 96.4   | 94.6   |      |
|          |       | deg | 0.0   | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |      |
| 4        | 0.1   |     |       | 100.0 | 100.0 | 100.0                 | 100.0 | .     | .     | .                | .      | .      | .      | .      | .      | .      | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  | 100.0  |        |      |
|          | deg   | 0.0 | .     | .     | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |        |      |
| total    |       |     |       | 98.9  | 98.5  | 98.5                  | 98.6  | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | 98.9   | 98.5   | 98.5   | 98.6   | 98.9   | 98.5   | 98.5   | 98.6   |      |
|          |       |     |       |       |       | Exclusive Std-VAR     |       |       |       | Exclusive LA-VAR |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |
|          |       |     |       |       |       | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ |        |        |        |        |        |        |        |        |        |        |        |        |      |
| 0.2      | 100   | .   |       |       |       | 38.9                  | 32.9  | 32.2  | 32.3  | 26.4             | 21.4   | 21.5   | 23.7   |        |        |        |        |        |        |        |        |        |        |        |        |      |
|          | 200   | .   |       |       |       | 79.0                  | 75.3  | 76.2  | 77.1  | 58.9             | 51.5   | 51.5   | 53.5   |        |        |        |        |        |        |        |        |        |        |        |        |      |
|          | 400   | .   |       |       |       | 98.8                  | 98.5  | 98.5  | 98.6  | 90.2             | 87.7   | 86.5   | 87.2   |        |        |        |        |        |        |        |        |        |        |        |        |      |

Table 2s (Empirical size in Case 2,  $p = 2$ )

| $T$ | $r$   | %    | $s$         | %            | Sequential procedures |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |
|-----|-------|------|-------------|--------------|-----------------------|-------|-------|-------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--|--|--|--|
|     |       |      |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ | $WG_0$ | $WG_a$ | $WG_b$ | $WG_c$ | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |  |  |  |  |
| 100 | 1     | 0.7  | full<br>deg | 0.0<br>100.0 | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | 8.1    | 8.1    | 5.4    | 5.4    | 13.5   | 5.4    | 2.7    | 5.4    |  |  |  |  |
|     | 2     | 96.8 | full<br>deg | 5.1<br>94.9  | 20.5                  | 15.7  | 8.0   | 6.0   | 9.2              | 3.3    | 3.2    | 2.6    | 8.8    | 6.1    | 3.6    | 3.1    | 9.8    | 4.0    | 3.5    | 2.8    | 9.4    | 6.6    | 3.8    | 3.3    |  |  |  |  |
|     | 3     | 2.3  | full<br>deg | 85.2<br>14.8 | 35.7                  | 23.5  | 11.2  | 8.2   | 5.9              | 0.0    | 5.9    | 0.0    | 17.6   | 5.9    | 5.9    | 5.9    | 31.3   | 20.0   | 10.4   | 7.0    | 33.0   | 20.9   | 10.4   | 7.8    |  |  |  |  |
|     | 4     | 0.2  |             |              | 40.0                  | 30.0  | 10.0  | 0.0   | .                | .      | .      | .      | .      | .      | .      | .      | 40.0   | 30.0   | 10.0   | 0.0    | 40.0   | 30.0   | 10.0   | 0.0    |  |  |  |  |
|     | total |      |             |              | 25.2                  | 18.2  | 9.0   | 6.4   | 9.2              | 3.4    | 3.3    | 2.6    | 8.9    | 6.1    | 3.6    | 3.2    | 10.3   | 4.4    | 3.7    | 2.9    | 10.1   | 7.0    | 3.9    | 3.4    |  |  |  |  |
| 200 | 1     | 0.0  | full<br>deg | .            | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |  |  |  |  |
|     | 2     | 98.0 | full<br>deg | 2.5<br>97.5  | 14.8                  | 23.0  | 16.4  | 9.8   | 7.4              | 4.3    | 4.4    | 4.1    | 7.2    | 5.8    | 5.4    | 4.4    | 7.5    | 4.8    | 4.7    | 4.2    | 7.4    | 6.2    | 5.7    | 4.5    |  |  |  |  |
|     | 3     | 1.9  | full<br>deg | 84.9<br>15.1 | 31.6                  | 31.6  | 21.5  | 16.5  | 7.1              | 7.1    | 7.1    | 7.1    | 7.1    | 0.0    | 0.0    | 0.0    | 28.0   | 28.0   | 19.4   | 15.1   | 28.0   | 26.9   | 18.3   | 14.0   |  |  |  |  |
|     | 4     | 0.1  |             |              | 50.0                  | 0.0   | 16.7  | 0.0   | .                | .      | .      | .      | .      | .      | .      | .      | 50.0   | 0.0    | 16.7   | 0.0    | 50.0   | 0.0    | 16.7   | 0.0    |  |  |  |  |
|     | total |      |             |              | 22.2                  | 25.6  | 18.4  | 12.1  | 7.4              | 4.3    | 4.4    | 4.1    | 7.2    | 5.7    | 5.4    | 4.4    | 8.0    | 5.2    | 5.0    | 4.4    | 7.8    | 6.6    | 5.9    | 4.7    |  |  |  |  |
| 400 | 1     | 0.0  | full<br>deg | .            | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      |  |  |  |  |
|     | 2     | 98.6 | full<br>deg | 1.8<br>98.2  | 16.1                  | 14.9  | 13.8  | 10.3  | 5.7              | 4.0    | 4.5    | 4.3    | 5.4    | 5.2    | 5.3    | 4.3    | 5.9    | 4.2    | 4.7    | 4.4    | 5.6    | 5.3    | 5.4    | 4.4    |  |  |  |  |
|     | 3     | 1.3  | full<br>deg | 84.6<br>15.4 | 38.2                  | 20.0  | 14.5  | 21.8  | 20.0             | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 35.4   | 20.0   | 15.4   | 21.5   | 35.4   | 20.0   | 15.4   | 21.5   |  |  |  |  |
|     | 4     | 0.1  |             |              | 25.0                  | 25.0  | 0.0   | 0.0   | .                | .      | .      | .      | .      | .      | .      | .      | 25.0   | 25.0   | 0.0    | 0.0    | 25.0   | 25.0   | 0.0    | 0.0    |  |  |  |  |
|     | total |      |             |              | 24.7                  | 17.1  | 13.7  | 14.4  | 5.7              | 4.0    | 4.6    | 4.3    | 5.4    | 5.2    | 5.3    | 4.3    | 6.3    | 4.4    | 4.8    | 4.6    | 6.0    | 5.5    | 5.6    | 4.6    |  |  |  |  |
|     |       |      |             |              | Exclusive Std-VAR     |       |       |       | Exclusive LA-VAR |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |
|     |       |      |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |
| 100 | .     |      |             |              | 18.1                  | 12.6  | 5.1   | 5.4   | 9.1              | 3.4    | 3.3    | 2.6    |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |
| 200 | .     |      |             |              | 14.1                  | 12.2  | 8.4   | 6.5   | 7.5              | 4.4    | 4.5    | 4.1    |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |
| 400 | .     |      |             |              | 12.2                  | 11.1  | 9.8   | 7.0   | 5.7              | 4.0    | 4.5    | 4.3    |        |        |        |        |        |        |        |        |        |        |        |        |  |  |  |  |

Table 2p (Empirical power in Case 2,  $p = 2$ )

| $\delta$ | $T$ | $r$   | %     | $s$  | %     | Sequential procedures |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |
|----------|-----|-------|-------|------|-------|-----------------------|-------|-------|-------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|------|------|--|
|          |     |       |       |      |       | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ | $WG_0$ | $WG_a$ | $WG_b$ | $WG_c$ | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |      |      |      |  |
| 0.1      | 100 | 1     | 0.9   | full | 0.0   | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | 4.3    | 4.3    | 2.2    | 4.3    | 0.0    | 2.2    | 13.0   | 6.5  |      |      |  |
|          |     |       |       | deg  | 100.0 | .                     | .     | .     | .     | 4.3              | 4.3    | 2.2    | 4.3    | 0.0    | 2.2    | 13.0   | 6.5    |        |        |        |        |        |        |        |        |      |      |      |  |
|          |     | 2     | 96.7  | full | 4.9   | 8.5                   | 3.0   | 5.5   | 9.8   | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 10.1   | 8.7    | 8.6    | 10.0   | 11.2   | 8.8    | 11.0 | 10.7 |      |  |
|          |     |       |       | deg  | 95.1  | .                     | .     | .     | .     | 10.2             | 9.0    | 8.8    | 10.0   | 11.3   | 9.1    | 11.3   | 10.7   |        |        |        |        |        |        |        |        |      |      |      |  |
|          |     | 3     | 2.3   | full | 77.8  | 7.7                   | 15.4  | 14.3  | 13.2  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 6.8    | 12.0   | 11.1   | 10.3   | 6.0    | 12.0   | 12.0 | 10.3 |      |  |
|          |     |       | deg   | 22.2 | .     | .                     | .     | .     | 3.8   | 0.0              | 0.0    | 0.0    | 0.0    | 0.0    | 3.8    | 0.0    |        |        |        |        |        |        |        |        |        |      |      |      |  |
|          |     |       | 4     | 0.1  |       | 0.0                   | 0.0   | 0.0   | 0.0   | .                | .      | .      | .      | .      | .      | .      | .      | .      | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0  |      |      |  |
|          |     |       | total |      |       | 8.2                   | 6.4   | 7.9   | 10.7  | 10.1             | 8.9    | 8.7    | 9.9    | 11.1   | 8.9    | 11.3   | 10.6   |        | 10.0   | 8.8    | 8.6    | 9.9    | 10.9   | 8.8    | 11.1   | 10.6 |      |      |  |
|          |     | 200   | 1     | 0.0  | full  | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    | .    |      |  |
|          |     |       |       | deg  | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    | .    |      |  |
|          |     |       |       | 2    | 98.4  | full                  | 2.7   | 17.2  | 12.7  | 14.2             | 11.2   | .      | .      | .      | .      | .      | .      | .      | .      | .      | 15.9   | 13.6   | 13.8   | 14.4   | 17.8   | 15.8 | 16.6 | 17.9 |  |
|          |     |       |       |      |       | deg                   | 97.3  | .     | .     | .                | .      | 15.9   | 13.6   | 13.8   | 14.5   | 17.9   | 15.9   | 16.7   | 18.0   |        |        |        |        |        |        |      |      |      |  |
|          |     |       |       | 3    | 1.5   | full                  | 83.6  | 9.8   | 11.5  | 8.2              | 6.6    | .      | .      | .      | .      | .      | .      | .      | .      | .      | 11.0   | 12.3   | 9.6    | 8.2    | 13.7   | 13.7 | 11.0 | 9.6  |  |
|          |     |       |       |      | deg   | 16.4                  | .     | .     | .     | .                | 16.7   | 16.7   | 16.7   | 16.7   | 33.3   | 25.0   | 25.0   | 25.0   |        |        |        |        |        |        |        |      |      |      |  |
|          |     |       | 4     | 0.1  |       | 20.0                  | 20.0  | 20.0  | 20.0  | 20.0             | .      | .      | .      | .      | .      | .      | .      | .      | .      | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0   | 20.0 | 20.0 |      |  |
|          |     | total |       |      | 15.0  | 12.5                  | 12.5  | 10.0  | 15.9  | 13.6             | 13.8   | 14.5   | 17.9   | 15.9   | 16.7   | 18.1   |        | 15.8   | 13.6   | 13.7   | 14.3   | 17.8   | 15.8   | 16.6   | 17.7   |      |      |      |  |
|          | 400 | 1     | 0.0   | full | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |      |      |  |
|          |     |       | deg   | .    | .     | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    |      |      |  |
|          |     |       | 2     | 98.6 | full  | 1.5                   | 21.3  | 20.0  | 18.7  | 18.7             | .      | .      | .      | .      | .      | .      | .      | .      | .      | 33.9   | 31.4   | 30.8   | 31.9   | 39.6   | 36.4   | 35.9 | 37.8 |      |  |
|          |     |       |       |      | deg   | 98.5                  | .     | .     | .     | .                | 34.1   | 31.6   | 31.0   | 32.1   | 39.9   | 36.6   | 36.1   | 38.1   |        |        |        |        |        |        |        |      |      |      |  |
|          |     |       | 3     | 1.3  | full  | 87.7                  | 12.3  | 19.3  | 15.8  | 12.3             | .      | .      | .      | .      | .      | .      | .      | .      | .      | 13.8   | 20.0   | 15.4   | 12.3   | 15.4   | 21.5   | 16.9 | 13.8 |      |  |
|          |     |       |       | deg  | 12.3  | .                     | .     | .     | .     | 25.0             | 25.0   | 12.5   | 12.5   | 37.5   | 37.5   | 25.0   | 25.0   |        |        |        |        |        |        |        |        |      |      |      |  |
|          |     | 4     | 0.1   |      | 40.0  | 40.0                  | 40.0  | 40.0  | 40.0  | .                | .      | .      | .      | .      | .      | .      | .      | .      | 40.0   | 40.0   | 40.0   | 40.0   | 40.0   | 40.0   | 40.0   | 40.0 |      |      |  |
|          |     | total |       |      | 18.2  | 20.4                  | 18.2  | 16.8  | 34.1  | 31.5             | 31.0   | 32.0   | 39.9   | 36.6   | 36.1   | 38.0   |        | 33.7   | 31.2   | 30.6   | 31.6   | 39.3   | 36.2   | 35.6   | 37.5   |      |      |      |  |
|          |     |       |       |      |       | Exclusive Std-VAR     |       |       |       | Exclusive LA-VAR |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |
|          |     |       |       |      |       | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |
| 0.1      | 100 | .     |       |      |       |                       |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |
|          | 200 | .     | 9.4   | 7.3  | 9.7   | 9.5                   | 10.1  | 8.8   | 8.6   | 9.8              |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |
|          | 400 | .     | 14.8  | 11.0 | 14.4  | 14.5                  | 15.7  | 13.4  | 13.9  | 14.6             |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |  |

Table 2p - contd. (Empirical power in Case 2,  $p = 2$ )

| $\delta$ | $T$ | $r$   | %    | $s$         | %            | Sequential procedures |       |       |       |                  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |
|----------|-----|-------|------|-------------|--------------|-----------------------|-------|-------|-------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|------|------|------|
|          |     |       |      |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ | $WG_0$ | $WG_a$ | $WG_b$ | $WG_c$ | $CL_0$ | $CL_a$ | $CL_b$ | $CL_c$ | $CG_0$ | $CG_a$ | $CG_b$ | $CG_c$ |      |      |      |      |
| 0.2      | 100 | 1     | 1.3  | full<br>deg | 0.0<br>100.0 | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | 21.9   | 15.6   | 7.8    | 10.9   | 23.4   | 25.0   | 43.8   | 26.6 |      |      |      |
|          |     | 2     | 96.9 | full<br>deg | 4.9<br>95.1  | 29.4                  | 10.5  | 20.2  | 25.6  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | 28.6   | 22.2   | 22.1   | 25.4   | 33.5   | 25.4   | 30.0 | 30.3 |      |      |
|          |     | 3     | 1.7  | full<br>deg | 81.9<br>18.1 | 23.5                  | 25.0  | 25.0  | 27.9  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 21.7   | 21.7   | 20.5   | 22.9   | 20.5   | 20.5 | 21.7 | 24.1 |      |
|          |     | 4     | 0.1  |             |              | 28.6                  | 14.3  | 0.0   | 14.3  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 28.6   | 14.3   | 0.0    | 14.3   | 28.6   | 14.3 | 0.0  | 14.3 |      |
|          |     | total |      |             |              | 28.1                  | 13.7  | 20.8  | 25.9  | 28.4             | 22.6   | 22.0   | 25.1   | 33.5   | 26.1   | 30.6   | 30.4   | 28.4   | 22.1   | 21.9   | 25.2   | 33.1   | 25.3   | 30.0   | 30.1   |      |      |      |      |
|          | 200 | 1     | 0.0  | full<br>deg | .            | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    | .    |      |      |
|          |     | 2     | 98.5 | full<br>deg | 2.1<br>97.9  | 50.0                  | 35.6  | 39.4  | 42.3  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 58.2   | 49.0   | 49.7   | 52.4   | 64.1   | 57.6 | 58.5 | 62.1 |      |
|          |     | 3     | 1.4  | full<br>deg | 78.6<br>21.4 | 56.4                  | 40.0  | 41.8  | 34.5  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 48.6   | 34.3   | 35.7   | 30.0   | 61.4   | 47.1 | 48.6 | 42.9 |      |
|          |     | 4     | 0.1  |             |              | 85.7                  | 85.7  | 71.4  | 85.7  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 85.7   | 85.7   | 71.4   | 85.7   | 85.7 | 85.7 | 71.4 | 85.7 |
|          |     | total |      |             |              | 53.6                  | 39.2  | 41.6  | 41.6  | 58.2             | 49.2   | 49.8   | 52.5   | 64.4   | 58.2   | 59.0   | 62.6   | 58.1   | 48.8   | 49.5   | 52.1   | 64.1   | 57.5   | 58.4   | 61.9   |      |      |      |      |
|          | 400 | 1     | 0.0  | full<br>deg | .            | .                     | .     | .     | .     | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | .    | .    |      |      |
|          |     | 2     | 98.7 | full<br>deg | 1.8<br>98.2  | 81.8                  | 79.5  | 78.4  | 79.5  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 91.3   | 88.8   | 88.3   | 88.6   | 94.8   | 93.5 | 93.1 | 93.8 |      |
|          |     | 3     | 1.2  | full<br>deg | 79.0<br>21.0 | 75.5                  | 79.6  | 69.4  | 65.3  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 79.0   | 80.6   | 71.0   | 69.4   | 79.0   | 82.3 | 74.2 | 71.0 |      |
|          |     | 4     | 0.1  |             |              | 83.3                  | 83.3  | 83.3  | 83.3  | .                | .      | .      | .      | .      | .      | .      | .      | .      | .      | .      | 83.3   | 83.3   | 83.3   | 83.3   | 83.3   | 83.3 | 83.3 | 83.3 |      |
|          |     | total |      |             |              | 79.7                  | 79.7  | 75.5  | 74.8  | 91.5             | 89.0   | 88.5   | 88.8   | 95.0   | 93.8   | 93.4   | 94.0   | 91.2   | 88.7   | 88.1   | 88.4   | 94.6   | 93.4   | 92.9   | 93.5   |      |      |      |      |
|          |     |       |      |             |              | Exclusive Std-VAR     |       |       |       | Exclusive LA-VAR |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |
|          |     |       |      |             |              | $W_0$                 | $W_a$ | $W_b$ | $W_c$ | $WL_0$           | $WL_a$ | $WL_b$ | $WL_c$ |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |
| 0.2      | 100 | .     |      |             |              | 27.8                  | 18.7  | 26.3  | 27.5  | 28.6             | 22.7   | 22.1   | 25.5   |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |
|          | 200 | .     |      |             |              | 56.3                  | 43.6  | 52.9  | 54.5  | 58.3             | 48.9   | 49.8   | 52.3   |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |
|          | 400 | .     |      |             |              | 90.6                  | 86.4  | 88.4  | 90.4  | 91.4             | 88.9   | 88.4   | 88.7   |        |        |        |        |        |        |        |        |        |        |        |        |      |      |      |      |

Table 3 The Granger Causality Between Long-Term Interest Rates

(A) Test for non-Stationarity of interest rates

|     | Z-A test |
|-----|----------|
| USD | -3.294   |
| DEM | -3.493   |
| GBP | -3.772   |
| JPY | -3.493   |

(B) Estimated lag length of VAR 2

(C) Test statistics for  $\alpha'_{\perp}\mu = 0$  0.844

(D) Test for the number of cointegrating vectors

|       |                     |                     |            |            |
|-------|---------------------|---------------------|------------|------------|
| Eig.  | 0.032               | 0.019               | 0.010      | 0.003      |
| $H_0$ | $r = 0$             | $r \leq 1$          | $r \leq 2$ | $r \leq 3$ |
| trace | 67.562 <sup>a</sup> | 33.622 <sup>c</sup> | 14.250     | 3.610      |
| lmax  | 33.940 <sup>a</sup> | 19.372              | 10.640     | 3.610      |

(E) Standardized adjustment coefficients  $\alpha'$

|       |       |       |        |
|-------|-------|-------|--------|
| 0.051 | 0.031 | 0.042 | -0.059 |
|-------|-------|-------|--------|

(F) Standardized cointegrating vectors  $\beta'$

|       |        |       |        |       |
|-------|--------|-------|--------|-------|
| 0.844 | -0.122 | 0.013 | -0.194 | 0.485 |
|-------|--------|-------|--------|-------|

(G) Test statistics for Granger non-causality

| to: | from: | USD                 | DEM   | GBP                | JPY   |
|-----|-------|---------------------|-------|--------------------|-------|
| USD |       | .                   | 0.319 | 4.398              | 2.130 |
| DEM |       | 49.706 <sup>a</sup> | .     | 9.318 <sup>a</sup> | 1.060 |
| GBP |       | 16.362 <sup>a</sup> | 0.020 | .                  | 1.425 |
| JPY |       | 2.461               | 0.606 | 0.648              | .     |

from: USD DEM to GPB JPY 19.974<sup>a</sup>

from: GBP JPY to USD DEM 15.342<sup>b</sup>

Figure 2 The Granger Causality at 5% Significance Level

