

# Resource Augmenting Technological Progress and Sustainable Development\*

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## Abstract

This paper constructs a three-sector growth model with non-renewable environmental resource and a resource augmenting technological progress, and investigates the relation between the sustainability of resource use and growth of the nations. When the resource augmenting technological progress arises, it is shown that, if the agent is patient, then resource extraction is reduced. We can also prove that, in the opposite preference case, resource use is promoted. These results present a significant policy implication for environmental conservation.

**Keywords:** Non-renewable resource; Resource augmenting technological progress; Sustainable development.

**JEL classification numbers:** O13, O41, Q32.

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# 1 Introduction

One of the main developments of modern macroeconomics is the attempt to explain economic growth, rather than to assume it. A literature on new growth theory (endogenous growth theory) provides significant implications for illuminating long-run growth of the economy.<sup>1</sup> To endogenize growth process, the literature has been concentrate on endogenous technological progress which is represented by innovation through research and development (R&D) activities, the accumulation of human capital, technological externality through learning-by-doing and so on. An essential result derived from the literature is that, under appropriate conditions, permanent (sustainable) growth is possible. Can this result be consistent with the existence of a limited stock of natural and environmental resources? If so, there would be no long-run trade-off between basic goods production and the stock of natural and environmental resources.<sup>2</sup>

The answer to the above question has been provided by many environmental economists who addressed the issue by adopting one or another among the several specifications proposed by the endogenous growth literature. For instance, many researchers have replied to the question by employing the endogenous accumulation processes of human or knowledge capital to the neoclassical growth model with environmental assets. In general, natural resources are classified into two types; i.e. “renewable resource” and “non-renewable resource”. In the following, we will review the previous works which explicitly introduce knowledge capital as a productive factor to the model with natural resource (renewable resource or non-renewable resource). Those relate to the present analysis directly.

As an important research stream, there are growth models with renewable environmental resource. Bovenberg and Smulders (1995; 1996) employ the theoretical framework proposed by Lucas (1988) where the accumulation of knowledge capital is the engine of economic growth and construct a two-sector growth model consists of a consumption good sector and an R&D sector. The latter sector generates knowledge about pollution-augmenting techniques. Under this framework, they show that better environmental quality improves factor productivity in the consumption good sector, and therefore tighter environmental policy leads to sustainable growth.

Another important branch is models with non-renewable resource. A pioneering model of Takayama (1980) emphasizes non-rival technological progress as the growth engine and investigates an optimal growth problem in which advancement of the state of technological knowledge is achieved only by engaging scarce resources in some positive quantities. As a result, the efficiency of human resources in the research sector must be sufficiently large to maintain sustainable growth of per capita consumption as well as to ensure the optimal growth

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<sup>1</sup>See for example Barro and Sala-i-Martin (1995).

<sup>2</sup>This issue is carefully studied in Beltratti (1997) and Carraro (1998).

problem.

In a recent paper, Barbier (1999) combines the influential models of Stiglitz (1974) and Romer (1990), and also examines the contribution of non-renewable resource to growth through innovation. Under existing resource availability constraints on R&D activity, he shows that it is possible to achieve a constant level of per capita consumption in the long-run, in other words, endogenous growth can overcome resource scarcity.

As we briefly mentioned above, there has been focus on the interaction between long-run economic growth and various environmental problems. In particular, the efficient energy utilization of non-renewable resources, like an exhaustible resource, has become an important problem. The aim of this paper is to reveal the role of knowledge capital for environmental improvement to resolve such a problem. Concretely, we investigate that the effect resource augmenting technological progress has on macroeconomic performances.

The present model is captured as a modified version of Stiglitz (1974) model. However, we introduce a resource augmenting technological progress, which is accelerated by the accumulation of environmental knowledge capital, into the Stiglitz model. The modeling strategy on environmental knowledge is very similar to the educational knowledge (human capital) model of Lucas (1988) and the R&D knowledge model of Romer (1990). We should notice that there is the fundamental difference between (environmental) knowledge and human capital occurs from rivalry and excludability. In contrast with human capital, disembodied environmental knowledge may be non-rival because that it can be spread freely over economic activities of arbitrary scale and may in some circumstances be non-excludable.

Briefly summarized, Bovenberg and Smulders (1995; 1996) and Barbier (1999) focus their attention on the relation between environmental R&D aspect and growth, while Takayama (1980) concentrates on resource saving labor input for environmental conservation. The main difference to existing contributions in the literature is that we investigate the role of the accumulation of environmental knowledge through non-paid labor input (acquiring environmental knowledge and information through education for environmental improvement) for sustainable development containing environmental conservation.<sup>3</sup>

The remainder of this article is organized in the following manner. Section 2 constructs a three-sector growth model with non-renewable environmental resource and the resource augmenting technological progress. In Section 3, we investigate the relation between the sustainability of resource use and growth of the nations. Furthermore, in Section 4 and 5, we analyze not only the effect of changes of various parameters on the growth rates of output, knowledge capital and resource use but the response of the level variables to the technological

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<sup>3</sup>The feature of this non-paid labor input provides the following important implications in the market economy. In this case, the agent cannot obtain factor payment from the activity of environmental knowledge accumulation. That is, the agent has no incentive to spend his time for the knowledge acquisition. See Section 6.

change. Section 6 provides the potential policy implications obtained from the results and refers to the market economy case of this model. Section 7 concludes.

## 2 The Model

We present in this section a three-sector growth model with basic goods production, resource extraction and knowledge capital production for environmental improvement. That is, this type of knowledge has the resource augmenting nature. The present model is captured as a modified version of Stiglitz (1974). However, we introduce the resource augmenting technological progress, which is accelerated by the accumulation of “environmental” knowledge capital, into the Stiglitz model. The modeling strategy on environmental knowledge is very similar to the educational (human capital) knowledge model of Lucas (1988) and the R&D knowledge model of Romer (1990). Population is assumed to be constant and is normalized to unity. Therefore, all economic variables in this paper are denoted by the lower-case letters.

### 2.1 Production Technologies

In the basic goods sector, output ( $y$ ) depends on physical capital ( $k$ ), the share of its time ( $u$ ) that the labor force ( $l$ ) spends in goods production, and the use of non-renewable resource ( $e$ ) with the level of resource augmenting environmental knowledge ( $a$ ).<sup>4</sup> This production function is assumed to exhibit constant returns to scale:

$$y = k^\alpha (ul)^\beta (a^\epsilon e)^{1-\alpha-\beta}, \quad \alpha, \beta > 0, \quad \alpha + \beta < 1, \quad \epsilon \in (0, 1], \quad (1)$$

where  $u \in [0, 1]$ . Basic goods are used for consumption and for investment in physical capital goods:

$$\dot{k} = y - c, \quad k(0) = k_0 > 0, \quad (2)$$

where  $c$  denotes a consumption, and the initial level of physical capital ( $k_0$ ) is given.<sup>5</sup>

There is a single representative agent endowed with one unit of lifetime. Since the time share  $u$  is devoted to basic goods production, then the remaining share  $1 - u$  is devoted to the acquisition of environmental knowledge. Following Lucas (1988), Romer (1990) and the modern formulation in the environmental literature; examples include Scholz and Ziemes (1999) and Schou (2000), we assume that the evolution of environmental knowledge depends linearly on the time fraction on activities for further knowledge acquisition and on the knowledge level already attained:

$$\dot{a} = \delta(1 - u)a, \quad \delta > 0, \quad a(0) = a_0 > 0, \quad (3)$$

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<sup>4</sup>From now on we will suppress the time argument when not needed for clarity.

<sup>5</sup>The variable with a dot denotes the derivative with respect to time.

where  $\delta$  is the efficiency parameter in the knowledge creation sector, and  $a_0$  is given. We should notice that there is the fundamental difference between (environmental) knowledge and human capital occurs from rivalry and excludability. In contrast with human capital, disembodied environmental knowledge may be non-rival because that it can be spread freely over economic activities of arbitrary scale and may in some circumstances be non-excludable.<sup>6</sup> As is well known, this type of a linear formulation exhibits ‘endogenous growth’.

The formulation of resource extracting activity is identical with Stiglitz (1974). That is, non-renewable resource is extracted at a rate  $e$  at every moment in time and the remaining resource stock is represented by  $s$ . The resource flow  $e$  is an input for basic goods production.

$$\dot{s} = -e, \quad s(0) = s_0 > 0, \quad (4)$$

where  $s_0$  is given. We follow Stiglitz (1974) and abstract from extraction costs as well as its uncertainty.

A path  $(y, c, u, e, k, a, s)_{t=0}^{\infty}$  is called feasible if: (a)  $k, a$  and  $s$  are continuous functions of  $t$ ; (b)  $y, c, u$  and  $e$  are piecewise continuous functions of  $t$ ; (c) the path satisfies Eq.(1) for all  $t \geq 0$ , and it satisfies Eqs.(2) and (4) for all  $t \geq 0$ , except at points of discontinuity of  $c, u$  and  $e$ ; and (d) for all  $t \geq 0$ , the path satisfies the non-negativity constraints

$$c, u, e \geq 0 \quad (5)$$

(for the control variables) and for all  $t \geq 0$

$$k, a, s \geq 0 \quad (6)$$

(for the state variables). These conditions Eqs.(4)-(6) on a feasible path imply the restriction

$$\int_0^{\infty} e(t)dt \leq s(0) \quad (7)$$

showing the finite upper bound on the cumulative extraction of resource over the infinite future.

## 2.2 Tastes

The utility of the representative infinitely-lived agent only depends on consumption. Assume utilitarian preferences with a constant rate of time preference  $\rho$ . Let instantaneous utility be  $U(c) = \frac{c^{1-\theta}-1}{1-\theta}$  where  $\theta > 0$  is a constant, the numerical value of the elasticity of marginal utility with respect to consumption.

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<sup>6</sup>For more detailed discussion, see Hosoya (2003, Ch.1).

The intertemporal utility function then is

$$\begin{aligned} \max_c \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} \exp(-\rho t) dt, \quad \rho > 0, \theta \neq 1, \\ \max_c \int_0^\infty \ln c \times \exp(-\rho t) dt, \quad \rho > 0, \theta = 1. \end{aligned} \quad (8)$$

### 3 The Optimal Solution

An omniscient social planner will want to maximize the utility of the representative household. Our problem is to choose  $c$ ,  $u$  and  $e$  so as to maximize the integral sum in Eq.(8) subject to Eqs.(2)-(4) and the usual non-negativity conditions including Eq.(6). In order to find the solution to this problem, we set up the current-value Hamiltonian  $\mathcal{H}$ :

$$\mathcal{H} \equiv \frac{c^{1-\theta} - 1}{1-\theta} + \lambda_1 \{k^\alpha u^\beta (a^\epsilon e)^{1-\alpha-\beta} - c\} + \lambda_2 \{\delta(1-u)a\} - \lambda_3 e, \quad (9)$$

where  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are the shadow values of physical capital, knowledge capital and non-renewable resource, respectively. As noted before, we set  $l = 1$  in the Hamiltonian.

Necessary first-order conditions for an interior optimal solution with respect to the three control variables  $c \geq 0$ ,  $u \in [0, 1]$  and  $e \geq 0$ , and three state variables  $k$ ,  $a$  and  $s$  are:

$$c^{-\theta} = \lambda_1, \quad (10)$$

$$\lambda_1 \beta \frac{y}{u} = \lambda_2 \delta a, \quad (11)$$

$$\lambda_1 (1 - \alpha - \beta) \frac{y}{e} = \lambda_3, \quad (12)$$

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - \alpha \frac{y}{k}, \quad (13)$$

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \frac{\lambda_1}{\lambda_2} \epsilon (1 - \alpha - \beta) \frac{y}{a} - \delta(1-u), \quad (14)$$

$$\frac{\dot{\lambda}_3}{\lambda_3} = \rho, \quad (15)$$

and three transversality conditions: <sup>7</sup>

$$\lim_{t \rightarrow \infty} \lambda_1(t) k(t) \exp(-\rho t) = 0,$$

$$\lim_{t \rightarrow \infty} \lambda_2(t) a(t) \exp(-\rho t) = 0,$$

$$\lim_{t \rightarrow \infty} \lambda_3(t) s(t) \exp(-\rho t) = 0.$$

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<sup>7</sup>In Appendix, it is proved that a balanced growth path is locally saddle point stable.

We examine a situation with balanced growth rates, defined as such a situation where all variables grow with constant (possibly zero or negative) rates. Along the balanced growth path (henceforth abbreviated BGP),  $y$ ,  $k$  and  $c$  must necessarily have the same growth rate, which we call  $g$ .<sup>8</sup> Also,  $a$  and  $e$  must be constant along the BGP, which we call  $g_a$  and  $g_e$ , respectively.

As will be explained in Appendix, differentiating with respect to time in the goods production function of Eq.(1) in the steady-state gives

$$g = \frac{\epsilon(1 - \alpha - \beta)}{1 - \alpha}g_a + \frac{1 - \alpha - \beta}{1 - \alpha}g_e. \quad (16)$$

Eq.(16) says that the growth rate of the economy  $g$  is represented by linear combination of  $g_a$  and  $g_e$ .

In Appendix, it is also shown that the steady-state growth rate for the production of basic goods (which is equal to the growth rate of physical capital and consumption) is

$$g = \frac{\delta\epsilon(1 - \alpha - \beta) - \rho(1 - \alpha)}{\theta(1 - \alpha)}. \quad (17)$$

Following Hartwick (1977), we define sustainable development as a situation that positive growth is attained, and this paper focuses attention on this case. So, from Eq.(17), we obtain Endogenous Growth Condition (henceforth abbreviated EGC) that assures sustainable development:

$$\delta\epsilon(1 - \alpha - \beta) - \rho(1 - \alpha) > 0. \quad (18)$$

The growth rate for knowledge capital is

$$g_a = \frac{\delta\epsilon(1 - \alpha - \beta)\{\beta + \theta(1 - \alpha - \beta)\} - \beta\rho(1 - \alpha)}{\theta\epsilon(1 - \alpha)(1 - \alpha - \beta)}. \quad (19)$$

The growth rates are derived under the assumption that we are looking at an interior path, which implies that labor force spends time in both the accumulation of knowledge capital and the production of basic goods. This will be secured by the following parameter restriction, which we impose:

$$\delta\epsilon(1 - \theta)(1 - \alpha - \beta) < \rho(1 - \alpha). \quad (20)$$

We call this condition Inner-Solution Condition (henceforth abbreviated ISC).

ISC in Eq.(20) ensures that the utility integral will converge and that the transversality conditions are fulfilled, and that the growth rate of resource extraction will be negative, as it must be along the BGP; in Appendix, it is shown that this growth rate will be:

$$g_e = \frac{\delta\epsilon(1 - \theta)(1 - \alpha - \beta) - \rho(1 - \alpha)}{\theta(1 - \alpha)}. \quad (21)$$

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<sup>8</sup>From now on we use the balanced growth path and the steady-state interchangeably as equivalent term.

As the basic property of Eq.(21), we get the following results.

**Lemma 1** At the BGP, the following holds: (a)  $g_s = g_e < 0$ , (b)  $e(0) = -g_e s(0)$  and (c)  $\lim_{t \rightarrow \infty} s(t) = 0$ .  $\square$

**Proof:** Consider the BGP situation. (a) From Eq.(4),  $g_s = -\frac{e}{s}$ ; differentiating with respect to time gives

$$\dot{g}_s = (g_e - g_s) \frac{e}{s} = 0,$$

by definition of the BGP. Hence,  $g_s = g_e$ . For any constant  $g_e$ , we have  $\int_0^\infty e(t) dt = \int_0^\infty e(0) \exp(g_e t) dt$ . If  $g_e \geq 0$ , Eq.(7) would be violated; hence,  $g_e < 0$  and

$$\int_0^\infty e(t) dt = -\frac{e(0)}{g_e}.$$

(b) By Eq.(4),  $\frac{s(0)}{s(0)} = -\frac{e(0)}{s(0)} = g_e$ , by applying (a) for  $t = 0$ ; hence,  $e(0) = -g_e s(0)$ . Finally, the solution for Eq.(4) can be written  $s(t) = s(0) \exp(g_s t)$ . Then, since  $g_s$  is a negative constant,  $s(t) \rightarrow 0$  for  $t \rightarrow \infty$ .  $\blacksquare$

Finally, Eq.(16) and Lemma 1 determine the sign of the growth rate of knowledge capital,  $g_a$ . That is to say,

**Lemma 2** In the steady-state, the growth rate of knowledge capital  $g_a$  is always positive.  $\square$

**Proof:** Applying the fact that EGC in Eq.(18) assures positive value of the economic growth rate and Lemma 1 to the linear combination of Eq.(16) reveals that the sign of  $g_a$  must be positive.  $\blacksquare$

## 4 Growth Effects: Comparative Statics

This section analyzes how changes of various parameters affect the growth rates of output, knowledge capital and resource use. The parameters we take up here are  $\epsilon$ ,  $\delta$ ,  $\theta$  and  $\rho$ . However, we focus on the effects of changes of  $\epsilon$  and  $\delta$  among four parameters in this paper.<sup>9</sup>

The effects of a rise in the efficiency of resource augmenting environmental knowledge,  $\epsilon$ , on  $g$ ,  $g_a$  and  $g_e$  will be:

$$\frac{\partial g}{\partial \epsilon} = \frac{\delta(1 - \alpha - \beta)}{\theta(1 - \alpha)} > 0, \quad (22)$$

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<sup>9</sup>We investigated how changes of  $\theta$  and  $\rho$  affect each growth rate. As for these results, we summarize in Table 1.



$$\frac{\partial g_a}{\partial \epsilon} = \frac{\beta \rho}{\epsilon(1 - \alpha - \beta)} > 0, \quad (23)$$

$$\frac{\partial g_e}{\partial \epsilon} = \frac{\delta(1 - \theta)(1 - \alpha - \beta)}{\theta(1 - \alpha)} \gtrless 0 \quad \text{as } \theta \lesseqgtr 1. \quad (24)$$

Eqs.(22) and (23) say that if  $\epsilon$  increases, the growth rates of output and knowledge capital rise. The sign of Eq.(24) depends on the size of  $\theta$ , that is to say, the sign is positive or negative as  $\theta$  is smaller than, or larger than unity. This means that if  $\theta$  is smaller than unity, in other words, the agent is patient, an increase in  $\epsilon$  will lead a more reduction of resource extraction. On the other hand, this also means that in the opposite preference case, that is, if the agent is impatient, resource use is promoted.

If the agent is patient, he will intend to invest more in knowledge capital accumulation. In this case, a rise in the efficiency of resource augmenting environmental knowledge implies that it is more efficient for him to produce basic goods by the accumulation of environmental knowledge rather than by the direct use of exhaustible resource. Hence, he will have an incentive to shift his precious time from basic goods production to the acquisition of environmental knowledge. Consequently, the growth rate of resource use declines and that of knowledge capital rises, and these effects result in a rise in the growth rate of output.

In the opposite case (the case that the agent is not patient), there is no incentive for him to reduce the use of natural resource, because it is more efficient to carry out goods production with natural resource rather than with knowledge capital. Needless to say, he has the incentive to increase his time devoted to the acquisition of knowledge, regardless of his preference is patient or impatient.<sup>10</sup> Therefore, both of the growth rates of natural resource use and knowledge capital increase.

The growth rates of output, knowledge capital and resource use will always be affected in the same direction if the efficiency parameter in the knowledge sector,  $\delta$ , become larger, namely that  $\frac{\partial g}{\partial \delta} > 0$ ,  $\frac{\partial g_a}{\partial \delta} > 0$  and  $\frac{\partial g_e}{\partial \delta} \gtrless 0$ , if  $\theta$  is smaller than, or larger than unity. Such the mechanism is the same as the case  $\epsilon$  increases.

## Implications from the Model

Table 1 displays the results obtained in this subsection. Note that for notational convenience in Table 1, we denote the substantial change of  $g_e$  is represented with a minus sign ( $-g_e$ ), because the sign of it is always negative from Lemma 1.

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<sup>10</sup>Generally, the patient agent would make a more investment comparing the agent is impatient.

<<<Inserting Table 1>>>

As mentioned above, the aim of this paper is to investigate the relation between the sustainability of resource use and economic growth, so we summarize these results from the viewpoint of sustainable development. Due to the definition on sustainable development we noted earlier, which means the attainment of positive growth in this paper, the results obtained above are classified broadly into the following two cases; first, the case that higher sustainable development is achieved; second, the case that the degree of it falls, although sustainable development is still accomplished.

Based on these aspects, we can get the following conclusions: (i) the shock that shifts the time from the activity of producing basic goods to the acquisition of environmental knowledge (a rise in the degree of resource augmenting environmental knowledge and an increase in the efficiency parameter in the knowledge creation sector) enables to promote the accumulation of environmental knowledge and to attain a higher growth rate; (ii) the shock that shifts the time from the acquisition of environmental knowledge to the activity of producing basic goods (an increase in the inverse of the elasticity of marginal utility with respect to consumption and a rise in the subjective rate of time preference) prevent the accumulation of environmental knowledge and lower the degree of positive growth. The conclusion (ii) is precisely corresponding to the case mentioned above that the degree of sustainable development falls.

## 5 What happens to the level variable to the technological shock?

This section provides an analysis which focuses on the response of the level variable to the technological shock. By using a four-quadrant diagram, we investigate that the effect of a rise in the knowledge efficiency  $\epsilon$  in goods production has on the major level variables  $y$ ,  $c$ ,  $e$ ,  $k$ ,  $s$  and  $u$ . This analysis will lead us to a better understanding about the relation between the sustainability of natural resource use and growth of the nations. To this end, new stationary variables are introduced into the present model. As a result, we can confirm the response of the major variables through observing changes in these stationary variables.

As will be defined in Appendix, we employ the following new stationary variables along the BGP:  $z_1 = \frac{y}{k}$ ,  $z_2 = \frac{c}{k}$  and  $z_3 = \frac{e}{s}$ , where each variable of the RHS denotes the ratio of resource utilization to the resource stock, or the rate of resource utilization. So, using Eqs.(B10), (B12) and (B13), let us derive the

loci for  $g_{z_1} = 0$ ,  $g_{z_2} = 0$  and  $g_{z_3} = 0$ . Consequently we have

$$\hat{z}_1 = \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha(1 - \alpha)}, \quad (25)$$

$$\hat{z}_2 = \left(1 - \frac{\alpha}{\theta}\right) \hat{z}_1 + \frac{\rho}{\theta}, \quad (26)$$

$$\hat{z}_2 = \hat{z}_3 + \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha}. \quad (27)$$

To obtain the  $g_u = 0$  locus, in addition, applying Eq.(27) to Eq.(B11) yields

$$\hat{z}_3 = \frac{\delta\epsilon(1 - \alpha - \beta)}{\beta} \hat{u}. \quad (28)$$

By employing Eqs.(25)-(28), we can draw the four-quadrant diagram.<sup>11</sup> For the values of  $\alpha$  and  $\theta$ , three possible cases are obtained: (Case I)  $\alpha < 1 < \theta$ , (Case II)  $\alpha < \theta < 1$  and (Case III)  $\theta < \alpha < 1$ .

**Case I (Small intertemporal elasticity):** Figure 1 shows Case I. We should first note the basic properties of this case as follows. If the initial levels of  $\frac{y}{k}$  and  $\frac{c}{k}$  are to be selected low enough relative to their equilibrium values, then the economy will increase the rate of resource utilization so as to raise the long-term equilibrium values of  $\hat{z}_1^*$  and  $\hat{z}_2^*$ . In the opposite initial situation, by reducing the rate of resource utilization, the economy will converge the lower long-run values of  $\hat{z}_1^*$  and  $\hat{z}_2^*$ . Next to investigate the response of the economy to the technological shock. When arising the technological shock in goods production through a rise in the efficiency of environmental knowledge, then the long-run equilibrium values  $\hat{z}_1^*$ ,  $\hat{z}_2^*$  and  $\hat{z}_3^*$  increase, respectively. On the other hand, the time share  $\hat{u}^*$  used to goods production decreases. Since the knowledge efficiency is increased in the production sector, the agent will make a more investment in knowledge capital accumulation. However, in this case, there is no incentive which boldly reduces the use of natural resource for the impatient agent. Due to the fact that the use of environmental resource is promoted, the economy described here is not environmental-friendly. In the view point of sustainable development, this result will probably be unfavorable. As a result, we obtain  $[\hat{z}_1^* (\uparrow), \hat{z}_2^* (\uparrow), \hat{z}_3^* (\uparrow), \hat{u}^* (\downarrow)]$ . Note that Figure 1 is justified for

<sup>11</sup>In this connection, we can calculate the equilibrium values of  $(\hat{z}_2, \hat{z}_3, \hat{u})$  under the given value  $\hat{z}_1$  (which is equal to its equilibrium value  $\hat{z}_1^*$ ). These are listed below:

$$\begin{aligned} \hat{z}_2^* &= \left(1 - \frac{\alpha}{\theta}\right) \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha(1 - \alpha)} + \frac{\rho}{\theta}, \\ \hat{z}_3^* &= \frac{\rho}{\theta} - \frac{\alpha\delta\epsilon(1 - \theta)}{\alpha(1 - \alpha)\theta}, \\ \hat{u}^* &= \left(1 - \frac{\alpha}{\theta}\right) \frac{\beta}{\alpha(1 - \alpha)} + \frac{\beta\rho}{\delta\epsilon(1 - \alpha - \beta)\theta} - \frac{\beta}{\alpha}, \end{aligned}$$

where the asterisk (\*) denotes the equilibrium value.

example under the following standard parameter values for some parameters:  $(\alpha, \beta, \theta, \rho, \epsilon, \delta) = (0.2, 0.6, 1.05, 0.02, 0.3, 0.5)$ .<sup>12</sup>

<<<Inserting Figure 1>>>

**Case II (Moderate intertemporal elasticity):** This case is depicted in Figure 2. The dynamic behavior of this case is almost identical with Case I. For the technological shock,  $\hat{z}_1^*$  and  $\hat{z}_2^*$  increase, respectively. On the other hand, both of  $\hat{z}_3^*$  and  $\hat{u}^*$  decrease. The agent in this case will increase the investment to knowledge capital creation and will act so that exhaustible resource use may be reduced, because that his preference has a relatively patient feature. Such the strategies are reflected in the movements of  $\hat{z}_3^*$  and  $\hat{u}^*$ . Needless to say, since the technological shock through a rise in the knowledge efficiency has a positive impact on goods production, then the rates of resource utilization  $\hat{z}_1^*$  and  $\hat{z}_2^*$  increase through a rise in the levels of output ( $y$ ) and consumption ( $c$ ).<sup>13</sup> To achieve a better situation for sustainable development, this case will be one of the desirable cases. Changes in four variables are listed as follows: [ $\hat{z}_1^*$  ( $\uparrow$ ),  $\hat{z}_2^*$  ( $\uparrow$ ),  $\hat{z}_3^*$  ( $\downarrow$ ),  $\hat{u}^*$  ( $\downarrow$ )]. Note that Figure 2 is justified for example under the following parameter values:  $(\alpha, \beta, \theta, \rho, \epsilon, \delta) = (0.2, 0.6, 0.5, 0.02, 0.3, 0.5)$ . The value for  $\theta$  satisfies our inequality condition. Other parameters are unchanged from Case I.

<<<Inserting Figure 2>>>

**Case III (Large intertemporal elasticity):** Figure 3 represents Case III. This case is different from the previous cases in that the dynamics of the model changes significantly, and therefore presents an important implication for the issues of sustainable development or environmental conservation. As well as Case I and II, let us investigate the economic response to the positive technological

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<sup>12</sup>Suppose that the environmental component has a large impact on the production activity of basic goods (see Eq.(1)), the parameterization on  $\alpha$  and  $\beta$  will be valid in view of the standard literature (see for example King and Rebelo, 1990). Next, the value for  $\theta = 1.05$  satisfies our inequality condition noted above, and  $\rho = 0.02$  is often employed in the growth literature (see for example Barro and Sala-i-Martin, 1995, Ch.5). As for the remaining parameters ( $\epsilon$  and  $\delta$ ), it is very difficult for us to find the appropriate consensus values in the relevant fields. The evidence for  $\epsilon$ , in particular, has never been reported in the literature so far. Therefore, as a tentative assumption, we suppose that the efficiency of environmental knowledge in goods production is not so large, and set the value of  $\epsilon = 0.3$ . Finally, we need to set the value of  $\delta$ . As explained in the earlier section, environmental knowledge we defined in this paper was obviously different from human capital. However, suitable parameter value for the efficiency of knowledge capital production is not easily found in the literature. On the other hand, according to the growth literature with human capital, we can confirm that the efficiency value for human capital production varies considerably depending on model specification or author's objective. In view of this point, by referring the case of human capital based growth, we assume tentatively  $\delta = 0.5$  for analytical convenience.

<sup>13</sup>If the agent has a considerably patient preference, level of consumption does not necessarily rise to the positive technological shock (see Case III).

shock arisen in the goods production sector. A rise in the degree of the knowledge efficiency on environmental knowledge leads to an increase in the rate of resource utilization on  $\hat{z}_1^*$  through an increase in  $y$ . On the other hand, the most patient agent of three possible cases will be reluctant to consume and will devote more of his precious time to accumulate environmental knowledge, because that the smaller  $\theta$  leads to the more slowly marginal utility falls as consumption increases and so the more willing the agent is to allow its consumption to vary over time. That is, both of  $\hat{z}_2^*$  and  $\hat{u}^*$  decrease. In comparison with the previous cases, one of the notable features of this case is a reduction in the equilibrium value for  $\hat{z}_2^*$  after the technological shock. Then, the agent also takes the scarcity of exhaustible resource into account, and intends to save natural resource use. Such a patient behavior of the agent is represented by change in  $\hat{z}_3^*$ . Under positive growth guaranteed, this case corresponds to the economy which enhances the sustainability of exhaustible resource at the expense of consumption level to some degree. From the view points of sustainable development or tighter environmental policy, the present case is most environmental-friendly. In summary,  $[\hat{z}_1^* (\uparrow), \hat{z}_2^* (\downarrow), \hat{z}_3^* (\downarrow), \hat{u}^* (\downarrow)]$ . Note that Figure 3 is justified for example under the following parameter values:  $(\alpha, \beta, \theta, \rho, \epsilon, \delta) = (0.2, 0.6, 0.15, 0.035, 0.3, 0.5)$ . The value for  $\theta$  satisfies our inequality condition. Finally, as for  $\rho$ , we have changed the value into 0.035 from 0.02. Such a new value is also included in the relevant range for  $\rho$ . Other parameters are unchanged from Case I. <sup>14</sup>

<<<Inserting Figure 3>>>

Throughout this section, we focus on the effects a rise in the knowledge efficiency in goods production have on the major level variables or the rates of resource utilization. It is interesting to note that the same results we obtained above apply to the case of a rise in the efficiency of environmental knowledge production. Similar properties were observed in the analyses of the growth effects developed in Section 4.

## 6 Discussion

Based on the results obtained in the previous analyses (Section 4 and 5), which support the achievement of sustainable development, we will discuss in this section the implications for environmental policies.

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<sup>14</sup>To assure the existence of the equilibrium, in this case, the equilibrium value for  $\hat{z}_2^*$  must be larger than the intercept of  $g_{z_3} = 0$  locus. This implies

$$\hat{z}_2^* = \left(1 - \frac{\alpha}{\theta}\right) \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha(1 - \alpha)} + \frac{\rho}{\theta} > \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha}.$$

Under the relevant parameter set noted above,  $(\alpha, \beta, \theta, \rho, \epsilon, \delta) = (0.2, 0.6, 0.15, 0.035, 0.3, 0.5)$ , this inequality condition is certainly satisfied:  $\left(\hat{z}_2^*, \frac{\delta\epsilon(1 - \alpha - \beta)}{\alpha}\right) = (0.171, 0.15)$ .

Recall that, following Hartwick (1977), we defined sustainable development as the accomplishment of positive growth with environmental factors. Considering this definition from the viewpoint of the sustainability of non-renewable resources which should be emphasized from the analytical purpose in this paper, the following additional definitions are derived: (i) sustainable development means the situation that positive growth is attained and non-renewable resource use is economized; (ii) in addition to (i), the situation that further restraint of non-renewable resource utilization is promoted. Note that the latter situation defined (ii) is more environmental-friendly than the former situation (i), so we assume that the latter situation corresponds to a **better** sustainable development.

Reconsidering the results obtained in the previous analyses in view of the definition (ii), we show that both conclusions presented by the result (i) in Section 4 and Case II and III in Section 5, respectively, support the better sustainable development defined above. Furthermore, it is also shown that both a rise in the knowledge efficiency ( $\epsilon$ ) in the goods sector and an increase in the production efficiency ( $\delta$ ) in the knowledge sector contribute to achieve the better sustainable development. Broadly speaking, it could be said that both of these effects represent the positive technological shock.

Then, what kinds of environmental policies enable environmental-friendly sustainable development? As mentioned earlier, knowledge capital ( $a$ ) is non-rival and non-excludable. In consideration of such the features of knowledge capital, one of the desirable environmental policies would be the public promotion of environmental education program or the public provision of the information for scarce natural resource.<sup>15</sup> Concretely, the following environmental policies would be emerged. First, we want to propose further improvement and promotion of environmental education in a schooling program at the stages of school education. In fact, for example, environmental education has been a part of the schooling program in Japan for several years. However, education on environmental problems is nothing but one of the integrated studies contained some topics such as welfare, health and information. In order to understand various environmental problems more deeply, we would expect the complete introduction of environmental education to the schooling program. Second, it is also essential to promote environmental education for firms or organization. For instance, this activity would lead firms or organization to construct a rich environmental management system conforms to the requirements of ISO 14001.

As a result, if such environmental policies permeate among community including consumers and firms, they would make it possible to form the basis of environmental conservation. This ideal process is precisely the philosophy of the Law for the Promotion of Environmental Education (tentative) will be enforced in Oct. 2003 in Japan, and corresponds to the role of knowledge capital in the present paper.

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<sup>15</sup>See for example World Bank (2000) indicates the promotion of environmental education pushes environmental standards higher.

Finally, we should note the optimal problem in a decentralized market economy. The critical difference between the social planning and the decentralized economy is the following two points. First, the agent in this economy will underestimate an intertemporal spillover effect on  $a$ , since environmental knowledge capital has non-rival and non-excludable features. That is, the accumulation process of knowledge capital is external factor for the agent's economic behavior. Second, the knowledge acquisition for the agent is non-paid educational activity. Because of these features, the agent has no incentive to spend his lifetime for the knowledge acquisition, in other words, he devotes all time to the basic goods production. So, he wants to maximize Eq.(8) subject to Eqs.(2) and (4). Consequently the goods production function will be changed into  $y = k^\alpha e^{1-\alpha}$ .<sup>16</sup> Therefore, the present decentralized market economy corresponds to Stiglitz (1974) economy. For the reasons stated above, we would not mention the decentralized economy.<sup>17</sup>

## 7 Concluding Remarks

In order to investigate the relation between the sustainability of resource use and growth of the nations, we have developed an endogenous growth model incorporating environmental knowledge capital and non-renewable natural resource. Based on the analyses of growth and level effects (in Section 4 and 5), we proposed the following important implications. The accumulation of environmental knowledge yields sustainable growth regardless of natural resource constraint; i.e. such the knowledge can overcome resource scarcity. In addition, under parametrically plausible situation, resource preservation and economic development can be compatible, and therefore it implies that sustainable development with environmental factor is feasible in our model. When we focus on the impact of resource augmenting technological progress, the patient agent attempts to reduce natural resource extraction. While the opposite preference agent promotes resource use.

From the view points of environmental policies, we emphasized the need for publicly promoting environmental education program or providing the information for scarce natural resource in order to achieve a true sustainable development. Such the activities should be performed using every opportunity including

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<sup>16</sup>In the command optimum, since labor force is normalized to unity, the production function is  $y = k^\alpha u^\beta (a^\epsilon e)^{1-\alpha-\beta}$ . On the other hand, in the market economy, the agent does not take into account the evolution of environmental knowledge in Eq.(3), and spends his all time to produce basic goods (i.e.  $u = 1$ ). Then, we can get  $a(t) = a(0) \exp\{\delta(1-u)\}t$  from Eq.(3). Since  $u = 1$ , the original functional form of goods production will be modified  $y = k^\alpha \{a(0)^\epsilon e\}^{1-\alpha}$  (where  $a(0)$  is a positive constant). For simplicity, assuming  $a(0) = 1$ , the production function takes the Cobb-Douglas form for physical capital and non-renewable resource. In view of these points, the goods production function eventually reduces to  $y = k^\alpha e^{1-\alpha}$ .

<sup>17</sup>For decentralized economy, see Stiglitz (1974).

education for school, firms and organization.

Finally, our model should be extended in several ways. In this paper, we focused only on the public aspects of knowledge; i.e. non-rival and non-excludable features on environmental knowledge capital. Environmental knowledge with such the features certainly exists in the real world. To resolve a large number of environmental problems exist in the world, environmental knowledge accumulation will play a significant role. However, such a type of knowledge is also accumulated through profit seeking R&D activities, in particular “non-excludable” knowledge becomes “excludable” when it is used for production with a patent system (e.g. Scholz and Ziemes 1999). Therefore, we would need to investigate carefully the role of environmental knowledge capital for sustainable development in a more broad sense. The task is our future research direction.



# Appendix

## A. Derivation of the balanced growth rates

Along the BGP,  $y$ ,  $k$  and  $c$  must necessarily have the same growth rate, which we denote  $g$ . Also, if  $a$  grows with a constant rate,  $u$  must be constant. Using these results, differentiating logarithmically with respect to time in Eqs.(10)-(12) in the steady-state yields

$$-\theta g = g_{\lambda_1}, \quad (A1)$$

$$g_{\lambda_1} + g = g_{\lambda_2} + g_a, \quad (A2)$$

$$g_{\lambda_1} + g - g_e = g_{\lambda_3}. \quad (A3)$$

Differentiating with respect to time in the production function of Eq.(1) in the steady-state gives Eq.(16):

$$g = \frac{\epsilon(1 - \alpha - \beta)}{1 - \alpha} g_a + \frac{1 - \alpha - \beta}{1 - \alpha} g_e.$$

To eliminate the growth rate of co-state variables in Eq.(A3), substituting Eqs.(A1) and (15) into Eq.(A3) gives the following relation:

$$(1 - \theta)g = g_e + \rho. \quad (A4)$$

By applying  $u = 1 - \frac{g_a}{\delta}$  obtained from Eq.(3) into  $g_{\lambda_2} = \rho - \frac{\delta\epsilon(1-\alpha-\beta)}{\beta}u - \delta(1-u)$  obtained from Eqs.(11) and (14), we can obtain

$$g_{\lambda_2} = \rho - \frac{\delta\epsilon(1 - \alpha - \beta)}{\beta} \left(1 - \frac{g_a}{\delta}\right). \quad (A5)$$

Substituting  $g_{\lambda_2} = (1-\theta)g - g_a$  derived from Eqs.(A1) and (A2) into Eq.(A5) yields the following Eq.(A6):

$$(1 - \theta)g = \rho - \frac{\delta\epsilon(1 - \alpha - \beta)}{\beta} \left(1 - \frac{g_a}{\delta}\right). \quad (A6)$$

Eqs.(16) , (A4) and (A6) are three differential equations for three unknown variables,  $g$ ,  $g_e$  and  $g_a$ . Solving the system and after some arrangements, we can get Eqs.(17), (19) and (21).

From Eq.(17), the following condition must be met to achieve sustainable development

$$\frac{\delta\epsilon(1 - \alpha - \beta) - \rho(1 - \alpha)}{\theta(1 - \alpha)} > 0. \quad (A7)$$

As the denominator is positive, the condition for sustainable development becomes Eq.(18) in the text.

Next, labor must spend time in both knowledge capital accumulation and the production of basic goods to ensure inner-solution. It means that the growth rate of knowledge capital must be more than 0 (in the case of  $u = 1$ ) and be less than  $\delta$  (in the case of  $u = 0$ ), that is to say,  $0 < g_a < \delta$ . From Lemma 2, positive definiteness of the growth rate of knowledge capital in the steady-state is assured, so ISC becomes

$$0 < \frac{\delta\epsilon(1-\alpha-\beta)\{\beta+\theta(1-\alpha-\beta)\}-\beta\rho(1-\alpha)}{\theta\epsilon(1-\alpha)(1-\alpha-\beta)} < \delta. \quad (A8)$$

From Eq.(A8), we can obtain Eq.(20).

## B. Proof that the optimal path is locally saddle point stable

Define the following variables that will be constant along the BGP:

$$z_1 \equiv \frac{y}{k}, \quad (B1)$$

$$z_2 \equiv \frac{c}{k}, \quad (B2)$$

$$z_3 \equiv \frac{e}{s}. \quad (B3)$$

Then,

$$g_k = z_1 - z_2, \quad (B4)$$

$$g_{z_1} = g_y - z_1 + z_2, \quad (B5)$$

$$g_{z_2} = g_c - z_1 + z_2, \quad (B6)$$

$$g_{z_3} = g_e + z_3. \quad (B7)$$

From the goods production function we can get

$$g_y = \alpha g_k + \beta g_u + \epsilon(1-\alpha-\beta)g_a + (1-\alpha-\beta)g_e. \quad (B8)$$

We can obtain  $g_e = g_y - \alpha z_1$  from Eqs.(12), (13) and (15), and  $g_u = g_y - \alpha z_1 + \frac{\delta\epsilon(1-\alpha-\beta)}{\beta}u$  from Eqs.(11), (13) and (14), respectively. Applying Eqs.(3), (B1) and these equations obtained above into Eq.(B8), and arranging it gives

$$g_y = \alpha z_1 + \frac{\delta\epsilon(1-\alpha-\beta)}{\alpha}. \quad (B9)$$

Substitute Eq.(B9) into Eq.(B5) and  $g_u = g_y - \alpha z_1 + \frac{\delta\epsilon(1-\alpha-\beta)}{\beta}u$  obtained above yields

$$g_{z_1} = (\alpha - 1)z_1 + \frac{\delta\epsilon(1-\alpha-\beta)}{\alpha}, \quad (B10)$$

$$g_u = \delta\epsilon(1-\alpha-\beta) \left( \frac{\beta + \alpha u}{\alpha\beta} \right) - z_2. \quad (B11)$$

From Eqs.(10), (13) and (B6), we can derive the following relation:

$$g_{z_2} = \left(\frac{\alpha}{\theta} - 1\right) z_1 - \frac{\rho}{\theta} + z_2. \quad (B12)$$

Finally, from Eq.(B7) and  $g_e = g_y - \alpha z_1$ , we can get

$$g_{z_3} = \frac{\delta\epsilon(1 - \alpha - \beta) - \alpha z_2}{\alpha} + z_3. \quad (B13)$$

The dynamical system  $(z_1, z_2, z_3, u)$  is completely described by Eqs.(B10) , (B12) , (B13) and (B11). Along the BGP,  $g_{z_1} = g_{z_2} = g_{z_3} = g_u = 0$ , so that  $z_1 = \hat{z}_1$  (where a hat denotes the steady-state value of a corresponding variable), etc. We can form the Jacobian, evaluated at the steady-state:

$$J = \begin{bmatrix} \frac{\partial z_1}{\partial z_1} & \frac{\partial z_1}{\partial z_2} & \frac{\partial z_1}{\partial z_3} & \frac{\partial z_1}{\partial u} \\ \frac{\partial z_2}{\partial z_1} & \frac{\partial z_2}{\partial z_2} & \frac{\partial z_2}{\partial z_3} & \frac{\partial z_2}{\partial u} \\ \frac{\partial z_3}{\partial z_1} & \frac{\partial z_3}{\partial z_2} & \frac{\partial z_3}{\partial z_3} & \frac{\partial z_3}{\partial u} \\ \frac{\partial u}{\partial z_1} & \frac{\partial u}{\partial z_2} & \frac{\partial u}{\partial z_3} & \frac{\partial u}{\partial u} \end{bmatrix} = \begin{bmatrix} (\alpha - 1)\hat{z}_1 & 0 & 0 & 0 \\ \left(\frac{\alpha_1}{\theta} - 1\right)\hat{z}_2 & \hat{z}_2 & 0 & 0 \\ 0 & -\hat{z}_3 & \hat{z}_3 & 0 \\ 0 & -\hat{u} & 0 & \frac{\delta\epsilon(1-\alpha-\beta)}{\beta}\hat{u} \end{bmatrix}.$$

We can verify that

$$Det[J] = \frac{\hat{z}_1 \hat{z}_2 \hat{z}_3 \hat{u} \delta\epsilon (\alpha - 1) (1 - \alpha - \beta)}{\beta} < 0, \quad (B14)$$

$$Trace[J] = (\alpha - 1)\hat{z}_1 + \hat{z}_2 + \hat{z}_3 + \frac{\delta\epsilon(1 - \alpha - \beta)}{\beta}\hat{u} > 0, \quad (B15)$$

altogether implying that there must be one-negative and three-positive eigenvalues, so that the dynamical system is saddle point stable around the steady-state.

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Parameter	$g$	$-g_e$	$g_a$
$\epsilon$	+	- (or +)	+
$\delta$	+	- (or +)	+
$\theta$	-	+	-
$\rho$	-	+	-

Table 1: The Effects of Various Parameters on Each Growth Rate

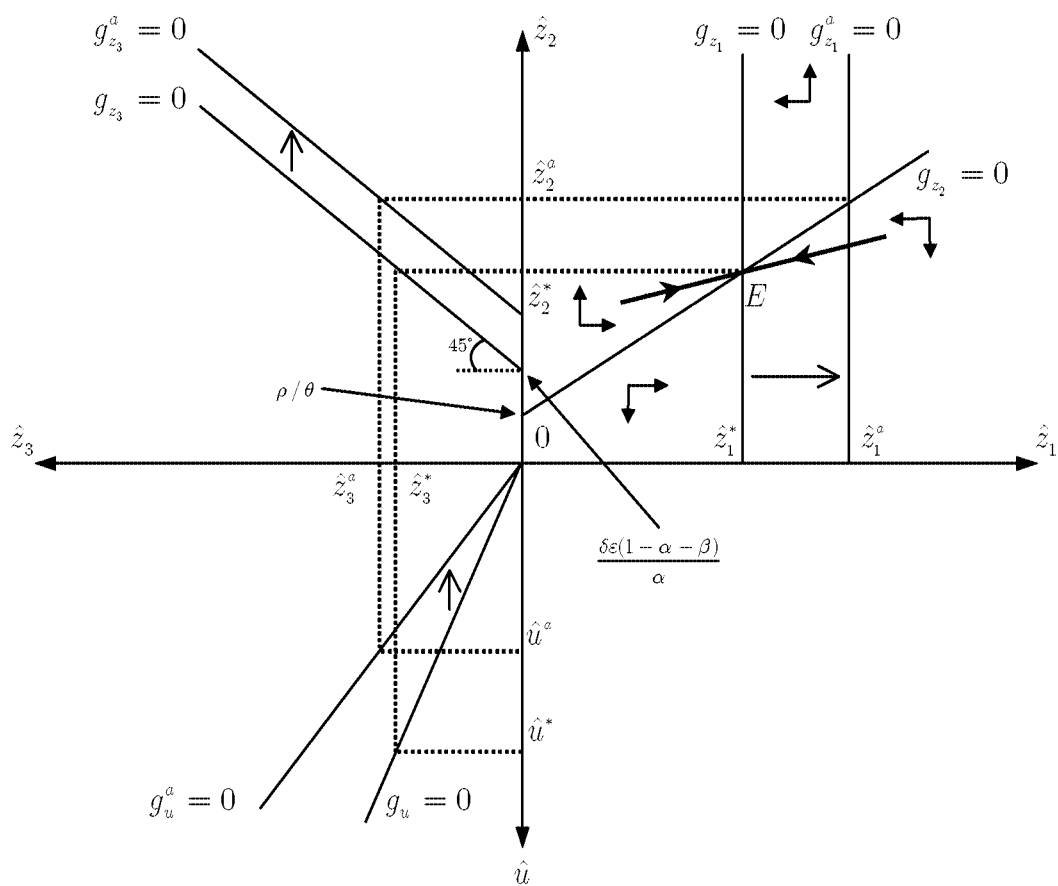


Figure 1: A Four-Quadrant Diagram (Case I)

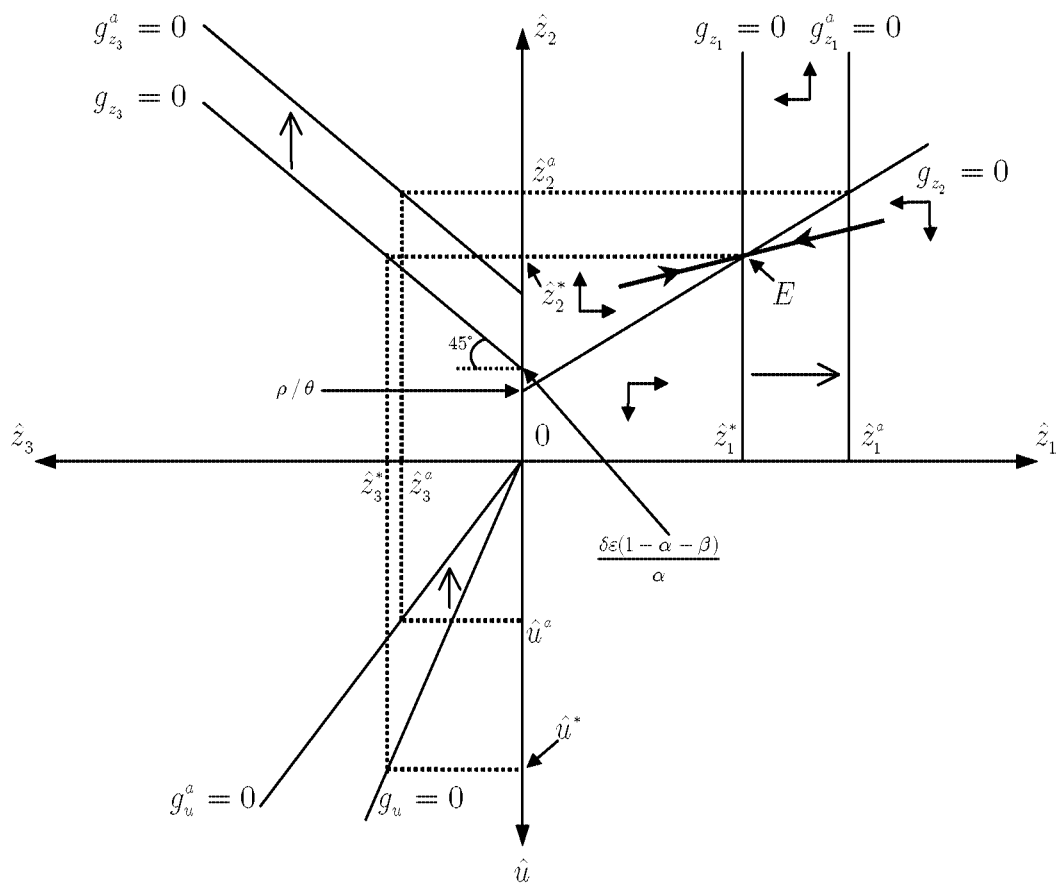


Figure 2: A Four-Quadrant Diagram (Case II)



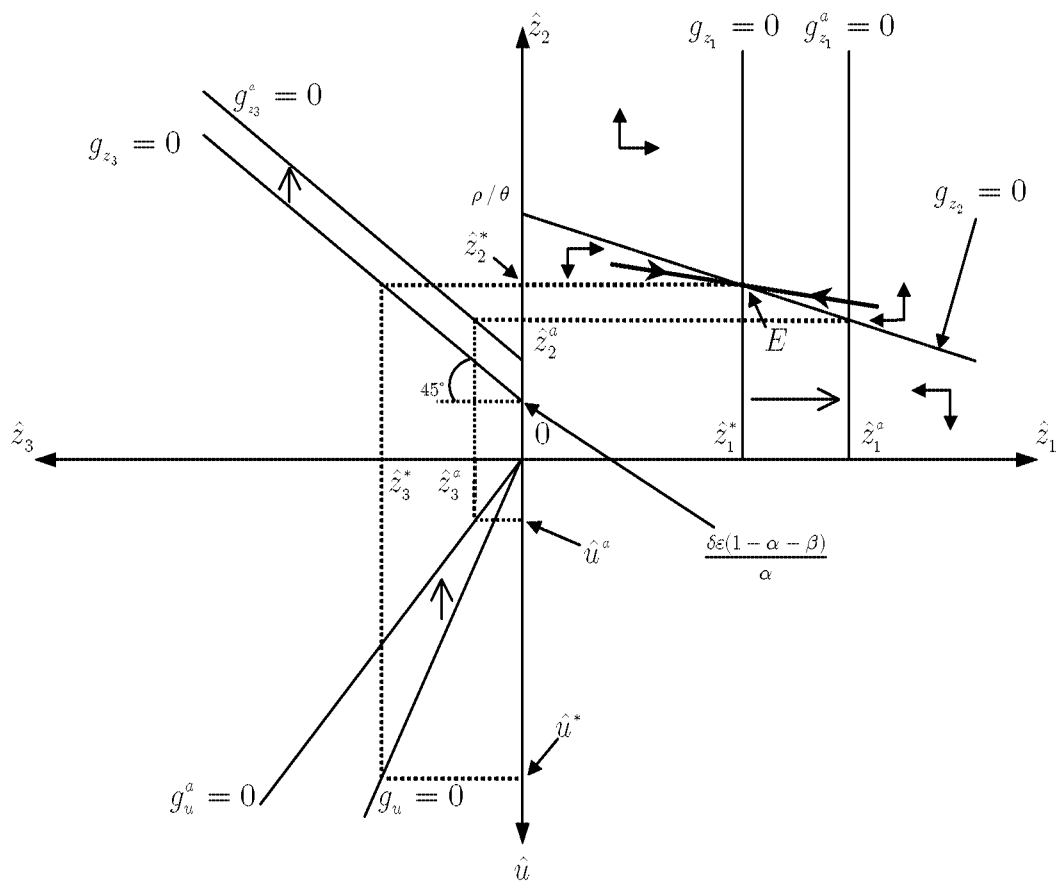


Figure 3: A Four-Quadrant Diagram (Case III)