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Tomoaki Yamada

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Hi-Stat
Institute of Economic Research
Hitotsubashi University
2-1 Naka, Kunitatchi Tokyo, 186-8601 Japan
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Income Risk, Consumption Inequality, and Macroeconomy in Japan*

Tomoaki YAMADA†

Rissho University

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Abstract

In this paper, using an OLG model with heterogeneous households, we investigate economic inequality in the recent decades in Japan. We decompose the causes of economic inequality into macroeconomic factors and a demographic factor, and demonstrate that the earning inequality in the model replicates the actual evolution of inequality in Japan. Based on a counterfactual simulation, we demonstrate that time-varying macroeconomic factors play an important role in the evolution of economic inequality. In particular, we show that the low growth rate of total factor productivity in the 1990s in Japan limited the dispersion of economic inequality.

Keywords: Income risk, Consumption inequality, Population aging

JEL Classification: E21, D11, D31, D91

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†Faculty of Economics, Rissho University: 4-2-16 Osaki Shinagawa-ku, Tokyo 141-8602, Japan. E-mail: tyamada@ris.ac.jp. Phone: +81-3-5487-3239.

1 Introduction

The life cycle permanent income hypothesis (LC-PIH) is a key concern when considering the consumption of households. According to the LC-PIH, income inequality measured at one period does not necessarily reflect economic inequality over the life cycle. Since income inequality includes temporary variation in income, even if income inequality rises, consumption inequality may not rise when households are prepared for such a temporary income shock. Therefore, to investigate economic inequality, we need to simultaneously consider the characteristics of income inequality and consumption inequality. Households face various idiosyncratic risks on labor income, such as career success/failure, a bonus cut, and unemployment, all of which generate income inequality.[1] In addition to income risk, the macroeconomic environment, including technical change and factor prices, may also affect economic inequality through households' decision making.

In this paper, we consider the following questions: what kinds of factors affect the evolution of economic inequality and how big their impacts are? In addition to the idiosyncratic income risks that households face, many factors cause macroeconomic inequality. To investigate the mechanism of economic inequality, we decompose the causes into two factors: (i) macroeconomic factors and (ii) the demographic factor. The first factor to affect economic inequality is time-varying macroeconomic variables, such as the total factor productivity (TFP) growth rate, labor share, and interest rate.[2] The

[1] The characteristics of income risks have direct implications for policy making. If the income shock is temporary, opening insurance markets or risk sharing within the family may be included in economic policy. On the other hand, if the shock is permanent, a different policy may be required. For example, Genda et al. (2007) show that cohorts who enter the economy in recession receive a negative shock on life time-income. This is a typical permanent income shock, which cannot be shared through market transaction. In such a case, to improve household welfare, the government needs to introduce a redistribution policy.

[2] Ríos-Rull and Santaeulàlia-Llopis (2008) showed the labor share as volatile and countercyclical with the TFP. They find that the bivariate specification of productivity shock (TFP) and redistributive shock (labor share), generate the observed dynamics of the business cycle in the US. We include the time-

Japanese economy is a good example in which to consider the relationship between macroeconomic factors and economic inequality, because Japan has experienced large booms and major recessions. Moreover, there have been several substantial changes in macroeconomics variables. For example, as in Table 1, the TFP growth rate in Japan in the 1990s was extremely low. Hayashi and Prescott (2002) demonstrate that a standard neoclassical growth model with calibrated parameters could explain Japan's deep recession, including the so-called lost decade of the 1990s.[3] Many subsequent papers have been published that investigate the lost decade using a dynamic general equilibrium framework. Chen et al. (2006,2007) and Braun et al. (2007) construct a general equilibrium model and explain the history of Japanese saving rates, based on macroeconomic factors and demographic change. If macroeconomic factors explain the change in the saving rate, then the same may be true of economic inequality. Thus, we examine economic inequality to explain the Japanese economy using the same approach. Second, empirical researches, such as Ohtake and Saito (1998), have pointed out that the dispersion of economic inequality in the Japanese economy in 1980s is largely due to population aging. As we show later, income and consumption inequality rises by age. Therefore, if the proportion of middle and old households increase due to aging, then observed total economic inequality, measured by variance of logarithm or Gini coefficient, also appears to increase. Even when considering economic inequality with a dynamic general equilibrium model, we cannot ignore the effect of the demographic factor.

Our analysis is at the cross roads of two kinds of literature. The first is quantitative macroeconomics research on the Japanese economy after Hayashi and Prescott (2002). As stated above, Hayashi and Prescott (2002) show, using a standard neoclassical growth

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varying labor share as macroeconomic factors, to consider whether the reallocation of labor earners and capital earners affect economic inequality.

[3]In Kehoe and Prescott (2007), and in addition to Hayashi and Prescott (2002), many other researchers investigate deep recessions in several countries, based on the dynamic general equilibrium framework. See also Cole and Ohanian (1999).

model, that the time variation of TFP growth, and the reduction in work hours, have a great impact on the lost decade. Moreover, Chen et al. (2006,2007) and Braun et al. (2007) extend their results to explain the history of Japan's saving rate. We consider our research as an extension of this literature, because it investigates the second moment in the model. In the existing literature, many authors using the general equilibrium model explain macroeconomic variables, such as GDP or the saving rate (*first moments*) of the Japanese economy. We consider economic inequality (variance or *second moments*) using quantitative analysis. The second is empirical research on household consumption/saving decision. In particular, our approach is similar to that of Heathcote et al. (2004,2008), who investigated the link between income risk and consumption inequality in the US. Since there are many researches on this topic, particularly in the US., we review this literature in the next section. Generally, modeling a stochastic income risk process requires detailed microdata for a long period.[4] If panel data is available, it will be possible to investigate the nature of economic inequality, and, in particular, to determine whether the idiosyncratic risks are affected by business cycles. Heathcote et al. (2004) decompose the property of income shock, using the Panel Study of Income Dynamics (PSID). However, there are no comparable panel data for Japan.[5] Instead, our approach in this paper is as follows. Based on a dynamic general equilibrium model in which the LC-PIH holds, we generate simulated economic inequality paths after controlling some factors. In other words, we decompose the income/consumption inequality into *explained* and *unexplained* components, through counterfactual simulations.

To include the impact of demographic change, we construct an overlapping generations (OLG) model with long-living households. The details of the model are as

[4] An incomplete list includes Abowd and Card (1989), Attanasio and Davis (1996), Blundell and Preston (1998), Blundell, Pistaferri, and Preston (2006), Guvenen (2007), Heathcote et al. (2004,2005,2008), and Storesletten et al. (2004b).

[5] Using long-term income tax statistics, Moriguchi and Saez (2007) investigate Japanese income inequality over 100 years. Since they depend on income tax statistics, they were unable consider consumption inequality.

follows. There are many households with different age groups, who make decisions on consumption and saving. In each cohort, a continuum of households exists and they face idiosyncratic income risk. Although they are identical before entering the economy, idiosyncratic income risks occur. As a result, in their middle and old age, the households differ in their wealth, labor supply, and consumption. Aggregating the heterogeneous households, we consider a general equilibrium of the model. By including a demographic structure and exogenous macroeconomic variables, we calibrate our model to the Japanese economy and use it as an experimental tool.

Our results are as follows. First, using the OLG model with calibrated parameters, the macroeconomic variables such as the interest and saving rate (first moments), and income/consumption inequality over life cycle, are well replicated between 1980 and 2000. Second, our model explain the time path of earning inequality in recent decades. Third, it is difficult to explain consumption inequality using the model, particularly in the 1980s. On the basis of counterfactual experiments, we find that earning, income, and consumption inequalities are reduced by the low TFP growth rate in the lost decade, although the legally restricted reduction in work hours in late the 1980s increases income inequality. Fourth, although the demographic factor gives rise to the positive trend of earning inequality, as empirical research shows, the macroeconomic factors also contribute to a positive trend of inequality. Lastly, we find that preference change, and the credibility of the social security system, may be causes of recent rises in consumption inequality.

The remainder of this paper is organized as follows. In Section 2, before constructing our dynamic general equilibrium model, we begin by reviewing empirical facts in Japan, and we define the income risks, that are applied in our model. In Section 3, we construct an OLG model with heterogeneous households and idiosyncratic income risks. In Section 4, we calibrate the parameters for the Japanese economy. In Section 5, we discuss income and consumption inequality in Japan that is based on the numerical results. In Section

6, we confirm the sensitivity of our analysis. Section 7 concludes the paper.

2 Empirical Facts on Earning, Income and Consumption Inequality in Japan

2.1 Economic Inequality in Japan

Before constructing an OLG model, we explain economic inequality in Japan, and review the research on the relationship between income risk and consumption inequality in the literature. In this paper, we consider two aspects of economic inequality: the life cycle dimension and the time series dimension. As a life cycle dimension, we focus on earning, income, and consumption inequality by age, and as a time series dimension we consider the transitional dynamics of earning, income, and consumption inequality over time.

Given the limited availability of micro data, there are not many empirical studies on economic inequality in Japan. In particular, there exist few available data set for consumption inequality. Ohtake and Saito (1998) and Abe and Yamada (2006) estimate the variance of log income and consumption, based on *The National Survey of Family Income and Expenditure* (NSFIE), which is conducted every five years by the Statistical Bureau of the Japanese Government. The NSFIE is a large survey with a sample size of about 50,000, which makes it one of the largest household surveys in the world.[6] Ohtake and Saito (1998) used the surveys of 1979, 1984, and 1989, and Abe and Yamada (2006) used those of 1984, 1989, 1994, and 1999.

[6] Ohtake and Saito (1998) used samples of households with two or more members, engaged in non-agricultural sectors, and heads aged 22–75. Abe and Yamada (2006) used the samples of households with two or more members, male household heads aged 25-70, engaged in non-agricultural sectors or self-employed or a company director, and the household who have fewer than nine households members. Each sample contains over 40,000 households. For detailed description of the NSFIE, see Abe and Yamada (2006).

We denote income for individual i at age j as y_j^i, and also denote the household's observable variables as $X_{i,t}$.[7] Then, we define the stochastic component of income e_j^i, as follows:

$$\ln y_{j,t}^i = \beta_{0,t} + f(X_{i,t}, \beta_{1,t}) + e_j^i.$$

In Panel (a) of Figure 1, from Abe and Yamada (2006), we show the variance of residuals obtained from the income regression stated above, which represents the life cycle dimension of income risk. In Panel (b) of Figure 1, we also plot the age profile of consumption inequality, constructed in the same way. From a life cycle point of view, income inequality rises by age in Japan. This observation is consistent with the empirical research by Storesletten et al. (2004a,b), who show that income and consumption inequality rises by age in the US. However, the shapes of the two age-variance profiles are not similar each other. Following Storesletten et al. (2004a), the variance of log-income increases almost linearly in the US, and the consumption variance profile is concave over age. In Japan, however, the consumption variance profile is convex over age: the variance of consumption is flat, or rather decreases, when households are young, and rises sharply after middle age.

Panel (c) and Panel (d) of Figure 1 show the time path of income and consumption inequality between 1979 and 1999, and indicate that the economic inequality of the time series dimension in Japan has also risen in recent decades.[8] To focus on the trend of economic inequality, we plot the percentage deviation from the mean. Using the same data from the NSFIE for 1984, 1989, 1994, and 1999, Kohara and Ohtake (2006) also estimate before-tax income and consumption inequality over time.[9] Recent empirical

[7]The control variables include dummy variables for the number of household members, area information, employment status of the household head, the number of employed household members, the industry the household head is employed in, and the type of job. For more detailed information, see Table 1 in Abe and Yamada (2006).

[8]Ohtake and Saito (1998) find that a large fraction of rise in income inequality in 1980s is due to population ageing.

[9]The definition of income used by Kohara and Ohtake (2006) includes, in addition to the earnings,

studies in the US, such as Krueger and Perri (2005) and Heathcote et al. (2004,2008) find that income inequality rises sharply in the 1980s, and stays higher level in the 1990s. Interestingly, the consumption inequality does not increases as much as income inequality. Compared with the estimation by Heathcote et al. (2004,2008), the rise of income inequality compares favorably with that in the US, and the positive trend of consumption inequality is slightly more moderate in Japan.

2.2 Idiosyncratic Income Risk

The estimation of income risks is extensively examined in the literature.[10] Since panel data over a long period, such as the PSID, are available in the US, many researchers estimate income risks by specifying a stochastic process. For example, Storesletten et al. (2004b) estimate the income risk by developing a generalized method of moments (GMM) estimator and a variation in the cross-sectional variance between households.

Storesletten et al. (2004a,b) decompose the residual e_j^i into three factors: (a) a fixed effect α^i, (b) a persistent shock η_j^i, and (c) a transitory shock z_j^i. They estimate the variances of these shocks $\{\sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$ and a persistence parameter of the shock ρ from the following equations:

$$e_j^i = \xi_t^f \alpha^i + z_j^i + \xi_t^t \varepsilon_j^i, \;\; \alpha \sim \mathcal{N}\left(0, \sigma_\alpha^2\right), \;\; \varepsilon_j \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \tag{1}$$

$$z_j^i = \rho z_{j-1}^i + \xi_t^p \eta_j^i, \;\; \eta_j \sim \mathcal{N}(0, \sigma_{\eta,j}^2), \tag{2}$$

where $\{\xi_t^f, \xi_t^p, \xi_t^t\}$ are time-varying loading factors for measuring the size of each shock over the business cycle.[11]

interest and dividend income. Since the interest and dividend income in the NSFIE in the sample is too small, Kohara and Ohtake (2006) compute the capital income from asset holdings (bank deposit) and the average interest rate.

[10] The research described below focuses on the *natures* of the income process, not on the *causes* of income inequality. With regard to the causes of income dispersion, for example, Acemoglu (2002) emphasizes the role of technological change.

[11] It would be possible to include time effects and cohort effects in equation (1). Heathcote et al. (2005)

Using the PSID, Storesletten et al. (2004b) showed that the autocorrelation coefficient ρ is at a highly persistent level of 0.95 in the US, which is consistent with the findings by Deaton and Paxson (1994). Heathcote et al. (2004) estimates the size of the time-varying factors $\{\xi_t^f, \xi_t^p, \xi_t^t\}$. Ohtake and Saito (1998) show that the variance of the logarithm of income increases with age in Japan as in the US. Moreover, the slope of the variance also increases over age 50. This observation implies that the autocorrelation coefficient ρ may be over one in Japan. In fact, Abe and Yamada (2006) estimate the stochastic process (2) using the variance structure in Japan, and find that it is over one.

2.3 LC-PIH and Consumption Inequality

Following the income risk specification of equations (1) and (2), it is well known from the LC-PIH that each shock has different implications for consumption inequality. If fixed effects such as educational background are large, there may be high consumption inequality because such a shock is uninsurable through the saving or insurance markets. On the contrary, a transitory shock has little effect on consumption inequality because of consumption smoothing over a period, if insurance markets function effectively or the amount of precautionary saving is sufficient.[12]

Following these ideas, Heathcote et al. (2004) investigate the relationship between income risk and consumption inequality in the US, using a dynamic general equilibrium framework. In the US, there was a sharp rise in income inequality in the 1980s. However, according to the Consumer Expenditure Survey, consumption inequality did not increase

show that the time effects largely account for the observed trends in inequality in the US, whereas the cohort effects are less important.

[12] If insurance markets are complete, the marginal utilities of all households coincide, and the growth rates of the individual's consumption are proportional to aggregate consumption. Based on this theoretical result, Hayashi, Altonji, and Kotlikoff (1996) test the complete market hypothesis in the U.S. Attanasio and Davis (1996) reveal that the insurance market works if the shock is temporary, but that infrequent shocks are not shared. Moreover, Kohara et al. (2002) test the complete market hypothesis in Japan.

significantly in the same period. On the basis of the income risk model stated above, these facts can be explained by the increase in the size of the transitory shock ξ_t^t. If the transitory shock is large, income inequality appears to rise, although the corresponding consumption inequality does not change from the LC-PIH. On the other hand, the period of rising consumption inequality can be explained by the increase in the size of the fixed effect or the persistent shock, i.e., $\{\xi_t^f, \xi_t^p\}$. From the same observation, Blundell et al. (2008) show that income dispersion and lack of dispersion in consumption inequality stem from the partial insurance of permanent shocks. Krueger and Perri (2005) show that, even if the income risk disperses in the model with limited commitment, consumption inequality does not necessarily disperse in the model, or in empirical studies.

These studies emphasis the link between income risk and consumption inequality. However, they do not consider the importance of macroeconomic factors such as TFP growth. In this paper, given the idiosyncratic income risk in Japan, we consider whether such macroeconomic factors generate economic inequality in recent decades in Japan.

3 Overlapping Generations Model

We used an OLG model with a continuum of heterogeneous households, developed by Aiyagari (1994) and extended to a life cycle model by Huggett (1996).

3.1 Demographics

We considered the OLG model with long-living households.[13] Households enter the economy at age 20 and live to a maximum age of 100. However, they face mortality risk and may die before the age of 100. $\mu_{j,t}$ denotes the population at age $j \in \mathcal{J} =$

[13]Our model includes population dynamics and the time variation of total factor productivity. Thus, to numerically solve the model, we need to remove the trends of all macroeconomic variables by the growth rates. For details, see Appendix.

$\{0, \ldots, 100\}$ at time t. In the next period, a fraction of the households $(1 - \phi_{j,t})$ dies and exits the economy. Thus, the size of each cohort evolves as follows:

$$\mu_{j+1,t+1} = s_{j,t}\mu_{j,t}$$

We denote the population growth rate of age 0 from time t to $t + 1$ as ψ_t. Thus, the new population at period $t + 1$ is determined from $\mu_{0,t+1} = (1 + \psi_t)\mu_{0,t}$. Because households are in their childhood at $j = 0, 1, \ldots, 19$, they do not engage in consumption or employment, but they are included in the population dynamics for computing the future fertility rate.

3.2 Household Behavior

A household that enters the economy at time t elastically supplies labor until the age of 65.[14] The utility function of the household that entered the economy at age 20 in period t is as follows:[15]

$$U_t = E_{20,t}\left\{\sum_{j=20}^{100} \beta^{j-20}\left(\prod_{i=20}^{j-1}\phi_{i,t}\right)\frac{\left[c_{j,t}^{\sigma}(\bar{h}_t - h_{j,t})^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}\right\}, \; s_{19,t} = 1,$$

[14] Braun et al. (2007) consider time-variation in the family scale in their model. Moreover, Heathcote et al. (2008) include education and intra-family time allocation in their model, and investigate wage inequality. We assume that the household consists of the male household head, and we omit the female labor supply and family structure from the model. Therefore, as will be discussed later, the parameters in our model will be calibrated for the husband's labor. The survival probability is also taken from that of the male.

[15] Some empirical researches report that, based on microeconomic data, labor-leisure choice of households is consistent with a separable utility function between consumption and leisure (See Browining, et al., 1999). However, because we consider a growth economy, in order to maintain consistency of the macroeconomic data, we need to adopt the Cobb-Douglas utility function. As is well known, the aggregate labor supply has no trend even if the economy grows. If we adopt a separable utility function with consumption and leisure, the aggregate labor supply declines as the economy grows. The only exception is the separable type with log utility function on consumption. We explore the separability on our results in Section 6.

where $\beta > 0$ is a discount factor, γ represents a parameter for intertemporal elasticity of substitution, σ is a parameter for the share of consumption and leisure, $\bar{h}_t$ is the time endowment, and $h_{j,t} \in [0, \bar{h}_t]$ is labor supply at age j. We assume that the time endowment depends on period t, because we want to consider the effect of the reduction in work hours (referred as *jitan*) in the late 1980s and early 1990s.

Each household faces idiosyncratic risks with regard to labor skill. The labor skill realized at age j is denoted by $e_j \in E$ and it e_j is assumed to follow the stochastic process of equations (1) and (2). Labor skill, which consists of the fixed effect, the persistent shock and the transitory shock, represents the state of a household, $s_j = \{\alpha, z, \varepsilon\}$. On the other hand, because average earning is observed to be hump-shaped across age groups, the average productivity of labor for each age is denoted as $\{\kappa_j\}_{j=20}^{65}$. Therefore, when a household supplies hours $h_{j,t}$, its earning before tax is $y_{j,t} = w_t \kappa_j e_j h_{j,t}$, where w_t is the wage rate of the macroeconomy.

The government provides social security benefits through a constant payroll tax on labor earnings, and retired households receive social security benefit. The payroll tax rate is τ_t^{ss}, and the retired households receive a constant amount $\varphi_t w_t H_t$. As will be explained later, the population-adjusted average earning of workers is denoted by $w_t H_t$, and φ_t is a replacement rate. Thus, we assume that all retired households receive a constant fraction of the working household's labor income. Since the households face a mortality risk, they may die with accidental bequests. We assume, for simplicity, that the government levies 100% tax on all accidental bequests, and distributes it equally to all households. The redistributed accidental bequest is denoted as b_t. We impose a capital income tax τ_t^{cap} on the interest rate.

The budget constraints of workers and retirees are as follows:

$$
\begin{aligned}
c_{j,t} + a_{j+1,t+1} &= (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\text{ss}}) w_t \kappa_j e_j h_{j,t}, && : \text{Worker} \\
c_{j,t} + a_{j+1,t+1} &= (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + \varphi_t w_t H_t, && : \text{Retiree}
\end{aligned}
$$

where r_t is the interest rate at time t. We impose the liquidity constraint $a_{j,t} \geq 0$ on saving $a_{j,t}$.

3.3 Firm's Behavior and Factor Prices

The aggregate production function is a Cobb-Douglas type function:

$$Y_t = A_t K_t^{\theta_t} H_t^{1-\theta_t}$$

where A_t is the time-varying total factor productivity (TFP) at time t, K_t is aggregate capital, and H_t is aggregate labor, which coincides with the average labor income of workers. θ_t is capital share, which is also dependent on time t. We assume that the sequence of TFP is deterministic and perfectly forecastable. Thus, there is no aggregate risk in the model.

Because of the idiosyncratic income risk, households of the same cohort may have different wealth holdings and different labor supply, depending on the realization of the income shocks. Denote the density function of age j in time t with asset holding a and the idiosyncratic income state s as $\Phi_t(a, s, j)$. Then aggregate capital K_t and aggregate labor H_t are integral to all households as follows:

$$K_t = \sum_{j=20}^{100} \mu_{j,t} \int a_{j,t} d\Phi_t(a, s, j), \ H_t = \sum_{j=20}^{65} \mu_{j,t} \int \kappa_j e_j h_{j,t} d\Phi_t(a, s, j). \quad (3)$$

Aggregate capital depreciates at the rate δ_t. Then, the interest rate r_t and wage w_t of time t are determined as follows:

$$r_t = \theta_t A_t (K_t/H_t)^{\theta_t - 1} - \delta_t, \ w_t = (1 - \theta_t) A_t (K_t/H_t)^{\theta_t}. \quad (4)$$

3.4 Government's Budget Constraints

In our model, the government has three roles: (1) collecting payroll tax for social security and managing the social security system, (2) collecting capital income tax and using it for government expenditure, and (3) distributing accidental bequests. The payroll tax for social security is endogenously determined, depending on the replacement rate φ_t. Thus, the government budget must satisfy the following equation:

$$\sum_{j=20}^{65} \mu_{j,t} \int \tau_t^{\text{ss}} w_t \kappa_j e_j h_{j,t} d\Phi_t(a,s,j) = \sum_{j=66}^{100} \mu_{j,t} \varphi_t w_t H_t. \tag{5}$$

Government expenditure, which does not yield utility, is based on the capital income tax.[16] Thus, government expenditure must satisfy the following condition:

$$G_t = \sum_{j=20}^{100} \mu_{j,t} \int \tau_t^{\text{cap}} r_t a_{j,t} d\Phi_t(a,s,j). \tag{6}$$

Accidental bequests are taxed away and redistributed among all households equally:

$$b_t = \sum_{j=20}^{100} \int (1-\phi_{j,t}) a_{j,t} d\Phi_t(a,s,j). \tag{7}$$

3.5 Definition of a Competitive Equilibrium

The definition of the competitive equilibrium is as follows.

Definition (Recursive Competitive Equilibrium): Given the paths of TFP $\{A_t\}$, the time endowment $\{\bar{h}_t\}$, the capital share $\{\theta_t\}$, the depreciation rate $\{\delta_t\}$, the capital income tax rate $\{\tau_t^{\text{cap}}\}$, and the replacement rate $\{\varphi_t\}$, the recursive competitive equilibrium is a set of the policy functions $\{g_t^c, g_t^h, g_t^a\}$, aggregate capital $\{K_t\}$, aggregate labor $\{H_t\}$, factor prices $\{r_t, w_r\}$, payroll tax rates $\{\tau_t^{\text{ss}}\}$, and the accidental bequest $\{b_t\}$ which satisfy the following conditions:

- A Household's Optimality: Given factor prices $\{r_t, w_r\}$ and payroll tax rates $\{\tau_t^{\text{ss}}\}$, the household maximizes expected utility, and the functions $\{g_t^c, g_t^h, g_t^a\}$ are the associated policy functions.

[16] In the specification, an increase in the capital income tax rate directly implies the reduction of household utility. However, welfare comparison is beyond the scope of our purpose. Thus, our specification does not cause any problem in our analysis. The capital income tax rate plays a crucial role on matching the saving rate of the model to the data.

- A Firm's Optimality: Factor prices are competitively determined by equation (4).
- Market Clearing: The market clearing conditions of equation (3) are satisfied.[17]
- The Government's Budget: The government budget constraints (5) and (6) are satisfied.
- Accidental Bequest: Accidental bequests are redistributed as in (7).
- Transition Law of Motion: The distribution function $\Phi_t(a, s, j)$ transits consistently with the policy functions.

Following Conesa and Krueger (1999), we compute initial and final stationary states as a first step, and then we compute the transition path between them.

4 Calibration

4.1 Fundamental Parameters

The purpose of this paper is to investigate the implications of macroeconomic factors, a demographic factor, and income risk factors for economic inequality in Japan, by replicating the Japanese economy from 1980 to 2000. As a target, we choose the model parameters for the Japanese economy in 1980.[18]

Although we assume that households must retire at age 65, the actual mandatory retirement age in Japan in 1980 was around 60. However, because of the steep decline in average labor productivity after 60, many households voluntarily retire around 60. The available time endowment for labor supply $\bar{h}_t$ is assumed that, in the early 1980s, all households have 16 hours $\times$ 5.5 days $\times$ 4 weeks $\times$ 12 months. Hayashi and Prescott

[17] According to Walras' law, a good market clears if the remaining capital and labor markets clear.

[18] For robustness, we also computed the transition path between 1970 and 2000, and found that the difference does not change our results.

(2002) show that, since a reduction in work hours was introduced by law in the late 1980s, the average work hours per week fell from 44 hours to 40. To replicate this fact, we assume that the time endowment reduces from 1988 to 1993 and after 1993 all households worked five days per week, i.e., 16 hours $\times$ 5 days $\times$ 4 weeks $\times$ 12 months. We set the share parameter for consumption and leisure at $\sigma = 0.55$, which implies that average work hours per year become $2000 - 2200$ hours in our model.

We set the intertemporal elasticity parameter γ at 2. In this calibration, the intertemporal elasticity of substitution becomes 0.5, and the relative risk aversion is $1 - \sigma(1 - \gamma) = 1.55$. These values are within the plausible range in the existing literature on the real business cycle. The discount factor β is chosen to be $\beta = 0.9871$ for matching the capital-output ratio of the model between 1980 and 2000 to the actual data. Following Oshio and Yashiro (1997), the replacement rate of social security in Japan in the 1990s was about 40% . Thus, we set the replacement parameter φ_t to match this fact.

4.2 Income Risk and Age-Efficiency Profile

As stated in Section 2, the idiosyncratic skill shock e_j follows equations (1) and (2). The income risk parameters $\{\sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$ are calibrated to match the cross-section variance profiles of the logarithms of income in Panel (a) of Figure 1. Based on equations (1) and (2), the variances of the fixed effect and the transitory shock determine the intercept of the profile, and the slope of the profile is determined by the variance of the persistent shock and the persistence parameter.[19]

Since the age profile of variance of log consumption is convex over age, Abe and Yamada (2006) report that they do not reject the possibility of $\rho \geq 1$. However, since it would be difficult to compute the model with $\rho \geq 1$, we assume that the autocorrelation coefficient is very close to one, and approximate it to the Markov chain. We choose

[19]For details, see Storesletten et al. (2004a,b).

$\rho = 0.98$ and $\sigma_{\eta 20} = 0.05$ and assume that the standard deviation of the persistent shock increases by 0.0005. After the specification of the AR(1) process, we approximate the process by the seven-state Markov chain from Tauchen's (1986) method. The standard deviation of the fixed effect σ_α and the transitory shock σ_ε are set at 0.25 and 0.03, respectively, because the variance of log-income at age 25 is approximately 0.1. Both are approximated by two-states as $\{e^{-\sigma}, e^{\sigma}\}$. To make the model tractable, we assume that the idiosyncratic income risk parameters are constant for all time. Although this may seem a strong assumption, Ohtake and Saito (1998) and Abe and Yamada (2006) report that the variance profiles of log income/consumption over life cycle do not change significantly from 1984 to 1999. In Section 5, we will discuss whether this calibration really explains Japanese inequality.

The average productivity profile over age $\{\kappa_j\}$ corresponds to the average hourly wage for each age. Following Hansen (1993) and Braun et al. (2007), and using the *Basic Survey in Wage Structure* by the Ministry of Health, Labour, and Welfare, we compute the average hourly wage (efficiency-unit wage) for all age classes. Table 2 shows the average hourly wages for workers in five-year age groups. We compute $\{\kappa_j\}$ by smoothing the average hourly wage. Since we assume that the male head of the household supplies labor, the average hourly wage is based on the male's wage. We use the average wage profile in the 1990s.

4.3 Demographic Structure

We choose the demographic parameters to replicate the actual and projected population dynamics. The National Institute of Population and Social Security Research (NIPSSR) provides recent population projections from 2006 to 2055. We set the survival probability $\{\phi_{j,t}\}_{t=2006}^{2055}$ from the life table estimated using the NIPSSR's estimates in 2005, and $\{\phi_{j,t}\}_{t=2000}^{2005}$ are taken from the NIPSSR's estimates in 2002. The survival probabilities from 1980 to 1999 are extrapolated from the survival probability of the life table.[20] Since population growth $\{\psi_t\}$ in our model is represented by the growth rate of 0-year old children, we use the ratio of the projected population of new born people between period t and $t+1$. The population projection displays *three* variants of estimation: low, medium, and high variants for fertility and mortality rates.[21] Therefore, there are nine variations in all. We use medium variants of both the fertility and mortality rates.[22] The calibrated population growth rates between 1980 and 2005 are those of the realized values.

The population distribution in the model moves from 1980 to 2055 following the projected and the realized values, and the population growth rate is assumed to converge to zero after the transition period. However, the convergence rate is slow, and it takes approximately 100 years to reach a new stationary population distribution. Thus, we choose 2200 as the final stationary state. Following Braun et al. (2007), we assume that the population growth rate converges to zero, $\psi_t = 0$, between 2056 and 2065.

There arises the problem of how to choose an initial population distribution in the initial stationary state. The actual population distribution in 1980 does not appear to be stationary because of the existence of the baby-boomer generations. However, a population distribution is required to compute the initial stationary state. For simplicity, we assume that the households in our model *believe* that the actual population in 1980 is stationary.

[20] Another way of obtaining the survival probability of realized years is to calculate them from the population distribution of recent years. However, since the new population distribution includes immigrants, the population size of age $j+1$ at period $t+1$ may be larger than that of age j at period t. This contradicts the model's assumption. Thus, we use the life table.

[21] For details, see the following website: http://www.ipss.go.jp/syoushika/tohkei/Popular/Popular2008.asp?chap=0.

[22] The differences between each projection make little difference to the income and consumption inequality of our model between 1980 to 2000.

4.4 Macroeconomic Factors

Finally, we need exogenously given macroeconomic variable paths. For the purpose of comparison, the paths of all macroeconomic factors are taken from Hayashi and Prescott (2002).[23] The TFP factor growth rates $A_{t+1}^{1/(1-\theta_{t+1})}/A_t^{1/(1-\theta_t)} = 1 + g_t$ between 1980 to 2000 are estimated by Hayashi and Prescott (2002), and re-estimated from Hayashi and Prescott's (2002) data if the capital share is time-varying.[24] Although the TFP factor growth after 2000 could be estimated by macro data, we unfortunately have no data on economic inequality in Japan after 2000. Therefore, we simply assume that the TFP factor growth rate converges after 2000. The converged TFP factor growth rate is the average TFP growth rate of 1960–2000, which is 2%, following Braun et al. (2007). The TFP level A_{1980} is normalized to make the equilibrium wage w_{1980} equal to one. The capital share $\{\theta_t\}$, the depreciation rate $\{\delta_t\}$, and the capital income tax $\{\tau_t^{\text{cap}}\}$ are also taken from the estimated data of Hayashi and Prescott (2002).[25] These values are assumed to converge to their average in the 1990s for ten years after 2000.[26] All calibrated parameters are summarized in Table 3.

[23] Data and the details are available from http://www.e.u-tokyo.ac.jp/˜hayashi/Hayashi-Prescott1.htm.

[24] There is some skeptism about low TFP growth in the 1990s. Kawamoto (2004) points out that the estimation by Hayashi and Prescott (2002) does not necessarily imply the pure effect of *productivity*. Moreover, Chari, Kehoe, and McGrattan (2007) demonstrate that a liquidity constraint on investment, for example, may lead to a decline in the TFP that apparently does not reflect productivity. Therefore, it may not be correct to consider time-varying TFP as a macroeconomic productivity shock. However, for our purposes, it does not matter, because the TFP movement in our model may cause inequality whether or not there is actually a productivity shock. We will consider the interpretation of the TFP shock on economic inequality in our next research. Our calibration targets, such as interest rate, are taken from the estimation data of Hayashi and Prescott (2002). Thus, for consistency, for all macroeconomic variables we use Hayashi and Prescott (2002).

[25] Hayashi and Prescott (2002) define the capital income tax as direct tax on corporate income, 50% of indirect business tax, and 8% of operating surplus.

[26] We confirm that a small change in the converged points does not change our results.

5 Quantitative Results

5.1 Macroeconomic Variables: First Moments

Before discussing economic inequality after 1980, we confirm whether the average paths, such as interest rate and saving rate in the model, replicates the Japanese economy. Panel (a) of Figure 2 plots the after-tax interest rate in the model and the data. Since the capital income tax rate, taken from Hayashi and Prescott (2002), is over 40%, the before-tax interest rate is over 10% in the late 1980s. Compared with the data, the endogenously determined interest rate closely replicates the data, although the model falls slightly below the data in the early 1990s. Panel (b) of Figure 2 plots the capital-output ratio K/Y. The model replicates the flat capital-output ratio in the 1980s and the capital deepening process in the lost decade.

In addition to the time variation of the TFP growth rate, Hayashi and Prescott (2002) emphasizes the effect of the reduction in work hours in the late 1980s when explaining macroeconomic dynamics in Japan. Panel (c) of Figure 2 represents yearly work hours. Because the level of work hours is slightly higher in the model than in the data, we normalize the work hour of year 1980 to one. Since the available time endowment $\bar{h}_t$ is assumed to decline from 1988 to 1993, the endogenously determined work hours decreases during this period. According to Hayashi and Prescott (2002), since the work hours per week were reduced from 44 hours/week to 40 hours/week, the reduction amounts to about 9.1%. In our model, since there is a corresponding reduction of about 10.4%, our model closely replicates the reduction of work hours. In addition, the work hours of the model after 1993 also trace the data. However, because we assume perfect foresight, the work hours in the model increase between 1985 and 1988. For the purpose of intertemporal substitution, households increase labor supply during the period, since they predict the future reduction in time endowment. Thus, for the period of before time reduction, the time path of the model deviates from the data.

Lastly, we focus on the saving rate of the data and the model. Chen et al. (2006,2007) and Braun et al. (2007) show that the time variation of TFP growth and population dynamics are crucial contributing factors in explaining the Japanese saving rate after WW II. Compared to Braun et al. (2007), we extend the model by including idiosyncratic income risk.[27] We plot the time series of the national saving rate in Panel (d) of Figure 2. The model appears to explain the saving rate before the mid-1990s very clearly. However, there is a discrepancy between the data and model in the period 1995–1997.[28] In conclusion, using the overlapping generations model with idiosyncratic income risk and exogenously given macroeconomic factors, the dynamic general equilibrium model can explain the data very clearly.

5.2 Income and Consumption Inequality over Life Cycle

Figure 3 plots the variance of logarithm of income and consumption over age given a calibrated parameter set of idiosyncratic income risks $\{\rho, \sigma_\alpha^2, \sigma_{\eta,j}^2, \sigma_\varepsilon^2\}$. As a target of the calibration, we also plot the variance of log income and consumption in 1984 and 1999 provided by Abe and Yamada (2006). From Panel (a) of Figure 3, we confirm that our numerical model closely replicates the income inequality.

From Panel (a) of Figure 3, it appears that the variance of log income increases by age, and that the slope also increases after 50. Our model clearly explains the income inequality of the cross-sectional dimension, except for the sharp rise after age 55. In contrast, in Panel (a) of Figure 3, the consumption inequality in old age is small in the model, although its consumption inequality around ages 25–40 matches the data, closely. Thus, the idiosyncratic income risks in the model are effectively shared by saving, which

[27] In contrast, Braun et al. (2007) consider family structure in their analysis.

[28] Most Asian countries, including Japan, experienced a financial crisis in 1997. In such a period, many Japanese households may face a serious liquidity constraint that may not be well represented in our model. From a simulation study, Ogawa (2007) shows that the saving rate declined by several percentage points as a result of liquidity constraints.

implies that consumption inequality becomes slightly lower. Although it is difficult to explain the consumption inequality of life cycle aspect perfectly, the overall level of the consumption variance, and the slopes of the profiles in the model, is similar to those in the estimates.

5.3 Evolution of Earning and Income Inequality

We plot earning and income inequality in the model between 1980 and 2004 in Panels (a) and (b) of Figure 4. To compute economic inequality, in Section 2, we used the variance of the logarithm of variables. For comparison, we also plot the evolution of the variance of log income and consumption, estimated from Ohtake and Saito (1998) and Kohara and Ohtake (2006). It is difficult to match exactly the *level* of these variances because of the definitions of consumption and income.[29] Therefore, we use the time paths of the percentage deviation from mean. To compare our model with Kohara and Ohtake's (2006) result, the definition of *income* in our model adds interest income $(1-\tau_t^{\text{cap}})r_t a_{j,t}$ to labor earning $y_{j,t}$ (Figure 4(b)). The *earning* inequality in Panel (a) of Figure 4 is the variance of the logarithm of labor earning $y_{j,t}$.

Based on two previous researches, we know that the variance of log income has a positive trend since 1980. Although the definition of income inequality used by Kohara and Ohtake (2006) is not consistent with our model, the slopes of the data and the model are close to each other in the earning inequality path. Panel (a) of Figure 4 shows that the simulation data between 1984 and 1999 show a strong positive trend; thus the evolution of earning inequality matches the data in this period. Because the interest

[29] For example, family structure, female labor supply, and difference between durables and nondurables are omitted from the model. Moreover, we do not consider a progressive income tax. Thus, it is impossible to compare the data and the model exactly, because of the differences in definition. In the model, the average of variance of log income is about 0.39, and for the corresponding data it is 0.32. The average of the variance of log consumption in the data is about 0.25, whereas the corresponding value in the model is only 0.21.

rate in this period is near zero, due to the zero interest rate policy adopted by the Bank of Japan, the income inequality estimated by Kohara and Ohtake (2006) may be close to the earning inequality. If so, our model explains the transition of earning inequality in Japan in the 1990s. The model fails, however, to explain the rise of inequality in the early 1980s. One possible reason for the failure to match wage inequality is that, in the model, the households can predict the reduction of the time endowment.[30] Thus, households prepared for the reduction by increasing labor supply in the early 1980s. This implies a concentration of high labor supply and small earning inequality.

Income inequality is shown in Panel (b) of Figure 4. Although both the model and the data show a positive trend from the 1980s to 1990s, the inequality in the data rises sharply compared to that in the model. The positive trend of income inequality in the model starts to decline in the mid 1990s. In this period, the corresponding saving rate in the model is higher than in the data. In Japan in the late 1990s, many households suffered from the serious recession, and decumulated their wealth. As a result, the income inequality in the data in this period may show dispersion.

5.4 Evolution of Consumption Inequality

Panel (c) of Figure 4 shows the transition path of consumption inequality, and also depicts the data obtained by Ohtake and Saito (1998), Kohara and Ohtake (2006), and the simulated model.[31,32] This figure also plots the deviation from mean. According to

[30]Another route that may affect household's decision on labor and consumption is the wealth effect of the time endowment. Our specification of the time-varying time endowment process implies the reduction of total wealth for workers, and this changes decisions on labor and consumption. However, it is difficult to remove the wealth effect from our model.

[31]As noted above, it is very difficult to define consumption precisely, because we need to consider family structure, durable goods, housing, etc. For details on the definition of consumption, see Ohtake and Saito (1998) and Kohara and Ohtake (2006).

[32]We define consumption inequality in the model for households aged between 20 to 80. Since the NSFIE surveys are based on household heads, there are very few samples in which the household head

previous studies, consumption inequality also has a positive trend, although the path is flatter than that of income inequality. In our simulation, and excluding the rise between the late 1980s and the early 1990s, consumption inequality does not appear to have a positive trend. This seems surprising because both population aging and the macroeconomic variables changed in this period, and, as a result, earning inequality fluctuated. This implies that households in the model carry out very effective consumption smoothing, and that the idiosyncratic shocks are shared through saving, which is also observed in Figure 3. Figure 3(b) shows that the age-profile of log consumption variance in the model is flatter than in the data. As a result, the effect of population aging may be underestimated. Although earning inequality rises from the 1980s to mid-1990s in the model, it does not lead to consumption inequality. Thus inequality in the 1980s is not clearly explained by the model.

From the above, we can safely say that the dynamic general equilibrium model clearly explains income and earning inequality in Japan. However, unlike these two inequalities, the consumption inequality of the data is difficult to match with that of the model. Since consumption inequality has direct implications for economic welfare, we need to consider the omitted factors on consumption inequality. One possible explanation is that the size parameter of each shock $\{\xi_t^f, \xi_t^p, \xi_t^t\}$ has changed through time, and the uninsurable component of the idiosyncratic income risk has become larger.

5.5 On the Effect of Macroeconomic Factors: A Counterfactual Simulation

Since our model is complex, it is difficult to analyze it theoretically. Instead, using counterfactual simulations, we consider how the income and consumption inequalities change if the macroeconomic factors such as the TFP factor growth rate and time endowment, are constant over time.

is over 80. We confirm that including households aged over 80 in the model does not change our results.

In Figure 5, we depict the evolution of the earning and consumption inequalities, if the TFP factor growth rates are constant at 2%, or if time endowment $\{\bar{h}_t\}$ is constant at the average of the 1990s.[33] In keeping with Hayashi and Prescott (2002), the TFP growth rate and the reduction in work hours both have a large impact on the path. From Panel (a) of Figure 5, we find that, if the time endowment is constant over time, the rise in earning inequality between 1988 and 1993 is small, and the path over the entire period is flat. If the TFP growth rate is constant then, except for the early 1980s, earning inequality becomes higher in comparison to the benchmark case. Therefore, these two macroeconomic factors have significant effect not only on the first moment, e.g., GDP or saving rate, but also on the second moment of the economy. We also examine the cases of constant capital share, capital income tax, and depreciation rate path, but their effect is very small.

We compare consumption inequality in Panel (b) of Figure 5. The reduction in work hours has a large impact on consumption inequality. The sharp rise of consumption inequality between the late 1980s and the early 1990s is due to the reduction in work hours. The reduction of the time endowment implies that there is less room for smoothing marginal utility by changing leisure, and this leads to consumption inequality. If the TFP growth rate is constant at 2%, then consumption inequality in the period of the so-called lost decade is higher than in the benchmark case. Thus, the low TFP growth rate in the lost decade implies low earning and consumption inequality.

The mechanism behind the impact of macroeconomic factors on inequality is as follows. Figures 1(a) and 1(b) show that income and consumption inequality over the life cycle are almost constant, i.e., ergodic, over time. On the other hand, the time path of income/consumption inequality changes significantly. If the macroeconomic factors are constant over time and there is no demographic change, then the time path of

[33] We omit the counterfactual simulation of the income inequality path because the economic implications of macroeconomic factors are similar to those of earning inequality.

income/consumption inequality is flat over time, because lifetime inequality is ergodic in the model. On the contrary, if the TFP growth rate is high in a period, some cohorts decide to increase work hours and consumption, depending on their remaining lifetime. The growth rates of work hours and consumption are not the same, due to different age or different state, and as a result, the fluctuating TFP growth rate leads to the dynamics of macroeconomic inequality, even if life cycle inequality is the same for all cohorts.

5.6 Demographics versus Macroeconomic Variables

Next, we discuss the effects of the demographic factor and the macroeconomic factors on economic inequality. The label denoted as "Population" in Figure 6 is the counterfactual simulation with constant population distribution, but the macroeconomic variables are exogenously given calibrated parameters. The label denoted as "Macro" is the simulated results in which the macroeconomic variables are constant over time, but the population is aging.

If there is no population aging, income inequality flattens over the period: income inequality declines slightly in the 1980s, whereas earning inequality becomes more U-shaped. These imply that the rise in income and earning inequality in the 1990s is partially due to the population aging effect. However, even without population aging, there remains a positive trend of earning and income inequality. In contrast, if all the macroeconomic factors are constant over time, earning inequality peaks in the mid-1990s, and declines in the late 1990s. In conclusion, both the aging and the macroeconomic factors create a positive trend of earning inequality. In particular, the increase in income and earning inequality in the late 1990s is due to macroeconomic factors. Consumption inequality, in Panel (c) of Figure 6, also reveals the importance of the macroeconomic factors. Without macroeconomic fluctuations, the time path of consumption inequality becomes hump-shaped at the peak in the mid-1990s. The time path becomes very smooth and declines after the peak. On the other hand, consumption inequality rises sharply after 1990 in the simulation labeled "Population." Thus, the positive trend of consumption inequality is also explained by aging *and* by the macroeconomic factors.

6 Sensitivity Analysis

6.1 Separability of Utility Function

Our specification of utility function is standard in the macroeconomics literature, because we used the Cobb–Douglas form. However, because we employ a time varying time endowment, the nonseparability of the marginal utility of consumption and leisure may cause problems when considering consumption inequality. Note that, because we consider a growth economy, the utility function needs to be consistent with no aggregate labor trend. To investigate the robustness of our findings, we consider the separable utility function. If the parameter γ is equal to 1, then the utility function becomes as follows:

$$u(c_{j,t}, \bar{h}_t - h_{j,t}) = \sigma \log c_{j,t} + (1-\sigma)\log(\bar{h}_t - h_{j,t}).$$

In this form, due to its separability, leisure has no effect on the marginal utility of consumption.[34] In addition to the log case, we consider the case of $\gamma = 4$.

In Figures 7(a) and 7(b), we plot the simulated earning and consumption inequality with the log utility and $\gamma = 4$. If the temporary utility function is of log-type, then the earning and consumption inequality profiles become steeper. Thus, compared to the benchmark case in Figure 4, the matching of the model to the data may improve, especially in consumption. In the benchmark case, a rise of consumption inequality in the data cannot be explained well by the model. On the contrary, in the log utility case, the sharp rise in consumption inequality in the 1990s is closer to the data. Against that, high gamma implies a less positive trend of inequality. Because the high and low gamma cases consistently move in all cases, the separability does not have a significant effect in our analysis.

[34] For more general separable utility forms, see King et al. (1998) and Basu and Kimball (2002).

6.2 On the Effect of Social Security System

Lastly, we confirm whether our specification of social security is significant when considering the time path of economic inequality. In Figures 7(c) and 7(d), we consider a reduction by half of the replacement rate compared to the benchmark case, i.e., $\varphi = 0.2$. Our specification of the social security form has redistribution and insurance effects, and thus reduces economic inequality. Panel (c) of Figure 7 shows that earning inequality does not change even if the replacement rate is small. Therefore, such a social security reform does not change the distribution of earnings and work hours significantly. In contrast, the time path of consumption decreases in the 1980s and increases after the mid 1990s. This is interesting, because recently it has been widely recognized that rapid aging worsens the sustainability and credibility of the social security system in Japan. If households predict that the social security payments that they expect to receive will become smaller, then the consumption inequality will disperse, even if the corresponding earning inequality does not change, compared to the benchmark case.

7 Concluding Remarks

In this paper, we investigated the evolution of economic inequality in Japan between 1980 and 2000, using a dynamic general equilibrium model. To analyze the inequality, we consider the quantitative impacts of macroeconomic factors and a demographic factor. In addition to such macroeconomic variables as the interest rate and the saving rate, our model could explain time variation of the variance of log earning and consumption; thus, an OLG model with heterogeneous households would be a suitable benchmark for understanding the transitional dynamics in the Japanese economy. In this respect, we consider our result an extension of the result obtained by Hayashi and Prescott (2002), which explains the causes of the lost decade using a standard neoclassical growth model.

Using counterfactual simulation, we analyzed the reason why economic inequality in Japan has risen. We found that the time variation of the TFP growth, and the reduction in work hours, have implications not only for the macroeconomic variables but also for economic inequality. In particular, lower TFP growth rate in the 1990s leads to low economic inequality. Moreover, we found that, even without demographic factors, such as aging, and the constant idiosyncratic income risk factors, earning and income inequality declined because of the macroeconomic factors. Based on our analysis, preference change and distrust of the social security system in Japan may also contribute to the deterioration of consumption inequality.

There still remain unexplained components on the time path of earning and consumption inequality. This unexplained component should be regarded as the changes brought about by fixed, persistent, and transitory effects. In addition, we do not consider the reason why earnings risk and inequality rises. For welfare analysis, we need to consider the causes of rising earning and consumption inequality: for example, skill-biased technological change or human capital may explain the causes. Kawamoto (2005) points out that the decline of TFP in 1990s is not due to the low productivity of the Japanese economy, but to inefficient resource allocation between productive and unproductive sectors. Based on this view, we need to consider sector specific allocation problems when we consider economic inequality in the Japanese economy. These are topics for future research.

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A Computation Procedure

A household's optimization problem is as follows:

$$V_{j,t}(a,s) = \max \left\{ u(c_{j,t}, \bar{h}_t - h_{j,t}) + \phi_{j,t}\beta E V_{j+1,t+1}(a', s') \right\}, \tag{8}$$

subjec to

$$c_{j,t} + a_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}})r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\text{ss}})w_t \kappa_j e_j h_{j,t},$$

$$c_{j,t} + a_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}})r_t)(a_{j,t} + b_t) + \varphi_t w_t H_t.$$

Because microeconomic variables are not affected by the population trend, we detrend equation (8) using the TFP factor growth rate alone. We define $c_{j,t}/A_t^{1/(1-\theta_t)} = \tilde{c}_{j,t}$, $a_{j,t}/A_t^{1/(1-\theta_t)} = \tilde{a}_{j,t}$, and $h_{j,t} = \tilde{h}_{j,t}$. Then, the normalized Bellman equation becomes as follows:

$$v_{j,t}(\tilde{a}, s) = \max \left\{ u(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) + \phi_{j,t}\tilde{\beta}_t E v_{j+1,t+1}(\tilde{a}', s') \right\}, \tag{9}$$

subject to

$$\tilde{c}_{j,t} + (1 + g_t)\tilde{a}_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}})r_t)(\tilde{a}_{j,t} + \tilde{b}_t) + (1 - \tau_t^{\text{ss}})\tilde{w}_t \kappa_j e_j \tilde{h}_{j,t},$$

$$\tilde{c}_{j,t} + (1 + g_t)\tilde{a}_{j+1,t+1} = (1 + (1 - \tau_t^{\text{cap}})r_t)(\tilde{a}_{j,t} + \tilde{b}_t) + \tilde{\varphi}\tilde{w}_t \tilde{H}_t$$

where $\tilde{\beta}_t = \beta(1 + g_t)^{\sigma(1-\gamma)}$ and $\tilde{b}_t = b_t/A_t^{1/(1-\theta_t)}$.

Computing a stationary state is a direct extension of Storesletten et al.'s (2004a) model. Among the many available procedures for computing the policy function, we apply the Endogenous Gridpoint Method (EGM) by Carroll (2006), because it is a safe and relatively fast method.[35]

After the computation of the stationary state in 1980 and 2200, we compute the transitional path between the stationary states. The basic idea here is the same as in Conesa and Krueger (1999).

[35] For details on the endogenous gridpoint method with endogenous labor supply, see appendix in Krueger and Ludwig (2006) and Barillas and Fernández-Villaverde (2006).

1. Given an exogenous path of $\{\tilde{\varphi}, g_t, \theta_t, \delta_t, \bar{h}_t, \tau_t^{\text{cap}}\}_{t=1980}^{2200}$, guess an equilibrium sequence of $\{r_t, \tilde{w}_t, \tau_t^{\text{ss}}, \tilde{H}_t, \tilde{b}_t\}_{t=1980}^{2200}$, which is needed to solve a household's problem.[36] We assume that the replacement rate, TFP growth rate, capital share, depreciation rate, time endowment, and capital income tax are all perfectly foreseen and exogenously given.

2. Because we have the policy function of the final stationary state in 2200, we compute a sequence of policy functions using the EGM by backward induction.

3. Given the policy functions, compute the distribution function from 1980 onwards and compute aggregate variables, $\{\tilde{K}_t, \tilde{H}_t, r_t, \tilde{w}_t\}_{t=1980}^{2200}$.

4. Check whether each market clearing condition and government budget balances are satisfied. If these are not in equilibrium, up-date the price sequences and repeat steps $2-3$.[37]

5. If all markets clear in all periods, stop computation.

[36] For simplicity, we start a linear case.

[37] There are many efficient methods of updating the price sequence. For example, Krueger and Ludwig (2006) and Ludwig (2006) use a modified version of the Gauss-Zeidel method to compute the transition path.

	$70s$	$80s$	$90s$
TFP Factor Growth Rate (%)	1.40	2.68	0.57
Depreciation Rate (%)	10.19	8.97	8.40
Capital Share	0.3512	0.3536	0.3627
Capital Income Tax Rate (%)	41.55	47.68	44.92

Table 1: Macroeconomic Variables in Japan

Age	Hourly Wage	Age	Hourly Wage
20–24	1,349	45–49	3,075
25–29	1,777	50–54	3,145
30–34	2,187	55–59	2,797
35–39	2,548	60–64	1,923
40–44	2,842	65–	1,617

Table 2: Average Hourly Wage for Each Age Group; Yen

	Value	Target Statistics and Previous Research
	Stationary State in 1980	
β	0.9871	$K/Y \approx 1.74$
γ	2	Frisch Elasticity ≈ 0.67
σ	0.55	Average Hours Worked $\approx$ 2,200h
φ	0.4	Ohio and Yashiro (1997)
$\bar{h}_{1980}$	$4,224$	16h $\times$ 5.5days $\times$ 4weeks $\times$ 12month
θ_{1980}	0.3452	Hayashi and Prescott (2002)
δ_{1980}	0.0949	Hayashi and Prescott (2002)
τ^{cap}_{1980}	0.4636	Hayashi and Prescott (2002)
	Stationary State in 2200	
β	0.9871	Constant over time
γ	2	Frisch Elasticity ≈ 0.63
σ	0.55	Average Hours Worked $\approx$ 2,000h
φ	0.4	Ohio and Yashiro (1997)
$\bar{h}_{2200}$	$3,840$	16h $\times$ 5days $\times$ 4weeks $\times$ 12month
θ_{2200}	0.362	Average of 1990-2000, Hayashi and Prescott (2002)
δ_{2200}	0.083	Average of 1990-2000, Hayashi and Prescott (2002)
τ^{cap}_{2200}	0.450	Average of 1990-2000, Hayashi and Prescott (2002)

Table 3: Calibrated Parameters in 1980 and 2200

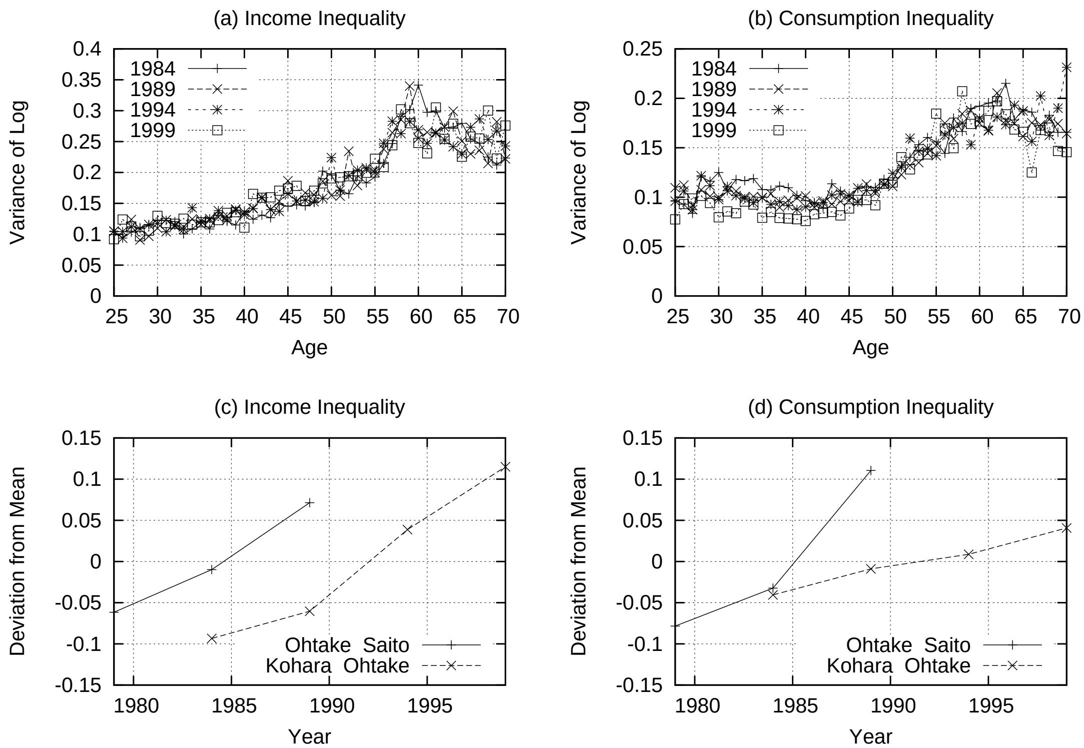

Figure 1: Economic Inequality in Japan

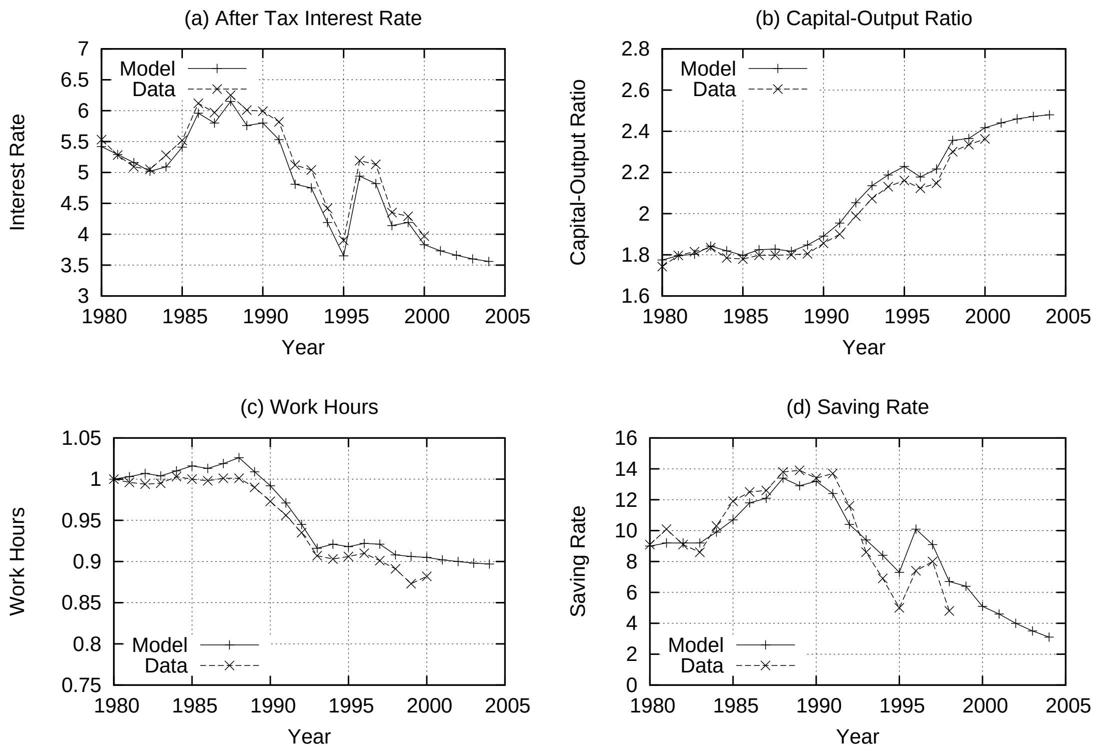

Figure 2: Paths of Macroeconomic Variables

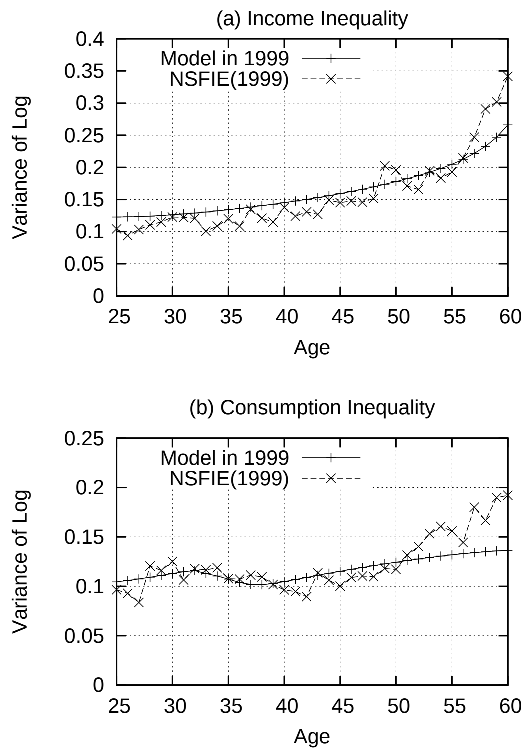

Figure 3: Cross Section Variance Profiles

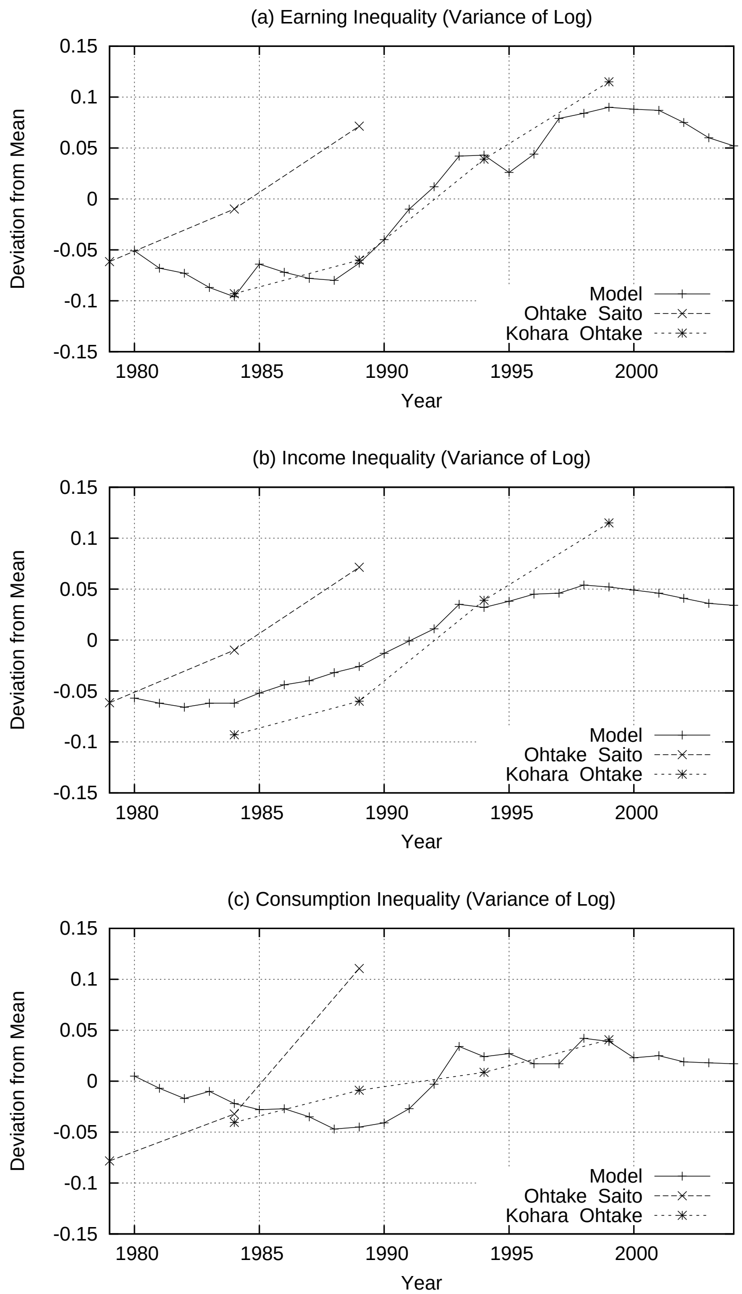

Figure 4: Inequality in Earning, Income, and Consumption: Model versus Data

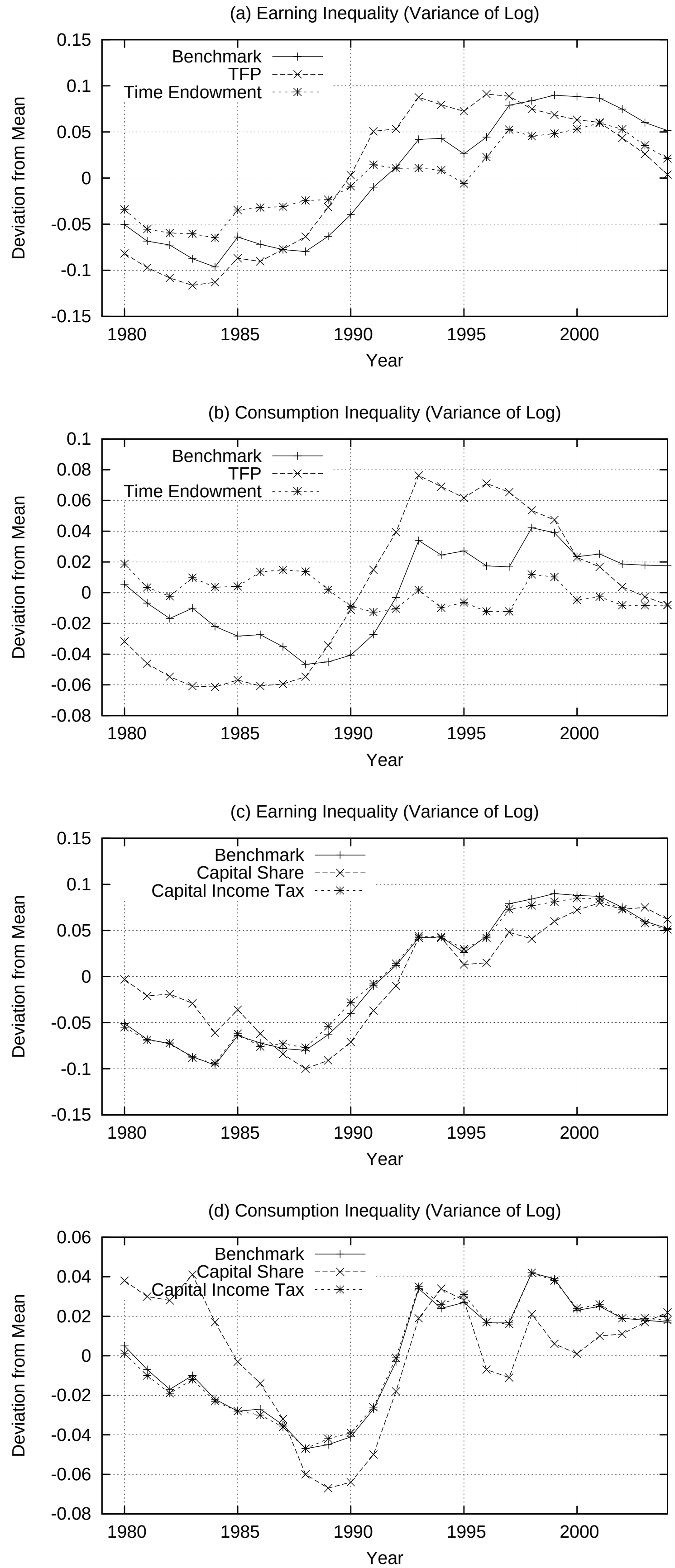

Figure 5: Counterfactual Simulation: Macroeconomic Effect

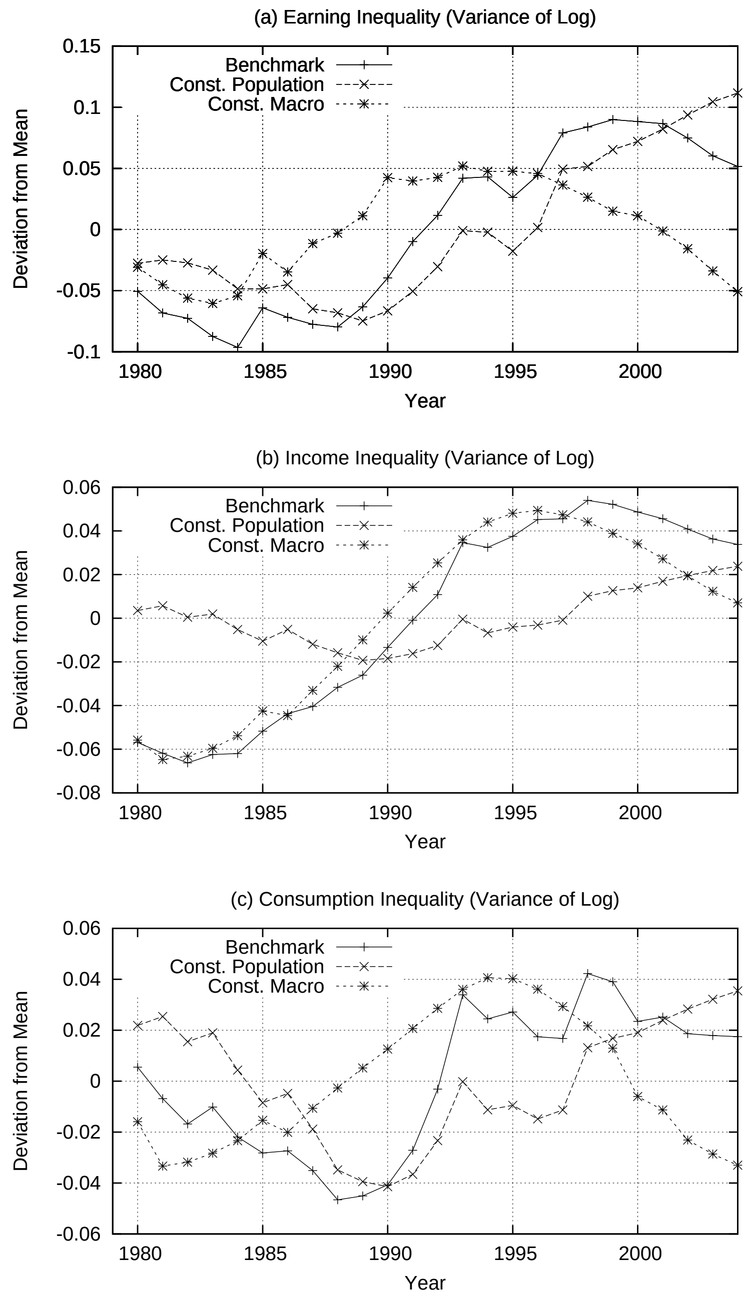

Figure 6: Counterfactual Simulation: Demographics versus Macroeconomic Variables

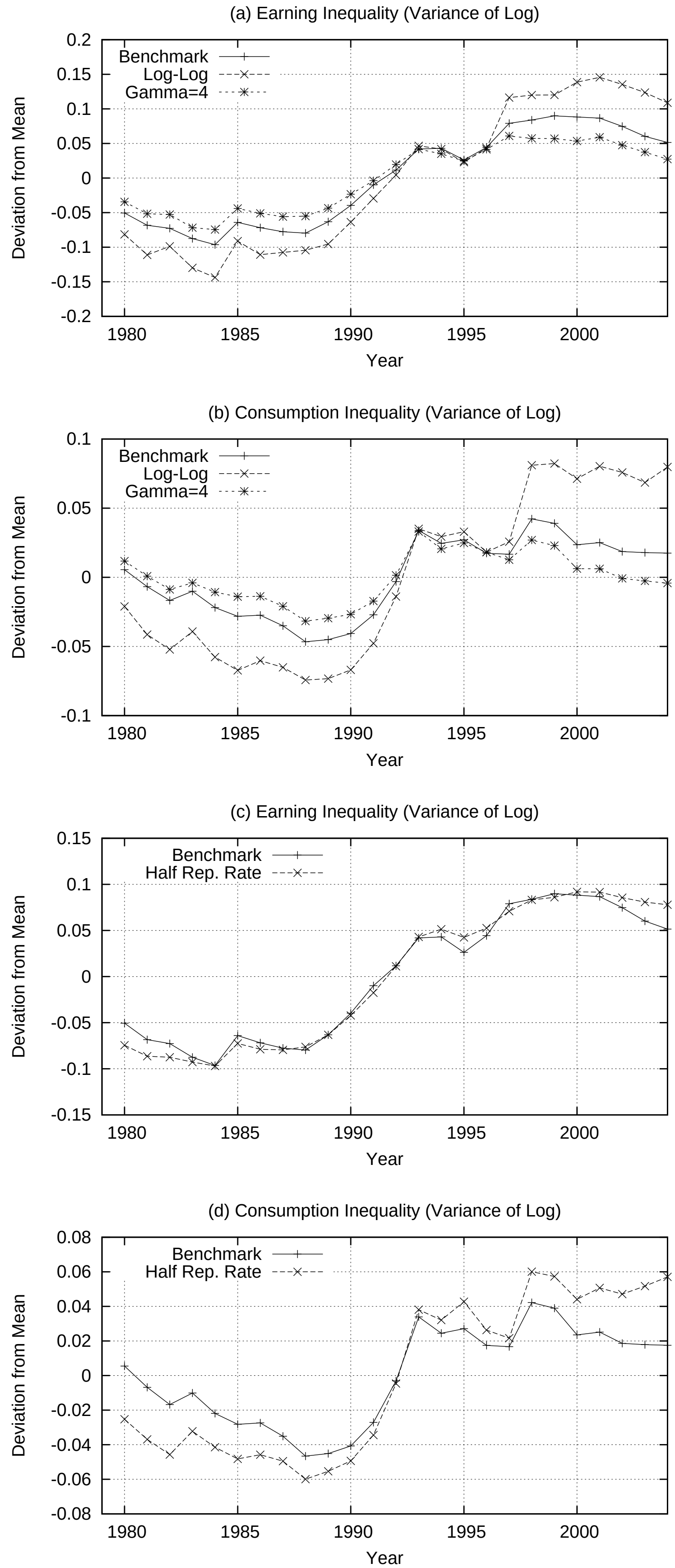

Figure 7: Robustness Analysis: Utility Function and Replacement Rate