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Risks after Disasters: A Note on the Effects of Precautionary Saving on Equity Premiums

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Risks after disasters: A note on the effects of precautionary saving on equity premiums *

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Abstract

This paper studies the effects on equity premiums of "risks after disasters", which are defined as a sharp rise in volatility of real per capita GDP growth rates immediately following disasters. This paper makes three contributions. First, we analytically demonstrate that if and only if the degree of relative prudence is higher than 2, risks after disasters decrease equity premiums. Second, we find that the differences between equity premiums with and without risks after disasters are quantitatively significant. Third, equity premiums are still higher in the case of disaster than without a disaster.

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1 Introdctuion

Many researchers pay serious attention to the potential impacts of disasters on equity premiums. Disasters are defined as events such as wars, severe depressions and natural disasters, which are infrequent but significantly reduce real per capita GDP growth rates. Rietz (1988) and Barro (2006) argued that representative agent models generate large equity premiums, if disasters are independently and identically distributed (hereafter, i.i.d.). On the other hand, Gourio (2008) examined the fact that historically observed disasters tended to be followed by sharp rises in real per capita GDP growth rates (hereafter, "recoveries"). Then he demonstrated that disasters with possible recoveries generated small equity premiums, if the degree of intertemporal elasticity of substitutions (hereafter, IES) was high. However, as discussed in a later section, historically observed disasters tended to be followed by persistent declines in real per capita GDP growth rates as well as by recoveries. In other words, disasters tended to be followed by sharp rises in volatility of real per capita GDP growth rates (hereafter, "risks after disasters").

This paper studies the effects of such risks after disasters on equity premiums. Then it demonstrates that precautionary saving, rather than IES, plays an important role in determining the equity premiums. In particular, using a three-period Lucas tree model, we analytically demonstrate that risks after disasters decrease equity premiums if and only if the degree of relative prudence (hereafter, RP)[1] is higher than 2. This result suggests that ignoring risks after disasters causes computed equity premiums to be too large. In fact, introducing risks after disasters into Barro's disaster model, we find that the differences between equity premiums with and without risks after disasters are quantitatively significant. However, we also find that the equity premiums are still higher than in the case where no disaster occurs.

This paper is organized as follows. Section 2 constructs a three-period Lucas tree economy and analyzes the effects of risks after disasters on equity premiums. Section 3 introduces risks after disasters into Barro's disaster model and explores quantitative effects of precautionary saving on equity premiums. Section 4 offers a conclusion.

[1]RP is a measure of precautionary saving proposed by Kimball (1990).

2 A three-period economy with risks after disasters

In order to analyze the effects of risks after disasters on equity premiums, we construct a three-period Lucas tree economy. In this economy, there are two assets. One is an equity share in a Lucas tree, which produces a single perishable consumption good as a dividend, and the other is a risk-free asset. In period 2, the normal state (n) occurs with a probability of $1-p$ and the disaster state (d) occurs with a probability of p. Once the normal state is realized in period 2, there is no uncertainty in period 3. On the other hand, if the disaster state is realized in period 2, the dividend in period 3 is uncertain. The parameters y_2^n and y_3^n represent the dividends of the Lucas tree in periods 2 and 3, when the normal state is realized in period 2, whereas y_2^d and $\tilde{y}_3^d$ represent the dividends in periods 2 and 3, when the disaster state is realized in period 2. We assume that $y_2^n > y_2^d$. $\tilde{y}_3^d$ is a random variable, where $\bar{y} \equiv E[\tilde{y}_3^d]$ and $\delta^2 \equiv E[(\tilde{y}_3^d - \bar{y})^2]$. The parameter δ represents risks after disasters[2]. $E[\cdot]$ denotes the mathematical expectation operator conditional on information available in period 1. The parameter y_1 represents the dividend of the Lucas tree in period 1. We assume that there are no idiosyncratic risks.

The parameters x_1 and a_1 represent agents' holdings of the equity share and the risk-free asset in period 1. P_1 and Q_1 represent the prices of the equity and the risk-free asset in period 1. x_t^s and a_t^s represent agents' holdings of the equity share and the risk-free assets in period $t = 2, 3$ and state $s = n, d$, and P_2^s and Q_2^s represent the prices of the equity and the risk-free asset in period 2 and state $s = n, d$.

The representative agent maximizes his/her expected utility subject to budget constraints:

$$u(c_1) + (1-p)\Big\{u(c_2^n) + u(c_3^n)\Big\} + p\Big\{u(c_2^d) + E[u(\tilde{c}_3^d)]\Big\}$$

The parameter c_1 represents consumption in period 1 and c_t^s represents consumption in period $t = 2, 3$ and state $s = n, d$. We assume that the periodic utility $u(\cdot)$ has positive first and negative second derivatives; that is, $u' > 0$ and $u'' < 0$. For simplicity, we ignore the subjective time preference. The budget constraints in periods 1, 2, and 3 are as follows: $c_1 + P_1 x_2 + Q_1 a_2 = (P_1 + y_1)x_1 + a_1$,

[2]While we assume that the level of the dividend in period 3 after the disaster state is uncertain, we can interpret that the growth rate is stochastic because the dividend growth is defined as $\frac{\tilde{y}_3^d}{y_2^d}$.

$c_2^s + P_2^s x_3^s + Q_2^s a_3^s = (P_2^s + y_2^s)x_2 + a_2$ for $s = n, d$, and $c_3^n = y_3^n x_3^n + a_3^n$ and $\tilde{c}_3^d = \tilde{y}_3^d x_3^d + a_3^d$. Then first-order conditions determine the asset prices, as follows:

$$P_1 = (1-p)(P_2^n + y_2^n)\frac{u'(c_2^n)}{u'(c_1)} + p(P_2^d + y_2^d)\frac{u'(c_2^d)}{u'(c_1)},\ P_2^n = y_3^n \frac{u'(c_3^n)}{u'(c_2^n)},\ P_2^d = E\Big[\tilde{y}_3^d \frac{u'(\tilde{c}_3^d)}{u'(c_2^d)}\Big],$$
$$Q_1 = (1-p)\frac{u'(c_2^n)}{u'(c_1)} + p\frac{u'(c_2^d)}{u'(c_1)},\ Q_2^n = \frac{u'(c_3^n)}{u'(c_2^n)},\ \text{and } Q_2^d = E\Big[\frac{u'(\tilde{c}_3^d)}{u'(c_2^d)}\Big].$$

Hereafter, scaling $u'(\cdot)$ so that, without loss of generality, $u'(y_1) = 1$.

Market clearing conditions are $x_1 = x_t^s = 1$, and $a_1 = a_t^s = 0$ for $t = 2, 3$ and $s = n, d$. Then, consumption always equals dividend: $c_1 = y_1$, $c_2^s = y_2^s$, for $s = n, d$, $c_3^n = y_3^n$, and $\tilde{c}_3^d = \tilde{y}_3^d$. Expected equity returns and risk-free rates in period 1 are: $R_e = \frac{(1-p)(P_2^n + y_2^n) + p(P_2^d + y_2^d)}{P_1}$ and $R_f = \frac{1}{Q_1}$. We define the expected equity premium as $\Pi \equiv \frac{Re}{Rf}$. Then equilibrium equity premiums are defined as follows[3]:

$$\Pi \equiv \frac{A + \alpha \hat{P}}{B + p\hat{P}}, \tag{1}$$

where, $A \equiv \big[(1-p)^2 + (1-p)p\frac{u'(y_2^d)}{u'(y_2^n)}\big] y_3^n u'(y_3^n) + (1-p)^2 y_2^n u'(y_2^n) + (1-p)p y_2^n u'(y_2^d) + (1-p)p y_2^d u'(y_2^n) + p^2 y_2^d u'(y_2^d)$, $\alpha \equiv (1-p)p\frac{u'(y_2^n)}{u'(y_2^d)} + p^2$, $B \equiv (1-p)y_3^n u'(y_3^n) + (1-p)y_2^n u'(y_2^n) + p y_2^d u'(y_2^d)$, and $\hat{P} \equiv E[\tilde{y}_3^d u'(\tilde{y}_3^d)]$. $\hat{P}$ is the equity price in the disaster state, P_2^d, multiplied by marginal utility in the disaster state, $u'(y_2^d)$. However, we hereafter refer to $\hat{P}$ as the equity price in the disaster state.

Below, we analyze the effects of an increase in the risks after disasters, δ, on the equity price, $\hat{P}$, and the equity premium, Π.

Lemma 1 *When the degree of relative prudence, $-\bar{y}\frac{u'''(\bar{y})}{u''(\bar{y})}$ is higher (lower) than 2, $\hat{P}$ is an increasing (decreasing) function of δ, for small δ.*

Proof. Taking a Taylor series expansion of $\hat{P}$ around $\bar{y}$ gives[4]:

$$\hat{P} \simeq \bar{y}u'(\bar{y}) + \Big\{u''(\bar{y}) + \frac{\bar{y}}{2}u'''(\bar{y})\Big\}\delta^2 + o(\tilde{y}^3).$$

Differencing $\hat{P}$ with respect to δ yields $\frac{\partial \hat{P}}{\partial \delta} = 2\big\{u''(\bar{y}) + \frac{\bar{y}}{2}u'''(\bar{y})\big\}\delta$. Therefore: $-\bar{y}\frac{u'''(\bar{y})}{u''(\bar{y})} > (<)2$ implies $u''(\bar{y}) + \frac{\bar{y}}{2}u'''(\bar{y}) > (<)0$, thus $\frac{\partial \hat{P}}{\partial \delta} > (<)0$. *(Q.E.D.)*

[3] Derivations of equation (1) are described in the Appendix.
[4] $o(\tilde{y}^3)$ refers to a higher-order term.

Lemma 1 implies that there are two opposite effects of risks after disasters on the equity price, $\hat{P}$. On the one hand, risks after disasters lower equity prices because of risk aversion ($u''(\bar{y})$). On the other hand, risk after disasters raises equity prices because of precautionary saving ($\frac{\bar{y}}{2}u'''(\bar{y})$). Thus, $u''(\bar{y}) + \frac{\bar{y}}{2}u'''(\bar{y})$ determines the total effects of risks after disasters on equity prices, $\hat{P}$. In fact, if RP, $-\bar{y}\frac{u'''(\bar{y})}{u''(\bar{y})}$, proposed by Kimball (1990) is higher than 2, $u''(\bar{y}) + \frac{\bar{y}}{2}u'''(\bar{y}) > 0$ holds.

Lemma 2 Π *is a decreasing function of* $\hat{P}$.

Proof. Differencing Π with respect to $\hat{P}$ yields:

$$\frac{\partial \Pi}{\partial \hat{P}} = \frac{\alpha B - pA}{(B + p\hat{P})^2}. \tag{2}$$

Expanding the sign condition, $\alpha B - pA$, yields[5]:

$$\alpha B - pA = p(1-p)\left[\left\{(1-p)\frac{u'(y_2^n)}{u'(y_2^d)} - (1-2p)\right\}u'(y_2^n) - pu'(y_2^d)\right]\left(y_3^n\frac{u'(y_3^n)}{u'(y_2^n)} + y_2^n\right). \tag{3}$$

Thus, $\frac{u'(y_2^n)}{u'(y_2^d)} < 1$ implies that the term in the bracket is negative. Now, we assume that $y_2^n > y_2^d$, $\frac{u'(y_2^n)}{u'(y_2^d)} < 1$ always holds because of concavity of utility functions. Thus, $\frac{\partial \Pi}{\partial \hat{P}} < 0$. *(Q.E.D.)*

Lemma 2 argues that a rise in the equity prices in the disaster state implies that an equity share becomes less risky and lowers the equity premiums. This is intuitive because increases in $\hat{P}$ mitigate capital losses on the occurrence of disasters, when consumption is low and marginal utility is high.

We thus have a key proposition of this paper.

Proposition 1 *: When the degree of relative prudence,* $-\bar{y}\frac{u'''(\bar{y})}{u''(\bar{y})}$, *is higher (lower) than 2,* Π *is a decreasing (increasing) function of* δ, *for small* δ.

Proof. From Lemmas 1 and 2, $\frac{\partial \Pi}{\partial \delta} = \frac{\partial \Pi}{\partial \hat{P}}\frac{\partial \hat{P}}{\partial \delta} < (>)0$. *(Q.E.D.)*

Proposition 1 suggests that precautionary saving plays an important role in determining equity premiums in the context of risks after disasters. In particular, proposition 1 suggests that when RP is higher than 2, ignoring risks after disasters

[5] Derivations of equation (3) are described in the Appendix.

makes calibrators overestimate the equity premiums. If RP is lower than 2, the opposite occurs.

Proposition 1 holds in the case of possible recoveries, as discussed in Gourio (2008), because Proposition 1 is independent of the magnitude relationship between $\bar{y}$ and y_2^d. In addition, we can easily show that Proposition 1 holds in the case of government default as discussed in Barro (2006)[6].

Finally, we offer some examples of well-known utility functions.

Example 1 *The quadratic utility* implies that third derivatives are zero; that is, there is no precautionary saving. Thus, risks after disasters always increase equity premiums.

Example 2 *The HARA (Hyperbolic Absolute Risk Aversion) utility* is usually represented as $u(c) = \frac{1-\eta}{\eta}\left(\frac{\lambda c}{1-\eta} + \frac{\chi}{\lambda}\right)^{\eta}$ with $\lambda > 0$ and $\chi > 0$. Then, $RP \equiv c\lambda\frac{2-\eta}{1-\eta}\left(\frac{\lambda c}{1-\eta} + \frac{\chi}{\lambda}\right)^{-1}$ and $RRA = c\lambda\left(\frac{\lambda c}{1-\eta} + \frac{\chi}{\lambda}\right)^{-1}$. Thus, $RP > 2$ implies that $RRA > 2\frac{1-\eta}{2-\eta}$ in the case of $\eta < 1$.

Example 3 *In the case of CRRA (Constant Relative Risk Aversion) utility,* $RP = 2$ implies that the degree of relative risk aversion (hereafter, RRA) equals 1, which is the case of the log utility. That is, when a calibrator uses the CRRA utility with a moderate degree of RRA, ignoring the risks after disasters caused computed equity premiums to be too large.

However, Proposition 1 in this section is based on Taylor expansion arguments for a small δ. In section 3, then, we conduct calibration exercises and evaluate quantitatively the effects of empirically plausible degree of risks after disasters on equity premiums.

3 Calibration: Barro model with risks after disasters

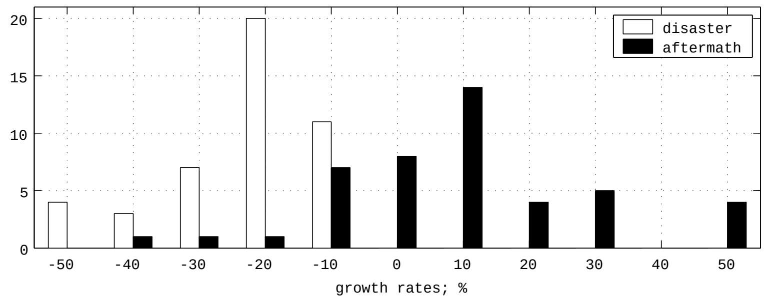
To explore the quantitative effects of precautionary saving on equity premiums, we introduce risks after disasters into Barro's disaster model. Barro's disaster model is the infinitely lived representative agent model with CRRA utility.

[6] Barro (2006) computed equity premiums using equity returns and defaultable government bond returns instead of risk-free rates.

To conduct calibration exercises, we begin by characterizing empirically plausible degrees of risks after disasters from cross-country evidence of real per capita GDP growth rates in disasters presented by Barro (2006, Table I, p.828-829). Duration of disasters is distributed between one and eight years with a mode of three years. Then, we define three years from the beginning of the disaster as a disaster period and the succeeding three years as an aftermath period. Figure 1 shows a histogram of real per capita GDP growth rates during the disaster and the aftermath period, where the growth rates in the disaster period are lower than -10%[7].

From Figure 1, we find that growth rates in the disaster period are distributed between -50% and -10% with a mode of -20%. On the other hand, growth rates in the aftermath period are distributed between -40% and 50% with a mode of 10%. Thus, we find that disasters tended to be followed by persistent declines in real per capita GDP growth as well as recoveries. In other words, disasters tended to be followed by a sharp rise in risks after disasters. In fact, the average growth rate in the aftermath period is 10.06% and the standard deviation is 21.73%.

Figure 1. Cross-countries distribution of real per-capita GDP growth



We introduce risks after disasters into Barro's disaster model. Consumption growth follows:

$$\begin{aligned}\Delta \log c_t &= \mu + (\sigma + \delta s_t)\epsilon_t, \text{ with probability } 1-p \\ &= \mu + \sigma\epsilon_t + \log(1-b), \text{ with probability } p\end{aligned}$$

where the parameter ϵ_t represents i.i.d. standard normal random variables with a distribution of N(0,1) and b represents the size of disasters. If the previous period

[7]From Maddison's (2003) data, we limit our study to 46 events in 27 countries.

is a disaster, $s_t = 1$, otherwise, $s_t = 0$. That is, risks after disasters are $\sigma + \delta$ while risks in normal times are σ. If $\delta = 0$, the model is the same as that of Barro. Because it is difficult to obtain an analytical solution in the case of $\delta \neq 0$, we must use a numerical technique to compute equity premiums.

Following Barro, we specify the calibration parameters as presented in Table I. In our setup, since the duration of a disaster is stochastic, we do not use the historical distribution of b but the single value $b = 43.15\%$ in order to replicate Barro's equity premium if there are no risks after disasters. From Figure 1, since standard deviation of consumption growth in the aftermath period is 21.73%, we compute equity premiums when δ is between 3% and 23% with an increment of 5%. Table I reports results from the default considered equity premiums.

Table I. Equity-risk premiums (the case of bond default) (%)

	no disasters	disasters					
Total risks after disasters: $\delta + \sigma$		2	5	10	15	20	25
RRA: $\gamma = 4$	0.16	3.53	3.48	3.18	2.95	2.43	1.67
RRA: $\gamma = 1$	0.04	0.37	0.37	0.39	0.41	0.45	0.49

subjective time preference $\rho = 0.03$, average growth $\mu = 2.5\%$,
standard deviation of consumption growth in normal times $\sigma = 2\%$,
the probability of occurrence of disasters $p = 1.7\%$,
and the default probability of government bonds in disaster $q = 40\%$.

When RRA $= 4$, the equity premiums equal 0.16% in the case where no disaster occurs, 5.71% in the case of disaster, and 3.53% in the case of disaster with bond default. Risks after disasters lower equity premiums because of strong precautionary saving. In particular, the default considered equity premium is 2.43% when $\sigma + \delta = 20\%$ and 1.67% when $\sigma + \delta = 25\%$. These values indicate that the differences in the equity premiums with and without risks after disasters are quantitatively significant. In addition, we find that the larger risks after disasters imply smaller equity premiums. Thus, these calibration exercises demonstrate that we must pay serious attention to potential impacts of risks after disasters. However, we also find that equity premiums are much higher in the case of disasters than the case where no disasters occur. Table I also reports the results in the case of log utility, where RP equals 2. In this case, while risks after disasters always increase equity premiums, magnitudes are very low.

4 Conclusion

This paper studies the effects of risks after disasters on equity premiums. On the one hand, we demonstrate that ignoring risks after disasters causes computed equity premiums to be too large, if and only if RP is higher than 2. In addition, we find that the differences in equity premiums with and without risks after disasters are quantitatively significant. In fact, many calibration exercises in asset pricing literature tended to employ CRRA utility with RRA higher than 2, and some empirical researchers found that RP is higher than 2[8]. Thus, we must pay serious attention to potential impacts of risks after disasters. On the other hand, the equity premiums are much higher than in the case where no disaster occurs. Thus, these results are quantitatively instructive because disaster models proposed by Rietz and Barro are potentially important. Therefore, future research should investigate whether historically observed disasters were empirically or statistically consistent with historically observed equity premiums and other asset prices.

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[8]For example, Merrigan and Normadin (1996) showed that the estimated RP took a value of around 2 in U.K. Hori and Shimizutani (2006) estimated that RP is around 4 in Japan. On the other hand, Dynan (1993) found that the estimated RP is very low in U.S.

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Appendix

Derivation of the equation (1) We describe the derivation of the equation (1) in detail. Using the definition of asset prices and the first order conditions, the equity premium is written as:

$$\begin{aligned}\Pi &\equiv \frac{R_e}{R_f} \\ &= \frac{(1-p)(P_2^n + y_2^n) + p(P_2^n + y_2^n)}{P_1} Q_1\end{aligned}$$

$$= \frac{\left((1-p)\left\{y_3^n \frac{u'(y_3^n)}{u'(y_2^n)} + y_2^n\right\} + p\left\{E\left[\tilde{y}_3^d \frac{u'(\tilde{y}_3^d)}{u'(y_2^d)}\right] + y_2^d\right\}\right)\left((1-p)\frac{u'(c_2^n)}{u'(c_1)} + p\frac{u'(c_2^d)}{u'(c_1)}\right)}{(1-p)\left\{y_3^n \frac{u'(y_3^n)}{u'(y_2^n)} + y_2^n\right\}\frac{u'(y_2^n)}{u'(y_1)} + p\left\{E\left[\tilde{y}_3^d \frac{u'(\tilde{y}_3^d)}{u'(y_2^d)}\right] + y_2^d\right\}\frac{u'(y_2^d)}{u'(y_1)}}$$

As discussed above, we assume that $u'(y_1) = 1$. Then, the denominator is written as:

$$\begin{aligned}&(1-p)\left\{y_3^n \frac{u'(y_3^n)}{u'(y_2^n)} + y_2^n\right\}u'(y_2^n) + p\left\{E\left[\tilde{y}_3^d \frac{u'(\tilde{y}_3^d)}{u'(y_2^d)}\right] + y_2^d\right\}u'(y_2^d) \\ =\; &(1-p)y_3^n u'(y_3^n) + (1-p)y_2^n u'(y_2^n) + pE\left[\tilde{y}_3^d u'(\tilde{y}_3^d)\right] + py_2^d u'(y_2^d)\end{aligned}$$

Thus, the denominator is written as $B + p\hat{P}$, where

$$\begin{aligned}B &\equiv (1-p)y_3^n u'(y_3^n) + (1-p)y_2^n u'(y_2^n) + py_2^d u'(y_2^d) \\ \hat{P} &\equiv E[\tilde{y}_3^d u'(\tilde{y}_3^d)].\end{aligned}$$

On the other hand, the numerator is written as:

$$\Big((1-p)\Big\{y_3^n\frac{u'(y_3^n)}{u'(y_2^n)}+y_2^n\Big\}+p\Big\{E\Big[\tilde{y}_3^d\frac{u'(\tilde{y}_3^d)}{u'(y_2^d)}\Big]+y_2^d\Big\}\Big)\Big((1-p)u'(y_2^n)+pu'(y_2^d)\Big)$$

$$\begin{aligned}
= \quad & (1-p)^2\Big\{y_3^n u'(y_3^n)+y_2^n u'(y_2^n)\Big\}+p(1-p)\Big\{E\Big[\tilde{y}_3^d\frac{u'(\tilde{y}_3^d)}{u'(y_2^d)}\Big]u'(y_2^n)+y_2^d u'(y_2^n)\Big\}\\
& +(1-p)p\Big\{\frac{u'(y_2^d)}{u'(y_2^n)}y_3^n u'(y_3^n)+y_2^n u'(c_2^d)\Big\}+p^2\Big\{E\big[\tilde{y}_3^d u'(\tilde{y}_3^d)\big]+y_2^d u'(y_2^d)\Big\}
\end{aligned}$$

$$\begin{aligned}
= \quad & \Big\{(1-p)^2+(1-p)p\frac{u'(y_2^d)}{u'(y_2^n)}\Big\}y_3^n u'(y_3^n)+(1-p)^2 y_2^n u'(y_2^n)+p(1-p)y_2^d u'(y_2^n)\\
& +(1-p)py_2^n u'(y_2^d)+p^2 y_2^d u'(y_2^d)+\Big\{p^2+p(1-p)\frac{u'(y_2^n)}{u'(y_2^d)}\Big\}E\Big[\tilde{y}_3^d u'(\tilde{y}_3^d)\Big]
\end{aligned}$$

Thus, the numerator is written as $A+\alpha\hat{P}$, where

$$\begin{aligned}
A \quad &\equiv \quad \Big\{(1-p)^2+(1-p)p\frac{u'(y_2^d)}{u'(y_2^n)}\Big\}y_3^n u'(y_3^n)+(1-p)^2 y_2^n u'(y_2^n)\\
& \qquad +(1-p)py_2^n u'(y_2^d)+(1-p)py_2^d u'(y_2^n)+p^2 y_2^d u'(y_2^d)\\
\alpha \quad &\equiv \quad (1-p)p\frac{u'(y_2^n)}{u'(y_2^d)}+p^2.
\end{aligned}$$

As a result, we derive equation (1), that is, $\Pi\equiv\frac{A+\alpha\hat{P}}{B+p\hat{P}}$.

Derivation of the equation (3) We describe the derivation of the equation (3) in detail.

$$\begin{aligned}
& \alpha B-pA\\
= \quad & \Big\{(1-p)p\frac{u'(y_2^n)}{u'(y_2^d)}+p^2\Big\}\Big\{(1-p)y_3^n u'(y_3^n)+(1-p)y_2^n u'(y_2^n)+py_2^d u'(y_2^d)\Big\}\\
- \quad & p\Big[\Big\{(1-p)^2+(1-p)p\frac{u'(y_2^d)}{u'(y_2^n)}\Big\}y_3^n u'(y_3^n)+(1-p)^2 y_2^n u'(y_2^n)\\
& +(1-p)py_2^n u'(y_2^d)+(1-p)py_2^d u'(y_2^n)+p^2 y_2^d u'(y_2^d)\Big]
\end{aligned}$$

$$
\begin{aligned}
= \quad & \Big\{(1-p)^2 p\frac{u'(y_2^n)}{u'(y_2^d)} + p^2(1-p) - p(1-p)^2 - (1-p)p^2\frac{u'(y_2^d)}{u'(y_2^n)}\Big\} y_3^n u'(y_3^n) \\
& + \Big\{(1-p)^2 p\frac{u'(y_2^n)}{u'(y_2^d)} + p^2(1-p) - p(1-p)^2\Big\} y_2^n u'(y_2^n) - (1-p)p^2 y_2^n u'(y_2^d)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & (1-p)p\Big\{(1-p)\frac{u'(y_2^n)}{u'(y_2^d)} - (1-2p)u'(y_2^n) - pu'(y_2^d)\Big\} y_3^n \frac{u'(y_3^n)}{u'(y_2^d)} \\
& +(1-p)p\Big\{(1-p)\frac{u'(y_2^n)}{u'(y_2^d)} - (1-2p)u'(y_2^n) - pu'(y_2^d)\Big\} y_2^n
\end{aligned}
$$

Thus, the sign condition is written as equation (3),

$$
\alpha B - pA = p(1-p)\Big[\Big\{(1-p)\frac{u'(y_2^n)}{u'(y_2^d)} - (1-2p)\Big\}u'(y_2^n) - pu'(y_2^d)\Big]\Big(y_3^n\frac{u'(y_3^n)}{u'(y_2^n)} + y_2^n\Big).
$$

When the term in the bracket is negative,

$$
\begin{aligned}
& \Big\{(1-p)\frac{u'(y_2^n)}{u'(y_2^d)} - (1-2p)\Big\}u'(y_2^n) - pu'(y_2^d) < 0 \\
\Leftrightarrow \quad & (1-p)\Big(\frac{u'(y_2^n)}{u'(y_2^d)}\Big)^2 - (1-2p)\frac{u'(y_2^n)}{u'(y_2^d)} - p < 0
\end{aligned}
$$

Therefore, if $\frac{u'(y_2^n)}{u'(y_2^d)} < 1$, the term in bracket is negative, which implies that the sign condition is negative.