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A New Approach to Estimating Tax Interactions in Fiscal Federalism*

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Abstract

The purpose of this paper is to propose a new approach to empirically analyze the existence of strategic interactions of taxation between state governments (horizontal) and between state and federal governments (vertical) using gasoline and cigarette taxation in the U.S. I explicitly estimate the structural parameters of consumer's utility and state government's objective functions. The slopes of the reaction functions, which represent the strategic interactions of state government taxation policies, are then computed given the estimated structural parameters. Empirical results show that contrary to the existing literature, there is very little horizontal tax interaction in both the gasoline and cigarette cases. On the other hand, there is a moderate positive vertical tax interaction for both gasoline and cigarette taxes and the scale is larger in the case of cigarette taxes. Furthermore, the value and sign of the slopes of the reaction function are very different across states. This suggests a new policy implication: as state governments respond differently to federal government fiscal policy, uniform fiscal policy is not appropriate for welfare maximization of the nation.

JEL classification : H11, H21, H71, H73, H77
Keywords: tax interaction, cross border shopping

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1 Introduction

The purpose of this paper is to explicitly estimate the structural parameters of consumer and government behavior, and to examine the existence of strategic interactions of taxation between state governments and between state and federal governments. The existence of strategic interactions between governments is evaluated by computing the slope of the reaction function given the estimated parameters. If the slopes of the reaction functions between state governments are positive, state governments' tax policies are strategic complements and a state government raises (reduces) its tax rate if other state governments raise (reduce) their tax rates. On the other hand, if the slopes are negative, tax policies are strategic substitutes and a state government reduces its tax rate if other state governments raise them. The intuition of strategic complements is that if other state governments raise their tax rate, a state government can raise its tax rate to increase its revenue without the fear of losing tax base or if other state governments reduce their tax rate, a state government needs to reduce its tax rate to protect its tax base. On the other hand, the intuition of strategic substitutes is that if other state governments raise their tax rate, a state government reduces its tax rate to attract tax resources from other states.

Horizontal commodity tax interaction happens when state governments compete against each other for tax resources, and each government reduces its tax rate to attract tax resource from other states. In this sense, the mobility of tax resources; i.e. cross border shoppers, is a crucial factor of horizontal commodity tax interaction, and the scale of horizontal tax interaction depends on the mobility of tax resources. The tax rate in equilibrium is generally less than optimal when state government taxes are strategic complements. On the other hand, vertical commodity tax interaction results from the situation that state and federal governments share a common tax base and that either or both governments ignore the fact that their tax would shrink the tax base of the other government. In this case, the tax elasticity of tax base; i.e. consumer's price elasticity of demand, is a crucial factor for the intensity of vertical commodity tax interaction. Tax rates in equilibrium tend to be higher than optimal especially when state governments' taxes are strategic complements. Consequently, if there is a tax interaction, there is a possibility that both tax rate and the amount of public goods are not optimal, and tax coordination or intergovernmental transfer is necessary to raise the total welfare (Boadway and Keen (1996), Hoyt (2001), and Lucas (2004)). These fiscal policies depend on the scale and direction of this tax externality, and it is difficult to know whether tax rates in equilibrium are lower or higher than optimal. Therefore, estimating the direction and the level of strategic interaction of taxes between governments becomes a very important policy question for countries under fiscal federalism, where both federal government and state governments co-exist.

In this paper, I use a structural approach to estimate the strategic interactions in tax policies. I first estimate the parameters of the household's utility function in a model of optimal consumption and cross border shopping. Then, using the estimated parameters of the household's specific utility function, I estimate the objective function of benevolent state governments in a model of optimal taxation[1]. Finally, based on the estimated structural parameters of the individuals and state governments, I derive the slope of the reaction function of each state's tax with respect to other states, and federal government tax changes.

There is already a large body of literature[2] that discusses both vertical and horizontal strategic interactions of taxation, both theoretically and empirically. Besley and Rosen (1998) theoretically and empirically examine vertical excise tax externality, i.e. strategic interactions between state and federal government excise taxes. They find that the theory of optimal consumer and government behavior does not put any restriction on the sign of the slope of the reaction function. Empirically, from their regression analysis, they find that the federal tax rate has a positive effect on state taxes for both gasoline and cigarette taxes.

Devereux et al (2007) extend the work of Besley and Rosen (1998) to include horizontal strategic interactions in their model, i.e. strategic tax interactions between state governments. In order to analyze both horizontal and vertical tax interactions, they use a weighted matrix to approximate the complex strategic interaction between state governments. That is, they estimate a linear model where the dependent variable is state taxes and independent variables include the weighted average of other states' taxes, the federal tax and other socio-economic variables. The results show that for the cigarette tax, the coefficient of the weighted average state tax rate is estimated to be significantly positive but the coefficient of the federal tax rate is insignificant. For the gasoline tax, the former is insignificant and the latter is positive and weakly significant. Devereux et al (2007) argue that the difference in the estimated strategic interactions of gasoline and cigarette taxes could be attributed to the difference in the characteristic of the good,

[1] I do not study the strategic behavior of the federal government in this paper.

[2] In theoretical papers, both Mintz & Tulkens (1986) and Kanbur & Keen (1993) study horizontal commodity tax competition. Keen and Kotsogiannis (2002) analyze both horizontal and vertical capital tax competition . Lockwood (2001) summarize previous commodity tax literatures. In empirical papers, Esteller-More and Sole-Olle(2001) examine vertical and horizontal tax competition on income and sales tax in U.S. Nelson (2002) considers horizontal tax competition on exercise taxes in U.S. Hayashi and Boadway (2001) analyze vertical tax competition on corporate income tax in Canada.

such as the difference in price elasticity of demand and transportation cost[3].

While the above regressions based on the approach with and without the weighted matrix have made us aware of the importance of the strategic interactions in taxation, I argue that there are several difficulties in interpreting the estimation results, especially for the results that includes horizontal tax interaction where a weighted matrix is used.

First, the theory of state tax policy predicts that the slopes of the reaction functions, which measure the response of own taxes to the marginal change of other states' or federal taxes, depend on several variables, which are: the difference between the own state tax rate and that of all the other states, transportation costs, own and other states' population, demand and price elasticity of demand. However, conventional construction of the weighted matrix allows the slope of the reaction function to depend on only one variable and also assumes the sign of the slope is the same across states. Hence, the interaction terms of taxes and the variables not included in the weighted matrix are omitted from the independent variables, resulting in omitted variable bias. The direction and the magnitude of the bias are likely to depend on which variable is included in the weighted matrix. I suspect this is the reason why the results are not robust to the specification of the empirical model; i.e. different studies that use different variables in the construction of the weighted matrix often obtain very different parameter estimates of tax interaction[4].

Second, I argue that the weighted matrix approach is a too simple approximation of the Nash equilibrium of state and federal governments' strategic taxation game. This is because the weighted matrix approach is a linear approximation around a symmetric Nash equilibrium[5] and only applicable when state governments are symmetric and consumer's utility function is Quasi-linear. Hence, the estimation result based on the weighted matrix approach is reliable only if the equilibrium is very close to being symmetric, i.e. if the states are very similar to each other. But the data show that states have very different populations and distance to each other, and that the weighted matrix approach is a simple approximation to the Nash equilibrium. Also, the Quasi-linear utility function means that demand is independent of income and this is a strong

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[3] "When individual demand for the good is relatively price-inelastic, and incentives for inter state arbitrage are strong [because of lower transportation cost], the tax set in any state is likely to be strongly positively responsive to taxes set in neighboring states, but unresponsive to the federal tax. Conversely, when individual demand for the good is relatively price-elastic, and incentives for inter-state arbitrage are weak, the tax set in any state is likely to be unresponsive to taxes set in neighboring states, and responsive to the federal tax, although this response may be positive or negative. As argued below, the first case describes the market for cigarettes in the US well, and the second case the market for gasoline." ; extract from Devereux et at (2007) pp.452 line 16-24.

[4] Refer to Madiès Thierry et al (2004)

[5] Refer to the Proposition 3 in Devereux et al (2007)

assumption considering consumption behavior varies across different income levels[6].

Lastly, results of the previous papers are not consistent with the typical idea of the relationship between price elasticity of demand and the scale of tax interaction. Generally, in a Ramsay optimal taxation context, I would expect the government to avoid levying a heavier tax rate on the good whose price elasticity of demand is high to avoid losing tax base. Therefore, the slope of the reaction function should be small in the good whose price elasticity of demand is high. Nevertheless, both Besley and Rosen (1998) and Devereux et al (2007) report that the value of the slope of the reaction functions between state and federal government is larger in the gasoline case than in the cigarette case, in spite of the fact that the price elasticity of demand of gasoline is higher than that of cigarettes.

In this paper, I take a structural approach to analyze tax interactions. I first solve and estimate a model of optimal consumption and cross border shopping behavior of individuals, similar to the one analyzed by Devereux et al (2007). In this first stage, I recover the parameters of the representative consumer's utility function. In contrast to the weighted matrix approach, our estimation is based on the full solution of consumer's behavior subject to taxes. Hence, I take into account all the important factors that determine optimal consumption, such as differences in own state and other states' tax rate, transportation costs, population, demand and price elasticity of demand which affect state and federal taxation via consumer's optimal behavior. I next estimate the parameter of the state government objective function by estimating the state government's first order condition with respect to taxes. After all the key structural parameters are estimated, I compute the slopes of the reaction functions and evaluate the strategic interaction between governments. Notice that the slope of the reaction function is derived from state government's first order condition, which maximizes the welfare of the representative household. This method fully captures the effect of other state or federal tax changes on consumers' cross border shopping and also takes into account the nonlinear functional forms of the reaction function.

The estimation results are the follows. First, the slope of the reaction functions between state governments of both gasoline and cigarette taxes, which describes the horizontal tax interaction, is positive but very small. The reason why this value is small is that the share of gasoline and cigarette consumption to total income is small and the percentage of cross border shopping is estimated to be very small. Second, the slope of the reaction function between state and federal governments, which describes the vertical tax interaction, is positive, and the value is larger for the cigarette tax than for

[6] This is based on the data of U.S. Bureau of Labour Statistics.

the gasoline tax. This result supports the Ramsay idea of the relationship between price elasticity of demand and tax interaction intensity. Third, the value of the slope of the reaction function of the tax interaction is positive on average, but its value and sign are very different among states and for some states the sign becomes negative. This is in contrast to the results from the weighted matrix estimation, where the slopes have the same sign for all states and only change linearly with variables of the weighted matrix, such as population, distance, or the border population density. I also identify the structure of the slope of the reaction function. The scale of the slope of the reaction function mainly depends on the share of commodity consumption to total income and the share of own state consumption in horizontal tax externality case, while price elasticity of demand and after tax price are important factors for vertical tax externality case, which are all different among states. This result casts some doubt on the validity of previous results which were obtained by assuming that the sign of the slope of the reaction function is the same among states, and the value of the slopes depends on only one factor.

The paper proceeds as follow. In section 2, I explain how to evaluate tax interaction using a reaction function. In section 3, I introduce the model of household consumption and government taxation and spending. In section 4, I provide details about the estimation strategy, and section 5 explains the data. The 6th section discusses the results of the empirical analysis and section 7 explains the intuition of them. Section 8 discusses the relation with previous papers and section 9 concludes.

2 General Framework of Tax Interaction

In this section, I briefly review the model of Devereux et al (2007).

Suppose there are two state governments, i and j who levy a excise tax on a good for which cross border shopping is possible. Assume that the state government i is Leviathan[7], who maximizes the total tax revenue R_i. Total tax revenues is composed of tax rate t_i and tax base $X_i(t_i, t_j, d_{ij}, n_i, n_j)$, where t_j is another state's tax rate, and d_{ij} is the distance between state i and j, measuring the transportation cost of cross border shopping, and n_i, n_j are the population of state i and j. Tax base X_i can be divided into two components; per consumer demand $x_i(t_i)$ and the number of people who purchase the good in state i $s_i(t_i, t_j, d_{ij}, n_i, n_j)$. Then, the state government

[7] Here, I assume that state government is Leviathan only for explanation since the model is simple.

i 's problem is

$$\underset{t_i}{Max} \quad R_i = t_i X_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)$$

$$\text{where} \quad X_i = x_i\left(t_i\right) s_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)$$

The first order condition for maximization is

$$\frac{\partial R_i}{\partial t_i} = X_i\left(t_i, t_j, d_{ij}, n_i, n_j\right) + t_i \frac{\partial X_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)}{\partial t_i} = 0 .$$

Next, I derive the reaction function of state i 's tax in response to changes in state j 's tax. It is

$$\frac{\partial t_i}{\partial t_j} = -\frac{\partial^2 R_i}{\partial t_i \partial t_j} \bigg/ \frac{\partial^2 R_i}{\partial t_i^{\ 2}}$$

where

$$\frac{\partial^2 R}{\partial t_i \partial t_j} = \left\{ x_i\left(t_i\right) + t_i \frac{\partial x_i\left(t_i\right)}{\partial t_i} \right\} \frac{\partial s_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)}{\partial t_j} + x_i\left(t_i\right) \frac{\partial s_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)}{\partial t_i \partial t_j}$$

The denominator, which is a second derivative with respect to its own tax rate is $\frac{\partial^2 R}{\partial t_i^{\ 2}} < 0$ and the sign of the reaction function depends on the sign of the numerator.

From the expression of the numerator, it is clear that per consumer demand and the price elasticity of per consumer demand enter in the reaction function. In addition, distance, which is related to transportation cost, and population affect the number of people who purchase the good $s_i\left(t_i, t_j, d_{ij}, n_i, n_j\right)$. Furthermore, both own tax and that of the other state enter in the reaction function as well. Many of these determinants of the slope of the reaction function are not included in the conventional weighted matrix specification. Moreover, we can see from the numerator that except for a very specific model specification and parameter values, the reaction function is a fundamentally nonlinear function of tax rate, and linear regression might not be appropriate.

To derive a specific expression of the reaction function, we need to give a specific functional form for household and government problem. In the next section, I will construct a more specific model of cross border shopping that I will then estimate.

3 Model Setting

There are N states ($i = 1,2,\cdots N$), and a federal government. Federal and state governments levy a commodity tax on a good x, and use this tax revenue to finance a public good G. G represents the "per capita amount of the public good" in this model, and the context of the public good is different for each private good. In another words, I consider per capita highway expenditure as a public good for gasoline consumption case and per capita health expenditure as a public good for cigarette consumption case. This is because gasoline tax revenue and cigarette tax revenue are kinds of earmarked revenue for highway and health expenditure. I denote y to be the other composite consumption good. I also denote the tax for state i as t_i and the federal tax rate T. They are both assumed to be per unit taxes. Then, the after tax commodity price in state i can be expressed as $P_i = p_i + t_i + T$, where p_i is the before tax price. State i has population n_i, and people can choose to cross border shop for the good that is taxed. State governments only consider the welfare of households in their own region, and the federal government's purpose is to maximize the total welfare of people in the nation. I assume that state and federal governments are Nash Competitors, and state government determine their tax rate and public good with other state and federal governments tax policy as given. I do not discuss federal government's behavior or consider the case where state government is Leviathan. Next, I describe the household's problem.

3.1 The Household's Problem

A household in state i has income I_i, and gets utility from consumption of good x, the composite good y and the public good G. The household can buy good x either in her own state or in a neighboring state. In the household cross border shops, I assume that the transportation cost is independent of the amount of consumption. The price of the composite good y is assumed to be unity for simplicity. I omit any public good from federal government in this section for simplicity because, given the assumption of additive separability of the utility of the private and public good, it will not affect the cross border shopping.

Utility Function and Demand Function

The utility function of a household in state A who chooses to purchase good x in state i is expressed as follows[8]:

$$U_A^i = \alpha_A \log(x_A^i - r_x) + (1-\alpha_A)\log y_A^i - \beta \cdot d_{Ai} + \phi_A \cdot G_A$$

where r_x is the subsistence level of the good x, and d_{Ai} is the distance between state A and state i. The household chooses x and y so as to maximize the above utility subject to the following budget constraint.

$$(p_i + t_i + T)x_A^i + y_A^i = I_A$$

The parameter α_i corresponds to the income share the household spends on the good x above the minimum consumption level r_x. β measures the transportation cost. ϕ_i is a weight between private good and public good utility. I assume that the value of α_i and ϕ_i are the same for people in the same state but different across states. I allow heterogeneity for β within state by assuming β to be distributed randomly across households. The minimum consumption level r_x is assumed to be the same for all states.

The solution of the above problem gives us the following demand for good x and y

$$x_A^i = \frac{\alpha_A I_A}{(p_i + t_i + T)} + (1-\alpha_A)r_x, \quad y_A^i = (1-\alpha_A)\{I_A - (p_i + t_i + T)r_x\} \tag{1}$$

Substituting them into the utility function, I derive the indirect utility function as follows.

$$\begin{aligned} &V_A^i(P_i, I_A, d_{Ai}, G_A) \qquad (2) \\ &= \alpha_A \log \alpha_A + (1-\alpha_A)\log(1-\alpha_A) - \alpha_A \log(p_i + t_i + T) + \log(I_A - (p_i + t_i + T)r_x) - \beta \cdot d_{Ai} \\ &+ \phi_A G_A \end{aligned}$$

[8] I choose Stone-Geary utility function in my model for the following two reasons. First, according to previous papers, a price elasticity of demand for gasoline and cigarettes is about -0.8~1, and -0.5 for each. Stone-Geary utility function is flexible to these values of price elasticity of demand. Second, this Stone-Geary utility function fit well with per capita consumption data of cigarette and gasoline. Other utility function, for example Cobb-Douglas utility function and Quasi-Linear utility function do not satisfy these two points.

Next, I derive the proportion of consumers who cross border shop. Since the utility from a public good is exogenous and does not depend on cross-border shopping, I exclude it from the indirect utility function. Furthermore, I also add a random component to the indirect utility function, which measures the unobserved utility the consumer gets from shopping in state i. Then, the indirect utility function becomes

$$V_A^i(P_i, I_A, d_{Ai}, \varepsilon_{Ai}) = -\alpha_A \log(p_i + t_i + T) + \log(I_A - (p_i + t_i + T) r_x) - \beta \cdot d_{Ai} + \varepsilon_{Ai} \qquad (2')$$

where ε_{Ai} is an error term if people in state A choose state i for shopping. I assume that people only cross border shop in neighboring states that share the same border with their own state. Suppose that state A is surrounded by states B and C, and that people in state A make a choice among three states A, B and C for shopping. Then a household chooses the state to shop that gives the highest indirect utility. That is, if a household in state A chooses state A for shopping, it means $V_A^A > V_A^B, V_A^C$. The share of households in state A that purchase products in their own state A is equal to the probability that state A is chosen for shopping among these three states. If the error term ε_{Ai} is independent and identically distributed with an extreme value distribution, the probability that state A is chosen by households in state A can be expressed as

$$s_A^A = \frac{\exp\{(-\alpha_A \log(p_A + t_A + T)) + \log\{I_A - (p_A + t_A + T) - \beta \cdot d_{AA}\} / h\}}{\sum_{i=A}^{C} \exp\{(-\alpha_A \log(p_i + t_i + T) + \log\{I_i - (p_i + t_i + T)\} - \beta \cdot d_{Ai}) / h\}} \qquad (3)$$

where s_i^j is the share of households in state i who shop in state j, and h is a standard error of the random component ε_{Ai}. It can be calculated given the parameters α_i, β, r_x and h and data on income, after tax price and distance. Remember that households can cross border shop only in neighboring states, and if state i does not share the border with state j, both s_i^j and s_j^i are zero.

Price Elasticity of Demand

From the demand function, the price elasticity of demand becomes

$$\varepsilon = \frac{\partial x}{\partial P}\frac{P}{x} = \frac{-\alpha_i \cdot I_i}{\alpha_i I_i + P_i(1-\alpha_i)r_x} \qquad \text{where} \quad P_i = p_i + t_i + T \tag{4}$$

From this equation, it is clear that the model restricts the price elasticity of demand to lie between -1 and 0, ($-1 < \varepsilon < 0$). Most estimates of price elasticity of demand in the previous literature satisfy the above restriction. Given the parameters α_i, r_x and data for income and after tax price, price elasticity of demand can be easily derived.

3.2 State Government's Problem

I assume that the state government is benevolent and maximizes the aggregate indirect utility of all households in the state.[9]

$$W_A = \int V_A^{i^*}\left(P_{i^*}, I_A, d_{Ai^*}, \varepsilon_{Ai^*}\right) d\varepsilon$$

where i^* is the optimal choice of states that a household in state A goes to shop. Given that the unobserved utility term ε_{ij} is assumed to be i.i.d extreme valued, the above integral can be expressed analytically as follows (N_A are neighboring states for state A).

$$W_A = \log\left\{\sum_{i=A}^{N_A} \exp\left[\alpha_A \log\left(x_A^i - r_x\right) + (1-\alpha_A) y_A^i - \beta \cdot d_{Ai} + \phi \cdot G_A\right]\right\}^{10}$$

The state government's budget constraint is as follows.

$$G_A = TR_{GA} + TR_{OA} + g_A{}^{11}$$

where TR_{GA} is per capita revenue from the gasoline tax (in cigarette case, it is per

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[9] I assume that state governments determine gasoline and cigarette tax rates separately. It is impossible to include both gasoline and cigarette consumption in one utility function because it prevents estimating the share function s_i^j separately for both goods.

[10] Please refer to Rust (1987), pp.1012.

[11] Some people might think that federal transfer depends on the federal tax rate, and that the amount g_A is also a function of federal tax rate T to derive the reaction function, especially for the gasoline tax case. When I research the history, there are several times when the federal government raise gasoline tax rate but most of increased tax revenue is used to finance other things, like war expenditures, decreasing fiscal deficit and so on. Therefore, the increase in the federal gasoline tax rate does not necessarily mean increase in federal grant and I assume that increasing federal tax rate will not affect the federal grant for simplicity.

capita revenue from the cigarette tax), TR_{OA} is per capita tax revenue from other sources and g_A is a per capita grant from the federal government. Gasoline tax revenue TR_{GA} can be expressed as follows.

$$TR_{GA} = \frac{1}{n_A}\left(n_A s_A^A x_A^A + n_B s_B^A x_B^A + n_C s_C^A x_C^A\right)t_A .$$

Where the term in parenthesis is the tax base, i.e. the amount of gasoline that is purchased in state A. Notice that the tax base consist of not only households in state A but also households in neighboring states B and C that decide to purchase gasoline in state A. It is also important to notice that the gasoline tax revenue not only depends on per capita consumption x_i^A, but also on the number of shoppers from state i, $n_i s_i^A$, and per unit tax t_A. Next, I derive the first order condition of the optimal taxation.

First Order Condition

The state government A determines the optimal tax rate to maximize W_A with other state and federal governments' tax policy as given. The first order condition with respect to its state tax t_A is,

$$\frac{\partial W_A}{\partial t_A} = \left(\frac{\alpha_A}{x_A^A - r_x}\frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A}\frac{\partial y_A^A}{\partial t_A}\right)s_A^A + \phi_A \frac{\partial TR_{GA}}{\partial t_A} = 0 \tag{5}$$

$$= -\left(\frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\}(p_A + t_A + T)}\right)s_A^A + \phi_A \frac{\partial TR_{GA}}{\partial t_A} = 0$$

where

$$\frac{\partial TR_{GA}}{\partial t_A} = n_A s_A^A \frac{\partial x_A^A}{\partial t_A} t_A + n_B s_B^A \frac{\partial x_B^A}{\partial t_A} t_A + n_C s_C^A \frac{\partial x_C^A}{\partial t_A} t_A$$

$$+ n_A\left(s_A^A + \frac{\partial s_A^A}{\partial t_A} t_A\right)x_A^A + n_B\left(s_B^A + \frac{\partial s_B^A}{\partial t_A} t_A\right)x_B^A + n_C\left(s_C^A + \frac{\partial s_C^A}{\partial t_A} t_A\right)x_C^A = 0$$

Reaction Function

The reaction function is derived from differentiating the first order condition above with respect to the tax rate. The slope of the reaction function measuring the effect of state B taxes change on state A government's tax is

$$\frac{\partial t_A}{\partial t_B} = -\frac{\partial^2 W_A}{\partial t_A \partial t_B} \bigg/ \frac{\partial^2 W_A}{\partial {t_A}^2} \tag{6}$$

and the slope of the reaction function measuring the effect of federal tax change on taxes of state A[12] is

$$\frac{\partial t_A}{\partial T} = -\frac{\partial^2 W_A}{\partial t_A \partial T} \bigg/ \frac{\partial^2 W_A}{\partial {t_A}^2} \tag{7}$$

Where (for more details, see Appendix A)

$$\frac{\partial^2 W_A}{\partial {t_A}^2} = \left(\frac{-\alpha_A}{\left(x_A^A - r_x\right)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-\left(1-\alpha_A\right)}{{y_A^A}^2} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial {t_A}^2} \right) s_A^A$$
$$- \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial p_A^S} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial {t_A}^2} \right) < 0$$

$$\frac{\partial^2 W_A}{\partial t_A \partial t_B} = - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial t_B} + \phi \left(\frac{\partial TR_{GA}^2}{\partial t_A \partial t_B} \right)$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \left(\frac{-\alpha_A}{\left(x_A^A - r_x\right)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right) \left(\frac{\partial x_A^A}{\partial T} \right) + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A \partial T} + \frac{-\left(1-\alpha_A\right)}{{y_A^A}^2} \left(\frac{\partial y_A^A}{\partial t_A} \right) \left(\frac{\partial y_A^A}{\partial T} \right) + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A \partial T} \right) s_A^A$$
$$- \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right)$$

Strategic Interactions between Governments

I would like to explain where the strategic interactions of taxation between governments are represented in the reaction function. In the horizontal tax interaction case, state governments compete for cross border shoppers to increase tax revenue for public goods, and how much cross border shoppers are sensitive to the tax rate change of other state governments is an important factor. In the model, the term $\frac{\partial s_A^A}{\partial t_B}$ represents this sensitivity which shows up in the term $\frac{\partial^2 W_A}{\partial t_A \partial t_B}$ and $\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B}$, and the scale of

[12] If the state government is Leviathan, $\frac{\partial t_A}{\partial t_B} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} \bigg/ \frac{\partial^2 TR_{GA}}{\partial {t_A}^2}$ and $\frac{\partial t_A}{\partial T} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \bigg/ \frac{\partial^2 TR_{GA}}{\partial {t_A}^2}$ for each.

horizontal tax interaction depends on this factor. If this value is small, the scale of horizontal tax interaction is small and vice versa.

On the other hand, in the vertical tax interaction case, state and federal governments share a common tax base and how much this tax base (consumer's demand) is sensitive to tax rate change of the federal government is a crucial factor. In the equation, the term $\frac{\partial x_A^A}{\partial T} = \frac{x_A^A}{P^2}\varepsilon$ represents the tax elasticity of tax base which enter in the term $\frac{\partial^2 W_A}{\partial t_A \partial T}$ and $\frac{\partial^2 TR_{GA}}{\partial t_A \partial T}$. In short, the price elasticity of demand and after tax price are key factors for vertical tax interaction. I will return to this issue again in section 7 and section 8.2.

4 Empirical Analysis

4.1 Estimating the Parameters of the Household Utility Function

Moment Condition

One difficulty in estimating the parameters of consumers' utility is that the data on how much gasoline or cigarettes are consumed by households in different states are unavailable at the state level[13]. The only available data are total sales, tax revenue, per unit tax rate and population in each state level. In other words, I do not know how much tax revenue comes from in-state consumers or out of state consumers. Considering this data restriction, I match the data of total sales in each state with the predicted total sales based on the model. For example, consider the case where there are only 3 states: state A, B and C, and they are all neighbors to each other. Then, the predicted total sales in state A,B and C can be expressed as follows.

$$C_A = n_A \cdot s_A^A \cdot x_A^A + n_B \cdot s_B^A \cdot x_B^A + n_C \cdot s_C^A \cdot x_C^A$$

$$C_B = n_A \cdot s_A^B \cdot x_A^B + n_B \cdot s_B^B \cdot x_B^B + n_C \cdot s_C^B \cdot x_C^B$$

$$C_C = n_A \cdot s_A^C \cdot x_A^C + n_B \cdot s_B^C \cdot x_B^C + n_C \cdot s_C^C \cdot x_C^C$$

13 I confirmed this point with the U.S. Bureau of Labour Statistics.

I then assume that the actual total sales C_i^d are the sum of the predicted total sales C_i plus an error term e_{C_i}. That is,

$$C_i^d = C_i + e_{C_i}, i = A, B, C$$

The Moment Condition, which minimize the difference between total sales in the data C_i^d and the total sales predicted based on the model C_i is

$$E\left(C_i - C_i^d \mid Z_i\right) = E\left(e_{C_i} \mid Z_i\right) = 0 \tag{8}$$

where Z_i is a vector of instruments. From this Moment Condition, I can estimate the parameters of the household utility function α_i, β, r_x and h which show up in the share function s_i^j and demand function x_i^j.

<u>Endogeneity Issues</u>

The tax policy of the state government creates a potential endogeneity problem in the above moment condition estimation in equation (8). Since the state i government maximizes welfare taking into account the consumer's behavior, its tax rate should be a function of the demand, and the error term should affect its tax. Hence, the error term e_{C_i} and tax rate t_A will be correlated, resulting in the bias of the coefficient estimates. To deal with this issue, I use IV method and use some instrument variables; i.e. for gasoline consumption, constant term, per capita federal grants to highway departments, population, the share of gasoline revenue to highway expenditure, the amount of CO2 emission and for cigarette consumption, constant term, per capita federal grants to health departments, population, the share of cigarette tax revenue to highway expenditure and the percentage of smokers in the state. These variables are related with state tax policy but I believe that it is reasonable to assume that they are not related to the error term, i.e. the unobservable component of the state gasoline and cigarette sales.

Next, I discuss in more detail the parameterization of the empirical model. The parameter α_i means how much share of their income a household spends on the consumption of the good x. This is likely to depend on the household's preferences and economic conditions. Hence, I assume that α_i for gasoline consumption is a linear function of log population density and log per capita income, and α_i for cigarette consumption is a linear function of log ratio of females in population and log per capita income[14]. That is,

$$\alpha_i = \alpha_0 + \alpha_1\left[\log(density) - \overline{\log(density)}\right] + \alpha_2\left[\log(income) - \overline{\log(income)}\right] \text{ for gasoline}$$

$$\alpha_i = \alpha_0 + \alpha_1\left[\log(female) - \overline{\log(female)}\right] + \alpha_2\left[\log(income) - \overline{\log(income)}\right] \text{ for cigarettes}$$

It is also natural to think that the cost of cross border shopping is different between people who live in the center of the state and people who live along the border of the state. In order to fully deal with this issue, one needs to accurately measure the geography of each state and the distribution of consumers over its area, which is not feasible. Instead, I address the issue by applying the idea of "the random coefficient model" from Bajari et al (2007) and Berry et al (1995), and allow the transportation cost parameter β to take different values for different households in the same state, but restrict the distribution of β to be same across states. I assume that $\beta = \eta\beta^*$ where β^* is taken to be chi-squared distributed with one degree of freedom. The parameter η is estimated. The minimum consumption r_x and the standard error h are assumed to be the same for all states.

4.2 Estimating the Parameter of State Government Objective Function

After estimating the parameters of the household utility, $\hat{\alpha}_i, \hat{\eta}$, $\hat{r}_x$ and $\hat{h}$, I then estimate the remaining parameter of the state government objective function ϕ_i, which determines the weight between utility from private goods and utility from public goods. It is estimated using the first order condition of the state government choosing the optimal tax level as a moment condition. That is,

$$\frac{\partial W_i}{\partial t_i} = -\left(\frac{\alpha_i I_i + (1-\alpha_i) r_x (p_i + t_i + T)}{\{I_i - (p_i + t_i + T) r_x\}(p_i + t_i + T)}\right) s_i^i + \phi_i \frac{\partial TR_{Gi}}{\partial t_i} = 0 \qquad (9)$$

Similar to the idea of the parameter α_i, I consider the parameter ϕ_i is likely to depend

14 I also subscribe from the average, which is represented by the bar term.

on the economic environment of each state. Therefore, I assume that ϕ_i is a linear function of income, population, the share of gasoline or cigarette tax revenue to the highway or health expenditure for both gasoline and cigarette cases.

$$\phi = \phi_0 + \phi_1 \cdot (income)^2 + \phi_2 \cdot (income) + \phi_3 \cdot (pop) + \phi_4 \cdot (share)$$

I estimate ϕ using the above moment condition in equation (9), given the parameter of household utility and data on price, per unit tax, income, and population. As instrumental variables, I use constant term, the previous year's per capita federal grant to highway department, CO2 emission and per capita car registrations for the gasoline case, while for the cigarette case, I use constant term, the previous year's per capita federal grant to health department, the percentage of smokers and the number of deaths caused by cancer.

5 Data

I use data on 48 U.S. states from 1999 to 2002. I exclude Hawaii and Alaska since both do not share the border with other states. I downloaded the unit tax rate of gasoline from the webpage of Federal Highway Administration (U.S Department of Highway). I used the gasoline price and consumption data in official Energy Statistics from the U.S. Government, which can be obtained from the website of Energy Information Administration. For cigarettes, I use cigarette price, tax rate, tax revenue, and consumption from the Report "The Tax Burden on Tobacco" by Orzechowski and Walker[15]. There are two candidates for state consumption data. One is consumption data itself, and another is calculated by dividing total tax revenue by unit tax price. I find that the original consumption data seems to be more accurate because of its small variance of per capita consumption across states. I use population and per capita disposal income data from the Bureau of Economic Analysis, and per capita government expenditure for highway and health from the U.S Census of Bureau. Both per capita federal grants to highway and health departments are available from Statistics of Abstract (National data book from U.S. Census of Bureau). I derive the population density by dividing the population by the land area which is available from Statistics of Abstract. Ratio of

[15] This report is available by request. Please refer http://www.srnt.org/pubs/nl_05_06/spotlight.html.

female to total population is accessible from the webpage of Center of Disease Control and Prevention. CO2 emission is obtained from the webpage of U.S. Environment Protection Agency. The number of car registrations is available from the webpage of Federal Highway Administration. The percentage of smokers and the number of the deaths caused by cancer come from Statistics of Abstract. I also computed the distance data from Google map. For the estimation, I used the real data by regarding the 1999 year data as index (=100). The consumption price index data for this realization is available from the webpage of Bureau of Labor Statistics. Details about data resources are explained in Appendix B.

6 Estimation Results

6.1 Moment Estimation

All the parameter estimates for household utility are shown in Table 1. Recall that α_i measures the share of private good consumption to total income after excluding minimum amount of consumption r_x. η measures the disutility from transportation costs. High transportation costs discourage consumer from purchasing goods in other states. r_x measures the minimum amount of consumption. h is the standard error of the random component ε_i^j.

Table 1.

$$\alpha_i = \alpha_0 + \alpha_1\left[\log(density) - \overline{\log(density)}\right] + \alpha_2\left[\log(income) - \overline{\log(income)}\right] \text{ for gasoline}$$

$$\alpha_i = \alpha_0 + \alpha_1\left[\log(female) - \overline{\log(female)}\right] + \alpha_2\left[\log(income) - \overline{\log(income)}\right] \text{ for cigarettes}$$

	Gasoline	Cigarette
α_0	0.0221	0.00521
α_1	-0.00297	0.0999
α_2	-0.0201	-0.00786
r_x	101.1	31.94
η	0.808	0.601
h	0.398	0.451
Demand price elasticity	-0.795	-0.529

From Table 1, for gasoline the coefficient α_1 is estimated to be -0.00297. This means that people in states where population densities are high spend lower share of their income on gasoline, which seems to be reasonable, since the high population density states would be more urban. Furthermore, the coefficient α_2 is estimated to be -0.0201. This means that states that have higher per household income spend lower share of their income on gasoline, which again seems reasonable. The total share of gasoline consumption to income, including the minimum consumption r_x is calculated to be 0.0271 on average. This is very close to the value 0.0275 in the data.

The parameter value of r_x is 101.1. The per capita demand for gasoline is 495 gallon on average in states, where the minimum amount is 294 in New York and the maximum amount is 690 in Wyoming. Considering these numbers, I believe the value of r_x to be reasonable. η is estimated to be 0.808. This parameter is used to explain the relative importance of cross border shopping. The estimated average share of households that purchase products in their own state is 96.1%. That is, about 4% of people do cross border shopping across states, which I believe to be sensible. Using the model and the parameter estimates, I also calculated the price elasticity of demand, which is -0.795 on average. This is close to the values obtained in the literature, which range from -0.8 to -1.

Next, I discuss the estimation results when data on cigarette consumption are used. α_1 is estimated to be 0.0999. Interestingly, this means that states with higher female populations consume more cigarettes. This result is acceptable considering the recent trend[16] of larger decline in the percentage of male smokers than female smokers and the previous papers' result that the price elasticity of cigarette demand is less for females than for males. α_2 is estimated to be -0.00786, which again means that higher income states spend a lower share of their income on cigarettes. Again this is consistent with the literature on smoking in health economics and in consumption estimation. The total share of cigarette consumption to income including the minimum consumption r_x is 0.00963 on average. This is close to the share of cigarette consumption 0.0115 in the data.

r_x is estimated to be 31.94[17]. The average per capita demand for cigarette is 84 packs per year. The minimum per capita demand is 35 in California and the maximum is 156 in New Hampshire. Considering these values, I again believe the estimated value of

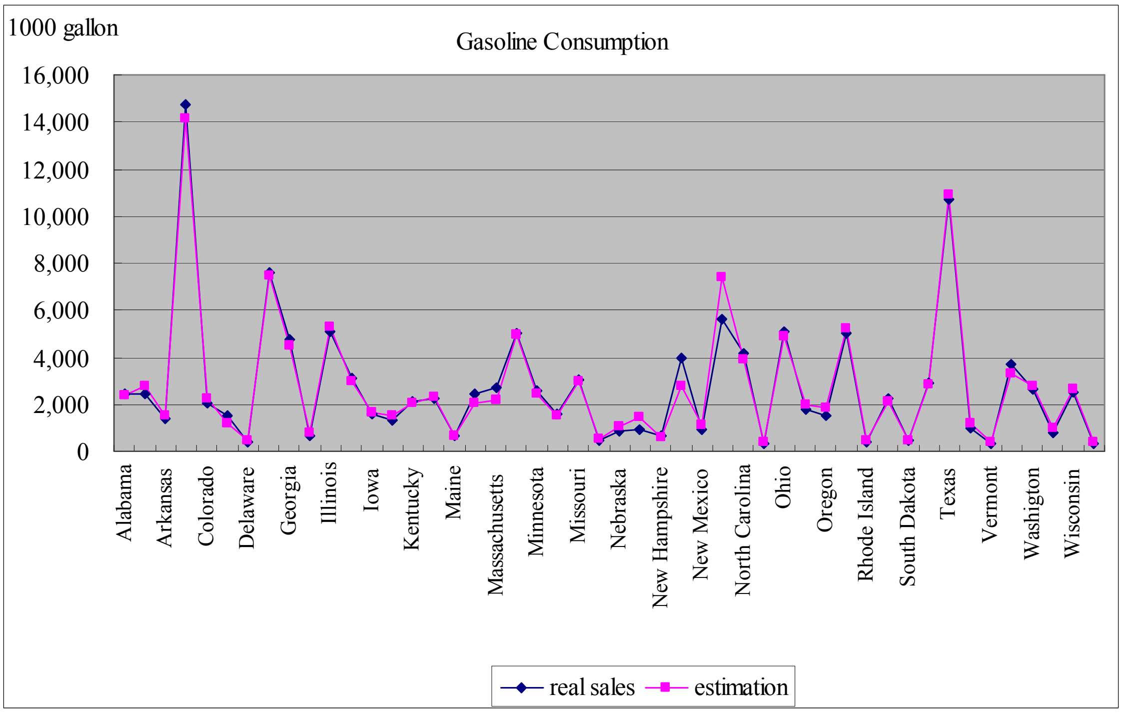
16 McGinnis, M., (1987) "TOBACCO AND HEALTH: Trends in Smoking and Smokeless Tobacco Consumption in the United States.", Annual Review Public Health, 8, pp.442-467

17 The idea of "minimum consumption" for cigarette consumption seems odd considering that not everyone consumes cigarette, and only about 20% people smoke. The value r_x is a kind of number when we assume that everyone smokes, and this amount is the minimum amount of consumption of the representative person. Please refer Appendix F for the case where there are both smokers and non smokers.

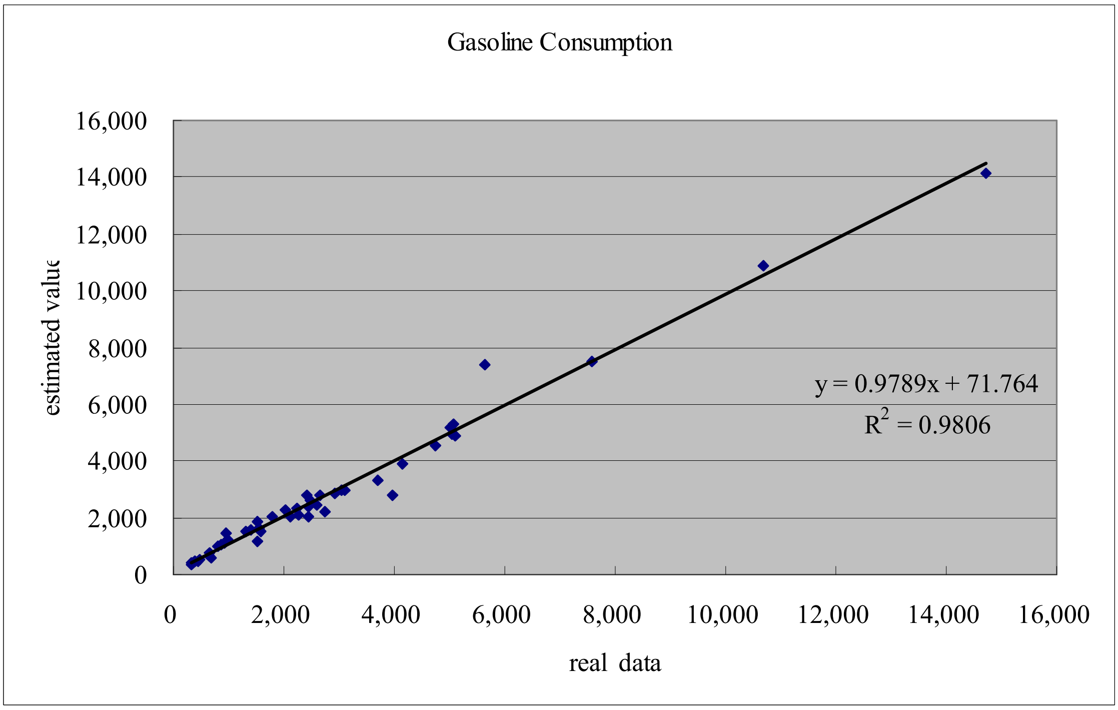
r_x to be reasonable. η is estimated to be 0.601. As before, I can determine whether this value is reasonable from the value of the share function. The estimated average share of within state consumption is 95.1%. That is, about 5% of people cross border shop for cigarettes. Flennor (1998) shows that the percentage of cross border purchases of cigarettes was approximately 3.6% in 1997. Considering the recent increase of cigarette prices and tax rates from 1997, I believe the value 5% to be consistent with Flennor's result. I also compute the price elasticity of demand to be -0.529, which is close to the value -0.5 obtained in the literature.

To see how well the model fits the actual data, I compare the real total sales and total sales predicted based on my model. Graph 1a and Graph 1b compare real data and estimated value for the gasoline consumption. In Graph 1a, we can see that estimated value fit very well across 48 states except New York. Graph 1b shows the correlation between the estimated value and the real data, and the value of the correlation is 0.979, which is very close to 1, and R-squares of the linear regression line is 0.9806. When I draw the same Graph excluding New York, the correlation is 0.9631 and R-squares is 0.9899. The estimated values are in almost perfect fit with real data. Notice that per capita demand for gasoline is extremely low in New York. On average, per capita gasoline consumption across states is around 500 gallon, but in New York it is less than 300 gallon. Even though geographic factor like population density is taken into account for household preference, the availability of public transportation in New York is not captured in the model. Otherwise, the model fits to the data very well in other states, in spite of its rather parsimonious parameterization.

Graph 1a: Average (4years) total gasoline sales

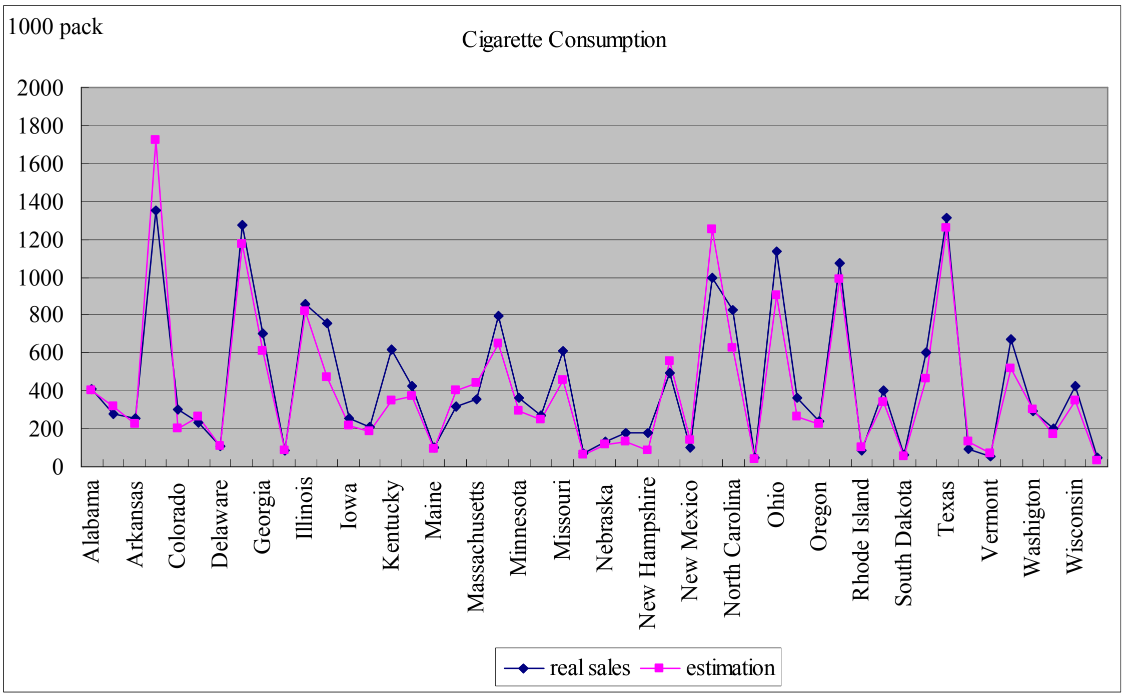


Graph 1b: The relationship between real data and estimated value (Gasoline)



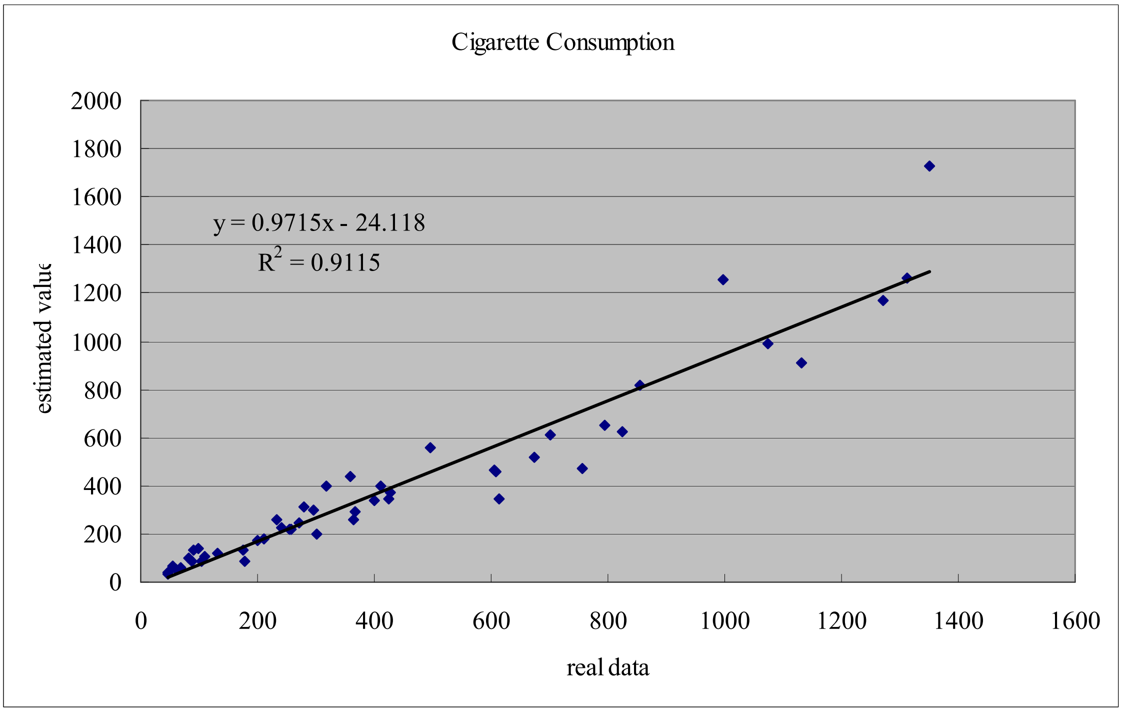
Graph 2a and Graph 2b compare predicated and the actual state level cigarette consumption. Graph 2a shows that the estimated value fit very well with the real data in most states. The estimated value noticeably exceeds the actual ones in California and New York. Graph 2b shows the correlation between estimated value and the actual data. The correlation between them is 0.9715 and R-squares of the regression line is 0.915. If I exclude New York and California, the correlation is 0.8547 and R-squares is 0.9475, i.e., the model fitness improves. The reason why the model fails to fit for the California and New York data is that per capita cigarettes consumption in the two states are extremely low. In California, per capita cigarette consumption is 35 packs and in New York it is 46 packs, whereas the average per capita consumption across states is 80 packs. The reason why cigarette consumption is so low in these two states is that their policy makers are known to be aggressive in reducing smoking of the younger generations and show great concern about the health problems associated with smoking. They increased cigarette tax rates drastically and spent much money on anti-smoking programs[18]. In this model these policy differences across states are not taken into account. Nevertheless the estimated value fit very well with actual data on the whole, and these two graphs confirm that my model captures consumer's consumption behavior very well.

Graph 2a: Average (4years) total cigarette sales



18 Refer to the website of Campaign for tobacco-free kids. http://www.tobaccofreekids.org/reports/settlements/

Graph 2b: The relationship between real data and estimated value (Cigarette)



To sum up, in my estimated model, households use about 3% of their income on gasoline consumption, and 4% of households cross the state border to purchase gasoline. Similarly, about 1% of income is used for cigarette consumption, and 5% of households cross the border to buy cigarettes. It is also important to notice that η is estimated to be larger for gasoline than for cigarettes, which results in households cross border shopping more for cigarettes than for gasoline. I consider the above result to be reasonable since the transportation costs of gasoline should be higher than those of cigarettes.

6.2 Estimation of First Order Condition of State Government

The parameter ϕ is the weight between utility from the private good and utility from the public good, and this parameter is estimated from the first order condition of state government with respect tax rate. The results are shown in Table 2.

Table 2.

$$W_A = \log\left\{\sum \exp\left[\alpha_A \log\left(x_A^i - r_x\right) + (1-\alpha_A) y_A^i - \eta \cdot \beta^* \cdot d_{Ai} + \phi \cdot G_A\right]\right\}$$

$$\phi = \phi_0 + \phi_1 \cdot (income)^2 + \phi_2 \cdot (income) + \phi_3 \cdot (pop) + \phi_4 \cdot (share)$$

	Gasoline	Cigarette
Constant	0.0001392	0.0001284
Income ^2	8.03E-08	8.11E-08
Income	-5.99E-06	-5.71E-06
Pop	-6.66E-08	5.78E-08
Share	6.55E-06	5.17E-06
Time dummy 1999	5.48E-07	
Time dummy 2000	-1.23E-06	
Time dummy 2001	-7.07E-07	

The coefficients of income ϕ_1 and ϕ_2 is estimated as 8.03 e-08 and -5.99 e-06 for gasoline and 8.11 e-08 and -5.71 e-06 for cigarettes[19]. By calculating, the total effect of income is turned out o be negative, and this means that the higher income states weight utility from the private good higher than utility from the public good. This result seems natural, since in richer states, the private sector offers similar or alternative services in place of public services, resulting in lower marginal benefit from government public services. On the other hand, the coefficient of population ϕ_3 is estimated as -6.66 e-08 (negative) for gasoline and 5.78 e-08 (positive) for cigarette. This result implies that for highway expenditure, the scale economy works, and the larger the number of population, the less the amount of per capita highway expenditure, and for health expenditure, there is a congestion cost, and the larger the number of population, the more the amount of per capita health expenditure is necessary. The coefficient of the share is 6.55 e-06 for gasoline and 5.17 e-06 for cigarette, and both are positive. It means that if the share of tax revenue to the expenditure is larger, state government put higher weight for the utility from public good, which is sensible.

19 This value appears to be very small, but the unit is changeable.

6.3 Reaction Function

Given the model and the estimated parameters, per unit taxes and other variables in the data, I compute the slope of the reaction function following equations (6) and (7). Both the horizontal and vertical reaction functions for each of the 48 states are derived. Notice that in my model, households can do cross border shopping only in neighboring states, and the value of the slope of reaction function between state governments is 0 if two states are not neighbors or states do not compete for the same cross border shoppers. The average slope[20] of the horizontal reaction function between state governments is 0.000377 for gasoline tax and 0.000241 for cigarette tax (for more details, see Appendix C). Those results imply that there is almost no horizontal tax interaction among state governments. One reason for this result is that the share of gasoline or cigarette consumption to income is very small; 3% for the former and 1% for the latter. This small share will not give households enough incentive for cross border shopping. Also, from the data, state sales and state population roughly correspond, and only a very small fraction of households is estimated to cross border shop (4% for gasoline and 5% for cigarettes). Since cross border shopping is the only reaction to taxes in other states, the small horizontal reaction seems to be reasonable. It is also important to notice that the value of the slope of the reaction function between non-neighboring states is not always estimated to be zero. This is because tax changes in non-neighboring states can have an effect through cross-border shopping by consumers who live in states between those two[21].

In contrast, the value of the slope of the vertical reaction function between state and federal governments is much higher (for more details, see Appendix D). The average value is 0.242 for gasoline tax and 0.265 for cigarette tax. This means that state and federal taxes are strategic complements. An increase in federal tax reduces the tax base of the state government and makes it necessary for state governments to increase taxes to pay for the spending of public goods. The true criterion for the tax externality is the absolute value of the slope of reaction function. The average of the absolute value is 0.243 for the gasoline tax and 0.296 for the cigarette tax. These results demonstrate that the scale of vertical externality is larger for the cigarette tax than for the gasoline tax.

In Graph 3, we plot the slope of the reaction function for both gasoline and cigarette taxes against change in federal tax. From Graph 3, it is clear that the value of the slope of reaction function is larger for the cigarette tax than for the gasoline tax. Graph 4 of

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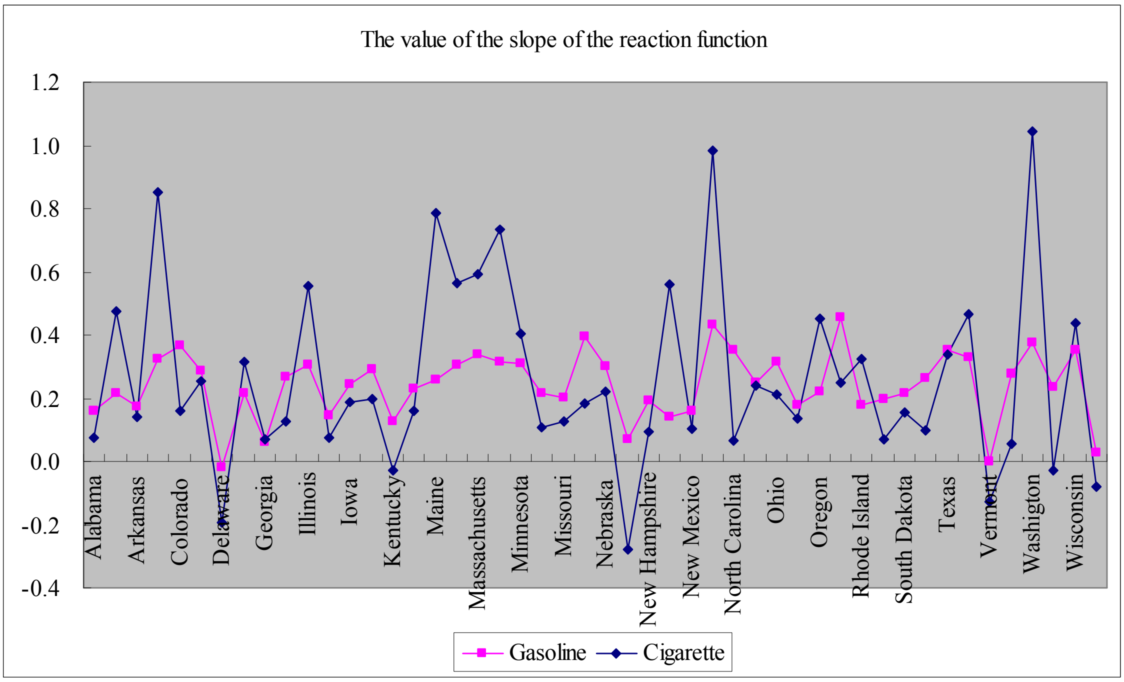
20 The average is the average value for neighbors which take non zero value.

21 This is the case where non neighbors state compete for the same cross border shoppers.

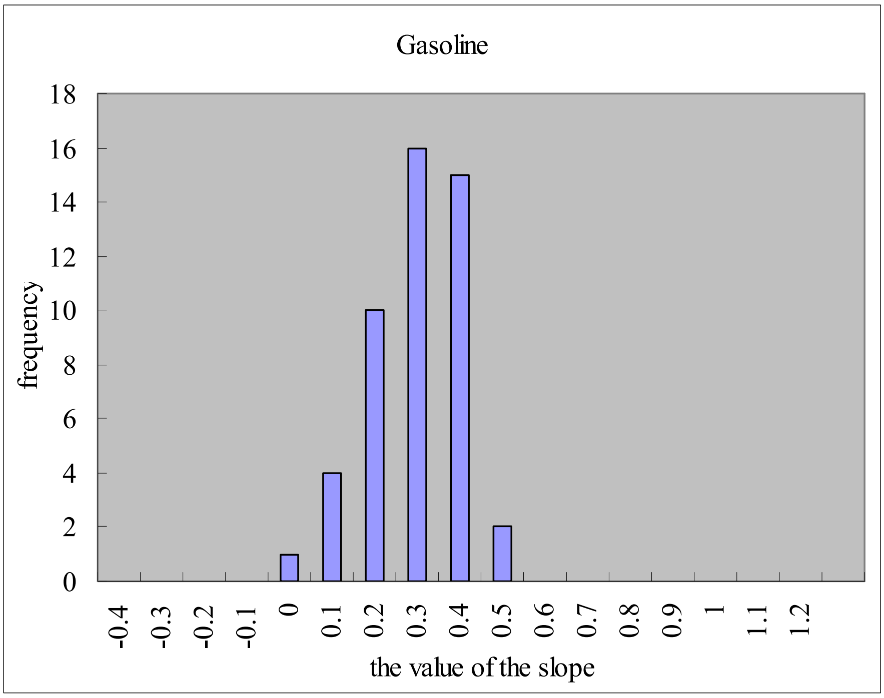
the histogram also shows that the scale and variance of the slopes are different between the gasoline and cigarette taxes. This result is consistent with the general idea of the relationship between price elasticity of demand and intensity of tax interaction. Generally, governments are reluctant to levy a heavier tax rate on a good whose price elasticity of demand is high to avoid losing tax base. For those goods whose price elasticity of demand is high, the consumer's demand changes drastically with the change in tax rate. Hence, the response of state government tax policies to other state or federal tax changes must be greater for a good whose price elasticity of demand is low since state governments do not need to be afraid of losing tax base even though they change their own tax rate following the other government's taxation change. Also, we can see a high positive correlation between the two slopes from Graph 3. I will explain this correlation by analyzing the relation between the scale of the vertical externality and the price elasticity of demand or after tax price in section 8.2.

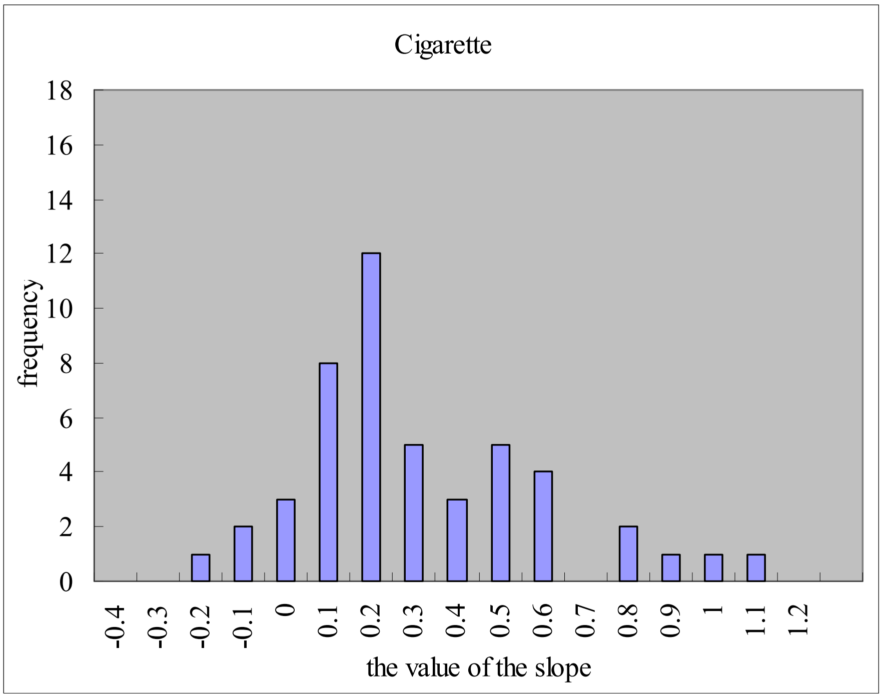
Graph 3: The value of the slope of the reaction function (Vertical Externality case)



It is also important to notice that even though on average, the slopes of the reaction function are positive, in some states, the slopes are negative. These results underscore my main point that the slopes of the reaction function are highly nonlinear functions of variables such as the share of consumption to income α_i, price elasticity of demand ε, after tax price P_i, the share function s_i^j and income I_i. As these variables show sizeable variation across states, it is very natural that the slopes of the reaction function vary across states in ways that cannot be approximated well by the weighted matrix, which imposes the same sign and scale on the slope of the vertical reaction function.

Graph 4: Histogram of the value of the slope of the reaction function (Vertical Externality case)





7 Intuitions

In this section, I would like to explain the factors which determine the sign and the scale of the slope of the reaction function based on the model and the intuition of it.

Horizontal tax interaction is attributed to consumers' cross border shopping and the scale of the slope of the horizontal reaction function is mainly determined by the share of private good consumption to income and the share function. We have seen from the estimation results that the slope of the horizontal reaction function is estimated to be small. The model indicates that the slope of the reaction function depends crucially on the cross border shopping behavior of households, since that is the only way that tax changes of other states affect consumers. We now present how share of own state consumers of state A (s_A^A) change due to changes in taxes in a neighbor state B (t_B).

$$\frac{\partial s_A^A}{\partial t_B} = -\left(\frac{\alpha_A}{x_A^B - r_x} \frac{x_A^B}{P_B} \varepsilon + \frac{1-\alpha_A}{y_A^B} \frac{\partial y_A^B}{\partial t_B} \right) s_A^A s_A^B$$

We can see that it depends on the parameter α_A roughly measuring the share of private goods consumption to income, price elasticity of demand ε and share function s_A^A and s_A^B. First, the share of private good consumption to income α_A is small. The ratio of gasoline or cigarette consumption to income is 3% and 1% for each, and this small ratio does not give people enough motivation to cross border shop. In addition, only small percentage of people cross border shop in both gasoline (4%) and cigarette (5%) cases and the share function s_i^j (in this example case s_A^B) is very small. Therefore, few people in state A are affected by state government B's tax rate change. For these two reasons, the value of the slope of the horizontal reaction function is small in both gasoline and cigarette cases.

Vertical tax interaction results from the fact that federal and state governments share the common tax base and the scale of the slope of the vertical reaction function depends on the utility function, price elasticity of demand and after tax price. If the federal government increases its tax rate, households reduces their demand for the private good. Tax revenue in state A decreases, and utility from both the private good and the public good decline. If state government A increase its tax rate with the federal government, utility from the private good and the public good move in opposite directions. Households must reduce their demand more for the private good, and utility from the private good decreases further. On the other hand, tax revenue from the private good increases, and utility from the public good increases. For simplicity, I consider the no cross border shopping case and express the utility function as,

$$W = u(x) + f(G)$$

where x is private good and G is public good. The numerator of the vertical reaction function is expressed as follow.

$$\frac{\partial W}{\partial t_A \partial T} = \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial x}{\partial t_A}\right)\left(\frac{\partial x}{\partial T}\right) + \frac{\partial u}{\partial x}\left(\frac{\partial^2 x}{\partial t_A \partial T}\right) + \frac{\partial^2 f}{\partial G^2}\left(\frac{\partial G}{\partial t_A}\right)\left(\frac{\partial G}{\partial T}\right) + \frac{\partial f}{\partial G}\left(\frac{\partial^2 G}{\partial t_A \partial T}\right)$$

$$= \frac{\partial^2 u}{\partial x^2}\left(\frac{x}{P}\varepsilon\right)^2 + \frac{\partial u}{\partial x}\left(-2\frac{x}{P^2}\varepsilon\right) + \frac{\partial^2 f}{\partial G^2}\left(\frac{x \cdot t_A}{P}\varepsilon + x\right)\left(\frac{x \cdot t_A}{P}\varepsilon\right) + \frac{\partial f}{\partial G}\left(-2\frac{x \cdot t_A}{P^2}\varepsilon + \frac{x}{P}\varepsilon\right)$$

State Government A compares "the extent of change of disadvantage (additional decrease in utility from a private good)" which is represented by first and second term, and "the extent of change of advantage (increase in utility from a public good)" which is represented by third and fourth term, and tries to equalize these two values to maximize the welfare of people. The scale of state government A's response to federal government tax rate change hinges on the difference between these two scales in increasing its tax rate. If this difference is larger, the state government has to respond considerably to equalize the marginal benefit and cost of increasing the tax rate. Conversely, if this difference is small, the state government reacts little to the federal government's tax policy change. It is clear from this equation that this difference is mainly determined by the utility function ($u(x)$ and $f(G)$), price elasticity of demand (ε) and after tax price(P).

The sign of the slope of the reaction function depends on the relative scale of "advantage" and "disadvantage" of increasing the tax rate. In the horizontal tax interaction case, "advantage" is increased tax revenue to finance the public good and "disadvantage" is disutility from reducing consumption of the private good. If state government B increases its tax rate, some people not only in state A but also in other state shift the place for shopping from state B to A. Then, the tax base of state A expands, and if state government A raises its tax rate, tax revenue increases. On the other hand, if state government A increases tax rate at this time, not only people who originally purchase their own region's good but also people who stop cross border shopping to state B have to reduce consumption of the private good, and utility from the private good decreases. State government A has to compare this advantage and disadvantage. If the advantage is greater, the sign of the slopes is positive, and the state government increases its tax rate to increase tax revenue for the public good. If the disadvantage is greater, the sign of the slopes is negative, and the state government decreases its tax rate to protect utility from the private good.

Similarly, in the vertical tax interaction case, "advantage" is the increase in the utility from a public good and "disadvantage" is the disutility from additionally reducing private good consumption. If the federal government increases its tax rate, people reduce the demand for the private good and tax revenue in state A decreases. If state government A increases its tax rate, utility from the private good and the public good move in opposite directions. Households must reduce the demand for the private good and utility from the private good decreases further. On the other hand, tax revenue from the private good increases, and utility from the public good increases. If the scale change of advantage (utility from a public good) is greater than the scale change of disadvantage (utility from a private good), the sign of the slopes is positive and the state government increase its tax rate to finance public good. If the scale change of disadvantage is larger than that of advantage, the sign of the slopes is negative and the state government decreases its tax rate to protect utility from the private good. In summary, the share of consumption to total income and the percentage of cross border shopping are important factors for horizontal tax interaction while the utility function, price elasticity of demand and after tax price are important factors for vertical tax interaction.

8 Discussions: Comparison with Previous Papers

In this section, I would like to emphasize the contribution of this paper from two different aspects. One aspect is comparing the weighted matrix method with the structural estimation method. Another is clarifying the sign of the slope of the reaction function and the relationship between the scale of the slope of the reaction function and the price elasticity of demand or after tax price in the vertical tax interaction case. I refer to Besley and Rosen (1998) and Devereux et al (2007) for the first argument and Keen (1998) for the latter argument.

8.1 Comparison between the Weighted Matrix Method and the Structural Estimation Method

In previous papers, the weighted matrix method is commonly used for estimating horizontal tax interaction. The idea of the weighted matrix method is calculating a weighted average of other state tax rates using a weighted matrix and regressing each state's tax rate with this weighted average tax rate as an independent variable. In short, this method approximates the complex strategic interactions between state governments. Table 3 shows the comparison between previous papers and this paper's result.

Table 3.

	Besley & Rosen (98)	Devereux et al (07)			This Paper
Gasoline		Uniform	Neighbor	Density[22]	
State		0.131	0.191	-0.099	0.000377
Federal	0.413 ***	0.033	0.077	0.122**	0.242
Cigarette					
State		0.2	0.277***	0.156**	0.000241
Federal	0.277 ***	0.103	0.082	0.081	0.265

** mean 5% significant, *** means 1% significant

Results from previous papers show that (1) Devereux et al (2007) estimate the positive and significant horizontal tax externality in cigarette case, but not in gasoline case. (2) Besley and Rosen (1998) estimate the positive and significant vertical tax externality both in gasoline and cigarette, while Devereux et al (2007) find positive vertical externality only in gasoline case. The scale of vertical externality is bigger in gasoline case than in cigarette case. (3) The sign of the slope of the reaction function is all positive both in the horizontal and vertical externality case. On the other hand, my results derive different results. First, there is little horizontal tax externality in both the gasoline and the cigarette cases. Second, there is a positive vertical tax externality in both the gasoline and cigarette cases. The scale of the tax externality is larger in the cigarette case than the gasoline case, which meet the general idea that governments are reluctant to levy a higher tax rate on a good whose price elasticity is high. Third, the sign and value of the slope of the reaction function is very different across states, and some states take negative values.

[22] These are the factors of the weighted matrix.

There are some reasons why my results are different from previous papers. The time span for the empirical analysis is different[23]. Also, socio economic factors used as independent variables are different. But the most important difference is a method of estimation; the weighted matrix method or structural estimation. This weighted matrix method has some limitations. First, tax response depends only on one variable which is used as a factor of the weighted matrix, and the sign of the slope of the reaction function is assumed to be the same across states. Other important factors (difference between own state tax and that of other state, transportation cost, own and other state's population, demand and price elasticity of demand) are all excluded, resulting in bias and unstable results. Because of this instability, the results are very different, depending which variable is used for the weight. Second, the weighted matrix approach is a poor approximation of the Nash equilibrium of state and federal governments' strategic taxation game. This is because the weighted matrix is a linear approximation around a symmetric Nash equilibrium which is applicable only if state governments are symmetric and the consumers' utility function is Quasi-linear. Hence, the estimation result based on the weighted matrix is reliable only if the equilibrium is very close to being symmetric, i.e. if the states are very similar to each other, and people's demand for the private good is independent to income, which is not true. This results in a misspecification problem.

To demonstrate the problem of the weighted matrix method, I simulate state tax rates based on my model under the condition of no cross border shopping, and replicate the weighted matrix method following Devereux et al (2007)[24]. The simulated data[25] fit well with real state tax rate (please refer to Appendix E) and this supports that my model is appropriate. The estimation result is shown in Table 4. The estimation result demonstrates the significant horizontal tax externality both for the gasoline and cigarette cases. These results are surprising, since state tax rates are simulated under the condition that there is no cross border shopping, and the state government determines its tax rate without taking into account other state's taxation. From this analysis, it is no exaggeration to say that the estimated coefficient does not necessarily mean the slope of the reaction function and that the weighted matrix method is not appropriate for assessing tax externality.

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[23] Besley and Rosen (1998) paper use data from 1975 to 1989 while Devereux et al (2007)'s paper use data from 1977 to 1997. My paper's time span is 1999 to 2002.

[24] I am grateful for Michael Devereux for letting me use his data.

[24] I am grateful for Michael Devereux for letting me use his data.

[25] Of course, the coefficients used for simulation are different from the results in Table 2 since I assume no cross border shopping. But the coefficients estimated under no cross border shopping are almost the same as those of Table 2.

Table 4.

	Simulated Data (No Cross Border Shopping)		
Gasoline	Uniform	Neighbor	Density
State	-5.84***	0.00874	0.246***
	(-4.20)	(1.14)	(3.58)
Federal	13.72***	1.12***	0.822***
	(4.62)	(3.91)	(3.07)
Cigarette			
State	-13.55***	0.248**	0.0806
	(-8.12)	(2.48)	(1.37)
Federal	0.629	-0.233	-0.227
	(1.56)	(-0.52)	(-0.51)

The value in parentheses is t statistics. ** means 5% significant, *** means 1% significant.

Contrary to these limitations, my method has the following virtues. First, my estimation is based on an optimal behavior of household consumption and state government's welfare maximization and fully captures all the important factors for taxation in the model. In addition, the slope of the reaction function is computed directly from the first order condition of the state government, and non linear functional form is taken into account. All the slopes of the reaction functions of state governments are derived for each state and federal government, and different values and signs are allowed across states. Concretely, my structure estimation method overcomes all the problems of the previous weighted matrix method, and my results are more appropriate considering this analysis.

8.2 The Reaction Function in the Vertical Tax Interaction Case

Keen (1998)'s paper examines vertical tax interaction and analyzes the sign of the slope of the reaction function. According to his explanation, the sign of the slope of the reaction function depends on the demand function in the Leviathan case. If the demand function is log convex in after tax price, the sign is negative and, if not, the sign is positive. This idea is consistent with my paper. My demand function is log convex in after tax price and if I calculate the slope of the reaction function in the Leviathan case

$\frac{\partial t_A}{\partial T} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \Big/ \frac{\partial^2 TR_{GA}}{\partial {t_A}^2}$, the value becomes negative. Also, under the extreme assumption of no cross border shopping, this value becomes almost -1/2. This is calculated by using the equation (C') in Appendix G. On the other hand, Keen explains that the sign of the slope of the reaction function is positive in benevolent government case. The author believes that the cost of additional reduction of utility from the private good is less than the benefit of increase in utility from the public good when both federal and state governments increase their tax rate, and that state governments increase their tax rate to finance public good when the federal government increase its tax rate. As I argued before, these results do not hold in my model. The sign depends on the relative scale of "advantage (utility increase from the public good)" and " disadvantage (utility decrease from the private good)" in increasing tax rate, which hinges on utility function, price elasticity of demand and after tax price, and some state take negative value. My results show that some states' slope of the reaction function is positive, and these state governments increase their tax rate when the federal government raises its tax rate, while in some states the slope of the reaction function is negative, and these state governments decrease their tax rate when the federal government raises its tax rate.

Next, I clarify the relationship between the scale of the slope of the vertical reaction function and price elasticity of demand (ε) or after tax price (P). First, I derive the slope of the vertical reaction function $\frac{\partial t_A}{\partial T} = -\frac{\partial^2 W_A}{\partial t_A \partial T} \Big/ \frac{\partial^2 W_A}{\partial {t_A}^2}$ in the equation (C) in Appendix G under the assumption of no cross border shopping. This is an extreme assumption but as we discussed earlier, does not deviate much from the estimated model, and the analysis under this simplification makes it easier to get some sense of how the scale and direction of tax interaction is determined. From the specification of the slope of the reaction function, it is clear that the slope depends on price elasticity of demand and after tax price. The derivative of the equation (C) with respect to price elasticity of demand (ε) is negative[26] and the derivative of equation

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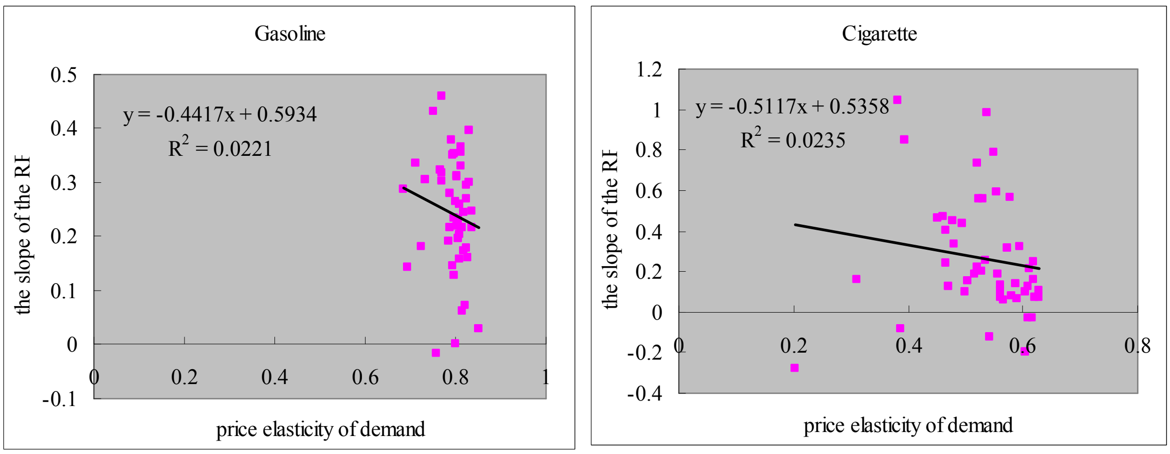
[26] the value is $\dfrac{\dfrac{-2P}{(P+t_A\varepsilon)}}{\left\{-2-\left(\dfrac{x}{x-r_x}+2\dfrac{P-t_A}{P+t_A}\right)\varepsilon\right\}^2} < 0$. This value is derived assuming that x and P is constant for simplicity.

(C) with respect to after tax price (P) is positive[27].

In short, the slope of the reaction function is negatively related to price elasticity of demand and positively related to the after tax price. If price elasticity of demand is large, consumers' response in demand to tax rate change is large, and state government is reluctant to change it tax rate to avoid losing tax base. This is why there is a negative relationship between price elasticity of demand and the scale of tax interaction. Furthermore, if after tax price is high, the price elasticity of demand becomes small[28]. If price elasticity of demand is low, state government's response to other state governments' tax change become large, and this is why there is a positive relation between after tax price and the scale of tax externality. This idea is consistent with my results, as shown in Graphs 5 and 6, where I present the estimated price elasticity of demand, after tax price and the slope of the reaction function for each state.

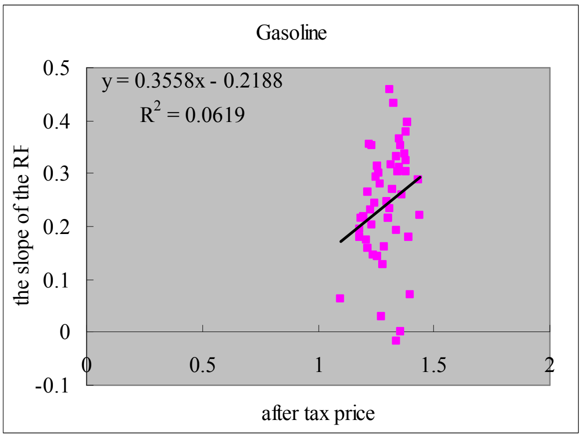
Graph 5: The relation between the slope of the reaction function and price elasticity of demand

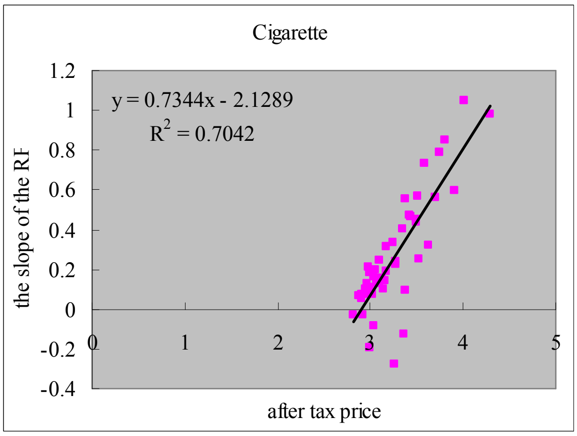


27 the value is $\dfrac{\dfrac{2t_A}{(P+t_A\varepsilon)^2}}{\left\{-\dfrac{2}{\varepsilon}-\dfrac{x}{x-r_x}-2\left(\dfrac{P-t_A}{P+t_A}\right)\right\}^2}>0$. This value is derived assuming that ε and x is constant for simplicity.

28 Please refer to equation (4).

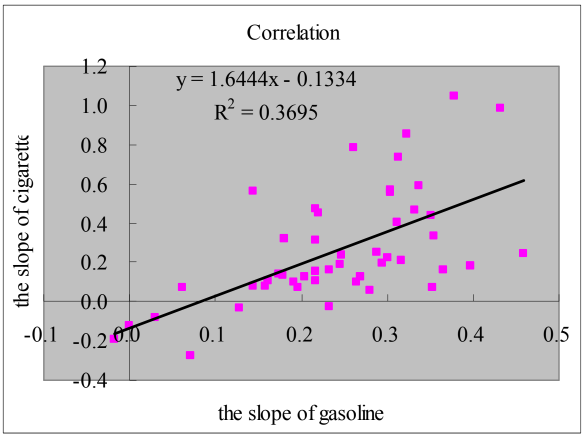
Graph 6: The relation between the slope of the reaction function and after tax price

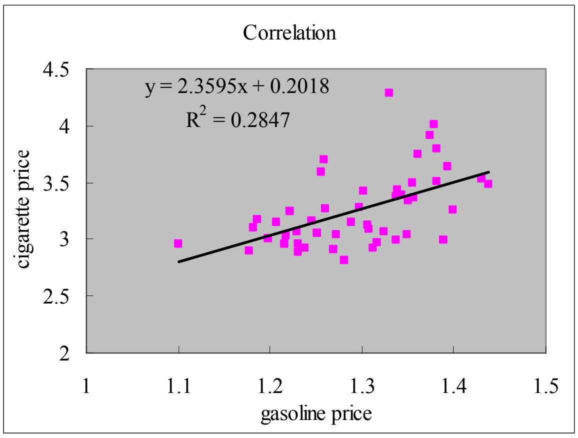




Comparing the gasoline and cigarette tax, the price elasticity of demand is larger in the gasoline case than in the cigarette case, and the scale of vertical externality is larger in the cigarette case than in the gasoline case. I also show a strong correlation between gasoline and cigarette cases in Graph 4, and it is due to a high positive correlation between after price tax of gasoline and cigarettes, as Graph 7 shows.

Graph 7: The correlation between gasoline case and cigarette case





From this argument, I can conclude that the price elasticity of demand and after tax price are important factors for vertical externality.

9 Conclusions

In this paper, I propose a structural estimation approach to analyze the question of whether vertical and horizontal tax interaction exist for gasoline and cigarette taxes. I estimate the structural parameters of the household utility function as well as the percentage of cross border shopping. Given the parameters of the household utility function, I recover the parameter of the objective function of the benevolent state government. Using all the estimated structural parameters, I compute the value of the slope of reaction function for each state, which represents strategic interaction of taxation between governments.

From this analysis, I obtained the following results. First, the estimated value of the slope of the horizontal reaction function between state governments is very small. That is, in contrast to past literature using the weighted matrix approach, we only estimate very small tax interactions between state governments. This is because both gasoline and cigarettes' consumption share to total income is very small, and the percentage of cross border shopping is estimated to be very low. Second, the value of the slope of the reaction function of state tax on federal tax is positive on average for both gasoline and cigarette taxes. That is, the state government tax reacts positively to a federal government increase in tax rate. The value of the slope of the reaction function is estimated to be greater for cigarette tax than for gasoline tax. This result is consistent with the idea of Ramsay tax. Third, even though on average, the slope of the reaction function is positive for both taxes, these values are very different among states, and even negative in some states. I also find that the important factors affecting the slope of the horizontal reaction function are the share of gasoline or cigarette consumption to total income and the percentage of cross border shopping, while the price elasticity of demand and the after tax price are important for the slope of the vertical reaction function. Finally, I generate artificial data of optimal taxes based on a model without cross-border shopping, and use these data to estimate the horizontal and vertical tax interactions using the weighted matrix approach. The estimation results imply strong and significant horizontal tax interaction, even though the model does not have horizontal tax interaction.

The results I obtained are in sharp contrast to those of the previous literature, for example, Besley and Rosen (1998) and Devereux et al (2007). They estimate a positive and significant horizontal tax interaction for cigarette tax and obtained similar estimates for vertical tax interaction for both cigarettes and gasoline taxes. The slope of the vertical reaction function is estimated to be bigger for gasoline tax than for cigarette tax.

This result is inconsistent with the Ramsay's idea of the negative relationship between price elasticity of demand and the scale of tax interaction. Also I find high degree of differences across states in the slopes of the vertical reaction function, which is very interesting, considering the common assumption adopted in the literature that the degree of vertical tax interaction is the same across states. Lastly, the simulation and the estimation exercise using the weighted matrix approach raises an important issue about the possibility of misspecification of the weighted matrix approach. On the other hand, it is clear that structural estimation approach adopted in this paper requires strong and restrictive functional form assumptions on the utility function of consumers and the objective function of the government. In that sense, I believe that the structural approach work as a useful complement to the conventional weighted matrix approach in pointing out possible direction for improvements in specification of the linear model. Also, this paper is a complement of Keen's (1998) paper, and analyzes the sign and the structure of the reaction function in vertical interaction case.

The estimation result has an important policy implication. The different value and sign of the slope of the reaction function tell us that state governments respond to federal government tax policies differently, and that the federal government should not use the same policy for all state to maximize the total welfare in the nation. This also implies that it could be potentially interesting for researchers using nonstructural approaches such as weighted matrix methods to adopt random coefficients estimation techniques or quantile regression techniques to capture heterogeneities of vertical tax interaction. I believe an important topic for future research for both structural and nonstructural analysis is to investigate further the difference in how each state government reacts to federal government policy. This will help federal government to better understand the effect of its tax policy at the state level.

Appendix A: The factor of the reaction function

$$\frac{\partial^2 W_A}{\partial t_A{}^2} = \left(\frac{-\alpha_A}{\left(x_A^A - r_x\right)^2}\left(\frac{\partial x_A^A}{\partial t_A}\right)^2 + \frac{\alpha_A}{x_A^A - r_x}\frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-(1-\alpha_A)}{y_A^{A\,2}}\left(\frac{\partial y_A^A}{\partial t_A}\right)^2 + \frac{1-\alpha_A}{y_A^A}\frac{\partial^2 y_A^A}{\partial t_A{}^2}\right) s_A^A$$

$$-\left(\frac{\alpha_A}{x_A^A - r_x}\frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A}\frac{\partial y_A^A}{\partial t_A}\right)\frac{\partial s_A^A}{\partial t_A} + \phi\left(\frac{\partial^2 TR_{GA}}{\partial t_A{}^2}\right) < 0$$

$$= \left(\frac{\alpha_A I_A^2 - 2\alpha_A I_A (p_A + t_A + T) r_x - (1-\alpha_A)(p_A + t_A + T)^2 r_x{}^2}{\{I_A - (p_A + t_A + T) r_x\}^2 (p_A + t_A + T)^2}\right) s_A^A$$

$$- \frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\}(p_A + t_A + T)}\frac{\partial s_A^A}{\partial t_A} + \phi\left(\frac{\partial^2 TR_{GA}}{\partial t_A{}^2}\right) < 0$$

where

$$\frac{\partial^2 TR_{GA}}{\partial t_A{}^2} = n_A s_A^A \frac{\partial^2 x_A^A}{\partial t_A{}^2} t_A + n_B s_B^A \frac{\partial^2 x_B^A}{\partial t_A{}^2} t_A + n_C s_C^A \frac{\partial^2 x_C^A}{\partial t_A{}^2} t_A$$

$$+ 2n_A\left(s_A^A + \frac{\partial s_A^A}{\partial t_A} t_A\right)\frac{\partial x_A^A}{\partial t_A} + 2n_B\left(s_B^A + \frac{\partial s_B^A}{\partial t_A} t_A\right)\frac{\partial x_B^A}{\partial t_A} + 2n_C\left(s_C^A + \frac{\partial s_C^A}{\partial t_A} t_A\right)\frac{\partial x_C^A}{\partial t_A}$$

$$+ n_A\left(2\frac{\partial s_A^A}{\partial t_A} + \frac{\partial^2 s_A^A}{\partial t_A{}^2} t_A\right) x_A^A + n_B\left(2\frac{\partial s_B^A}{\partial t_A} + \frac{\partial^2 s_B^A}{\partial t_A{}^2} t_A\right) x_B^A + n_C\left(2\frac{\partial s_C^A}{\partial t_A} + \frac{\partial^2 s_C^A}{\partial t_A{}^2} t_A\right) x_C^A < 0$$

$$\frac{\partial^2 W_A}{\partial t_A \partial t_B} = -\left(\frac{\alpha_A}{x_A^A - r_x}\frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A}\frac{\partial y_A^A}{\partial t_A}\right)\frac{\partial s_A^A}{\partial t_B} + \phi\left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B}\right)$$

$$= -\left(\frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\}(p_A + t_A + T)}\right)\frac{\partial s_A^A}{\partial t_B} + \phi\left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B}\right)$$

where

$$\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} = n_A \frac{\partial s_A^A}{\partial t_B}\frac{\partial x_A^A}{\partial t_A} t_A + n_B \frac{\partial s_B^A}{\partial t_B}\frac{\partial x_B^A}{\partial t_A} t_A + n_C \frac{\partial s_C^A}{\partial t_B}\frac{\partial x_C^A}{\partial t_A} t_A$$

$$+ n_A\left(\frac{\partial s_A^A}{\partial t_B} + \frac{\partial^2 s_A^A}{\partial t_A \partial t_B} t_A\right) x_A^A + n_B\left(\frac{\partial s_B^A}{\partial t_B} + \frac{\partial^2 s_B^A}{\partial t_A \partial t_B} t_A\right) x_B^A + n_C\left(\frac{\partial s_C^A}{\partial t_B} + \frac{\partial^2 s_C^A}{\partial t_A \partial t_B} t_A\right) x_C^A$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \left(\frac{-\alpha_A}{\left(x_A^A - r_x\right)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-\left(1-\alpha_A\right)}{{y_A^A}^2} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial {t_A}^2} \right) s_A^A$$

$$- \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right)$$

$$= \left(\frac{\alpha_A I_A^2 - 2\alpha_A I_A \left(p_A + t_A + T\right) r_x - \left(1-\alpha_A\right)\left(p_A + t_A + T\right)^2 {r_x}^2}{\left\{I_A - \left(p_A + t_A + T\right) r_x\right\}^2 \left(p_A + t_A + T\right)^2} \right) s_A^A$$

$$- \frac{\alpha_A I_A + \left(1-\alpha_A\right) r_x \left(p_A + t_A + T\right)}{\left\{I_A - \left(p_A + t_A + T\right) r_x\right\}\left(p_A + t_A + T\right)} \frac{\partial s_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right)$$

where

$$\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} = n_A s_A^A \frac{\partial^2 x_A^A}{\partial t_A \partial T} t_A + n_B s_B^A \frac{\partial^2 x_B^A}{\partial t_A \partial T} t_A + n_C s_C^A \frac{\partial^2 x_C^A}{\partial t_A \partial T} t_A$$

$$+ n_A \left(s_A^A + \left(\frac{\partial s_A^A}{\partial t_A} + \frac{\partial s_A^A}{\partial T} \right) t_A \right) \frac{\partial x_A^A}{\partial t_A} + n_B \left(s_B^A + \left(\frac{\partial s_B^A}{\partial t_A} + \frac{\partial s_B^A}{\partial T} \right) t_A \right) \frac{\partial x_B^A}{\partial t_A} + n_C \left(s_C^A + \left(\frac{\partial s_C^A}{\partial t_A} + \frac{\partial s_C^A}{\partial T} \right) t_A \right) \frac{\partial x_C^A}{\partial t_A}$$

$$+ n_A \left(\frac{\partial s_A^A}{\partial T} + \frac{\partial^2 s_A^A}{\partial t_A \partial T} t_A \right) x_A^A + n_B \left(\frac{\partial s_B^A}{\partial T} + \frac{\partial^2 s_B^A}{\partial t_A \partial T} t_A \right) x_B^A + n_C \left(\frac{\partial s_C^A}{\partial T} + \frac{\partial^2 s_C^A}{\partial t_A \partial T} t_A \right) x_C^A$$

Appendix B: Data sources

Data	resouce	webpage
Gasoline unit tax rate	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
Gasoline price	Energy Information	http://www.eia.doe.gov/emeu/states/_seds.html
Gasoline consumption	Energy Information	http://www.eia.doe.gov/emeu/states/_seds.html
Cigarette unit tax rate	Report:Tax Burden on Tabocco	http://www.srnt.org/pubs/nl_05_06/spotlight.html
Cigarette price	Report:Tax Burden on Tabocco	
Cigarette consumption	Report:Tax Burden on Tabocco	
Expenditure of State	U.S Census of Bureau	http://www.census.gov/govs/www/index.html
Per capita federal grant to Highway department	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
Per capita federal grant to Health department	Statistics of Abstract	http://www.census.gov/compendia/statab/
Population	Bureau of Economic Analysis	http://www.bea.gov/regional/spi/
Per capita disposal Income	Bureau of Economic Analysis	http://www.bea.gov/regional/spi/
Land area	Statistics of Abstract	http://www.census.gov/compendia/statab/
Ratio of female	Center of Disease Control and Prevention	http://wonder.cdc.gov/Bridged-Race-v2006.HTML
CO2 emission	U.S. Enviroment Protection	http://www.epa.gov/climatechange/emissions/state_energyco2inv.html
the number of car registration	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
the percentage of smoker	Statistics of Abstract	http://www.census.gov/compendia/statab/
the number of death cancer	Statistics of Abstract	http://www.census.gov/compendia/statab/
Consumer price index	Bureau of Labor Statistics	ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt
Distance between center of city	Google map	

Data summary statistics

Data	unit	Observation	Mean	Standard Deviation	Min	Max
Gasoline unit tax rate	$/gallon	192	0.20	0.04	0.075	0.32
Gasoline price	$/gallon	192	0.97	0.14	0.66	1.23
Gasoline consumption	gallon	192	493.7	61.7	293.8	690.4
Cigarette unit tax rate	$/pack	192	0.40	0.29	0.03	1.5
Cigarette price	$/pack	192	2.68	0.24	2.275	4.035
Cigarette consumption	pack	192	83.6	24.8	35.3	164.8
Highway Expenditure	$	192	330.5	109.6	154.9	739.2
Health Expenditure	$	192	115.2	65.3	44.9	416.2
Per capita federal grant to Highway department	$	192	142.9	60.5	51.9	323.5
Per capita federal grant to Health department	$	192	605.3	169.4	238.8	1294.6
Population	thousands	192	5843	6232	492	34526
Per capita disposal Income	$	192	24719	3421	18038	35868
Land area	square mile	192	168340	122466	4002	695621
Ratio of female	%	192	0.51	0.01	0.49	0.52
CO2 emission	millon metricc tons	192	37.2	40.9	1.6	223.2
the number of car registration	thousands	192	2791	3035	195.8	18400
the percentage of smoker	%	192	23.2	3.1	12.7	32.6
the number of death cancer	thousands	192	3.25	3.09	0.2	13.5
Distance between center of city	miles	1159	1692.5	991.4	66.2	4292

Appendix C: The value of the slope of the reaction function in the case of Horizontal Externality

Gasoline Case

Gasoline	Alabama	Arizona	Arkansas	California	Colorado	Connecticut	Delaware	Florida	Georgia	Idaho	Illinois	Indiana	Iowa	Kansas	Kentucky	Lousiana	Maine	Maryland	Massachus	Michigan	Minnesota	Mississippi	Missouri
Alabama	0	0	1.51E-04	0	0	0	0	1.89E-03	1.59E-03	0	0	0	0	0	6.87E-05	1.17E-04	0	0	0	0	0	2.27E-04	4.74E-05
Arizona	0	0	0	7.91E-04	1.02E-04	0	0	0	0	6.75E-05	0	0	0	0	0	0	0	0	0	0	0	0	0
Arkansas	2.29E-04	0	0	0	6.47E-05	0	0	0	9.20E-05	0	9.79E-05	0	8.93E-05	2.08E-04	1.48E-04	1.34E-03	0	0	0	0	0	7.18E-04	3.27E-04
California	0	3.46E-05	0	0	0	0	0	0	0	2.99E-05	0	0	0	0	0	0	0	0	0	0	0	0	0
Colorado	0	1.43E-04	5.90E-05	0	0	0	0	0	0	8.36E-05	0	0	3.47E-05	1.92E-04	0	0	0	0	0	0	0	0	1.57E-04
Conneticut	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.38E-03	0	0	0	0
Delaware	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.99E-03	0	0	0	0	0
Florida	1.40E-04	0	0	0	0	0	0	0	1.49E-04	0	0	0	0	0	0	0	0	0	0	0	0	5.09E-05	0
Georgia	6.78E-04	0	2.86E-05	0	0	0	0	7.95E-04	0	0	0	0	0	0	4.12E-05	0	0	0	0	0	0	1.08E-04	2.74E-05
Idaho	0	2.38E-04	0	4.35E-04	2.11E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Illinois	0	0	3.89E-05	0	0	0	0	0	0	0	0	1.75E-04	1.63E-04	4.72E-05	1.54E-04	0	0	0	0	1.57E-04	1.03E-04	0	1.68E-04
Indiana	0	0	0	0	0	0	0	0	0	0	3.83E-04	0	1.50E-04	0	5.22E-04	0	0	0	0	7.36E-04	0	0	2.24E-04
Iowa	0	0	9.30E-05	0	3.97E-05	0	0	0	0	0	7.74E-04	2.79E-04	0	2.02E-04	2.87E-04	0	0	0	0	2.15E-04	6.66E-04	0	5.85E-04
Kansas	0	0	2.43E-04	0	2.45E-04	0	0	0	0	0	1.38E-04	0	2.26E-04	0	7.81E-05	0	0	0	0	0	0	0	4.88E-04
Kentucky	8.48E-05	0	1.21E-04	0	0	0	0	0	1.08E-04	0	5.51E-04	1.02E-03	2.24E-04	5.45E-05	0	0	0	1.54E-04	0	4.18E-04	0	7.24E-05	3.61E-04
Lousiana	1.30E-04	0	7.25E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.94E-04	5.39E-05
Maine	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.04E-04	0	0	0	0
Maryland	0	0	0	0	0	0	4.54E-04	0	0	0	0	0	0	0	2.05E-04	0	0	0	0	0	0	0	0
Massachusetts	0	0	0	0	0	6.73E-04	0	0	0	0	0	0	0	0	0	0	7.35E-05	0	0	0	0	0	0
Michigan	0	0	0	0	0	0	0	0	0	0	1.50E-04	2.09E-04	7.82E-05	0	1.95E-04	0	0	0	0	0	7.52E-05	0	0
Minnesota	0	0	0	0	0	0	0	0	0	0	2.11E-04	0	2.67E-04	0	0	0	0	0	0	1.62E-04	0	0	5.97E-05
Mississipi	4.47E-04	0	6.82E-04	0	0	0	0	2.32E-04	3.65E-04	0	0	0	0	0	9.38E-05	1.13E-03	0	0	0	0	0	0	1.67E-04
Missouri	4.11E-05	0	1.85E-04	0	9.82E-05	0	0	0	5.04E-05	0	4.99E-04	2.29E-04	2.89E-04	2.66E-04	2.40E-04	4.24E-05	0	0	0	0	4.18E-05	9.08E-05	0
Montana	0	0	0	0	6.05E-05	0	0	0	0	3.56E-04	0	0	4.76E-05	0	0	0	0	0	0	0	1.60E-04	0	0
Nebraska	0	0	1.28E-04	0	4.01E-04	0	0	0	0	1.42E-05	2.86E-04	0	5.74E-04	1.04E-03	9.26E-05	0	0	0	0	0	1.72E-04	0	7.36E-04
Nevada	0	1.55E-03	0	7.54E-03	6.54E-05	0	0	0	0	2.66E-04	0	0	0	0	0	0	0	0	0	0	0	0	0
New Hampshire	0	0	0	0	0	3.42E-04	0	0	0	0	0	0	0	0	0	0	1.73E-03	0	1.44E-03	0	0	0	0
New Jersey	0	0	0	0	0	2.65E-04	2.65E-04	0	0	0	0	0	0	0	0	0	0	2.81E-04	2.26E-04	0	0	0	0
New Mexico	0	8.64E-04	8.11E-04	2.24E-04	3.42E-04	0	0	0	0	0	0	0	0	2.08E-04	0	7.38E-04	0	0	0	0	0	0	9.17E-05
New York	0	0	0	0	0	9.55E-06	1.17E-04	0	0	0	0	0	0	0	0	0	0	6.30E-05	4.71E-05	0	0	0	0
North Carolina	1.22E-04	0	2.41E-05	0	0	0	0	8.20E-05	3.85E-04	0	0	0	0	0	9.82E-05	0	0	1.39E-04	0	0	0	2.42E-05	2.32E-05
North Dakota	0	0	0	0	0	0	0	0	0	2.09E-04	0	0	8.13E-04	0	0	0	0	0	0	0	1.18E-03	0	0
Ohio	0	0	0	0	0	0	6.68E-05	0	0	0	1.18E-04	4.57E-04	0	0	2.27E-04	0	0	9.12E-05	0	5.09E-04	0	0	2.44E-05
Oklahoma	0	6.31E-05	7.40E-04	0	2.15E-04	0	0	0	0	0	5.97E-05	0	5.57E-05	2.68E-04	3.87E-05	6.66E-04	0	0	0	0	0	7.01E-05	2.59E-04
Oregon	0	6.77E-04	0	1.72E-03	0	0	0	0	0	5.24E-04	0	0	0	0	0	0	0	0	0	0	0	0	0
Pennsylvania	0	0	0	0	0	1.26E-04	1.57E-04	0	0	0	0	8.70E-05	0	0	1.09E-04	0	0	1.32E-04	1.12E-04	8.00E-05	0	0	0
Rhode Island	0	0	0	0	0	1.83E-03	0	0	0	0	0	0	0	0	0	0	0	0	4.67E-03	0	0	0	0
South Carolina	2.32E-04	0	0	0	0	0	0	1.91E-04	6.64E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0
South Dakota	0	0	0	0	1.93E-04	0	0	0	0	1.88E-04	1.69E-04	0	1.08E-03	1.99E-04	0	0	0	0	0	0	1.32E-03	0	3.34E-04
Tennessee	4.22E-04	0	1.96E-04	0	0	0	0	2.62E-04	5.62E-04	0	9.24E-05	7.22E-05	4.49E-05	4.78E-05	2.07E-04	1.24E-04	0	6.77E-05	0	0	0	2.70E-04	1.54E-04
Texas	0	1.11E-05	4.26E-05	0	2.20E-05	0	0	0	0	0	0	0	0	1.64E-05	0	4.32E-05	0	0	0	0	0	5.91E-05	2.31E-05
Utah	0	6.98E-04	0	3.20E-04	2.49E-04	0	0	0	0	2.23E-04	0	0	0	9.04E-05	0	0	0	0	0	0	0	0	0
Vermont	0	0	0	0	0	1.99E-03	0	0	0	0	0	0	0	0	0	0	2.50E-04	0	2.13E-03	0	0	0	0
Virginia	2.80E-05	0	2.53E-05	0	0	0	1.06E-04	0	2.22E-04	0	2.54E-05	3.63E-05	0	0	1.05E-04	0	0	1.51E-04	0	0	0	2.54E-05	4.70E-05
Washigton	0	0	0	1.04E-04	0	0	0	0	0	1.42E-04	0	0	0	0	0	0	0	0	0	0	0	0	0
West Virginia	0	0	0	0	0	0	4.98E-04	0	0	0	1.23E-04	8.03E-04	0	0	1.18E-03	0	0	8.63E-04	0	5.33E-04	0	0	1.03E-04
Wisconsin	0	0	0	0	0	0	0	0	0	0	3.61E-04	5.79E-04	4.21E-04	0	1.56E-04	0	0	0	0	6.40E-04	3.41E-04	0	2.68E-04
Wyoming	0	2.20E-04	0	0	3.83E-03	0	0	0	0	6.71E-04	0	0	2.35E-04	5.45E-04	0	0	0	0	0	0	7.67E-05	0	1.28E-04

Gasoline	Montana	Nebraska	Nevada	New Ham	New Jersey	New Mexi	New York	North Car	North Dak	Ohio	Oklahoma	Oregon	Pennsylvar	Rhode Isla	South Car	South Dak	Tennessee	Texas	Utah	Vermont	Virginia	Washington	West Virgi	Wisconsin	Wyoming
Alabama	0	0	0	0	0	0	0	1.70E-04	0	0	0	0	0	0	1.99E-04	0	3.89E-04	0	0	0	3.22E-05	0	0	0	0
Arizona	0	0	7.04E-04	0	0	1.13E-04	0	0	0	0	5.39E-05	4.12E-04	0	0	0	0	0	4.29E-05	8.21E-05	0	0	0	0	0	3.77E-05
Arkansas	0	7.61E-05	0	0	0	5.45E-04	0	5.08E-05	0	0	1.07E-03	0	0	0	0	0	3.39E-04	1.89E-03	0	0	4.41E-05	0	0	0	0
California	0	0	-1.32E-06	0	0	2.38E-05	0	0	0	0	0	1.19E-05	0	0	0	0	0	0	3.15E-05	0	0	2.13E-05	0	0	0
Colorado	1.40E-05	1.37E-04	5.25E-05	0	0	2.69E-04	0	0	0	0	2.55E-04	0	0	0	0	4.99E-05	0	1.19E-04	1.50E-04	0	0	0	0	0	1.43E-04
Conneticut	0	0	0	2.17E-04	7.23E-04	0	3.75E-03	0	0	0	0	0	5.90E-04	5.19E-04	0	0	0	0	0	9.88E-04	0	0	0	0	0
Delaware	0	0	0	0	3.17E-03	0	1.23E-03	0	0	5.22E-04	0	0	4.56E-03	0	0	0	0	0	0	0	6.04E-04	0	8.95E-04	0	0
Florida	0	0	0	0	0	0	0	4.00E-05	0	0	0	0	0	0	5.74E-05	0	1.01E-04	0	0	0	0	0	0	0	0
Georgia	0	0	0	0	0	0	0	3.11E-04	0	0	0	0	0	0	3.48E-04	0	2.18E-04	0	0	0	1.20E-04	0	0	0	0
Idaho	3.68E-04	1.95E-05	6.39E-04	0	0	0	0	0	1.11E-04	0	0	1.53E-03	0	0	0	1.23E-04	0	0	6.08E-04	0	0	1.61E-03	0	0	3.36E-04
Illinois	0	6.76E-05	0	0	0	0	0	0	0	1.12E-04	3.11E-05	0	0	0	0	1.90E-05	6.10E-05	0	0	0	1.76E-05	0	2.71E-05	1.59E-04	0
Indiana	0	0	0	0	0	0	0	0	0	7.31E-04	0	0	1.17E-04	0	0	0	6.72E-05	0	0	0	3.54E-05	0	2.49E-04	4.22E-04	0
Iowa	1.25E-05	2.70E-04	0	0	0	0	0	0	1.95E-04	0	7.58E-05	0	0	0	0	2.65E-04	7.78E-05	0	0	0	0	0	0	8.50E-04	6.42E-05
Kansas	0	4.10E-04	0	0	0	1.63E-04	0	0	0	0	4.51E-04	0	0	0	0	6.57E-05	9.27E-05	1.13E-04	6.54E-05	0	0	0	0	0	1.67E-04
Kentucky	0	4.49E-05	0	0	0	0	0	1.69E-04	0	8.24E-04	4.12E-05	0	2.13E-04	0	0	0	3.86E-04	0	0	0	2.46E-04	0	4.79E-04	1.65E-04	0
Lousiana	0	0	0	0	0	3.61E-04	0	0	0	0	6.33E-04	0	0	0	0	0	1.50E-04	1.31E-03	0	0	0	0	0	0	0
Maine	0	0	0	2.32E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.90E-04	0	0	0	0	0
Maryland	0	0	0	0	4.27E-04	0	2.21E-04	3.16E-04	0	2.38E-04	0	0	1.39E-03	0	0	0	1.22E-04	0	0	0	8.91E-04	0	4.87E-04	0	0
Massachusetts	0	0	0	2.48E-04	3.41E-04	0	9.32E-04	0	0	0	0	0	2.88E-04	2.65E-04	0	0	0	0	0	5.11E-04	0	0	0	0	0
Michigan	0	0	0	0	0	0	0	0	0	2.28E-04	0	0	7.28E-05	0	0	0	0	0	0	0	0	0	1.12E-04	1.09E-04	0
Minnesota	3.30E-05	8.37E-05	0	0	0	0	0	0	6.69E-05	0	0	0	0	0	0	9.64E-05	0	0	0	0	0	0	0	3.33E-04	1.64E-05
Mississipi	0	0	0	0	0	0	0	5.38E-05	0	0	9.67E-05	0	0	0	0	0	4.23E-04	3.69E-04	0	0	4.68E-05	0	0	0	0
Missouri	0	2.10E-04	0	0	0	3.52E-05	0	2.80E-05	0	3.35E-05	2.38E-04	0	0	0	0	5.41E-05	1.54E-04	7.82E-05	0	0	4.69E-05	0	3.26E-05	1.99E-04	1.92E-05
Montana	0	8.97E-05	1.40E-04	0	0	0	0	0	2.54E-04	0	0	1.39E-04	0	0	0	2.63E-04	0	0	2.04E-04	0	0	1.29E-04	0	0	1.88E-04
Nebraska	3.79E-05	0	0	0	0	1.23E-04	0	0	5.49E-05	0	4.75E-04	0	0	0	0	1.97E-04	1.08E-04	0	1.22E-04	0	0	0	0	1.35E-04	2.62E-04
Nevada	0	4.00E-05	0	0	0	1.83E-04	0	0	0	0	0	1.66E-03	0	0	0	0	0	0	3.32E-04	0	0	1.88E-04	0	0	9.42E-05
New Hampshire	0	0	0	0	0	0	4.27E-04	0	0	0	0	0	0	6.02E-04	0	0	0	0	0	6.40E-04	0	0	0	0	0
New Jersey	0	0	0	0	0	0	5.67E-04	0	0	1.19E-04	0	0	8.58E-04	0	0	0	0	0	0	2.40E-04	0	0	1.30E-04	0	0
New Mexico	0	1.10E-04	2.40E-04	0	0	0	0	0	0	0	1.10E-03	0	0	0	0	0	0	1.57E-03	3.98E-04	0	0	0	0	0	2.37E-04
New York	0	0	0	4.29E-05	1.18E-04	0	0	0	0	3.90E-05	0	0	1.25E-04	6.23E-05	0	0	0	0	0	6.06E-06	0	0	4.19E-05	0	0
North Carolina	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.44E-04	0	1.72E-04	0	0	0	2.81E-04	0	9.79E-05	0	0
North Dakota	4.08E-04	1.43E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	1.07E-03	0	0	0	0	0	0	0	6.56E-04	2.90E-04
Ohio	0	0	0	0	6.94E-05	0	5.23E-05	0	0	0	0	0	1.02E-04	0	0	0	4.26E-05	0	0	0	5.26E-05	0	1.52E-04	2.01E-04	0
Oklahoma	0	2.17E-04	0	0	0	5.98E-04	0	0	0	0	0	0	0	0	0	0	9.76E-05	1.65E-03	4.45E-05	0	0	0	0	0	7.80E-05
Oregon	3.75E-05	0	1.45E-03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8.12E-05	0	0	1.20E-03	0	0	2.29E-05
Pennsylvania	0	0	0	0	2.62E-04	0	2.32E-04	0	0	1.36E-04	0	0	0	0	0	0	0	0	0	1.18E-04	8.55E-05	0	1.50E-04	0	0
Rhode Island	0	0	0	8.59E-04	0	0	8.84E-04	0	0	0	0	0	0	0	0	0	0	0	0	4.04E-04	0	0	0	0	0
South Carolina	0	0	0	0	0	0	0	7.85E-04	0	0	0	0	0	0	0	0	3.82E-04	0	0	0	3.73E-04	0	0	0	0
South Dakota	4.33E-04	5.27E-04	0	0	0	0	0	0	1.14E-03	0	0	0	0	0	0	0	0	0	4.96E-05	0	0	0	0	7.36E-04	3.35E-04
Tennessee	0	3.89E-05	0	0	0	0	0	3.72E-04	0	6.16E-05	7.67E-05	0	0	0	2.98E-04	0	0	3.08E-05	0	0	2.69E-04	0	1.17E-04	0	0
Texas	0	0	0	0	0	1.54E-05	0	0	0	0	2.11E-05	0	0	0	0	0	8.65E-06	0	0	0	0	0	0	0	0
Utah	8.30E-05	1.18E-04	4.02E-04	0	0	4.30E-04	0	0	0	0	9.38E-05	1.22E-04	0	0	0	2.26E-05	0	0	0	0	0	5.39E-05	0	0	2.56E-04
Vermont	0	0	0	9.64E-04	1.32E-03	0	3.94E-03	0	0	0	0	0	1.11E-03	3.61E-04	0	0	0	0	0	0	0	0	0	0	0
Virginia	0	0	0	0	0	0	0	6.44E-04	0	7.25E-05	0	0	1.17E-04	0	2.78E-04	0	2.14E-04	0	0	0	0	0	1.29E-04	0	0
Washigton	2.52E-05	0	1.28E-04	0	0	0	0	0	0	0	0	3.71E-04	0	0	0	0	0	0	2.59E-05	0	0	0	0	0	1.59E-05
West Virginia	0	0	0	0	3.29E-04	0	2.45E-04	3.73E-04	0	1.94E-03	0	0	9.69E-04	0	0	0	3.50E-04	0	0	0	8.11E-04	0	0	0	0
Wisconsin	0	6.15E-05	0	0	0	0	0	0	1.16E-04	3.72E-04	0	0	0	0	0	1.60E-04	0	0	0	0	0	0	0	0	0
Wyoming	4.70E-04	7.35E-04	3.16E-04	0	0	6.07E-04	0	0	2.55E-04	0	3.89E-04	8.17E-05	0	0	0	4.27E-04	0	0	1.21E-03	0	0	7.84E-05	0	0	0

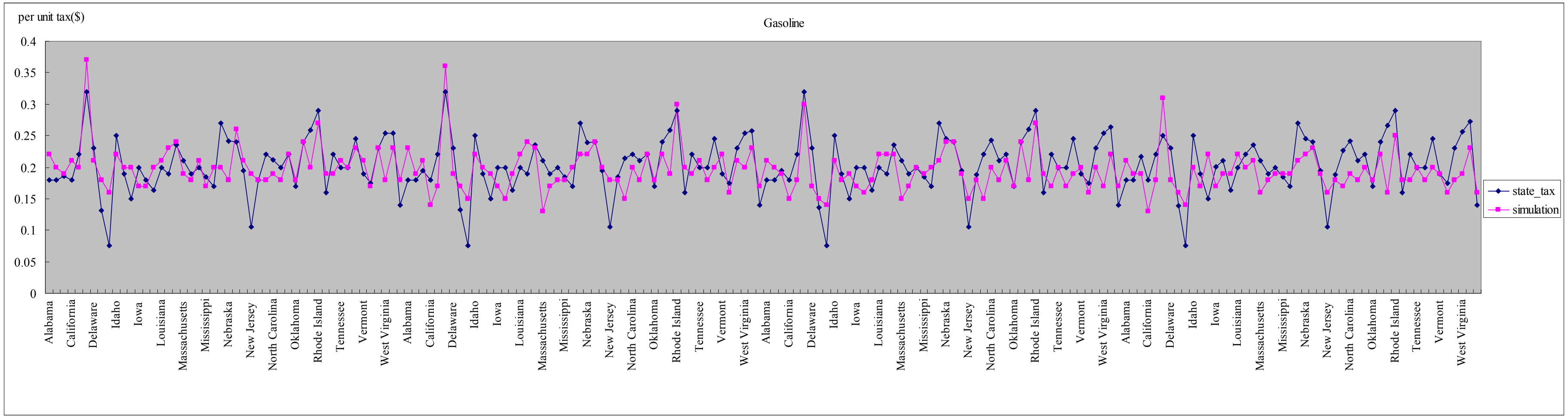
Cigarette Case

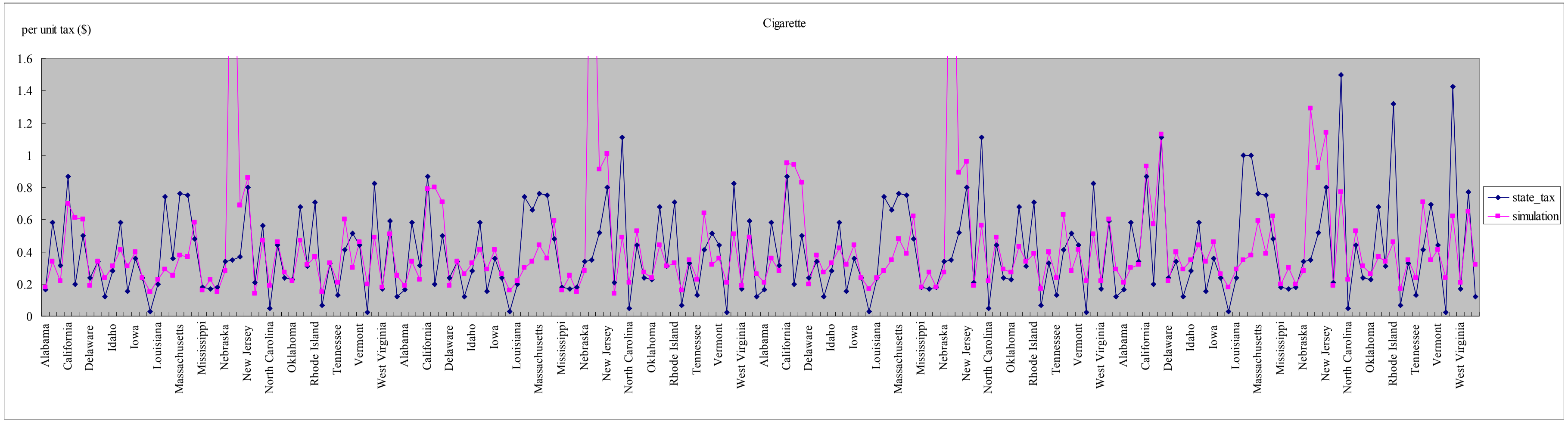
Cigarette	Alabama	Arizona	Arkansas	California	Colorado	Conneticut	Delaware	Florida	Georgia	Idaho	Illinois	Indiana	Iowa	Kansas	Kentucky	Lousiana	Maine	Maryland	Massachus	Michigan	Minnesota	Mississipi	Missouri	Montana	Nebraska	Nevada	New Ham	New Jersey	New Mexi	New York	North Caro	North Dak	Ohio	Oklahoma	Oregon	Pennsylvar	Rhode Isla	South Caro	South Dak	Tennessee	Texas	Utah	Vermont	Virginia	Washigton	West Virgi	Wisconsin	Wyoming
Alabama	0	0	7.72E-05	0	0	0	0	1.03E-03	8.08E-04	0	0	0	0	0	3.63E-05	6.23E-05	0	0	0	0	0	1.18E-04	2.48E-05	0	0	0	0	0	0	0	8.04E-05	0	0	0	0	0	0	8.36E-05	0	1.88E-04	0	0	0	1.89E-05	0	0	0	0
Arizona	0	0	0	6.46E-04	8.54E-05	0	0	0	0	3.67E-05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6.22E-04	0	0	6.97E-05	0	0	0	0	4.50E-05	4.04E-04	0	0	0	0	0	3.73E-05	2.77E-05	0	0	0	0	0	2.61E-05
Arkansas	1.59E-04	0	0	0	3.67E-05	0	0	0	5.60E-05	0	5.92E-05	0	5.31E-05	1.12E-04	1.04E-04	7.41E-04	0	0	0	0	0	5.17E-04	2.07E-04	0	4.58E-05	0	0	0	2.55E-04	0	3.84E-05	0	0	4.47E-04	0	0	0	0	0	2.33E-04	7.44E-04	0	0	3.48E-05	0	0	0	0
California	0	1.50E-05	0	0	0	0	0	0	0	1.92E-05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9.78E-05	0	0	2.21E-05	0	0	0	0	0	#######	0	0	0	0	0	0	2.22E-05	0	0	1.72E-05	0	0	0
Colorado	0	1.44E-04	7.63E-05	0	0	0	0	0	0	6.72E-05	0	0	3.26E-05	2.17E-04	0	0	0	0	0	0	0	0	1.81E-04	6.35E-06	1.28E-04	4.90E-05	0	0	3.00E-04	0	0	0	0	2.86E-04	0	0	0	0	3.61E-05	0	1.44E-04	1.09E-04	0	0	0	0	0	1.10E-04
Conneticut	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9.78E-04	0	0	0	0	0	0	0	1.83E-04	4.16E-04	0	2.54E-03	0	0	0	0	0	3.81E-04	4.24E-04	0	0	0	0	0	6.93E-04	0	0	0	0	0
Delaware	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.18E-03	0	0	0	0	0	0	0	0	0	1.89E-03	0	5.60E-04	0	0	2.41E-04	0	0	2.72E-03	0	0	0	0	0	0	0	3.49E-04	0	4.38E-04	0	0
Florida	7.64E-05	0	0	0	0	0	0	0	6.75E-05	0	0	0	0	0	0	0	0	0	0	0	0	3.54E-05	0	0	0	0	0	0	0	0	2.35E-05	0	0	0	0	0	0	3.06E-05	0	6.30E-05	0	0	0	0	0	0	0	0
Georgia	5.60E-04	0	1.92E-05	0	0	0	0	5.96E-04	0	0	0	0	0	0	3.04E-05	0	0	0	0	0	0	8.05E-05	1.95E-05	0	0	0	0	0	0	0	2.29E-04	0	0	0	0	0	0	2.61E-04	0	1.74E-04	0	0	0	8.72E-05	0	0	0	0
Idaho	0	9.47E-05	0	1.71E-04	1.03E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.55E-04	5.36E-06	2.75E-04	0	0	0	0	0	5.14E-05	0	0	8.20E-04	0	0	0	5.51E-05	0	0	2.43E-04	0	0	8.00E-04	0	0	1.56E-04
Illinois	0	0	2.51E-05	0	0	0	0	0	0	0	0	1.27E-04	8.14E-05	3.27E-05	1.12E-04	0	0	0	0	8.85E-05	5.42E-05	0	1.33E-04	0	4.00E-05	0	0	0	0	0	0	0	8.51E-05	2.02E-05	0	0	0	0	1.13E-05	4.22E-05	0	0	0	1.40E-05	0	2.05E-05	7.43E-05	0
Indiana	0	0	0	0	0	0	0	0	0	0	2.32E-04	0	7.62E-05	0	3.57E-04	0	0	0	0	3.40E-04	0	0	1.17E-04	0	0	0	0	0	0	0	0	0	4.93E-04	0	0	8.34E-05	0	0	0	3.52E-05	0	0	0	1.94E-05	0	1.72E-04	1.99E-04	0
Iowa	0	0	5.90E-05	0	1.75E-05	0	0	0	0	0	5.26E-04	1.99E-04	0	1.09E-04	2.20E-04	0	0	0	0	9.82E-05	2.86E-04	0	4.25E-04	4.99E-06	1.35E-04	0	0	0	0	0	0	7.57E-05	0	4.94E-05	0	0	0	0	1.02E-04	5.30E-05	0	0	0	0	0	0	4.52E-04	2.49E-05
Kansas	0	0	1.57E-04	0	1.02E-04	0	0	0	0	0	1.09E-04	0	1.38E-04	0	6.74E-05	0	0	0	0	0	0	0	4.36E-04	0	2.34E-04	0	0	0	7.59E-05	0	0	0	0	2.59E-04	0	0	0	0	3.00E-05	7.25E-05	6.62E-05	1.73E-05	0	0	0	0	0	5.16E-05
Kentucky	3.68E-05	0	5.13E-05	0	0	0	0	0	4.36E-05	0	2.43E-04	5.46E-04	9.70E-05	2.36E-05	0	0	0	6.52E-05	0	1.93E-04	0	3.17E-05	1.57E-04	0	1.95E-05	0	0	0	0	0	7.43E-05	0	5.38E-04	1.80E-05	0	1.21E-04	0	0	0	2.23E-04	0	0	0	1.08E-04	0	2.76E-04	7.28E-05	0
Lousiana	7.34E-05	0	2.73E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.50E-04	2.61E-05	0	0	0	0	0	1.33E-04	0	0	0	0	2.10E-04	0	0	0	0	0	8.40E-05	3.56E-04	0	0	0	0	0	0	0
Maine	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.72E-04	0	0	0	0	0	0	0	2.34E-05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.67E-04	0	0	0	0	0
Maryland	0	0	0	0	0	0	2.70E-04	0	0	0	0	0	0	0	1.09E-04	0	0	0	0	0	0	0	0	0	0	0	0	3.09E-04	0	1.38E-04	1.59E-04	0	1.70E-04	0	0	1.27E-03	0	0	0	6.19E-05	0	0	0	5.26E-04	0	2.87E-04	0	0
Massachusetts	0	0	0	0	0	6.52E-04	0	0	0	0	0	0	0	0	0	0	3.24E-05	0	0	0	0	0	0	0	0	0	4.45E-05	2.54E-04	0	7.97E-04	0	0	0	0	0	2.47E-04	2.22E-04	0	0	0	0	0	3.66E-04	0	0	0	0	0
Michigan	0	0	0	0	0	0	0	0	0	0	1.31E-04	2.34E-04	5.59E-05	0	2.46E-04	0	0	0	0	0	5.58E-05	0	0	0	0	0	0	0	0	0	0	0	3.12E-04	0	0	1.01E-04	0	0	0	0	0	0	0	0	0	1.56E-04	5.54E-05	0
Minnesota	0	0	0	0	0	0	0	0	0	0	1.57E-04	0	2.07E-04	0	0	0	0	0	0	1.09E-04	0	0	4.64E-05	1.83E-05	5.75E-05	0	0	0	0	0	0	2.35E-05	0	0	0	0	0	0	5.42E-05	0	0	0	0	0	0	0	2.63E-04	8.02E-06
Mississipi	2.60E-04	0	3.46E-04	0	0	0	0	1.25E-04	1.86E-04	0	0	0	0	0	5.09E-05	6.86E-04	0	0	0	0	0	0	8.06E-05	0	0	0	0	0	0	0	3.12E-05	0	0	4.03E-05	0	0	0	0	0	2.23E-04	1.93E-04	0	0	2.84E-05	0	0	0	0
Missouri	2.29E-05	0	8.60E-05	0	3.92E-05	0	0	0	2.56E-05	0	3.18E-04	1.24E-04	1.32E-04	8.56E-05	1.36E-04	2.05E-05	0	0	0	0	1.69E-05	4.59E-05	0	0	6.35E-05	0	0	0	1.58E-05	0	1.77E-05	0	1.92E-05	8.20E-05	0	0	0	0	2.07E-05	8.15E-05	3.45E-05	0	0	2.99E-05	0	1.90E-05	9.92E-05	6.29E-06
Montana	0	0	0	0	1.32E-05	0	0	0	0	1.25E-04	0	0	1.95E-05	0	0	0	0	0	0	0	6.41E-05	0	0	0	2.93E-05	5.34E-05	0	0	0	0	0	8.95E-05	0	0	5.26E-05	0	0	0	9.27E-05	0	0	6.66E-05	0	0	4.54E-05	0	0	6.30E-05
Nebraska	0	0	1.08E-04	0	1.56E-04	0	0	0	0	3.96E-06	2.23E-04	0	4.90E-04	6.79E-04	9.37E-05	0	0	0	0	0	1.10E-04	0	5.29E-04	1.60E-05	0	0	0	0	3.59E-05	0	0	2.13E-05	0	2.58E-04	0	0	0	0	1.04E-04	9.92E-05	0	3.60E-05	0	0	0	0	9.18E-05	6.99E-05
Nevada	0	1.30E-03	0	7.39E-03	4.88E-05	0	0	0	0	1.95E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	2.54E-05	0	0	0	0	1.47E-04	0	0	0	0	0	1.47E-03	0	0	0	0	0	0	2.33E-04	0	0	1.29E-04	0	0	6.53E-05
New Hampshire	0	0	0	0	0	4.81E-04	0	0	0	0	0	0	0	0	0	0	9.72E-04	0	1.84E-03	0	0	0	0	0	0	0	0	0	0	3.82E-04	0	0	0	0	0	0	8.73E-04	0	0	0	0	0	5.79E-04	0	0	0	0	0
New Jersey	0	0	0	0	0	2.52E-04	1.94E-04	0	0	0	0	0	0	0	0	0	0	2.50E-04	1.91E-04	0	0	0	0	0	0	0	0	0	0	4.70E-04	0	0	1.02E-04	0	0	8.55E-04	0	0	0	0	0	0	2.20E-04	0	0	1.11E-04	0	0
New Mexico	0	2.99E-04	4.16E-04	1.01E-04	9.31E-05	0	0	0	0	0	0	0	0	8.83E-05	0	3.79E-04	0	0	0	0	0	0	5.76E-05	0	2.47E-05	1.16E-04	0	0	0	0	0	0	0	5.09E-04	0	0	0	0	0	0	6.95E-04	1.52E-04	0	0	0	0	0	4.80E-05
New York	0	0	0	0	0	#######	1.80E-04	0	0	0	0	0	0	0	0	0	0	7.29E-05	4.81E-05	0	0	0	0	0	0	0	5.78E-05	1.50E-04	0	0	0	0	5.06E-05	0	0	1.72E-04	1.05E-04	0	0	0	0	0	-4.44E-05	0	0	5.43E-05	0	0
North Carolina	5.29E-05	0	1.23E-05	0	0	0	0	3.33E-05	1.76E-04	0	0	0	0	0	4.83E-05	0	0	6.12E-05	0	0	0	1.26E-05	1.26E-05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.45E-04	0	7.82E-05	0	0	0	1.60E-04	0	4.36E-05	0	0
North Dakota	0	0	0	0	0	0	0	0	0	1.24E-04	0	0	5.26E-04	0	0	0	0	0	0	0	7.48E-04	0	0	2.34E-04	6.97E-05	0	0	0	0	0	0	0	0	0	0	0	0	0	7.57E-04	0	0	0	0	0	0	0	4.51E-04	1.64E-04
Ohio	0	0	0	0	0	0	4.23E-05	0	0	0	4.78E-05	1.82E-04	0	0	1.04E-04	0	0	4.97E-05	0	1.72E-04	0	0	1.02E-05	0	0	0	0	3.67E-05	0	2.80E-05	0	0	0	0	0	5.71E-05	0	0	0	1.72E-05	0	0	0	2.29E-05	0	7.75E-05	7.05E-05	0
Oklahoma	0	3.26E-05	3.77E-04	0	8.44E-05	0	0	0	0	0	4.25E-05	0	3.96E-05	1.20E-04	3.26E-05	3.26E-04	0	0	0	0	0	4.54E-05	1.52E-04	0	9.68E-05	0	0	0	2.84E-04	0	0	0	0	0	0	0	0	0	0	7.11E-05	6.50E-04	1.08E-05	0	0	0	0	0	1.69E-05
Oregon	0	5.37E-04	0	1.16E-03	0	0	0	0	0	2.89E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	1.99E-05	0	1.06E-03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.95E-05	0	0	6.87E-04	0	0	1.25E-05
Pennsylvania	0	0	0	0	0	7.94E-05	1.32E-04	0	0	0	0	4.47E-05	0	0	5.62E-05	0	0	9.87E-05	6.39E-05	3.71E-05	0	0	0	0	0	0	0	1.97E-04	0	1.42E-04	0	0	6.89E-05	0	0	0	0	0	0	0	0	0	7.28E-05	5.37E-05	0	8.26E-05	0	0
Rhode Island	0	0	0	0	0	1.60E-03	0	0	0	0	0	0	0	0	0	0	0	0	3.58E-03	0	0	0	0	0	0	0	8.30E-04	0	0	6.61E-04	0	0	0	0	0	0	0	0	0	0	0	0	3.55E-04	0	0	0	0	0
South Carolina	9.61E-05	0	0	0	0	0	0	7.59E-05	2.84E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.05E-04	0	0	0	0	0	0	0	0	1.77E-04	0	0	0	1.82E-04	0	0	0	0
South Dakota	0	0	0	0	8.60E-05	0	0	0	0	8.51E-05	1.31E-04	0	6.29E-04	1.05E-04	0	0	0	0	0	0	7.08E-04	0	2.27E-04	1.95E-04	3.06E-04	0	0	0	0	0	0	5.49E-04	0	0	0	0	0	0	0	0	0	1.26E-05	0	0	0	0	4.40E-04	1.44E-04
Tennessee	2.15E-04	0	9.14E-05	0	0	0	0	1.22E-04	2.64E-04	0	4.34E-05	3.59E-05	2.08E-05	2.25E-05	1.23E-04	6.38E-05	0	3.25E-05	0	0	0	1.42E-04	7.24E-05	0	1.83E-05	0	0	0	0	0	1.80E-04	0	3.13E-05	3.48E-05	0	0	0	1.38E-04	0	0	1.42E-05	0	0	1.37E-04	0	6.06E-05	0	0
Texas	0	7.76E-06	5.33E-05	0	1.78E-05	0	0	0	0	0	0	0	0	1.21E-05	0	6.07E-05	0	0	0	0	0	6.21E-05	1.96E-05	0	0	0	0	0	1.35E-05	0	0	0	0	2.28E-05	0	0	0	0	0	8.34E-06	0	0	0	0	0	0	0	0
Utah	0	4.22E-04	0	1.88E-04	8.00E-05	0	0	0	0	9.06E-05	0	0	0	3.79E-05	0	0	0	0	0	0	0	0	0	4.64E-05	4.64E-05	2.35E-04	0	0	2.82E-04	0	0	0	0	3.74E-05	5.45E-05	0	0	0	7.76E-06	0	0	0	0	0	2.94E-05	0	0	8.93E-05
Vermont	0	0	0	0	0	1.73E-03	0	0	0	0	0	0	0	0	0	0	9.70E-05	0	1.57E-03	0	0	0	0	0	0	0	5.99E-04	9.08E-04	0	3.30E-03	0	0	0	0	0	8.70E-04	3.56E-04	0	0	0	0	0	0	0	0	0	0	0
Virginia	1.56E-05	0	1.39E-05	0	0	0	1.03E-04	0	1.02E-04	0	1.32E-05	1.82E-05	0	0	5.78E-05	0	0	1.54E-04	0	0	0	1.43E-05	2.65E-05	0	0	0	0	0	0	0	3.98E-04	0	3.84E-05	0	0	9.40E-05	0	1.30E-04	0	1.10E-04	0	0	0	0	0	9.27E-05	0	0
Washigton	0	0	0	1.20E-04	0	0	0	0	0	1.20E-04	0	0	0	0	0	0	0	0	0	0	0	0	0	2.60E-05	0	1.55E-04	0	0	0	0	0	0	0	0	4.56E-04	0	0	0	0	0	0	2.42E-05	0	0	0	0	0	1.68E-05
West Virginia	0	0	0	0	0	0	3.19E-04	0	0	0	4.77E-05	3.99E-04	0	0	6.22E-04	0	0	4.36E-04	0	2.47E-04	0	0	4.16E-05	0	0	0	0	1.66E-04	0	1.24E-04	1.35E-04	0	1.31E-03	0	0	5.72E-04	0	0	0	1.37E-04	0	0	0	3.75E-04	0	0	0	0
Wisconsin	0	0	0	0	0	0	0	0	0	0	3.49E-04	4.79E-04	2.91E-04	0	1.53E-04	0	0	0	0	4.56E-04	1.80E-04	0	2.27E-04	0	3.98E-05	0	0	0	0	0	0	6.03E-05	3.03E-04	0	0	0	0	0	9.18E-05	0	0	0	0	0	0	0	0	0
Wyoming	0	1.79E-04	0	0	2.43E-03	0	0	0	0	5.09E-04	0	0	1.90E-04	3.10E-04	0	0	0	0	0	0	5.49E-05	0	1.19E-04	3.49E-04	4.17E-04	2.67E-04	0	0	2.47E-04	0	0	1.79E-04	0	1.60E-04	6.44E-05	0	0	0	3.34E-04	0	0	7.05E-04	0	0	5.68E-05	0	0	0

Appendix D: The slope of the reaction function in the case of Vertical Externality

	Gasoline	Cigarette
Alabama	0.158	0.075
Arizona	0.216	0.474
Arkansas	0.173	0.141
California	0.323	0.852
Colorado	0.366	0.161
Conneticut	0.288	0.253
Delaware	-0.018	-0.194
Florida	0.217	0.316
Georgia	0.062	0.071
Idaho	0.269	0.128
Illinois	0.304	0.558
Indiana	0.144	0.076
Iowa	0.245	0.187
Kansas	0.294	0.198
Kentucky	0.128	-0.030
Lousiana	0.232	0.159
Maine	0.260	0.787
Maryland	0.304	0.567
Massachusetts	0.337	0.593
Michigan	0.314	0.734
Minnesota	0.311	0.406
Mississipi	0.217	0.107
Missouri	0.203	0.127
Montana	0.397	0.185
Nebraska	0.301	0.222
Nevada	0.072	-0.276
New Hampshire	0.192	0.096
New Jersey	0.143	0.560
New Mexico	0.161	0.105
New York	0.432	0.983
North Carolina	0.353	0.068
North Dakota	0.247	0.240
Ohio	0.317	0.212
Oklahoma	0.178	0.136
Oregon	0.220	0.450
Pennsylvania	0.458	0.248
Rhode Island	0.181	0.323
South Carolina	0.196	0.072
South Dakota	0.217	0.153
Tennessee	0.264	0.098
Texas	0.355	0.337
Utah	0.331	0.466
Vermont	0.000	-0.125
Virginia	0.280	0.056
Washigton	0.379	1.046
West Virginia	0.233	-0.028
Wisconsin	0.351	0.440
Wyoming	0.030	-0.081

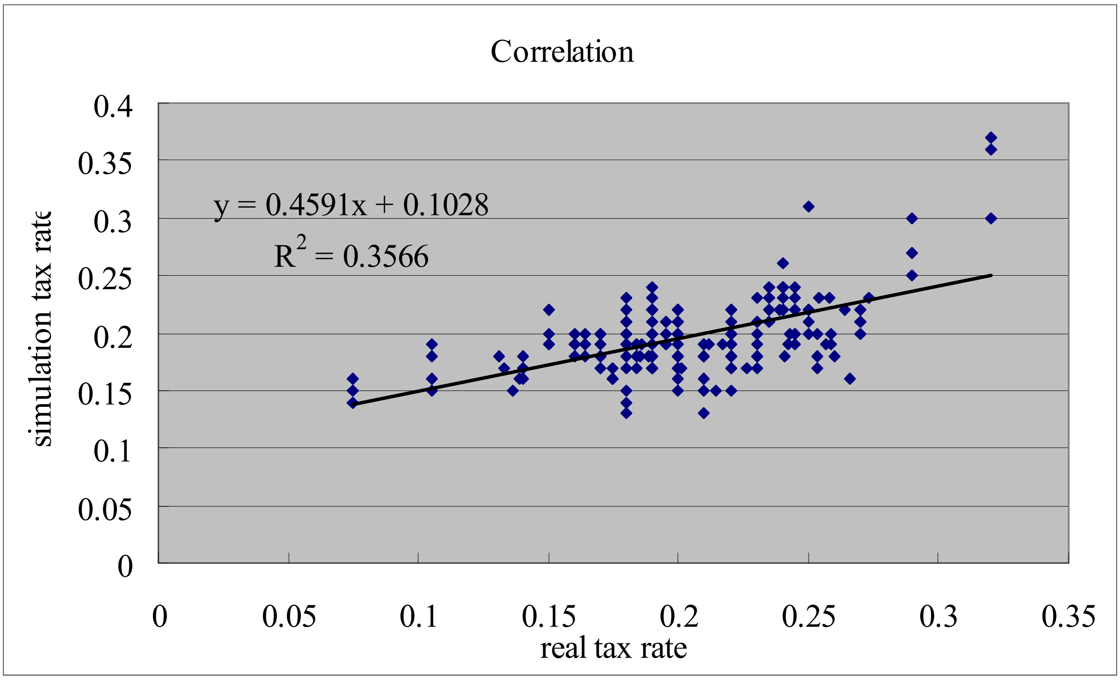
Appendix E: Real tax rate and Simulated tax rate



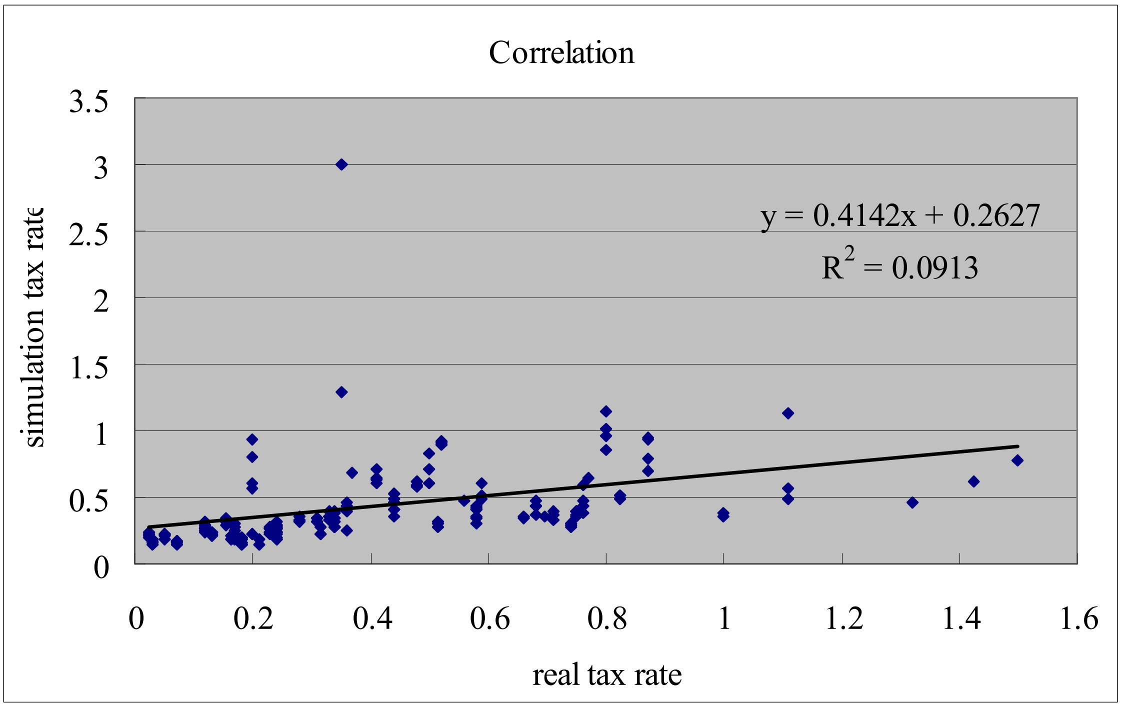


Correlation; Real tax rate and Simulated tax rate

Gasoline)



Cigarette)



Appendix F: Cigarette (Smoker case)

Since the assumption that all the people in state smoke is un realistic, I also analyze the case where there are non smokers and smokers, and state government maximize the total welfare of both people. The following is the result of this case; cigarette (smoker)

The estimated parameter (household utility function)

	Gasoline	Cigarette	Cigarette(smoker)
α_0	0.0221	0.00521	0.0263
α_1	-0.00297	0.0999	0.195
α_2	-0.0201	-0.00786	-0.0738
r_x	101.1	31.94	149.2
η	0.808	0.601	0.620
h	0.398	0.451	0.491
Share Function	0.961	0.951	0.950
Price elasticity of demand	-0.795	-0.529	-0.547

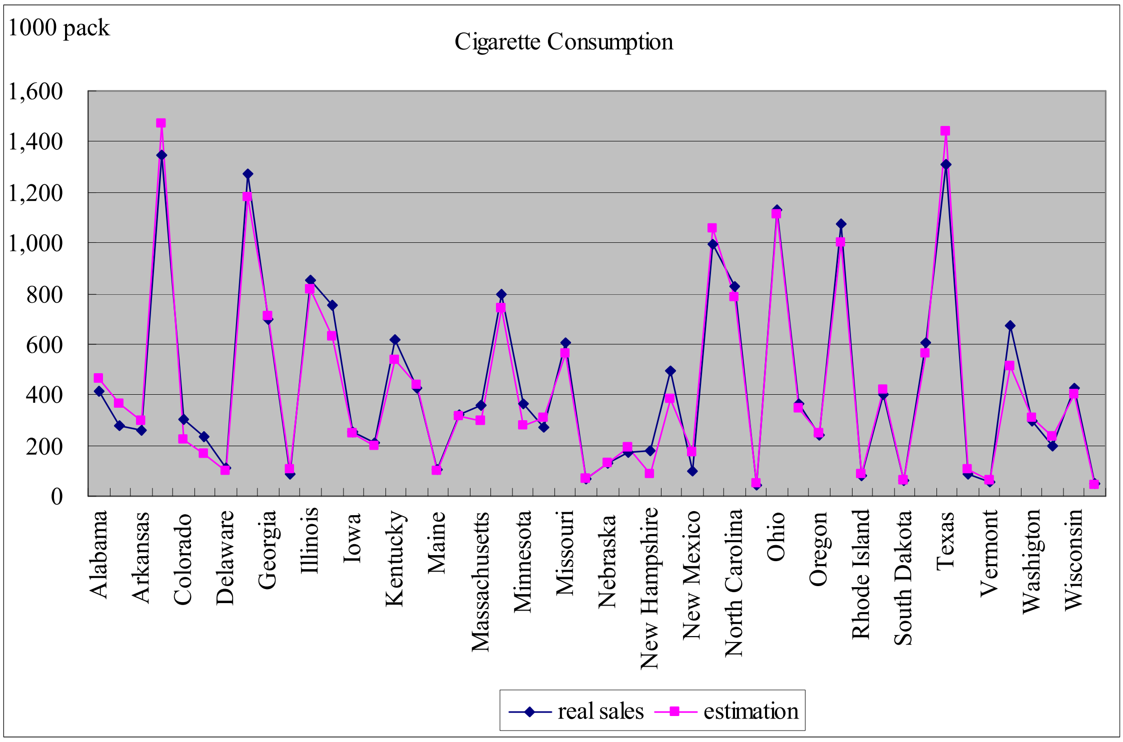
The estimated parameter (state government objective function)

	Gasoline	Cigarette	Cigarette(smoker)
Constant	0.0001392	0.0001284	0.0001303
Income ^2	8.03E-14	8.11E-14	8.27E-14
Income	-5.99E-09	-5.71E-09	5.79E-09
Pop	-6.66E-11	5.78E-11	7.52E-11
Share	6.55E-06	5.17E-06	6.16E-06
Time dummy 1999	5.48E-07		
Time dummy 2000	-1.23E-06		
Time dummy 2001	-7.07E-07		

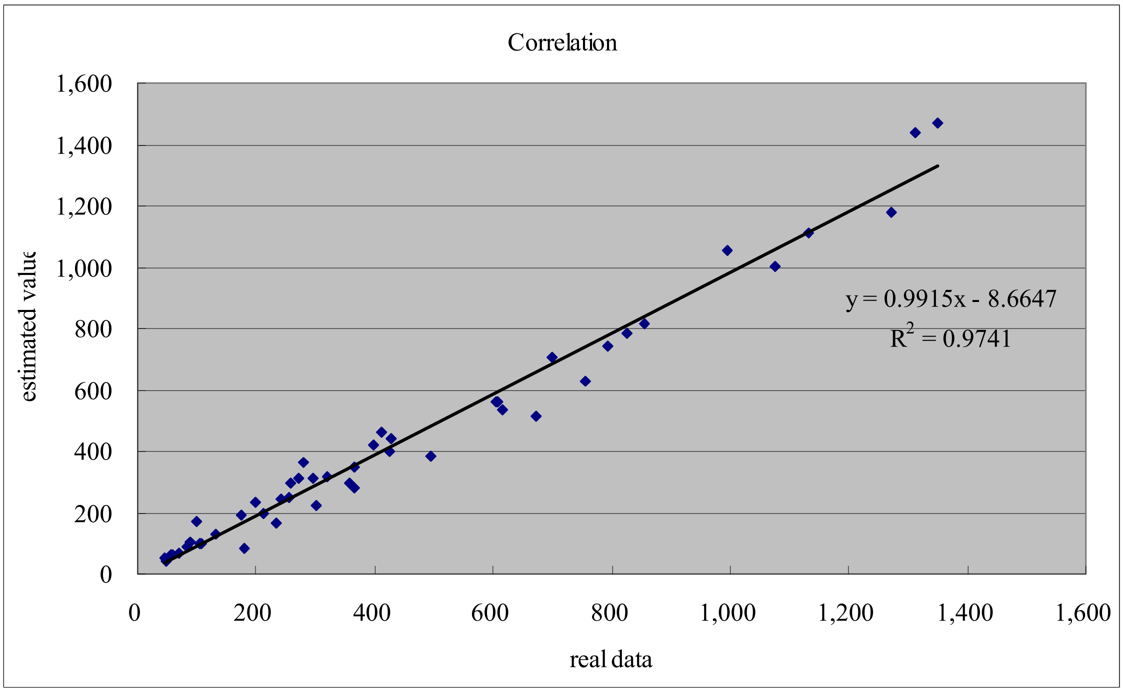
The value of the reaction function

	Gasoline	Cigarette	Cigarette(smoker)
Horizontal case	0.000377	0.000241	0.000935
Vertical case	0.242	0.265	0.194

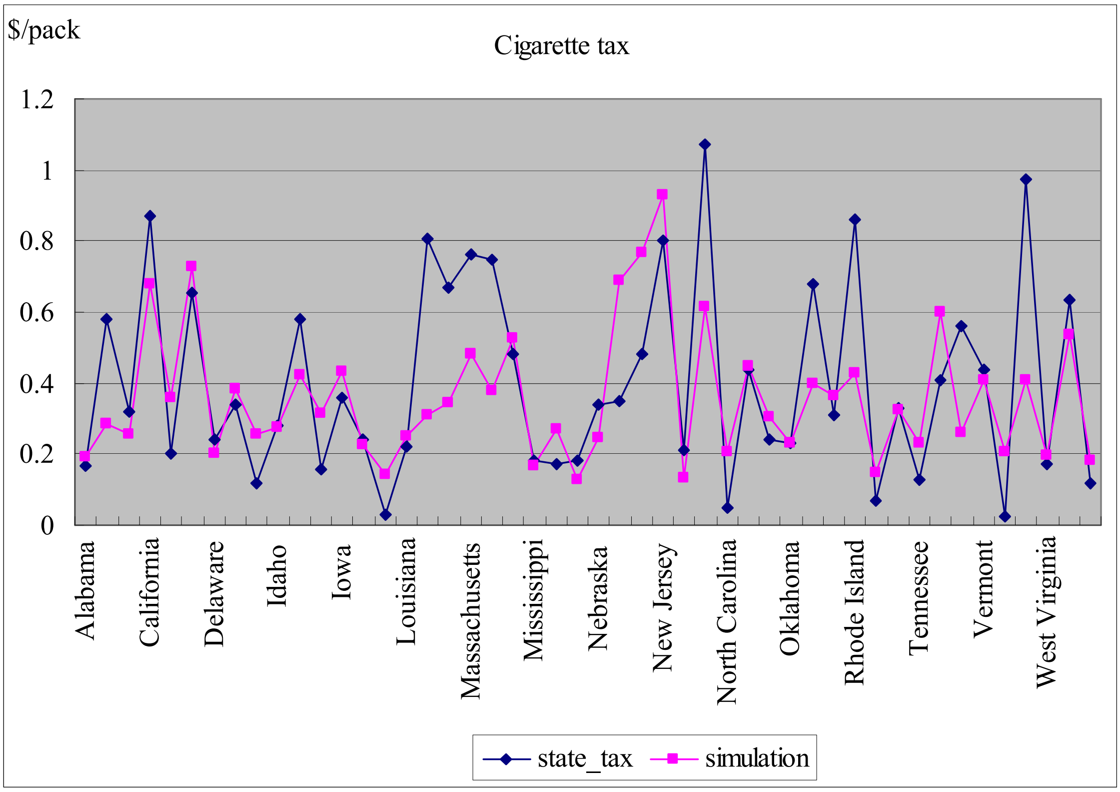
Total Consumption

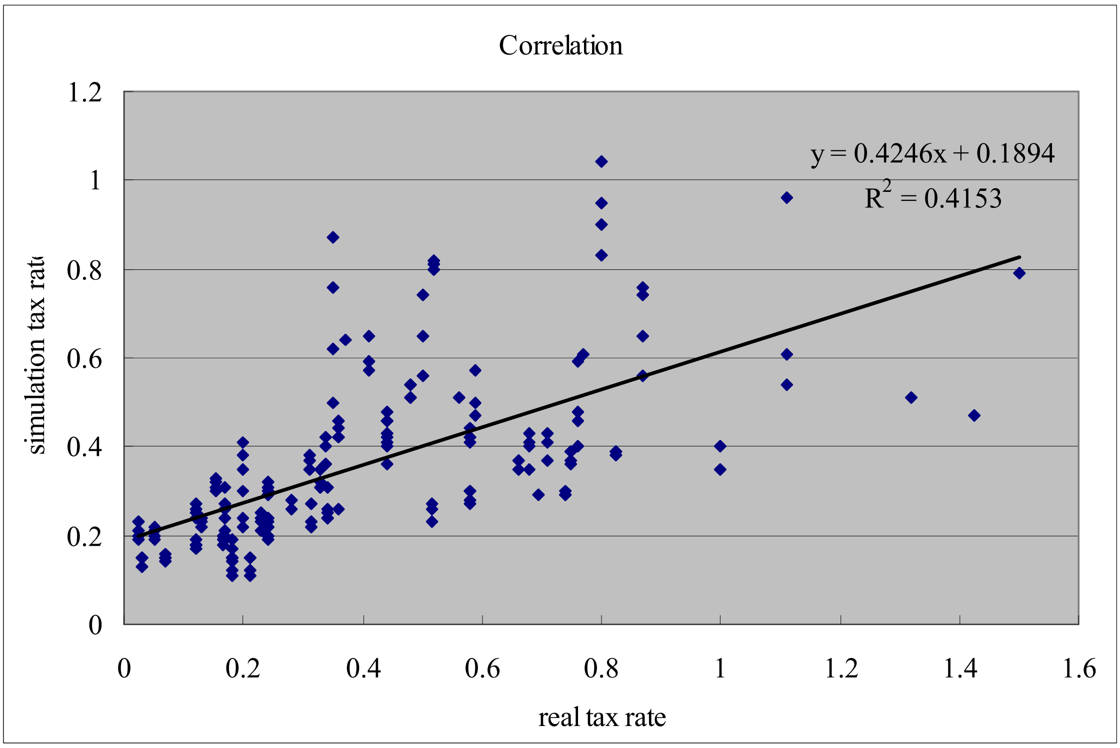


Correlation between real data and estimated value

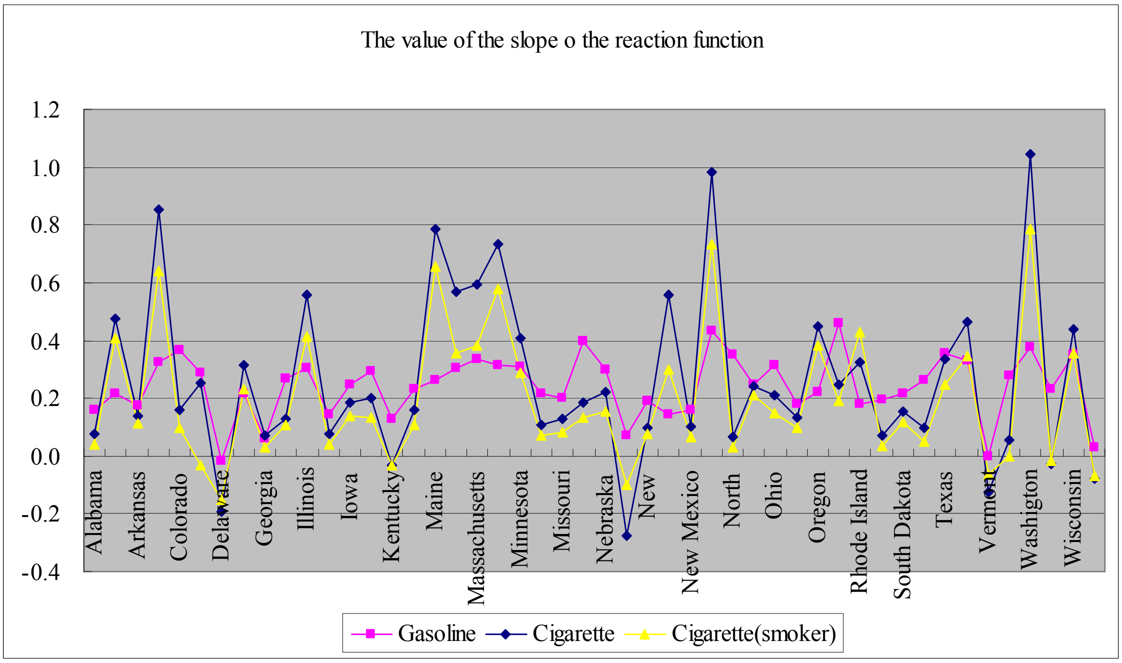


The relation between real tax rate and simulation tax rate





The value of the slope of the reaction function



Appendix G

Here, I would like to express the slope of the reaction function in the vertical externality case. For simplicity, I assume there is no cross border shopping; that is $s_i^i = 1$ and $s_i^j = 0$. This is an extreme example, but it gives a clear idea of the important factors for vertical tax competition. I estimated that the percentage of cross border shopping is around 4.2% for gasoline and 5% for cigarette, and this extreme assumption is not inappropriate. The numerator and denominator of the reaction function are expressed as follows.

$$\frac{\partial^2 W_A}{\partial t_A{}^2} = \gamma_A^2 s_A^A - \gamma_A^1 \frac{\partial s_A^A}{\partial t_A} + \phi \frac{\partial^2 TR_A}{\partial t_A{}^2} \tag{A}$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \gamma_A^2 s_A^A + \gamma_A^1 \frac{\partial s_A^A}{\partial T} + \phi \frac{\partial^2 TR_A}{\partial t_A \partial T} \tag{B}$$

where

$$\gamma_A^2 = \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A{}^2} - \frac{\alpha_A}{\left(x_A^A - r_x\right)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A{}^2} - \frac{1-\alpha_A}{y_A^{A2}} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 \right)$$

$$\text{or } = -2 \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \varepsilon - \frac{\alpha_A}{\left(x_A^A - r_x\right)^2} \frac{x_A^{A2}}{P_A{}^2} \varepsilon^2 - \frac{1-\alpha_A}{y_A^2} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2$$

$$\gamma_A^1 = \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right)$$

$$\text{or } = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A} \varepsilon + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A}$$

From the first order condition,

$$\phi = \frac{\gamma_A^1}{x_A^A \left\{ \varepsilon \frac{t_A}{P_A} + 1 \right\}}$$

If we omit the part of y_A^A term for simplicity,

$$\frac{\partial^2 W_A}{\partial t_A{}^2} = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \left(-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - 2\left(\frac{P_A - t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2 \right) \tag{A'}$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \left(-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - \left(\frac{P_A - 2t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2 \right) \tag{B'}$$

Then, the value of the slope of the reaction function becomes,

$$\frac{\partial^2 W_A}{\partial t_A \partial T} \Bigg/ \frac{\partial^2 W_A}{\partial t_A{}^2} = \frac{-2\varepsilon - \dfrac{x_A^A}{x_A^A - r_x} \varepsilon^2 - \left(\dfrac{P_A - 2t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2}{-2\varepsilon - \dfrac{x_A^A}{x_A^A - r_x} \varepsilon^2 - 2\left(\dfrac{P_A - t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2} \tag{C}$$

It is also interesting to see the value of the slope of the reaction function in vertical tax competition for the Leviathan case.

$$\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \Bigg/ \frac{\partial^2 TR_{GA}}{\partial t_A{}^2} = \frac{P_A - 2t_A}{-2(P_A - t_A)} \approx -\frac{1}{2} \tag{C'}$$

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