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Pension and Child Care Policies with Endogenous Fertility*

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Abstract

This paper describes how a child allowance policy and income transfer to older people policy alter fertility and economic growth under a pay-as-you-go pension system. Moreover, this paper presents ways to finance such policies: one for income taxation and the other for consumption tax. The results presented in this paper are as follows. First, the relation between fertility and economic growth depends on fertility. This relation is positive if fertility is at a certain level. However, if fertility is low or high, this relation is negative.

Second, a child allowance does not always raise the fertility rate. On the other hand, income transfer to older people raises the fertility rate. This result underscores that it is important to consider policies for older people when considering how to raise the fertility rate. Income transfer to older people financed by consumption taxation can achieve two goals: increasing fertility in a society with fewer children, and providing income security for older people.

JEL Classification:D10, H55, J13, J14, J18, J26

Keywords: Child allowance, Endogenous fertility, Pay-as-you-go pension system, Taxation

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1 Introduction

The total fertility rate (TFR) in Japan in 2007 was 1.34. Recent statistical projections related to fertility rates show a decreasing trend. A decreasing trend in the number of children holds not only in Japan, but also in other developed countries, as portrayed in Fig. 1.

[Insert Fig. 1 around here.]

The fertility in these countries is low, as presented in Fig. 1; however, these countries are roughly divisible into two groups: countries for which fertility continues to decrease and countries for which fertility has stopped decreasing and instead increases. Italy, Germany, and Japan belong to the former countries. France and Sweden are of the latter group of countries. Fertility stopped decreasing in France and Sweden because those governments came to spend funds to support family policies. Figure 2 portrays the transition of government expenditures related to family policy in developed countries.

[Insert Fig. 2 around here.]

As portrayed in Fig. 2, the ratio of expenditures for family policy to GDP is large in France and Sweden, where fertility has changed from negative to increase. However, it is low in Italy and Japan, where fertility continues to decrease. The government expenditure for family policies, which includes expenditures for child-care service or child allowance, is considered as public child care. We can say that increased fertility occurs if this expenditure level is high. For example, fertility and government expenditures for family policies in Sweden and France is higher than that in Japan.[1]. Therefore, it is necessary that the government give the child allowance actively as in France and Sweden, and thereby bears child-care costs publicly: Japan must increase the expenditures for family policies actively to raise the fertility rate in Japan.

Why must the government restore declining fertility? It must do so because the government is obligated to manage the social security system. Social security systems

[1]The child allowance level in France is higher than that in Japan. In Japan, the child allowance is given for children of elementary school age. In France, the child allowance is given for children less than 20 years old, except for the first-born child. The amount of child allowance is 10,000 yen per month in Japan. The amount in France is 19,000 yen for the second child (25,000 yen for the third child). In addition to this allowance, children more than 11 years old are given a higher allowance (Data: Cabinet Office, Government of Japan (2007): "White Paper on Birthrate-Declining Society")

such as those of health insurance and pension systems must be supported by younger generations. Unless the population of the young generation is sufficiently large, the social security system can not be maintained. Concretely, the government must collect a contribution from younger people to run the pay-as-you-go pension system. If the population of the younger generation is small, then the pension that older people receive under the constant contribution rate would be small, too. If the government were to fix the pension paid to older people, the burden of the younger people would become large. Therefore, unless the population is sufficiently large, the government can not afford to pay pension to older people sufficiently and maintain the system.

In the public pension system in Japan, benefits for the older people are financed not only by the contributions of younger people but also by tax revenues (National General Account).[2] In recent years, the burden of the national general account has tended to increase, as shown in Fig. 3.

[Insert Fig. 3 around here.]

Consequently, with the burden of the national general account, the government can pay some pension benefits for the older people even if the government can not collect contributions from younger people because of the small young population. However, the government must carry out policies to stem the decline in fertility to maintain a sustainable pension system. In carrying out such a policy, the government must consider the income of older people. However, it is considered that a tradeoff relation holds between the child allowance for younger people and the income transfer to older people. If the income transfer to older people increases, then younger people can not receive a sufficient child allowance; thereby, we can not expect increased fertility. On the other hand, if the child allowance for younger people increases, older people can not receive a sufficient income. This paper describes how the intergenerational balance of burden and benefit affects fertility and the economic growth rate; it also describes what policies are desirable in terms of stemming the decline in fertility or income transfer to older people under a model such as an intergenerational tradeoff.

[2]The national basic pension liability in Japan is shared by the national general account.

Some reports have described how fertility determines pay-as-you-go pension systems. Groezen, Leers and Meijdam (2003) and Hirazawa and Yakita (2008) show that the existence of market failure resulting from externality generates pay-as-you-go pension system failures.[3] Hirazawa and Yakita (2008) report that the dynamics of fertility might fluctuate because of existence of a pay-as-you-go pension system.[4] The analyses used in these studies assume not a closed economy, which considers capital accumulation, but rather a small open economy. Oshio (2001) and Yasuoka (2006) demonstrate whether the child allowance raises the fertility rate or not under a closed economy model. These studies show how the government collects revenue to spend for the child allowance in addition to the result derived by Groezen, Leers, and Meijdam (2003), who show that it is necessary to raise the fertility rate using the child allowance. Yasuoka (2006) shows that the government should collect revenues not by labor income taxation, which greatly decreases saving, i.e. the capital per capita, but instead by consumption taxation.

Some reports of the relevant literature analyze the pension system under a model without endogenous fertility (Br*ä*uninger (2005), Corneo and Marquart (2000), Ono (2007)). Br*ä*uninger (2005) and Corneo and Marquart (2000) show that a pay-as-you-go pension system imparts a negative effect on economic growth. On the other hand, Ono (2007) shows that an increase in the contribution rate of the pension raises the employment rate and stabilizes the economy.

Yasuoka and Goto (2009) show how fertility determines under a small open economy which adopts a pay-as-you-go pension system. Moreover, they describe how the child allowance affects fertility and the welfare, how it is financed to spend for the child allowance, and how both increased fertility by the child allowance and the direct income transfer to older people affects for the welfare of older people. They derive the result that the level of a consumption tax or child-care cost determines the dynamics of fertility: one for the convergence and the other for divergence. This paper describes a closed economy, which is not considered by Yasuoka and Goto (2009).

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[3] Groezen, Leers, and Meijdam (2003) show that fertility in a decentralized economy does not coincide with that of the command optimum. Then, the policy to increase the fertility is justified.

[4] Hirazawa and Yakita (2008) assume a fertility function, into which is inputted the child-care service and a parent's child-care time. With a pay-as-you-go pension system, a parent's child care fluctuates over time. If this effect is large, then fertility fluctuates, too.

Results of those analyses suggest the following conclusions. On the balanced growth path, the relation between fertility and the economic growth rate is positive if fertility is positive to some degree, but this relation becomes negative if fertility is either low or high. Moreover, a child allowance can not always raise the fertility rate. Nevertheless, an income transfer to older people by a consumption tax certainly raises the fertility rate. This result shows that increased lifetime income attributable to increased income transfer to older people can raise the fertility rate. However, even if fertility increases, it is possible to decrease the economic growth rate. Therefore, an increase in the tax rate to increase the income transfer brings about an increase in the income transfer in the short run. However, the income transfer and pension benefit decreases because of the decrease in the economic growth rate in the long run.

The remainder of this paper is presented as follows: Section 2 introduces the model and Section 3 presents a description of the equilibrium. Section 4 assesses child-care and transfer policies for older people. The final section presents conclusions of this study.

2 The Model

The model economy is based on a two-period (young and old) overlapping generations model. This economy has agents of three types: households, firms, and a government.

2.1 Households

Households experience two periods: young and old. During the young period, each household supplies labor to earn labor income. The household also raises children. These analyses assume that it is necessary for households (parents) to invest in child care to have children. Households have one unit of time, which is assumed to be supplied for labor inelasticity. During the old period, each household only consumes. The government runs a pay-as-you-go pension system and imposes labor income taxation and consumption taxation to provide a child allowance and income transfer to older people. Each household distributes its labor income among child-care goods and other consumption. Consequently, we obtain the following budget constraint.

$$(1+\tau_c)c_{1t} + \frac{(1+\tau_c)c_{2t+1}}{1+r_{t+1}} + (z_t - q_t)n_t = (1-\tau)(1-\phi n_t)w_t - \tilde{p}_t + \frac{p_{t+1} + \bar{n}_t\tilde{p}_{t+1}}{1+r_{t+1}} \quad (1)$$

The analyses presented in this paper assume that child-care goods are not taxed, and therefore do not affect fertility negatively.[5] Herein, $\tilde{p}_t$ represents contributions paid by younger people, where $\hat{p}$ represents the contribution rate, or the constant rate of income collected by the government. Running a pay-as-you-go pension, the pension which older people receive is $\bar{n}_t \tilde{p}_{t+1}$, where $\bar{n}_t$ represents an average fertility in t period, which means the intergenerational population size ratio, i.e. the share of younger people size N_{t+1} to older people size N_t in $t+1$ period ($\bar{n}_t = \frac{N_{t+1}}{N_t}$). The benefit rate is $\bar{n}_t \frac{w_{t+1}}{w_t}$. Furthermore, regarding this pension benefit, older people also receive an income transfer financed by taxation. The government imposes a consumption tax on consumption both by younger people and older people and a labor income tax on labor income. The consumption tax rate and labor income tax rate are denoted respectively as τ_c and τ. In addition, w_t and r_{t+1} respectively represent the wage rate and interest rate. Furthermore, ϕ denotes the child-care time per child, thereby labor time reduces $1 - \phi n_t$. With an increase in the number of children n_t, the labor time decreases.[6] In these analyses, z_t denotes the child-care goods cost per child. The child-care goods cost is assumed as $z_t = \hat{z} w_t$. If the income level becomes higher, then the child-care goods cost becomes higher, too. Assuming that q_t denotes the child allowance per child, it is assumed $q_t = \hat{q} w_t$, where $\hat{q}$ represents the benefit rate of the child allowance. Therefore, $z - q + \phi(1-\tau) w_t = (\hat{z} - \hat{q} + \phi(1-\tau)) w_t$ shows the cost of having a child. In addition, c_{1t} and c_{2t+1} respectively represent consumption by younger people and older people.

A household's utility function u_t is given as follows:[7]

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + \gamma \ln n_t, \ 0 < \alpha, \beta, \gamma < 1. \tag{2}$$

An individual chooses consumption during young and old life c_{1t}, c_{2t+1} and chooses the number of children n_t to maximize lifetime utility subject to the lifetime budget con-

[5]Even if child-care goods are subject to a consumption tax, the result derived in this paper holds. See Section 4 for the proof.

[6]This paper assumes that the child-care time per child is constant; thereby the child-care cost per child is also constant. Some earlier studies included the assumption that the child-care cost per child depends on the number of children (Zhang and Zhang (1998)). However, some studies assume constant child-care cost per child (Galor and Weil).

[7]This assumption is conventional in modeling of endogenous fertility: Eckstein and Wolpin (1985), Galor and Weil (1996) and Groezen, Leers, and Meijdam (2003)

straint. The optimal allocations are determined as

$$c_{1t} = \frac{1}{1+\tau_c}\frac{\alpha}{\alpha+\beta+\gamma}\left((1-\tau)w_t + \frac{p_{t+1}+\bar{n}_t\tilde{p}_{t+1}}{1+r_{t+1}} - \tilde{p}_t\right), \quad (3)$$

$$c_{2t+1} = \frac{1+r_{t+1}}{1+\tau_c}\frac{\beta}{\alpha+\beta+\gamma}\left((1-\tau)w_t + \frac{p_{t+1}+\bar{n}_t\tilde{p}_{t+1}}{1+r_{t+1}} - \tilde{p}_t\right), \quad (4)$$

$$n_t = \frac{\gamma}{\alpha+\beta+\gamma}\frac{(1-\tau)w_t + \frac{p_{t+1}+\bar{n}_t\tilde{p}_{t+1}}{1+r_{t+1}} - \tilde{p}_t}{z_t - q_t + \phi(1-\tau)w_t}. \quad (5)$$

The number of children (fertility) n_t decreases with increased child-care costs $(\hat{z}-\hat{q}+\phi(1-\tau))w_t$.

$$n_t = \frac{\gamma}{\alpha+\beta+\gamma}\frac{(1-\tau) - \hat{p} + \frac{p_{t+1}+\bar{n}_t\hat{p}w_{t+1}}{w_t(1+r_{t+1})}}{\hat{z}-\hat{q}+\phi(1-\tau)} \quad (6)$$

Under constant income, which older people receive $p_{t+1}+\bar{n}_t\hat{p}w_{t+1}$, an increase in the wage rate of w_t diminishes fertility for the following reason. An increase in w_t increases fertility because of increased income, however, an increase in child-care costs, which shows an opportunity cost to care for children (loss of wage income not to supply labor), decreases the fertility rate. In this model, the latter effect is greater than the former one.

2.2 Firms

The production function of final goods is given as a neoclassical constant-returns-to-scale function:

$$Y_t = K_t^{\theta}(A_tL_t)^{1-\theta}, \ 0<\theta<1, \quad (7)$$

where $A_t \equiv \frac{K_t}{L_t}\frac{1}{b}$ is assumed as described in Grossman and Yanagawa (1993). In addition, Y_t and K_t respectively represent the final goods and capital stock. Moreover, L_t denotes labor input, and N_t denotes younger population size. Because each labor time is $1-\phi n_t$, the labor input is given as$L_t=(1-\phi n_t)N_t$. Each firm determines the demand for capital stock and labor to maximize the profit under given A_t. Assuming perfect competition, the wage rate w_t and the interest rate r_t are

$$w_t = \frac{1-\theta}{b^{1-\theta}}\frac{k_t}{1-\phi n_t}, \quad (8)$$

$$1+r_t = \frac{\theta}{b^{1-\theta}}, \quad (9)$$

where $k_t \equiv \frac{K_t}{N_t}$ and the capital stock is fully depreciated in one period. Moreover, the interest rate is constant over time in this production function $(r_t = r_{t+1} = r)$. However, the wage rate w_t increases concomitantly with per-capita capital k_t. Then, the income per capita is defined as $y_t \left(\equiv \frac{Y_t}{N_t}\right)$ is $y_t = \frac{k_t}{b^{1-\theta}}$. Consequently, the growth rate of per-capita income defined by $\frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t}$ is equivalent to the growth rate of capital stock per capita.

2.3 Government

The government imposes labor income taxation at a tax rate τ and consumption at a tax rate τ_c to provide a child allowance and benefit for older people. The government budget constraint is

$$N_t q n_t + N_{t-1} p_t = N_t \tau(1 - \phi n_t) w_t + \tau_c (N_t c_{1t} + N_{t-1} c_{2t}). \tag{10}$$

Then, the per-capita benefit for older people is shown as follows.

$$p_{t+1} = n_t \left((\tau(1 - \phi n_{t+1}) - \hat{q} n_{t+1}) w_{t+1} + \frac{1}{\gamma} \frac{\tau_c}{1 + \tau_c} (\hat{z} - \hat{q} + \phi(1 - \tau))(\alpha w_{t+1} n_{t+1} + \beta(1 + r) w_t) \right). \tag{11}$$

The income transfer to older people p_{t+1} decreases if the benefit rate of child allowance $\hat{q}$ increases to raise the fertility rate. Furthermore, if the income tax rate increases, the income transfer to older people p_{t+1} increases. However, fertility might decrease because of a decrease in disposable income by an increase in the tax rate. Consequently, an increase in the income tax rate does not always increase the income transfer to older people.

3 Equilibrium

In this economy, the dynamics of capital stock k_t and fertility n_t specify the equilibrium. First, we derive the dynamics of capital stock k_t. These dynamics are represented by $K_{t+1} = N_t s_t$, where s_t denotes an individual's saving. Dividing this equation by N_t, then

$$k_{t+1} = \frac{s_t}{n_t}. \tag{12}$$

An individual saving s_t is as presented below.

$$\begin{aligned} s_t &\equiv (1 - \tau)(1 - \phi n_t) w_t - \tilde{p}_t - (z_t - q_t) n_t - (1 + \tau_c) c_{1t} \\ &= (1 - \tau) w_t - \tilde{p}_t - \frac{\alpha + \gamma}{\gamma} (z_t - q_t + \phi(1 - \tau) w_t) n_t. \end{aligned} \tag{13}$$

Therefore, the dynamics equation is as shown below.

$$
\begin{aligned}
k_{t+1} &= \frac{(1-\tau)w_t - \tilde{p}_t}{n_t} - \frac{\alpha+\gamma}{\gamma}(z_t - q_t + \phi(1-\tau)w_t) \\
&= \frac{1-\theta}{b^{1-\theta}} \frac{1}{1-\phi n_t} \left(\frac{(1-\tau)-\hat{p}}{n_t} - \frac{\alpha+\gamma}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) \right) k_t \qquad (14)
\end{aligned}
$$

Consequently, the economic growth rate $1+g_t \equiv \frac{y_{t+1}}{y_t}$ is

$$
1+g_t = \frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t} = \frac{1-\theta}{b^{1-\theta}} \frac{1}{1-\phi n_t} \left(\frac{(1-\tau)-\hat{p}}{n_t} - \frac{\alpha+\gamma}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) \right). \qquad (15)
$$

Second, the dynamics equation of fertility is shown as

$$
n_{t+1} = \frac{\frac{1+r}{1+\tau_c}\frac{\beta}{\gamma}(\hat{z} - \hat{q} + \phi(1-\phi))\frac{1-\phi n_{t+1}}{1-\phi n_t}\frac{k_t}{k_{t+1}} - (\hat{p}+\tau) - (1+r)(1-\phi n_{t+1})\frac{b^{1-\theta}}{1-\theta}}{\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) - \tau\phi - \hat{q}}. \qquad (16)
$$

Consequently, capital in the subsequent period k_{t+1} depends on k_t and n_t; fertility in the next period n_{t+1} is determined. Considering (15) and (16), we can show the dynamics of the model economy as only fertility dynamics:

$$
\begin{aligned}
&\frac{1}{1-\phi n_{t+1}} \left(\left(\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) - \tau\phi - \hat{q} \right) n_{t+1} + \hat{p} + \tau \right) \\
&\qquad = \left(\frac{\frac{1}{1+\tau_c}\frac{\beta}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau))}{\frac{(1-\tau)-\hat{p}}{n_t} - \frac{\alpha+\gamma}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau))} - 1 \right) \frac{\theta}{1-\theta}. \qquad (17)
\end{aligned}
$$

Then, the following shows the dynamics of n_t.

[Insert Fig. 4 around here.]

Figure 4-1 portrays the case of $\frac{dn_{t+1}}{dn_t} > 0$. The unique or two-steady-state situation exists. If the sign of $n_{t+1} = \frac{-\frac{\theta}{1-\theta} - (\hat{p}+\tau)}{\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau)) - \tau\phi - \hat{q} - \frac{\phi\theta}{1-\theta}}$ is negative, then a unique, unstable steady state exists. A stable steady state and an unstable steady state exist if this sign is positive. Figure 4-1 shows the condition of $\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) - \tau\phi - \hat{q} > 0$. With a two-steady-state equilibrium, if initial fertility is not large, then fertility converges to the steady state with low fertility. The fertility continues increasing if the initial fertility is larger than fertility at the unstable steady state. The fertility continues decreasing if the initial fertility is not large under the unique steady state.

However, if $\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) - \tau\phi - \hat{q} > 0$, then the dynamics of fertility fluctuates as depicted in Fig. 4-2. The stable condition $-1 < \frac{dn_{t+1}}{dn_t} < 0$ as the following equation decides which fertility converges or diverges.

$$
\frac{dn_{t+1}}{dn_t} = -\frac{\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau)) - \hat{q} - \phi\left(\tau + \frac{\phi\theta}{1-\theta}\left(1 - \frac{\frac{1}{1+\tau_c}\frac{\beta}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))}{\frac{(1-\tau)-p}{n} - \frac{\alpha+\gamma}{\gamma}}(\hat{z} - \hat{q} + \phi(1-\tau))\right)\right)}{\frac{(1-\phi n)\frac{(1-\tau)-\hat{p}}{n}\frac{1}{1+\tau_c}\frac{\beta}{\gamma}\frac{\theta}{1-\theta}(\hat{z}-\hat{q}+\phi(1-\tau))}{\left(\frac{(1-\tau)-p}{n} - \frac{\alpha+\gamma}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))\right)^2}}
$$

With $-1 < \frac{dn_{t+1}}{dn_t} < 1$, the steady state is stable. The dynamics converges with fluctuation if $-1 < \frac{dn_{t+1}}{dn_t} < 0$. The dynamics converges monotonically if $0 < \frac{dn_{t+1}}{dn_t} < 1$.

We show how the economic growth rate changes when fertility changes. Considering (15), we derive the following equation.

$$\frac{dg_t}{dn_t} = \frac{1-\theta}{b^{1-\theta}} \frac{1}{(1-\phi n_t)^2 n_t^2} \left((2\phi n_t - 1)((1-\tau) - \hat{p}) - \frac{\alpha+\gamma}{\gamma} \phi(\hat{z} - \hat{q} + \phi(1-\tau))n_t^2 \right). \tag{18}$$

The sign of $\frac{dg}{dn}$ is ambiguous; however, this sign is determined by the level of fertility. Defining $\hat{L} \equiv ((1-\tau) - \hat{p})(2\phi n - 1)$ and $\hat{R} \equiv \frac{\alpha+\gamma}{\gamma}(\hat{z} - \hat{q} + \phi(1-\tau))\phi n^2$, we show Fig. 5 under some parametric conditions.

[Insert Fig. 5 around here.]

The relation between fertility and the growth rate is not determined uniquely if two intersections exist. This relation is negative if fertility is either low or high. However, fertility of some positive and medium range makes this relation is positive. Then, the following proposition is established.

Proposition 1 Under some parametric conditions, the relation between fertility and economic growth rate is negative if fertility is low or high. On the other hand, if fertility is in some range, then this condition is positive.

This proposition does not always hold. For example, with large $\hat{z}$ or small $\hat{q}$, the intersection vanishes because $\hat{R}$ moves more above. Without the intersection, the relation between fertility and growth rate is always negative. With small $\hat{z}$ or large $\hat{q}$, this relation changes from negative to positive: this relation is determined by the level of child-care goods cost or the benefit rate of the child allowance. Child-care time ϕ does not always enlarge the area for which this relation is positive.

Therefore, if fertility continues increasing over time, it is possible that the economic growth rate changes a decrease to an increase and changes to a decrease thereafter. Conversely, if fertility continues decreasing over time, it is possible that the relation between fertility and the economic growth rate changes over time.

The fertility and growth rate at the steady state are shown, respectively, as

$$\frac{1}{1-\phi n}\left(\left(\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))-\tau\phi-\hat{q}\right)n+\hat{p}+\tau\right)$$
$$=\left(\frac{\frac{1}{1+\tau_c}\frac{\beta}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))}{\frac{(1-\tau)-\hat{p}}{n}-\frac{\alpha+\gamma}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))}-1\right)\frac{\theta}{1-\theta}, \tag{19}$$

$$1+g=\frac{1-\theta}{b^{1-\theta}}\frac{1}{1-\phi n}\left(\frac{(1-\tau)-\hat{p}}{n}-\frac{\alpha+\gamma}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))\right). \tag{20}$$

Considering (19), we derive fertility at the stable steady state.

$$L \equiv \frac{1}{1-\phi n}\left(n\left(\frac{\tau_c}{1+\tau_c}\frac{\alpha}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))-\tau\phi-\hat{q}\right)+\hat{p}+\tau\right),$$
$$R \equiv \left(\frac{\frac{1}{1+\tau_c}\frac{\beta}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))}{\frac{(1-\tau)-\hat{p}}{n}-\frac{\alpha+\gamma}{\gamma}(\hat{z}-\hat{q}+\phi(1-\tau))}-1\right)\frac{\theta}{1-\theta}.$$

We depict two figures because of the sign of L. The intersections show the stable steady states.

[Insert Fig. 6 around here.]

Figure 6-1 presents the possibility that two intersections show fertility at the steady state. However, we do not consider the intersection, which shows higher fertility because this is fertility at the unstable steady state.

4 Policy Effects

This section presents analysis of how an increase in the benefit rate of child allowance $\hat{q}$ or an increase in the income transfer to older people with an increase in the consumption tax rate τ_c or income tax rate τ affects fertility and the growth rate at the stable steady state.

4.1 Increase in the Benefit Rate of the Child Allowance $\hat{q}$

An increase in the benefit rate of the child allowance $\hat{q}$ under constant consumption tax rate τ_c and income tax rate τ decreases the income transfer to older people. However, if this increase brings about an increase in fertility or the growth rate, the income transfer to older people increases. Therefore, the older person income increases, too. As portrayed in Fig. 7, an increase in the benefit rate of the child allowance $\hat{h}$ does not increase fertility.

[Insert Fig. 7 around here.]

The economic growth rate also increases if an increase in the benefit rate of the child allowance increases fertility and the fertility is within some range. Then, the pension benefit that older people receive is large. Consequently, it is possible that the benefit for older people increases even if an increase in the benefit rate of child allowance produces an income transfer to older people directly in the short term. However, if fertility is low or high, this policy does not always increase pension benefits for older people because of a decrease in the economic growth rate. Conversely, an increase in the benefit rate of child allowance might increase pension benefits for older people because of the increased economic growth rate even if this policy decreases the fertility rate.

Moreover, this policy affects the economic growth rate directly. Therefore, this policy prevents a decrease in the economic growth rate even if this policy decreases the economic growth rate indirectly through increased fertility.

4.2 Increased Income Transfer Financed by an Increase in the Income Tax Rate τ

An increase in the income tax rate τ raises the ratio of income transfer to older people to total tax revenue. However, tax revenue is affected not only by the change of tax rate, but also by the change of the economic growth rate. It decreases not only the income transfer to older people but also the pension benefit for them if an increase in the income tax rate decreases the fertility rate. In reality, it is ambiguous whether fertility increases or decreases.

[Insert Fig. 8 around here.]

Therefore, this policy effect is nearly identical to the effect of an increase in the benefit rate of child allowance. However, an increase in the income transfer to older people financed by an increase in the income tax rate directly decreases the economic growth rate. Consequently, this policy sharply decreases the economic growth rate. If $(1-\tau)\left(1-\frac{\alpha+\gamma}{\gamma}\phi n\right) > 0$, i.e., if fertility is low, then the change of the income tax rate increases the economic growth rate directly. On the other hand, if fertility is higher than a certain level, then this change decreases the economic growth rate directly.

4.3 Increase in Income Transfer Financed by an Increase in the Consumption Tax Rate τ_c

An increase in the consumption tax rate τ_c raises the income transfer to older people. However, this policy brings about a result that differs from the cases of an increase in the benefit rate of child allowance or income transfer financed by an increase in the income tax rate. This policy certainly raises the fertility rate.

[Insert Fig. 9 around here.]

Increased income transfer to older people financed by an increase in the consumption tax rate raises the fertility rate. In addition to this effect, if fertility is in a certain range, this policy raises the economic growth rate, too. Then, both income transfer and the pension benefit for older people certainly increase. Moreover, the change of the consumption tax rate does not affect economic growth directly. The indirect effect occurs because the change of fertility derived by the change of the consumption tax rate affects economic growth. Consequently, the following proposition is established.

Proposition 2 On the balanced growth path, the increase in the benefit rate of the child allowance or income transfer financed by an increase in the income tax rate does not always raise the fertility rate. However, an income transfer financed by consumption tax surely raises the fertility rate.

Why does an increased income transfer financed by consumption tax raise the fertility rate? This answer is increased lifetime income. In OECD countries, the fertility rate is low, so governments seek to increase fertility. Simultaneously, the government must consider the income policy for older people. If the government adopts this policy, then two policy goals are achieved in an aging society with fewer children: increased fertility and an increase in the income transfer and pension benefits for older people. Policymakers are likely to consider an increase in the child allowance to raise the fertility rate, but this policy does not always raise the fertility rate. Because the positive effect that an increase in the child allowance raises the fertility rate and the negative effect that this brings about decrease in income transfer to older people, the decrease in lifetime income decreases the fertility rate.

5 Conclusions and Remarks

This paper presented an examination of policies related to child care and income transfer to older people with a pay-as-you-go pension system model introduced with endogenous fertility. The analyses described herein yielded the following results. First, if fertility is within a certain range under some parameter conditions, the relation between the economic growth rate and fertility is positive. However, if fertility is either lower or higher, this relation becomes negative. Moreover, with high child-care costs, the relation is always negative.

Second, policies to support an increase in income transfer to older people do not always increase the income of older people. An increase in income transfer financed by income taxation might increase the economic growth rate. However, this policy might decrease the fertility rate, thereby ultimately decreasing the income of older people. In contrast, an increase in income transfer to older people always raises the fertility rate. Then, if fertility is in some range, this policy increases the economic growth rate, so the income received by older people increases.

Finally, an increase in the child allowance does not always raise the fertility rate. Therefore, it is desirable to raise the fertility rate by a policy including increased income transfer financed by a consumption tax. Thereby, the government can achieve two policy goals: it can simultaneously increase fertility under a society with fewer children and secure the income level of older people.

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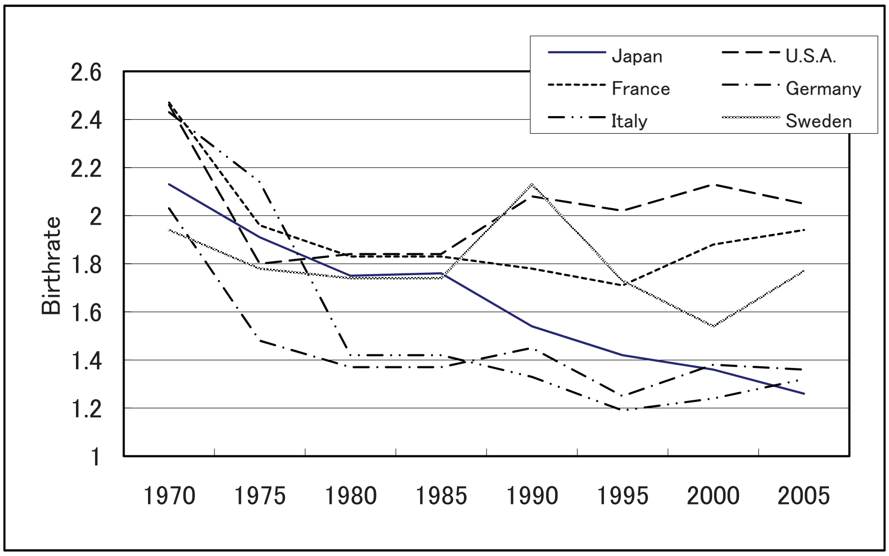

Fig. 1: Fertility Rates in Developed Countries (Data: Population Statistics (2008))

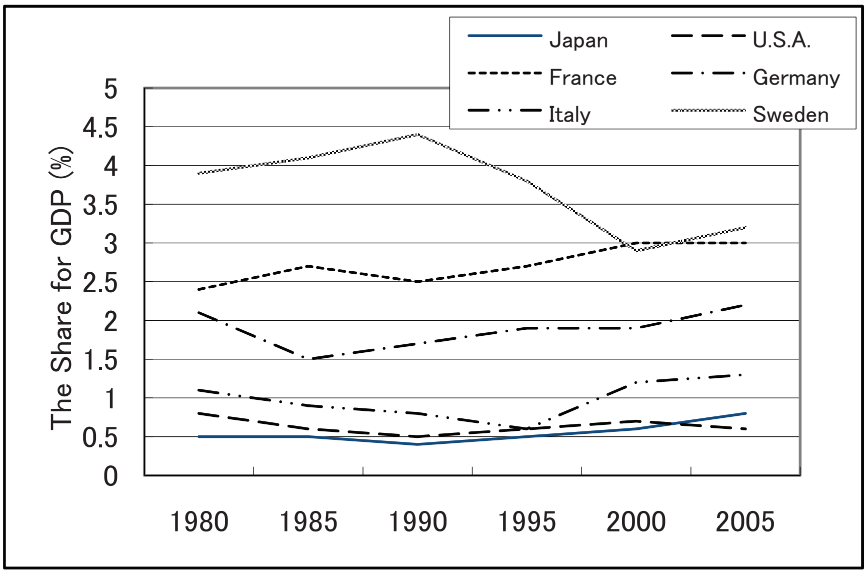

Fig. 2: Ratio of Social Expenditure on Family Policies to GDP (Data: Social Expenditure Database (2007))

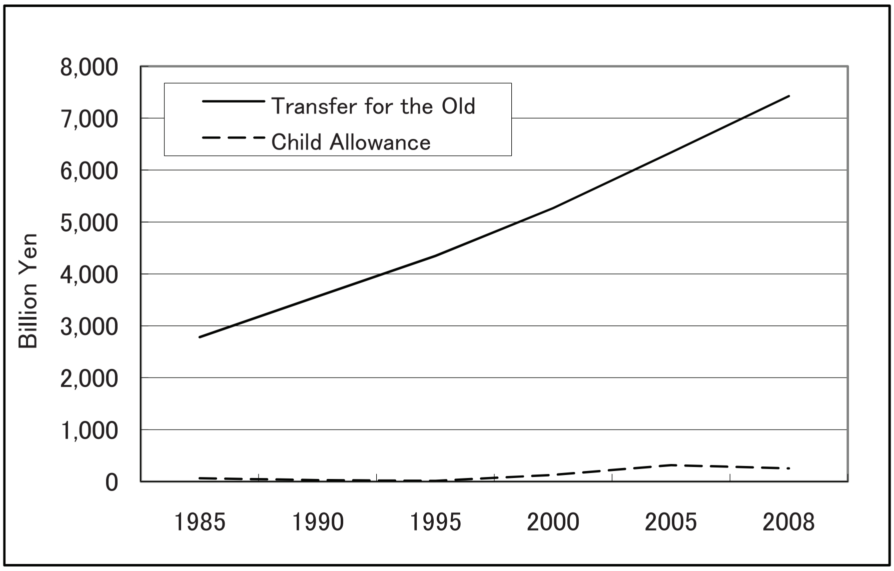

Fig. 3: Transfer for the Old and Child Allowance in the National General Account in Japan (Data: Annual Reports on Health and Welfare (2008))

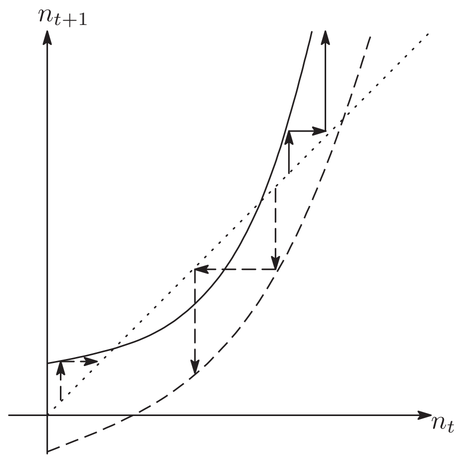

Fig. 4-1: Dynamics of Fertility (Monotonic Convergence)

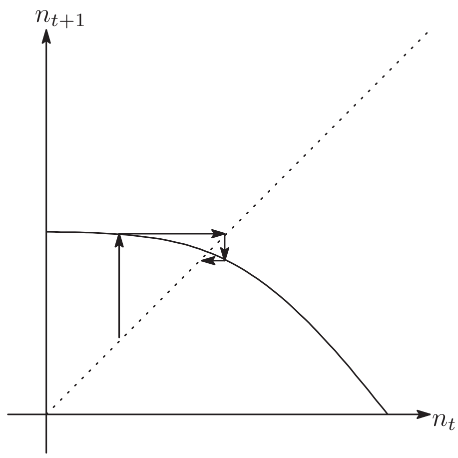

Fig. 4-2: Dynamics of Fertility (Convergence with Fluctuation)

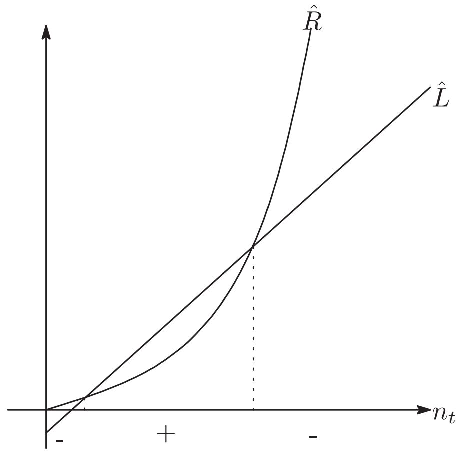

Fig. 5: Relation between the Fertility Rate and the Economic Growth Rate

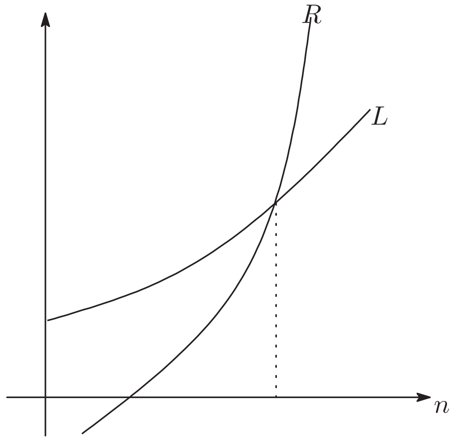

Fig. 6-1: Fertility at the Steady State

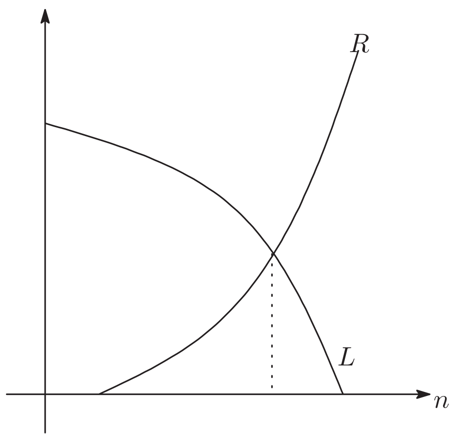

Fig. 6-2: Fertility at the Steady State

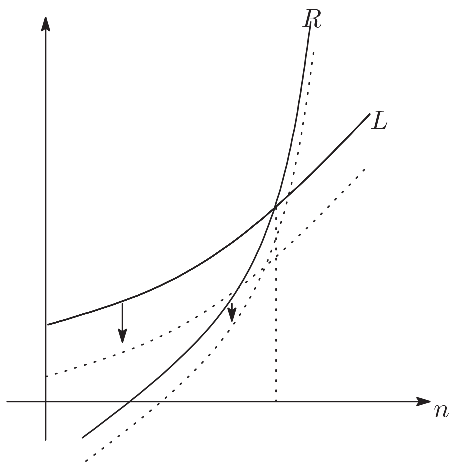

Fig. 7-1: Increase in the Child Allowance $\hat{q}$ and Fertility

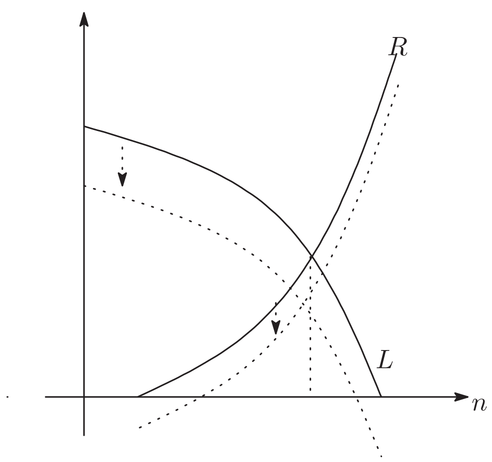

Fig. 7-2: Increase in the Child Allowance $\hat{q}$ and Fertility

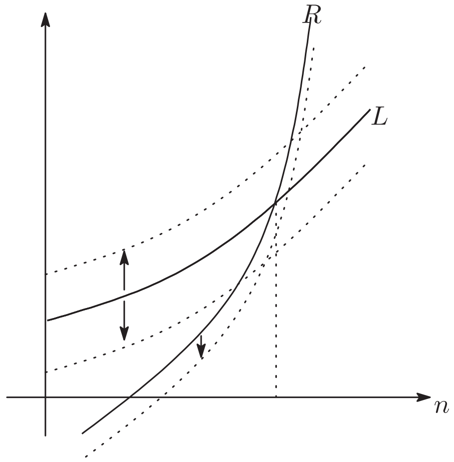

Fig. 8-1: Increase in the Labor Income Tax Rate τ and Fertility

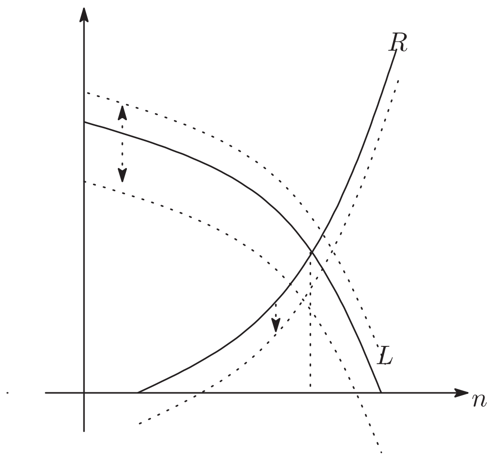

Fig. 8-2: Increase in the Labor Income Tax Rate τ and Fertility

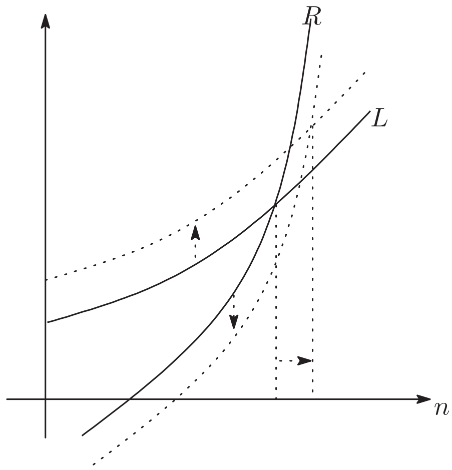

Fig. 9-1: Increase in the Consumption Income Tax Rate τ_c and Fertility

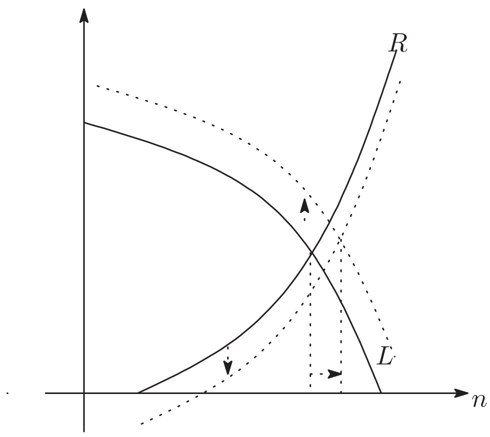

Fig. 9-2: Increase in the Consumption Income Tax Rate τ_c and Fertility