Hitotsubashi University

Global COE Hi-Stat Discussion Paper Series 066

Research Unit for Statistical and Empirical Analysis in Social Sciences (Hi-Stat)

Option Pricing Using Realized Volatility and ARCH Type Models

Toshiaki Watanabe
Masato Ubukata

April 2009

Hi-Stat
Institute of Economic Research
Hitotsubashi University
2-1 Naka, Kunitatchi Tokyo, 186-8601 Japan
http://gcoe.ier.hit-u.ac.jp

Hi-Stat Discussion Paper

Option Pricing Using Realized Volatility and ARCH Type Models

Toshiaki Watanabe*and Masato Ubukata†

April 2009

Abstract

This article analyzes whether daily realized volatility, which is the sum of squared intraday returns over a day, is useful for option pricing. Different realized volatilities are calculated with or without taking account of microstructure noise and with or without using overnight and lunch-time returns. The both ARFIMA and ARFIMAX models are employed to specify the dynamics of realized volatility. The former can capture the long-memory property and the latter can also capture the asymmetry in volatility depending on the sign of previous day's return. Option prices are derived under the assumption of risk-neutrality. For comparison, GARCH, EGARCH and FIEGARCH models are estimated using daily returns, where option prices are derived by assuming the risk-neutrality and by using the Duan (1995) method in which the assumption of risk-neutrality is relaxed. Main results using the Nikkei 225 stock index and its put options prices are: (1) the ARFIMAX model with daily realized volatility performs best, (2) applying the Bartlett adjustment to the calculation of realized volatility to take account of microstructure noise does not improve the performance while the Hansen and Lunde (2005a) adjustment without using overnight and lunch-time returns improves the performance, and (3) the Duan (1995) method does not improve the performance compared with assuming the risk neutrality.

JEL classification: C22; C52; G13

Keywords: ARFIMA, GARCH, Microstructure Noise, Option, Realized Volatility

*Institute of Economic Research, Hitotsubashi University, Email: watanabe@ier.hit-u.ac.jp

†Faculty of Economics, Osaka University, Email: ubukata@econ.osaka-u.ac.jp

1. Introduction

One of the most important variables in option pricing is the volatility of the underlying asset. While the well-known Black and Scholes (1973) model assumes that the volatility is constant, few would dispute the fact that the volatility changes over time. Many time series models are now available to describe the dynamics of volatility. One of the most widely used is the ARCH (autoregressive conditional heteroskedasticity) family including ARCH model by Engle (1982), GARCH (generalized ARCH) model by Bollerslev (1986) and their extensions.

The problem of using these models is that we must specify the model before estimating the volatility and the estimate of volatility depends on the specification of volatility dynamics. Recently, realized volatility has attracted the attentions of financial econometricians as an accurate estimator of volatility. Realized volatility is independent of the specification of volatility dynamics because it is simply the sum of squared intraday returns.

ARCH type models have already been applied to option pricing (Bollerslev and Mikkelsen, 1999; Duan, 1995). As far as we know, realized volatility has, however, not yet been applied to option pricing while it has been applied to volatility forecasting (Koopman et al. 2005) and Value-at-Risk (Giot and Laurent, 2004; Clements et al., 2008). This article applies realized volatility to the pricing of Nikkei 225 stock index options traded at Osaka Securities Exchange and compares its performance with that of using the ARCH family.

There are two problems in calculating realized volatility. First, realized volatility is influenced by market microstructure noise such as bid-ask spread and non-synchronous trading (Campbell et al., 1997). There are some methods available for mitigating the effect of microstructure noise on realized volatility (Aït-Sahalia et al., 2005; Bandi and Russell, 2006, 2008; Barndorff-Nielsen et al., 2007; Zhang, 2006; Zhang et al., 2005). It is worthwhile applying these methods and comparing the results. We use the method proposed by Hansen and Lunde (2006), who employ the Bartlett kernel to take account of the autocorrelation in intraday returns caused by microstructure noise. We analyze whether using this method may improve the performance of option pricing. Second, the Tokyo stock exchange, where the underlying asset of the Nikkei 225 stock index options are traded, opens only for 9:00–11:00 and 12:30–15:00. We cannot obtain high-frequency returns during the period when the market is closed. Adding the squares of overnight (15:00-9:30) and lunch-time (11:00-12:30) returns may make realized volatility noisy. Following Hansen and Lunde (2005a), we calculate realized volatility without overnight and lunch-time returns and multiply a constant such that the sample mean of daily realized volatility is equal to the sample variance of daily returns. We examine whether this method is effective in option pricing by comparing with simply adding the squares of overnight and lunch-time returns.

Many authors have documented that realized volatility follows a long-memory process (Andersen et al., 2001, 2003). We use the ARFIMA (autoregressive fractionally integrated moving average) model to describe the dynamics of realized volatility. It is also well known in stock markets that today's volatility is negatively correlated with yesterday's return. We also extend the ARFIMA model to take account of this asymmetry in volatility.

For ARCH type models, we use the simple GARCH model proposed by Bollerslev (1986), the EGARCH (exponential GARCH) model by Nelson (1991) that may capture the asymmetry in volatility and the FIEGARCH (fractionally integrated EGARCH) model by Bollerslev and Mikkelsen (1996) that may allow for the long-memory property of volatility.

We calculate option prices under the assumption of risk neutrality. Duan (1995) has developed a more general method for pricing options in ARCH type models, which does not assume risk neutrality. We also calculate option prices both assuming the risk neutrality and by using the Duan (1995) method.

Main findings are: (1) the ARFIMAX model with daily realized volatility performs best, (2) applying the Bartlett adjustment to the calculation of realized volatility to take account of microstructure noise does not improve the performance while the Hansen and Lunde (2005a) adjustment without using overnight and lunch-time returns improves the performance, and (3) the Duan (1995) method does not improve the performance compared with assuming the risk neutrality.

The article proceeds as follows. Section 2 explains realized volatility and ARFIMA(X) model to describe its dynamics. Section 3 explains ARCH type models used in this article. Section 4 explains how to calculate option prices using ARFIMA(X) model with daily realized volatility and ARCH type models with daily returns. Section 5 explains the data and Section 6 compares the performance of option pricing. Section 7 concludes.

2. Realized Volatility and ARFIMA(X) Model

We start with a brief review of realized volatility using the following diffusion process.

$$dp(s) = \mu(s)ds + \sigma(s)dW(s), \tag{1}$$

where s is time, $p(s)$ is the log-price, $W(s)$ is a standard Brownian motion, and $\mu(s)$ and $\sigma(s)$ are the drift and the volatility respectively, which may be time-varying but are assumed to be independent of $dW(s)$. In this article, we call $\sigma(s)$ or $\sigma^2(s)$ volatility interchangeably although $\sigma(s)$ is usually called volatility in the finance literature.

Then, the volatility for day t is defined as the integral of $\sigma^2(s)$ over the interval $(t-1, t)$ where $t-1$ and t represent the market closing time on day $t-1$ and t respectively, i.e.,

$$IV_t = \int_{t-1}^{t} \sigma^2(s) ds, \tag{2}$$

which is called integrated volatility.

The integrated volatility is unobservable, but if we have the intraday return data $\left(r_{t-1+1/n}, r_{t-1+2/n}, \ldots, r_t\right)$, we can estimate it as the sum of their squares

$$RV_t = \sum_{i=1}^{n} r_{t-1+i/n}^2, \tag{3}$$

which is called realized volatility. If the prices do not include any noise, realized volatility RV_t will provide a consistent estimate of IV_t, i.e.,

$$\operatorname*{plim}_{n\to\infty} RV_t = IV_t. \tag{4}$$

There are two problems in measuring realized volatility using equation (3). One problem is the presence of the microstructure noise in transaction prices. If there presents microstructure noise, equation (4) may not be true. The other problem is the presence of non-trading hours. The Tokyo Stock Exchange is open only for 9:00–11:00 (morning session) and 12:30–15:00 (afternoon session) except for the first and last trading days in every year, when it is open only for 9:00-11:00. It is impossible to obtain high-frequency returns for 15:00–9:00 (overnight) and 11:00–12:30 (lunch-time). We calculate several realized volatilities using different methods for dealing with overnight and lunch-time returns and microstructure noise and compare the results.

Suppose that we have overnight return $r_{t-1+\Delta^{(n)}}$, $n^{(m)}$ returns in the morning session $(r_{t-1+\Delta^{(n)}+\Delta}, \ldots, r_{t-1+\Delta^{(n)}+n^{(m)}\Delta})$, lunch time return $r_{t-1+\Delta^{(n)}+n^{(m)}\Delta+\Delta^{(l)}}$ and $n^{(a)}$ returns in the afternoon session $(r_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+1)\Delta}, \ldots, r_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+n^{(a)})\Delta})$ where $\Delta^{(n)}$, $\Delta^{(l)}$ and Δ denote the time interval for overnight, lunch-time and returns when the market is open and $\Delta^{(n)} + \Delta^{(l)} + (n^{(m)} + n^{(a)})\Delta = 1$. We first calculate realized volatility by summing the squares of all these returns.

$$
\begin{aligned}
RVN_t \quad = \quad & r^2_{t-1+\Delta^{(n)}} + \sum_{i=1}^{n^{(m)}} r^2_{t-1+\Delta^{(n)}+i\Delta} + r^2_{t-1+\Delta^{(n)}+n^{(m)}\Delta+\Delta^{(l)}} \\
& + \sum_{j=1}^{n^{(a)}} r^2_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+j)\Delta}. \qquad (5)
\end{aligned}
$$

This realized volatility may be subject to discretization error because the time interval $\Delta^{(n)}$ and $\Delta^{(l)}$ are long. Hansen and Lunde (2005a) propose to calculate realized volatility only when the market is open, which is denoted as $RV_t^{(o)}$, and multiply a constant c such that the sample mean of realized volatility is equal to the sample variance of daily returns, i.e.,

$$
RVHL_t = cRV_t^{(o)}, \quad c = \frac{\sum_{t=1}^T (R_t - \overline{R})^2}{\sum_{t=1}^T RV_t^{(o)}}, \qquad (6)
$$

where $(R_1, \ldots, R_T)$ is the sample of daily returns and $\overline{R}$ is the sample mean[1].

We calculate $RV_t^{(o)}$ in the following two ways. Omitting the microstructure noise, we first calculate $RV_t^{(o)}$ simply by adding the squares of returns when the market is open.

$$
RV_t^{(o)} = \sum_{i=1}^{n^{(m)}} r^2_{t-1+\Delta^{(n)}+i\Delta} + \sum_{j=1}^{n^{(a)}} r^2_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+j)\Delta}. \qquad (7)
$$

The obtained $RV_t^{(o)}$ is adjusted to $RVHL_t$ using equation (6).

The microstructure noise may produce autocorrelation in intraday returns. To take this autocorrelation into account, we also use the Bartlett adjustments to calculate $RV_t^{(o)}$.

$$
\begin{aligned}
& RV_t^{(o)}(q) \\
= \quad & \sum_{i=1}^{n^{(m)}} r^2_{t-1+\Delta^{(n)}+i\Delta} + 2\sum_{k=1}^{q} \omega_k \sum_{i=1}^{n^{(m)}-k} r_{t-1+\Delta^{(n)}+i\Delta} r_{t-1+\Delta^{(n)}+(i+k)\Delta} \\
& + \sum_{j=1}^{n^{(a)}} r^2_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+j)\Delta} \\
& + 2\sum_{l=1}^{q} \omega_l \sum_{j=1}^{n^{(a)}-l} r_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+j)\Delta} r_{t-1+\Delta^{(n)}+\Delta^{(l)}+(n^{(m)}+j+l)\Delta}, \qquad (8)
\end{aligned}
$$

where ω_j is the Bartlett coefficient defined as

$$
\omega_j = 1 - \frac{j}{q+1}, \quad j = 1, \ldots, q. \qquad (9)
$$

[1] See Martens (2002) and Hansen and Lunde (2005b) for the other methods.

Following equation (6), we adjust $RV_t^{(o)}(q)$ as follows.

$$RVHL_t(q) = cRV_t^{(o)}(q), \quad c = \frac{\sum_{t=1}^{T}(R_t - \overline{R})^2}{\sum_{t=1}^{T} RV_t^{(o)}(q)}. \tag{10}$$

We set $q = 1, 2$. In sum, we calculate four different realized volatilities: RVN, $RVHL$, $RVHL(1)$ and $RVHL(2)$.

Many researchers have documented that realized volatility may follow a long-memory process. Let $\rho(h)$ denote the h-th order autocorrelation coefficient of variable X. Then, X follows a short-memory process if $\sum_{h=0}^{\infty} |\rho(h)| < \infty$ and a long-memory process if $\sum_{h=0}^{\infty} |\rho(h)| = \infty$. A stationary ARMA model is a short-memory process. As h increases, the autocorrelation coefficient $\rho(h)$ of the long-memory process decays more slowly than that of the short-memory process. More specifically, the former decays hyperbolically and the latter decays geometrically.

The most widely used for a long-memory process is ARFIMA(p, d, q) model[2]

$$\phi(L)(1 - L)^d X_t = \theta(L) u_t, \quad u_t \sim NID(0, \sigma^2), \tag{11}$$

where L denotes the lag operator and $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ are the p-th and q-th order lag polynomials assumed to have all roots outside the unit circle. The order of integration d is allowed to take non-integer values. If $d = 0$, ARFIMA model collapses to stationary ARMA model and if $d = 1$, it becomes non-stationary ARIMA model. If $0 < d < 0.5$, X_t follows a stationary long-memory process and if $0.5 \leq d < 1$, X_t follows a non-stationary long-memory process. $(1 - L)^d$ may be written as follows.

$$(1 - L)^d = 1 + \sum_{k=1}^{\infty} \frac{d(d-1)\cdots(d-k+1)}{k!}(-L)^k. \tag{12}$$

We assume that u_t follows an independent normal distribution with zero mean and variance σ^2.

By setting $p = 0$ and $q = 1$, which are selected by SIC, and $X_t = \ln(RV_t) - \mu$ where μ is the unconditional mean of $\ln(RV_t)$, we consider the following model.

$$(1 - L)^d [\ln(RV_t) - \mu] = u_t + \theta u_{t-1}, \quad u_t \sim NID(0, \sigma^2). \tag{13}$$

We estimate parameters d, μ and θ jointly using the approximate maximum likelihood method (Beran, 1995), where it is assumed that $\ln(RV_t) = \mu$ $(t = 0, -1, \ldots)$. We can estimate σ^2 as the sample variance of residual.

[2] See Beran (1994) for the details of long-memory and ARFIMA model.

It is well-known that there is a negative correlation between today's return and tomorrow's volatility in stock markets. To take into account this phenomenon, we extend the above ARFIMA(0,d,1) model (13) to the following ARFIMA(0,d,1)-X model.

$$(1-L)^d \left[\ln(RV_t) - \mu_0 - \mu_1|R_{t-1}| - \mu_2 D^-_{t-1}|R_{t-1}|\right] = u_t + \theta u_{t-1}, \quad u_t \sim NID(0, \sigma^2), \quad (14)$$

where D^-_{t-1} is a dummy variable that takes one if the return on day $t-1$ is negative and zero otherwise.

We estimate parameters d, μ_0, μ_1, μ_2, θ and σ^2 using the same method as that for ARFIMA model. If the estimate of μ_2 has a statistically significant positive value, it is consistent with a well-known negative correlation between today's return and tomorrow's volatility in stock markets.

3. ARCH Type Models

We also estimate ARCH type models using daily returns. We define daily return as

$$R_t = \ln(S_t) - \ln(S_{t-1}), \quad (15)$$

where S_t is the closing price on day t.

We specify daily return as

$$R_t = E(R_t|\boldsymbol{I}_{t-1}) + \epsilon_t, \ \epsilon_t = \sigma_t z_t, \ z_t \sim NID(0,1), \quad (16)$$

where $E(R_t|\boldsymbol{I}_{t-1})$ is the expectation of R_t conditional on the information up to day $t-1$ and z_t is assumed to follow an independent standard normal distribution. Then, σ_t^2 is the variance of R_t conditional on the information up to day $t-1$. We will explain how to specify $E(R_t|\boldsymbol{I}_{t-1})$ later.

For volatility specification, we use three different ARCH type models. First is the GARCH model proposed by Bollerslev (1986). Specifically, we use the GARCH$(1,1)$ model

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2, \ \omega > 0, \ \beta, \alpha \geq 0, \quad (17)$$

where ω, β and α are parameters, which are assumed to be non-negative to guarantee that volatility is always positive. This model can capture the volatility clustering. Volatility is stationary if $|\beta + \alpha| < 1$, and the speed for which the shock to volatility decays becomes slower as $\beta + \alpha$ approaches to one.

As has already been mentioned, another well-known phenomenon in stock markets is volatility asymmetry, which cannot be captured by the above GARCH model. To capture this phenomenon, we also use the EGARCH model proposed by Nelson (1991). Specifically, we use the EGARCH$(1,0)$ model

$$\ln(\sigma_t^2) \quad = \quad \omega + \phi\left[\ln(\sigma_{t-1}^2) - \omega\right] + \theta z_{t-1} + \gamma\left(|z_{t-1}| - E\,|z_{t-1}|\right), \quad |\phi| < 1. \tag{18}$$

While the GARCH model specifies the process of σ_t^2, the EGARCH model specifies that of its logarithm. Thus, it does not require non-negativity constraints for parameters. If $\theta < 0$, it is consistent with the volatility asymmetry in stock markets. In this model, volatility is stationary if $|\phi| < 1$, and the speed for which the shock to volatility decays becomes slower as ϕ approaches to one. Since z_{t-1} is assumed to follow the standard normal distribution, $E\,|z_{t-1}| = \sqrt{2/\pi}$.

Neither GARCH nor EGARCH models allow volatility to have long-memory property. Hence, we also use the FIEGARCH model proposed by Bollerslev and Mikkelsen (1996). Since this model is an extension of the above EGARCH model to allow the long-memory of volatility, it can also capture the volatility asymmetry. We use the following FEGARCH$(1,d,0)$ model.

$$(1-\phi L)(1-L)^d\left[\ln(\sigma_t^2) - \omega\right] = \theta z_{t-1} + \gamma\left(|z_{t-1}| - E\,|z_{t-1}|\right), \quad |\phi| < 1. \tag{19}$$

Similarly to the EGARCH model, it is consistent with the volatility asymmetry in stock markets if $\theta < 0$. As for d, the same argument as that for the ARFIMA model holds.

FIGARCH (Baillie et al., 1996) and FIAPGARCH (Tse, 1998) models can also take into account the possibility that the volatility follows a long-memory process. These models, however, has some drawbacks. First, the variance of return will be infinite even though $0 < d < 0.5$ (Schoffer, 2003). Second, the parameter constraints to guarantee that the volatility is always positive are complicated (Conrad and Haag, 2006). Thus, we do not use these models in this article.

We estimate parameters in GARCH, EGARCH and FIEGARCH models using the maximum likelihood method[3].

[3]See Taylor (2001) for the estimation method for FIEGARCH model.

4. Option Pricing

We first calculate option prices under the assumption of risk neutrality. If the traders are risk neutral, the expected return may be represented by

$$E(R_t|\boldsymbol{I}_{t-1}) = r - d - \frac{1}{2}\sigma_t^2, \tag{20}$$

where r and d are continuously compounded risk-free rate and dividend rate.

The price of European option will be equal to the discounted present value of the expectation of option prices on the expiration date. For example, the price of European put option with the exercise price K and the maturity τ is given by

$$P_T = \exp(-r\tau)E\left[\text{Max}(K - \tilde{S}_{T+\tau}, 0)|\boldsymbol{I}_T\right], \tag{21}$$

where $\tilde{S}_{T+\tau}$ is the price of the underlying asset on the expiration date $T+\tau$.

We cannot evaluate this expectation analytically if the volatility of the underlying asset follows ARFIMA(X) or ARCH type models. We calculate option prices by simulating $\tilde{S}_{T+\tau}$ from ARFIMA(X) or ARCH type models. Suppose that $(S_{T+\tau}^{(1)}, \ldots, S_{T+\tau}^{(m)})$ are simulated. Then, (21) may be calculated as follows.

$$P_T \approx \exp(-r\tau)\frac{1}{m}\sum_{i=1}^{m}\text{Max}(K - S_{T+\tau}^{(i)}, 0). \tag{22}$$

We set $m = 10000$. For variance reduction, we used the control variate and the Empirical Martingale Simulation proposed by Duan and Shimonato (1998) jointly.

Duan (1995) relaxed the assumption of risk neutrality to derive option prices when the price of underlying asset follows ARCH type models. We also use this method. Following Duan (1995), we set

$$E(R_t|\boldsymbol{I}_{t-1}) = r - d - \frac{1}{2}\sigma_t^2 + \lambda\sigma_t, \tag{23}$$

where $\lambda\sigma_t$ captures the risk premium.

Unless the traders are risk neutral, we must convert the physical measure P into the risk neutral measure Q and evaluate the expectation in equation (21) under the risk neutral measure Q. Duan (1995) makes the following assumptions on Q, called local risk-neutral valuation relationship (LRNVR).

1. $R_t|I_{t-1}$ follows a normal distribution under the risk neutral measure Q.

2. $E^Q[\exp(R_t)|I_{t-1}] = \exp(r-d)$.

3. $Var^Q[R_t|I_{t-1}] = Var^P[R_t|I_{t-1}]$ a.s.

Under assumptions 1 and 2, daily returns under the risk neutral measure Q must be represented by

$$R_t = r - d - \frac{1}{2}\sigma_t^2 + \xi_t, \quad \xi_t = \sigma_t v_t, \quad v_t \sim NID(0,1). \tag{24}$$

Comparing equation (24) with equations (16) and (23) leads to

$$\epsilon_t = \xi_t - \lambda\sigma_t, \tag{25}$$

$$z_t = v_t - \lambda. \tag{26}$$

Since assumption 3 means that volatilities are the same between P and Q, all we have to do for volatility is to substitute equations (25) or (26) into ϵ_t in the GARCH volatility equation or z_t in the EGARCH and FIEGARCH volatility equations. For example, the GARCH(1,1) volatility equation will be

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha(\xi_{t-1} - \lambda\sigma_{t-1})^2, \ \omega > 0, \ \beta, \alpha \geq 0. \tag{27}$$

Equations (24) and (27) constitute the GARCH(1,1) model under Q. Hence, we can evaluate the option prices as follows.

[1] Estimate the parameters λ, ω, β and α in the GARCH(1,1) model under P that consists of equations (16), (23) and (17).

[2] Simulate $\tilde{S}_{T+\tau}$ using the GARCH(1,1) model under Q that consists of equations (24) and (27) by setting the parameters λ, ω, β and α equal to their estimates in [1].

[3] Substitute $(S_{T+\tau}^{(1)}, \ldots, S_{T+\tau}^{(m)})$ simulated in [2] into equation (22) to obtain the option price.

Similarly, we can calculate the option price using the EGARCH and FIEGARCH models. The EGARCH(1,0) and FIEGARCH(1,d,0) volatility equations under Q will be

$$\ln(\sigma_t^2) = \omega + \phi\left[\ln(\sigma_{t-1}^2) - \omega\right] + \theta(v_{t-1} - \lambda) + \gamma\left(|v_{t-1} - \lambda| - \sqrt{2/\pi}\right), \tag{28}$$

$$(1 - \phi L)(1 - L)^d\left[\ln(\sigma_t^2) - \omega\right] = \theta(v_{t-1} - \lambda) + \gamma\left(|v_{t-1} - \lambda| - \sqrt{2/\pi}\right). \tag{29}$$

For comparison, we also calculate option prices using the Black-Scholes formula with volatility σ as the standard deviation of daily returns over the past 20 days.

5. Data

We analyze the Nikkei 225 stock index options traded at Osaka Securities Exchange. The underlying asset is the Nikkei 225 stock index, which is the average of the prices of 225 representative stocks traded at Tokyo Stock Exchange. The sample period is from May 29, 1996 to September 27, 2007. Following equation (15), we calculate the daily returns for the underlying asset as the log-difference of the closing prices of the Nikkei 225 index in consecutive days. Table 1(a) summarizes the descriptive statistics of the daily returns (%) for the full sample. The mean is not significantly different from zero. While the skewness is not significantly different from zero, the kurtosis is significantly above 3, indicating the well-known phenomenon that the distribution of the daily return is leptokurtic. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags. According to this statistic, the null hypothesis is not rejected at the 1% significance level although it is rejected at the 5% level. We do not consider autocorrelations in the daily return in the following analyses.

We calculate realized volatility using the Nikkei NEEDS-TICK data. This dataset includes the Nikkei 225 stock index for every minute from 9:01 to 11:00 in the morning session and from 12:31 to 15:00 in the afternoon session. Sometimes, the time stamps for the closing prices in the morning and afternoon sessions are slightly after 11:00 and 15:00 because the recorded time shows when the Nikkei 225 stock index is calculated. In such cases, we use all prices up to closing prices. Using these one-minute prices, we calculate four different realized volatilities RVN, $RVHL$, $RVHL(1)$ and $RVHL(2)$ defined by equations (5)–(10), where the adjustment coefficient c defined by equation (6) or (10) is calculated using the full sample.

Figure 1 plots these realized volatilities and Table 2 summarizes their descriptive statistics. The means of $RVHL$, $RVHL(1)$ and $RVHL(2)$ are the same because they are adjusted such that the mean of realized volatility is equal to the sample variance of daily returns. RVN is not adjusted and its mean is much lower than those of the others. Among $RVHL$, $RVHL(1)$ and $RVHL(2)$, $RVHL$ has the smallest standard deviation and $RVHL(2)$ has the largest standard deviation, but the difference is small. The standard deviation of RVN is much smaller than those of $RVHL$, $RVHL(1)$ and $RVHL(2)$. These results are confirmed by Figure 1. Figure 1(a) shows that the difference among $RVHL$, $RVHL(1)$ and $RVHL(2)$ is small while Figure 1(b) shows that $RVHL$ is larger on average and more volatile than RVN. The values of skewness and kurtosis indicate that the distributions of all realized volatilities are non-normal. LB(10) is so large that the null hypothesis of no autocorrelation is rejected. Table 1 (c) shows the descriptive statistics for log-realized volatilities. They are qualitatively the same as those of Table 1 (b) except skewness and kurtosis. While realized volatilities are positively skewed, log-realized volatilities are negatively skewed. The kurtosis of log-realized volatilities is much smaller than those of realized volatilities. The kurtosis of $\ln(RVHL)$, $\ln(RVHL(1))$ and $\ln(RVN)$ is not significantly above 3 at the 5% level. The distributions of log-realized volatilities are much closer to the normal distribution than those of realized volatilities. Thus, we use log-realized volatility as a dependent variable in ARFIMA model (13) and ARFIMAX model (14).

To measure the performance of option pricing, we also use prices of Nikkei 225 stock index options traded at Osaka Securities Exchange. Nikkei 225 stock index options are European options and their maturities are the trading days previous to the second Friday every month. For the Nikkei 225 stock index options, put options are traded more heavily than call options and the options with the maturity more than one month are not traded so much. Thus we concentrate on put options whose maturity is 30 days (29 days if the day when the maturity is 30 days is a weekend or holiday). On such days, we consider all put options with different exercise prices whose bid and ask prices are both available at the same time between 14:00 and 15:00. For each option, we use the average of bid and ask prices at the same time closest to 15:00 as the market price at 15:00. The reason why we use the average of bid and ask prices instead of transaction prices is that transaction prices are subject to market microstructure noise due to bid-ask bounce (Campbell et al., 1997).

We estimate ARFIMA and ARFIMAX models using 1200 daily realized volatilities ($RVHL$, $RVHL(1)$, $RVHL(2)$ and RVN) up to the day before the options whose maturity is one month are traded, where the adjustment coefficient c defined by equation (6) or (10) is calculated using the same 1200 realized volatilities with 1200 daily returns. We also estimate ARCH type models using the same 1200 daily returns with risk-free rate and dividend. As mentioned, the daily returns are calculated as the log difference of closing prices. We use CD rate as a risk-free rate and fix the annual dividend rate as 0.5% following Nishina and Nabil (1997). The first date when options whose maturity is one month are traded is April 11, 2001. We first estimate the parameters in the ARFIMA(X) and ARCH type models using 1200 daily realized volatilities and returns up to April 10, 2001, where we calculate the adjustment coefficient c using the same 1200 daily realized volatilities and returns. Then, given the obtained parameter estimates, we calculate the put option prices on April 11, 2001 using CD rate and the Nikkei 225 index at 15:00 on that date. The next date when options whose maturity is one month are traded is May 9, 2001. We first estimate the parameters in the ARFIMA(X) and ARCH type models using 1200 daily realized volatilities and returns up to May 8, 2001, where we calculate the adjustment coefficient c using the same 1200 daily realized volatilities and returns. Then, given the obtained parameter estimates, we calculate the put option prices on May 9, 2001 using CD rate and the Nikkei 225 index at 15:00 on that date. We repeat this procedure up to September 2007.

Figures 2 (a)–(d) plot the estimates of all parameters in all models for each of the above 78 iterations. Figures 2 (a) and (b) plot the estimates of parameters in ARFIMA and ARFIMAX models using $RVHL$ defined by equations (6) and (7). The estimates of d in ARFIMA and ARFIMAX models move around 0.5 and are above 0.5 in the latter half, indicating the long-memory and the possibility of non-stationarity of log-realized volatility. The estimates of μ_2 in ARFIMAX model are positive for all periods, indicating the well-known phenomenon of a negative correlation between today's return and tomorrow's volatility. Figures 2 (c)–(e) plot the estimates of parameters in ARCH type models using daily returns. The sum of the estimates of β and α in the GARCH model and the estimates of ϕ in the EGARCH model are close to 1 for all periods, indicating the well-known phenomenon of volatility clustering. These models, however, do not allow for the long-memory of volatility. The estimates of d in FIEGARCH model are more volatile than those of ARFIMA(X) model. They move around 0.5 in the first half while they move up to 0.54 and down to 0 in the latter half. These results provide evidence that a structural change may occur during our sample period, but we leave it for the future research. The estimates of θ in EGARCH and FIEGARCH models are negative for all periods, indicating a negative correlation between today's return and tomorrow's volatility.

6. Results

To measure the performance of option pricing, we use four loss functions, MAE (Mean Absolute Error), RMSE (Root Mean Square Error), MAPE (Mean Absolute Percentage Error) and RMSPE (Root Mean Square Percentage Error) defined as

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}\left|\tilde{P}_i - P_i\right|, \quad \text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\tilde{P}_i - P_i\right)^2},$$

$$\text{RMAPE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\tilde{P}_i - P_i}{\tilde{P}_i}\right|, \quad \text{RMSPE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\tilde{P}_i - P_i}{\tilde{P}_i}\right)^2}.$$

where N is the number of put options used for evaluating the performance, $\tilde{P}_i$ is the price of the ith put option calculated by each model and P_i is its market price.

Following Bakshi, Cao and Chen (1997), we classify put options into four categories such as DITM (deep-in-the-money), ITM (in-the-money), ATM (at-the-money), OTM (out-of-the-money) and DOTM (deep-out-of-the-money) using the moneyness which is the ratio of the underlying asset price over the exercise price. Table 3 shows this classification. We examine the performance in each category as well as in total.

Table 4 shows the values of loss functions for ARCH type models with daily returns, ARFIMA(X) models with $RVHL$, and BS model. In total, the ARFIMAX model performs best no matter which loss function is used. It is also true for ATM and ITM. In OTM, ARFIMAX model performs best for RMSE while ARFIMA model performs best for the other loss functions. In DOTM, ARFIMAX model performs best for RMSPE and MAPE while FIEGARCH model performs best for the other loss functions. In DITM, GARCH or EGARCH models perform best. Although there are some exceptions depending on moneyness and loss function, we may conclude that the ARFIMAX model performs best.

Table 5 shows the values of loss functions for ARFIMAX model with four different realized volatilities $RVHL$, $RVHL(1)$, $RVHL(2)$ and RVN. In total, $RVHL$ is the best for RMSE, MAE and RMSPE while $RVHL(1)$ is the best for MAPE but the difference between $RVHL$ and $RVHL(1)$ is minor. It is true for most loss functions and moneyness. This result means that the Bartlett adjustment to take account of microstructure noise does not improve the performance of option pricing. On the other hand, in total, the loss functions of RVN have much larger values than those of $RVHL$, $RVHL(1)$ and $RVHL(2)$ and it is true for some loss functions and moneyness. This result means that it is better to use the Hansen and Lunde (2005a) adjustment without adding the squares of overnight and lunch-time returns.

So far, we assumed the risk neutrality. As explained in Section 4, Duan (1995) has proposed a method for GARCH option pricing relaxing this assumption. We also apply this method to GARCH, EGARCH and FIEGARCH models. Table 6 shows the result. The values of loss functions using this method are not different so much from those assuming the risk neutrality. This result means that the Duan (1995) method does not improve the performance of option pricing compared with assuming the risk neutrality.

7. Conclusions

This article compares the performance of option pricing among the ARFIMA(X) model with daily realized volatility and ARCH models with daily returns. Main results are: (1) the ARFIMAX model with daily realized volatility performs best, (2) the Bartlett adjustment to take account of microstructure noise does not improve the performance while the Hansen and Lunde (2005a) adjustment without adding the squares of overnight and lunch-time returns improves the performance, and (3) the Duan (1995) method does not improve the performance compared with assuming the risk neutrality.

Several extensions are possible. First, we only used the Bartlett adjustment to take account of microstructure noise. Some other methods are available such as the selection of optimal time interval of intraday returns (Aït-Sahalia et al., 2005; Bandi and Russell, 2006, 2008) and two (multi) scale estimator (Zhang et al., 2005; Zhang, 2006) and the resampling (Barndorff-Nielsen et al., 2007). It is worthwhile applying these methods and comparing the results. Second, we did not consider jumps in returns. It is important to take account of jumps to calculate realized volatility (Barndorff-Nielsen and Shephard, 2002a, 2004). Third, we only used ARFIMA(X) model for realized volatility. There are other models available such as HAR (heterogeneous interval autoregressive) model (Corsi, 2004) and UC (unobserved components) model (Barndorff-Nielsen et al., 2004; Barndorff-Nielsen and Shephard, 2001, 2002b; Nagakura and Watanabe, 2009). It is worthwhile applying these models and comparing the performance.

Acknowledgements

Financial support from the Ministry of Education, Culture, Sports, Science and Technology of the Japanese Government through Grant-in-Aid for Scientific Research (No.18203901) and the Global COE program "Research Unit for Statistical and Empirical Analysis in Social Sciences" at Hitotsubashi University is gratefully acknowledged. Thanks are also due to the participants in Conference "Recent Developments in Finance and Econometrics" at Ryukyu University and APFA7 & Tokyo Tech-Hitotsubashi Interdisciplinary Conference "New Approaches to the Analysis of Large-Scale Business and Economic Data" at Tokyo Institute of Technology for valuable comments. All remaining errors are solely our responsibility.

References

Aït-Sahalia, Y., Mykland, P.A., Zhang, L., 2005. How often to sample a continuous-time process in the presence of market microstructure noise. Review of Financial Studies 18, 351–416.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics 61, 43–76.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579–625.

Baillie, R.T., Bollerslev, T., Mikkelsen, H.O., 1996. Fractionally integrated generalized autoregressive conditionally heteroskedasticity. Journal of Econometrics 74, 3–30.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

Bandi, F.M., Russell, J.R., 2006. Separating microstructure noise from volatility. Journal of Financial Economics 79, 655–692.

Bandi, F.M., Russell, J.R., 2008. Microstructure noise, realized variance, and optimal sampling. Review of Economic Studies 75, 339–369.

Barndorff-Nielsen, O.E., Nielsen, B., Shephard, N., Ysusi, C., 2004. Measuring and forecasting financial variability using realized variance. In: Harvey, A.C., Koopman S.J., Shephard N. (Eds.), State Space and Unobserved Components Models: Theory and Applications. Chapter 10, pp.205–235. Cambridge University Press, Cambridge.

Barndorff-Nielsen, O.E., Shephard, N., 2001. Non-Gaussian OU based models and some of their uses in financial economics (with discussion). Journal of the Royal Statistical Society. Series B 63, 167–241.

Barndorff-Nielsen, O.E., Shephard, N., 2002a. Estimating quadratic variation using realized volatility. Journal of Applied Econometrics 17, 457–477.

Barndorff-Nielsen, O.E., Shephard, N., 2002b. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society. Series B 64, 253–280.

Barndorff-Nielsen, O.E., Shephard, N., 2004. Power and bipower variation with stochastic volatility and jumps (with discussion). Journal of Financial Econometrics 2, 1–37, Cambridge University Press.

Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A., Shephard, N., 2007. Subsampling realized kernels. Available at SSRN: http//ssrn.com/abstract=927483.

Beran, J., 1994. Statistics for Long-Memory Processes. Chapman & Hall, New York.

Beran, J., 1995. Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models. Journal of the Royal Statistical Society. Series B 57, 659–672.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–654.

Bollerslev, T., 1986, Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307–327.

Bollerslev, T., Mikkelsen, H.O., 1996. Modeling and pricing long memory in stock market volatility. Journal of Econometrics 73, 151–184.

Bollerslev, T., Mikkelsen, H.O., 1999. Long-term equity anticipation securities and stock market volatility dynamics. Journal of Econometrics 92, 75–99.

Campbell, J.Y., Lo, A.W., MacKinlay, A.C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton.

Clements, M.P., Galvão, A.B. and Kim, J.H., 2008. Quantile forecasts of daily exchange rate returns from forecasts of realized volatility. Journal of Empirical Finance 15, 729–750.

Conrad, C., Haag, B.R., 2006. Inequality constraints in the fractionally integrate GARCH model. Journal of Financial Econometrics 4, 413–449.

Corsi, F., 2004. A simple long memory model of realized volatility. Available at SSRN: http://ssrn.com/abstract=626064.

Diebold, F.X., 1988. Empirical Modeling of Exchange Rate Dynamics. Springer-Verlag, Berlin.

Duan, J.-C., 1995. The GARCH option pricing model. Mathematical Finance 5, 13–32.

Duan, J.-C., Shimonato, J.-G., 1998. Empirical Martingale simulation for asset prices. Management Science 44, 1218–1233.

Engle, R.F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007.

Giot, P., Laurent, S., 2004. Modelling daily value-at-risk using realized volatility and ARCH type models. Journal of Empirical Finance 11, 379–398.

Hansen, P.R., Lunde, A., 2005a. A forecast comparison of volatility models: does anything beat a GARCH(1,1)? Journal of Applied Econometrics 20, 873–889.

Hansen, P.R., Lunde, A., 2005b. A realized variance for the whole day based on intermittent high-frequency data. Journal of Financial Econometrics 3, 525–554.

Hansen, P.R., Lunde, A., 2006. Realized variance and market microstructure noise. Journal of Business & Economic Statistics 24, 127–161.

Koopman, S.J., Jungbacker, B., Hol, E., 2005. Forecasting daily variability of the S&P 100 stock index using historical, realized and implied volatility measurements. Journal of Empirical Finance 12, 445–475.

Martens, M., 2002. Measuring and forecasting S&P 500 index-futures volatility using high-frequency data. Journal of Futures Markets 22, 497–518.

Nagakura, D., Watanabe, T., 2009. A state space approach to estimating integrated variance and microstructure noise component. Global COE High-Stat Discussion Paper Series 055, Hitotsubashi University.

Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: a new approach. Econometrica 59, 247–370.

Nishina, K., Nabil, M.N., 1997. Return dynamics of Japanese stock index options. Japanese Economic Review 48, 43–64.

Schoffer, O., 2003. HY-A-PARCH: A stationary A-PARCH model with long memory. Technical Report 4003, Fachbereich Statitik, Universität Dortmunt.

Taylor, S.J., 2001. Consequences for option pricing for a long memory in volatility. Working Paper 2001/017, Lancaster University Management School.

Tse, Y.K., 1998. The conditional heteroskedasticity of the Yen-Dollar exchange rate. Journal of Applied Econometrics 13, 49–55.

Zhang, L., 2006. Efficient estimation of stochastic volatility using noisy observations: a multi-scale approach. Bernoulli 12, 1019–1043.

Zhang, L., Mykland, P.A., Aït-Sahalia, Y., 2005. A tale of two time scales: determining integrated volatility with noisy high-frequency data. Journal of the American Statistical Association 100, 1394–1411.

Table 1

Descriptive statistics of daily returns

Mean	-0.0095
	(0.0270)
Standard Deviation	1.4261
Min	-7.2340
Max	7.6605
Skewness	-0.0616
	(0.0464)
Kurtosis	4.9003
	(0.0927)
LB(10)	18.69

The numbers in parentheses are standard errors. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags.

Table 2

Descriptive statistics of daily realized volatilities

(a) Daily realized volatilities

	$RVHL$	$RVHL(1)$	$RVHL(2)$	RVN
Mean	2.0331	2.0331	2.0331	1.1366
	(0.0306)	(0.0314)	(0.0336)	(0.0163)
Standard Deviation	1.6144	1.6602	1.7760	0.8611
Min	0.1700	0.1389	0.0948	0.0774
Max	26.2450	23.6369	21.8362	11.5341
Skewness	3.5715	3.2168	3.2811	2.8283
	(0.0464)	(0.0464)	(0.0464)	(0.0464)
Kurtosis	33.9767	24.2832	22.6734	21.3087
	(0.0927)	(0.0927)	(0.0927)	(0.0927)
LB(10)	1862.35	1880.76	1716.72	1737.20

$RVHL$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and without the Bartlett adjustment. $RVHL(q)$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and using the Bartlett adjustment with lag-length q. RVN is the realized volatility calculated simply using the squares of overnight returns and lunch-time returns without the Bartlett adjustment. The numbers in parentheses are standard errors. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags.

(b) Log daily realized volatilities

	$\ln(RVHL)$	$\ln(RVHL(1))$	$\ln(RVHL(2))$	$\ln(RVN)$
Mean	0.4594	0.4481	0.4214	−0.1099
	(0.0137)	(0.0140)	(0.0146)	(0.0163)
Standard Deviation	0.7230	0.7375	0.7725	0.7048
Min	−1.7722	−1.9743	−2.3555	−2.5592
Max	3.2675	3.1628	3.0836	2.4453
Skewness	−0.2055	−0.2010	−0.1915	−0.2098
	(0.0464)	(0.0464)	(0.0464)	(0.0464)
Kurtosis	3.0190	3.1633	3.2741	3.1395
	(0.0927)	(0.0927)	(0.0927)	(0.0927)
LB(10)	6347.39	5772.89	5271.76	4738.12

$RVHL$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and without the Bartlett adjustment. $RVHL(q)$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and using the Bartlett adjustment with lag-length q. RVN is the realized volatility calculated simply using the squares of overnight returns and lunch-time returns without the Bartlett adjustment. The numbers in parentheses are standard errors. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags.

Table 3

Moneyness of put options

	S/K	< 0.91	deep-in-the-money (DITM)
$0.91 <$	S/K	< 0.97	in-the-money (ITM)
$0.97 <$	S/K	< 1.03	at-the-money (ATM)
$1.03 <$	S/K	< 1.09	at-the-money (OTM)
$1.09 <$	S/K		deep-out-of-the-money (DOTM)

S =price of underlying asset and K =exercise price.

Table 4

Put option pricing performance using different models

	DOTM	OTM	ATM	ITM	DITM	Total
Sample size	269	102	115	99	145	730
RMSE						
GARCH	26.112	54.245	73.784	71.062	49.788*	51.935
EGARCH	23.727	57.713	77.483	69.895	50.204	52.743
FIEGARCH	22.388*	50.265	67.407	65.271	50.293	48.308
ARFIMA	26.129	48.442	64.183	66.471	51.232	48.466
ARFIMAX	25.513	47.465*	63.321*	65.245*	51.322	47.820*
BS	32.531	68.651	96.101	76.962	52.183	62.029
MAE						
GARCH	11.359	35.794	59.921	50.821	38.082*	33.083
EGARCH	11.561	43.465	65.403	50.182	38.164	35.022
FIEGARCH	9.839*	35.291	55.689	45.015	38.227	31.027
ARFIMA	10.700	26.777*	48.077	45.034	39.112	29.134
ARFIMAX	10.516	27.035	47.730*	44.224*	39.054	28.926*
BS	13.973	45.020	68.107	49.908	39.000	36.684
RMSPE						
GARCH	0.889	0.617	0.291	0.091	0.021	0.599
EGARCH	1.698	0.851	0.318	0.088	0.020*	1.086
FIEGARCH	1.516	0.643	0.268	0.081	0.021	0.958
ARFIMA	0.659	0.327*	0.207	0.078	0.022	0.427
ARFIMAX	0.643*	0.328	0.204*	0.076*	0.022	0.418*
BS	0.805	0.527	0.263	0.088	0.024	0.538
MAPE						
GARCH	0.665	0.430	0.218	0.065	0.015	0.351
EGARCH	1.036	0.603	0.244	0.063	0.015*	0.516
FIEGARCH	0.828	0.463	0.206	0.057	0.015	0.413
ARFIMA	0.578	0.248*	0.158	0.055	0.015	0.283
ARFIMAX	0.564*	0.253	0.157*	0.054*	0.015	0.278*
BS	0.729	0.442	0.209	0.061	0.016	0.375

This is calculated using $RVHL$ with the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and without the Bartlett adjustment. * indicates the best model which minimizes the loss function.

Table 5

Put option pricing performance using different realized volatilities

	DOTM	OTM	ATM	ITM	DITM	Total
Sample size	269	102	115	99	145	730
RMSE						
$RVHL$	25.513*	47.465*	63.321*	65.245*	51.322	47.820*
$RVHL(1)$	25.533	48.197	64.222	65.991	51.298*	48.161
$RVHL(2)$	26.099	50.656	65.546	67.926	51.626	49.622
RVN	33.917	72.502	75.034	69.487	51.395	56.787
MAE						
$RVHL$	10.516	27.035*	47.730*	44.224	39.054	28.926*
$RVHL(1)$	10.380*	27.295	48.332	44.767	39.031	29.077
$RVHL(2)$	10.502	28.708	49.845	46.265	39.315	29.817
RVN	16.995	48.187	48.753	39.513*	38.622*	33.706
RMSPE						
$RVHL$	0.643	0.328*	0.204	0.076	0.022	0.418*
$RVHL(1)$	0.639*	0.345	0.208	0.077	0.022	0.418
$RVHL(2)$	0.665	0.372	0.215	0.079	0.022	0.437
RVN	0.875	0.446	0.182*	0.074*	0.021*	0.562
MAPE						
$RVHL$	0.564	0.253*	0.157	0.054	0.015	0.278
$RVHL(1)$	0.551*	0.262	0.158	0.054	0.015*	0.275*
$RVHL(2)$	0.563	0.282	0.163	0.056	0.016	0.283
RVN	0.861	0.407	0.140*	0.046*	0.015	0.406

$RVHL$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and without the Bartlett adjustment. $RVHL(q)$ is the realized volatility calculated using the Hansen and Lunde (2005a) adjustment without overnight and lunch-time returns and using the Bartlett adjustment with lag-length q. RVN is the realized volatility calculated simply using the squares of overnight returns and lunch-time returns without the Bartlett adjustment. * indicates the best model which minimizes the loss function.

Table 6

Put option pricing performance of ARCH type models assuming the risk-neutrality and using the Duan (1995) method

	DOTM	OTM	ATM	ITM	DITM	Total
Sample size	269	102	115	99	145	730
RMSE						
GARCH						
Risk neutral	25.889	53.891	73.216	70.538	49.701	51.601
Duan	26.112	54.245	73.784	71.062	49.788	51.935
EGARCH						
Risk neutral	23.727	57.713	77.483	69.895	50.204	52.743
Duan	23.920	57.739	77.517	70.047	50.295	52.831
FIEGARCH						
Risk neutral	22.388	50.265	67.407	65.271	50.293	48.308
Duan	22.419	49.747	65.562	63.739	50.624	47.625
MAE						
GARCH						
Risk neutral	11.359	35.794	59.921	50.821	38.082	33.083
Duan	11.326	35.652	59.489	50.118	38.042	32.880
EGARCH						
Risk neutral	11.561	43.465	65.403	50.182	38.164	35.022
Duan	11.706	43.341	65.352	50.200	38.236	35.067
FIEGARCH						
Risk neutral	9.839	35.291	55.689	45.015	38.227	31.027
Duan	9.788	34.563	53.426	42.929	37.708	30.164

"Risk neutral" shows the results assuming the risk-neutrality, which are the same as those in Table 3. "Duan" shows the ones using the Duan (1995) method without assuming the risk-neutrality.

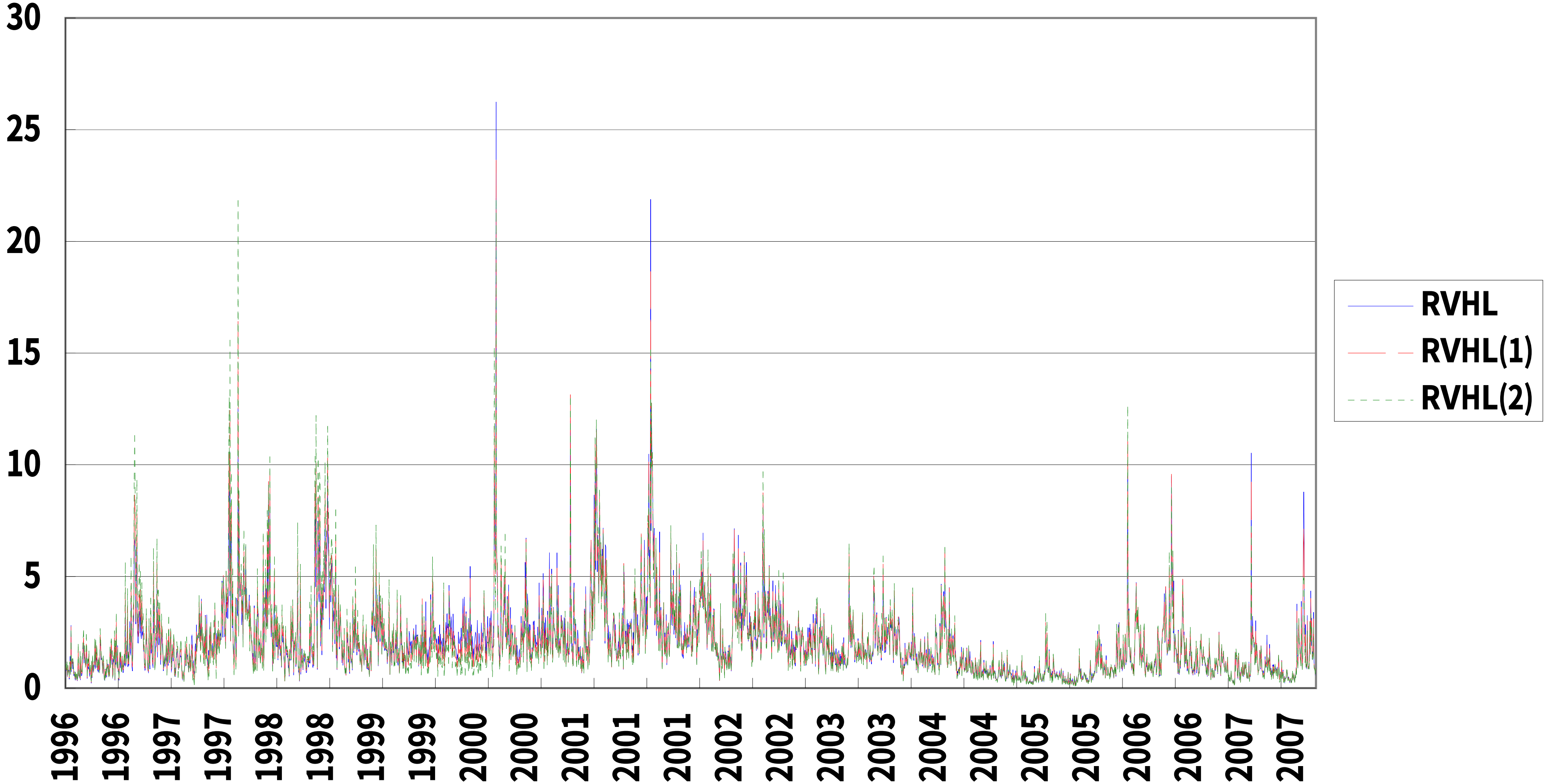
Table 6 (Continued)

Put option pricing performance of ARCH type models assuming the risk-neutrality and using the Duan (1995) method

	DOTM	OTM	ATM	ITM	DITM	Total
Sample size	269	102	115	99	145	730
RMSPE						
GARCH						
Risk neutral	0.889	0.617	0.291	0.091	0.021	0.599
Duan	0.944	0.624	0.290	0.090	0.021	0.630
EGARCH						
Risk neutral	1.698	0.851	0.318	0.088	0.020	1.086
Duan	1.790	0.849	0.319	0.088	0.021	1.139
FIEGARCH						
Risk neutral	1.516	0.643	0.268	0.081	0.021	0.958
Duan	1.328	0.624	0.258	0.078	0.020	0.846
MAPE						
GARCH						
Risk neutral	0.665	0.430	0.218	0.065	0.015	0.351
Duan	0.686	0.431	0.217	0.064	0.015	0.359
EGARCH						
Risk neutral	1.036	0.603	0.244	0.063	0.015	0.516
Duan	1.068	0.599	0.244	0.063	0.015	0.527
FIEGARCH						
Risk neutral	0.828	0.463	0.206	0.057	0.015	0.413
Duan	0.798	0.449	0.197	0.054	0.015	0.398

"Risk neutral" shows the results assuming the risk-neutrality, which are the same as those in Table 3. "Duan" shows the ones using the Duan (1995) method without assuming the risk-neutrality.

Fig 1: Realized Volatility
(a) With or without the Bartlett adjustment



(b) With or without the Hansen and Lunde (2005a) adjustment

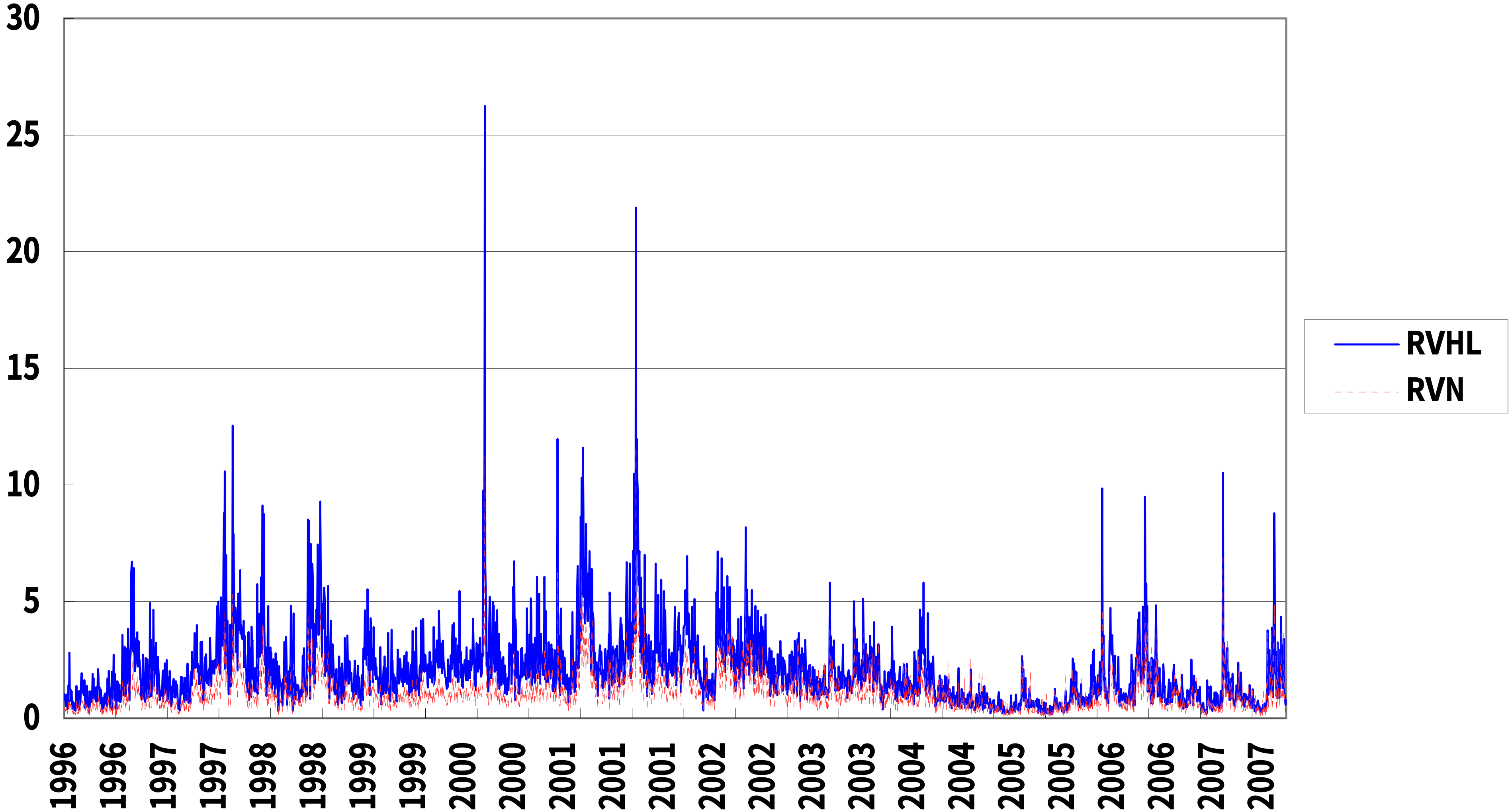
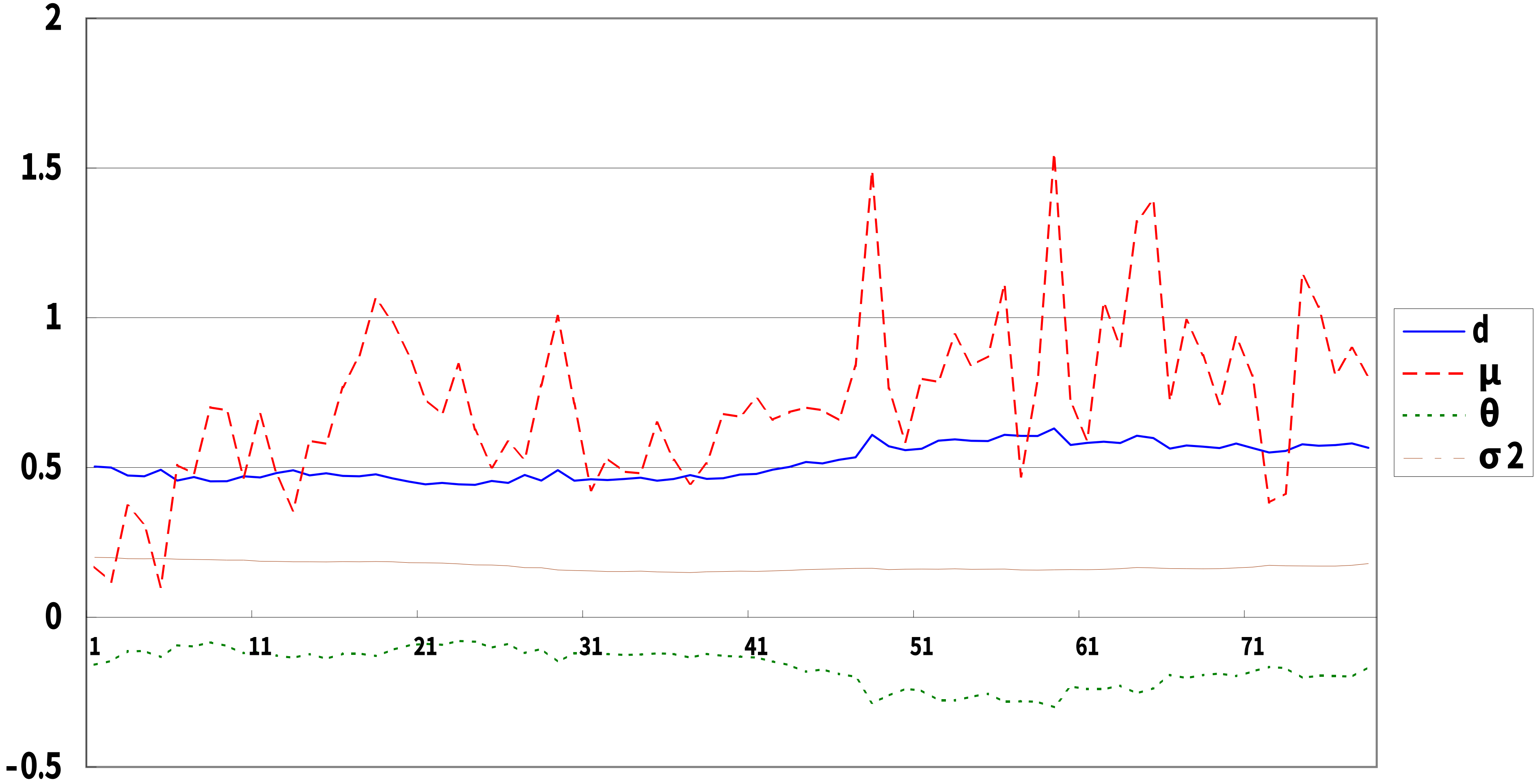
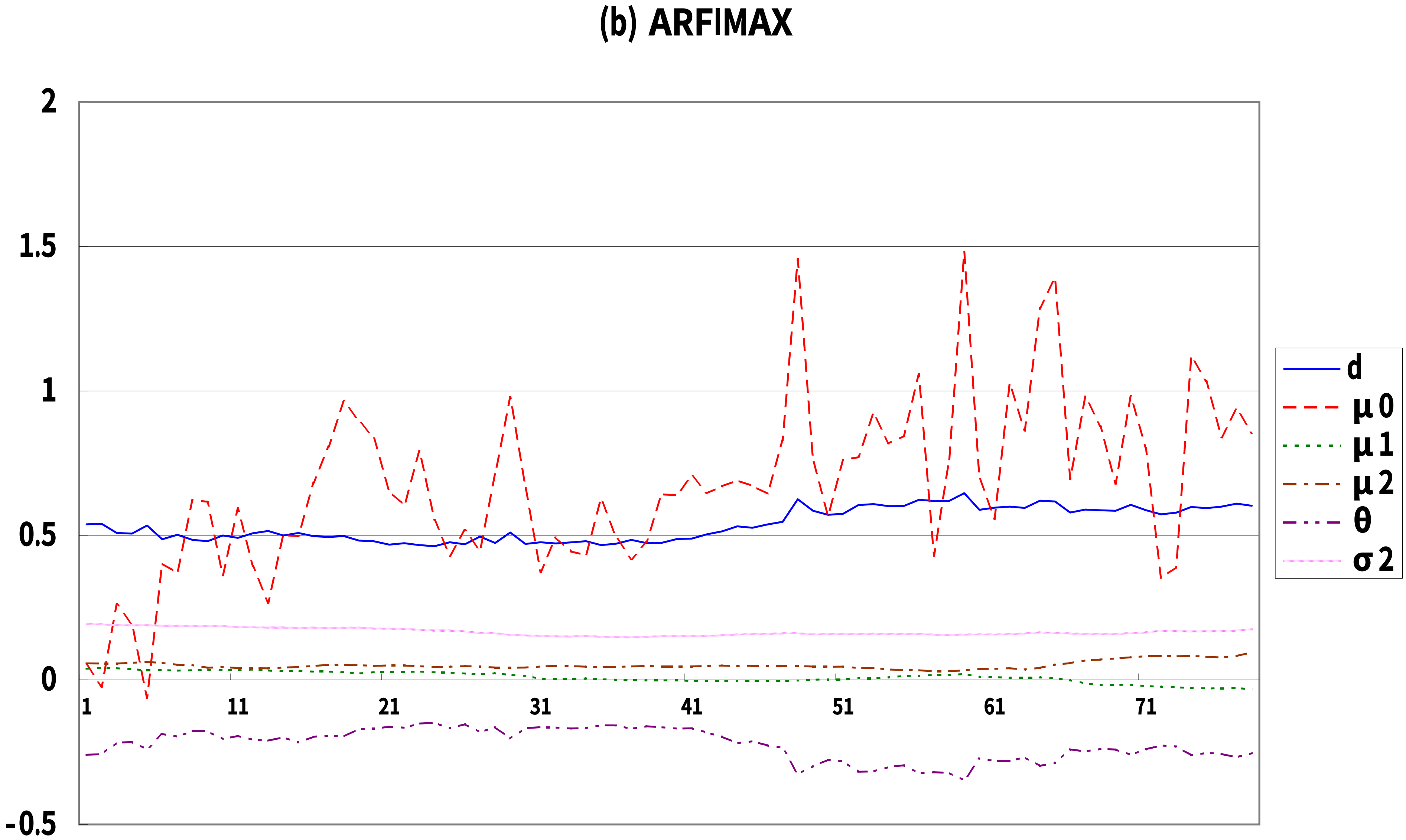
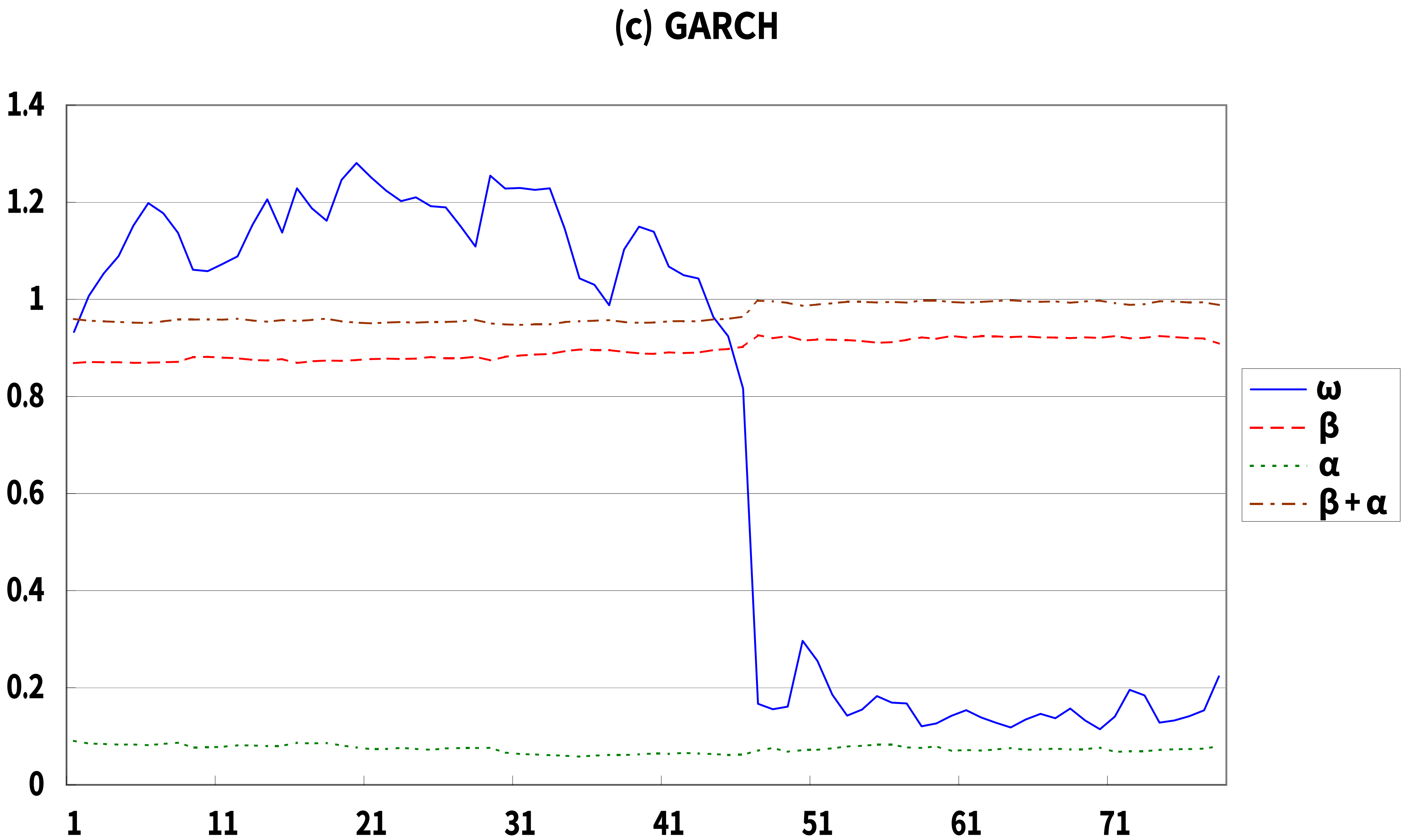


Fig 2. Parameter Estimates
(a) ARFIMA

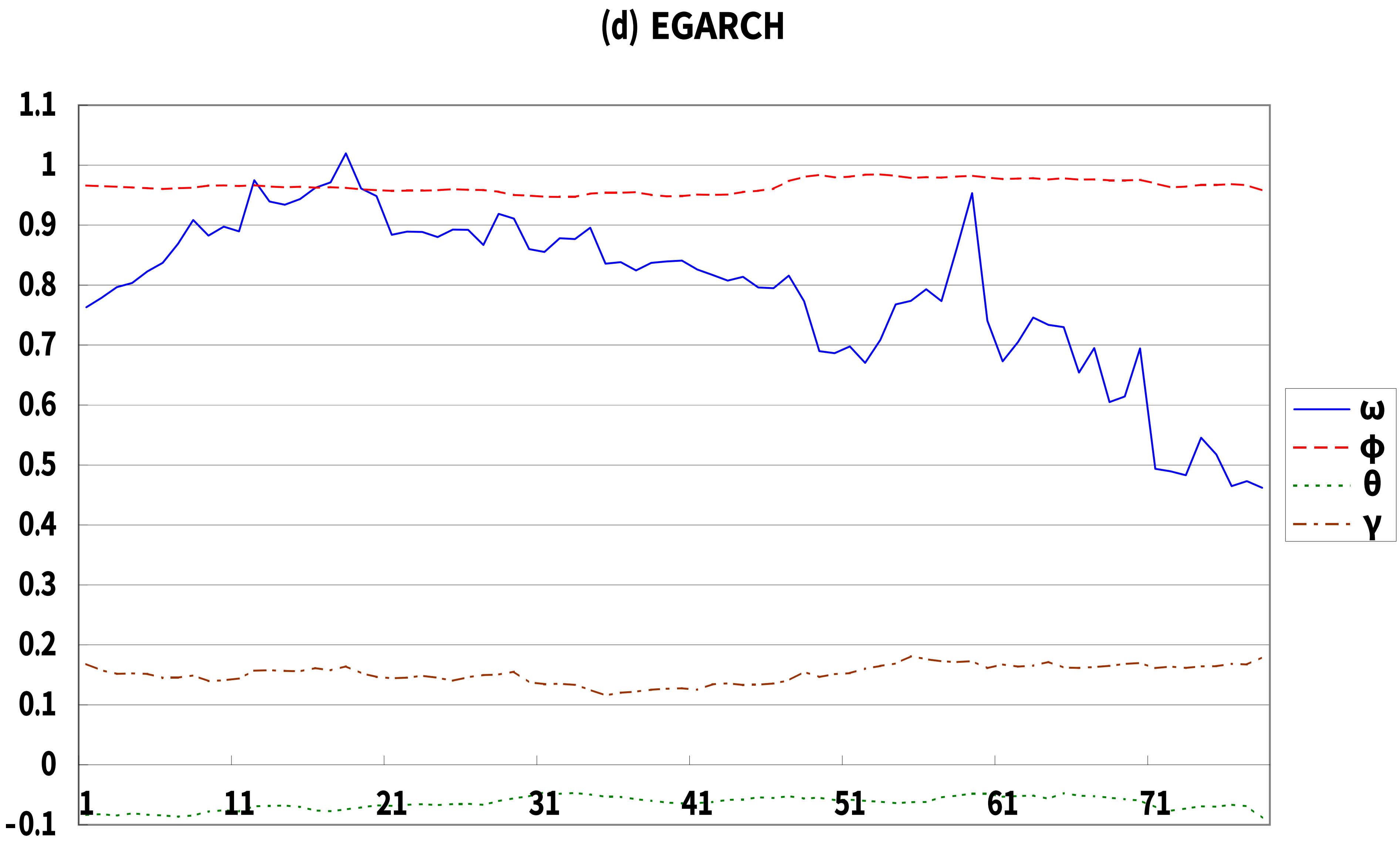


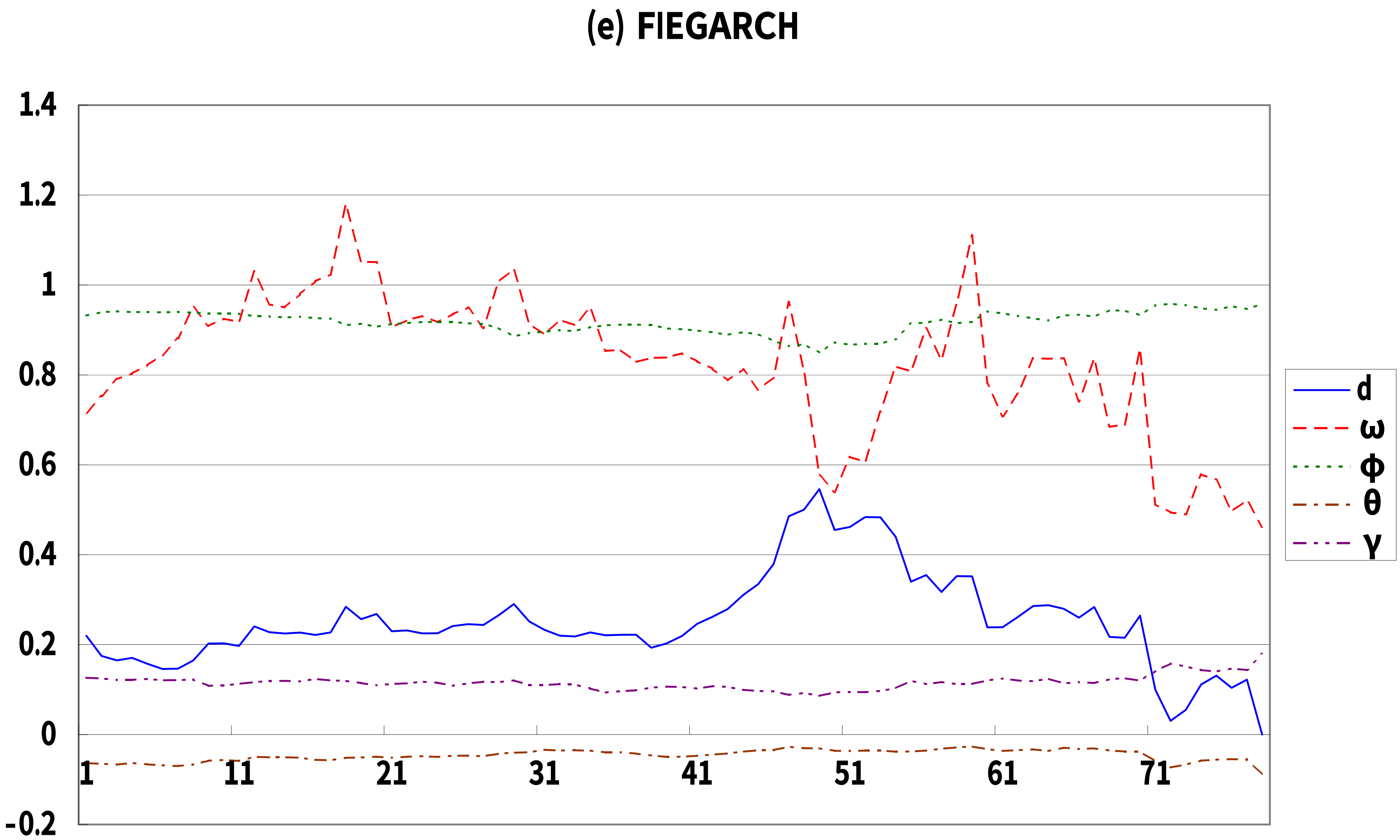


(b) ARFIMAX
2
1.5
1
0.5
0
-0.5
1
11
21
31
41
51
61
71
d
μ 0
μ 1
μ 2
θ
σ 2



(c) GARCH
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
11
21
31
41
51
61
71
ω
β
α
β + α


(d) EGARCH
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
1
11
21
31
41
51
61
71
ω
φ
θ
γ



(e) FIEGARCH
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1
11
21
31
41
51
61
71
d
ω
φ
θ
γ