Discussion Paper # 2009-13

Information Advantage in Cournot Oligopoly with Separable Information, or Non-differentiable Inverse Demand

by
Ori Haimanko

September 2009

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Abstract

Einy et al (2002) showed that information advantage of a firm is rewarded in any equilibrium of an incomplete information Cournot oligopoly, provided the inverse demand function is differentiable and monotonicaly decreasing, and costs are affine. We extend this result in two directions. We show first that a firm receives not less than its rival even if that firm's information advantage is only regarding payoff-relevant data, and not necessarily payoff-irrelevant "sunspots". We then show that there is at least one equilibrium which rewards firm's information advantage even with non-differentiable, but concave, inverse demand function. Under certain conditions, these results hold even with always non-negative inverse demand functions.

Keywords: Oligopoly, Incomplete Information, Information advantage, Bayesian Cournot, Equilibrium, Sunspots, Non-differentiability, Inverse demand.

Journal of Economic Literature Classification Numbers: C72, D43, L13.

*JSPS Fellow, Graduate School of Economics, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8601, Japan; and Department of Economics, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel. E-mail: orih@bgu.ac.il

1 Introduction

It has been well known ever since Blackwell (1953) that, in a decision problem (i.e., a one-person game) with incomplete information, better information on the environment yields higher value of the problem. It is also well known that similar statements do not apply in general to games with more than one player (see, e.g., Hirshleifer (1971)): improving the information of one player while fixing the information endowments of the rest may lead in equilibrium to lower expected payoff to that player. A big body of literature has been dedicated to identifying classes of games where the equilibrium payoff of a player is monotonically increasing/decreasing with an improvement of his information endowment, i.e., where the *value of information* is positive/negative. (See, e.g., Bassan et al (2003), Kamien et al (1990), Neyman (1991), Lehrer et al (2006), in the context of general games; and Gal-Or (1985,1986), Raith (1996), Sakai (1985), Shapiro (1986), and Vives (1984,1988) in the context of oligopolistic markets.) There has also been a strand of research devoted to a closely related question, of whether a player who is better informed than some of his rivals, in a game where the players are symmetric in all but their information endowments, receives in equilibrium more than his less-informed rivals; in other words, whether *information advantage* is rewarded in equilibrium. (See, e.g., Milgrom and Weber (1986) in the context of auctions; Einy et al (2002) in the context of Cournot oligopoly; Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994) in the context of exchage economies.)

Einy et al (2001) showed that information advantage of a firm is rewarded in any equilibrium of an incomplete information Cournot oligopoly, under very light assumptions on the demand (commonplace in the literature): the inverse demand function needs to be differentiable and monotonicaly decreasing. Cost functions, however, must to be affine; despite this limitation, the result is nonetheless of great appeal because of the generality of the inverse demand functions that it admits.

This work contains two generalizations of the information advantage result of Einy et al (2009). Our first generalization pertains to the definition of information advantage itself. The classical notion of player (firm) i being better informed than j involves a *global* comparison of players' information: at any state of nature, i must know at least what j knows at that state. However, if i has a better knowledge only of *directly payoff-relevant* data, we may still deem him better informed than j. Does the result of Einy et al (2002) still obtain under this weaker notion of better information?

We answer this question in the affirmative in Section 3, under certain conditions (that are quite general if there are just two firms). We assume that each state of nature is *separable* into two components, one that conveys information about market fundamentals (the demand and costs), and one that does not (i.e., it is a "sunspot" – see, e.g., Aumann et al (1988) – which may be related to political events and decisions that do not immediately affect the business climate). Note that firms may try to condition their strategies on the sunspot component, thereby affecting other firms' expectations and ultimately the outcome of the game. Thus, although not containing payoff-relevant information, the presense of sunspots may have a significant effect on equilibrium outcomes. Our result is, however, that when firm i is *at least as well informed about the market fundamentals as* firm j (i.e., i is at least as well informed as j about the payoff-relevant components of the sates of nature), the information advantage is rewarded in equilibrium as in Einy et al (2002) – see Theorem 1.

Our Section 4 is dedicated to a different generalization of the information advantage result: we consider *non-differentiable* inverse demand functions. With non-differentiable but concave inverse demand functions the equilibrium may not be unique even in the complete information case with symmetric firms, which shows that not every equilibrium reflects information advantage (or symmetry). However, we show that under general conditions there *is* at least one equilibrium that does it – see Theorem 2.

Finally, Section 5 considers oligopolies with always non-negative prices. It was shown in Einy et al (2009) that Bayesian Cournot equilibrium may fail to exist when the inverse demand functions are always non-negative (as they are in reality), and that "truncating" the inverse demand function at zero so as to make it always non-negative may significantly change strategic considerations in the model. We show, however, that the conclusions of Theorems 1 and 2 remain valid even with always non-negative inverse demand functions, under conditions for equilibrium existence set forth in Einy et al (2009).

2 Cournot Competition with Incomplete Information

Consider an industry where a set of firms, $N = \{1, 2, ..., n\}$, compete in the production of a homogeneous good. There is uncertainty about the market demand and the production costs. This uncertainty is described by a finite set Ω of *states of nature*, together with a probability measure μ on Ω, which represents the *common prior belief* of the firms about the distribution of the realized state. The information of the firms about the state of nature may be incomplete: the *private information* of firm $i \in N$ is given by a partition Π^i of Ω into disjoint sets. For any $\omega \in \Omega$, $\Pi^i(\omega)$ denotes the *information set* of i given ω, that is, the element of Π^i that contains ω. W.l.o.g., we assume that μ has full support on Ω, that is, $\mu(\Pi^i(\omega)) > 0$ for every $i \in N$ and $\omega \in \Omega$. If, for $i, j \in N$, $\Pi^j \subset \Pi^i$, we shall say that firm i is *at least as well informed* as firm j.

If $q^i(\omega)$ denotes the quantity of the good produced by firm i in state $\omega \in \Omega$, and $Q(\omega) \equiv \sum_{i=1}^n q^i(\omega)$ is the aggregate output in ω, then the *profit* of firm i in ω is given by

$$u^i\left(\omega, \left(q^1(\omega), ..., q^n(\omega)\right)\right) = q^i(\omega) P(\omega, Q(\omega)) - c(\omega) q^i(\omega)$$

where $P(\omega, \cdot)$ is the *inverse demand function* in ω, and $c(\omega)$ is the constant *marginal cost* of firm i in ω. (Thus, the firms are symmetric in all but their information.)

We assume throughout that:

(i) For every $\omega \in \Omega$, $c(\omega) > 0$.

(ii) For every $\omega \in \Omega$, $P(\omega, \cdot)$ is non-increasing, and for every $\omega \in \Omega$ there exists a level of aggregate output $0 < \overline{Q}(\omega) < \infty$ such that for every $Q < \overline{Q}(\omega)$

$$P(\omega, Q) > 0,$$

and

$$P\left(\omega, \overline{Q}(\omega)\right) = 0$$

if $\overline{Q}(\omega) < \infty$. We refer to $\overline{Q}(\omega)$ as the horizontal *demand intercept* in ω.[1]

A (pure) *strategy* for firm i is a function $q^i : \Omega \to \mathbb{R}_+$ that specifies its output in every state of nature, subject to *measurability with respect to i's private information* (i.e., q^i is constant on every information set of firm i). The set of strategies of firm i will be denoted by Σ^i. Given a strategy profile $q = (q^1, ..., q^n) \in \prod_{j=1}^n \Sigma^j$ the *expected profit* of firm i is

$$U^i(q) = E\left[u^i\left(\cdot, \left(q^1(\cdot), ..., q^1(\cdot)\right)\right)\right].$$

[1] A demand intercept arises in standard complete information models with a linear or concave inverse demand function. Existence of a demand intercept is consistent with (and usually implied by) Novshek's condition – see Remark 5.1 in Novshek (1985).

A strategy profile $q_* \in \prod_{j=1}^n \Sigma^j$ a *(pure strategy Bayesian) Cournot equilibrium*, if no firm finds it profitable to unilaterally deviate to another strategy, i.e., if for every $i \in N$ and $q^i \in \Sigma^i$

$$U^i(q_*) \geq U^i\left(q_* \mid q^i\right), \tag{1}$$

where $(q_* \mid q^i)$ stands for the profile of strategies which is identical to q_* in all but the ith strategy, which is replaced by q^i. This is equivalent to requiring

$$E\left(u^i(\cdot, q_*(\cdot)) \mid \Pi^i(\omega)\right) \geq E\left(u^i\left(\cdot, \left(q_* \mid q^i\right)(\cdot)\right) \mid \Pi^i(\omega)\right) \tag{2}$$

for every $\omega \in \Omega$. Here $E(g(\cdot) \mid A)$ stands for the expectation of a random variable g conditional on event A.

Remark 1. Note that, for any Cournot equilibrium q_* and any $i \in N$,

$$q_*^i(\cdot) \leq \max_{\omega \in \Omega} \overline{Q}(\omega), \tag{3}$$

since otherwise a firm could deviate to the strategy of zero output in all sates of nature, and save its costs.

3 Separable Information and Information Advantage

It was shown in Einy et al (2002) that the information advantage of a firm is rewarded in any equilibrium of an oligopoly, assuming only monotonicity and differentiability of the inverse demand function (the costs must be linear, however). The notion of information advantage of firm i over firm j is straightforward. Firm i needs to be better informed, *globally*, than firm j : *any* information set of i must be contained in some information set of j, which is equivalently expressed by the inclusion $\Pi^j \subset \Pi^i$.

A weaker notion of information advantage of i over j can be contemplated, whereby firm i has a better knowledge of only *payoff-relevant* data (that determines the inverse demand and costs). At the same time, the firm need not be better informed (globally), i.e., it may be the case that $\Pi^j \not\subseteq \Pi^i$ and in particular i may have less knowledge than j of all other, *payoff-irrelevant*, data. We will introduce this notion fromally in Section 3.2.

3.1 An Auxiliary result: Uniqueness of Cournot Equilibrium

Consider the following condition on the inverse demand, which is akin to the collation of (2) and (3) in Theorem 3 in Novshek (1985):

(A) For every $\omega \in \Omega$, $P(\omega, \cdot)$ is twice continuously differentiable and satisfies

$$QP''(\omega, Q) + P'(\omega, Q) \leq 0 \tag{4}$$

for every $Q \in \mathbb{R}_+$. (At $Q = 0$ we have in mind the right-side derivatives of P and P'.)

Inequality (4) in condition (A) is equivalent to the requirement that the marginal revenue of a firm be decreasing in the aggregate output of the other firms. It is satisfied, e.g., by all decreasing, concave, and twice continuously differentiable inverse demand functions.

Our auxiliary result, Proposition 1, gives a sufficient condition for existence and uniqueness of Cournot equilibrium. In addition to (A), we assume that there are *two types* of information endowments of firms, as is formally stated in (B):

(B) The set N of firms can be partitioned into two disjoint sets, K and M, such that $1 \in K$, $2 \in M$, and such that $\Pi^i = \Pi^1$ for every $i \in K$, $\Pi^j = \Pi^2$ for every $j \in M$.

Note that (B) is satisfied trivially in a duopoly.

Proposition 1. Consider an oligopoly satisfying conditions (i), (ii), (A), and (B), and such that for every $\omega \in \Omega$, $P'(\omega, \cdot) < 0$. Then it has a *unique* Cournot equilibrium q_* which, moreover, has the equal treatment property: $q_*^i = q_*^1$ for every $i \in K$, $q_*^j = q_*^2$ for every $j \in M$.

Proof. By Theorem 1A in Einy et al (2009), the oligopoly has at least one Cournot equilibrium. We will show that it is unique.

Let q_* be a Cournot equilibrium, and pick a firm i. Since

$$E\left(q^i(\cdot)\, P\left(\cdot, \sum_{j \neq i} q_*^j(\cdot) + q^i(\cdot)\right) - c(\cdot) q^i(\cdot) \mid \Pi^i(\omega)\right) \tag{5}$$

is maximized (and in particular locally maximized) at $q^i = q^i_*$ for every $\omega \in \Omega$, the Kuhn-Tucker conditions are satisfied:

$$E\left(q^i_*(\cdot)\, P'(\cdot, Q_*(\cdot)) + P(\cdot, Q_*(\cdot)) - c(\cdot) \mid \Pi^i(\omega)\right) = 0 \tag{6}$$

for every ω in which $q^i_* > 0$, and

$$E\left(q^i_*(\cdot)\, P'(\cdot, Q_*(\cdot)) + P(\cdot, Q_*(\cdot)) - c(\cdot) \mid \Pi^i(\omega)\right) \leq 0 \tag{7}$$

for every ω in which $q^i_* = 0$.

Note that for each $\omega \in \Omega$ the function

$$F(q, Q) = qP'(\omega, Q) + P(\omega, Q) - c(\omega)$$

is decreasing in q and non-increasing in Q when $q \leq Q$. Indeed, $\frac{\partial F}{\partial q} = P'(\omega, Q) < 0$ by assumption, and $\frac{\partial F}{\partial Q} = qP''(\omega, Q) + P'(\omega, Q) \leq 0$ as follows from $P(\omega, \cdot)$ being decreasing and condition A. Now suppose that q_* and q_{**} are two Cournot equilibria. That F is decreasing in q and non-increasing in Q implies that one cannot have

$$\left(q^i_*, Q_*\right) < \left(q^i_{**}, Q_{**}\right) \text{ or } \left(q^i_*, Q_*\right) > \left(q^i_{**}, Q_{**}\right)$$

(inequality in both coordinates and *strict* inequality in the first coordinate) on any atom $\Pi^i(\omega)$ of Π^i. This is because otherwise conditions (6) and (7) would not hold simultaneously for $\max\left(\left(q^i_*, Q_*\right), \left(q^i_{**}, Q_{**}\right)\right)$. To summarize, any firm's equilibrium strategy and the aggregate output in equilibrium cannot move in the same direction:

$$\left(q^i_*, Q_*\right) \not< \left(q^i_{**}, Q_{**}\right) \text{ and } \left(q^i_*, Q_*\right) \not> \left(q^i_{**}, Q_{**}\right) \tag{8}$$

on *any* element of Π^i.

We will next show that every Cournot equilibrium satisfies the equal treatment property. Indeed, if q_* is a Cournot equilibrium, and $q^i_* \neq q^j_*$ where i and j are firms of the same type, then consider an n-tuple q_{**} obtained from q_* by interchanging i and j. Clearly, q_{**} is also a Cournot equilibrium. However, if $\Pi^i(\omega) \in \Pi^i$ is a set on which w.l.o.g. $q^i_* > q^j_* = q^i_{**}$, then the obvious fact that $Q_* = Q_{**}$ leads to contradiction with (8). Thus, the equal treatment property holds in any Cournot equilibrium.

Now suppose that q_* and q_{**} are Cournot equilibria in the oligopoly. We will show that they coincide. Indeed, if $q_* \neq q_{**}$, consider

$$\Delta \equiv \max\left[\max_{\omega\in\Omega}\left(|K|\cdot\left|q_*^1(\omega)-q_{**}^1(\omega)\right|\right), \max_{\omega\in\Omega}\left(|M|\cdot\left|q_*^2(\omega)-q_{**}^2(\omega)\right|\right)\right] > 0. \tag{9}$$

Assume, w.l.o.g., that $\Delta = \max_{\omega\in\Omega}\left(|K|\cdot|q_*^1(\omega)-q_{**}^1(\omega)|\right)$, and that let $\omega_0\in\Omega$ be a state of nature where this maximum is obtained. W.l.o.g.,

$$q_*^1 > q_{**}^1 \text{ on } \Pi^1(\omega_0). \tag{10}$$

But then, on $\Pi^1(\omega_0)$,

$$Q_* - Q_{**} = |K|\left(q_*^1 - q_{**}^1\right) + |M|\left(q_*^2 - q_{**}^2\right)$$

by the equal treatment property shown above, and $|K|(q_*^1-q_{**}^1)+|M|(q_*^2-q_{**}^2)$ is a non-negative function on $\Pi^1(\omega_0)$ by the choice of ω_0. Thus $Q_* \geq Q_{**}$ on $\Pi^1(\omega_0)$, which together with (10) contradict (8). We conclude that $q_*^1 = q_{**}^1$.□

3.2 Separable Information

Assume that the set of states of nature Ω is a product set $\Omega_1\times\Omega_2$, and that the common prior μ is a product measure $\mu_1\times\mu_2$. The probability space (Ω_1,μ_1) will be regarded as the space of *payoff-relevant states*, that contain all information pertaining to the market fundamentals (the demand and firms' costs), while (Ω_2,μ_2) will be regarded as the space of *payoff-irrelevant states* that represents all other uncertainty that there might be ("sunspots"). Accordingly, the (state-dependent) inverse demand and cost depend on $\omega_1\in\Omega_1$ but not on $\omega_2\in\Omega_2$:

$$P((\omega_1,\omega_2),Q) = P((\omega_1,\omega_2'),Q) \tag{11}$$

and

$$c(\omega_1,\omega_2) = c(\omega_1,\omega_2') \tag{12}$$

for every $\omega_1\in\Omega_1$, every $\omega_2,\omega_2'\in\Omega_2$, and every $Q\geq 0$. Denote the expressions in (11) and (12) by $P_1(\omega_1,Q)$ and $c_1(\omega_1)$, respectively.

Assume further that each agent i's information about the realized payof-relevant state is given by a partition Π_1^i of Ω_1, and about the payoff-irrelevant state – by a partition Π_2^i of Ω_2. Thus, if $\omega = (\omega_1, \omega_2) \in \Omega$ is realized, i's information set $\Pi^i(\omega)$ is the product set $\Pi_1^i(\omega_1) \times \Pi_2^i(\omega_2)$, where $\Pi_k^i(\omega_k)$ is the element of Π_k^i of Ω_k that contains ω_k.

An oligopoly O conforming to such a description will be called an *oligopoly with separable information*: information can be decomposed into the payoff-relevant component and the payoff-irrrelevant component.

The functions $P_1(\cdot, \cdot)$, $c_1(\cdot)$, and the partitions $\{\Pi_1^i\}_{i \in N}$ can be viewed as the inverse demand and the marginal cost, and the information endowments of firms, respectively, in a *restricted oligopoly* O_1 associated with O, where the uncertainty is described by the space of payoff-relevant states (Ω_1, μ_1). Compared with the oligopoly O with separable information, in the restricted oligopoly O_1 the firms possess the same payoff-relevant information, but are ignorant of the data which is not payoff-relevant.

Note that any oligopoly can be represented as the one with separable information: any Ω is isomorphic to the product $\Omega_1 \times \Omega_2$ where $\Omega_1 = \Omega$ and Ω_2 is a singleton. However, it is easy to think of oligopolies with separable information where a non-trivial decomposition $\Omega = \Omega_1 \times \Omega_2$ is present inherently. There may be uncertainty about the market demand and costs, represented by Ω_1, but also incomplete information on variables other than the market fundamentals (such as political events and decisions that do not immediately affect the business climate). These payoff-irrelevant signals, or "sunspots", on which firms may try to condition their strategies, thereby affecting other firms' expectations and ultimately the outcome of the game, are represented by Ω_2.

If i,$j \in N$ and

$$\Pi_1^j \subset \Pi_1^i, \tag{13}$$

we will say that firm i is *at least as well informed about the market fundamentals as* firm j. This is an extension of the standard version of possessing a better information, whereby the inclusion in (13) would need to hold for the information partitions Π^j and Π^i of Ω. However, according to (13), the better knowledge should only pertain to market fundamentals, as represented by partitions of Ω_1 induced by Π^j and Π^i. Firm i with a better information about the market fundamentals may, according to our definition, have little or no knowledge of the realized state in Ω_2, even if j always has this knowledge.

3.3 Information advantage

According to our main result of this section, a firm which is at least as well informed about the market fundamentals as a firm of another type, receives at least as much in the unique Cournot equilibrium.

Theorem 1. Consider an oligopoly O with separable information such that conditions (i), (ii), (A), and (B) are satisfied, and for every $\omega \in \Omega$, $P'(\omega, \cdot) < 0$. Assume that firm 1 is at least as well informed about the market fundamentals as firm 2. Then in the unique equilibrium q_*,

$$U^1(q_*) \geq U^2(q_*) . \tag{14}$$

Proof. By Proposition 1, the restricted oligopoly O_1 possesses a unique equilibrium q_{*1}, which moreover satisfies the equal treatement property. For every $\omega = (\omega_1, \omega_2) \in \Omega$ and $i \in N$, denote

$$\overline{q}^i_*(\omega_1, \omega_2) \equiv q^i_{*1}(\omega_1) . \tag{15}$$

We claim that $\overline{q}_* = (\overline{q}^1_*, ..., \overline{q}^n_*) \in \prod_{j=1}^n \Sigma^j$ is an equilibrium in the original oligopoly O. Indeed, let $\omega' = (\omega'_1, \omega'_2) \in \Omega$, $i \in N$, and $q^i \in \Sigma^i$. By (15) and Π^i-measurability of q^i, the strategy profile $(\overline{q}_* \mid q^i)(\omega_1, \omega_2)$ does not depend on ω_2 provided $(\omega_1, \omega_2) \in \Pi^i_1(\omega'_1) \times \Pi^i_2(\omega'_2)$, and thus neither does the state-dependent payoff $u^i((\cdot,\cdot), (\overline{q}_* \mid q^i)(\cdot,\cdot))$. Let q^i_1 be a strategy of firm i in O_1, defined by $q^i_1(\omega_1) \equiv q^i(\omega_1, \omega'_2)$ for $\omega_1 \in \Pi^i_1(\omega'_1)$ and arbitrarily elsewhere. It follows that

$$E\left(u^i\left((\cdot,\cdot)\left(\overline{q}_* \mid q^i\right)(\cdot,\cdot)\right) \mid \Pi^i_1(\omega'_1) \times \Pi^i_2(\omega'_2)\right) \tag{16}$$

$$= E\left(u^i\left((\cdot,\omega'_2), \left(\overline{q}_* \mid q^i\right)(\cdot,\omega'_2)\right) \mid \Pi^i_1(\omega'_1) \times \Pi^i_2(\omega'_2)\right) \tag{17}$$

$$= E_{\mu_1}\left(u^i_1\left(\cdot, \left(q_* \mid q^i_1\right)(\cdot)\right) \mid \Pi^i_1(\omega'_1)\right), \tag{18}$$

where $E_{\mu_1}(g(\cdot) \mid A)$ stands for the expectation of a random variable g on Ω_1 conditional on event A, with respect to the probability measure μ_1. Since q_{*1} is an equilibrium in O_1, it follows from (16)-(18) and (2) that $\overline{q}_*$ is an equilibrium in O. By Proposition 1, $\overline{q}_* = q_*$ (the *unique* equilibrium in O).

Since $\Pi^2_1 \subset \Pi^1_1$, by Theorem 1 of Einy et al (2002) applied to the oligopoly O_1:

$$E_{\mu_1}\left(u^1_1(\cdot, q_{*1}(\cdot))\right) \geq E_{\mu_1}\left(u^2_1(\cdot, q_{*1}(\cdot))\right) .$$

But

$$E_{\mu_1}\left(u_1^i\left(\cdot, q_{*1}\left(\cdot\right)\right)\right) = E\left(u^i\left(\left(\cdot,\cdot\right), \overline{q}_*\left(\cdot,\cdot\right)\right)\right) = U^1\left(q_*\right)$$

for every $i \in N$, and thus (14) follows. □

4 Non-differentiable Market Demand and Information Advantage

With non-differentiable but concave demand function, Cournot equilibium exists in the oligopoly, although it may not be unique, even with complete information. There may be equilibria that do not reward the information advantage of a better-informed firm, but we will show that there *is* at least one equilibrium that does reflect information advantage.

The following example demonstrates non-uniqueness of the equilibrium in a complete information duopoly with a concave inverse demand:

Example. Assume that there are two firms that face the inverse demand function given by

$$P\left(Q\right) = \begin{cases} 1, & \text{if } Q \leq 0.99; \\ 100(1-Q), & \text{if } 0.99 < Q \leq 1; \\ 0, & \text{if } Q > 1, \end{cases}$$

and have the marginal cost of 0.001. This is a *symmetric* duopoly with multiple Cournot equilibria. In particular, in addition to the symmetric equilibrium $(0.495, 0.495)$ there is a continuum of asymetric ones: every pair $(\varepsilon, 0.99-\varepsilon)$ is a Cournot equilibrium for $\varepsilon \in [0.02, 0.97]$.

This example shows that, without a differentiability assumption on the inverse demand, we cannot expect the expected equilibrium profits of firms to reflect firms' information advantage, as in Einy et al (2002). Here, despite having the same information on the environtment, a firm may have a smaller, or a larger, profit than its rival. It is, however, obvious, that there *is* an equilibrium that reflects the information symmetry in the game, which is the symmetric $(0.495, 0.495)$. Our aim is to show that, in a general oligopoly with asymmetric information, concave inverse demand and linear costs, there will always be *at least one* equilibrium rewarding a firm with information advantage.

Let $P(\cdot,\cdot)$ be a state dependent inverse demand function satisfying:
(C) For every $\omega \in \Omega$, $P(\omega,\cdot)$ is a continuous and concave function.

Theorem 2. Consider an oligopoly O that satisfies conditions (i), (ii), and (C). Assume that firm $1 \in N$ is at least as well informed as firm $2 \in N$: $\Pi^2 \subset \Pi^1$. Then there exists a Cournot equilibrium q_* in which

$$U^1(q_*) \geq U^2(q_*). \tag{19}$$

Proof. Given any $\varepsilon > 0$, consider a real-valued function P_ε defined on $\Omega \times \mathbb{R}_+$ by

$$P_\varepsilon(\omega, Q) = \frac{1}{\varepsilon}\int_0^\varepsilon P_\varepsilon(\omega, Q + x)\, dx.$$

Since $P(\omega,\cdot)$ is differentiable almost everywhere, being a concave function by (C), $P_\varepsilon(\omega,\cdot)$ is (continuously) differentiable everywhere, and concave. Moreover, for every $\omega \in \Omega$,

$$\lim_{\varepsilon \to 0} P_\varepsilon(\omega,\cdot) = P(\omega,\cdot) \tag{20}$$

pointwise; the convergence is uniform on any given interval $[0, z]$, and in particular for $z = Z \equiv \max_{\omega\in\Omega} \overline{Q}(\omega)$.

Consider an oligopoly O_ε which is identical to the given oligopoly O in all but the inverse demand function, which is P_ε. As in the proof of Theorem 1B in Einy et al (2009), oligopoly O_ε has a Cournot equilibrium; denote one such equilibrium by $q_{\varepsilon *}$. Since all conditions of Theorem 1 in Einy et al (2002) are satisfied by O_ε (in particular, the inverse demand function P_ε is differentiable and nonincreasing), the information advantage of firm 1 is rewarded in $q_{\varepsilon *}$:

$$U_\varepsilon^1(q_{\varepsilon *}) \geq U_\varepsilon^2(q_{\varepsilon *}), \tag{21}$$

where U_ε^i stands for the expected payoff function of firm i in O_ε.

Consider now the sequence $\left\{q_{\frac{1}{n}*}\right\}_{n=1}^{\infty}$. If we identify the set of strategy profiles with $\mathbb{R}_+^{\Pi^1\times\ldots\times\Pi^n}$, by Remark 1 each $q_{\frac{1}{n}*}$ is a point in the compact cube $[0, Z]^{\Pi^1\times\ldots\times\Pi^n}$, where (recall) $Z \equiv \max_{\omega\in\Omega} \overline{Q}(\omega)$. The sequence $\left\{q_{\frac{1}{n}*}\right\}_{n=1}^{\infty}$ thus has a convergent subsequence (w.l.o.g., the sequence itself) with a limit q_*. We shall show that q_* is an equilibrium in the oligopoly O.

The convergence in (20) is uniform on $[0, Z]$, and hence $\lim_{\varepsilon \to 0} U^i_\varepsilon(q) = U^i(q)$ uniformly in $q \in [0, Z]^{\Pi^1 \times \ldots \times \Pi^n}$, for every $i \in N$. Since

$$\left| U^i_{\frac{1}{n}} \left(q_{\frac{1}{n}*} \right) - U^i(q_*) \right| \leq$$

$$\left| U^i_{\frac{1}{n}} \left(q_{\frac{1}{n}*} \right) - U^i \left(q_{\frac{1}{n}*} \right) \right| + \left| U^i \left(q_{\frac{1}{n}*} \right) - U^i(q_*) \right|,$$

and U^i is continuous on $\mathbb{R}_+^{\Pi^1 \times \ldots \times \Pi^n}$, it follows that

$$\lim_{\varepsilon \to 0} U^i_{\frac{1}{n}} \left(q_{\frac{1}{n}*} \right) = U^i(q_*) \tag{22}$$

for every $i \in N$. Similarly, given $i \in N$ and $q^i \in [0, Z]^{\Pi^i}$ (viewed as a subset of Σ^i)

$$\lim_{\varepsilon \to 0} U^i_{\frac{1}{n}} \left(q_{\frac{1}{n}*} \mid q^i \right) = U^i \left(q_* \mid q^i \right). \tag{23}$$

From the assumption of $q_{\frac{1}{n}*}$ being an equilibrium in $O_{\frac{1}{n}}$ and (1), it follows that for every $i \in N$ and $q^i \in [0, Z]^{\Pi^i}$

$$U^i(q_*) \geq U^i \left(q_* \mid q^i \right). \tag{24}$$

Furthermore, since firms have positive marginal costs, for every $i \in N$ and $q^i \in \Sigma^i$

$$U^i \left(q_* \mid q^i \right) \leq U^i \left(q_* \mid \min \left(q^i, Z \right) \right),$$

and $\min(q^i, Z) \in [0, Z]^{\Pi^i}$. Thus (24) in fact holds for every $i \in N$ and every $q^i \in \Sigma^i$, showing that q_* is an equilibrium in the oligopoly O.

Now (21) and (22) imply (19). □

5 Information advantage in oligopolies with always non-negative prices

In both Theorems 1 and 2 of the previous section Cournot equilibrium did exist under conditions (A) or (C). However, uder these conditions, the inverse demand function typically becomes negative for sufficiently large levels of aggregate output. (This is also the case in the complete information setting – see remark 5.1 in Novshek (1985).) If the inverse demand is then *truncated* to rule out non-negative prices, existence of a Cournot equilibrium cannot be guaranteed even in duopolies with linear demand functions, as seen in examples 1 and 2 in Einy et al (2009).

In this section we adopt the Einy et al (2009) condition for equilibrium existence with *non-negative* inverse demand functions, and present the corresponding versions of Theorems 1 and 2.

Consider a non-negative inverse demand function P, i.e., a function that satisfies $P(\omega, Q) \geq 0$ for all $\omega \in \Omega$ and $Q \in \mathbb{R}_+$. Since for every $\omega \in \Omega$, $P(\omega, \cdot)$ is non-increasing by assumption (ii), for $Q \geq \overline{Q}(\omega)$ we have

$$P(\omega, Q) = 0. \tag{25}$$

The analogs of Novshek's condition (A), or the concavity condition (C), will now be used in conjunction with the requirement that the inverse demand be a non-negative function. These conditions must now be restated in the form that makes them consistent with (25). (In what follows, the conditions and assumptions on derivatives of $P(\omega, \cdot)$ refer to *one-sided* derivatives at the endpoints of the interval $[0, \overline{Q}(\omega)]$.[2])

(A') For every $\omega \in \Omega$, $P(\omega, \cdot)$ is a *non-negative* function, that is twice continuously differentiable on $[0, \overline{Q}(\omega)]$ and satisfies $QP''(\omega, Q) + P'(\omega, Q) \leq 0$ for every $Q \in [0, \overline{Q}(\omega)]$.

(C') For every $\omega \in \Omega$, $P(\omega, \cdot)$ is a *non-negative* function, that is continuous and concave on $[0, \overline{Q}(\omega)]$, with a finite left-hand derivative at $\overline{Q}(\omega)$.

The following condition is a slightly strenghtened version of a condition used in Einy et al (2009), which in conjunction with (A) or (C) guarantees exitence of equilibrium with always non-negative prices.

(D) There exists a profile of state-dependent thresholds of output $\overline{q} \in \prod_{i=1}^{n} \Sigma^i$ such that for every $\omega \in \Omega$

$$\sum_{i=1}^{n} \overline{q}^i(\omega) \leq \overline{Q}(\omega), \tag{26}$$

[2] The function $P(\omega, \cdot)$ need not (and typically *will not*) be differentiable at $\overline{Q}(\omega)$.

and for every strategy profile $q \in \prod_{i=1}^{n} \Sigma^i$ with $q^i \nleq \overline{q}^i$ for some $i \in N$ there exists a strategy $r^i \leq \overline{q}^i$ such that[3]

$$U^i(q) < U^i(q \mid r^i). \tag{27}$$

Intuitively, condition D implies that each firm i does not want to produce too much, since by reducing its output below the level $\overline{q}^i$ its expected profit increases.

Theorem 1'. Consider an oligopoly O with separable information such that conditions (i), (ii), (A'), (B), and (D) are satisfied, and for every $\omega \in \Omega$, $P'(\omega, \cdot) < 0$ on $[0, \overline{Q}(\omega)]$. Assume that firm 1 is at least as well informed about the market fundamentals as firm 2. Then in the unique equilibrium q_*,

$$U^1(q_*) \geq U^2(q_*). \tag{28}$$

Proof: For every $\omega \in \Omega$ consider a function $P_-(\omega, Q)$ which is identical to $P(\omega, \cdot)$ on $[0, \overline{Q}(\omega)]$, but is defined by

$$P_-(\omega, Q) \equiv P\left(\omega, \overline{Q}(\omega)\right) + P'\left(\omega, \overline{Q}(\omega)\right) \cdot \left(Q - \overline{Q}(\omega)\right) + \frac{1}{2} P''\left(\omega, \overline{Q}(\omega)\right) \cdot \left(Q - \overline{Q}(\omega)\right)^2$$

on $\left(\overline{Q}(\omega), \infty\right)$. Note that oligopoly O_- which is identical to the given oligopoly O in all but the inverse demand function, which is P_-, satisfies all the conditions of Theorem 1. Moreover, since $P_-(\omega, \cdot) \leq P(\omega, \cdot)$ on $\left(\overline{Q}(\omega), \infty\right)$, the oligopoly O_- also satisfies (D), and O and O_- have the same set of Bayesian equilibria. Thus, the conclusion of Theorem 1 applies to the oligopoly O. $\square$

Theorem 2'. Consider an oligopoly O that satisfies conditions (i), (ii), (C'), and (D). Assume that firm $1 \in N$ is at least as well informed as firm $2 \in N$: $\Pi^2 \subset \Pi^1$. Then there exists a Cournot equilibrium q_* in which

$$U^1(q_*) \geq U^2(q_*).$$

[3] Here and henceforth, we use the notation $h \leq g$ (for $h, g : \Omega \to \mathbb{R}_+$) if and only if $h(\omega) \leq g(\omega)$ for every $\omega \in \Omega$.

Proof: Repeat the arguments of the proof of Theorem 1', with the following exception: for every $\omega \in \Omega$, the function $P_{-}(\omega, Q)$ is defined by

$$P_{-}(\omega, Q) \equiv P\left(\omega, \overline{Q}(\omega)\right) + P'\left(\omega, \overline{Q}(\omega)\right) \cdot \left(Q - \overline{Q}(\omega)\right)$$

on $\left(\overline{Q}(\omega), \infty\right)$; $P'\left(\omega, \overline{Q}(\omega)\right)$ stands for the left-hand side derivative of P at $\overline{Q}(\omega)$, well defined by assumption (C'). $\square$

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