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Nash Implementation in Production Economies with Unequal Skills: A Complete Characterization*

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Abstract

In production economies with unequal labor skills, where the planner is ignorant to the set of feasible allocations in advance of production, the paper firstly introduces a new axiom, *Non-manipulability of Irrelevant Skills* (**NIS**), which together with *Maskin Monotonicity* constitute the necessary and sufficient conditions for Nash implementation. Secondly, the paper defines natural mechanisms, and then fully characterizes Nash implementation by natural mechanisms, using a slightly stronger variation of **NIS** and *Supporting Price Independence*. Following these characterizations, it is shown that there is a Maskin monotonic allocation rule which is not implementable when information about individual skills is absent. In contrast, many fair allocation rules, which are known to be non-implementable in the present literature, are implementable by the natural mechanisms.

JEL Classification Numbers: C72, D51, D78, D82

Keywords: Unequal labor skills; Nash implementation; Non-manipulability of Irrelevant Skills

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1 Introduction

In this paper, Nash implementation of desired resource allocations is discussed in production economies with possibly unequal labor skills. A typical example of such economies is a *fishery*, where mechanism design for Nash implementation is of practical interest: in fisheries, each individual's free operation may lead to an overexploitation of resources with little regard to future sustainability, and so many countries with the fishery as one of their major industries work on resource management, and employ incentive schemes to control individuals' operation. For instance, in Norway, the harvesting of marine resources is regulated to ensure that the capacity of the (source) stocks are able to renew themselves.[1][2][3] Given that the total allowable catch (TAC) in the Barents Sea is allocated through negotiations under international agreements, the country's quotas are distributed among different groups of fishermen, and then subdivided and allocated among fishing boats in each group, which constitutes desired resource allocations. With this in mind, the *resource management mechanism* in Norway is expected to implement such allocations of fishing by monitoring and punishing each fishing boat's overexploitation of resources.

Most of the vast literature on implementation theory presumes that the social planner cannot know each individual's preference but knows the set of feasible alternatives. In economic environments, however, there are many examples of resource allocation problems in which each individual's private information consists of not only her preferences but also her endowments and/or human capital. In such problems, as Jackson (2001) pointed out, the planner may not know in advance the set of feasible alternatives (feasible allocations), since it is endogenously fixed due to individuals' strategies on

[1] Sustainable management requires knowledge of the size of the stocks, their age composition, their distribution, and the environment in which they live. Every year, data from Norwegian scientific surveys and from fishermen are compared with data from other countries (Norwegian marine scientists cooperate closely with researchers from other countries, especially Russia) and assessed by the International Council for Exploration of the Sea (ICES).

[2] This is in accordance with international agreements including the 1982 UN Law of the Sea Convention, the 1995 UN Fish Stocks Agreement and the 1995 FAO Code of Conduct for Responsible Fisheries.

[3] Recently, the ecosystem approach is increasingly being applied to Norwegian fisheries management. It not only takes into account how harvesting affects fish stocks, but also how the fisheries affect the marine environment for living marine resources in general.

how to utilize their own endowments or human capital, which the social planner may have no control over. Given this setting, it is necessary to extend the classical framework of implementation theory into the framework with endogenous feasible allocations that allows each individual to misrepresent not only her preference but also her endowments or human capital.

It was Hurwicz et al. (1995) which provided a systematic analysis of endogenous feasible allocations in Nash implementation under economic environments, and there is also some literature such as Tian and Li (1995); Hong (1995); Tian (1999, 2000, 2009) which addresses the above issue in designing a mechanism to implement a specific social choice correspondence (SCC) like the Walrasian solution. In these works, each individual is allowed to *understate* (or withhold) her own material endowments, but she is not allowed to overstate them, since the planner is assumed to require individuals to "place the claimed endowments on the table" [Hurwicz et al. (1995)].

In production economies with unequal skills, however, one of the essential features is that each individual is allowed to not only understate but also *overstate* her endowment of labor skill, since here the planner cannot require individuals to place the claimed endowments of their skills on the table in advance of production. Among several papers[4] on implementation in production economies, there are a few works such as Yamada and Yoshihara (2007, 2008) which address this essential feature, and then discuss implementation under some stringent restriction on available mechanisms.

In contrast to the above literature, this paper firstly imposes no restriction on the available class of mechanisms and then provides a general (necessary and sufficient) characterization of Nash-implementable (efficient) SCCs in production economies with unequal skills. It is *Monotonicity* (**M**) [Maskin (1999)] which is the necessary and sufficient condition for Nash implementation in production economies if the endowments of skills are known to the planner, though this characterization no longer holds if they are unknown. Thus, this paper introduces a new condition, called *Non-manipulability of Irrelevant Skills* (**NIS**), which together with **M** fully characterize Nash-implementable SCCs in such problems. The axiom **NIS** requires independence of a particular change in individual skills, which is weak enough in the

[4]In addition to the above mentioned literature, for instance, Suh (1995), Yoshihara (1999), Kaplan and Wettstein (2000), and Tian (2009) have proposed simple or natural mechanisms to implement particular SCCs, whereas Shin and Suh (1997) and Yoshihara (2000) have discussed characterizations of SCCs implementable by simple or natural mechanisms.

sense that any efficient SCC satisfies it in economies with strictly concave production functions, but it is by no means trivial. Actually, as shown below, there is an economically meaningful SCC, called the *maximal workfare solution*, which satisfies **M**, but not **NIS** in economies with linear production functions. Thus, though this solution is implementable in the classical framework where the endowments of skills are known to the planner, it is non-implementable in the extended framework where they are unknown.

Secondly, this paper defines a class of *natural mechanisms* applied to these economies, and then provides a full characterization of SCCs implementable by them. To define what natural mechanisms are in these economies, it is worth mentioning the practical applicability of the above mentioned resource management mechanism in Norway, which implements desired allocations of fishing to preserve the living marine resources for future sustainability, while letting the fishing boats operate freely within their quotas and claim as their due share what they produced.[5] Taking this property as primarily relevant in production economies, the paper considers *labor sovereignty* [Kranich (1994); Gotoh et al. (2005)] as a condition of natural mechanisms. This requires each individual to have the right of choosing her own labor hours, and the outcome functions of such mechanisms simply distribute the produced output to agents, according to the information they provided and the record of their labor hours completed. The paper also introduces *forthrightness* [Dutta et al. (1995); Saijo et al. (1996, 1999)], and defines natural mechanisms as *feasible* ones satisfying these two conditions and having strategy spaces of *price-quantity annoucements*.

Note that the conditions of natural mechanisms are slightly weaker than that of the restricted mechanisms (we call *simple mechanisms* here) discussed in Yamada and Yoshihara (2007). This is because one peculiar condition imposed on simple mechanisms is not present in natural mechanisms. Correspondingly, the necessary and sufficient conditions for naturally imple-

[5] In more detail, this mechanism requires ocean-going vessels to install and use satellite-based tracking equipment that enables the authorities to continually monitor their operations, though the volume of each individual boat's catch is not easily identified. Therefore, the adherence to the government imposed harvest quota cannot reasonably be measured by the government for each individual fisher's catch, but rather measured by the increase or decrease of the remaining fish resources. The following quota is then adjusted accordingly up or down relevant to this change: if a decrease in the amount of the resource is confirmed, then catch of the fish in question is partially or totally prohibited for a length of time.

mentable SCCs in this paper are *Supporting Price Independence* (**SPI**) [Gaspart (1998); Yoshihara (1998)] and *Non-manipulability of Irrelevant Skills** (**NIS***) which is slightly stronger than **NIS**. Note that **SPI** and **NIS*** are slightly weaker than **SPI** and *Independence of Unused Skills* (**IUS**) [Yamada and Yoshihara (2007)], where the latter two conditions characterize simple implementation as in Yamada and Yoshihara (2007). Surprisingly, though most of the well-known *fair allocation rules* are non-implementable in Yamada and Yoshihara (2007), they become implementable by the natural mechanisms. Actually, as it is shown below, any *efficient SCC* satisfying *non-discrimination* [Thomson (1983)] also satisfies **NIS***, though many of them do not satisfy **IUS**, so that many of such SCCs are implementable by the natural mechanisms.

The model is defined in Section 2. In section 3 and 4 we respectively provide characterizations of Nash implementation and natural implementation, and in section 5 we give some examples of implemantable and non-implementable SCCs. Concluding remarks appear in Section 6.

2 The Basic Model

There are two goods, one of which is an input (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[6] There is a set $N = \{1, \ldots, n\}$ of agents, where $2 \leq n < +\infty$. Each agent i's consumption is denoted by $z_i = (x_i, y_i)$, where x_i denotes her labor time, and y_i the amount of her output. All agents face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, and so have the same consumption set $Z \equiv [0, \bar{x}] \times \mathbb{R}_+$.

Each i's preference is defined on Z and represented by a utility function $u_i : Z \to \mathbb{R}$, which is continuous and quasi-concave on Z, and strictly monotonic (decreasing in labor time and increasing in the share of output) on $\overset{\circ}{Z} \equiv [0, \bar{x}) \times \mathbb{R}_{++}$.[7] We use $\mathcal{U}$ to denote the class of such utility functions. Moreover, we impose an additional condition on $\mathcal{U}$ as follows:

Assumption 1: $\forall i \in N$, $\forall z_i \in \overset{\circ}{Z}$, $\forall z'_i = (x'_i, 0) \in Z$, $u_i(z_i) > u_i(z'_i)$.

Each i has a **labor skill** $s_i \in \mathbb{R}_{++}$. The universal set of skills for all

[6] The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.
[7] The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

agents is denoted by $\mathcal{S} = \mathbb{R}_{++}$.[8] The labor skill $s_i \in \mathcal{S}$ is i's **effective labor supply** per hour measured in efficiency units. It can also be interpreted as i's **labor intensity** exercised in production.[9] Thus, if the agent's **labor time** is $x_i \in [0, \bar{x}]$ and her labor skill $s_i \in \mathcal{S}$, then $s_i x_i \in \mathbb{R}_+$ denotes the agent's **effective labor contribution** to production measured in efficiency units. The production technology is a function $f : \mathbb{R}_+ \to \mathbb{R}_+$, that is continuous, strictly increasing, concave, and such that $f(0) = 0$. For simplicity, we fix f. Thus, an economy is a pair of profiles $\boldsymbol{e} \equiv (\boldsymbol{u},\boldsymbol{s})$ with $\boldsymbol{u}= (u_i)_{i\in N} \in \mathcal{U}^n$ and $\boldsymbol{s}= (s_i)_{i\in N} \in \mathcal{S}^n$. Denote the class of such economies by $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$.

Given $\boldsymbol{s}\in \mathcal{S}^n$, an allocation $\boldsymbol{z}= (x_i, y_i)_{i\in N} \in Z^n$ is **feasible for $\boldsymbol{s}$** if $\sum y_i \leq f(\sum s_i x_i)$. Denote by $Z(\boldsymbol{s})$ the set of feasible allocations for $\boldsymbol{s}\in \mathcal{S}^n$. Given $\boldsymbol{s}\in \mathcal{S}^n$, a feasible allocation $\boldsymbol{z}\in Z(\boldsymbol{s})$ is **interior** if $z_i \in \overset{\circ}{Z}$ for all $i \in N$. Denote by $\overset{\circ}{Z}(\boldsymbol{s})$ the set of interior feasible allocations for $\boldsymbol{s}\in \mathcal{S}^n$. An allocation $\boldsymbol{z}= (z_i)_{i\in N} \in Z^n$ is **Pareto efficient for $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$** if $\boldsymbol{z}\in Z(\boldsymbol{s})$ and there does not exist $\boldsymbol{z}' = (z_i')_{i\in N} \in Z(\boldsymbol{s})$ such that for all $i \in N$, $u_i(z_i') \geq u_i(z_i)$, and for some $i \in N$, $u_i(z_i') > u_i(z_i)$. Let $P(\boldsymbol{e})$ denote the set of Pareto efficient allocations for $\boldsymbol{e} \in \mathcal{E}$. A **social choice correspondence** (SCC) or **solution** is a correspondence $\varphi : \mathcal{E} \twoheadrightarrow Z^n$ such that for each $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\varphi(\boldsymbol{e}) \subseteq \overset{\circ}{Z}(\boldsymbol{s}) \cap P(\boldsymbol{e})$. Given φ, $\boldsymbol{z}\in Z^n$ is **φ-optimal for $\boldsymbol{e} \in \mathcal{E}$** if $\boldsymbol{z}\in \varphi(\boldsymbol{e})$.

Let A_i, for each $i \in N$, denote the **strategy space** of agent i. We call $a_i \in A_i$ a **strategy of agent** $i \in N$, and $\boldsymbol{a} \in A \equiv \times_{i\in N} A_i$ a **strategy profile**. For any $\boldsymbol{a} \in A$ and $i \in N$, let $\boldsymbol{a}_{-i}$ be the list $(a_j)_{j\in N\setminus\{i\}} \in \times_{j\in N\setminus\{i\}} A_j$ of elements of the profile $\boldsymbol{a}$ for all agents except i. Denote the set of such $\boldsymbol{a}_{-i}$ by A_{-i} for each $i \in N$. Given a list $\boldsymbol{a}_{-i} \in A_{-i}$ and a strategy $a_i \in A_i$ of agent i, we denote by $(a_i, \boldsymbol{a}_{-i})$ the profile consisting of these a_i and $\boldsymbol{a}_{-i}$. A **mechanism** or **game form** γ is a pair $\gamma = (A, h)$, where $h : A \to Z^n$ is the outcome function such that, for each $\boldsymbol{a} \in A$, $h(\boldsymbol{a}) = (h_i(\boldsymbol{a}))_{i\in N} \in Z^n$. Let

[8] For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

[9] It might be more natural to define labor skill and labor intensity in a discriminative way: for example, if $\overline{s}_i \in \mathcal{S}$ is i's labor skill, then i's labor intensity is a variable s_i, where $0 < s_i \leq \overline{s}_i$. In such a formulation, we may view the amount of s_i as being determined endogenously by the agent i. In spite of this more natural view, we will assume in the following discussion that the labor intensity is a constant value, $s_i = \overline{s}_i$, for the sake of simplicity. The main theorems in the following discussion would remain valid with a few changes in the settings of the economic environments even if the labor intensity were assumed to be varied.

$h_i(A_i, \boldsymbol{a}_{-i}) \equiv \{h_i(a_i, \boldsymbol{a}_{-i}) \mid a_i \in A_i\}$. Denote the unversal set of such game forms by Γ.

Given $\gamma \in \Gamma$, for each economy $\boldsymbol{e} \in \mathcal{E}$, a (non-cooperative) game is given by $(N, \gamma, \boldsymbol{e})$. Fixing the set of players N, we simply denote a game $(N, \gamma, \boldsymbol{e})$ by $(\gamma, \boldsymbol{e})$. Given a game $(\gamma, \boldsymbol{e})$, a profile $\boldsymbol{a}^* \in A$ is a (**pure-strategy**) **Nash equilibrium of** $(\gamma, \boldsymbol{e})$ if for each $i \in N$ and each $a_i \in A_i$, $u_i(h_i(\boldsymbol{a}^*)) \geq u_i\left(h_i\left(a_i, \boldsymbol{a}^*_{-i}\right)\right)$. Let $NE(\gamma, \boldsymbol{e})$ denote the set of Nash equilibria of $(\gamma, \boldsymbol{e})$. An allocation $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in Z^n$ is a **Nash equilibrium allocation of** $(\gamma, \boldsymbol{e})$ if there exists $\boldsymbol{a} \in NE(\gamma, \boldsymbol{e})$ such that $h(\boldsymbol{a}) = \boldsymbol{z}$. Let $NA(\gamma, \boldsymbol{e})$ denote the set of Nash equilibrium allocations of $(\gamma, \boldsymbol{e})$. A mechanism $\gamma \in \Gamma$ **implements** φ **in Nash equilibria** if for each $\boldsymbol{e} \in \mathcal{E}$, $NA(\gamma, \boldsymbol{e}) = \varphi(\boldsymbol{e})$. An **SCC** φ **is implementable** if there exists a mechanism $\gamma \in \Gamma$ which implements φ in Nash equilibria.

Among various types of mechanisms in Γ, we are interested in mechanisms having the property of **labor sovereignty** [Kranich (1994); Gotoh et. al (2005)],[10] which says that every agent can choose freely her own labor time. As such, we focus on the following types of mechanisms. For each $i \in N$, let her strategy space be $A_i \equiv M_i \times [0, \bar{x}]$, with generic element (m_i, x_i). Note that here M_i stands for an abstract general message space as in classical mechanisms, while the members of $[0, \bar{x}]$, which represent i's choice of labor time as part of her observable action, are also considered as a strategic variable for i. Let $y \in \mathbb{R}_+$ be the *total output the coordinator observes after production*. Then, a **sharing mechanism** is a function $g : A \times \mathbb{R}_+ \rightarrow \mathbb{R}^n_+$ such that for each $(\boldsymbol{m}, \boldsymbol{x}) \in A$ and each $y \in \mathbb{R}_+$, $g(\boldsymbol{m}, \boldsymbol{x}, y) = \boldsymbol{y}$ for some $\boldsymbol{y} \in \mathbb{R}^n_+$. A sharing mechanism g is **feasible** if for each $(\boldsymbol{m}, \boldsymbol{x}) \in A$ and each $y \in \mathbb{R}_+$, $\sum g_i(\boldsymbol{m}, \boldsymbol{x}, y) \leq y$. We denote by $\mathcal{G}$ (*resp.* $\mathcal{G}^*$) the class of all sharing (*resp.* feasible sharing) mechanisms. In the following discussion, we assume that the production technology function f is known and the total output after production is observable to the coordinator. Thus, for each $\boldsymbol{s} \in \mathcal{S}^n$ and each $\boldsymbol{x} \in [0, \bar{x}]^n$, $y = f(\sum s_j x_j)$ is known to the coordinator after production, without the true information about $\boldsymbol{s}$.[11] Then, $g \in \mathcal{G}^*$ implies

[10] The previous mechanisms such as Suh (1995), Yoshihara (1999, 2000a), Tian (2000) do not have this property.

[11] Since the coordinator also knows f and $\boldsymbol{x}$, he can figure out that the true skill profile belongs to the hyperplane $\left\{\boldsymbol{s} \in \mathcal{S}^n \mid \boldsymbol{s}\cdot\boldsymbol{x} = f^{-1}(y)\right\}$. However, the exact location of the true skill profile in this hyperplane cannot be figured out. Note that, to see which of the feasible allocations are true φ-optimal allocations, one needs to know the information of the true skill profile.

that for each $\boldsymbol{s} \in \mathcal{S}^n$ and each $(\boldsymbol{m}, \boldsymbol{x}) \in A$, $(\boldsymbol{x}, g(\boldsymbol{m}, \boldsymbol{x}, f(\sum s_j x_j))) \in Z(\boldsymbol{s})$. In the following discussion, for each $g \in \mathcal{G}$, we simply write a value of g as $g(\boldsymbol{m}, \boldsymbol{x})$ instead of $g(\boldsymbol{m}, \boldsymbol{x}, f(\sum s_j x_j))$ except for when we define new mechanisms in $\mathcal{G}$.

Given $g \in \mathcal{G}$ (*resp.* $g \in \mathcal{G}^*$), a **sharing game** (*resp.* **feasible sharing game**) is defined for each economy $\boldsymbol{e} \in \mathcal{E}$ as a non-cooperative game $(N, A, g, \boldsymbol{e})$. Fixing the set of players N and their strategy sets A, we simply denote a sharing game (*resp.* feasible sharing game) $(N, A, g, \boldsymbol{e})$ by $(g, \boldsymbol{e})$.

Given a profile $(\boldsymbol{m}, \boldsymbol{x}) \in A$, let $(m_i', \boldsymbol{m}_{-i}, x_i', \boldsymbol{x}_{-i}) \in A$ be another strategy profile that is obtained by replacing the i-th component (m_i, x_i) of $(\boldsymbol{m}, \boldsymbol{x})$ with (m_i', x_i'). A profile $(\boldsymbol{m}^*, \boldsymbol{x}^*) \in A$ is a (**pure-strategy**) **Nash equilibrium of** $(g, \boldsymbol{e})$ if for each $i \in N$ and each $(m_i, x_i) \in A_i$, $u_i(x_i^*, g_i(\boldsymbol{m}^*, \boldsymbol{x}^*)) \geq u_i\left(x_i, g_i\left(m_i, \boldsymbol{m}_{-i}^*, x_i, \boldsymbol{x}_{-i}^*\right)\right)$. Let $NE(g, \boldsymbol{e})$ denote the set of Nash equilibria of $(g, \boldsymbol{e})$. An allocation $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in Z^n$ is a **Nash equilibrium allocation of** $(g, \boldsymbol{e})$ if there exists $\boldsymbol{m} \in M$ such that $(\boldsymbol{m}, \boldsymbol{x}) \in NE(g, \boldsymbol{e})$ and $\boldsymbol{y} = g(\boldsymbol{m}, \boldsymbol{x})$, where $\boldsymbol{x} = (x_i)_{i \in N}$ and $\boldsymbol{y} = (y_i)_{i \in N}$. Let $NA(g, \boldsymbol{e})$ denote the set of Nash equilibrium allocations of $(g, \boldsymbol{e})$.

3 Implementation: A General Characterization

We introduce two axioms as necessary conditions for Nash implementation. The first axiom is the well-known monotonicity condition. Given $u_i \in \mathcal{U}$ and $z_i \in Z$, let $L(z_i, u_i) \equiv \{z_i' \in Z \mid u_i(z_i') \leq u_i(z_i)\}$ be **the weakly lower contour set for** u_i **at** z_i. Then:

Monotonicity (M): *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}), \boldsymbol{e}' = (\boldsymbol{u}', \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, *if* $L(z_i, u_i) \subseteq L(z_i, u_i')$ *for each* $i \in N$, *then* $\boldsymbol{z} \in \varphi(\boldsymbol{u}', \boldsymbol{s})$.

This condition is slightly weaker than *Maskin Monotonicity* (Maskin 1999), since in the latter condition, the lower contour set of each agent is defined over the set of feasible allocations.

The second axiom is relevant to the change in individual skills.

Non-manipulability of Irrelevant Skills (NIS): *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, *for each* $\boldsymbol{e}' = (\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ *where* $s_i' = s_i$ *for each* $i \in N$ *with* $x_i > 0$, *if* $\boldsymbol{z} \in P(\boldsymbol{e}') \backslash \varphi(\boldsymbol{e}')$, *then for each* $i \in N$ *with* $s_i' \neq s_i$, *there is no* $z_i' \in Z$ *such that* $(z_i', \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{e}')$.

That is, suppose that the current economy changes, due to the change of someone's skill, where this agent supplies no labor hour in the current allocation, but this allocation is still efficient after this change of economy. Then, if this allocation is no longer φ-optimal, we can find no other φ-optimal allocation just by replacing this agent's consumption bundle.

If the profile of individual skills is known to the planner, then the framework can be reduced to the classical framework of Maskin (1999). In this case, it follows from Sjöström (1991) that, assuming that any SCC selects interior feasible allocations, an SCC is implementable if and only if it satisfies **M**, which is true even in $n = 2$. In contrast, if the profile of individual skills may be unknown to the planner, this characterization no longer holds. However, even in such an extended framework, we show that, under **Assumption 1**, an SCC is implementable if and only if it satisfies **M** and **NIS**.

Theorem 1: *If an SCC* φ *is* implementable*, then* φ *satisfies* **M** *and* **NIS**.

Proof. Let φ be an implementable SCC. Then, there exists a mechanism $\gamma = (A, h)$ such that for any $\boldsymbol{e}= (u_i, s_i)_{i \in N} \in \mathcal{E}$, $NA(\gamma, \boldsymbol{e}) = \varphi(\boldsymbol{e})$. The necessity of **M** is shown as usual, so we focus on the necessity of **NIS**.

For each $\boldsymbol{e}= (\boldsymbol{u}, \boldsymbol{s})$, $\boldsymbol{e}' = (u_i, s'_i)_{i \in N} \in \mathcal{E}$, and each $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, let $s_j = s'_j$ for each $j \in N$ with $x_j > 0$. Moreover, let $\boldsymbol{z} \in P(\boldsymbol{e}') \backslash \varphi(\boldsymbol{e}')$. Suppose for some $i \in N$ with $s'_i \neq s_i$, there exists $z'_i \in Z$ such that $(z'_i, \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{e}')$. Since $\boldsymbol{z} \in P(\boldsymbol{e}')$ and $(z'_i, \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{e}') \subseteq P(\boldsymbol{e}')$, $u_i(z_i) = u_i(z'_i)$ holds. By implementability of φ, there exist $\boldsymbol{a} \in NE(\gamma, \boldsymbol{e})$ and $\boldsymbol{a}' \in NE(\gamma, \boldsymbol{e}')$ such that $h(\boldsymbol{a}) = \boldsymbol{z}$ and $h(\boldsymbol{a}') = (z'_i, \boldsymbol{z}_{-i})$. By definition of Nash equilibrium, it follows that for any $j \neq i$, $h_j(A_j, \boldsymbol{a}_{-j}) \subseteq L(z_j, u_j)$ and $h_j\left(A_j, \boldsymbol{a}'_{-j}\right) \subseteq L(z_j, u_j)$; and for i, $h_i(A_i, \boldsymbol{a}_{-i}) \subseteq L(z_i, u_i)$ and $h_i\left(A_i, \boldsymbol{a}'_{-i}\right) \subseteq L(z'_i, u_i)$. Since $u_i(z_i) = u_i(z'_i)$ holds, $L(z_i, u_i) = L(z'_i, u_i)$. The last equation implies that $\boldsymbol{a} \in NE(\gamma, \boldsymbol{e}')$, and so $\boldsymbol{z} \in NA(\gamma, \boldsymbol{e}')$, which is a contradiction from implementability of φ, since $\boldsymbol{z} \notin \varphi(\boldsymbol{e}')$. Thus, φ satisfies **NIS**. ■

Theorem 2: *Let Assumption 1 hold. Then, if an SCC* φ *satisfies* **M** *and* **NIS***, then* φ *is* implementable *by a sharing mechanism.*

Let us construct a feasible sharing mechanism, which is used in the proof of Theorem 2 in Appendix A. Given $\boldsymbol{x} \in [0, \bar{x}]^n$ and $i \in N$, let $\pi(\boldsymbol{x}_{-i}) \equiv \max\left\{\frac{x_j + \bar{x}}{2} \mid x_j < \bar{x} \text{ for } j \neq i\right\}$. We construct the following two auxiliary outcome functions:

- Let $g^{\boldsymbol{y}}$ be such that for each $\boldsymbol{s} \in \mathcal{S}^n$, each $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, and each $i \in N$,
$$g_i^{\boldsymbol{y}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) = \begin{cases} f\left(\sum s_k x_k\right) & \begin{array}{l}\text{if } x_i = \pi(\boldsymbol{x}_{-i}) \text{ and} \\ y_i > \max\left\{f\left(\sum \sigma_k \bar{x}\right),\ \max\{y_j \mid j \neq i\}\right\},\end{array} \\ 0 & \text{otherwise.}\end{cases}$$
- Let $g^{\boldsymbol{\sigma}}$ be such that for each $\boldsymbol{s} \in \mathcal{S}^n$, each $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, and each $i \in N$,
$$g_i^{\boldsymbol{\sigma}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) = \begin{cases} f\left(\sum s_k x_k\right) & \text{if } x_i = 0,\ y_i = 0, \text{ and } \sigma_i > \sigma_j \text{ for each } j \neq i, \\ 0 & \text{otherwise.}\end{cases}$$

The function $g^{\boldsymbol{y}}$ assigns all of the produced output[12] to only one agent who provides the maximal *positive* amount, but less than $\bar{x}$, of labor time and reports a maximal demand for the output. The funciton $g^{\boldsymbol{\sigma}}$ assigns all of the produced output to only one agent who demands no output, reports the highest skill, and does not work.

As a preliminary step, let $p_\alpha(x_i; \boldsymbol{x}_{-i}, \boldsymbol{s}) \equiv \lim_{x_i' \to x_i} \frac{f\left(\sum_{j\neq i} s_j x_j + s_i x_i'\right) - f\left(\sum_{j\neq i} s_j x_j + s_i x_i\right)}{s_i x_i' - s_i x_i} s_i$, where $\alpha =$ '+' if $x_i' > x_i$; and $\alpha =$'−' if $x_i' < x_i$. Given $(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \mathcal{U}^n \times \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let

$$\begin{aligned} & N(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \\ \equiv\ & \left\{i \in N \mid \exists (x_i', y_i') \in Z \text{ s.t. } \left((x_i', \boldsymbol{x}_{-i}), \left(y_i', \boldsymbol{y}_{-i}\right)\right) \in \varphi(\boldsymbol{u},\boldsymbol{\sigma})\right\}. \end{aligned}$$

Note that if $i \in N(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$, then it might be the case that there are multiple (σ_i', x_i', y_i') such that $\left((x_i', \boldsymbol{x}_{-i}), \left(y_i', \boldsymbol{y}_{-i}\right)\right) \in \varphi(\boldsymbol{u}, (\sigma_i', \boldsymbol{\sigma}_{-i}))$. Then, let

$$(\sigma_i^\mu, x_i^\mu, y_i^\mu) \equiv \arg \inf_{\left\{(\sigma_i', x_i', y_i') \mid \left((x_i', \boldsymbol{x}_{-i}), (y_i', \boldsymbol{y}_{-i})\right) \in \varphi\left(\boldsymbol{u}, (\sigma_i', \boldsymbol{\sigma}_{-i})\right)\right\}} \sigma_i' x_i'.$$

Note that $y_i^\mu - p_-(x_i^\mu; \boldsymbol{x}_{-i}, (\sigma_i^\mu, \boldsymbol{\sigma}_{-i}))\, x_i^\mu \leq y_i' - p_-(x_i'; \boldsymbol{x}_{-i}, (\sigma_i', \boldsymbol{\sigma}_{-i}))\, x_i'$ holds for any (σ_i', x_i', y_i') with $\left((x_i', \boldsymbol{x}_{-i}), \left(y_i', \boldsymbol{y}_{-i}\right)\right) \in \varphi(\boldsymbol{u}, (\sigma_i', \boldsymbol{\sigma}_{-i}))$.

Denote the upper boundary of $L(z_i, u_i)$ by $\partial L(z_i, u_i)$. We define $g^* \in \mathcal{G}$ with $M_i \equiv \mathcal{U}^n \times \mathcal{S} \times \mathbb{R}_+$, with generic element $(\boldsymbol{u}^i, \sigma, y)$, for each $i \in N$, as follows:

For each $\boldsymbol{s} \in \mathcal{S}^n$ and each $\boldsymbol{\tau} = \left((\boldsymbol{u}^i)_{i\in N}, \boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}\right) \in (\mathcal{U}^n)^n \times \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$,
Rule 1: if $f\left(\sum \sigma_k x_k\right) = f\left(\sum s_k x_k\right)$, and

[12] We implicitly assume that the mechanism coordinator can hold all of the produced output after the production process, although he may not monitor that process perfectly.

1-1: there exists $\boldsymbol{u} \in \mathcal{U}^n$ such that $\boldsymbol{u}^i = \boldsymbol{u}$ for each $i \in N$ and
1-1-a): if $(\boldsymbol{x},\boldsymbol{y}) \in \varphi(\boldsymbol{u},\boldsymbol{\sigma})$, then $g^*(\boldsymbol{\tau}) = \boldsymbol{y}$,
1-1-b): if $(\boldsymbol{x},\boldsymbol{y}) \in P(\boldsymbol{u},\boldsymbol{\sigma}) \backslash \varphi(\boldsymbol{u},\boldsymbol{\sigma})$, then $g^*(\boldsymbol{\tau}) = \mathbf{0}$,
1-2: there exists $j \in N$ such that $\boldsymbol{u}^i = \boldsymbol{u}$ for each $i \neq j$, $(\boldsymbol{x},\boldsymbol{y}) \notin \varphi(\boldsymbol{u}^j,\boldsymbol{\sigma})$, and
1-2-a): if $j \in N(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$, then $g_i^*(\boldsymbol{\tau}) = 0$ for each $i \neq j$, and $g_j^*(\boldsymbol{\tau}) = \begin{cases} y_j'' & \text{if } y_j > f\left(\sum \sigma_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$
where y_j'' is given by
$$\left(x_j, y_j''\right) \in \begin{cases} \partial L\left(z_j', u_j\right) & \text{if } x_j > 0 \\ \left\{\left(0, y_j^{\mu} - p_{-}\left(x_j^{\mu}; \boldsymbol{x}_{-j}, \left(\sigma_j^{\mu}, \boldsymbol{\sigma}_{-j}\right)\right) x_j^{\mu}\right)\right\} & \text{otherwise,} \end{cases}$$
for $z_j' = (x_j', y_j')$ with $\left(\left(x_j', \boldsymbol{x}_{-j}\right), \left(y_j', \boldsymbol{y}_{-j}\right)\right) \in \varphi(\boldsymbol{u},\boldsymbol{\sigma})$,
1-2-b): if $j \notin N(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ and there exists $(x_j', y_j') \in Z$ such that $\left(\left(x_j', \boldsymbol{x}_{-j}\right), \left(y_j', \boldsymbol{y}_{-j}\right)\right) \in P(\boldsymbol{u}^j,\boldsymbol{\sigma})$, then $g_i^*(\boldsymbol{\tau}) = 0$ for each $i \neq j$, and
$$g_j^*(\boldsymbol{\tau}) = \begin{cases} f\left(\sum s_k x_k\right) & \text{if } x_j > 0 \text{ and } y_j > f\left(\sum \sigma_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$
1-3: in any other case, $g^*(\boldsymbol{\tau}) = g^{\boldsymbol{y}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$,

Rule 2: if $f\left(\sum \sigma_k x_k\right) \neq f\left(\sum s_k x_k\right)$, then $g^*(\boldsymbol{\tau}) = g^{\boldsymbol{\sigma}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$.

Note that this g^* is not feasible, due to **Rule 1-2-a)**.

Corollary 1: *Let Assumption 1 hold. Then, an SCC φ is* implementable *if and only if φ satisfies* **M** *and* **NIS**.

4 Implementation by Natural Sharing Mechanisms

We define Nash implementation by natural sharing mechanisms. The set of price vectors is the unit simplex $\Delta \equiv \{p = (p_x, p_y) \in \mathbb{R}_+ \times \mathbb{R}_+ \mid p_x + p_y = 1\}$, where p_x represents the price of labor (measured in efficiency units) and p_y the price of output.

Definition 1: *A vector $p \in \Delta$ is an* efficiency price *for $\boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \in Z^n$ at $\boldsymbol{e} = (\boldsymbol{u},\boldsymbol{s}) \in \mathcal{E}$ if*
(i) *for each $x' \in \mathbb{R}_+$, $p_y f(x') - p_x x' \leq \sum (p_y y_i - p_x s_i x_i)$;*

(ii) *for each* $i \in N$ *and each* $z_i' \in Z$, *if* $u_i(z_i') \geq u_i(z_i)$, *then* $p_y y_i' - p_x s_i x_i' \geq p_y y_i - p_x s_i x_i$.

The set of efficiency prices for $\boldsymbol{z}$ at $\boldsymbol{e}$ is denoted by $\Delta^P(\boldsymbol{e}, \boldsymbol{z})$. Given $\boldsymbol{s} \in \mathcal{S}^n$, $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \in Z^n$, and $p \in \Delta$, let $B(p, s_i, z_i) \equiv \{z_i' \in Z \mid p_y y_i' - p_x s_i x_i' \leq p_y y_i - p_x s_i x_i\}$.

Definition 2: *An SCC* φ *is* naturally implementable, *if there exists a feasible sharing mechanism* $g \in \mathcal{G}^*$ *with* $A_i = \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$ *(*$\forall i \in N$*) such that*:

(i) g implements φ in Nash equilibria;
(ii) g *is* forthright: *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{e})$, *there exists* $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ *such that* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{y}$ *with* $\boldsymbol{\rho} = (\rho_i)_{i \in N} = (p, \ldots, p)$;
(iii) *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, *if* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{y}$ *such that* $\boldsymbol{\rho} = (\rho_i)_{i \in N} = (p, \ldots, p) \in \left(\Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})\right)^n$, *then for each* $i \in N$ *and each* $(\rho_i', \sigma_i', x_i', y_i') \in A_i$,

$$\left(x_i', g_i\left(\left(\rho_i', \sigma_i', x_i', y_i', \boldsymbol{\rho}_{-i}, \boldsymbol{s}_{-i}, \boldsymbol{x}_{-i}, \boldsymbol{y}_{-i}\right), f\left(\sum_{j \neq i} s_j x_j + s_i x_i'\right)\right)\right) \in B(p, s_i, z_i).$$

Let us call a feasible sharing mechanism satisfying Definition 2 a *natural sharing mechanism.*

Two new axioms are introduced as necessary conditions for natural implementation. As a preliminary step, given $p \in \Delta$ and $(\boldsymbol{s}, \boldsymbol{z}) \in \mathcal{S}^n \times Z^n$, let $P^{-1}(p, \boldsymbol{s}, \boldsymbol{z}) \equiv \left\{\boldsymbol{u} \in \mathcal{U}^n \mid \boldsymbol{z} \in P(\boldsymbol{u}, \boldsymbol{s}) \text{ and } p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})\right\}$.

Supporting Price Independence (SPI) [Yoshihara (1998); Gaspart (1998)]**:** *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, *there exists* $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $\mathbf{e}' = (\boldsymbol{u}', \boldsymbol{s}) \in \mathcal{E}$, *if* $p \in \Delta^P(\mathbf{e}', \boldsymbol{z})$, *then* $\boldsymbol{z} \in \varphi(\mathbf{e}')$.

Let $\Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \equiv \left\{p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z}) \mid \forall \boldsymbol{u}' \in \mathcal{U}^n \text{ s.t. } p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{z}), \boldsymbol{z} \in \varphi(\boldsymbol{u}', \boldsymbol{s})\right\}$ for $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s})$.

Non-manipulability of Irrelevant Skills* **(NIS*):** *For each* $\mathbf{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, *there exists* $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $\mathbf{s}' \in \mathcal{S}^n$ *where* $s_i' = s_i$ *for each* $i \in N$ *with* $x_i > 0$, *and each* $\boldsymbol{u}' \in P^{-1}(p, \boldsymbol{s}', \boldsymbol{z})$, *if* $\boldsymbol{z} \in P(\boldsymbol{u}', \boldsymbol{s}') \setminus \varphi(\boldsymbol{u}', \boldsymbol{s}')$, *then for each* $i \in N$ *with* $s_i' \neq s_i$, *there is no* $z_i' \in Z$ *such that* $(z_i', \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{u}', \boldsymbol{s}')$.

Note that **NIS*** implies **NIS**.

Theorem 3: *If an SCC* φ *is* naturally implementable*, then* φ *satisfies* **SPI** *and* **NIS****.*

Proof. Let φ be an *SCC* that is naturally implementable. Then, there exists $g \in \mathcal{G}^*$ that satisfies conditions (i)-(iii) in Definition 2. The necessity of **SPI** is shown as in Yamada and Yoshihara (2007).

Show the necessity of **NIS***. Given $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ and $\boldsymbol{e}' = (\boldsymbol{u}', \boldsymbol{s}') = (u_i', s_i')_{i \in N} \in \mathcal{E}$, let $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in \varphi(\boldsymbol{e})$, $s_i' = s_i$ for each $i \in N$ with $x_i > 0$, and there exist $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}', \boldsymbol{z})$. Suppose that $\boldsymbol{z} \in P(\boldsymbol{e}') \setminus \varphi(\boldsymbol{e}')$ and there exists $z_j' \in Z$ such that $(z_j', \boldsymbol{z}_{-j}) \in \varphi(\boldsymbol{e}')$. Since φ satisfies **SPI**, $\boldsymbol{z} \in P(\boldsymbol{e}') \setminus \varphi(\boldsymbol{e}')$ implies that there exists $j \in N$ with $x_j = 0$ and $s_j' \neq s_j$. From (ii), for $\boldsymbol{\rho} = (\rho_i)_{i \in N}$ with $\rho_i = p$ for each $i \in N$, $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$ and $g(\boldsymbol{\tau}) = \boldsymbol{y}$. Therefore, from (iii), $g_i\left((\tau_i^*, \boldsymbol{\tau}_{-i}), f\left(\sum_{k \neq i} s_k x_k + s_i x_i^*\right)\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i (x_i^* - x_i)\right\}$ for each $i \in N$ and each $\tau_i^* \in A_i$. Moreover, from (ii), $\boldsymbol{\tau}' = (\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}', \boldsymbol{y}') \in NE(g, \boldsymbol{e}')$ and $g(\boldsymbol{\tau}') = \boldsymbol{y}'$, where $x_i' = x_i$ and $y_i' = y_i$ for any $i \neq j$, and from (iii), $g_i\left((\tau_i^*, \boldsymbol{\tau}'_{-i}), f\left(\sum_{k \neq i} s_k' x_k' + s_i' x_i^*\right)\right) \leq \max\left\{0, y_i' + \frac{p_x}{p_y} s_i' (x_i^* - x_i')\right\}$ for each $i \in N$ and each $\tau_i^* \in A_i$.

Note that, for any $i \neq j$, $g_i(\boldsymbol{\tau}') = y_i$ and $g_i\left((\tau_i^*, \boldsymbol{\tau}'_{-i}), f\left(\sum_{k \neq i} s_k' x_k' + s_i' x_i^*\right)\right) \leq \max\left\{0, y_i + \frac{p_x}{p_y} s_i (x_i^* - x_i)\right\}$ for each $\tau_i^* \in A_i$. For j, $g_j\left((\tau_j, \boldsymbol{\tau}'_{-j}), f\left(\sum_{k \neq j} s_k' x_k'\right)\right) = g_j\left((\tau_j, \boldsymbol{\tau}_{-j}), f\left(\sum_{k \neq j} s_k x_k\right)\right) = y_j$ and $g_j\left((\tau_j^*, \boldsymbol{\tau}'_{-j}), f\left(\sum_{k \neq j} s_k x_k + s_j' x_j^*\right)\right) \leq \max\left\{0, y_j' + \frac{p_x}{p_y} s_j' \left(x_j^* - x_j'\right)\right\}$ for each $\tau_i^* \in A_i$. Note that, since $\boldsymbol{z} \in P(\boldsymbol{e}') \setminus \varphi(\boldsymbol{e}')$ and $(z_j', \boldsymbol{z}_{-j}) \in \varphi(\boldsymbol{e}')$, $u_j'(z_j) = u_j'(z_j')$ holds. Moreover, since $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}', \boldsymbol{z})$, $y_j = y_j' + \frac{p_x}{p_y} s_j' (x_j - x_j') = y_j' - \frac{p_x}{p_y} s_j' x_j'$. In summary, these arguments imply that $(\tau_j, \boldsymbol{\tau}'_{-j}) = \boldsymbol{\tau} \in NE(g, \boldsymbol{e}')$ and $g\left((\tau_j, \boldsymbol{\tau}'_{-j}), f\left(\sum_{k \neq j} s_k' x_k'\right)\right) = g\left(\boldsymbol{\tau}, f\left(\sum_{k \neq j} s_k x_k\right)\right) = \boldsymbol{y}$. Hence, $\boldsymbol{z} \in NA(g, \boldsymbol{e}') = \varphi(\boldsymbol{e}')$, which is a contradiction. Thus, $\boldsymbol{z} \in P(\boldsymbol{e}') \setminus \varphi(\boldsymbol{e}')$ implies that there is no $z_j' \in Z$ such that $(z_j', \boldsymbol{z}_{-j}) \in \varphi(\boldsymbol{e}')$. ■

Theorem 4: *Let Assumption 1 hold. Then, if an SCC* φ *satisfies* **SPI** *and* **NIS****, then* φ *is* naturally implementable*.*

Given $p \in \Delta$ and $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $\varphi^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \equiv \left\{\boldsymbol{u} \in \mathcal{U}^n \mid (\boldsymbol{x},\boldsymbol{y}) \in \varphi(\boldsymbol{u},\boldsymbol{\sigma}) \text{ and } p \in \Delta^{SPI}(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})\right\}$. Given $p \in \Delta$ and $(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \equiv \{i \in N \mid \exists (x'_i, y'_i) \in [0,\bar{x}] \times \mathbb{R}_+$ s.t. $\varphi^{-1}\left(p,\boldsymbol{\sigma},(x'_i,\boldsymbol{x}_{-i}),\left(y'_i,\boldsymbol{y}_{-i}\right)\right) \neq \varnothing\}$.

Given a strategy profile $(\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ such that for each j, $\rho^j = p$, $i \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ is called a "*potential deviator,*" for the following reason. Suppose $\varphi^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) = \varnothing$ and $N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \neq \varnothing$. The first equation implies that $(\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ is inconsistent with φ. The second equation $N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \neq \varnothing$ implies that there is an agent i who can switch his strategy to another one $(\rho^i,\sigma_i,x'_i,y'_i)$ so that the new profile $\left(\boldsymbol{\rho},\boldsymbol{\sigma},(x'_i,\boldsymbol{x}_{-i}),\left(y'_i,\boldsymbol{y}_{-i}\right)\right)$ is consistent with φ. That is, it may be this agent i who makes the current strategy profile $(\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ inconsistent with φ. This means that $i \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ is a "potential deviator."

We define $g^{**} \in \mathcal{G}^*$ with $M_i \equiv \Delta \times \mathcal{S} \times \mathbb{R}_+$ for each $i \in N$, as follows:

For each $\boldsymbol{s} \in \mathcal{S}^n$ and each $\boldsymbol{\tau} = (\boldsymbol{\rho},\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \in \Delta^n \times \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$,

Rule 1: if $f(\sum \sigma_k x_k) = f(\sum s_k x_k)$, and

1-1: there exists $p \in \Delta$ such that $\rho^i = p$ for each $i \in N$ and

1-1-a): if $\varphi^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \neq \varnothing$, then $g^{**}(\boldsymbol{\tau}) = \boldsymbol{y}$,

1-1-b): if $\varphi^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) = \varnothing$ and $P^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) \neq \varnothing$, then $g^{**}(\boldsymbol{\tau}) = \mathbf{0}$,

1-2: there exist $j \in N$ and $p \in \Delta$ such that $\rho^i = p$ for each $i \neq j$, $\varphi^{-1}(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y}) = \varnothing$, and

1-2-a): if $j \in N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$, then $g^*_i(\boldsymbol{\tau}) = 0$ for each $i \neq j$, and $g^{**}_j(\boldsymbol{\tau}) =$

$$\begin{cases} \max\left\{0, \min\left\{y'_j + \frac{p_x}{p_y}\left(\sigma_j x_j - \sigma_j x'_j\right), f\left(\sum s_k x_k\right)\right\}\right\} & \text{if } y_j > f\left(\sum \sigma_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$

for (x'_j, y'_j) with $\varphi^{-1}\left(p,\boldsymbol{\sigma},\left(x'_j,\boldsymbol{x}_{-j}\right),\left(y'_j,\boldsymbol{y}_{-j}\right)\right) \neq \varnothing$,

1-2-b): if $j \notin N(p,\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$ and there exists $\left(x'_j, y'_j\right) \in [0,\bar{x}] \times \mathbb{R}_+$ such that $P^{-1}\left(p,\boldsymbol{\sigma},\left(x'_j,\boldsymbol{x}_{-j}\right),\left(y'_j,\boldsymbol{y}_{-j}\right)\right) \neq \varnothing$, then $g^{**}_i(\boldsymbol{\tau}) = 0$ for each $i \neq j$, and

$$g^{**}_j(\boldsymbol{\tau}) = \begin{cases} f\left(\sum s_k x_k\right) & \text{if } x_j > 0 \text{ and } y_j > f\left(\sum \sigma_k \bar{x}\right) \\ 0 & \text{otherwise,} \end{cases}$$

1-3: in any other case, $g^{**}(\boldsymbol{\tau}) = g^{\boldsymbol{y}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$,

Rule 2: if $f(\sum \sigma_k x_k) \neq f(\sum s_k x_k)$, then $g^{**}(\boldsymbol{\tau}) = g^{\boldsymbol{\sigma}}(\boldsymbol{\sigma},\boldsymbol{x},\boldsymbol{y})$.

This mechanism works well even in economies of *two agents.*

Corollary 2: *Let Assumption 1 hold. Then, an SCC φ is* naturally implementable *if and only if φ satisfies* **SPI** *and* **NIS***.

If each agent i can control her contribution by selecting $\widetilde{s}_i \in [0, s_i]$, Corollary 2 still applies. Though we only focus on Nash implementation, it can be shown, by slightly reformulating g^{**} as a two-stage mechanism, that any *SCC* is *triply* naturally implementable in Nash, strong Nash, and subgame perfect equilibria if and only if it satisfies **SPI** and **NIS***.

In the above characterization, the mechanism g^{**} satisfies neither the *balancedness* [Hurwicz et al. (1995)] nor the *best response property* [Jackson et. al (1994)]. However, as shown in Appendix (B), no *SCC* is implementable by balanced natural sharing mechanisms. This suggests that there is a trade-off between labor sovereignty and balancedness of mechanisms. Given the primary relevance of the former condition, the latter condition is not required in this paper. Also, as shown in Appendix (C), no *SCC* is implementable by a natural sharing mechanism satisfying the best response property.

Finally, let us define Nash implementation by simple mechanisms.

Definition 3: *An SCC* φ *is* simply implementable, *if there exists a feasible sharing mechanism* $g \in \mathcal{G}^*$ *with* $A_i = \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$ $(\forall i \in N)$ *such that* ***Definition 2****-(i), (ii), and (iii), and*
(iv) *for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$*, if* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e})$*, then for each* $\boldsymbol{e}' = (\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ *where* $s'_i = s_i$ *for each* $i \in N$ *with* $x_i > 0$, $[(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y}) \in NE(g, \boldsymbol{e}')$ *and* $g(\boldsymbol{\rho}, \boldsymbol{s}', \boldsymbol{x}, \boldsymbol{y}) = g(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})]$.

This additional condition (iv) requires that the sharing of outputs is independent of the skill parameters stated by "non-working" agents. It simplifies the sharing process, but it is not necessarily an indispensable condition.

The next axiom was introduced by Yamada and Yoshihara (2007) for simple implementation.

Independence of Unused Skills (IUS) [Yamada and Yoshihara (2007)]: *For each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ *and each* $\boldsymbol{z} \in \varphi(\boldsymbol{e})$*, there exists* $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z})$ *such that for each* $\boldsymbol{e}' = (\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ *where* $s'_i = s_i$ *for each* $i \in N$ *with* $x_i > 0$*, if* $p \in \Delta^P(\boldsymbol{e}', \boldsymbol{z})$*, then* $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$*.*

Note that **SPI** and **IUS** imply **SPI** and **NIS***, though **IUS** and **NIS*** are independent of each other.

Using this condition, Yamada and Yoshihara (2007) gave the characterization for simple implementation as follows:

Proposition 1: *Let Assumption 1 hold. Then, an SCC* φ *is* simply implementable *if and only if* φ *satisfies* **SPI** *and* **IUS***.*

More precisely, Yamada and Yoshihara (2007) characterized *triple implementation by simple mechanisms* in Nash, strong, and subgame perfect equilibria. However, in this model of economies, triple implementation by simple mechanisms is equivalent to simple implementation, because both are characterized by **SPI** and **IUS**.

5 Applications

As discussed in the present literature such as Maskin (1999) and Yamada Yoshihara (2007), there are well-known SCCs which satisfy **M** and/or **SPI**. Thus, to see which SCCs are implementable and/or implementable by natural mechanisms, it is sufficient to examine which of such SCCs satisfies **NIS** and/or **NIS***.

We show that most of *equitable SCC*s are non-implementable by simple mechanisms. Remember that in production economies with unequal skills, the *no-envy and efficient solution* [Foley (1967)] is not well-defined, though the no-envy principle is compatible with **M**, while the egalitarian-equivalence [Pazner and Schmeidler (1978)] is incompatible. Thus, we may need to consider a weaker version of the no-envy principle for defining equitable SCCs. As one such examples, consider the *equal-opportunity-for-budget-set* (**EOB**) principle, which is a condition for the *basic income* policy (Van Parijs 1995), and can be formulated as follows:

Set-inclusion Undomination (SIU):[13] *For each* $\boldsymbol{e}=(\boldsymbol{u},\boldsymbol{s})\in\mathcal{E}$ *and each* $\boldsymbol{z}\in\varphi(\boldsymbol{e})$, *there exists* $p\in\Delta^P(\boldsymbol{e},\boldsymbol{z})$ *such that for each* $i,j\in N$, *neither* $B(p,s_i,z_i)\subsetneq B(p,s_j,z_j)$ *nor* $B(p,s_i,z_i)\supsetneq B(p,s_j,z_j)$.

Any SCC satisfying the no-envy principle also satisfies **SIU**. Pareto efficiency and **SIU** are compatible. For instance, the *$\widetilde{u}$-reference welfare equivalent budget solution* [Fleurbaey and Maniquet (1996)] satisfies **SIU**.

Definition 4: *An SCC is the* $\widetilde{u}$-reference welfare equivalent budget solution $\varphi^{\widetilde{u}\text{-}RWEB}$ *if for each* $\boldsymbol{e}=(\boldsymbol{u},\boldsymbol{s})\in\mathcal{E}$, $\boldsymbol{z}\in\varphi^{\widetilde{u}\text{-}RWEB}(\boldsymbol{e})$ *implies that* $\boldsymbol{z}\in P(\boldsymbol{e})$; *and there exists* $p=(p_x,p_y)\in\Delta^P(\boldsymbol{e},\boldsymbol{z})$ *for* $\boldsymbol{z}$ *at* $\boldsymbol{e}=(\boldsymbol{u},\boldsymbol{s})\in\mathcal{E}$ *such that for any* $i,j\in N$, $\max_{z'\in B(p,s_i,z_i)}\widetilde{u}(z')=\max_{z'\in B(p,s_j,z_j)}\widetilde{u}(z')$.

[13] Van Parijs (1995) formulated the EOB principle as **Undominated Diversity** [Parijs (1995)], which is stronger than SIU.

The following corollary shows that $\varphi^{\tilde{u}\text{-}RWEB}$ is not simply implementable:

Corollary 3: *No SCC satisfying* **SIU** *is* simply implementable.

Proof. W.l.o.g., suppose that φ satisfies **SPI**. Let $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ and $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ such that for some $i \in N$, $x_i = 0$. Let $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})$ and for any $j \in N$, neither $B(p, s_i, z_i) \subsetneq B(p, s_j, z_j)$ nor $B(p, s_i, z_i) \supsetneq B(p, s_j, z_j)$.

If $s_i > \min_{k \in N}\{s_k \mid k \in N\}$, then consider $(\boldsymbol{u}, \boldsymbol{s}')$ such that $\boldsymbol{s}'_{-i} = \boldsymbol{s}_{-i}$ and $s'_i = \min_{k \in N}\{s_k \mid k \in N\}$. Let $\min_{k \in N}\{s_k \mid k \in N\} = s_j$. Then, $p \in \Delta^P(\boldsymbol{u}, \boldsymbol{s}', \boldsymbol{z})$, but $B(p, s'_i, z_i) \subsetneq B(p, s_j, z_j)$. Thus, $\boldsymbol{z} \notin \varphi(\boldsymbol{u}, \boldsymbol{s}')$, which implies that φ dose not satisfy **IUS**.

If $s_i \leq \min_{k \in N}\{s_k \mid k \in N\}$, then consider $(\boldsymbol{u}, \boldsymbol{s}')$ such that $\boldsymbol{s}'_{-i} = \boldsymbol{s}_{-i}$ and $s'_i > \min_{k \in N}\{s_k \mid k \in N\} \setminus \{s_i\}$. Consider $\boldsymbol{u}' \in \mathcal{U}^n$ such that $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}', \boldsymbol{z})$ with $\boldsymbol{u}'_{-i} = \boldsymbol{u}_{-i}$. Then, $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{z})$, which implies that $\boldsymbol{z} \in \varphi(\boldsymbol{u}', \boldsymbol{s})$, because φ satisfies **SPI**. Note that there exists $\min_{k \in N}\{s_k \mid k \in N\} \setminus \{s_i\} = s_j$. Then, $B(p, s'_i, z_i) \supsetneq B(p, s_j, z_j)$, so that $\boldsymbol{z} \notin \varphi(\boldsymbol{u}', \boldsymbol{s}')$. As $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}', \boldsymbol{z})$, this implies that φ dose not satisfy **IUS**. ■

Thus, if **SIU** is requested as a minimal condition of equity, then no equitable *SCC* is simply implementable. However, as we see, the condition of simple mechanisms is stringent, and it seems sufficient to consider natural mechanisms. Actually, in contrast to the case of simple implementation, there are many equitable *SCC*s which are naturally implementable.

To see this, let φ be an *SCC*. This φ meets *non-discrimination* if, for any $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, any $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s})$, and any $\boldsymbol{z}' \in P(\boldsymbol{u}, \boldsymbol{s})$ such that $u_i(z_i) = u_i(z'_i)$ for each $i \in N$, $\boldsymbol{z}' \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ holds. We have:

Lemma 1: *If an SCC satisfies* non-discrimination, *then it satisfies* **NIS***.

Proof. Let φ be an *SCC* not satisfying **NIS***. For each $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s})$, $\boldsymbol{e}' = (u'_i, s'_i)_{i \in N} \in \mathcal{E}$, and each $\boldsymbol{z} \in \varphi(\boldsymbol{e})$, let $s_j = s'_j$ for each $j \in N$ with $x_j > 0$, and let there exist $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}', \boldsymbol{z})$. Moreover, let $\boldsymbol{z} \in P(\boldsymbol{e}') \setminus \varphi(\boldsymbol{e}')$. Suppose for some $i \in N$ with $s'_i \neq s_i$, there exists $z'_i \in Z$ such that $(z'_i, \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{e}')$. Since $\boldsymbol{z} \in P(\boldsymbol{e}')$ and $(z'_i, \boldsymbol{z}_{-i}) \in \varphi(\boldsymbol{e}') \subseteq P(\boldsymbol{e}')$, $u'_i(z_i) = u'_i(z'_i)$ holds. Thus, since φ satisfies non-discrimination, $\boldsymbol{z} \in \varphi(\boldsymbol{e}')$, which demonstrates a contradiction. ■

Note that there is an *SCC* which does not satisfy non-discrimination, but satisfies **NIS***. For instance, the *proportional solution* [Roemer and Silvestre (1993); Roemer (1996; Chapter 5)] is such an *SCC*.

Corollary 4: *Any SCC satisfying* non-discrimination *is* implementable *if it satisfies* **M.** *Moreover, it is* naturally implementable *if it satisfies* **SPI**.

Thus, an equitable solution satisfying non-discrimination is naturally implementable if it satisfies **SPI**. There are many such *SCC*s, one such example is $\varphi^{\widetilde{u}\text{-}RWEB}$.

Is there an *SCC* which satisfies **M** but does not satisfy **NIS**? First of all, let us consider the case that the production function f is strictly concave. Then:

Lemma 2: *Let f be strictly concave. Then, any SCC satisfies* **NIS***.

Proof. Take any $\boldsymbol{e}= (\boldsymbol{u}, \boldsymbol{s})$, $\boldsymbol{e}' = (\boldsymbol{u}', \boldsymbol{s}') \in \mathcal{E}$ such that (i) $s_j = s'_j$ for each $j \in N\backslash \{1\}$, and $s_1 \neq s'_1$. Let $\boldsymbol{z}\in \varphi(\boldsymbol{e})$ with $z_1 = (0, y_1)$, and let there exist $p \in \Delta^P(\boldsymbol{e}, \boldsymbol{z}) \cap \Delta^P(\boldsymbol{e}', \boldsymbol{z})$. Suppose $\boldsymbol{z}\in P(\boldsymbol{e}') \backslash \varphi(\boldsymbol{e}')$. Then, given $\boldsymbol{z}_{-1}$, there is no other consumption bundle z'_1 such that $(z'_1, \boldsymbol{z}_{-1}) \in P(\boldsymbol{e}')$ holds. This is because f is strictly concave, so that for any efficiency price $p \in \Delta^P(\boldsymbol{e}', \boldsymbol{z})$, z_1 is the unique intersection point of $\partial B(p, s'_1, z_1)$ and the set $\left\{(x, y) \in Z \mid y = f\left(\sum_{j\neq 1} s_j x_j + s'_1 x\right)\right\}$. Thus, φ satisfies **NIS***. ■

Corollary 5: *Let f be strictly concave. Then, an SCC is implementable if and only if it satisfies* **M.** *Moreover, it is* naturally implementable *if and only if it satisfies* **SPI**.

Second, consider the case that the production function f is not strictly concave. In this case, we can find an *SCC* which satisfies **M** but does not satisfy **NIS**. Given $\boldsymbol{s}\in \mathcal{S}^n$, there exists an agent whose skill level is the lowest at $\boldsymbol{s}$ within the population. Denote such an agent at $\boldsymbol{s}$ by $i(\boldsymbol{s})$. Then:

Definition 5: *An SCC is the* maximal workfare solution φ^{WF} *if for each* $\boldsymbol{e}= (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, $\boldsymbol{z}\in \varphi^{WF}(\boldsymbol{e})$ *implies that there exists an efficiency price $p \in \Delta$ of $\boldsymbol{z}$ at $\boldsymbol{e}$ such that* $\boldsymbol{z} \in \arg\max_{z'\in P(e)} p_y y'_{i(s)} - p_x s_{i(s)} x'_{i(s)}$ *and there is no* $z''_{i(s)} \in Z$ *with* $x''_{i(s)} > x_{i(s)}$ *and* $\left(z''_{i(s)}, \boldsymbol{z}_{-i(s)}\right) \in P(\boldsymbol{e})$.

To see an implication of this solution, let us assume that f is linear. Then, let p^* be the efficiency price of any Pareto efficient allocation, which has the property that $\frac{p^*_y}{p^*_x} = \frac{f(x)}{x}$ holds for any $x > 0$. Given this p^*, if $\left(\left(x_{i(s)}, y_{i(s)}\right), \boldsymbol{z}_{-i(s)}\right), \left(\left(0, y^*_{i(s)}(\boldsymbol{e})\right), \boldsymbol{z}_{-i(s)}\right) \in P(\boldsymbol{e})$ with $y_{i(s)} = y^*_{i(s)}(\boldsymbol{e}) +$

$\frac{p_x^*}{p_y^*} s_{i(s)} x_{i(s)}$ are such that $u_{i(s)}\left(x_{i(s)}, y_{i(s)}\right) = u_{i(s)}\left(0, y_{i(s)}^*\left(\boldsymbol{e}\right)\right)$, then φ^{WF} never selects $\left(\left(0, y_{i(s)}^*\left(\boldsymbol{e}\right)\right), \boldsymbol{z}_{-i(s)}\right)$, since the welfare payment $y_{i(s)}^*\left(\boldsymbol{e}\right)$ via φ^{WF} is to urge the lowest skill agent to work. In other words, φ^{WF} provides the lowest skill agents with the maximal welfare payment if and only if they work as much as possible, which is the reason why we call φ^{WF} the *workfare* solution.

It is easy to see that φ^{WF} does not satisfy non-discrimination. Moreover:

Lemma 3: *Let f be linear. Then, φ^{WF} is an SCC which satisfies* **M**, *but does not satisfy* **NIS**.

Proof. It is easy to check that φ^{WF} satisfies **M**. Let us check that φ^{WF} does not satisfy **NIS**. Take $\boldsymbol{e}= (\boldsymbol{u}, \boldsymbol{s})$, $\boldsymbol{e}' = (\boldsymbol{u}, \boldsymbol{s}') \in \mathcal{E}$ such that (i) $s_j = s_j'$ and $s_1 < s_1' < s_j$ for each $j \in N \setminus \{1\}$; (ii) there is $\boldsymbol{z} \in P\left(\boldsymbol{e}\right)$ such that $\boldsymbol{z} = \arg\max_{z' \in P(e)} p_y y_1' - p_x s_1 x_1'$ and $z_1 = (0, y_1)$, where $\frac{p_y}{p_x} = \frac{f(x)}{x}$ holds for any $x > 0$, by linearity of f. Without loss of generality, let this z_1 be the unique solution to maximize $u_1\left(z\right)$ subject to $z \in B\left(p, s_1, z_1\right)$. Such uniqueness is guaranteed if s_1 is sufficiently small. Furthermore, (iii) let us assume under $\boldsymbol{e}'$ that there is an interval $[0, x_1']$ such that for any $x \in [0, x_1']$, $\left(x, y_1 + \frac{p_x}{p_y} s_1' x\right) \in \arg\max_{z \in B\left(p, s_1', z_1\right)} u_1\left(z\right)$.

By definition, $\boldsymbol{z} \in \varphi^{WF}\left(\boldsymbol{e}\right)$. Note $\boldsymbol{z} \in P\left(\boldsymbol{e}'\right)$. Then, $z_1 \in \arg\max_{z'' \in P(e')} p_y y_1'' - p_x s_1' x_1''$. To see this, let us take any $z_1^* \equiv \left(x_1^*, y_1^* + \frac{p_x}{p_y} s_1' x_1^*\right) \in \arg\max_{z'' \in P(e')} p_y y_1'' - p_x s_1' x_1''$, and suppose $y_1^* > y_1$. Let $\boldsymbol{z}^* \in P\left(\boldsymbol{e}'\right)$, whose 1st component is z_1^*. Then, $\left(\left(0, y_1^*\right), \boldsymbol{z}_{-1}^*\right) \in P\left(\boldsymbol{e}'\right)$ is also Pareto efficient for $\boldsymbol{e}$, which implies that $\boldsymbol{z} \notin \varphi^{WF}\left(\boldsymbol{e}\right)$, thus a contradiction. Therefore, $y_1^* = y_1$ holds, which implies the desired result.

By (iii), $z_1' \equiv \left(x_1', y_1 + \frac{p_x}{p_y} s_1' x_1'\right) \in \arg\max_{z'' \in P(e')} p_y y_1'' - p_x s_1' x_1''$, and $x_1' > 0$, which implies that $\boldsymbol{z} \notin \varphi^{WF}\left(\boldsymbol{e}'\right)$ whereas $\left(z_1', \boldsymbol{z}_{-1}\right) \in \varphi^{WF}\left(\boldsymbol{e}'\right)$. Thus, φ^{WF} does not satisfy **NIS**. ■

Corollary 6: *Let f be linear. Then, φ^{WF} is not implementable.*

Note that φ^{WF} is Nash-implementable if skills are not private information, since φ^{WF} satisfies **M**. Thus, Corollary 6 implies that, introducing the private information of skills makes the set of implementable *SCC*s properly shrink.

6 Concluding Remarks

We have characterized Nash implementation in production economies with unequal labor skills. Firstly, without any restriction of available mechanisms, we have characterized the class of efficient SCCs which are Nash-implementable in such economies. Then, we have seen that Maskin monotonicity alone is no longer sufficient for Nash implementation, and there is a Maskin-monotonic efficient SCC which is not implementable in these economies. Secondly, we have defined natural mechanisms in these economies, and then characterized the class of efficient SCCs which are Nash-implementable by natural mechanisms. By this characterization, many efficient and equitable SCCs are shown to be naturally implementable, though most of them were known to be non-implementable by simple mechanisms in the present literature. The definition of natural mechanisms in this paper allows that the mechanisms satisfy neither the balancedness nor the best response property. However, the loss of the former property is inevitable when we are interested in mechanisms having the labor sovereign property, whereas the loss of the latter property is also inevitable for implementation by natural mechanisms.

This paper presumes a simple setting of one-input and one-output economies with unequal skills among agents, and workability of the constructed mechanisms depends on this simple model. However, the main conclusions of this paper can be generalized to more complicated models of multi-input and multi-outputs economies with unequal skills, albeit at the cost of a substantial increase in unessential technicalities.

7 Appendix A

1. Proof of Theorem 2.

Lemma A1: *Let Assumption 1 hold. Let $g^* \in \mathcal{G}$ be as above. Given $(\boldsymbol{u},\boldsymbol{s}) \in \mathcal{E}$, let $\left((\boldsymbol{u}^i)_{i \in N}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}\right) \in (\mathcal{U}^n)^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ be a Nash equilibrium of $(g^*, \boldsymbol{u}, \boldsymbol{s})$ such that $f\left(\sum \sigma_k x_k\right) = f\left(\sum s_k x_k\right)$. Then, for each $i \in N$ with $x_i > 0$, $\sigma_i = s_i$.*

This proof is given as of Lemma 1 in Yamada and Yoshihara (2007).

Lemma A2: *Let Assumption 1 hold. Then, g^** implements *any SCC φ satisfying* **M** *and* **NIS** in Nash equilibria.

Proof. Let φ be an SCC satisfying **M** and **NIS**. Let $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$.

(1) First, we show that $\varphi(\boldsymbol{e}) \subseteq NA(g^*, \boldsymbol{e})$. Let $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = \left((\boldsymbol{u}^i)_{i \in N}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}\right) \in (\mathcal{U}^n \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$ be such that $\boldsymbol{u}^i = \boldsymbol{u}$ for each $i \in N$. Then, $g^*(\boldsymbol{\tau}) = \boldsymbol{y}$ from Rule 1-1. Suppose $j \in N$ deviates to $\tau'_j = \left(\boldsymbol{u}^{j\prime}, s'_j, x'_j, y'_j\right) \in \mathcal{U}^n \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$. From Assumption 1 and the continuity of utility functions, if $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$, it implies the worst outcome for j.

If τ'_j induces Rule 2, then $x'_j > 0$ and $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$. If τ'_j induces Rule 1-3, then either $\left(\left(x'_j, \boldsymbol{x}_{-j}\right), \left(y'_j, \boldsymbol{y}_{-j}\right)\right) \in \varphi\left(\boldsymbol{u}^j, \left(s'_j, \boldsymbol{s}_{-j}\right)\right)$ or $x'_j = 0$ and $s'_j \neq s_j$. The former implies $s'_j = s_j$ and $y'_j \leq f\left(\sum_{i \neq j} s_i \overline{x} + s'_j \overline{x}\right)$. Thus, in either case, $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$. If τ'_j induces Rule 1-2-b, then $x_j = 0$ and $s'_j \neq s_j$. Thus, $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$.

If τ'_j induces Rule 1-2-a), then either $x'_j > 0$ or $x'_j = 0$ and $s'_j \neq s_j$. In the former case, since $s'_j = s_j$, Rule 1-2-a) implies that $\left(x'_j, g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right)\right) \in L(z_j, u_j)$. In the latter case, there exists $\left(\sigma_j^\mu, x_j^\mu, y_j^\mu\right)$ such that $\left(\left(x_j^\mu, \boldsymbol{x}_{-j}\right), \left(y_j^\mu, \boldsymbol{y}_{-j}\right)\right) \in \varphi\left(\boldsymbol{u}, \left(\sigma_j^\mu, \boldsymbol{s}_{-j}\right)\right)$ and $y_j^\mu - p_-\left(x_j^\mu; \boldsymbol{x}_{-j}, \left(\sigma_j^\mu, \boldsymbol{s}_{-j}\right)\right) x_j^\mu \leq y_j - p_-(x_j; \boldsymbol{x}_{-j}, \boldsymbol{s}) x_j$ hold. Let $p = (p_x, p_y)$ be the efficiency price which supports $\boldsymbol{z}$ as a φ-optimal allocation at $\boldsymbol{e}$. Then, $y_j - \frac{p_x}{p_y} s_j x_j \geq y_j - p_-(x_j; \boldsymbol{x}_{-j}, \boldsymbol{s}) x_j$, so that $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = y_j^\mu - p_-\left(x_j^\mu; \boldsymbol{x}_{-j}, \left(\sigma_j^\mu, \boldsymbol{s}_{-j}\right)\right) x_j^\mu \leq y_j - \frac{p_x}{p_y} s_j x_j$. This implies $\left(x'_j, g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right)\right) \in L(z_j, u_j)$. Finally, if τ'_j induces Rule 1-1, then $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = y'_j = f(\sum_{i \neq j} s_i x_i + s_j x'_j) - \sum_{i \neq j} y_i$. Thus, since $\boldsymbol{z} \in P(\boldsymbol{e})$, $u_j\left(x'_j, y'_j\right) \leq u_j(z_j)$. In summary, j has no incentive to switch to τ'_j.

(2) Second, show $NA(g^*, \boldsymbol{e}) \subseteq \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = \left((\boldsymbol{v}^i)_{i \in N}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}\right) \in NE(g^*, \boldsymbol{e})$.

Suppose that $\boldsymbol{\tau}$ induces Rule 2. Then, either $N^0(\boldsymbol{x}) \equiv \{i \in N \mid x_i = 0\} = \varnothing$ or $N^0(\boldsymbol{x}) \neq \varnothing$. If $N^0(\boldsymbol{x}) = \varnothing$, then for each $i \in N$, $g_i^*(\boldsymbol{\tau}) = 0$. Then, if for each $k \in N$, $\sum_{i \neq k} \sigma_i x_i = \sum_{i \neq k} s_i x_i$, then $(n-1) \cdot (\sum \sigma_i x_i) = (n-1) \cdot (\sum s_i x_i)$, which contradicts from Rule 2. Thus, for some $j \in N$, $\sum_{i \neq j} \sigma_i x_i \neq \sum_{i \neq j} s_i x_i$. If j switches to $\tau'_j = \left(\boldsymbol{v}^{j\prime}, \sigma'_j, x'_j, y'_j\right)$ with $\sigma'_j > \max\{\sigma_i \mid i \neq j\}$, $x'_j = 0$, and $y'_j = 0$, then $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) > 0$ under Rule 2.

Let $N^0(\boldsymbol{x}) \neq \varnothing$ with $\# N^0(\boldsymbol{x}) \geq 2$. Then, for each $j \in N^0(\boldsymbol{x})$, if j's deviating strategy τ'_j is such that for each $i \neq j$, $\sigma'_j > \sigma_i$ and $\left(x'_j, y'_j\right) = (0, 0)$, then $g_j^*\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = f(\sum s_k x_k)$ under Rule 2.

Let $\# N^0(\boldsymbol{x}) = 1$ and $\# N \backslash N^0(\boldsymbol{x}) \geq 2$. Then, there exists $j \in N \backslash N^0(\boldsymbol{x})$ such that $\sum_{i \in N \backslash (N^0(\boldsymbol{x}) \cup \{j\})} \sigma_i x_i \neq \sum_{i \in N \backslash (N^0(\boldsymbol{x}) \cup \{j\})} s_i x_i$. Thus, j can switch to τ_j' such that $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) > 0$ under Rule 2. This can be shown in a similar way to the case of $N^0(\boldsymbol{x}) = \varnothing$.

Let $N^0(\boldsymbol{x}) = \{i\}$ and $N \backslash N^0(\boldsymbol{x}) = \{j\}$. If $y_i > 0$, then switching i's strategy to $\sigma_i' > \sigma_j$, $x_i' = 0$, and $y_i' = 0$ implies $g_i^*\left(\boldsymbol{v}^{i\prime}, \sigma_i', x_i', y_i', \boldsymbol{v}^j, \sigma_j, x_j, y_j\right) = f\left(s_j x_j\right)$ under Rule 2. If $y_i = 0$, then switching j's strategy to $\boldsymbol{v}^{j\prime} = \boldsymbol{v}^i$, $\sigma_j' = s_j$, $x_j' = \frac{\overline{x}}{2}$, and $y_j' > f\left(s_j\overline{x} + \sigma_i\overline{x}\right)$ implies $g_j^*\left(\boldsymbol{v}^i, \sigma_i, x_i, y_i, \boldsymbol{v}^{j\prime}, \sigma_j', x_j', y_j'\right) = f\left(s_j x_j'\right)$ under Rule 1-3. In summary, $\boldsymbol{\tau}$ does not induce Rule 2.

Suppose that $\boldsymbol{\tau}$ induces Rule 1-2 or 1-3. Then, there exists $j \in N$ such that $g_j^*(\boldsymbol{\tau}) = 0$. By Lemma A1, $\sigma_j = s_j$ or $x_j = 0$. Suppose $\boldsymbol{\tau}$ induces Rule 1-2. Then, $g_j^*(\boldsymbol{\tau}) = 0$ implies that $y_j \leq f\left(\sum \sigma_k \overline{x}\right)$. Then, j can either deviate to Rule 1-3 with $\sigma_j' = s_j$, $x_j' = \pi\left(\boldsymbol{x}_{-j}\right) < \overline{x}$, and $y_j' > \max\left\{f\left(\sum_{i \neq j} \sigma_i \bar{x} + \sigma_j' \bar{x}\right), \max\left\{y_i \mid i \neq j\right\}\right\}$ or get $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) > 0$ under Rule 1-2 by $y_j' > f\left(\sum_{i \neq j} \sigma_i \bar{x} + \sigma_j' \bar{x}\right)$. Suppose $\boldsymbol{\tau}$ induces Rule 1-3. Then, there exists τ_j' such that $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) > 0$ under Rule 1-3. In summary, $\boldsymbol{\tau}$ induces neither Rule 1-2 nor 1-3.

Suppose that $\boldsymbol{\tau}$ induces Rule 1-1-b). Then, $g^*(\boldsymbol{\tau}) = \mathbf{0}$. Then, some $j \in N$ can deviate to induce Rule 1-2, so that $g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right) > 0$, which is a contradiction.

Thus, $\boldsymbol{\tau}$ induces Rule 1-1-a), and $g^*(\boldsymbol{\tau}) = \boldsymbol{y}$. By definition of Rule 1-1-a), $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma})$ where $\boldsymbol{u}' = \boldsymbol{v}^i$ for all $i \in N$. Since $\boldsymbol{\tau} \in NE(g^*, \boldsymbol{e})$, $\sigma_i = s_i$ holds for any $i \in N$ with $x_i > 0$ by Lemma A1. Assume, without loss of generality, that there exists at most one unique individual j such that $x_j = 0$. Let us consider the following two cases below:

Case 1: Let $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}', \boldsymbol{s})$. Then, we can show that $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{s})$. Suppose that $(\boldsymbol{x}, \boldsymbol{y}) \notin \varphi(\boldsymbol{u}', \boldsymbol{s})$. Then, for the individual $j \in N$ with $x_j = 0$, $\sigma_j \neq s_j$. Then, if j takes the strategy $\tau_j' = \left(\boldsymbol{u}', s_j, x_j', y_j'\right)$ with $x_j' > 0$, $y_j' > f\left(\sum s_k \overline{x}\right)$, then **NIS** implies $j \notin N(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, so that Rule 1-2-b) can be applied. Then, if $x_j' > 0$ is sufficiently small, $u_j\left(x_j', g_j^*\left(\tau_j', \boldsymbol{\tau}_{-j}\right)\right) = u_j\left(x_j', f\left(\sum_{i \neq j} s_i x_i + s_j x_j'\right)\right) > u_j(0, y_j) = u_j\left(x_j, g_j^*(\boldsymbol{\tau})\right)$. This implies $(\boldsymbol{x}, \boldsymbol{y}) \notin NA(g^*, (\boldsymbol{u}, \boldsymbol{s}))$, which is a contradiction. Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{s})$. Then, $(\boldsymbol{x}, \boldsymbol{y}) \in NA(g^*, (\boldsymbol{u}, \boldsymbol{s}))$ implies that $L\left((x_i, y_i), u_i'\right) \subseteq L\left((x_i, y_i), u_i\right)$ holds for each $i \in N$ by Rule 1-2-a). Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ by **M**.

Case 2: Let $(\boldsymbol{x}, \boldsymbol{y}) \notin P(\boldsymbol{u}', \boldsymbol{s})$. Since $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}', \boldsymbol{\sigma})$, we have for the individual $j \in N$ with $x_j = 0$, $\sigma_j < s_j$. Then, there exists $\left(u_j'', \boldsymbol{u}_{-j}'\right) \in \mathcal{U}^n$ such that $L\left((x_i, y_i), u_i''\right) \supsetneq L\left((x_i, y_i), u_i'\right)$ and $(\boldsymbol{x}, \boldsymbol{y}) \in P\left(\left(u_j'', \boldsymbol{u}_{-j}'\right), \boldsymbol{s}\right)$. Let

$\boldsymbol{u}'' \equiv \left(u''_j, \boldsymbol{u}'_{-j}\right)$. Note that by **M**, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}'', \boldsymbol{\sigma})$. Then, by Rule 1-1-a), for $\boldsymbol{\tau}'' = \left((\boldsymbol{v}''^i)_{i \in N}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}\right)$ with $\boldsymbol{v}''^i = \boldsymbol{u}''$ $(\forall i \in N)$, $g^*(\boldsymbol{\tau}'') = \boldsymbol{y}$, which implies $\boldsymbol{\tau}'' \in NE(g^*, \boldsymbol{e})$. Suppose $(\boldsymbol{x}, \boldsymbol{y}) \notin \varphi(\boldsymbol{u}'', \boldsymbol{s})$. Then, by the same argument as in Case 1, j can induce Rule 1-2-b), so that $(\boldsymbol{x}, \boldsymbol{y}) \notin NA(g^*, (\boldsymbol{u}, \boldsymbol{s}))$, a contradiction. Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}'', \boldsymbol{s})$. Then, $\boldsymbol{\tau}'' \in NE(g^*, \boldsymbol{e})$ and $(\boldsymbol{x}, \boldsymbol{y}) \in NA(g^*, (\boldsymbol{u}, \boldsymbol{s}))$ imply that $L((x_i, y_i), u''_i) \subseteq L((x_i, y_i), u_i)$ holds for each $i \in N$ by Rule 1-2-a). Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ by **M**. ■

Proof of Theorem 2. From **Lemma A2**, we obtain the desired result. ■

2. Proof of Theorem 4.

Lemma A3: *Let Assumption 1 hold. Let $g^{**} \in \mathcal{G}^*$ be as previously defined. Given $(\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$, let $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}) \in \Delta^n \times \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ be a Nash equilibrium of $(g^{**}, \boldsymbol{u}, \boldsymbol{s})$ such that $f(\sum \sigma_k x_k) = f(\sum s_k x_k)$. Then, for each $i \in N$ with $x_i > 0$, $\sigma_i = s_i$.*

The proof of Lemma A3 is given in a similar way to that of Lemma A1.

Lemma A4: *Let Assumption 1 hold. Then,* g^{**} implements *any SCC φ satisfying* **SPI** *and* **NIS*** in Nash equilibria.

Proof. Let φ be a SCC satisfying **SPI** and **NIS***. Let $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$.

(1) First, we show that $\varphi(\boldsymbol{e}) \subseteq NA(g^{**}, \boldsymbol{e})$. Let $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in (\Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$ be such that $\rho^i = p = (p_x, p_y)$ for each $i \in N$ and $p \in \Delta^{SPI}(\boldsymbol{u}, \boldsymbol{s}, \boldsymbol{z})$. Then, $g^{**}(\boldsymbol{\tau}) = \boldsymbol{y}$ from Rule 1-1. Suppose $j \in N$ deviates to $\tau'_j = \left(\rho^{j\prime}, \sigma'_j, x'_j, y'_j\right) \in \Delta \times \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$.

If τ'_j induces Rule 1-1-b), Rule 1-3, or Rule 2, then $g^{**}_j\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$, as similarly shown in the corresponding part of the proof of Lemma A2. If τ'_j induces Rule 1-2-b), then $\sigma'_j = s_j$ does not hold, since $N\left(p, \left(\sigma'_j, \boldsymbol{s}_{-j}\right), \left(x'_j, \boldsymbol{x}_{-j}\right), \left(y'_j, \boldsymbol{y}_{-j}\right)\right) = N\left(p, \boldsymbol{s}, \left(x'_j, \boldsymbol{x}_{-j}\right), \left(y'_j, \boldsymbol{y}_{-j}\right)\right) = N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ and $j \in N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$. Thus, if τ'_j induces Rule 1-2-b), $\sigma'_j \neq s_j$ and $x'_j = 0$ hold, so that $g^{**}_j\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) = 0$. If τ'_j induces Rule 1-2-a) or Rule 1-1, then $g^{**}_j\left(\tau'_j, \boldsymbol{\tau}_{-j}\right) \leq y_j + \frac{p_x}{p_y}\left(s_j x'_j - s_j x_j\right)$. In summary, j has no incentive to switch to τ'_j.

(2) Second, show $NA(g^{**}, \boldsymbol{e}) \subseteq \varphi(\boldsymbol{e})$. Let $\boldsymbol{\tau} = (\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g^{**}, \boldsymbol{e})$.

Note that $\boldsymbol{\tau}$ can induce neither of Rule 1-1-b), Rule 1-2, Rule 1-3, or Rule 2, which is shown by almost the same way as the corresponding cases of Rule 1-1-b), Rule 1-2, Rule 1-3, and Rule 2 in the proof of Lemma A2.

Thus, $\boldsymbol{\tau}$ induces Rule 1-1-a), and $g^{**}(\boldsymbol{\tau}) = \boldsymbol{y}$. By definition of Rule 1-1-a), there exists $\boldsymbol{u}' \in \mathcal{U}^n$ such that for each $i \in N$, $\rho^i = p \in \Delta^{SPI}(\boldsymbol{u}', \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y})$ and $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{\sigma})$. Since $\boldsymbol{\tau} \in NE(g^{**}, \boldsymbol{e})$, $\sigma_i = s_i$ holds for any $i \in N$ with $x_i > 0$ by Lemma A3. Assume, without loss of generality, that there exists at most one unique individual j such that $x_j = 0$. Let us consider the following two cases below:

Case 1: Let $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}', \boldsymbol{s})$. Then, we can show that $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{s})$. Suppose that $(\boldsymbol{x}, \boldsymbol{y}) \notin \varphi(\boldsymbol{u}', \boldsymbol{s})$. Then, for the individual $j \in N$ with $x_j = 0$, $\sigma_j \neq s_j$. In this case, $j \notin N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ follows from **NIS***. This is because, first of all, by **SPI**, $(\boldsymbol{x}, \boldsymbol{y}) \notin \varphi(\boldsymbol{u}', \boldsymbol{s})$ together with $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ imply that $p \notin \Delta^{SPI}(\boldsymbol{u}'', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ holds for any $\boldsymbol{u}''$ with $p \in \Delta^P(\boldsymbol{u}'', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$. In fact, if $p \in \Delta^P(\boldsymbol{u}', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ and $p \in \Delta^{SPI}(\boldsymbol{u}'', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ for some $\boldsymbol{u}''$, then $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{s})$ by **SPI**, which is a contradiction. Thus, $\varphi^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \varnothing$. Hence, for any $\boldsymbol{u}'' \in P^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, $(\boldsymbol{x}, \boldsymbol{y}) \notin \varphi(\boldsymbol{u}'', \boldsymbol{s})$. Then, it follows from **NIS*** that, for any $\boldsymbol{u}'' \in P^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, there is no $(x_j', y_j') \in Z$ such that $((x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j})) \in \varphi(\boldsymbol{u}'', \boldsymbol{s})$. Suppose that there is $(x_j', y_j') \in Z$ such that $((x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j})) \in \varphi(\boldsymbol{u}'', \boldsymbol{s})$ and $\boldsymbol{u}'' \notin P^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$. Then, consider $\boldsymbol{u}''' = (u_j''', \boldsymbol{u}_{-j}'') \in \mathcal{U}^n$ such that $p \in \Delta^P(\boldsymbol{u}''', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \cap \Delta^P(\boldsymbol{u}''', \boldsymbol{s}, (x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j}))$. By **SPI**, $((x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j})) \in \varphi(\boldsymbol{u}''', \boldsymbol{s})$. However, since $\boldsymbol{u}''' \in P^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, $\varphi^{-1}(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = \varnothing$ and **NIS*** imply that $((x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j})) \notin \varphi(\boldsymbol{u}''', \boldsymbol{s})$, which is a contradiction. Thus, $j \notin N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$ holds.

Given this argument, if j takes the strategy $\tau_j' = (p, s_j, x_j', y_j')$ with $y_j' > f(\sum s_k \overline{x})$, then $j \notin N(p, \boldsymbol{s}, (x_j', \boldsymbol{x}_{-j}), (y_j', \boldsymbol{y}_{-j})) = N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, so that Rule 1-2-b) is induced. If $x_j' > 0$ is sufficiently small, then $u_j\left(x_j', f\left(\sum_{k \neq j} s_k x_k + s_j x_j'\right)\right) > u_j(0, y_j)$, which implies $u_j(x_j', g_j^{**}(\boldsymbol{\tau}_{-j}, \tau_j')) > u_j(x_j, g_j^{**}(\boldsymbol{\tau}))$, a contradiction from $\boldsymbol{\tau} \in NE(g^{**}, \boldsymbol{e})$. Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}', \boldsymbol{s})$. Note that $B(p, s_i, (x_i, y_i)) \subseteq L((x_i, y_i), u_i)$ holds for each $i \in N$ with $x_i > 0$ by Rule 1-2-a). Moreover, if it does not hold $B(p, s_j, (x_j, y_j)) \subseteq L((x_j, y_j), u_j)$ for $j \in N$ with $x_j = 0$, then j can choose the strategy (p, s_j, x_j', y_j') with sufficiently small $x_j' > 0$ and $y_j' > f(\sum s_k \overline{x})$, so that $(x_j', g_j^{**}(\tau_j', \boldsymbol{\tau}_{-j})) \notin L((x_j, y_j), u_j)$ under Rule 1-2-a), which is again a contradiction from $\boldsymbol{\tau} \in NE(g^{**}, \boldsymbol{e})$. Thus, $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}, \boldsymbol{s})$, and $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ by **SPI**.

Case 2: Let $(\boldsymbol{x}, \boldsymbol{y}) \notin P(\boldsymbol{u}', \boldsymbol{s})$. Since $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}', \boldsymbol{\sigma})$, for the individual $j \in N$ with $x_j = 0$, we have $\sigma_j < s_j$. Then, there exists $(u_j'', \boldsymbol{u}_{-j}') \in \mathcal{U}^n$ such that $L((x_i, y_i), u_i'') \supsetneq L((x_i, y_i), u_i')$ and $(\boldsymbol{x}, \boldsymbol{y}) \in P((u_j'', \boldsymbol{u}_{-j}'), \boldsymbol{s})$. Let

$\boldsymbol{u}'' \equiv \left(u''_j, \boldsymbol{u}'_{-j}\right)$. Note that since φ satisfies **SPI** which implies **M**, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}'', \boldsymbol{\sigma})$. Then, by the same argument as in Case 1, $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}'', \boldsymbol{s})$. Since $(\boldsymbol{x}, \boldsymbol{y}) \in NA(g^{**}, (\boldsymbol{u}, \boldsymbol{s}))$ with $(\boldsymbol{\rho}, \boldsymbol{\sigma}, \boldsymbol{x}, \boldsymbol{y}) \in NE(g^{**}, \boldsymbol{e})$, we have, by Rule 1-1-a) and 1-2-a), that $(\boldsymbol{x}, \boldsymbol{y}) \in P(\boldsymbol{u}, \boldsymbol{s})$ with p as the corresponding efficiency price. Thus, since $p \in \Delta^{SPI}(\boldsymbol{u}'', \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y})$, we have $(\boldsymbol{x}, \boldsymbol{y}) \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ by **SPI**. ■

Proof of Theorem 4. By the construction of g^{**}, it is a feasible sharing mechanism having the property of forthrightness. Thus, it satisfies Definition 2. From **Lemma A4**, we obtain the desired result. ■

8 Appendix B

Given $\boldsymbol{s} \in \mathcal{S}^n$, a feasible allocation $(\boldsymbol{x}, \boldsymbol{y}) \in Z(\boldsymbol{s})$ for $\boldsymbol{s}$ is *balanced* if $\sum y_i = f(\sum s_i x_i)$. Given $\boldsymbol{s} \in \mathcal{S}^n$, $(\boldsymbol{x}, \boldsymbol{y}) \in Z^n$, and $i \in N$, let $Z_i(\boldsymbol{s}, (\boldsymbol{x}, \boldsymbol{y}), p) \equiv \left\{(x'_i, y'_i) \in Z \mid \varphi^{-1}\left(p, \boldsymbol{s}, (x'_i, \boldsymbol{x}_{-i}), (y'_i, \boldsymbol{y}_{-i})\right) \neq \varnothing\right\}$.

Condition PQP: *For each $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ and each $(p, \boldsymbol{x}, \boldsymbol{y}) \in \Delta \times Z^n$ such that $f(\sum s_j x_j) \neq \sum y_j$ and $N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = N$, there exists a* balanced *feasible allocation $\boldsymbol{z}^* \in Z(\boldsymbol{s})$ such that:*
(1) *for all $i \in N$, $x^*_i = x_i$;*
(2) *for all $i \in N$, $z^*_i \in \bigcap_{(x'_i, y'_i) \in Z_i(s, (x,y), p)} B(p, s_i, (x'_i, y'_i))$;*
(3) *if there exists $\boldsymbol{u}^* \in \mathcal{U}^n$ such that for all $i \in N$,*
$\bigcap_{(x'_i, y'_i) \in Z_i(s, (x,y), p)} B(p, s_i, (x'_i, y'_i)) \subseteq L(z^*_i, u^*_i)$, *then $\boldsymbol{z}^* \in \varphi(\boldsymbol{u}^*, \boldsymbol{s})$.*

Proposition B1: *If an SCC φ is* implementable by a balanced natural sharing mechanism, *then φ satisfies* **PQP**.

Proof. Given $\boldsymbol{s} \in \mathcal{S}^n$, let $(p, \boldsymbol{x}, \boldsymbol{y}) \in \Delta \times Z^n$ be such that $N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = N$. Then, for each $i \in N$ and each $(x'_i, y'_i) \in Z_i(\boldsymbol{s}, (\boldsymbol{x}, \boldsymbol{y}), p)$, let $\boldsymbol{a}^i \in A$ be such that $a^i_i \equiv (p, s_i, x'_i, y'_i)$ and $a^i_j \equiv (p, s_j, x_j, y_j)$ for any $j \neq i$. Then, for any $\boldsymbol{u} \in \varphi^{-1}\left(p, \boldsymbol{s}, (x'_i, \boldsymbol{x}_{-i}), (y'_i, \boldsymbol{y}_{-i})\right)$, $g(\boldsymbol{a}^i) = \left(y'_i, \boldsymbol{y}_{-i}\right)$ and $\left((x'_i, \boldsymbol{x}_{-i}), (y'_i, \boldsymbol{y}_{-i})\right) \in NA(g, (\boldsymbol{u}, \boldsymbol{s}))$ hold by Definition 2-(ii). Then, by Definition 2-(iii), for each $i \in N$ and each $(\rho''_i, \sigma''_i, x''_i, y''_i) \in A_i$,

$$\left(x''_i, g_i\left(\left((\rho''_i, \sigma''_i, x''_i, y''_i), \boldsymbol{a}^i_{-i}\right), f\left(\sum_{j \neq i} s_j x_j + s_i x''_i\right)\right)\right) \in B(p, s_i, (x'_i, y'_i)).$$

Let $\boldsymbol{z}^* \equiv \left((x_i)_{i \in N}, g(\boldsymbol{a}^*)\right)$ for $\boldsymbol{a}^* = (p, s_i, x_i, y_i)_{i \in N}$. Then, $\boldsymbol{z}^* \in Z(\boldsymbol{s})$ is balanced, since g is a balanced feasible sharing mechanism. Moreover, from the property of $\boldsymbol{a}^* \in A$, it follows that $z_i^* = \left(x_i, g\left(a_i^*, \boldsymbol{a}_{-i}^i\right)\right) \in B(p, s_i, (x_i', y_i'))$ for all $(x_i', y_i') \in Z_i(\boldsymbol{s}, (\boldsymbol{x}, \boldsymbol{y}), p)$ and all $i \in N$. Finally, if there exists $\boldsymbol{u}^* \in \mathcal{U}^n$ such that for all $i \in N$, $\cap_{(x_i', y_i') \in Z_i(s,(x,y),p)} B(p, s_i, (x_i', y_i')) \subseteq L(z_i^*, u_i^*)$, then $\boldsymbol{a}^* \in NE(g, (\boldsymbol{u}^*, \boldsymbol{s}))$. Thus, by implementability, $\boldsymbol{z}^* \in \varphi(\boldsymbol{u}^*, \boldsymbol{s})$. ■

Proposition B2: *There is no SCC φ which satisfies* **PQP***.*

Proof. Take any φ and an economy $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$ such that $s_i = 1$ for all $i \in N$. Moreover, let $N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = N$. Suppose that there exists $\boldsymbol{z}^* \in Z(\boldsymbol{s})$ which satisfies the requirements of Condition **PQP**.

Fristly, consider $\sum y_i > f\left(\sum x_i\right)$. In this case, for each $i \in N$, $y_i^* \leq y_i' + \frac{p_x}{p_y}(x_i^* - x_i')$ for $(x_i', y_i') \in Z_i(\boldsymbol{s}, (\boldsymbol{x}, \boldsymbol{y}), p)$. Note that $x_i^* = x_i$ for each $i \in N$. Note also that there is some x_N such that p being the supporting price of $(x_N, f(x_N))$, and moreover, for each $i \in N$, $y_i' + \frac{p_x}{p_y}(x_i^* - x_i') = \left[f(x_N) - \sum_{h \neq i} y_h\right] + \frac{p_x}{p_y}\left(x_i - x_N + \sum_{h \neq i} x_h\right)$. The latter equation follows from the fact that, for each $i \in N$, $y_i' = f\left(\sum_{h \neq i} x_h + x_i'\right) - \sum_{h \neq i} y_h$ and $\left[f(x_N) - \sum_{h \neq i} y_h\right] + \frac{p_x}{p_y}\left(x_i - x_N + \sum_{h \neq i} x_h\right) = \left[f\left(\sum_{h \neq i} x_h + x_i'\right) - \sum_{h \neq i} y_h\right] + \frac{p_x}{p_y}(x_i - x_i')$. Thus, $y_i^* \leq f(x_N) - \sum_{h \neq i} y_h + \frac{p_x}{p_y}\left(x_i - x_N + \sum_{h \neq i} x_h\right)$ for each $i \in N$. Let us sum this up over i, so that $\sum y_i^* \leq n\left[\left(f(x_N) - \sum y_i\right) + \frac{p_x}{p_y}\left(\sum x_i - x_N\right)\right] + \sum y_i$. Since $\sum y_i^* = f\left(\sum x_i\right)$ by the balanced feasible allocation of $\boldsymbol{z}^*$, $f\left(\sum x_i\right) - \sum y_i \leq n\left[\left(f(x_N) - \sum y_i\right) + \frac{p_x}{p_y}\left(\sum x_i - x_N\right)\right]$. Note, in general, $f(x_N) + \frac{p_x}{p_y}\left(\sum x_i - x_N\right) \geq f\left(\sum x_i\right)$. Now, let f be such that $f(x_N) + \frac{p_x}{p_y}\left(\sum x_i - x_N\right)$ and $f\left(\sum x_i\right)$ are sufficiently close or even equal. For instance, the latter is available if f is linear in the interval $[\min\{\sum x_i, x_N\} - \varepsilon, \max\{\sum x_i, x_N\} + \varepsilon]$ for some $\varepsilon > 0$. Thus, we have $f\left(\sum x_i\right) - \sum y_i \leq n\left[\left(f(x_N) - \sum y_i\right) + \frac{p_x}{p_y}\left(\sum x_i - x_N\right)\right] \approx n\left[f\left(\sum x_i\right) - \sum y_i\right]$. However, $f\left(\sum x_i\right) - \sum y_i > n\left[f\left(\sum x_i\right) - \sum y_i\right]$ holds from $\sum y_i > f\left(\sum x_i\right)$ and $n \geq 2$, which is a contradiction.

Secondly, consider $\sum y_i < f\left(\sum x_i\right)$. In this case, we see that even if $\sum y_i < f\left(\sum x_i\right)$ for $N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = N$, there is always another $N(p, \boldsymbol{s}, \boldsymbol{x}', \boldsymbol{y}') = N$ such that $\sum y_i' > f\left(\sum x_i'\right)$ for $n = 2$. By $N(p, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) = N$, for each $i \in N$, there is $(x_i', y_i') \in Z_i(\boldsymbol{s}, (\boldsymbol{x}, \boldsymbol{y}), p)$, so that $y_i' = f(x_h + x_i') - y_h$. Then,

$y'_i + y'_j > f\left(x'_i + x'_j\right)$, and $N\left(p, \boldsymbol{s}, \boldsymbol{x}', \boldsymbol{y}'\right) = N$ still holds. Then, we can apply the argument for the case of $\sum y_i > f\left(\sum x_i\right)$. ■

9 Appendix C

Definition C1: *A natural sharing mechanism* $g \in \mathcal{G}^*$ *has* the best response property, *if for each* $\boldsymbol{e} = (\boldsymbol{u}, \boldsymbol{s}) \in \mathcal{E}$*, each* $(\boldsymbol{\rho}, \boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}) \in A$*, and each* $i \in N$*, there is* $(\rho_i^*, \sigma_i^*, x_i^*, y_i^*) \in A_i$ *such that for each* $(\rho'_i, \sigma'_i, x'_i, y'_i) \in A_i$*,* $\left(x'_i, g_i\left(\left(\rho'_i, \sigma'_i, x'_i, y'_i, \boldsymbol{\rho}_{-i}, \boldsymbol{s}_{-i}, \boldsymbol{x}_{-i}, \boldsymbol{y}_{-i}\right), f\left(\sum_{j \neq i} s_j x_j + s_i x'_i\right)\right)\right) \in L\left(\left(x_i^*, y_i^*\right), u_i\right)$*, where* $y_i^* = g_i\left(\left(\rho_i^*, \sigma_i^*, x_i^*, y_i^*, \boldsymbol{\rho}_{-i}, \boldsymbol{s}_{-i}, \boldsymbol{x}_{-i}, \boldsymbol{y}_{-i}\right), f\left(\sum_{j \neq i} s_j x_j + s_i x_i^*\right)\right)$*.*

Proposition C1: *If a natural sharing mechanism* $g \in \mathcal{G}^*$ *implements an SCC satisfying* **SPI** *and* **NIS****, it does not have the* best response property*.*

Proof. Let φ be an SCC satisfying **SPI** and **NIS*** which has the following property: for some $\boldsymbol{s} = (s_i)_{i \in N} \in \mathcal{S}^n$ and $\boldsymbol{s}^* = \left(s_j^*, \boldsymbol{s}_{-j}\right) \in \mathcal{S}^n$ and for some $\boldsymbol{z} = (x_i, y_i)_{i \in N} \in Z(\boldsymbol{s}) \cap Z(\boldsymbol{s}^*)$ with $x_j = 0$, there exists $p \in \Delta$ such that $\varphi^{-1}(p, \boldsymbol{s}, \boldsymbol{z}) \neq \varnothing$, $\varphi^{-1}(p, \boldsymbol{s}^*, \boldsymbol{z}) = \varnothing$, and $P^{-1}(p, \boldsymbol{s}^*, \boldsymbol{z}) \neq \varnothing$. Let $g \in \mathcal{G}^*$ be a natural sharing mechanism which implements φ. Take any $\boldsymbol{u} \in \mathcal{U}^n$ such that $p \in \Delta^P((\boldsymbol{u}, \boldsymbol{s}), \boldsymbol{z}) \cap \Delta^P((\boldsymbol{u}, \boldsymbol{s}^*), \boldsymbol{z})$. Then, $\boldsymbol{z} \in \varphi(\boldsymbol{u}, \boldsymbol{s})$ and $\boldsymbol{z} \in P(\boldsymbol{u}, \boldsymbol{s}^*) \backslash \varphi(\boldsymbol{u}, \boldsymbol{s}^*)$. Since g implements φ, $\boldsymbol{z} \in NA(g, (\boldsymbol{u}, \boldsymbol{s}))$ and $\boldsymbol{z} \notin NA(g, (\boldsymbol{u}, \boldsymbol{s}^*))$. By Definition 2-(ii), $\boldsymbol{\tau} = (p, s_i, x_i, y_i)_{i \in N} \in NE(g, (\boldsymbol{u}, \boldsymbol{s})) \backslash NE(g, (\boldsymbol{u}, \boldsymbol{s}^*))$. To guarantee the last equation, $j \in N$ has to have an alternative strategy $\tau'_j = \left(p'^j, s'_j, x'_j, y'_j\right)$ in the game $(g, (\boldsymbol{u}, \boldsymbol{s}^*))$ such that $\left(x'_j, g\left(\tau'_j, \boldsymbol{\tau}_{-j}, f\left(\sum_{k \neq j} s_k x_k + s_j^* x'_j\right)\right)\right) \in U(z_j, u_j)$, where $U(z_j, u_j)$ is the strictly upper-contour set of u_j at z_j. The same argument should apply to any other $\boldsymbol{u}' \in \mathcal{U}^n$ with $p \in \Delta^P((\boldsymbol{u}', \boldsymbol{s}^*), \boldsymbol{z})$. Denote the attainable set of g for j at $\boldsymbol{\tau}$ under $\boldsymbol{s}^*$ by $g_j(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*) \equiv \left\{\left(x'_j, g_j\left(\tau'_j, \boldsymbol{\tau}_{-j}, f\left(\sum_{k \neq j} s_k x_k + s_j^* x'_j\right)\right)\right) \in Z \mid \tau'_j = \left(p'^j, s'_j, x'_j, y'_j\right) \in A_j\right\}$. Note that, for each $\boldsymbol{u}' \in \mathcal{U}^n$ with $p \in \Delta^P((\boldsymbol{u}', \boldsymbol{s}^*), \boldsymbol{z})$, $g_j(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*) \cap U\left(z_j, u'_j\right) \neq \varnothing$. In contrast, for any $\tau'_j \in A_j$ with $x'_j = 0$, $g_j\left(\tau'_j, \boldsymbol{\tau}_{-j}, f\left(\sum_{k \neq j} s_k x_k\right)\right) \leq y_j$ holds, since $\boldsymbol{z} \in NA(g, (\boldsymbol{u}, \boldsymbol{s}))$ and $\boldsymbol{\tau} \in NE(g, (\boldsymbol{u}, \boldsymbol{s}))$. Thus, for each $\boldsymbol{u}' \in \mathcal{U}^n$ with $p \in \Delta^P((\boldsymbol{u}', \boldsymbol{s}^*), \boldsymbol{z})$, $\left(x'_j, y'_j\right) \in g_j(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*) \cap U\left(z_j, u'_j\right)$ implies $x'_j > 0$ and $y'_j > y_j$.

Consider a sequence $\left\{u_j^t\right\} \subset \mathcal{U}$ such that $p \in \Delta^P\left(\left(\left(u_j^{t'}, \boldsymbol{u}'_{-j}\right), \boldsymbol{s}^*\right), \boldsymbol{z}\right)$ for each $u_j^{t'} \in \left\{u_j^t\right\}$ and $U\left(z_j, u_j^t\right) \supset U\left(z_j, u_j^{t+1}\right)$ for each t. For each

$u_j^{t'} \in \{u_j^t\}$, let us take $\left(x\left(u_j^{t'}\right), y\left(u_j^{t'}\right)\right) \in Z$ such that $\left(x\left(u_j^{t'}\right), y\left(u_j^{t'}\right)\right) \in g_j\left(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*\right) \cap U\left(z_j, u_j^{t'}\right)$. Then, by the property of the sequence $\{u_j^t\}$, $x\left(u_j^{t'}\right) \to 0$ as $t' \to \infty$ while $\lim_{t' \to \infty} y\left(u_j^{t'}\right) > y_j$ holds. The former follows from $\cap_{\boldsymbol{u}' \in \mathcal{U}^n : p \in \Delta^P((\boldsymbol{u}', \boldsymbol{s}^*), \boldsymbol{z})} U\left(z_j, u_j'\right) = \{(0, y') \mid y' > y_j\}$. The latter follows from the following reason: if $\lim_{t' \to \infty} y\left(u_j^{t'}\right) \leq y_j$ holds, then there is $u_j'' \in \mathcal{U}$ such that $p \in \Delta^P\left(\left(\left(u_j'', \boldsymbol{u}_{-j}'\right), \boldsymbol{s}^*\right), \boldsymbol{z}\right)$ and $g_j\left(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*\right) \cap U\left(z_j, u_j''\right) = \varnothing$, which is a contradiction. Thus, $\lim_{t' \to \infty}\left(x\left(u_j^{t'}\right), y\left(u_j^{t'}\right)\right) \notin g_j\left(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*\right)$, since $\lim_{t' \to \infty} x\left(u_j^{t'}\right) = 0$ and $g_j\left(\tau_j', \boldsymbol{\tau}_{-j}, f\left(\sum_{k \neq j} s_k x_k\right)\right) \leq y_j$ for any $\tau_j' \in A_j$ with $x_j' = 0$. This implies that $g_j\left(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*\right)$ is not closed. Then, consider $u_j^* \in \mathcal{U}$ such that, for $\left(0, y_j^*\right)$ with $y_j^* \equiv \lim_{t' \to \infty} y\left(u_j^{t'}\right)$, the corresponding indifference curve of u_j^* at $\left(0, y_j^*\right)$ has no intersection with $g_j\left(A_j, \boldsymbol{\tau}_{-j}; \boldsymbol{s}^*\right)$. In this case, there is no best response strategy for j against $\boldsymbol{\tau}_{-j}$, thus g does not have the best response property. ■

10 References

Dutta, B., Sen, A., and Vohra, R. (1995): Nash implementation through elementary mechanisms in economic environments, *Economic Design* **1**, pp. 173-204.

Fleurbaey, M. and Maniquet, F. (1996): Fair allocation with unequal production skills: the no-envy approach to compensation, *Mathematical Social Sciences* **32**, pp. 71-93.

Gaspart, F. (1998): Objective measures of well-being and the cooperative production problem, *Social Choice and Welfare* **15**, pp. 95-112.

Gotoh, R., Suzumura, K., and Yoshihara, N. (2005): Extended Social Ordering Functions for Rationalizing Fair Game Forms in the sense of Rawls and Sen, *International Journal of Economic Theory* **1**, pp. 21-41.

Hong, L. (1995): Nash implementation in production economies, *Economic Theory* **5**, pp. 401-417.

Hurwicz, L., Maskin, E., and Postlewaite A. (1995): Feasible Nash implementation of social choice rules when the designer does not know endowments or production sets, in Ledyard , J. (ed.), *The economics of informational decentralization: Complexity, efficiency, and stability*, Essays in Honor of Stanley Reiter, pp. 367-433. Amsterdam: Kluwer Academic Publishers.

Jackson, M. O. (2001): A crash course in implementation theory, *Social Choice and Welfare* **18**, pp. 655-708.

Jackson, M. O., Palfrey, T., and Srivastava, S. (1994): Undominated Nash implementation in bounded mechanisms, *Games and Economic Behavior* **6**, pp. 474-501.

Kaplan, T. and Wettstein, D. (2000): Surplus sharing with a two-stage mechanism, *International Economic Review* **41**, pp. 399-409.

Kranich, L. (1994): Equal division, efficiency, and the sovereign supply of labor, *American Economic Review* **84**, pp. 178-189.

Maskin, E. (1999): Nash equilibrium and welfare optimality, *Review of Economic Studies* **66**, pp. 23-38.

Roemer, J. and Silvestre, J. (1993): The proportional solution for economies with both private and public ownership, *Journal of Economic Theory* **59**, pp. 426-444.

Saijo, T., Tatamitani, Y., and Yamato, T. (1996): Toward natural implementation, *International Economic Review* **37**, pp. 949-980.

Saijo, T., Tatamitani, Y., and Yamato, T. (1999): Characterizing natural implementability: the fair and Walrasian correspondences, *Games and Economic Behavior* **28**, pp. 271-293.

Shin, S. and Suh, S.-C. (1997): Double implementation by a simple game form in the commons problem, *Journal of Economic Theory* **77**, pp. 205-213.

Sjöström, T. (1991): On the necessary and sufficient conditions for Nash implementation, *Social Choice and Welfare* **8**, pp. 333-340.

Suh, S.-C. (1995): A mechanism implementing the proportional solution, *Economic Design* **1**, pp. 301-317.

Tian, G. (1999): Double implementation in economies with production technologies unknown to the designer, *Economic Theory* **13**, pp. 689-707.

Tian, G. (2000): Incentive mechanism design for production economies with both private and public ownerships, *Games and Economic Behavior* **33**, pp. 294-320.

Tian, G. (2009): Implementation in economies with non-convex production technologies unknown to the designer, *Games and Economic Behavior* **66**, pp. 526-545.

Tian, G. and Li, Q. (1995): On Nash-implementation in the presence of withholding, *Games and Economic Behavior* **9**, pp. 222-233.

Thomson, W. (1983): Equity in exchange economies, *Journal of Economic Theory* **29**, pp. 217-244.

Van Parijs, P. (1995): *Real freedom for all*, Oxford: Oxford University Press.

Yamada, A. and Yoshihara, N. (2007): Triple implementation by sharing mechanisms in production economies with unequal labor skills, *International Journal of Game Theory* **36**, pp. 85-106.

Yamada, A. and Yoshihara, N. (2008): A mechanism design for a solution to the tragedy of commons, *Review of Economic Design* **11**, pp. 253-270.

Yoshihara, N. (1998): Characterizations of the public and private ownership solutions, *Mathematical Social Sciences* **35**, pp. 165-184.

Yoshihara, N. (1999): Natural and double implementation of public ownership solutions in differentiable production economies, *Review of Economic Design* **4**, pp. 127-151.

Yoshihara, N. (2000): A characterization of natural and double implementation in production economies, *Social Choice and Welfare* **17**, pp. 571-599.