Demographic Change, Intergenerational Altruism, and Fiscal Policy
－ A Political Economy Approach －[1]

24 November 2010

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Abstract

Our study employs an OLG model under which political strengths of different generations (the working and retirees with and without children) determine the distribution of the fiscal burden between the generations, including the future generation. We investigate the relationship between the extent of intergenerational altruism, the political regime, and the intergenerational distribution of the fiscal burden.

We show that if the working generation were to care more about the utility of the retirees (their parents), cooperation between the working and retirees with children would be possible, changing the political outcome. As a result, the tax burden of the working generation would decrease and its members would be better off. Lowering the voting age and having parents vote on behalf of their children would also result in the same shift, but for higher levels of intergenerational altruism and the working generation's political power. The resulting shift would lower the tax burden and give higher utility to the working generation.

Key words: Public debt, public deficit, OLG models, intergenerational altruism, Demeny voting method

JEL Classification: D64, E60, H63

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1. Introduction

This study employs an OLG model in which allocation of the fiscal burden among generations, including future generations, is determined by political power. We demonstrate how a change in intergenerational altruism and political power can change the political regime and outcome. This will reconfigure the allocation of the tax burden and, therefore, the utility of each generation.

The Japanese outstanding public debt (as a proportion of the GDP) is now the largest among developing countries, primarily because of the changing age structure of the population. The situation is expected to worsen, with social security payments continuing to increase at the rate of about 1 trillion yen per year. There needs to be a drastic change in public financing if Japan's fiscal sustainability is to continue into the future.

According to Masujima, Shimazawa, and Murakami (2009), if the current situation were to continue, the lifetime net public burden (lifetime net public burden (life time tax – life transfers) divided by lifetime wages) of the generation aged 90+ will be -7% while that of the future generation will be 51.4%. This means that the lifetime public burden of the future generations will be 60 percentage points more than that of the 90+ generation. This intergenerational inequality needs to be corrected.

However, correction of this inequality and the fiscal policy change required have proved extremely difficult. One source of the difficulty may be the relative political influence of the retired and working generations, due to factors such as voter turnout. If each generation votes to increase net government transfer, such as through higher pensions and lower taxes, resistance to reducing pensions or lessening the tax burden of the working generation may be a reflection of the older generation having greater political influence. In fact, the numbers of the retired generation are increasing while the size of the younger generation is decreasing as a result of lower fertility. This means that the relative proportion of retired people among all voters is increasing while that of the younger generation is decreasing. The political influence of the older generation is even more pronounced due to the difference in voter turnout (Figure 1). The older generation can use the political process to shift the burden to the working generation.

In addition, the life-cycle hypothesis suggests that because the time horizons of the retirees are shorter, they are subject to stronger incentives to issue public debt and leave the liability to future generations. This, together with their relatively greater political influence, can explain why public debt continues to grow and political will to change the situation is lacking.

The situation may change if we take into account the fact that retirees may have children. The children of these retirees are the current working generation, and their children are the grandchildren of the retirees. Intergenerational altruism may make the retirees act in a way calculated to benefit the working or future generations even if such a choice may reduce the level of their own consumption. On the other hand, such intergenerational altruism may not be enough if the retirees without children outnumber those with children. They may maneuver the political situation into an outcome that increases the burden of the working and future generations. However, even in such a scenario, it may be possible for them to form a political coalition with the working generation, ensuring that the working generation is better off (Working generation and Retirees with children Cooperation Regime, Section 2). This suggests that while the current electoral system does not include the future generation in the process (because of the minimum voting age), the future generation's well-being will be reflected in the process through the actions of their parents, if intergenerational altruism exists. Of course, in order that they cooperate, there must be sufficient benefit for retirees with children from making the future generations better off. If there is no such benefit, the political outcome will be determined by the Retirees Cooperation Regime (see Section 2). The extent of intergenerational altruism will determine which of these potential coalitions come into being.

Since the 1990s, both empirical and theoretical political economic approaches have been taken with regard to the politics of public finance (Alesina and Perotti 1998, Persson and Tabellini 2000, Shi and Svensson 2006, to name a few). Several sources of political factors have been identified: (1) the political cycle of fiscal policy generated by the reelection motive in politicians and a change in the majority party (Rogoff 1990, Kneebone and McKenzie 2001, Foucault et al. 2008), (2) a change of government and strategic motive (Persson and Svensson 1989, Tabellini and Alesina 1990, Crain and Tollison 1993), and (3) the common pool problem (Alesina and Drazen 1991, Ihori and Itaya 2001). The common pool problem has been identified as an important political source of overspending (resulting in a negative fiscal budget). Income inequality and racial bias (Woo 2003) and the relationship between federal and state (central and local) governments (Rodden 2002, Doi and Ihori 2002, Schaltegger and Feld 2009) have also been suggested as significant factors.

There has been no study of the relationship between the political regime and fiscal policy, to the best of our knowledge. In this paper, we use the OLG framework with two generations (working and retired) and three groups (working, retired with children, and retired without children). The three groups are political voting blocks. The framework allows us to analyze the relationship between the political regime, defined by the relative political power of voting groups, and the political outcome, which determines fiscal policy. The fiscal policy defines tax and transfers, including pensions.

In Section 2, we present an OLG model with two generations and three voting blocks and characterize the possible political regimes and outcomes. Section 3 consists of a simple analysis by simulation and application to the current situation in Japan. We summarize the results and discuss questions for future research in Section 4.

2. Model

(1) Household

There are two generations involved in each period t (t=0, 1, 2,⋯), working generation *t* and retired generation *t*－*1* (which was the working generation in period t). Working generation t earns a lifetime wage W_t, has lifetime consumption C_t and pays tax $T_t(t)$ in period *t*. Lifetime consumption C_t is the sum of consumption while working and after retirement. We assume that the wage is exogenous and the interest rate is zero. The lifetime consumption of working generation *t* will be

$$C_t(t) = W_t - T_t(t), \tag{1}$$

where $T_t(s) \equiv \theta_t(s)W_t$ defines the lifetime tax burden rate of working generation *t* in the periods.

On the other hand, the retirees *t-1* in period *t* can recover some of the tax paid in period *t-1* by issuing bonds. That is to say, retirees *t-1* can reduce their lifetime tax burden to $T_{t-1}(t)$ in period *t*. We can define generation *t-1*'s "profit" in period t as $\varphi_t \equiv T_{t-1}(t-1) - T_{t-1}(t)$. Of course, if this generation must shoulder a larger tax burden in period *t*, it will be making a negative profit (a loss), $\varphi_t \equiv T_{t-1}(t-1) - T_{t-1}(t) < 0$. Accordingly, retired generation *t-1* must revise its lifetime consumption in period *t* as well,

$$C_{t-1}(t) = (W_{t-1} - T_{t-1}(t-1)) + \varphi_t, \tag{2}$$

where generation *t-1*'s lifetime tax burden rate in period *t*, $\theta_{t-1}(t)$, is defined as $T_{t-1}(t) \equiv \theta_{t-1}(t)W_{t-1}$

(2) Government budget constraint

For ease of analysis, we assume there is no government expenditure (other than transfer) so that all debt incurred in a period is assumed to be repaid in the following period. Denoting the (planned) tax on generation *t+1* in period *t* as $T_{t+1}(t) \equiv \theta_{t+1}(t)W_{t+1}$, public debt as D_t, and population size as N_t, we have the following government budgetary constraint.

$$D_t = \mathrm{N}_{t-1}(T_{t-1}(t) - T_{t-1}(t-1)) + N_t T_t(t) + N_{t+1}T_{t+1}(t) \tag{3}$$

$$\Leftrightarrow \quad \frac{D_t}{\mathrm{N}_{t-1}W_{t-1}} + \theta_{t-1}(t-1) = \theta_{t-1}(t) + n_t\theta_t(t)G + n_{t+1}n_t\theta_{t+1}(t)G^2$$

where $n_t \equiv N_t / N_{t-1}$ is the population growth rate and G is the rate of wage increase. Debt for this period will be whatever is not paid from the debt of the previous period,

$$D_{t+1} = D_t - \left[N_{t-1}(T_{t-1}(t) - T_{t-1}(t-1)) + N_t T_t(t)\right] \tag{4}$$

(3) Household Utility

In addition, females aged around 40 can give birth to children and men over 40 can have children. Therefore, we assume that the working generation *t* is made up of people in the working period, which means it includes people who have not finished having children. We are not able to categorize (exactly) members of the working generation into groups of those who have children and those who do not. The retirees, on the other hand, can be divided into two groups, those with children and those without. We assume proportion $\pi_t > 0.5$ have children (See Figure 2). Thus, retirees are heterogeneous, while the working generation is homogeneous.

We define generation *j*'s utility from lifetime consumption as $v_j = \log C_j$ and assume that parents and children are mutually altruistic. We can define the utilities of working generation *t* and retires *t-1* with and without children.

First, we define the utility function of retirees *t*－*1* with children,

$$U_{t-1}^{child} = \log\left[1-\theta_{t-1}(t)\right] + \delta\log\left[1-\theta_t(t)\right] + \pi_t\delta^2\log\left[1-\theta_{t+1}(t)\right], \tag{5}$$

Where δ measures how much parents care about their children (forward altruism). The first term is utility derived from their own consumption, the second term from that of their children (the working generation), and third term from that of their grandchildren (the future generation). Additionally, the future generation is made up of the children of the working generation, and only proportion π_t of them have children.

Next, we define the utility of retirees without children,

$$U_{t-1}^{nc} = \log\left[1-\theta_{t-1}(t)\right] \tag{6}$$

This is equivalent to equation (6), less the last two terms, which represented utility from children and grandchildren's consumption.

Finally, we define the utility of the working generation,

$$U_t = \sigma\log\left[1-\theta_{t-1}(t)\right] + \log\left[1-\theta_t(t)\right] + \pi_t\delta\log\left[1-\theta_{t+1}(t)\right], \tag{7}$$

where σ measures how much a child cares about its parents' utility (backward altruism). The first term is utility from parent's consumption, the second term is utility from own consumption, and the third term is utility from children's consumption.

(4) Objective function of the political process

Let us consider the extent of political influence, as in, for instance, turnout at an election, in each period for each generation. We will denote by $s_k(t)$ the extent of generation *k*'s political activism in period *t*. We now define the total political power for each group. Group 1 consists of retirees with children, whose total political power is $V_1 \equiv \pi_{t-1} s_{t-1}(t) N_{t-1}$. Group 2 are the retirees without children, and the group's total political power is $V_2 \equiv (1-\pi_{t-1}) s_{t-1}(t) N_{t-1}$. Finally, Group 3 consists of the working generation, the total political power of which is $V_3 \equiv s_t(t) N_t$.

We assume that the political objective is to maximize the utility of the group *j* (*j* = 1, 2, 3) that has the majority. Thus, if Group 1 (retirees with children) is in the majority, the objective function will be U_{t-1}^{child}, which we will call case 1.

1) "*Retirees with children Independent Majority Regime*"

$\pi_{t-1} s_{t-1}(t) N_{t-1} > (1-\pi_{t-1}) s_{t-1}(t) N_{t-1} + s_t(t) N_t$ holds, and

$$W_t \ (case1) = U_{t-1}^{child} \tag{8}$$

Since we assumed that the majority of retirees have children, $\pi_{t-1} > 0.5$, it will always be the case that $V_1 > V_2$, meaning that Group 2 will never be the majority by itself. The only other possibility is that of Group 3 obtaining a majority; we will refer to this situation as case 2.

2) "*Working generation Independent Majority Regime*"

$s_t(t) N_t > s_{t-1}(t) N_{t-1}$ holds, and

$$W_t(case2) = U_t \tag{9}$$

Now we consider a situation in which none of the three groups can obtain the majority by itself. A group will form a coalition with another group that will enable it to attain the highest level of own utility (defined by equations (5)－(7)). The objective function when a coalition achieves majority will be the weighted average of its coalition members, where the weights reflect total political power. Denoting the utility of group *k* by U_k, the objective function will be

$$W_t = V_j U_j + V_i U_i$$

Note that it is very unlikely that there will be a coalition of the working generation and retirees without children (Groups 2 and 3), given the utility functions defined by equation (5)－(7) [2]. We only need to consider the following two cases with cooperation (See Figure 3).

3) *"Retirees Cooperation Regime"*

$s_t(t)N_t > s_{t-1}(t)N_{t-1}$ and $\pi_{t-1}s_{t-1}(t)N_{t-1} < (1-\pi_{t-1})s_{t-1}(t)N_{t-1} + s_t(t)N_t$ holds, and

$$W_t(case3) = s_{t-1}(t)\pi_{t-1}N_{t-1}U_{t-1}^{child} + s_{t-1}(t)(1-\pi_{t-1})N_{t-1}U_{t-1}^{nc} \tag{10}$$

$$\propto \pi_{t-1}U_{t-1}^{child} + (1-\pi_{t-1})U_{t-1}^{nc}$$

4) *"Working generation + Retirees with children Cooperation Regime"*

$(1-\pi_{t-1})s_{t-1}(t)N_{t-1} < \pi_{t-1}s_{t-1}(t)N_{t-1} + s_t(t)N_t$ holds, and

$$W_t(case4) = s_{t-1}(t)\pi_{t-1}N_{t-1}U_{t-1}^{child} + s_t(t)N_tU_t \tag{11}$$

$$\propto \pi_{t-1}U_{t-1}^{child} + \rho_t n_t U_t$$

where $\rho_t \equiv s_t(t)/s_{t-1}(t)$ is the relative political influence of the working generation as compared to that of retirees. Note that the utility of retirees with children must be higher in case 4 than in case 3 so that the objective function of case 4 may be actually as defined by equation (11). That is, the following relationship must hold in addition:

$$U_{t-1}^{child}(case4) > U_{t-1}^{child}(case3) \tag{12}$$

3. Simple Analysis and Application

(1) Simple Analysis: $\pi_t = 1$, $n_t = n < 1$ and $\rho_t = 1$

There are no retirees without children, so the objective function is determined by the relative size of the working generation and the group of retirees (who all have children). Since, $N_{t-1} > N_t$ holds, case 1 will be true, and the objective function is given by (8).

The allocation for each generation in period *t* can be arrived at by solving the following constrained optimization problem defined by equations (3) and (8),

[2] A coalition of the working generation and retirees without children requires the following two conditions in order to hold: 1) The utility of the working generation from cooperating with the retirees without children is higher than from cooperating with retirees with children, and 2) The utility of retirees without children will be higher from cooperating with retirees with children than from cooperating with the working generation. We show in Section 3 that there were no values of s、δ、σ that satisfied both conditions at once.

$$\max U_{t-1}^{child}\text{, subject to }\ \theta_{t-1}(t)+\theta_t(t)nG+\theta_{t+1}(t)n^2G^2=\frac{D_t}{\mathrm{N}_t W_t}nG+\theta_{t-1}(t-1)$$

Specifically, we obtain the following conditions on $(\theta_{t-1}(t),\theta_t(t),\theta_{t+1}(t))$, where λ is the Lagrangean multiplier.

$$\frac{1}{1-\theta_{t-1}(t)}=\lambda$$

$$\frac{\delta}{1-\theta_t(t)}=\lambda nG$$

$$\frac{\delta^2}{1-\theta_{t+1}(t)}=\lambda n^2G^2$$

Solving the system of equations, we obtain

$$\theta_{t-1}(t)=1-\frac{1}{\lambda} \tag{13}$$

$$\theta_t(t)=1-\frac{\delta}{\lambda nG}$$

$$\theta_{t+1}(t)=1-\frac{\delta^2}{\lambda n^2G^2}$$

$$1/\lambda=\frac{1+nG+n^2G^2-\dfrac{D_t}{\mathrm{N}_t W_t}nG-\theta_{t-1}(t-1)}{1+\delta+\delta^2}$$

Using equations (4) and (13), we obtain the dynamic equation for outstanding public debt $d_t\equiv D_t/(N_tW_t)$ and the lifetime tax burden rate of the working generation *t* in period *t*,

$$\begin{bmatrix} d_{t+1} \\ \theta_t(t) \end{bmatrix}=\left(1-\frac{\delta^2}{n^2G^2}\frac{1+nG+n^2G^2}{1+\delta+\delta^2}\right)\begin{bmatrix} 1 \\ 1 \end{bmatrix}+\begin{bmatrix} \dfrac{\delta^2}{nG(1+\delta+\delta^2)} & \dfrac{\delta^2}{n^2G^2(1+\delta+\delta^2)} \\ \dfrac{\delta}{(1+\delta+\delta^2)} & \dfrac{\delta}{nG(1+\delta+\delta^2)} \end{bmatrix}\begin{bmatrix} d_t \\ \theta_{t-1}(t-1) \end{bmatrix} \tag{14}$$

Ignoring the upper bounds of d_t and $\theta_t(t)$ for the present, we can derive the following statements from equation (14).

1) If $n>\dfrac{\delta+\delta^2}{(1+\delta+\delta^2)G}$ then outstanding public debt d_t will converge (to a value),

2) If $n < \frac{\delta + \delta^2}{(1+\delta+\delta^2)G}$ then outstanding public debt d_t will diverge.

By taking the upper bounds of d_t and $\theta_t(t)$ into account, we can derive the following proposition from equation (13).

Proposition If $n < \Gamma(\delta) \equiv \frac{\delta + \delta^2}{(1+\delta+\delta^2)G}$, then the political game defined in Section 2 is unsustainable.

(2) Application to Japan

Assuming $\pi_{t-1} < 1$, we can apply the framework presented in Section 2 to Japan. We divide the population from age 20 to age 89 into two groups: working generation *t* (20 to 54 year olds) and retired generations *t*—*1* (55 to 89 year olds). The minimum voting age in Japan is 20. Using the figures published by the Ministry of Internal Affairs and Communication (Population by Age, 2008), we get approximations $N_t = 58{,}000{,}000$ and $N_{t-1} = 45{,}000{,}000$, and $n_t = 1.29$. The Population Projection by National Institute of Population and Social Security Research estimates (mid-level estimation as of 2006) that population growth up to 2100 is - 0.7% per annual, which implies $n_{t+1} = (1-0.007)^{35} = 0.78$.

Given these values, we can identify the following two Independent Majority Regimes (cases 1 and 2) according to relative political influence $\rho_t \equiv s_t(t)/s_{t-1}(t)$, independent of values δ and σ from equations (8) and (9).

1) $\rho_t < (2\pi_{t-1} - 1)/n_t = 0.31$: ***"Retirees with children Independent Majority Regime"***

2) $\rho_t > 1/n_t = 0.776$: ***"Working generation Independent Majority Regime"***

For other values ($0.31 < \rho_t < 0.776$) it will be either case 3 (Retirees Cooperation Regime) or case 4 (Working generation and Retirees with children Cooperation Regime), according to conditions (10) and (12).

We can arrive at the allocation determined by the political process by solving a constrained optimization problem as we did to get (13). We optimize equation (3) subject to the constraints given by (10) or (11).

3) When the constraint is equation (10),

$$1 - \theta_{t-1}(t) = \frac{h_t}{1 + A_t n_t G + A_t b_t n_t n_{t+1} G^2} \tag{15}$$

$$1-\theta_t(t)=\frac{A_t h_t}{1+A_t n_t G+A_t b_t n_t n_{t+1}G^2}$$

$$1-\theta_{t+1}(t)=\frac{A_t b_t h_t}{1+A_t n_t G+A_t b_t n_t n_{t+1}G^2}$$

$$h_t \equiv 1+n_t G+n_t n_{t+1}G^2-\frac{D_t}{N_t \mathrm{W}_t}n_t G-\theta_{t-1}(t-1)$$

$$A_t=\frac{\pi_{t-1}\delta}{n_t G} \text{ and } b_t=\frac{\delta\pi_t}{n_{t+1}G}$$

4) When the constraint is equation (11),

$$1-\theta_{t-1}(t)=\frac{h_t}{1+a_t n_t G+a_t b_t n_t n_{t+1}G^2} \tag{16}$$

$$1-\theta_t(t)=\frac{a_t h_t}{1+a_t n_t G+a_t b_t n_t n_{t+1}G^2}$$

$$1-\theta_{t+1}(t)=\frac{a_t b_t h_t}{1+a_t n_t G+a_t b_t n_t n_{t+1}G^2}$$

$$a_t=\frac{[\rho_t n_t+\pi_{t-1}\delta]}{[\rho_t \sigma n_t+\pi_{t-1}]n_t G}$$

Given condition (12), we see that it will be Retirees Cooperation Regime or Working generation and Retirees with children Cooperation Regime, according to the relative size of equation (5) determined by (15) and (16). The condition depends on values of δ, σ, and ρ_t, which is too complicated to analyze analytically.

We resort to simulation for the characterization. We calculate condition (5) for values of δ, σ, and ρ_t by increment of 0.05 for ranges $0<\delta<1$, $0<\sigma<1$, and $0.31<\rho_t<0.77$. The 2005 White Paper on the National Lifestyle (Cabinet Office) states that the proportion of households of 20-49 year olds with children was 69.4% in 1980 and 53.2% in 2000. Thus, we assume the proportion of retirees with children to be π_{t-1}=0.7 and that of the working generation with children to be π_t = 0.53. The outstanding public debt as proportion of GDP was 190% in 2009. We use this, divided by 35 years (age group in a generation) for $D_t/(N_t W_t)$ in equation (15). We assume $G=1$ and $\theta_{t-1}(t-1)=0.25$.

The simulation results are summarized in Table 1. The blank cell denotes the Retirees Cooperation Regime. For instance, the Retirees Cooperation Regime will be selected when (δ, σ) = (0.5, 0.5) (blank cell), independent of the value of ρ_t.

A number in each cell denotes the upper bound of ρ_t for a Working generation and Retirees with children Cooperation Regime to come into existence. For instance, the number 0.5 appears in cell (δ , σ) = (0.7, 0.7), meaning that for (δ , σ) = (0.7, 0.7), the Working generation and Retirees with children Cooperation Regime will exist when $\rho_t < 0.5$ and the Retirees Cooperation Regime will when $\rho_t > 0.5$.

According to the simulation, if the value for Japan is (ρ_t , δ , σ) = (0.45, 0.8, 0.4), the Retirees Cooperation Regime is in existence. However, we need to be more careful about parameter σ , which appears only in the working generation's utility function (7). The parameter is not relevant for the utility functions of the retirees, (5) and (6).

Table 1 shows that the Working generation and Retirees with children Cooperation Regime will prevail if the parameter value σ is equal to or greater than 0.55. The simulation also shows that $\theta_t(t)$ declines and the utility of the working generation increases as σ increases above 0.55 (See Table 2).

This is because the tax burden $\theta_{t-1}(t)$ of the retirees with children will be heavier if they cooperate with the working generation than if they cooperate with retirees without children, when the working generation's value for retirees' (parents') utility very low. Even if σ is as high as 0.55, the Retiree Cooperation Regime will be selected if ρ_t is 0.6 and not 0.45. As with the previous situation, if the relative political influence of the working generation is greater, then the tax burden $\theta_{t-1}(t)$ of the retirees with children will be larger if they cooperate with the working generation than if they cooperate with retirees without children.

Thus, if the situation in Japan is (ρ_t , δ , σ) = (0.6, 0.8, 0.4), then we need to increase σ to make it equal to or above 0.6. That is, by increasing σ , caring more about the well-being of parents, to levels determined by ρ_t , the working generation can cooperate with the retirees with children and be better off themselves.

(3) Effect of Demeny voting

In this section, we will examine the implications of the so-called Demeny voting. Demeny proposed that children (all those currently below the voting age) be given a vote but parents be made to vote on their behalf (Demeny 1986, Aoki and Vaithianathan 2009)。

We use parameter ξ to denote the extent of extension of voting rights to children (how low the minimum age should be). We adjust equation (11) accordingly, and the political objective function becomes,

$\pi_{t-1}s_{t-1}(t)N_{t-1} < (1-\pi_{t-1})s_{t-1}(t)N_{t-1} + s_t(t)N_t(1+\xi n_{t+1})$ holds, and

$$W_t(case5) = \pi_{t-1}U_{t-1}^{child} + \rho_t n_t(1+\xi n_{t+1})U_t \tag{17}$$

If the Retirees Cooperation Regime is currently prevailing in Japan, we will call the situation given by equations (17) and (18) the Demeny voting + Working generation and Retirees with children Cooperation Regime.

$$U_{t-1}^{child}(case5) > U_{t-1}^{child}(case3) \tag{18}$$

As in the previous section, we will find the values of intergenerational altruism and relative political influence that may change the regime from that of Retirees Cooperating to that of Demeny voting + Working generation and Retirees with children Cooperating. The results of the simulation are summarized in Table 3. The parameters (δ, σ, ρ_t) were increased in increments of 0.02. We assumed the voting age was lowered to 10, meaning that there would be 10 new age groups. Since both the working and retired generation each contain 35 age groups (20 to 54 and 55 to 89), we set parameter ξ to 10/35.

We can see from Table 3 that if the current situation in Japan is summarized by (ρ_t, δ, σ) = (0.45, 0.8, 0.4), then the society will become one with Demeny voting + Working generation and Retirees with children Cooperating when σ is equal or greater to 0.96. We can also see from the calculation results presented in Table 4 that when the regime switches at σ = 0.96, the working generation's lifetime tax burden rate $\theta_t(t)$ decreases and its utility increases.

4. Summary and Future Research

We examined an OLG with two generations, the working and the retired, divided into three groups, working, retired with children, and retired without children, where allocation of government funding is determined by the relative political power (votes) of the group. We examined how the majority or majority coalition depends on political power and the extent of intergenerational altruism.

We observed that by caring more about their parents (more backward altruism), the working generation can change the political outcome from that of Retirees Cooperating (retirees with and without children voting together) to that of Working generation and Retirees with children Cooperating. The tax burden of the working generation will be lighter, and the working generation will be better off.

We undertook simulation of a situation in which the voting age is lowered to 10 but parents vote on behalf of voters under 20 (Demeny voting). The regime change from Retirees Cooperating (retirees with and without children voting together) to Working generation and Retirees Cooperating will occur for higher levels of intergenerational altruism and with less relative political influence of the working generation. The switch will lighten the tax burden of the working generation and increase their utility.

There are several assumptions that we hope to relax in future research. We assumed that retirees without children did not care about other generations at all, while retirees with children care not only about their children but also about their grandchildren. We also assumed that population growth rate, wages, and interest rates are exogenous.

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Appendix

We use equation (14) to define vector $\alpha \equiv (\alpha_1, \alpha_2)$,

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \left(1 - \frac{\delta^2}{n^2G^2}\frac{1+nG+n^2G^2}{1+\delta+\delta^2}\right)\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}\dfrac{\delta^2}{nG(1+\delta+\delta^2)} & \dfrac{\delta^2}{n^2G^2(1+\delta+\delta^2)}\\ \dfrac{\delta}{(1+\delta+\delta^2)} & \dfrac{\delta}{nG(1+\delta+\delta^2)}\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} \tag{A1}$$

We can solve for this vector,

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \left(1 - \frac{\delta^2}{n^2G^2}\frac{1+nG+n^2G^2}{1+\delta+\delta^2}\right)\begin{bmatrix}1-\dfrac{\delta^2}{nG(1+\delta+\delta^2)} & -\dfrac{\delta^2}{n^2G^2(1+\delta+\delta^2)}\\ -\dfrac{\delta}{(1+\delta+\delta^2)} & 1-\dfrac{\delta}{nG(1+\delta+\delta^2)}\end{bmatrix}^{-1}\begin{bmatrix}1\\1\end{bmatrix}$$

$\Leftrightarrow$

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \frac{1}{\Delta}\left(1 - \frac{\delta^2}{n^2G^2}\frac{1+nG+n^2G^2}{1+\delta+\delta^2}\right)\begin{bmatrix}1-\dfrac{\delta}{nG(1+\delta+\delta^2)} & \dfrac{\delta^2}{n^2G^2(1+\delta+\delta^2)}\\ \dfrac{\delta}{(1+\delta+\delta^2)} & 1-\dfrac{\delta^2}{nG(1+\delta+\delta^2)}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

where $\Delta = 1 - \dfrac{\delta}{nG(1+\delta+\delta^2)} - \dfrac{\delta^2}{nG(1+\delta+\delta^2)}$.

By subtracting (A1) from (14), we get,

$$\begin{bmatrix}d_{t+1}-\alpha_1\\ \theta_t(t)-\alpha_2\end{bmatrix} = \begin{bmatrix}\dfrac{\delta^2}{nG(1+\delta+\delta^2)} & \dfrac{\delta^2}{n^2G^2(1+\delta+\delta^2)}\\ \dfrac{\delta}{(1+\delta+\delta^2)} & \dfrac{\delta}{nG(1+\delta+\delta^2)}\end{bmatrix}\begin{bmatrix}d_t-\alpha_1\\ \theta_{t-1}(t-1)-\alpha_2\end{bmatrix} \tag{A2}$$

Obtain the eigen value Ω and vector Ξ of the matrix as follows is straightforward:

$\Omega_1 = 0$, $\Omega_2 = \dfrac{\delta+\delta^2}{nG(1+\delta+\delta^2)}$

$\Xi_1 = \begin{bmatrix}\delta\\ nG\end{bmatrix}$, $\Xi_2 = \begin{bmatrix}1\\ -nG\end{bmatrix}$

We define the initial value by,

$$\begin{bmatrix} d_1-\alpha_1 \\ \theta_0(t)-\alpha_2 \end{bmatrix} = b_1\Xi_1 + b_2\Xi_2 \tag{A3}$$

$\Leftrightarrow$

$$\begin{bmatrix} \delta & 1 \\ nG & -nG \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} d_1-\alpha_1 \\ \theta_0(t)-\alpha_2 \end{bmatrix}$$

$\Leftrightarrow$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{nG(1+\delta)}\begin{bmatrix} nG & 1 \\ nG & -\delta \end{bmatrix}\begin{bmatrix} d_1-\alpha_1 \\ \theta_0(t)-\alpha_2 \end{bmatrix}$$

Substituting (A3) in (A2) yields,

$$\begin{bmatrix} d_{t+1}-\alpha_1 \\ \theta_t(t)-\alpha_2 \end{bmatrix} = b_1\left(\frac{\delta+\delta^2}{nG(1+\delta+\delta^2)}\right)^t \begin{bmatrix} \delta \\ nG \end{bmatrix}$$

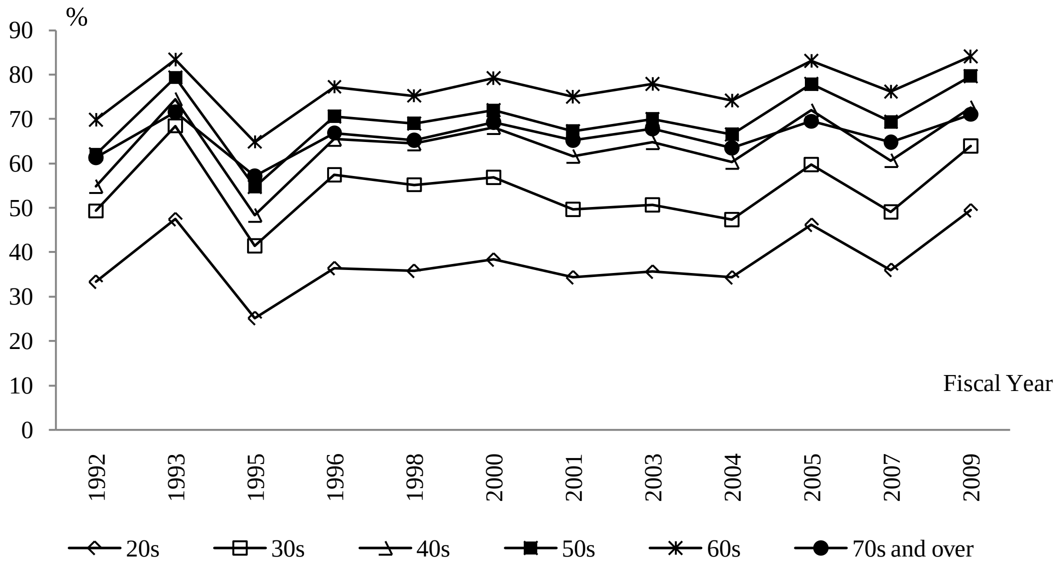
Thus if $\Omega_2 < 1$, then

$$\begin{bmatrix} d_{t+1} \\ \theta_t(t) \end{bmatrix} \to \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \text{ as } t \to \infty.$$

If $\Omega_2 > 1$, then the following relationship holds:

$$\begin{bmatrix} d_{t+1} \\ \theta_t(t) \end{bmatrix} \to \begin{bmatrix} \infty \\ \infty \end{bmatrix} \text{ as } t \to \infty.$$

Figure 1: Election turnout rate by age group



Source: “The Association for Promoting Fair Elections” http://www.akaruisenkyo.or.jp/

Figure 2: The model's generational structure

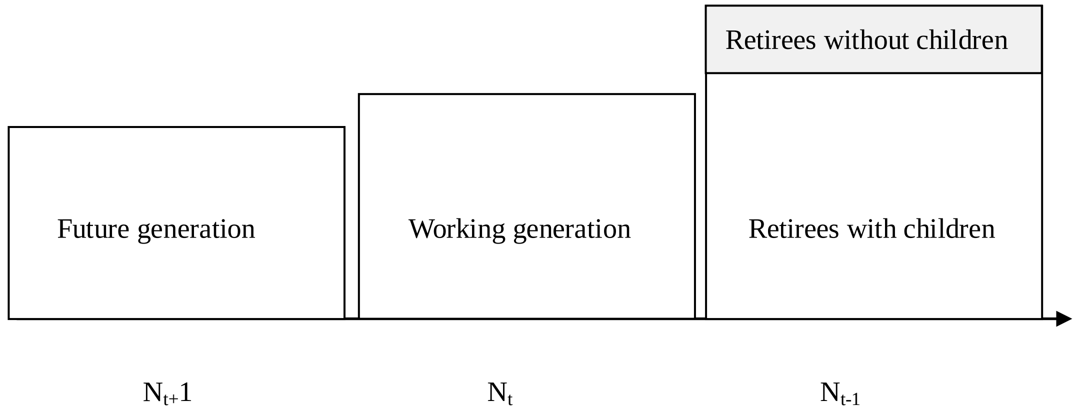


Figure 3: Cooperation among generations

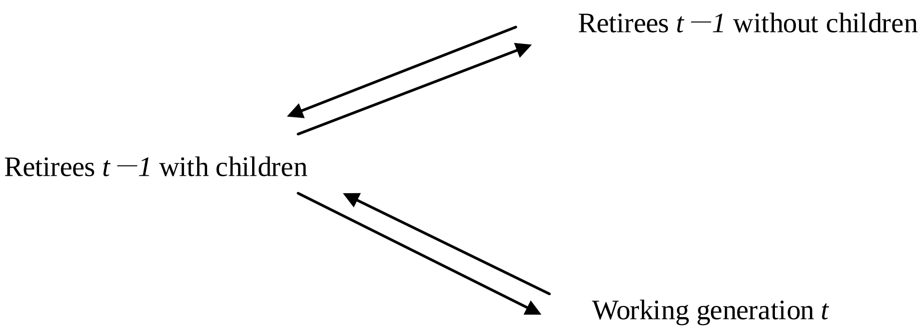


Table 1: "Retirees Cooperation Regime" vs " Working generation and Retirees with children Cooperation Regime"

		σ																			
		0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
	0.05																				
	0.1																				
	0.15																				
	0.2																				
	0.25																				
	0.3																				
	0.35																				
	0.4																				
	0.45																				
δ	0.5																				0.35
	0.55																	0.35	0.4	0.45	0.55
	0.6															0.35	0.4	0.45	0.55	0.7	0.75
	0.65													0.35	0.4	0.45	0.55	0.7	0.75	0.75	0.75
	0.7											0.35	0.4	0.45	0.5	0.6	0.75	0.75	0.75	0.75	0.75
	0.75										0.35	0.4	0.45	0.55	0.7	0.75	0.75	0.75	0.75	0.75	0.75
	0.8									0.35	0.4	0.5	0.6	0.7	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.85							0.35	0.35	0.4	0.5	0.6	0.7	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.9							0.35	0.4	0.5	0.6	0.7	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.95						0.35	0.4	0.5	0.55	0.7	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	1					0.35	0.4	0.45	0.55	0.65	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75

Note: A blank cell means that the "Retirees Cooperation Regime" will be selected. The above figures represent the upper bounds of $\rho_{t,}$ under which "Working generation and Retirees with children Cooperation Regime" will be selected. For example, the figure 0.5 of (δ, σ) = (0.5, 0.5) means that "Working generation and Retirees with children Cooperation Regime" will be selected if $\rho_t < 0.5$ and "Retirees Cooperation Regime" will be selected if $\rho_t > 0.5$.

Table 2: "Retirees Cooperation Regime"⇒"Working generation and Retirees with children Cooperation Regime"
Lifetime tax rate and Utility for each generation by change of σ : (ρ_t, δ) = (0.45, 0.8)

	σ										
	0.4	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Lifetime tax rate of Retired generation ($\theta_{t-1}(t)$)	-0.656	-0.148	-0.168	-0.187	-0.206	-0.225	-0.243	-0.261	-0.278	-0.296	-0.312
Lifetime tax rate of Working generation ($\theta_t(t)$)	0.280	0.004	0.014	0.025	0.035	0.045	0.055	0.065	0.075	0.084	0.093
Lifetime tax rate of Future generation ($\theta_{t+1}(t)$)	0.610	0.460	0.466	0.471	0.477	0.482	0.488	0.493	0.498	0.503	0.508
Utility of Retired generation with children (U_{t-1}^{child})	-0.078	-0.074	-0.069	-0.065	-0.061	-0.058	-0.055	-0.052	-0.050	-0.049	-0.047
Utility of Retired generation without children (U_{t-1}^{nc})	0.504	0.138	0.155	0.171	0.187	0.203	0.218	0.232	0.245	0.259	0.272
Utility of Working generation (U_t)	-0.526	-0.189	-0.187	-0.184	-0.179	-0.174	-0.167	-0.158	-0.149	-0.138	-0.127

Table 3: "Retirees Cooperation Regime" vs. "Demeny voting + Working generation and Retirees with children Cooperation Regime"

		σ												
		0.02 - 0.76	0.78	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98	1
	0.02 - 0.74													
	0.76													0.4
	0.78												0.44	0.76
	0.8											0.5	0.76	0.76
	0.82										0.76	0.76	0.76	0.76
	0.84									0.76	0.76	0.76	0.76	0.76
δ	0.86								0.54	0.76	0.76	0.76	0.76	0.76
	0.88							0.48	0.76	0.76	0.76	0.76	0.76	0.76
	0.9						0.44	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	0.92					0.4	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	0.94				0.36	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	0.96			0.34	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	0.98			0.52	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76
	1		0.42	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76

Note: A blank cell means that "Retirees Cooperation Regime" will be selected. The above figures represent the upper bounds of ρ_t, under which "Demeny voting + Working generation and Retirees with children Cooperation Regime" will be selected. For example, the figure 0.48 of (δ, σ) = (0.88, 0.88) means that "Demeny voting + Working generation and Retirees with children Cooperation Regime" will be selected if $n_t < 0.48$ and "Retirees Cooperation Regime" will be selected if $\rho_t > 0.48$.

Table 4 : "Retirees Cooperation Regime"⇒"Demeny voting +Working generation and Retirees with children Cooperation Regime"
Lifetime tax rate and Utility for each generation by change of σ : (ρ_t , δ) = (0.45, 0.8)

	σ			
	0.4	0.96	0.98	1
Lifetime tax rate of Retired generation ($\theta_{t-1}(t)$)	-0.656	-0.281	-0.289	-0.297
Lifetime tax rate of Working generation ($\theta_t(t)$)	0.280	0.006	0.000	-0.007
Lifetime tax rate of Future generation ($\theta_{t+1}(t)$)	0.610	0.461	0.458	0.454
Utility of Retired generation with children (U_{t-1}^{child})	-0.078	0.033	0.047	0.060
Utility of Retired generation without children (U_{t-1}^{nc})	0.504	0.248	0.254	0.260
Utility of Working generation (U_t)	-0.526	-0.030	-0.010	0.010