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THE VALUATION AND REDISTRIBUTION EFFECT OF THE KOREA NATIONAL PENSION

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Abstract

This paper evaluates the Korea National Pension (KNP) and investigates its redistribution effects. The educational level is used as a proxy for mortality and various socioeconomic factors are considered. The financial and utility-based analyses reveal strong progressive redistribution with income level. Also, the utility-based analysis indicates significant progressive redistribution with non-pension asset level but no significant redistribution with the educational level. Generally, the KNP is extremely valuable and its value seems higher with a pre-existing private annuity especially for the poor. Finally, when people are assumed to spend at least the minimum consumption level, it becomes more beneficial.

Keywords: Korea National Pension; redistribution; annuity evaluation; annuity equivalent wealth; money's worth ratio; bequest
JEL Classification: C61, D91, J26, I38

I. *Introduction*

The national pension was introduced in Korea in 1988. Since then, it has experienced phenomenal growth and is recognized as one of the four largest public pension funds in the world (www.nps.or.kr). Currently, nearly 40% of the nation's total population is insured under the KNP scheme, and it has firmly settled as the prime social welfare system for the aged and

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needy. Further, according to the social survey by the Statistics Korea (KOSTAT) in 2010, 38.5% of baby boomers born between 1955 and 1963 have the KNP as the sole source of their retirement income (www.kostat.go.kr). It is scarcely known outside Korea that the speed of aging in Korea is actually the fastest among Organization for Economic Co-operation and Development (OECD) member-countries even exceeding that of Japan. According to the KOSTAT, the percentage of the population that is over 65 years of age was 7.2% in 2000 and is expected to rise to 15.1% in 2020. This rapidly aging population induces an increase in the longevity risk, i.e., the risk of outliving one's resources (MacMinn et al. 2006; Stallard 2006). Hence, the KNP is a critical source of protection for Korean retirees against the longevity risk, and also it is a social security system with income redistribution effects through its progressive benefit formula.

Many studies have established the progressive redistribution effect of the KNP. However, some recent studies (e.g., Lee, 2006; Chung and Lee, 2008) that reflect heterogeneity in mortality have shown that the progressive redistribution is reduced or even that the regressive redistribution is manifested. However, most previous studies have investigated the redistribution effect of the KNP based on financial measurements such as 'benefit/tax ratio' (e.g., Chung and Lee, 2008; Kim, 2004; Lee, 2006) and have ignored the insurance value of the KNP. In other words, they have not included the utility value of longevity insurance in their annuity valuation framework. Meanwhile, a large body of the US-based literature (e.g., Brown, 2001; Mitchell et al., 1999) has focused on measuring the insurance value of annuitization only for representative groups without bequest options. The effect of the bequest motive on the annuitization of wealth is somewhat controversial. For instance, some studies argue that it is not quite significant (Brown, 2001; Hamermesh, 1984; Hurd, 1987; King and Dicks-Mireaux, 1982), while others suggest that the motive have a significant impact on the value of annuitization (Bernheim, 1991; Davidoff et al., 2005; Warshawsky, 1988). However, the bequest motive is found to be an important factor that affects utility levels for elderly Koreans (Chung, 2002). Moreover, since the intergenerational relationship between children and their parents in Korea is more normative and obligatory than that in other western countries including US, Koreans tend to have strong bequest motives (Cho, 2003; Shin et al., 1997; Song, 2009). Therefore, incorporation of the bequest motive in the annuity valuation framework should be useful and valuable for the Korean population and for other peoples that possess similar characteristics.

This study assesses the redistribution effect of the KNP with the rigorous framework that reflects realities of Korean life. First, this study evaluates the value of the KNP for retired couples in the framework that incorporates the utility value of longevity insurance and various levels of intensity of couples' bequest motives. Second, we analyze the redistribution effect of the KNP by examining the pension value across various socioeconomic groups including the educational level, income, assets, pre-existing private annuities, and risk when the bequest motive is included in the valuation model. While analyzing, we use the educational level as a proxy for mortality in order to classify the population. We use a life cycle-based optimization model, and utilize the dynamic programming (DP) technique as a solution procedure to measure the utility value of an annuity. In particular, by incorporating the bequest motive of couples, this study extends and upgrades the previous annuity valuation model called the annuity equivalent wealth (AEW) that was developed by Brown (2001) and Brown and Poterba (2000).

II. *An Overview of the Prior Literature*

1. Valuation of National Pension

The KNP is a life annuity and provides some protection to retirees against the risk of outliving their accumulated assets. The insurance value of pensions has been considered in several pension valuation studies regarding some developed countries including the US (e.g., Brown, 2003; Gong and Webb, 2008) and Switzerland (e.g., Butler and Teppa, 2007). Hence, it should be included in the valuation framework of the KNP.

The valuation of annuities has been one of the main issues in the study of annuities, and several attempts have been made to evaluate annuities based purely on financial measurements. For instance, many actuarial and insurance papers use the internal rate of return (IRR) types of concepts to evaluate annuities and other forms of life insurance (i.e., Broverman, 1986). Similarly, Mielvsky (2005) suggests a measure called implied longevity yield (ILY), which is equal to the IRR over a fixed deferral period that an individual would have to earn on their investable wealth if she decides to self-annuitize using a systematic withdrawal plan. Another commonly used financial measurement for measuring the value of an annuity is the money's worth ratio (MWR), which is defined as the ratio of the expected net present value of all the payouts to the premium paid for the annuity (Brown, 2007; Friedman and Warshawsky, 1988, 1990; Mitchell et al., 1999; Warshawsky, 1988).

However, a purely financial measurement ignores the insurance value that individuals derive from the elimination of longevity risk; hence, in order to assess the welfare effect of differential mortality, one must include heterogeneous mortality into a utility-based model (Brown 2003). To overcome the limitation posed by purely financial measurement, several studies use a utility-based optimization model with the framework of a life-cycle model of consumption to measure the value of annuities. In this type of model, the valuation of annuities is usually attempted under varying conditions of the risk aversion, group-specific mortality rate, time preference rate, and pre-existing annuity. Most of the previous studies that consider a utility-based optimization model use numerical optimization techniques to calculate either the wealth equivalent of an annuity or the AEW (Gong and Webb, 2008). Brown and Poterba (2000) examine joint-life annuity products for married couples and analyze the potential utility that an actuarially fair annuity can provide for couples. Their results indicate that for any given level of risk aversion, married couples will place a lower valuation than single individuals on annuitization. They also point out the importance of such analysis because most potential buyers are married.

Abel and Warshawsky (1988), Bernheim (1991), and Davidoff et al. (2005) claim that the bequest motive may be one of the reasons for the annuity puzzle; that is, the bequest motive has a negative impact on the valuation of annuities. Cocco et al. (2005) point out that the introduction of a bequest motive has a relatively stronger effect after retirement than before retirement. Also, Brown and Poterba (2000) notice the importance of calibrating the effect of bequest motives on the demand for joint-and-survivor annuities. They expect that annuities will be less valuable if couples value wealth that is left to their heirs. Brown (2003) suggests that even though the economics profession does not have a consensus about the importance of bequests or how to model them, a study that furthers an understanding of the value of annuities with bequest options will be useful.

Some studies specifically deal with the importance of the bequest motive in the Korean population. While Chung (2002) notes that traditional life cycle models generally do not include the bequest motive, she develops two separate life cycle models for the pre-retirement and post-retirement periods, respectively. By analyzing a survey of 324 Korean retirees through the models, she concludes that the bequest motive is one of the significant factors that determine the utility level of individuals. Also, Song (2009) analyzes the 2004 Korea National Survey of the Actual Conditions and Welfare Demand of the Elderly and provides evidence that 88.5% of Korean retirees over 60 years of age want to save more money in order to receive better care from their descendents. She argues that the holding of bequeathable wealth implies the expectation of better attitudes on the part of descendents towards their parents and also that the Korean elderly possess the exchange (or strategic) bequest motives that were suggested by Bernheim et al. (1985). Song (2009) also examines the 2005 Korea National Panel Survey of Security for the Aged and argues that when a family has one additional child, the possibility of buying a private lifetime annuity decreases by 1.1%, which basically shows the existence of somewhat strong bequest motives among the Korean people. Also, many Korean parents want to leave their house for their heirs as inheritance or help their children buy a house (Nam 2006; Yoon 2005). We may conclude from previous studies that due to the strong influence of tradition, Koreans have a strong tendency to save money to bequeath it to their children and in exchange, expect care-giving from their children.

Few studies have specifically dealt with the utility-based valuation of annuities for the Korean population. Yuh and Yang (2009) use the AEW to measure the value of life-time annuities for the Korean population. Their results indicate that the value of a life-time annuity is high regardless of the gender and degree of risk aversion, and decreases as the level of pre-existing annuities increases. However, they do not use cohort-specific life tables, and their analysis is confined to single individuals without bequest motives.

2. The Distributional Effects of National Pension

There is considerable literature that evaluates the distributional effects of the US social security system in purely financial terms (e.g., Coronado, Fullerton, and Glass, 2000; Gustman and Steinmeier, 2001; Liebman, 2002). They all find that mandatory annuitization reduces the overall progressivity of the system since households with high lifetime incomes tend to live longer. In other words, mandatory annuitization such as social security and most defined benefit pensions redistributes wealth from those who die young and are disproportionately male and/or less well educated toward those with low mortality and who are disproportionately female and/or college educated (Gong and Webb, 2008).

Brown (2003) is the first to extend the analysis of the AEW to explain the redistribution effects of wealth due to so-called mandatory annuitization such as Social Security. He finds that the degree of the redistribution that arises from a mandatory annuity program is substantially lower on a utility-adjusted basis than when evaluated on purely financial terms, such as the MWR of an actuarially fair annuity. Specifically, he shows that quite a large redistribution exists when measured on a financial basis and often is away from economically disadvantaged groups and toward groups that are better off financially. Furthermore, he finds that far less redistribution appears to exist when evaluating on a utility-adjusted basis. Recently, Gong and

Webb (2008) extend the work of Brown (2003), which relates to an AEW analysis of the distribution effect of wealth, by including married couples and pre-existing annuities. Using the Health and Retirement Study (HRS) data and considering longevity risk-pooling within marriages and pre-existing annuity wealth, they find a significant redistribution away from disadvantaged groups in expected-utility terms: a significant minority will perceive themselves as suffering a loss from mandatory annuitization.

There are some Korean studies which estimate the distributional consequences of mandatory annuitization in expected utility terms, but almost all studies deal with them using purely financial measurements such as benefit/tax ratio. Most studies using the financial measurements confirm that in the KNP system, redistribution appears to exist across income groups showing that the financial measurements decreases as the income level increases (e.g., Kwon, 2005; Seok and Kim, 2002; Tchoi, 1999). For instance, Tchoi (1999) examines the redistribution effect of the national pension across different income groups, and estimates 5.06, 2.32, and 1.63 for low income, middle income, and high income groups respectively. In the meantime, Lee (2006) investigates the distributional effects of the national pension considering mortality differences by income and generation, and finds that heterogeneous mortality moderately reduces the redistributive effects of the national pension system. However, Lee (2006) also uses a financial measurement which is the benefit/tax ratio to measure the value of the KNP. Very recently, Yang et al. (2010) study the KNP with single individuals without considering the bequest motives and find that the regressive redistribution is somewhat small under a utility-based measurement. Further, Lee (2006) and Yang et al. (2010) are limited to the case of single individuals instead of couples and do not consider the bequest motivation.

III. *Methodology*

We begin by describing the KNP payouts to individuals depending upon their marital status. Then, we discuss the cohort-specific life table that we apply to evaluate the financial and utility value of the KNP. Also, we introduce the MWR, which is a commonly used measurement to financially evaluate the KNP and illustrate our life-cycle model, which has the objectives of maximizing the utility from consumption and bequests. We then introduce a method to calculate the AEW of the KNP for couples in Korea. The AEW analysis that we use is similar to those in Brown (2001; 2003), Brown and Poterba (2000), Gong and Web (2008), Yang et al. (2010), and Yuh and Yang (2009). However, we extend the previous models by considering couples instead of singles and including the bequest motive of couples. All the monetary values in the model are converted to USD using an exchange rate of USD 1 = 1,000 KRW for simplicity and ease of exposition[1].

1. The KNP Payouts

We assume that a forty-year-old Korean male who was born in 1970 is insured under the KNP[2] and has a dependent wife. We also assume that he started contributing to the KNP when

[1] As of January 1, 2006, USD 1 = 1,013 KRW.
[2] According to the KOSTAT, more than 78% of heads of households were males in Korea in 2006.

he was thirty years old and did that for thirty years. The couple will start receiving pension payouts when he is 65 years old[3]. As long as both husband and wife are alive, the monthly pension payout, A_t, which will be received by the couple is determined by the formula from the KNP (www.nps.or.kr) as follows. For easy comparison, we follow the notation used in the KNP website.

$$A_t=\left[\frac{1.8(A+B)P1}{P}+\frac{1.5(A+B)P2}{P}+\frac{1.485(A+B)P3}{P}+\cdots+\frac{1.215(A+B)P21}{P}+\frac{1.2(A+B)P22}{P}\right]\left(\frac{1+0.05n}{12^2}\right)+\frac{214.86}{12}. \tag{1}$$

In Eq. (1), A is the average monthly income of all the insurers for the last three years and B is a particular individual's average monthly income during his/her period of contribution. Also, n represents the total number of additional periods in months after 20 years of contributing to the KNP. Since we assume that the individual has contributed for thirty years, the total number of months of contribution is $P=30\times 12=360$ and so, n should be 120 months. Also, $P1$ denotes the total number of months between years 2000 and 2007, inclusive, and is $8\times 12=96$ months. Similarly, each of $P2$, $P3$,..., $P21$ represents the number of months existing in years 2008, 2009,..., 2027, respectively, and they are all 12 months. Finally, $P22$ denotes the number of months existing in years 2028 and 2029, and is $2\times 12=24$ months. Since we do not know the future value, A, for the individual, we use $A=1,750,959$ KRW, which is about 1,751 USD that the KNP uses in their latest example to illustrate its policy for the Korean public (www.nps.or.kr).

When a dependent dies, Eq. (1) changes slightly and becomes the same as Eq. (1) without the last term, which is $214.86/12=17.905$. When the primary benefactor dies, Eq. (1) changes more significantly as in Eq. (2), and this amount is paid out to a surviving dependent.

$$A_t=\left[\frac{1.8(A+B)P1}{P}+\frac{1.5(A+B)P2}{P}+\frac{1.485(A+B)P3}{P}+\cdots+\frac{1.215(A+B)P21}{P}+\frac{1.2(A+B)P22}{P}\right]\left(\frac{1+0.05n}{12^2}\right)\times 0.6+\frac{214.86}{12}. \tag{2}$$

2. Classification of the Population and Life Table

1 Classification of the population with different mortality rates

This study classifies the Korean population into four different groups by the educational level, viz., elementary school graduates, middle school graduates, high school graduates, and college graduates. Further, we use the educational level as a proxy for mortality to classify the population in Korea. The people who have a higher socioeconomic level are known to have a lower mortality rate than the people with a lower socioeconomic level. Among various

[3] According to the KNP scheme, a person starts receiving the benefit at age 60 in 2007, and the commencement of benefit reception will increase and eventually correspond to age 65 in year 2033.

socioeconomic variables, income level has been found to be a strong factor affecting the mortality rate in Korea as other countries (Jeong, 2009; Kang et al., 2004). More strictly speaking, the level of lifetime financial resources available to an individual is a more accurate measure to use, but solid measures of lifetime resources are not always available. Thus, one widely used measure is the current income of an individual or family, but this is found to be a poor measure of lifetime resources (Brown, 2003; Lee, 2006). A better alternative to the current income seems to be educational attainment, simply education which is a reasonable proxy for lifetime resources because more highly educated individuals have, on average, higher lifetime incomes (Brown, 2003; Son, 2004; Liebman, 1999; Lee, 2006; Kang et al., 2008). In addition, education is a predetermined variable for most retired individuals.

A significant negative correlation between education and mortality has been well documented in literature (Deaton and Paxon, 2001; Kitagawa and Hauser, 1973; Lantz et al., 1998). Further, the association of education with mortality seems somewhat stronger in Korea than in more developed countries like UK and US (Son, 2004). Specifically, some studies suggest that higher mortality rates relate to a lower educational level for most causes of death, and that mortality ratios for elementary versus university education were 5.11 for men, and 3.42 for women in Korea (e.g., Kang et al., 2004; Son, 2004). These factors are the primary motivation for using the educational level as a measure of economic status in this study.

2 Cohort-specific life table

We calculate cohort-specific mortality rates for people who were born in 1970. Also, for the same population, we calculate group-specific cohort mortality rates for groups that are differentiated by gender and educational level.

In order to obtain the AEW, it is critical to use a cohort-specific life table instead of a regular life table for a certain year. Since it was impossible to obtain raw cohort mortality rate data for age-specific groups in Korea, we used past population census data and the future population data of the target cohort from the KOSTAT to estimate the cohort-specific mortality rates. Fortunately, the data were available for age-specific cohorts and thus, we could estimate mortality rates for the target cohort. Further, some more estimation of mortality rates was necessary due to the absence of the estimated age-specific population data after the year, 2050[4].

After we obtain the mortality rates for men and women in the target cohort, we smooth the data by using a nonlinear model for the age-specific mortality rates regarding each group. As suggested in Brown et al. (2002), we apply the Gompertz-Makeham survival function to obtain fitted estimates for the mortality rates for a particular group regarding a specific age. The three parameters used in the Gompertz-Makeham survival function are estimated by using nonlinear least squares regression. This approach guarantees that the fitted mortality rates are monotonic functions of the age.

With the fitted mortality rates for particular groups, we construct group-specific cohort mortality rates by using the method in Brown et al. (2002). To justify the method, Brown et al. (2002) made two assumptions. First, the ratio of a group's age-specific mortality to that of the population as a whole is an accurate portrayal of the ratio in the entire population. Second,

[4] We used the estimation method that is used by the KOSTAT. The KOSTAT assumes that $q_x = q_{x-1}e^{ax+b}$ for successive mortality rates after the age of 85, where a and b are estimated through regression. We use this method to calculate the mortality rate after the year 2050.

TABLE 1. MORTALITY RATES BY GENDER AND EDUCATIONAL LEVEL

Age	All		Educational level							
			Male				Female			
	Male	Female	Elementary	Middle	High	College	Elementary	Middle	High	College
40	0.0033	0.0010	0.0170	0.0088	0.0037	0.0021	0.0060	0.0026	0.0012	0.0008
45	0.0037	0.0011	0.0141	0.0079	0.0034	0.0024	0.0051	0.0023	0.0011	0.0008
50	0.0044	0.0013	0.0121	0.0075	0.0035	0.0029	0.0042	0.0021	0.0011	0.0008
55	0.0056	0.0016	0.0113	0.0079	0.0041	0.0038	0.0037	0.0021	0.0012	0.0010
60	0.0077	0.0022	0.0118	0.0094	0.0056	0.0054	0.0036	0.0025	0.0016	0.0014
65	0.0114	0.0036	0.0141	0.0126	0.0085	0.0084	0.0044	0.0036	0.0026	0.0022
70	0.0178	0.0064	0.0190	0.0186	0.0144	0.0139	0.0067	0.0062	0.0048	0.0041
75	0.0289	0.0124	0.0285	0.0294	0.0257	0.0238	0.0121	0.0120	0.0099	0.0082
80	0.0480	0.0248	0.0458	0.0487	0.0475	0.0418	0.0242	0.0247	0.0214	0.0171
85	0.0807	0.0503	0.0769	0.0822	0.0888	0.0741	0.0514	0.0521	0.0470	0.0365
90	0.1352	0.1018	0.1316	0.1393	0.1639	0.1308	0.1101	0.1099	0.1028	0.0779
95	0.2230	0.2010	0.2235	0.2328	0.2906	0.2259	0.2292	0.2242	0.2169	0.1631

these ratios are constant over time. In this paper, we follow their method. For example, $q_{x,\,male}^{cohort,\,colleage\ graduate}$, which is a cohort, group-specific mortality rate (corresponding to Korean male college graduates), can be calculated as follows.

$$q_{x,\,male}^{cohort,\,colleage\ graduate} = q_{x,\,male}^{cohort}\left(\frac{q_{x,\,male}^{fitted,\,colleage\ graduate}}{q_{x,\,male}}\right)$$

where $q_{x,\,male}$ is the mortality rate of Korean men in the life table of 2005 and $q_{x,\,male}^{colleage\ graduate}$ is the mortality rate of Korean male college graduates in the life table of 2005[5]. Through this approach, we calculate the group-specific cohort mortality rates for elementary-school graduates, middle-school graduates, high-school graduates, and college graduates for both men and women.
We included our estimated mortality rates by education and gender in Table 1. To see the pattern of the data more clearly, we also included the graph which depicts changes of mortality rates by education and ages for Korean male (Figure 1). The graph shows different patterns of mortality rates according to ages by education. We intentionally excluded ages more than 85 in order to see the different patterns more clearly.

3. Financial Measurement: MWR

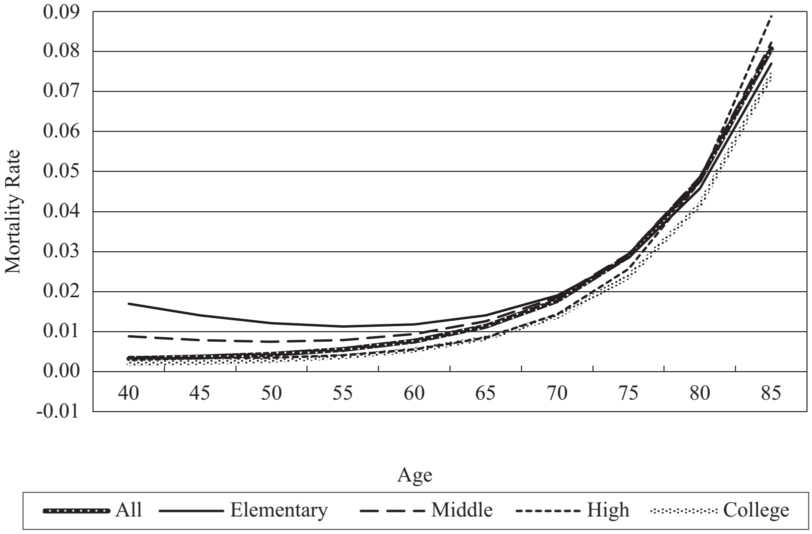
By using the annual contribution to the KNP, Eqs. (1) and (2), and mortality rates[6], we calculate the MWR, which is a commonly-used financial measurement to evaluate the annuity (Brown, 2007). In this paper, we use the MWR to financially evaluate the KNP. To calculate the MWR, we first calculate the net present value (NPV) of the total contribution to the KNP and the NPV of the total pension payout. Then, the MWR can be simply defined as follows:

$$\mathrm{MWR} = \frac{\text{Expected NPV of total pension payout}}{\text{NPV of total contribution to KNP}}.$$

[5] These data are either obtained or purchased from the KOSTAT. They show the number of people with a specific educational background who died during the year 2005.

[6] The probability of survival can be easily calculated from the mortality rate.

FIG. 1. MORTALITY RATES BY EDUCATIONAL LEVEL (MALE)



Note that the numerator is the sum of all future pension payments, weighted by the probability that an individual will be alive to receive them and discounted back to the present (or the time of retirement) under a risk-free interest rate. Similarly, the denominator is the sum of all contributions to the KNP and discounted to the present (or the time of retirement) under a risk-free interest rate. If the MWR is less than 1.0, then we may say that an individual or couple who are insured by the KNP on average will receive less in pension payments than they contributed in premiums.

4. Utility-based Measurement: AEW

We introduce a multi-period optimization model to calculate the AEW. Originally, the AEW is the amount of wealth that an individual (or couple) would need in the absence of an actuarially fair annuity market in order to achieve the same utility level that s/he (resp., they) earns when such markets are available (Brown and Poterba, 2000). In this paper, we measure the AEW for the KNP instead of an actuarially fair annuity. We assume that a couple has a non-annuitized net wealth W_0 and the KNP at the time of retirement. We also assume that they may or may not have pre-existing annuities. In order to include the situation where an individual can bequeath to his/her descendants upon death, we modify the consumption-based utility function in the optimization model. For simplicity, we assume that the utility function that is applied for the bequest is identical to the utility function that is applied for the investor's own consumption when alive, as in Cocco et al. (2005). Further, to make the model more realistic, we add one other constraint regarding the minimum annual consumption. Also, we assume that all the monetary values in the model are in real terms[7].

[7] For a description of the optimization model, see Appendix A.

1 Solution procedure: DP

Generally, it is difficult to obtain a closed-form solution for AEW (i.e., α) in a multi-period setting with liquidity constraints that are imposed by the annuity structure. In such cases, one way to solve for α is to use the DP techniques; we use the DP to solve for the optimal consumption path and bequest amount[8]. To apply the DP technique, we use a recursively defined value function $V_t\ (W_t)$ where W_t denotes the non-pension assets at time t[9].

2 Calculation of the AEW

We assume that a couple has a non-pension wealth of $\overline{W_1}$ at the time of retirement. Also, we assume that the NPV (at the time of retirement) of the total contribution to the KNP is $\overline{W_2}$. For notational convenience, we let $W^* = \overline{W_1} + \overline{W_2}$. If the couple is not insured by the KNP, then $W_0 = W^*$ and $A_t = 0$ for $t=0, 1, ..., T-age+1$ where age is the retirement age.

To calculate α, we first find the maximum utility, V^*, for the case where the couple has the KNP and $W_0 = \overline{W_1}$. Then, we solve for the case where the KNP is not available. In other words, we solve the same optimization problem with the constraints, $A_t = 0$ for $t=0, 1, ..., T-age+1$. Then, we solve for the additional wealth, ΔW, which is required for the couple in addition to W^* in order to achieve the utility value, V^*. We can define this mathematically as follows (Brown, 2003):

$$V\ (W^* + \Delta W | A_t = 0,\ \forall t) = V^*.$$

In order to obtain ΔW, we also use the DP repeatedly by applying a series of ΔW's. Finally, the AEW is obtained as:

$$\alpha = \frac{W^* + \Delta W}{W^*}.$$

3 Pre-existing private annuity

A couple may already have wealth in the form of a life annuity that is paid out to the couple or surviving individual during the retirement period. If the household receives this type of income from a pre-existing annuity such as a private annuity payout, then $\overline{W_1}$ should be modified to consider the total value of the pre-existing annuity. In this case, we calculate the NPV of the pre-existing annuity by summing up all the payouts from the annuity discounted by the real interest rate and probability of survival. Brown and Poterba (2000) call this the expected present discounted value (EPDV), which can be defined as follows for the case of a single individual:

$$EPDV = \sum_{t=1}^{T} \frac{\overline{A_t} P_t}{(1+r)^t},$$

where $\overline{A_t}$ is the annuity payout from the pre-existing annuity and P_t is the probability of

[8] In this paper, we develop a computer code by using the programming language, C, to solve the problem. The program is run on a PC with an Intel® Xeon® CPU E5504, a clock-speed of 2 GHz, and 2.99 GB RAM.

[9] For detailed descriptions of the value functions and the DP procedure, see Appendix B.

survival. Then, we subtract the EPDV from $\overline{W_1}$ when the household has a pre-existing annuity. For the analysis, we assume that the household may hold a certain percentage of $\overline{W_1}$ as the pre-existing annuity. In this case, the constraints, (A-4), in the optimization model should be changed as follows:

$$W_{t+1}=(W_t-C_t+A_t+\overline{A_t})(1+r), \quad \textit{for } t=0, 1, \ldots, T-age+1.$$

For simplicity, we assume that pre-existing private annuity is paid out to couples when they are 65 years old which is the same age when the KNP starts paying out their first pension payment to the couples.

5. Assumptions and Parameters

In this section, we discuss the assumptions and parameters used in this study.

1 Target cohort and retirement age

Couples with four different educational levels are considered[10]. Specifically, the cohort we analyze is forty-year-old in 2005 and varies by educational level (elementary school, middle school, high school, and college). For simplicity, we assume that the ages of the husband and wife are identical[11] and they have the same educational level[12]. The retirement age is set to 65 because for this cohort, the KNP is supposed to pay out the first pension payment when they are 65 years old.

2 Life tables

We use the group-specific cohort mortality rates for elementary school graduates, middle school graduates, high school graduates, and college graduates for both men and women. See Table 1 for estimated mortality rates used in the paper.

3 CRRA (constant relative risk aversion, β)

We utilize $\beta=1$ to represent the case of low risk-aversion and $\beta=3$ to represent the case of high risk-aversion. The case of $\beta=2$ can be considered an intermediate case between the two extremes.

4 Non-pension assets, pre-existing private annuities, and minimum consumption

We analyze the AEW results under various values of the pre-existing annuity ratio, i.e., the proportion of pre-existing annuity assets relative to the total net wealth at the time of

[10] According to KOSTAT, for those people who are between 60 and 69 years old, around 70% of their households have married couples in 2005.

[11] According to KOSTAT census data between years 2006 and 2009, the same age marriage was the most popular with slight margin. During this period, couples with the same age comprise about 16%. Also, couples with 1~2 year difference (husband is older) comprise about 26% and couples with 3~5 year difference (husband is older) comprise about 28%.

[12] According to the 2000 Korea census, about 69% of the couples belong to the cohort born in the 1970s have the same educational level. Specifically, 36.73% of couples have high school diplomas, 31.17% have college degrees, 0.73% have middle school diplomas, and 0.16% have elementary school diplomas as their same and final educational level. (Lee, 2010).

TABLE 2. MONTHLY INCOME, CONTRIBUTION TO THE KNP, AND PENSION PAYOUT (UNIT: USD)

Income percentile	Monthly income	Contribution to KNP (9%)	Monthly pension payout (w/ dependent)	Monthly pension payout (w/o dependent)	Monthly pension payout (surviving dependent)
10%	490	44.1	428.28	410.38	264.13
30%	1,660	149.4	642.54	624.63	392.68
50%	2,400	216.0	778.05	760.14	473.99
70%	3,280	295.2	939.20	921.29	570.68
90%	4,740	426.6	1,206.56	1,188.66	731.10

retirement: 0%, 25%, and 50%[13]. We consider three different levels of the net wealth for the age of 65, viz., \$73, 000 (25th percentile), \$163, 000 (50th percentile), and \$331, 000 (75th percentile). The data are based on Korean household data in 2006 from the KOSTAT. Also, the minimum annual consumption level is set to \$8,000[14].

5 Contribution to the KNP and pension payouts

As mentioned earlier, we assume that a Korean male started contributing to the KNP at thirty years of age and did that for thirty years. For simplicity, we assume that he earns an identical income in real terms throughout his thirty years of employment[15]. We consider five different income levels, and they are 10th, 30th, 50th, 70th, and 90th percentiles in terms of the real income in 2006 from the KOSTAT. Pension payouts for each income level are calculated by using Eqs. (1) and (2); they are summarized in Table 2.

6 Interest rate, inflation rate, insurance transactions fee, and other parameters

This study follows the assumptions used in Brown (2003). We assume that the real interest rate, r, is 0.03 and the annual inflation rate, π, is 0.00[16]. In accordance with the value used in Brown (2003), we established the utility discount rate, ρ, as being 3%; further, the time preference is set to 3%. The parameter, b, controls the intensity of the bequest motive; different values for b ranging from 0 to 5 are considered. Also, the fraction of pension payment that will be paid to the survivor after the death of one member of the couple is determined in Eq. (2); for pre-existing private annuities, this number is set to 0.67 as in Brown and Poterba (2000). Also, regarding the pre-existing private annuity, we assume that the annuity is priced at an actuarially fair rate using uniform pricing. Also, following Brown and Poterba's (2000) assumptions, the degree of jointness, λ, and the relative weights of the husband's and wife's utilities in the household utility aggregate, φ, are set to 0 and 1, respectively. The survivor ratio

[13] For instance, the pre-existing annuity ratio is set to 50% in Brown and Poterba (2000).

[14] According to the Ministry for Health, Welfare, and Family Affairs (MIHWAF) in Korea, the actual minimum standard costs of living for two-person and one-person households were approximately \$8,400 and \$5,000, respectively in 2006.

[15] This assumption may look too simple. However, the main purpose of this assumption is not to model the life cycle of an individual but to classify the population according to income level. Further, the simulation runs only for the retirement period.

[16] This value is reasonable because the average rate of the 10-year Korea Treasury Bond (KTB) was 5.87% and the average annual growth rate in the Consumer Price Index (CPI) from 2000 to 2006 was 3.04%.

Table 3. The MWR Values for Various Income Percentiles and Educational Levels

Income percentile	All	Educational level Elementary school	Middle school	High school	College
10	5.5029	3.2255	4.4297	5.3817	6.1820
30	2.4355	1.4231	1.9574	2.3804	2.7355
50	2.0395	1.1904	1.6381	1.9929	2.2904
70	1.8011	1.0503	1.4460	1.7596	2.0225
90	1.6008	0.9327	1.2846	1.5637	1.7975

ϕ is not an important concern for our model because the KNP specifically provide how annuity is changed after a partner is passed away. Hence, only pre-existing private annuity needs to set this value. For all cases, we fix ϕ as 0.7 but we will provide some sensitivity analysis in Appendix C. Finally, the insurance transactions fee of a private annuity when annuitizing, M, is fixed at 5%[17].

IV. *Findings and Discussions*

1. Financial Analysis

We first calculate the MWR, and the results are presented in Table 3. The MWR values in Table 2 generally indicate that the KNP is financially valuable for people who are insured under the KNP because the MWR values are all greater than unity except for one case. Especially, the MWRs are extremely profitable for people who earn small incomes. Further financial analysis shows that the KNP is favorable to couples that earn less income, and it also reveals that couples with a higher educational level receive more benefits from the KNP. The first result indicates a strong progressive redistribution through the KNP system, which is due to the KNP payout formula that determines the amount of pension payout by combining an individual's own income level and the average income level over all those insured by the KNP. However, the second result indirectly implies that the progressive redistribution may be significantly reduced or even nullified if we opt in the educational level, which has strong correlations with the mortality rate and the non-pension wealth level[18]. For example, a couple with the 30th percentile income level and elementary school education has 1.4231. However, this value is smaller than 1.5637, which is for a couple with the 90th percentile income level and high-school education. Finally, we can see that the reduction in the MWR is the greatest between the 10th and 30th percentile income levels (Fig. 2). This figure also signifies that the MWRs are extremely valuable for people who earn very small incomes.

[17] The value of M is difficult to determine since each insurance company uses its own method for this calculation. We believe that 5% is about the average level in Korea.

[18] It is quite surprising to see the magnitude of difference in the MWR by the educational level, and it is all due to differences in the mortality rate during the retirement period.

FIG. 2. THE MWR VALUES FOR VARIOUS INCOME PERCENTILES AND EDUCATIONAL LEVELS

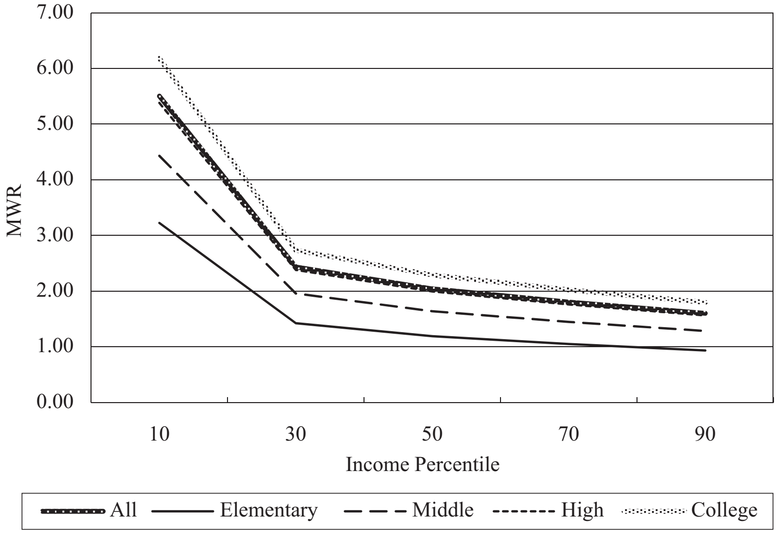


FIG. 3. THE AEW VALUES FOR THREE NON-PENSION ASSET LEVELS (1,000 USD) WHEN THERE IS NO BEQUEST MOTIVATION

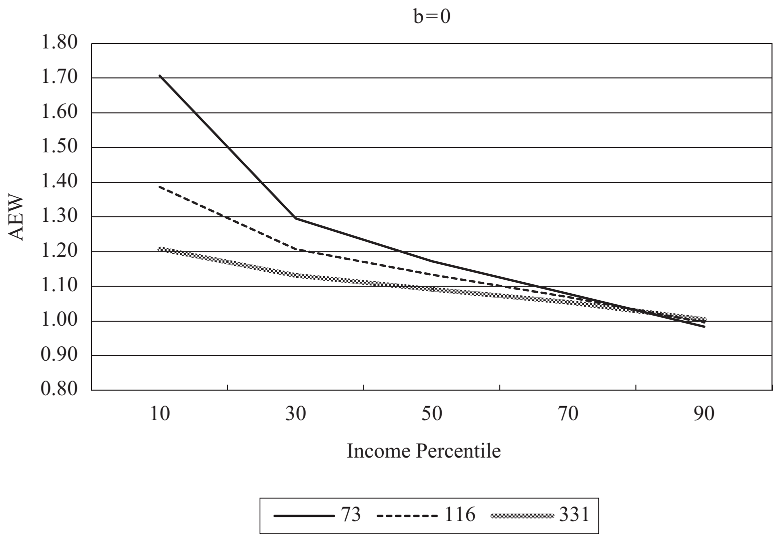


Table 4. Variation of the AEW with the Income Percentile, Bequest Motivation Level, and Non-pension Asset Level

Bequest motivation	Income percentile	Non-pension assets 73	116	331
	10	1.7068	1.3864	1.2081
	30	1.2952	1.2066	1.1314
$b=0$	50	1.1721	1.1332	1.0913
	70	1.0781	1.0688	1.0540
	90	0.9836	0.9957	1.0045
	10	1.6670	1.3642	1.1948
	30	1.2638	1.1841	1.1156
$b=1$	50	1.1434	1.1093	1.0757
	70	1.0512	1.0462	1.0369
	90	0.9587	0.9737	0.9866
	10	1.6505	1.3540	1.1893
	30	1.2496	1.1740	1.1085
$b=2$	50	1.1301	1.0986	1.0686
	70	1.0383	1.0359	1.0290
	90	0.9468	0.9634	0.9786
	10	1.6391	1.3474	1.1856
	30	1.2401	1.1671	1.1038
$b=3$	50	1.1213	1.0919	1.0638
	70	1.0298	1.0287	1.0237
	90	0.9388	0.9563	0.9732
	10	1.6298	1.3426	1.1829
	30	1.2324	1.1621	1.1002
$b=4$	50	1.1142	1.0870	1.0602
	70	1.0231	1.0234	1.0197
	90	0.9325	0.9510	0.9690
	10	1.6224	1.3384	1.1807
	30	1.2261	1.1578	1.0977
$b=5$	50	1.1084	1.0830	1.0574
	70	1.0176	1.0190	1.0166
	90	0.9274	0.9466	0.9658

2. Utility-based Analysis

The values of the AEW of a couple in relation to the income level, which ranges from 10% to 90%, non-pension asset level, which includes the 25th, 50th, and 75th percentile values, and the intensity level of the bequest motive, b, which ranges from 0 through to 5, are presented in Table 4. In Table 4, it is assumed that the CRRA is 1 and no pre-existing private annuity exists. A risk aversion of unity corresponds to log utility, a value that is often found to be the average risk aversion in many studies of consumption (Laibson et al., 1998).

Generally, the KNP is worthwhile in terms of the utility because the AEW values are greater than unity for all cases except when the income level is very high (90%). Without a bequest motivation, the AEW decreases as the income increases for a fixed non-pension asset level (Fig. 3). This indicates a progressive redistribution by the income level. However, even though it is difficult to compare directly the effects due to the inherent differences, we can

FIG. 4. THE AEW VALUES FOR SIX DIFFERENT INTENSITY LEVELS OF THE BEQUEST MOTIVE WHEN THE LEVEL OF NON-PENSION ASSETS IS FIXED

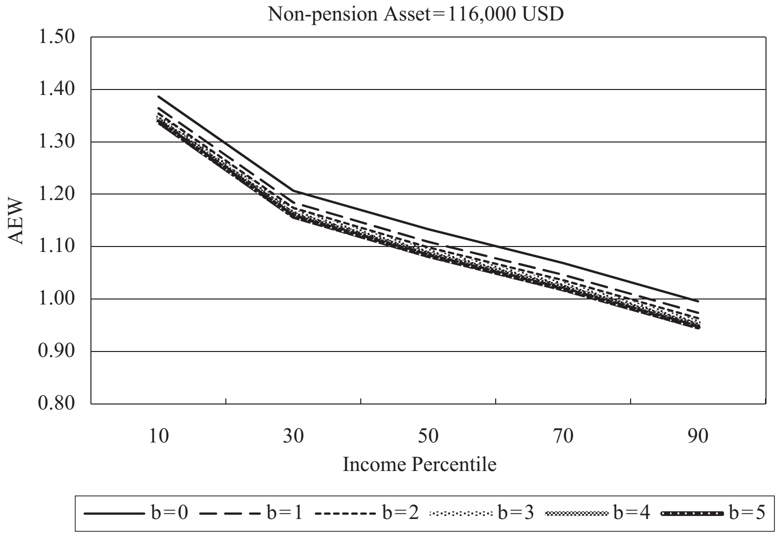


FIG. 5. VARIATION OF THE AEW WITH THE LEVELS OF NON-PENSION ASSETS (1,000 USD) AND BEQUEST INTENSITY

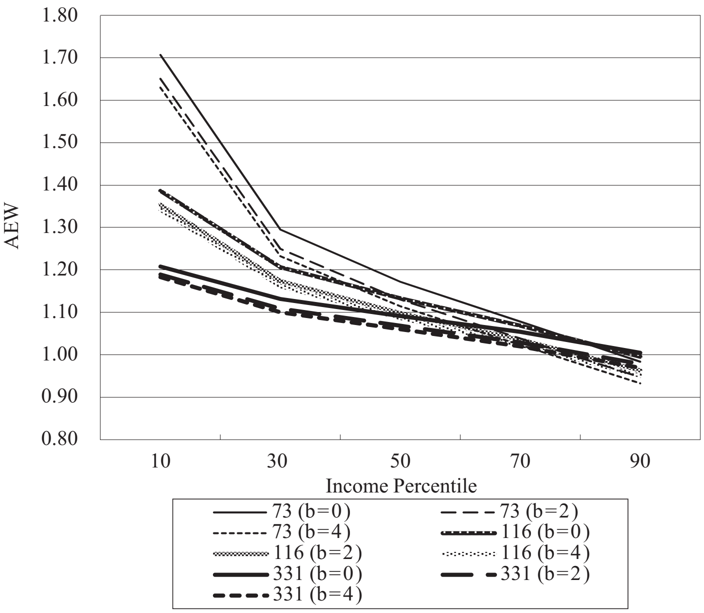


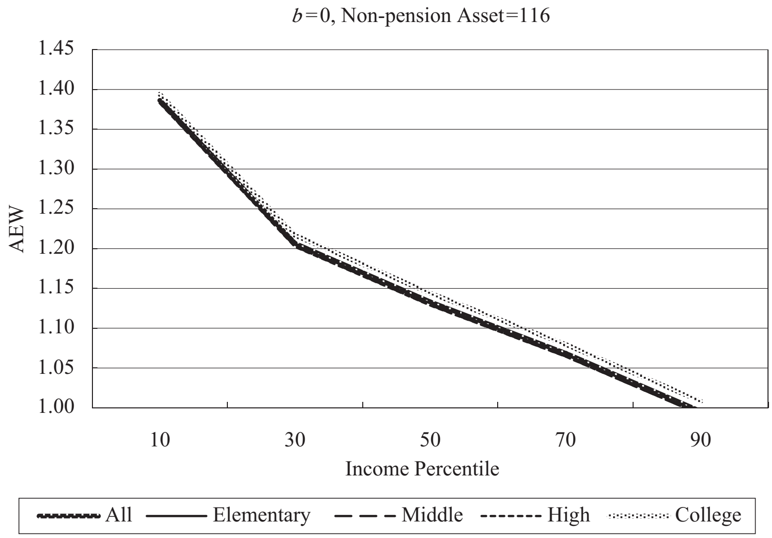
TABLE 5. THE AEW VALUES FOR VARIOUS EDUCATIONAL LEVELS, INCOMES, BEQUEST INTENSITY LEVELS, AND LEVELS OF NON-PENSION ASSETS

Non-pension assets (1,000 USD)	Bequest motivation	Income percentile	Educational level				
			All	Elementary School	Middle School	High School	College
73	$b=0$	10	1.7068	1.7042	1.7006	1.7000	1.7246
		30	1.2952	1.2927	1.2895	1.2900	1.3126
		50	1.1721	1.1696	1.1666	1.1674	1.1885
		70	1.0781	1.0756	1.0726	1.0737	1.0947
		90	0.9836	0.9812	0.9783	0.9794	1.0000
	$b=2$	10	1.6505	1.6477	1.6436	1.6470	1.6757
		30	1.2496	1.2472	1.2435	1.2470	1.2725
		50	1.1301	1.1278	1.1244	1.1278	1.1523
		70	1.0383	1.0361	1.0327	1.0363	1.0602
		90	0.9468	0.9448	0.9415	0.9451	0.9680
	$b=4$	10	1.6298	1.6275	1.6234	1.6280	1.6570
		30	1.2324	1.2304	1.2267	1.2315	1.2571
		50	1.1142	1.1123	1.1088	1.1137	1.1380
		70	1.0231	1.0212	1.0178	1.0228	1.0466
		90	0.9325	0.9308	0.9275	0.9326	0.9609
116	$b=0$	10	1.3864	1.3850	1.3833	1.3826	1.3942
		30	1.2066	1.2048	1.2027	1.2021	1.2169
		50	1.1332	1.1313	1.1290	1.1286	1.1441
		70	1.0688	1.0668	1.0646	1.0644	1.0798
		90	0.9957	0.9937	0.9914	0.9915	1.0084
	$b=2$	10	1.3540	1.3524	1.3504	1.3513	1.3664
		30	1.1740	1.1724	1.1701	1.1713	1.1875
		50	1.0986	1.0971	1.0948	1.0961	1.1137
		70	1.0359	1.0342	1.0317	1.0334	1.0511
		90	0.9634	0.9617	0.9591	0.9612	0.9793
	$b=4$	10	1.3426	1.3413	1.3391	1.3410	1.3561
		30	1.1621	1.1607	1.1584	1.1608	1.1771
		50	1.0870	1.0857	1.0833	1.0859	1.1024
		70	1.0234	1.0220	1.0193	1.0224	1.0403
		90	0.9510	0.9496	0.9469	0.9502	0.9685
331	$b=0$	10	1.2081	1.2074	1.2064	1.2056	1.2115
		30	1.1314	1.1304	1.1291	1.1282	1.1364
		50	1.0913	1.0903	1.0889	1.0881	1.0968
		70	1.0540	1.0528	1.0512	1.0504	1.0605
		90	1.0045	1.0029	1.0011	1.0004	1.0126
	$b=2$	10	1.1893	1.1885	1.1875	1.1876	1.1951
		30	1.1085	1.1075	1.1060	1.1064	1.1166
		50	1.0686	1.0676	1.0660	1.0665	1.0770
		70	1.0290	1.0278	1.0261	1.0267	1.0387
		90	0.9786	0.9774	0.9755	0.9764	0.9891
	$b=4$	10	1.1829	1.1822	1.1811	1.1819	1.1896
		30	1.1002	1.0994	1.0980	1.0991	1.1094
		50	1.0602	1.0594	1.0578	1.0591	1.0698
		70	1.0197	1.0188	1.0170	1.0185	1.0307
		90	0.9690	0.9680	0.9661	0.9679	0.9809

TABLE 6. VARIATION OF THE AEW WITH THE LEVEL OF PRE-EXISTING PRIVATE ANNUITIES

Non-pension assets (1,000 USD)	Bequest motivation	Income percentile	Pre-existing private annuities 0%	25%	50%
73	$b=0$	10	1.7068	1.8280	2.0514
		30	1.2952	1.3148	1.3494
		50	1.1721	1.1742	1.1837
		70	1.0781	1.0720	1.0712
		90	0.9836	0.9733	0.9679
	$b=2$	10	1.6505	1.7728	1.9876
		30	1.2496	1.2710	1.3064
		50	1.1301	1.1338	1.1441
		70	1.0383	1.0334	1.0348
		90	0.9468	0.9372	0.9327
	$b=4$	10	1.6298	1.7537	1.9650
		30	1.2324	1.2538	1.2884
		50	1.1142	1.1179	1.1276
		70	1.0231	1.0179	1.0199
		90	0.9325	0.9228	0.9183
116	$b=0$	10	1.3864	1.4802	1.6465
		30	1.2066	1.2300	1.2717
		50	1.1332	1.1396	1.1573
		70	1.0688	1.0668	1.0681
		90	0.9957	0.9849	0.9769
	$b=2$	10	1.3540	1.4483	1.6110
		30	1.1740	1.1976	1.2395
		50	1.0986	1.1067	1.1263
		70	1.0359	1.0344	1.0376
		90	0.9634	0.9545	0.9465
	$b=4$	10	1.3426	1.4358	1.5969
		30	1.1621	1.1857	1.2261
		50	1.0870	1.0944	1.1135
		70	1.0234	1.0216	1.0251
		90	0.9510	0.9424	0.9340
331	$b=0$	10	1.2081	1.2707	1.3846
		30	1.1314	1.1567	1.1990
		50	1.0913	1.1037	1.1260
		70	1.0540	1.0578	1.0640
		90	1.0045	0.9975	0.9897
	$b=2$	10	1.1893	1.2531	1.3650
		30	1.1085	1.1359	1.1783
		50	1.0686	1.0819	1.1032
		70	1.0290	1.0342	1.0408
		90	0.9786	0.9735	0.9656
	$b=4$	10	1.1829	1.2463	1.3570
		30	1.1002	1.1278	1.1699
		50	1.0602	1.0734	1.0942
		70	1.0197	1.0248	1.0312
		90	0.9690	0.9640	0.9556

FIG. 6. THE AEW VALUES FOR VARIOUS EDUCATIONAL LEVELS



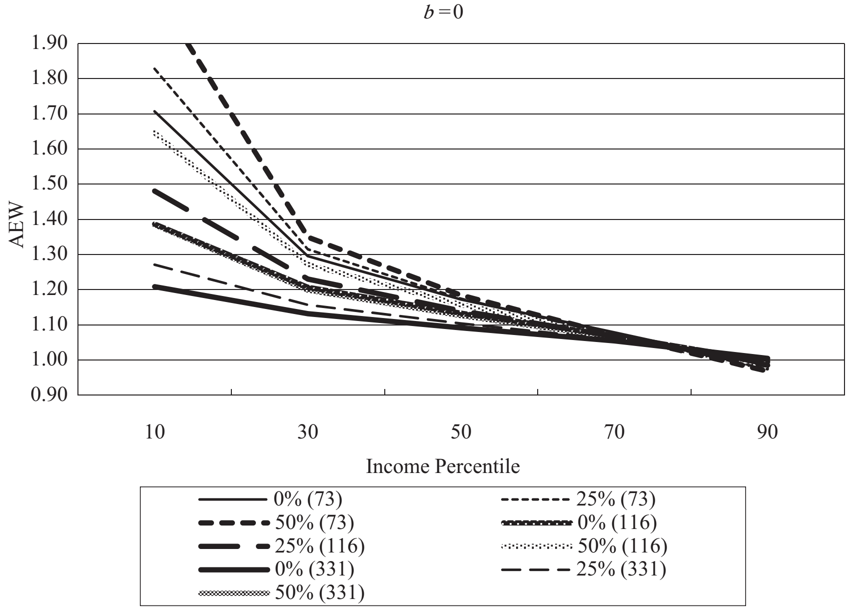
observe that the size of the effect is much bigger with the MWRs than with the AEWs. With regard to non-pension assets, the AEW decreases as the non-pension asset level increases for a fixed income level (Fig. 3). Consequently, there is a progressive redistribution by the non-pension asset level. Note that we cannot consider factors such as the non-pension asset level when we calculate the MWR. Regardless of the bequest intensity, the variation of the AEW values with the different non-pension asset levels diminishes as the income level increases, and when the income percentile is 90 (for all cases) or 70 (for $b=4$ and higher), the AEW for the smallest non-pension asset level is even slightly larger than in the other cases (Table 4). Under a fixed non-pension asset level, the reduction in the AEW is the greatest between the 10th and 30th percentile income levels and becomes smaller as the income increases (Fig. 5). Also, it is seen that the bequest effect is clear and consistent but its magnitude seems somewhat smaller than that of the income and non-pension assets (Fig. 5).

The AEW values by the educational level, bequest motive, income, and non-pension assets are presented in Table 5. Fig. 6 plots the AEW values when there is no bequest motive and the level of non-pension assets is fixed. Surprisingly, the AEW values do not show clear differences in relation to the educational background except for slightly higher values for college graduates. This result contrasts with that of the MWR, wherein the MWR values increase as the educational level increases. From this observation, we may conclude that in terms of the utility value, the regressive redistribution effect is not as strong as that estimated through the financial measurement. Brown (2003) points out that regardless of a specific group's mortality risk, annuity provides much utility by eliminating the risk of running resources down to a very low level in the event that one lives longer than expected. He also points out that even for high-mortality-risk groups, avoidance of low consumptions at the later part of their life implies huge utility gains. Therefore, quite large amount of utility gains are due to the role of annuity as longevity insurance which helps not only the low-mortality-risk groups but also the high-

TABLE 7. VARIATION OF THE AEW WITH THE CRRA

Non-pension assets (1,000 USD)	Bequest motivation	Income percentile	CRRA 1	CRRA 2	CRRA 3
73	$b=0$	10	1.7068	1.7439	1.7723
		30	1.2952	1.3292	1.3538
		50	1.1721	1.2026	1.2269
		70	1.0781	1.1089	1.1315
		90	0.9836	1.0137	1.0365
	$b=2$	10	1.6505	1.7140	1.7523
		30	1.2496	1.3014	1.3362
		50	1.1301	1.1789	1.2079
		70	1.0383	1.0848	1.1130
		90	0.9468	0.9898	1.0175
	$b=4$	10	1.6298	1.7034	1.7465
		30	1.2324	1.2937	1.3316
		50	1.1142	1.1715	1.2031
		70	1.0231	1.0777	1.1086
		90	0.9325	0.9830	1.0129
116	$b=0$	10	1.3864	1.4064	1.4206
		30	1.2066	1.2312	1.2474
		50	1.1332	1.1588	1.1750
		70	1.0688	1.0929	1.1098
		90	0.9957	1.0229	1.0416
	$b=2$	10	1.3540	1.3899	1.4094
		30	1.1740	1.2116	1.2351
		50	1.0986	1.1393	1.1626
		70	1.0359	1.0749	1.0958
		90	0.9634	1.0014	1.0268
	$b=4$	10	1.3426	1.3848	1.4064
		30	1.1621	1.2053	1.2321
		50	1.0870	1.1329	1.1596
		70	1.0234	1.0689	1.0930
		90	0.9510	0.9954	1.0232
331	$b=0$	10	1.2081	1.2189	1.2262
		30	1.1314	1.1462	1.1556
		50	1.0913	1.1067	1.1174
		70	1.0540	1.0717	1.0823
		90	1.0045	1.0251	1.0385
	$b=2$	10	1.1893	1.2094	1.2208
		30	1.1085	1.1345	1.1487
		50	1.0686	1.0942	1.1088
		70	1.0290	1.0584	1.0742
		90	0.9786	1.0089	1.0281
	$b=4$	10	1.1829	1.2061	1.2192
		30	1.1002	1.1306	1.1469
		50	1.0602	1.0904	1.1066
		70	1.0197	1.0541	1.0722
		90	0.9690	1.0038	1.0255

FIG. 7. VARIATION OF THE AEW WITH THE LEVEL OF PRE-EXISTING PRIVATE ANNUITIES AND NON-PENSION ASSETS (1,000 USD).



mortality-risk groups. Therefore, quite large amount of utility gains are due to the role of annuity as longevity insurance which helps not only the low-mortality-risk groups but also the high-mortality-risk groups. This result is consistent with that in Brown (2003).

We analyze the variation in the AEW with the level of the pre-existing private annuity (Table 6). The pre-existing private annuity is expressed as a percentage of the non-pension assets. As the percentage of pre-existing private annuity increases, the AEW also increases. In other words, a pre-existing annuity tends to improve the value of the KNP. These results are somewhat opposite to previous findings in several other studies (e.g., Brown and Poterba, 2000; Yuh and Yang, 2009). However, it seems that these results are due to the extremely high financial attractiveness of the KNP, especially when income level is low[19]. Certainly, we can find 'normal' results when the income is very high, say at the 70th and 90th percentile levels where the KNP is not as attractive as at the lower income level. In these cases, the AEW may decrease slightly as the pre-existing annuity increases (Fig. 7). This result implies that purchasing an additional private annuity instead of self-annuitization actually helps people in lower incomes and non-pension asset classes improve their satisfaction in their retirement.[20]

Fig. 7 plots the variation of the AEW with the levels of pre-existing private annuity and

[19] The similar results can be found in Yang et al. (2010).

[20] We run a sensitivity analysis with different utility discount rate. This extra result implies that more annuities (private, corporate, and etc.) are preferred when people put relatively more value on the consumption in the future (i.e., low utility discount rate). However, if people earn more value from the present consumption and consumption in near future (i.e., high utility discount rate), then they may not need extra annuities.

TABLE 8. THE AEW VALUES UNDER THE MINIMUM CONSUMPTION REQUIREMENT

Non-pension assets (1,000 USD)	Bequest motivation	Income percentile	Minimum consumption	
			No	Yes
73	$b=0$	10	1.7068	Infeasible
		30	1.2952	Infeasible
		50	1.1721	1.1935
		70	1.0781	1.1105
		90	0.9836	1.0151
	$b=2$	10	1.6505	Infeasible
		30	1.2496	Infeasible
		50	1.1301	1.1609
		70	1.0383	1.0625
		90	0.9468	0.9601
	$b=4$	10	1.6298	Infeasible
		30	1.2324	Infeasible
		50	1.1142	1.1321
		70	1.0231	1.0345
		90	0.9325	0.9377
116	$b=0$	10	1.3864	1.3901
		30	1.2066	1.2177
		50	1.1332	1.1484
		70	1.0688	1.0882
		90	0.9957	1.0169
	$b=2$	10	1.3540	1.3727
		30	1.1740	1.1907
		50	1.0986	1.1137
		70	1.0359	1.0466
		90	0.9634	0.9695
	$b=4$	10	1.3426	1.3557
		30	1.1621	1.1700
		50	1.0870	1.0922
		70	1.0234	1.0270
		90	0.9510	0.9526
331	$b=0$	10	1.2081	1.2102
		30	1.1314	1.1362
		50	1.0913	1.0973
		70	1.0540	1.0634
		90	1.0045	1.0157
	$b=2$	10	1.1893	1.1943
		30	1.1085	1.1129
		50	1.0686	1.0719
		70	1.0290	1.0315
		90	0.9786	0.9801
	$b=4$	10	1.1829	1.1843
		30	1.1002	1.1011
		50	1.0602	1.0609
		70	1.0197	1.0201
		90	0.9690	0.9692

non-pension assets when there is no bequest motive. It indicates that the effect of the pre-existing annuity is the greatest when the income level is low. However, as the income increases, the effect becomes very small and the pattern even becomes slightly reversed. This may imply that the rich are not better off by purchasing an additional private annuity.

We analyze our results with various levels of the CRRA; as expected, the results indicate that the AEW increases as the CRRA increases (Table 7). In other words, people with high risk averseness would value the KNP higher than those with low risk averseness. Even though the effect of the CRRAs is clear and consistent, the magnitude of the effect seems to be small and it may be due to the structure of the KNP[21].

Finally, we analyze the effect of the minimum annual consumption requirement. The results in Table 8 show that the AEW increases if couples are assumed to spend at least the minimum consumption level, which is 8,000 USD annually. We investigate the effect of this requirement because it may be unrealistic that people spend a lot at the beginning of their retirement but consume so small at the later part of their life. With this requirement, in the model couples should maintain at least the minimum consumption level annually, which represents a more realistic situation in real world. The results also show that those whose non-pension asset levels are at the 25th and 30th percentiles and whose incomes are lower cannot maintain the minimum annual consumption level with their disposable wealth (Table 8). This last observation may be inevitable for those who do not have more savings at the time of retirement. However, policymakers should keep this in mind when they improve the KNP in near future.

V. *Conclusions*

By incorporating the bequest motives that are found to be relatively strong for Korean retirees and couples as units of analysis, this study extends and improves the annuity valuation model used in previous studies. To the best of our knowledge, using both financial and utility-based measures with bequest motives, this paper is the first to provide empirical evidence of the redistribution effect of the KNP for couples. For this study, the educational level is used as a proxy for mortality to classify the population in Korea. A financial measurement, the MWR, and a utility-based measurement, the AEW, are utilized to evaluate the value of the KNP with various parameters and assumptions. Even though each of the measurements has its own characteristics, the AEW seems to provide a more comprehensive evaluation of the KNP because it takes into consideration couples' socioeconomic conditions such as the non-pension asset level, pre-existing private annuities, bequest motivation, etc.

This paper presents new evidence on the valuation and the extent of redistribution of the KNP. First, both the MWR and AEW analyses show strong progressive redistribution depending on the income level. Even though it is difficult to compare these two measurements directly, we can discern that the magnitude of the effect is much bigger with the MWR than with the AEW. Also, the result of the AEW analysis implies that a strong progressive redistribution effect is found in relation to the non-pension asset level. However, even though

[21] Table 1 in Brown and Poterba (2000) also indicates that their so-called REAL ANNUITIES where they assume zero inflation rates like us have small impact of different CRRAs on the AEW.

the results from the MWR analysis show strong regressive redistribution with respect to the educational level, the AEW analysis indicates no significant effect of redistribution with regard to the educational level. It indicates that the regressive distribution effect may not be as strong as we estimate based on the financial measurement; further, this finding is consistent with that in Brown (2003).

Second, in general, the KNP is extremely valuable in terms of both financial and utility-based measurements except for the people with very high income and a very strong bequest motive. Third, as the intensity of bequest motivation increases, the value of the KNP decreases and its effect is a clear and consistent. However, the magnitude of the effect may not be as large as that of income or non-pension asset. Fourth, pre-existing private annuities seem to enhance the value of the KNP and this pattern is more evident when the income level is low. Finally, if couples are assumed to spend at least the minimum annual consumption level, then the value of the KNP actually increases. Also, some low-income groups with low levels of non-pension assets (e.g., corresponding to the 25th or 30th percentile or less) and low income levels cannot meet the requirement with their disposable wealth.

The results of this study provide useful and specific insights for policymakers to re-design or improve the KNP scheme and policy formula. First, the progressive redistribution from the rich to the poor and from the more educated to the less educated should be encouraged to accomplish the progressivity of the KNP scheme. Regarding this matter, we may recommend that the KNP include heterogeneity in mortality by group with regard to, for instance, the educational level or other levels of wealth in the pension benefit formula. On the other hand, we may suggest a simpler solution. Observe in Table 6 that in reality most of population may be located on diagonal cells of the table where a high educational level means high income and a low educational level means low income due to a positive correlation between income and educational attainment. Also in Table 6, the values on the diagonal cells show strong progressive redistribution with respect to income especially for the case with low non-pension asset. Then, it may be true that the current progressive redistribution policy of the KNP based solely on income level is indeed a solid solution to help the poor and disadvantaged have better retirement. Second, in order to augment the value of the KNP benefit, other pensions such as a private pension or a corporate pension are strongly recommended for couples with low earnings potential. Third, some policy attention must be given to the people who cannot maintain the minimum consumption level especially during later part of their life. Ultimately, it would be necessary to design policies which help poor people maintain at least minimum consumption level and prevent these people from experiencing retirement ruin.

Appendix A. Description of the Life-cycle Optimization Model Used in the Simulations

We begin by formally describing an optimization model for an individual in the case of bequests. Then, we will extend the model to include the case of a couple.

$$Maximize_{\{C_t, D_t\}} \sum_{t=1}^{T-age+1} \left(\prod_{j=1}^{t-1} (1-q_j) \right) \left[\frac{(1-q_t)\, U(C_t)}{(1+\rho)^t} + \frac{b \cdot q_t U(D_t)}{(1+\rho)^t} \right] \tag{A-1}$$

Subject to

$$W_0 \ given \tag{A-2}$$

$W_t \geq 0, \quad for\ t=0, 1,..., T-age+1$ (A-3)

$W_{t+1}=(W_t-C_t+A_t)(1+r), \quad for\ t=0, 1,..., T-age+1$ (A-4)

$C_t \geq C_{\min}, \quad for\ t=0, 1,..., T-age+1$ (A-5)

$D_t \leq W_t, \quad for\ t=0, 1,..., T-age+1.$ (A-6)

In the above, q_t is the probability of dying during period t, time 0 is the time of retirement, and *age* is the retirement age. Also, W_t is the non-pension and non-annuitized wealth in period t, C_t is the annual consumption in period t, and A_t is the pension payment in period t. $C_{\min}$ is the minimum level of annual consumption and ρ is the utility discount rate. The one-period utility function, $U(C_t)$ (or $U(D_t)$), is utilized as the objective function in the model; $U(C_t)$ is a twice continuously differentiable, increasing, and strictly concave function. It is assumed that $U(C_t)$ is of a form that yields constant relative risk aversion. That is:

$$U(C_t)=\frac{C_t^{1-\beta}}{1-\beta}$$

where β is the CRRA (constant relative risk aversion). Greater is the CRRA, greater is the risk aversion.

First of all, the objective function, (A-1), is a summation of the expected-utility values over the planning horizon, viz., the duration of retirement. For each period, ρ is applied to ensure a relevant discount on the utility value for that period. The annual consumption, C_t in period t, is a decision variable in the model and is determined for each period. The constraint, (A-2), implies that the net wealth at the time of retirement is fixed as some constant. Constraints (A-3) mean that wealth cannot be negative at any point over the planning horizon. This kind of constraint also can be found in other research (Brown 2001; 2003; Brown and Poterba, 2000; Gong and Webb, 2008; Gupta and Li, 2007). The constraints, (A-4), imply that any positive wealth is invested in risk-free assets. Finally, the restrictions, (A-5), require that the minimum consumption level should be maintained in order to support a basic standard of living in each period. Without the constraints, unreasonably small annual consumption levels were obtained, especially towards the end of the lifetime.

Note that the parameter, b, controls the intensity of the bequest motive and is set to 0 when there is no bequest motive (Cocco et al., 2005). Also, D_t is the amount of bequests at the time of death, viz., period t.

When we consider a couple instead of an individual, we must use a different objective function in the optimization model. For that purpose, we follow the model used in Brown and Poterba (2000). Following Brown and Poterba (2000), we use the following utility function for the consumption portion of the optimization model:

$$U_c(C_t^m, C_t^f)=U_m(C_t^m+\lambda C_t^f)+\varphi U_f(C_t^f+\lambda C_t^m),$$

where λ is the degree of jointness or complementarity of consumption and φ is the parameter for determining the relative weights of the husband's and wife's utilities in the household utility aggregate. Also, C_t^m and C_t^f are the annual consumption by a husband and wife in period t, respectively.

Appendix B. Description of the DP Solution Procedure Used in the Simulations

Since we consider the case of a couple, we need a separate value function for the male and female. Each value function can be expressed as a recursively-defined Bellman equation (Brown, 2003). For a surviving husband and a surviving wife with a bequest motive, these value functions satisfy:

$$M_t(W_t)=Max_{\{C_t^m\}}U(C_t^m)+\frac{(1-q_{t+1}^m)}{(1+\rho)}M_{t+1}(W_{t+1})+\frac{b\cdot q_{t+1}^m}{(1+\rho)}U(W_{t+1}),$$

Table A-1. Variation of the AEW with the Degree of Jointness (CRRA = 2)

Degree of jointness	Income percentile	Non-pension asset 73	116	331
	10	1.7439	1.4064	1.2189
	30	1.3292	1.2312	1.1462
$\lambda = 0$	50	1.2026	1.1588	1.1067
	70	1.1089	1.0929	1.0717
	90	1.0137	1.0229	1.0251
	10	1.7384	1.4008	1.2152
	30	1.3243	1.2258	1.1421
$\lambda = 0.5$	50	1.1977	1.1537	1.1021
	70	1.1038	1.0886	1.0675
	90	1.0093	1.0180	1.0204
	10	1.7304	1.3968	1.2125
	30	1.3187	1.2213	1.1388
$\lambda = 1$	50	1.1936	1.1492	1.0987
	70	1.0992	1.0844	1.0640
	90	1.0047	1.0132	1.0163

$$F_t\,(W_t) = Max_{\{C_t^f\}} U(C_t^f) + \frac{(1-q_{t+1}^f)}{(1+\rho)} F_{t+1}(W_{t+1}) + \frac{b \cdot q_{t+1}^f}{(1+\rho)} U(W_{t+1})$$

The Bellman equation that defines the couple's value function with bequests in this setting is:

$$V_t\,(W_t) = Max_{\{C_t^m,\, C_t^f\}} \Big[U\,(C_t^m) + U\,(C_t^f)\Big] + \frac{(1-q_{t+1}^m)(1-q_{t+1}^f)}{(1+\rho)} V_{t+1}\,(W_{t+1}) + \frac{(1-q_{t+1}^m)q_{t+1}^f}{(1+\rho)} M_{t+1}(W_{t+1})$$

$$+ \frac{q_{t+1}^m\,(1-q_{t+1}^f)}{(1+\rho)} F_{t+1}(W_{t+1}) + \frac{b \cdot q_{t+1}^m \cdot q_{t+1}^f}{(1+\rho)} U(W_{t+1})$$

subject to constraints (A-2), (A-3), (A-4), (A-5), and (A-6).

Here, q_{t+1} is the one-period mortality probability, i.e., the probability of the individual's dying in period $t+1$ conditional upon his/her surviving through period t. The superscriptions m and f indicates a husband (male) and wife (female), respectively.

Through the above Bellman equation, a complete, multi-period optimization problem can be transformed into a series of simple, two-period, optimization problems, which can be solved numerically by solving a two-period problem in the backward sequence from the final period.

In order to solve a series of two-period optimization problems, we assume a discrete W_t space and solve each problem numerically. Also, a *stage* is a period (e.g., period t) and a *state* is one value from among the discretized W_t's.

Appendix C. Sensitivity Analysis on Degree of Jointness and Survivor Ratio

We perform sensitivity analysis on degree of jointness and survivor ratio analysis. First of all, the results with different degrees of jointness values are presented in Table A-1. Note that the CRRA is set to 2 because the AEW does not changes when CRRA $=$ 1 (Brown and Poterba, 2000). In general, the degree of jointness has small impact on the AEWs. The AEW values are slightly decreases as λ increases from 0 to 1.

The results of the AEW with different survivor ratios are presented in Table A-2. The AEW values are slightly decreased as the survivor ratio increases. It implies that increase of the survivor ratio actually

Table A-2. Variation of the AEW with Different Survivor Ratios

Survivor ratio	Income percentile	Non-pension asset 73	116	331
$\phi=0.5$	10	2.1056	1.7113	1.4589
	30	1.3730	1.3114	1.2562
	50	1.2019	1.1882	1.1736
	70	1.0849	1.0935	1.1044
	90	0.9782	0.9965	1.0232
$\phi=0.7$	10	2.0514	1.6465	1.3846
	30	1.3494	1.2717	1.1990
	50	1.1837	1.1573	1.1260
	70	1.0712	1.0681	1.0640
	90	0.9679	0.9769	0.9897
$\phi=1.0$	10	1.9943	1.5796	1.3080
	30	1.3240	1.2306	1.1454
	50	1.1643	1.1246	1.0785
	70	1.0564	1.0415	1.0229
	90	0.9567	0.9561	0.9566

decreases the annuity money from pre-existing private annuity and in turn, it decreases value of the AEW. Note that survivor ratio is only applicable to pre-existing private annuity because in case of the KNP, survivor ratio is specifically set in the pension benefit formulae.

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