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Abstract

This article applies the realized GARCH model, which incorporates the GARCH model with realized volatility (RV), to quantile forecasts of financial returns such as Value-at-Risk and expected shortfall. This model has certain advantages in the application to quantile forecasts because it can adjust the bias of RV casued by microstructure noise and non-trading hours and enables us to estimate the parameters in the return distribution jointly with the other parameters. Student's t- and skewed strudent's t-distributions as well as normal distribution are used for the return distribution. The EGARCH model is used for comparison. Main results for the S&P 500 stock index are: (1) the realized GARCH model with the skewed student's t-distribution performs better than that with the normal and student's t-distributions and the EGARCH model using the daily returns only, and (2) the performance does not improve if the realized kernel, which takes account of microstructure noise, is used instead of the plain realized volatility, implying that the realized GARCH model can adjust the bias of RV caused by microstructure noise.

JEL classification: C52; c53; G17

Key words: Expected shortfall; GARCH; Realized volatility; Skewed student's t-distribution; Value-at-Risk

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1 Introduction

Quantile forecasts of financial returns are important for financial risk management such as Value-at-Risk (VaR) and expected shortfall (ES). Few would dispute the fact that financial volatility changes over time and hence it is important to model the dynamics of volatility. One of the most widely used is the ARCH (autoregressive conditional heteroskedasticity) family including ARCH model by Engle (1982), GARCH (generalized ARCH) model by Bollerslev (1986) and their extensions. Recently, realized volatility has also attracted the attentions of financial econometricians as an accurate estimator of volatility.

There are two problems in calculating realized volatility. First, realized volatility is influenced by market microstructure noise such as bid-ask spread and non-synchronous trading (Campbell et al., 1997). There are some methods available for mitigating the effect of microstructure noise on realized volatility (Bandi and Russell, 2008, 2011; Barndorff-Nielsen et al., 2008, 2011; Zhang et al., 2005). Second, there are non-trading hours such as overnight and lunch-time, when we cannot obtain high-frequency returns. Adding the squares of overnight returns may make realized volatility noisy. Hansen and Lunde (2005a,b) propose a method for calculating realized volatility without overnight returns.

Hansen et al. (2011) have recently proposed to extend GARCH models incorporating them with realized volatility. Their models, which are called realized GARCH models, have certain advantages in quantile forecasts. First, they can adjust the bias of RV caused by microstructure noise and non-trading hours. Second, they enable us to estimate the parameters of return and volatility equations simultaneously. Thus, we can estimate the parameters of the return distribution jointly with the other parameters of the model. Takahashi et al. (2009) has extended the stochastic volatility (SV) model in the same direction. The estimation of the realized GARCH model is less time-consuming than that of the realized SV model because the former can be estimated by the maximum likelihood method while the latter requires more computer-intensive methods such as simulated maximum likelihood estimation via the importance sampling and Bayesian estimation via the MCMC. In this paper, we apply the realized GARCH model to quantile forecasts. GARCH models and RV have already been applied to quantile forecasts (Giot and Laurent, 2004; Watanabe and Sasaki, 2006; Clements et al., 2008) but this paper is the first to apply the realized GARCH model to quantile forecasts as far as I know.

In this article, we use the student's t- and skewed student's t-distributions as well as the normal distribution for the return distribution because it is straightforward to estimate the parameters in the student's t- and skewed student's t-distributions jointly with the other parameters in the realized GARCH model by the maximum likelihood method. If the realized GARCH model can adjust the bias of RV caused by microstructure noise correctly, we need not take the bias into account in calculating RV. To analyze whether it is true, we use the plain realized volatility which is the sum of intraday returns and the realized kernel proposed by Barndorff-Nielsen et al. (2008) to take account of microstructure noise. For comparison, we also use the EGARCH model proposed by Nelson (1991), which is estimated using daily returns only. The data we use are daily returns, realized volatility and realized kernel of the S&P 500. Main results are: (1) the realized GARCH model with the skewed student's t-distribution performs better than that with normal and student's t-distributions and the EGARCH model using the daily returns only, and (2) the performance does not improve if the realized kernel, which takes account of microstructure noise, is used instead of the plain realized volatility, implying that the realized GARCH model can adjust the bias caused by microstructure noise.

The article proceeds as follows. Section 2 reviews the realized GARCH model. Section 3 explains the method for forecasting the one-day ahead VaR and ES using the realized GARCH model. Section 4 explains the data and summarizes the empirical results. Conclusions and possible extensions are given in Section 5.

2 Realized GARCH Model

We start with a brief review of the realized GARCH model. Daily return R_t is specified as

$$R_t = \mathrm{E}(R_t|\boldsymbol{I}_{t-1}) + \epsilon_t,\ \ \epsilon_t = \sigma_t z_t,\ \ z_t \sim \text{i.i.d.}(0,1), \tag{1}$$

where $\mathrm{E}(R_t|\boldsymbol{I}_{t-1})$ is the expectation of R_t conditional on the information up to day $t-1$, σ_t^2 is the volatility, and z_t is the standardized error which follows an independent and identical distribution with mean 0 and variance 1. In what follows, we set $\mathrm{E}(R_t|\boldsymbol{I}_{t-1}) = 0$ because the null hypothesis of zero mean and that of no autocorrelations are not rejected in our empirical application. The distribution of z_t will be explained below.

For volatility specification, we use the simplest version of realized GARCH model:

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \gamma X_{t-1} \tag{2}$$

$$X_t = \mu + \varphi \ln \sigma_t^2 + \tau_1 z_t + \tau_2 (z_t^2 - 1) + u_t, \quad u_t \sim \text{i.i.d.N}(0, \sigma_u^2) \tag{3}$$

where X_t denotes the log of realized volatility.

Equation (2) specifies the dynamics of the true volatility σ_t^2. While GARCH models specify σ_t^2 as a function of the past values of σ_t^2 and ϵ_t (or z_t), the realized GARCH model specifies it as a function of the past values of σ_t^2 and X_t. Equation (3) is called measurement equation, which relates the realized volatility to the true volatility. If the realized volatility were an unbiased estimator of the true volatility, μ and φ would be 0 and 1 respectively. Realized volatility, however, has a bias caused by microstructure noise and non-trading hours. For example, New York Stock Exchange is open only for 6.5 hours within a day. Suppose that R_t and σ_t^2 are return and volatility for a whole day and RV_t is realized volatility calculated using the intraday returns only when the market is open. Then, we should expect $\mu < 0$ or $\varphi < 1$. Equation (3) assumes that X_t, i.e., the log of realized volatility, depends on the current value of z_t. If $\tau_1 < 0$, X_t will be larger when $z_t < 0$ than when $z_t > 0$, which will make the σ_{t+1}^2 larger when $z_t < 0$ through equation (2) if $\gamma > 0$. This is consistent with the well known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility.

The above model is the realzed GARCH(1,1) model. The realized GARCH(p,q) model replaces equation (2) with

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \ln \sigma_{t-i}^2 + \sum_{j=1}^{q} \gamma_j X_{t-j} \tag{4}$$

We also estimate the realized GARCH models of $(1,2)$, $(2,1)$ and $(2,2)$ but the performance of quantile forecasts does not change so much. Therefore, we explain the results of realized GARCH$(1,1)$ model in what follows.

The distribution of z_t is important in quantile forecasts. We use the standard normal, standardized student's t- and standardized skewed student's t-distributions for the standardized error term z_t in equation (1). The standardized version of the skewed student's t-distribution introduced by Fernández and Steel (1998) has the pdf:

$$f(z_t|\xi, \upsilon) = \begin{cases} \frac{2\xi}{\xi^2+1} s g[\xi(s z_t + m)|\upsilon] & \text{if } z_t < -\frac{m}{s} \\ \frac{2\xi}{\xi^2+1} s g[(s z_t + m)/\xi|\upsilon] & \text{if } z_t \geq -\frac{m}{s} \end{cases} \tag{5}$$

where $\upsilon > 2$ and $\xi > 0$. $g(\,\cdot\,|\upsilon)$ is a pdf of standardized student's t-distribution with degree of freedom υ. Parameters m and s^2 are the mean and the variance of the nonstandardized skewed student's t-distribution:

$$m = \frac{\Gamma\left(\frac{\upsilon-1}{2}\right)\sqrt{\upsilon-2}}{\sqrt{\pi}\Gamma\left(\frac{\upsilon}{2}\right)}\left(\xi - \frac{1}{\xi}\right), \quad s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2 \tag{6}$$

ξ and υ determine the skewness and kurtosis respectively. The skewness of z_t is zero if $\xi = 1$ and positive (negative) if $\xi > (<)1$. The kurtosis decreases as υ increases.

The likelihood of the realized GARCH model can easily be evaluated as

$$L = \prod_{t=1}^{T} h(R_t|X_1,\ldots,X_{t-1})l(X_t|X_1,\ldots,X_{t-1},R_t) \tag{7}$$

where $h(R_t|X_1,\ldots,X_{t-1})$ is the density determined by the distribution of z_t and $l(X_t|X_1, \ldots,X_{t-1},R_t)$ is the normal density with mean $\mu + \varphi\ln\sigma_t^2 + \tau_1 z_t + \tau_2(z_t^2 - 1)$ and variance σ_u^2. Given the initial value σ_0^2, we can calculate σ_t^2 by substituting $(\sigma_0^2, X_1,\ldots,X_{t-1})$ sequentially to equation (2). Hence, it is straightforward to evaluate $h(R_t|X_1,\ldots,X_{t-1})$. Given R_t and σ_t, we can calculate z_t as $z_t = R_t/\sigma_t$. Thus, it is also straightforward to evaluate $l(X_t|X_1,\ldots,X_{t-1},R_t)$. We set the initial value $\ln\sigma_0^2$ equal to the unconditional mean $(\omega+\gamma\mu)/(1-\beta-\gamma\varphi)$. We estimate the degree of freedom υ for the student's t-distribution or (ξ,υ) for the skewed student's t-distribution jointly with the parameters $(\omega,\ \beta,\ \gamma)$ in equation (2) and $(\mu,\ \varphi,\ \tau_1,\ \tau_2,\ \sigma_u^2)$ in equation (3) by the maximum likelihood method.

3 Value-at-Risk and Expected Shortfall

In this paper, we concentrate on long position. Then, one-day-ahead forecast for the VaR of the daily return R_t with probability α is defined as $\mathrm{VaR}_t(\alpha)$ satisfying

$$\Pr\left(R_t < \mathrm{VaR}_t(\alpha)|\boldsymbol{I}_{t-1}\right) = \alpha \tag{8}$$

The sample size of daily returns and realized volatility used in our empirical analysis is 3263. Using 1500 daily returns and realized volatilities, we calculate one-day-ahead forecasts of the VaR $(\mathrm{VaR}_{1501},\ldots,\mathrm{VaR}_{3263})$ as follows.

A1. Set $i = 1$.

A2. Estimate the parameters of the realized GARCH model using the sample $(R_i,\ldots,R_{1499+i}, X_i,\ldots,X_{1499+i})$ by the maximum likelihood method.

A3. Set the parameters (ω, β, γ) in equation (2) equal to their estimates obtained in A2. Then, calculate σ^2_{1500+i} by substituting σ^2_{1499+i} and X_{1499+i} to equation (2).

A4. Set the parameters υ or (υ, ξ) of the distribution of z_{1500+i} equal to their estimates in A2 if the distribution is student's t or skewed student's t. Then, obtain $z_{1500+i}(\alpha)$ satisfying $\Pr\left(z_{1500+i} < z_{1500+i}(\alpha)\right) = \alpha$ depending on the distribution.

A5. Set $\mathrm{VaR}_{1500+i}(\alpha) = \sigma_{1500+i} z_{1500+i}(\alpha)$.

A6. Set $i = i + 1$ and return to A1 if $i < 1763$ and end if $i = 1763$.

Using $(\mathrm{VaR}_{1501}, \ldots, \mathrm{VaR}_{3263})$ obtained by executing this algorithm, we calculate the empirical failure rate. Let N be the number of times when the VaR is violated, i.e., $R_t < \mathrm{VaR}_t$ for $i = 1501, \ldots, 3263$. Then, the empirical failure rate is defined as $N/1763$. Using the empirical failure rate, we apply the likelihood ratio (LR) test proposed by Kupiec (1995) to test the null hypothesis of $f = \alpha$, where f is the true failure rate. The LR statistic is:

$$\mathrm{LR} = 2\ln\left(\left(\frac{N}{1763}\right)^{N}\left(1 - \frac{N}{1763}\right)^{1763-N}\right) - 2\ln\left(\alpha^{N}(1-\alpha)^{1763-N}\right) \tag{9}$$

This LR statistic is asymptotically distributed as a $\chi^2(1)$ if the null hypothesis of $f = \alpha$ is true.

The problem of VaR is that it only measures a quantile of the distribution and hence ignores important information regarding the tails of the distribution beyond this quantile. We also use expected shortfall (ES), which is defined as the conditional expectation of the return given that it is beyond the VaR. The one-day-ahead forecast for the ES of the daily return R_t with probability α is defined as follows.

$$\mathrm{ES}_t(\alpha) = \mathrm{E}\left[R_t | R_t < \mathrm{VaR}_t(\alpha), \boldsymbol{I}_{t-1}\right] \tag{10}$$

Using $(\mathrm{VaR}_{1501}, \ldots, \mathrm{VaR}_{3263})$, we calculate $(\mathrm{ES}_{1501}, \ldots, \mathrm{ES}_{3263})$ as follows.

B1. Set $i = 1$.

B2. Simulate 10,000 sample for R_{1500+i} using equation (1) given σ_{1500+i} and distribution of z_{1500+i}.

B3. Calculate $\mathrm{ES}_{1500+i}(\alpha)$ as the average of the sample violating the VaR, i.e., $R_{1500+i} < \mathrm{VaR}_{1500+i}(\alpha)$.

B4. Set $i = i + 1$ and return to 1 if $i < 1763$ and end if $i = 1763$.

To backtest the predicted ES value with probability α, we use the measure proposed by Embrechts et al. (2004). The standard backtesting measure for expected shortfall estimates is

$$D_1(\alpha) = \frac{1}{x(\alpha)} \sum_{t \in \kappa(\alpha)} \delta_t(\alpha) \tag{11}$$

where $\delta_t(\alpha) = R_t - \mathrm{ES}_t(\alpha)$, $x(\alpha)$ is the number of days for which a violation of $\mathrm{VaR}(\alpha)$, i.e., $R_t < \mathrm{VaR}_t(\alpha)$ occurs and $\kappa(\alpha)$ is the set of days for which it happens.

Its weakness is that it depends strongly on the VaR estimates without adequately reflecting the correctness of these values. To correct for this, it is combined with a penalty

$$D_2(\alpha) = \frac{1}{y(\alpha)} \sum_{t \in \tau(\alpha)} \delta_t(\alpha) \tag{12}$$

where $y(\alpha)$ be the number of days $\delta_t(\alpha)$ is less than its α-quantile, and $\tau(\alpha)$ the set of days for which it happens.

The Embrechts et al. (2004) measure is given by

$$D(\alpha) = \left(|D_1(\alpha)| + |D_2(\alpha)|\right)/2 \tag{13}$$

A good estimation of expected shortfall will lead to a low value of $D(\alpha)$.

4 Empirical Application

4.1 Data

We use daily data on returns and realized volatilities of the S&P 500 stock index. The sample period is 1996/1/3–2009/2/27. These data are obtained from Oxford–Man Institute's Realized Library (Heber et al., 2009), where we can download two types of realized volatilities. One is the plain RV which is the sum of the intraday returns and the other is the realized kernel (RK) calculated using the method by Barndorff-Nielsen et al. (2008) to take account of microstructure noise. If the bias of RV caused by microstructure noise can be adjusted by the realized GARCH model, RK will not improve the performance of VaR and ES. To analyze whether this is true, we use the both RVs.

Figure 1 plots these data and Table 1 summarizes the descriptive statistics for the full sample. Table 1(a) shows the descriptive statistics of daily returns (%). The mean is not

Figure 1: Daily Rerurns, realized volatility and realized kernel of the S&P 500

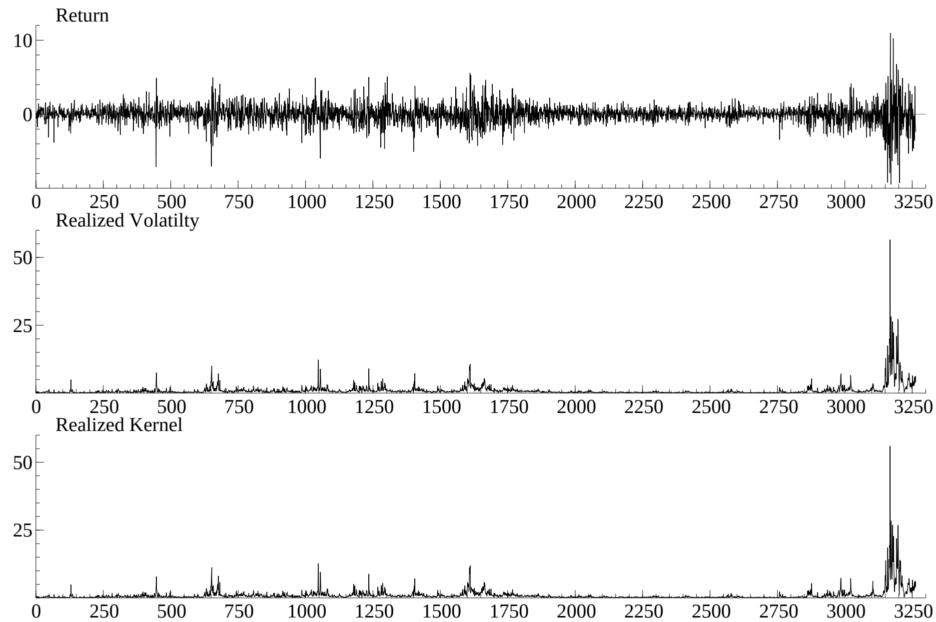


Table 1: Descriptive statistics for the full sample

Mean	St. dev.	Skewness	Kurtosis	JB	LB(10)
(a) Daily returns (%)					
0.005	1.313	−0.258	11.025	8791.47	16.27
(0.023)		(0.043)	(0.086)		
(b) Realized volatility					
0.964	2.089	10.976	202.19	5460146.73	329.39
(0.037)		(0.043)	(0.086)		
(c) Realized kernel					
1.003	2.141	10.406	180.99	4366158.90	343.24
(0.037)		(0.043)	(0.086)		
(d) Log realized volatility					
−0.657	0.993	0.556	3.825	260.69	6339.12
(0.017)		(0.043)	(0.086)		
(e) Log realized kernel					
−0.621	1.001	0.530	3.772	233.76	6475.19
(0.018)		(0.043)	(0.086)		

Sample period:1996/1/3–2009/2/27. Sample size:3263. The numbers in parentheses are standard errors. JB is the Jarque-Bera ststistic to test the null hypothesis of normality. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags.

significantly different from zero. LB(10) is the Ljung-Box statistic adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelations up to 10 lags. According to this statistic, the null hypothesis is not rejected at the 10% significance level. Thus, we set $\mathrm{E}(R_t|\boldsymbol{I}_{t-1}) = 0$ in equation (1). The skewness is significantly below zero and the kurtosis is significantly above 3, indicating the well-known phenomenon that the distribution of the daily return is leptokurtic. The Jarque-Bera (JB) statistic using the both skewness and kurtosis also rejects the null hypothesis of normality at the 1% significance level.

Table 1 (b)–(c) summarize the descriptive statistics of daily RV and RK. The values of skewness, kurtosis and JB statistic indicate that the distributions of RV and RK are non-normal. LB(10) is so large that the null hypothesis of no autocorrelation is rejected, which is consistent with the phenomenon called volatility clustering. Table 1 (d)–(e) show the descriptive statistics for log-RV and log-RK. Their distributions are much closer to the normal distribution than those of RV and RK but still non-normal. Thus, it might be better to assume a non-normal distribution also for u_t in equation (3) but we leave it for the future analysis.

4.2 Estimation results of the realized GARCH model

As explained in Section 3, we estimate the realized GARCH model using the 1500 daily returns and RV and then forecast one-day ahead VaR and ES given the parameter estimates. Table 2 summarizes the estimation results of the realized GARCH model with the normal, student's t- and skewed student's t- distributions for z_t using the first 1500 returns and realized volatility. The sample period is from 1996/1/3 to 2002/2/4. Table 2 (a) shows the results using RV. Judging from the likelihood values, the skewed student's t-distribution fits the data best. ξ in the skewed student's t-distribution is significantly below one, indicating a negative skewness of z_t. Figure 2 plots the pdf of the standard normal, standardized student's t- and standardized skewed student's t-distributions where the parameters (v, ξ) in the skewed student's t and v in the student's t are set equal to their estimates in Table 2 (a). The parameter estimates of the realized GARCH model do not depend on the distribution of z_t. The persistence in volatility can be measured by the estimates of $\beta + \gamma\varphi$, which is about 0.95 no matter which distribution is used for z_t. This result shows a well-known phenomenon of a high persistence in volatility. The estimate of μ is significantly below zero and that of φ is significantly above one at the 1% significance level, showing that the log-RV is a biased estimator of the true log-volatility. The estimate of τ_1 is significantly below zero, which is consistent with a well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility

Table 2: Estimation results of realized GARCH model for the first 1500 sample

(a) RV

ω	β	γ	μ	φ	τ_1	τ_2	σ_u	υ	ξ
Normal (Log-likelihood = -3440.53)									
0.258	0.587	0.275	-0.896	1.305	-0.198	0.058	0.518		
(0.024)	(0.031)	(0.022)	(0.051)	(0.076)	(0.014)	(0.007)	(0.009)		
Student's t (Log-likelihood = -3411.36)									
0.261	0.586	0.288	-0.870	1.252	-0.197	0.056	0.518	8.138	
(0.028)	(0.031)	(0.025)	(0.061)	(0.082)	(0.014)	(0.007)	(0.009)	(1.466)	
Skewed student's t (Log-likelihood = -3397.67)									
0.268	0.590	0.287	-0.920	1.245	-0.197	0.079	0.516	8.625	0.826
(0.028)	(0.030)	(0.024)	(0.061)	(0.076)	(0.014)	(0.009)	(0.009)	(1.670)	(0.029)

Sample period:1996/1/3–2002/2/24. Sample size:1500. The numbers in parentheses are standard errors.

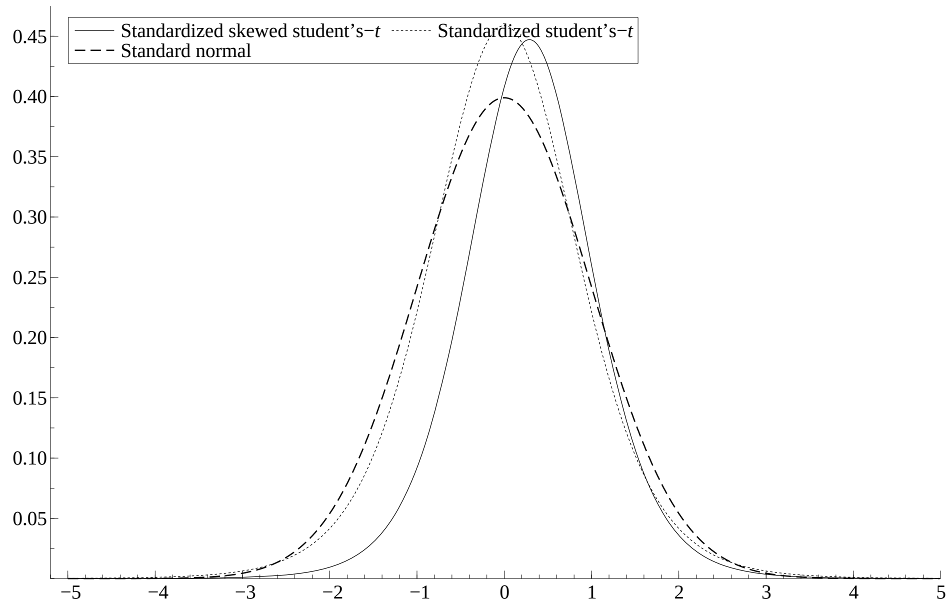
(b) RK

ω	β	γ	μ	φ	τ_1	τ_2	σ_u	υ	ξ
Normal (Log-likelihood = -3453.27)									
0.240	0.589	0.272	-0.843	1.315	-0.197	0.056	0.522		
(0.023)	(0.031)	(0.022)	(0.051)	(0.077)	(0.014)	(0.007)	(0.010)		
Student's t (Log-likelihood = -3424.16)									
0.242	0.588	0.284	-0.817	1.262	-0.196	0.054	0.523	8.142	
(0.027)	(0.031)	(0.025)	(0.062)	(0.084)	(0.015)	(0.007)	(0.010)	(1.481)	
Skewed student's t (Log-likelihood = -3410.54)									
0.249	0.592	0.284	-0.868	1.258	-0.197	0.077	0.520	8.608	0.826
(0.027)	(0.030)	(0.024)	(0.062)	(0.078)	(0.014)	(0.009)	(0.010)	(1.670)	(0.030)

Sample period:1996/1/3–2002/2/24. Sample size:1500. The numbers in parentheses are standard errors.

(Nelson, 1991). Figure 3 plots the news impact curve where the horizontal axis is z_{t-1} and the vertical axis is σ_t. Table 2 (b) summarizes the estimation results of the Realized GARCH model using RK. The results in Table 2(b) are almost the same as those in Table 2 (a).

Figure 2: Estimated pdf of z_t (Parameters υ in the student's t-distribution and (υ, ξ) in the skewed student's t-distribution are set equal to their estimates in Table 2 (a))



For comparison, we also calculate VaR and ES using the EGARCH model proposed by Nelson (1991):

$$\ln \sigma_t^2 = \omega + \phi(\ln \sigma_{t-1}^2 - \omega) + \theta z_{t-1} + \gamma(|z_{t-1}| - E(|z_{t-1}|) \qquad (14)$$

We use the standard normal, the standardized student's t and the standardized skewed student's t for the distribution of z_t.

Table 3 summarizes the estimation results of EGARCH model. Judging from the log-likelihood values, the skewed student's t-distribution fits the data best also in the EGARCH model.

4.3 Comparison using VaR and ES

Table 4 (a)–(b) show the empirical failure rates and the p-values for the Kupiec LR test for $\alpha = 1\%, 5\%, 10\%$ where RG(RV), RG(RK) and EG denote the realized GARCH model with

Figure 3: News impact curve (Parameters are set equal to their estimates in the skewed studen's-t distribution in Table 2 (a))

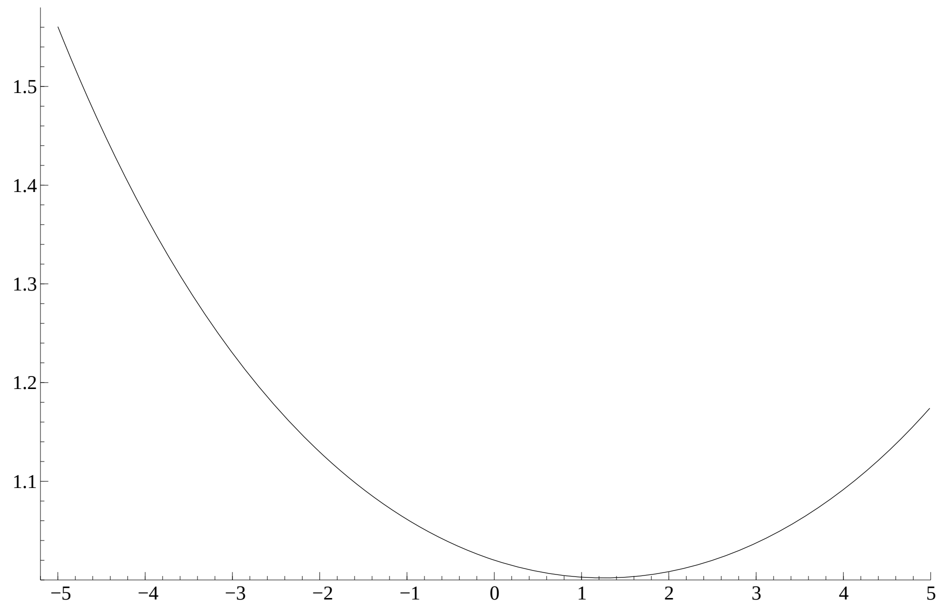


Table 3: Estimation results of EGARCH model for the first 1500 sample

ω	ϕ	θ	γ	υ	ξ
Normal (Log-likelihood = −2289.65)					
0.338	0.948	−0.165	0.101		
(0.067)	(0.011)	(0.019)	(0.020)		
Student's t (Log-likelihood = −2267.98)					
0.280	0.954	−0.158	0.099	8.730	
(0.079)	(0.011)	(0.022)	(0.022)	(1.695)	
Skewed student's t (Log-likelihood = −2261.16)					
0.297	0.955	−0.155	0.104	9.178	0.871
(0.081)	(0.010)	(0.02)	(0.022)	(1.929)	(0.032)

Sample period:1996/1/3–2002/2/24. Sample size:1500. The numbers in parentheses are standard errors.

Table 4: Results for VaR

(a) Empirical failure rate

α	10%	5%	1%
RG(RV)-n	10.323	5.332	1.645
RG(RV)-t	10.720	5.672	1.134
RG(RV)-skt	10.267	4.594	0.908
RG(RK)-n	10.380	5.615	1.645
RG(RK)-t	11.004	5.729	1.248
RG(RK)-skt	10.323	4.651	0.908
EG-n	10.040	5.445	1.872
EG-t	10.777	5.559	1.531
EG-skt	10.607	5.218	1.248

(b) p-values from the LR test

α	10%	5%	1%
RG(RV)-n	0.652	0.527	0.013*
RG(RV)-t	0.318	0.204	0.579
RG(RV)-skt	0.710	0.429	0.692
RG(RK)-n	0.597	0.245	0.013*
RG(RK)-t	0.166	0.170	0.314
RG(RK)-skt	0.652	0.497	0.692
EG-n	0.956	0.397	0.001**
EG-t	0.282	0.290	0.038*
EG-skt	0.400	0.676	0.314

The numbers in the table are p-values from the Kupic (1995) LR test calculated from the LR statistic (9). * and ** indicate that the null hypothesis of $f = \alpha$ is rejected at the 5% and 1% significance levels respectively.

RV, the realized GARCH model with RK and the EGARCH model and n, t and skt represent the normal, student's t- and skewed student's t-distributions for z_t. As can be seen from Table 4 (b), the null hypothesis of $f = \alpha$ is accepted for all models when $\alpha = 5\%, 10\%$. When $\alpha = 1\%$, the null hypothesis is rejected for RG(RV)-n, RG(RK)-n, EG-n and EG-t at the 5% significance level. Thus, we may conclude that it is not good to assume the normal distribution for z_t no matter which models are used. We cannot conclude which model performs best among RG(RV), RG(RK) and EG because the null hypothesis is accepted for all models and α if we use the skewed student's t-distribution.

Table 5: Results for ES

α	10%	5%	1%
RG(RV)-n	0.092	0.168	0.437
RG(RV)-t	0.070*	0.089	0.237
RG(RV)-skt	0.077	0.030*	0.087*
RG(RK)-n	0.110	0.174	0.443
RG(RK)-t	0.072	0.091	0.223
RG(RK)-skt	0.076	0.031	0.120
EG-n	0.136	0.240	0.456
EG-t	0.082	0.140	0.287
EG-skt	0.071	0.064	0.211

The numbers in the table are the value of $D(\alpha)$ defined by equation (13). * indicates the lowest value for each α.

In Table 5, we show the $D(\alpha)$ values defined by equation (13). As can be seen from the table, RG(RV)-skt gives the lowest values for $\alpha = 5\%, 1\%$. For $\alpha = 10\%$, RG(RV)-t gives the lowest value but it is not so much different from that of RG(RV)-skt. Hence, for the prediction of the expected shortfall of our test data, the realized GARCH model with the skewed student's t-distribution is superior to that with other distributions and the EGARCH model. RG(RV)-skt and RG(RK)-skt perform better than EG-skt for $\alpha = 5\%, 1\%$, indicating that using realized volatility improves the performance. The performance of RG(RV)-skt is almost the same as that of RG(RK)-skt for $\alpha = 10\%, 5\%$ and the former is superior to the latter for $\alpha = 1\%$, showing that the realized GARCH model can adjust the bias caused by the microstructure noise and hence we need not take the bias into account in calculating RV.

5 Conclusions and Extensions

This article applies the realized GARCH model to quantile forecasts such as VaR and ES. Using the daily returns, RV and RK of S&P 500 stock index, we find that the realized GARCH model with the skewed student's t-distribution performs better than that with the normal and student's t-distributions and the EGARCH model using the daily returns only and that the performance does not improve if the RK, which takes account of microstructure noise, is used instead of the plain RV.

Several extensions are possible. First, we used the normal, student's t- and skewed student's t-distributions for z_t. The normal inverse Gaussian (NIG) and generalized hyperbolic (GH) skew student's t-distributions have recently been applied to financial returns (Fosberg and Bollerslev, 2002; Aas and Haff, 2006). It is, however, difficult to estimate the parameters in these distributions by the maximum likelihood method (Aas and Haff, 2006). The joint estimation of the parameters in these distributions and the parameters in the realized GARCH model may be challenging. Using the empirical distribution of the standardized residuals or extreme value theory might improve the performance (Mancini and Trojani, 2011). Second, it is worthwhile using the other realized measures of volatility such as the realized range (Christensen and Podolskij, 2007; Martens and van Dijk, 2007) and the realized volatility from which significant jumps are removed (Barndorff-Nielsen and Shephard, 2004; Andersen et al., 2007). Third, realized SV model proposed by Takahashi et al. (2009) should also be applied to quantile forecasts, for example, using Bayesian estimation via MCMC. This method is time-consuming but enables us to estimate the parameters in the distribution and in the model jointly even if GH skew student's t-distribution is used for z_t (Nakajima and Omori, 2011). It also makes it possible to estimate the parameters and forecast VaR and ES jointly by sampling the parameters and the forecasts of VaR and ES jointly from their posterior distribution.

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