Default Risk on Government Bonds, Deflation, and Inflation

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Abstract

This paper analyzes the impact of deflation and inflation on the real interest rates of GBs using an overlapping generations model with the relationship between the real interest rate of GBs and the fiscal consolidation rule. We find that deflation may lower the real interest rate of GBs to the same level of public debt to capital, even if the fiscal consolidation rule is the same, as opposed to the conventional view that the real interest rate of GBs is determined independent of deflation if the Fisher equation holds. Our results are consistent with how the real interest rates of Japanese GBs react in periods of deflation.

This paper also addresses the impact of fiscal inflation (i.e., monetizing all parts of the GB's default using monetary policy). We calculate the expected fiscal inflation when the default rate in the event of fiscal consolidation is raised. The fiscal inflation may be extremely high if the extent of the required tax increase in fiscal consolidation is low. Initial inflation accelerates the expected fiscal inflation, but initial deflation suppresses it.

Key words: Overlapping generations model, real interest rate, fiscal consolidation rule, default risk, fiscal inflation

JEL codes: E17, H30, H5, H60, E62, H63

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1. Introduction

Although the fiscal burden has been increasing in all industrialized countries, Japan's debt-to-GDP ratio is the highest among developed nations, even beyond those of Italy and Greece, which have recently faced serious fiscal crises. The sustainability of the Japanese fiscal system is declining because of its low fertility rate and aging population, with the International Monetary Fund (2009) estimating that the gross public debt of Japan could reach 277% of GDP by 2016. In a country with such huge public debt, the interest rate on government bonds (GBs) would then rise to reflect this default risk. Manganelli and Wolswijk (2009) pointed out that within the GBs of EU countries, a well-established empirical pattern shows that interest rate spreads are expanding because of increasing public debt-to-GDP ratios and that these spreads could be interpreted as reflecting the risk of governments defaulting on their debt obligations. Codogno et al. (2003), Bernoth et al. (2004), and Akitobi and Stratmann (2008) also found the existence of spreads that may be interpreted as a risk premium.

By contrast, the interest rate on Japanese GBs is currently lower than those of other developed countries and it is declining even though Japanese public debt continues to increase. In order to solve this paradox, Oguro and Sato (2011) constructed an overlapping generations model within an endogenous and stochastic growth setting. They analyzed the relationship between the interest rate of GBs and the consolidation fiscal rule and made three key findings. First, the interest rate of GBs may be declining because public debt is accumulated relative to private capital (where the former crowds out the latter) as opposed to the conventional view that public debt accompanies a rise in the interest rate. Second, the fiscal consolidation rule plays a key role in determining the interest rate in equilibrium. Third, economic changes imply that the interest rate of GBs remaining relatively low may not continue in the future.

<< Insert Figure 1 about here. >>

Furthermore, Oguro and Sato (2011) abstracted the role of monetary policy (e.g., the effect of deflation and inflation on the “real” interest rate of GBs). If the Fisher equation proposed by Fisher (1930) holds, the real interest rate of GBs would be determined independent of deflation. However, according to the recent view of the New Keynesian model developed by Mankiw and Romer (1991), Walsh (2003), and Woodford (2003), inflation targeting policy affects real economic growth and the real interest rate. Therefore, if price dynamics is involved in the model proposed by Oguro and Sato (2011), new findings may be shown. As presented in Figure 1, the long-term real interest rate is stable, even though Japanese public debt continues to increase. In this mechanism, deflation may suppress the real interest rate of GBs because of the arbitrage between private capital and GBs, thereby pulling down the real return on capital.

Therefore, in the present study we provide a macroeconomic model in order to analyze the effect of deflation and inflation on the real interest rate of GBs by integrating price dynamics into the model of Oguro and Sato (2011). We find that deflation may lower the real interest rate of GBs to the same level of public debt to capital, even if the fiscal consolidation rule is the same, as opposed to the conventional view that the real interest rate of GBs is determined independent of deflation if the Fisher equation holds. Our results are consistent with how the real interest rate of Japanese GBs reacts in periods of deflation. This paper also addresses the impact of fiscal inflation (i.e., monetizing all parts of the GB’s default using monetary policy). We calculate expected fiscal inflation when the default rate in the event of fiscal consolidation is raised. Fiscal inflation may be extremely high if the extent of the required tax increase in fiscal consolidation is low. Initial inflation accelerates expected fiscal inflation but initial deflation suppresses it.

The remainder of the present paper is organized as follows. Section 2 describes the model. In section 3, we consider fiscal consolidation and establish the equilibrium of the real interest rate of GBs and the expected fiscal inflation. In section 4, we describe the parameters and scenarios for the presented simulation and discuss the implications of the results. Section 5 concludes.

2. Model Setting

2.1 Basic setting

Based on the setting proposed by Oguro and Sato (2011), we use an overlapping generations model with price dynamics (e.g., deflation and inflation). Specifically, we suppose that each generation contains a representative household who lives for two periods. Each period is composed of several stages. In stage 1, production shock is shown. The household of the young generation supplies labor in stage 2. Then, in stage 3, output is realized, wages are paid to the young and the return on capital is distributed to the old. The government collects taxes and repays public debt in stage 4. In stage 5, the young and old households consume the former also saving and choosing portfolio. Public debt and private capital are carried over into the next period. Our analysis has two steps. First, we establish intra-period or static equilibrium given the degree of public debt and capital carried over from the previous period. We then turn to their dynamics in which economic growth is endogenous and stochastic.

2.2 Production

We use Y_t to denote the aggregate real output at period *t* that is produced by a representative private firm. The production function of the economy is given as

$$Y_t = \varepsilon_t A K_t^{\mu} k_t^{\alpha} l_t^{1-\alpha} \tag{1}$$

where A (>0) is constant, μ>0, and 0<α<1. ε_t is the productivity shock. For the sake of simplicity, we assume that the shock is distributed according to the distribution function $F(\varepsilon_t)$ over the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$ with $E_t\varepsilon_t = 1$. k_t refers to private real capital that is invested in the previous period and l_t is labor supply per worker at period *t*. Given that the population of each generation is normalized to one, $L_t = 1 \times l_t$ becomes the total labor supply. K_t refers to the average real capital investment, which represents the external effect of capital accumulation. Following the literature on endogenous growth, this may be interpreted as knowledge spillover that generates a scale economy. In equilibrium, we have $k_t = K_t$

Suppose that the production is perfectly competitive. We can write the real wage and the real return on capital as

$$w_t = (1-\alpha)Y_t / l_t \quad ; \quad r_t = \alpha Y_t / k_t \tag{2}$$

Note that in determining (2), K_t is taken as given. With the price level at period *t* being denoted as P_t, we can get the "nominal" wage ($P_t w_t$) and the "nominal" return on capital ($P_t r_t$).

2.3 Household's problem

The household's lifetime utility is assumed to take the following form:

$$U_t = (c_t^y - \frac{l_t^{1+1/\delta}}{1+1/\delta})^\theta c_{t+1}^o \tag{3}$$

where θ>0 and δ>0. c_t^y denotes the young period real consumption, whereas c_t^o is the old period one. The second term in the first brackets is the disutility of labor supply. This enters the utility function so that labor supply responds to the after tax wage abstracting income effect. This simplification follows the literature on optimal income taxation (Greenwood et al, 1988). The first bracketed term of (3) can thus be interpreted as the net gain of the youth.

Eq. (3) implies that the household's preference is neutral to these risks. One may find it odd that risk and time preferences are separately defined. Our specification deviates from the standard setting, which assumes that the lifetime utility is additive over periods and over different states of the economy. The inter-temporal elasticity is therefore not tied to the inverse of the risk aversion in the present context. Eq. (3) is useful to isolate the household's portfolio choice between private capital and GB from the decision about total real saving s_t.

We now turn to the household's budget constraints. Denoting the price level at period *t* as P_t, the household's budget constraints for the young and old periods are given by

$$P_t c_t^y + P_t s_t = (1-\tau_t) P_t w_t l_t \equiv P_t \omega_t l_t \tag{4.1}$$

$$P_{t+1} \tilde{c}_{t+1}^o = q_t P_t s_t (1-\tilde{\zeta}_{t+1}) R_{t+1} + (1-q_t) P_{t+1} s_t \tilde{r}_{t+1} \tag{4.2}$$

where τ_t is wage income tax, ω_t is real wages after tax, R_t is the "nominal" interest rate of GBs (one plus) and q_t represents the share of GBs in the household's saving. ζ_{t+1} is the default rate, taking a value between zero and unity. The variables with tilde state that they are not known when saving at period *t*. R_t is determined at period *t* but with a default risk (ζ_{t+1}), the net return on GBs is not certain.[1] In (4.2), the first term of the right-hand side ($P_t s_t (1-\tilde{\zeta}_{t+1}) R_{t+1}$) is interpreted as the net return on GBs of the nominal saving ($P_t s_t$) and the second term of the right-hand side ($P_{t+1} s_t \tilde{r}_{t+1}$) as the return on capital of the nominal saving ($P_t s_t$), where $P_{t+1}\tilde{r}_{t+1}/P_t$ represents (one plus) "capital gain" per saving. Based on the hypothesis of the Fiscal Theory of Price Level developed by Leeper (1991), Woodford (1994, 1995), Sims (1994), and Cochrane (1998,

[1] In (4.2), we abstract idiosyncratic risk including the bankruptcy of the private capital. This presumes that the household can fully diversify such risk and that only the aggregate shock remains.

2010), we can interpret the default rate (ζ_{t+1}) as the effect of fiscal inflation ($\Delta\pi_{t+1}$). If the government chooses to monetize all parts of the default using monetary policy, $\tilde{\zeta}_{t+1} = \Delta\tilde{\pi}_{t+1}/(1+\Delta\tilde{\pi}_{t+1})$ and $\tilde{P}_{t+1} = P_{t+1}(1+\Delta\tilde{\pi}_{t+1})$ holds.[2]

In the young period, the household decides labor supply l_t and real saving s_t and chooses portfolio q_t in order to maximize

$$E_t U_t = (c_t^y - \frac{l_t^{1+1/\delta}}{1+1/\delta})^\theta E_t[c_{t+1}^o]$$

subject to Eq. (4), where the expectation is taken over ζ_{t+1} and r_{t+1}. The household's optimization yields the following:

$$l_t^* = \omega_t^{\ \delta} \tag{5.1}$$

$$s_t^* = \frac{\Psi_t}{1+\theta} \tag{5.2}$$

$$R_{t+1}(1-\frac{\pi_{t+1}}{1+\pi_{t+1}})E_t(1-\tilde{\zeta}_{t+1}) = E_t\tilde{r}_{t+1} \tag{5.3}$$

where

$$\Psi_t = \omega_t l_t - \frac{l_t^{1+1/\delta}}{1+1/\delta} = \left(1-\frac{\delta}{1+\delta}\right)\omega_t^{\ 1+\delta} \equiv \frac{1}{1+\delta}\omega_t^{\ 1+\delta}$$

$$\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$$

By (5.1), the wage elasticity of labor is constant at δ. The wage taxation becomes distorted because the elasticity is larger. Owing to the Cobb–Douglas specification, the real saving is a fixed share of real wage income net of labor disutility Ψ_t. The income effect and substitution effect offset one another as given in (5.2). Finally, (5.3) gives the arbitrage condition between private capital and GBs. Given that the household is risk-neutral, the arbitrage leads the expected return of both assets

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2 In the case, we obtain $\tilde{P}_{t+1}\tilde{c}_{t+1}^o = q_t P_t s_t R_{t+1} + (1-q_t)\tilde{P}_{t+1}s_t\tilde{r}_{t+1}$ from (4.2).

to be equated, which should be intuitive.

2.4 Market Equilibrium

This subsection considers market equilibrium given the prevailing fiscal policy. In every period, both labor and capital markets are cleared. Given ε_t and k_t, the equilibrium values of real wage and real return on private capital at period *t* are determined by substituting (5.1) into (2) so that

$$w_t \equiv \frac{\omega_t}{1-\tau_t} = \left((1-\alpha)(1-\tau_t)\varepsilon_t A K_t^{\ \mu} k_t^{\ \alpha}\right)^{1/(1+\alpha\delta)} \tag{6.1}$$

$$r_t = \frac{\alpha(\varepsilon_t A K_t^{\ \mu})^{(1+\delta)/(1+\alpha\delta)}}{k_t^{\ (1-\alpha)/(1+\alpha\delta)}}\left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} \tag{6.2}$$

Consider the external effect. In equilibrium, we have $K_t = k_t$, the capital investment in market being exactly equal to the average in the economy. In addition, we set the parameter associated with the externality so that the equilibrium real output is proportional to private capital. The following assumption is imposed:

(Assumption) $\quad \mu = \dfrac{1-\alpha}{1+\delta}$

Note $\mu = 1-\alpha$ if δ=0 or labor supply is completely inelastic, which was assumed by Romer (1990). Then, real output turns out to be

$$Y_t = \left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} \varepsilon_t^{\ (1+\delta)/(1+\alpha\delta)} (A)^{(1+\delta)/(1+\alpha\delta)} k_t \tag{7}$$

The above is familiar in the endogenous growth model, which yields constant real growth rate as a function of policy parameters. The real wage rate is linear with respect to k_t as well, whereas the real return on private capital turns out to be independent of k_t:

$$w_t \equiv \frac{\omega_t}{1-\tau_t} = \left((1-\alpha)(1-\tau_t)\varepsilon_t A\right)^{1/(1+\alpha\delta)} k_t^{\ 1/(1+\delta)} \tag{6.1'}$$

$$r_t = \alpha(\varepsilon_t A)^{(1+\delta)/(1+\alpha\delta)}\left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} \tag{6.2'}$$

Lastly, we turn to the capital market. Because of the closed economy, household savings must meet the demand of private firms and the government. Denoted by b_{t+1} is the real value of GBs issued at period *t* and repaid at *t*+1.[3] Given that the total real saving at period *t* is s_t, which is allocated between k_{t+1} and b_{t+1}, the equilibrium condition is expressed by

$$k_{t+1} + b_{t+1} = s_t^* = \frac{\left((1-\alpha)(1-\tau_t)\varepsilon_t A\right)^{(1+\delta)/(1+\alpha\delta)} k_t}{(1+\theta)(1+\delta)} \tag{8}$$

Manipulating the above establishes the dynamics of private capital accumulation as the following:

$$\left(1+\frac{b_{t+1}}{k_{t+1}}\right)\frac{k_{t+1}}{k_t} = \frac{\left((1-\alpha)(1-\tau_t)A\varepsilon_t\right)^{(1+\delta)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)} \tag{8'}$$

2.5 Government Budget

The government raises revenue by issuing GBs and taxing wage income. It then spends on debt repayment and the real value of public expenditure, the latter being denoted by G_t. G_t is assumed not to contribute to production (1) or directly enter the household's utility (3). This assumption is motivated to simplify our analysis, but it may be plausible when the government spending comprises mostly political rents or pork diverted to special interest groups. The fund flow of the government budget at period *t* is written as

$$P_t b_{t+1} = P_{t-1} R_t b_t - P_t\{T_t - G_t\} \tag{10}$$

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[3] We consider only a single period bond in order to abstract issues of bond maturity composition.

where $T_t = \tau_t w_t l_t = (1-\alpha)\tau_t Y_t$ and Y_t is given in (7). $P_t b_{t+1}$ denotes the nominal value of GBs issued at period *t*. In the other two terms, P_t is multiplied to convert them to nominal variables.

At this point, we distinguish the fiscal rule between the pre-fiscal consolidation and the fiscal consolidation regimes. This is denoted by $\Omega_t \equiv \{\tau_t, \lambda_t, \zeta_t\}$, which contains tax rate τ_t, government expenditure ratio λ_t, and default rate $\zeta_t = \Delta\tilde{\pi}_{t+1}/(1+\Delta\tilde{\pi}_{t+1})$, and may be state contingent in the consolidation regime. The fiscal rules are assumed to be public information, implying that they are incorporated into the pricing of GBs as discussed below. In the present mode, we instead take the pragmatic view that government policy is largely politically constrained, so it does not aim to optimize social welfare.

Let $\Omega^0 \equiv \{\tau, \lambda, \zeta\}$ with $\zeta = 0$. In the pre-consolidation regime, the government taxes wage income at the rate of $\tau_t = \tau$ and spends a given portion $\lambda_t = \lambda$ of the potential real output that calculates Y_t at the mean of ε_t, i.e., $\varepsilon_t = 1$ and at $\tau_t = \tau$, so that $G_t = \lambda \overline{Y}_t$ where

$$\overline{Y}_t = \big((1-\alpha)(1-\tau)\big)^{\delta(1-\alpha)/(1+\alpha\delta)} A^{(1+\delta)/(1+\alpha\delta)} k_t \tag{11}$$

G_t remains proportional to $\overline{Y}_t$ defined above in the consolidation regime as illustrated later. With (11) and (12), the real value of the primary surplus at period *t* is defined by

$$PS_t = T_t - G_t = k_t\big((1-\alpha)A\big)^{(1+\delta)/(1+\alpha\delta)} \Delta(\tau, \lambda, \varepsilon_t) \tag{12}$$

where

$$\Delta(\tau_t, \lambda_t, \varepsilon_t) \equiv \left(\tau_t (1-\tau_t)^{\delta(1-\alpha)/(1+\alpha\delta)} (\varepsilon_t)^{(1+\delta)/(1+\alpha\delta)} - \frac{\lambda_t}{1-\alpha}(1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} \right)$$

Substituting (12) into (10) and manipulating it establishes the dynamics of the public debt over periods:

$$\frac{b_{t+1}}{k_{t+1}}\frac{k_{t+1}}{k_t} = (1-\frac{\pi_t}{1+\pi_t})R_t \frac{b_t}{k_t} - \big((1-\alpha)A\big)^{(1+\delta)/(1+\alpha\delta)} n_t^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau, \lambda, \varepsilon_t) \tag{13}$$

where k_{t+1}/k_t is as given in (8').

Note that in the present economy, b_{t+1}/k_{t+1} as well as k_{t+1} serve as state variables that are determined at period *t* and carried over to period *t*+1. This affects the risk of fiscal consolidation at *t*+1, as discussed in section 3.

3. Equilibrium

3.1 Fiscal Sustainability

The fiscal rule $\Omega^0 \equiv \{\tau, \lambda, \zeta\}$ in the pre-consolidation regime does not ensure that public debt remains at a fiscally sustainable level. The tax rate may be too low and/or the expenditure ratio too high, which structurally generates a primary deficit, i.e., $\Delta(\tau, \lambda, \varepsilon_t) < 0$ for most of ε_t. Then, public debt may reach a level at which the status quo fiscal rule cannot be sustained.

Given the use of the overlapping generations model in this study, the capital market may not discipline government financing because households are not necessarily concerned about long-run fiscal sustainability. Unless ζ_{t+1}=1 for sure, with risk-neutral preferences, households are willing to purchase GBs as long as (5.3) holds, with the default risk compensated by a higher ex ante promised nominal interest rate.

In the present context, therefore, the government can access credit as long as the GB level does not exceed domestic saving with the interest rate fulfilling (5.3). Suppose, however, that the economy reaches $b_{t+2} = s_{t+1}$, that is, the domestic saving at period *t*+1 is fully absorbed by government borrowing. Given that the economy is closed, no private investment can take place, which implies that there is no production in the subsequent period or $Y_{t+2} = 0$ for all ε_{t+2}. Once this occurs, the government can find no resource for repayment. It then has to default on the debt so that ζ_{t+2}=1 is certain and that there is no return on GBs.[4] This in turn implies that the household ceases to lend to the government. The government is then forced to undertake fiscal consolidation without further borrowing. This entails tax increases, expenditure reductions, and the default on GBs.

Lemma 1: Full default is inevitable at period t+1 irrespective of ε_{t+2} when $b_{t+2} = s_{t+1}$.

3.2 Threshold

With $b_{t+2} = s_{t+1}$ or $k_{t+2} = 0$, we have $b_{t+1} / k_{t+2} = \infty$ at period t+1. Inserting this into (13) and manipulating it yields the following condition of the threshold level of ε_{t+1}, which the regime change arises:

$$R_{t+1}\frac{b_{t+1}}{k_{t+1}} = \frac{\left((1-\alpha)(1-\tau)A\right)^{(1+\delta)/(1+\alpha\delta)}}{(1-\dfrac{\pi_{t+1}}{1+\pi_{t+1}})}\left(\frac{(\hat{\varepsilon}_{t+1})^{(1+\delta)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)}+\frac{1}{1-\tau}\left(\tau\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}-\frac{\lambda}{1-\alpha}\right)\right) \tag{14}$$

The above defines the threshold $\hat{\varepsilon}_{t+1}$ implicitly as the function of the interest rate charged on b_{t+1} as well as the debt-to-capital ratio and demography: $\hat{\varepsilon}_{t+1} = \hat{\varepsilon}_{t+1}(R_{t+1}, Z_{t+1})$ where $Z_{t+1} = (b_{t+1} / k_{t+1}, \pi_{t+1})$. With R_{t+1} and b_{t+1} / k_{t+1}, $\hat{\varepsilon}_{t+1}$ increases so that fiscal consolidation is more likely to be in place, whereas it is lowered with π_{t+1}.

Lemma 2: Fiscal consolidation must occur at period t+1 when $\varepsilon_{t+1} \le \hat{\varepsilon}_{t+1}$

[4] However, return on private capital remains positive with the revenue maximizing tax rate being bounded by less than 100%.

The fiscal consolidation rule involves tax increases, expenditure reductions, and the default on GBs. We denote the state of the economy at period *t*+1 by $\Xi_{t+1} = (\varepsilon_{t+1}, b_{t+1}/k_{t+1}, \pi_{t+1})$. The fiscal rule is then expressed as $\Omega_{t+1} \equiv \Omega(\Xi_{t+1})$ with $b_{t+2} = 0$ that contains

$$\tau \le \tau_{t+1} = \tau(\Xi_{t+1}) \le \bar{\tau}\,,\ \ \lambda \ge \lambda_{t+1} = \lambda(\Xi_{t+1})\,,\ \ 0 \le \zeta_{t+1} = \zeta(\Xi_{t+1}) \le 1$$

where the default rate ($\zeta_t = \Delta\pi_{t+1}/(1+\Delta\pi_{t+1})$) fulfills[5]

$$(1-\zeta_{t+1})R_{t+1}\frac{b_{t+1}}{k_{t+1}} = \frac{\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}}{(1-\dfrac{\pi_t}{1+\pi_{t+1}})}\Delta(\tau_{t+1}, \lambda_{t+1}, \varepsilon_{t+1}) \tag{15}$$

The government cannot fully meet its obligation but repays its outstanding debt as much as possible out of the primary surplus, as illustrated in (15). Under the consolidation rule, tax rate, expenditure ratio, or default rate deviates from the initial levels. The fiscal rule can take the general form.

In the simulation shown in section 4, we specify the fiscal consolidation rule. Note that it takes only one period to restructure government finance. Given that no GBs are issued, the economy will return to the initial regime in the next period with no debt liability being carried over.

3.3 Nominal Interest Rate

Let us now turn to the nominal interest rate of GBs R_{t+1} that is settled at period *t*, accounting for the fiscal consolidation in the event of $\varepsilon_{t+1} \le \hat{\varepsilon}_{t+1}$. Recall the arbitrate condition (5.3) that equates return on GB and capital in the expected term. Manipulating this using (6.2) and (15) establishes the following:

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[5] Note that with fiscal inflation

$$\frac{\tilde{P}_{t+1}}{P_t} = \frac{P_{t+1}}{P_t}(1+\Delta\tilde{\pi}_{t+1}) = (1+\pi_{t+1})(1+\Delta\tilde{\pi}_{t+1}) = \left(1-\frac{\pi_{t+1}}{1+\pi_{t+1}}\right)^{-1}(1-\zeta_{t+1})^{-1}$$

$$(1-F(\hat{\varepsilon}_{t+1}))R_{t+1} = \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)} \left\{ \frac{\alpha}{1-\alpha} \frac{\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1})}{(1-\frac{\pi_{t+1}}{1+\pi_{t+1}})} - \frac{k_{t+1}}{b_{t+1}} \int^{\hat{\varepsilon}_{t+1}} \Delta(\tilde{\tau}_{t+1}, \tilde{\lambda}_{t+1}, \tilde{\varepsilon}_{t+1}) dF(\tilde{\varepsilon}_{t+1}) \right\} \quad (16)$$

where

$$\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1})$$
$$= \int^{\hat{\varepsilon}_{t+1}} (1-\tau_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)} \left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)} dF(\varepsilon_{t+1}) + (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} \int_{\hat{\varepsilon}_{t+1}} \left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)} dF(\varepsilon_{t+1})$$

and T_{t+1} denotes vector of τ_{t+1}

Note that $\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1})$ reflects the expected return on private capital.[6] It is clear to see that this is non-increasing with the threshold level given that $\tau \leq \tau_{t+1}$. This represents the perverse effect of wage tax increases under fiscal consolidation that discourages labor supply and, in turn, lowers the productivity of private capital. Eq. (16) yields the nominal interest rate of GBs as a function of the threshold, the debt-to-capital ratio, and the population: $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, Z_{t+1})$.

3.4 Interaction

There exists interaction between the threshold of fiscal consolidation $\hat{\varepsilon}_{t+1}$ and the nominal interest rate of GBs R_{t+1} defined by (14) and (16), respectively. By solving these equations, their equilibrium values can be obtained. Note that these are assessed from period *t* or an ex ante perspective when ε_{t+1} is not known and fiscal consolidation is not yet in place.

Proposition 1: Denoted by R^*_{t+1} and $\hat{\varepsilon}^*_{t+1}$ are the equilibrium levels of the nominal interest rate of GBs and the threshold of fiscal consolidation conditional upon b_{t+1}/k_{t+1} and the

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[6] $E_{t+1}r_{t+1} = \frac{\alpha}{1-\alpha}(A(1-\alpha))^{(1+\delta)/(1+\alpha\delta)}\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1})$

consolidation rule $\Omega_{t+1} = \Omega(\Xi_{t+1})$. These are given as solutions to the following equations:

$$R_{t+1}\frac{b_{t+1}}{k_{t+1}} = \frac{\left((1-\alpha)(1-\tau)A\right)^{(1+\delta)/(1+\alpha\delta)}}{(1-\frac{\pi_{t+1}}{1+\pi_{t+1}})}\left(\frac{(\hat{\varepsilon}_{t+1})^{(1+\delta)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau}\left(\tau\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)} - \frac{\lambda}{1-\alpha}\right)\right)$$

$$(1-F(\hat{\varepsilon}_{t+1}))R_{t+1} = \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\left\{\frac{\alpha}{1-\alpha}\frac{\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1})}{(1-\frac{\pi_{t+1}}{1+\pi_{t+1}})} - \frac{k_{t+1}}{b_{t+1}}\int^{\hat{\varepsilon}_{t+1}}\Delta(\tilde{\tau}_{t+1},\tilde{\lambda}_{t+1},\tilde{\varepsilon}_{t+1})dF(\tilde{\varepsilon}_{t+1})\right\}$$

In the above proposition, we do not preclude the case that there arise multiple equilibria, the two equations intersecting more than twice or the equilibrium diverges, $\hat{\varepsilon}^*_{t+1}$ reaching $\bar{\varepsilon}$ as shown by Oguro and Sato (2011). In addition, by inserting the solution into (15) and taking the expectation, we can obtain the following proposition:

Proposition 2: With the debt-to-capital ratio b_{t+1}/k_{t+1} and the consolidation rule $\Omega_{t+1} = \Omega(\Xi_{t+1})$, fiscal inflation that is calculated in the expected term is given as follows:

$$E_t[1+\Delta\pi_{t+1}] = \frac{(1-\frac{\pi_{t+1}}{1+\pi_{t+1}})R_{t+1}\frac{b_{t+1}}{k_{t+1}}}{\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}}\int^{\hat{\varepsilon}_{t+1}}\frac{dF(\tilde{\varepsilon}_{t+1})}{\Delta(\tilde{\tau}_{t+1},\tilde{\lambda}_{t+1},\tilde{\varepsilon}_{t+1})} + 1 - F(\tilde{\varepsilon}_{t+1}) \qquad (17)$$

4 Simulation

4.1 Parameter Setting and Scenarios

By using the simulation developed in section 3, the aim of this section is to analyze the impacts of deflation and inflation on the real interest rate of GBs and the threshold that the consolidation occurs given b_t/k_t at period *t*. Moreover, we calculate the fiscal inflation of (17) and the threshold that the consolidation occurs given b_t/k_t at period *t*. Our quantitative analysis does not intend to replicate any practice of economy. Rather, it aims to supplement our theoretical model, resolve the ambiguity of its results, and clarify its policy implications.

The parameters are specified in Table 1. ε_t distributes over [0.5–1.5] according to the inverse U-shaped density function with a mean of one. We set the tax rate to be relatively low (10%) and the expenditure rate at 10% of potential output as well. This implies that a primary deficit is likely to result unless ε_t is larger than the mean of one, so there exists the possibility that public debt is accumulated as consolidation risk is enhanced. The fiscal consolidation rule demands that expenditure be cut by 10% to $\lambda_{t+1} = 0.09$. The wage tax rate under consolidation is assumed to be increasing in ε_{t+1}, whereas it increases with b_{t+1}/k_{t+1}. This is specified as in Table 1. The consolidation rule relies on more tax increases for large debt to capital ratio, whereas the default rate ($\zeta(\Xi_{t+1}) = \Delta\pi_{t+1}/(1+\Delta\pi_{t+1})$) is raised when ε_{t+1} is small and so the economy is depressed. Such a presumption should be plausible.

The parameter *g* in the tax function refers to the extent of the required tax increase. The simulation sets two values for *g*: *g*=2 and *g*=5. A higher *g* value implies larger tax increases in fiscal consolidation, which in turn implies lower fiscal inflation. This is defined as residual by (17). By comparing the results of different levels of *g*, we can assess the effect of the fiscal rule on R^*_{t+1} and $\hat{\varepsilon}^*_{t+1}$ as well as the transition of the debt-to-capital ratio. In order to examine the impacts of deflation and inflation, we consider three cases: 1) zero inflation ($\pi_{t+1} = 0$), 2) 0.5% deflation per annum ($\pi_{t+1} = (1-0.005)^{30} - 1 = -0.139$), and 3) 0.5% inflation per annum ($\pi_{t+1} = (1+0.005)^{30} - 1 = 0.161$). Here, we take one period in our model to represent 30 years. Distinguished by the parameter *g* and the price dynamics, six scenarios are presented in Table 2.

<< Insert Tables 1 and 2 about here. >>

4.2 Results

In this section, we focus on the interior equilibrium in the simulation. The real interest rate of GBs for the six scenarios are shown as in Figures 2 and 3, where b_{t+1}/k_{t+1} is treated parametrically on horizontal axis.[7] Figure 2 is related to Scenarios 1 to 3 with *g*=2 and Figure 3 related to Scenarios 4 to 6 with *g*=5.

In Figure 2 with *g*=2, we take Scenario 1 with zero inflation ($\pi_{t+1}=0$). The real interest rate of GBs is gradually increasing with b_{t+1}/k_{t+1}. At b_{t+1}/k_{t+1}=0.45, the stable interior level of the real interest rate of GBs disappears. Scenario 1 is compared with Scenarios 2 and 3 in order to assess the impacts of deflation and inflation. The real interest rate of GBs in Scenario 2 with $\pi_{t+1}=-0.139$ and that in Scenario 3 with $\pi_{t+1}=0.161$ slightly differ from that in Scenario 1. In Scenario 2, the stable interior level of the real interest rate of GBs disappears at b_{t+1}/k_{t+1}=0.46 and in Scenario 3 it disappears at b_{t+1}/k_{t+1}=0.44.

We now turn to Figure 3 with *g*=5. In Scenarios 4 to 6, there exists a range in which the real interest rate of GBs shows downward sloping, confirming the theoretical hypothesis of Oguro and Sato (2011). Take Scenario 4 with zero inflation ($\pi_{t+1}=0$). The real interest rate of GBs initially declines with b_{t+1}/k_{t+1}. Its moderate downward trend continues until b_{t+1}/k_{t+1}=0.37 where the real interest rate of GBs takes its minimum value. The slope is then reversed, further increasing the debt-to-capital ratio and rapidly raising the real interest rate. At b_{t+1}/k_{t+1}=0.81, the stable interior level of the real interest rate of GBs disappears. Scenario 4 is also compared with Scenarios 5 and 6 in order to assess the impacts of deflation and inflation. The real interest rate of GBs in Scenario 6 with $\pi_{t+1}=0.161$ barely differs from that in Scenario 4 up to a low level of b_{t+1}/k_{t+1}. After b_{t+1}/k_{t+1}=0.2, however, the former begins to exceed the latter, and the difference between them begins to widen fast. Once the ratio goes beyond 0.74, Scenario 6 loses the interior equilibrium, whereas it remains in Scenario 4. In the former with $\pi_{t+1}=0.161$, inflation leads to a higher real

[7] The real interest rate of GBs is calculated from the nominal interest rate of GBs (R^*_{t+1}) and price dynamics (π_{t+1}).

interest rate of GBs than the latter when fiscal consolidation is implemented. Given that both scenarios impose the same extent of required tax increase (g=5) and the same expenditure reductions ($\lambda_{t+1} = 0.09$) in the event of consolidation, this implies that Scenario 6 with inflation experiences a higher default rate and consequently adds risk premium to the GBs. Turn to Scenario 5 with $\pi_{t+1} = -0.139$. Again, its real interest rate moves about the same amount as that in Scenario 4 when the public debt-to-capital ratio is very low. For b_{t+1}/k_{t+1} >0.19, the disparity becomes prominent, with the real interest rate of GBs in Scenario 5 staying lower than in Scenario 4. The former can then sustain the interior equilibrium for larger b_{t+1}/k_{t+1} compared with the latter. It can be thus concluded that deflation lowers the real interest rate of GBs and sustains the interior equilibrium.

<< Insert Figures 2 and 3 about here. >>

Consider the threshold of the regime change $\hat{\varepsilon}^{*}_{t+1}$. In all scenarios, this monotonically increases in b_{t+1}/k_{t+1} as shown in Figures 4 and 5. By comparing these figures with different consolidation rules, $\hat{\varepsilon}^{*}_{t+1}$ stays lower when the tax increase in the consideration is larger (i.e., g is high), reflecting a lower interest rate. The prospect for large tax increases in the event of fiscal restructuring, which contributes to lowering the default rate, only serves to mitigate consolidation risk, which should be intuitive. The risk is reflected in the GB premium, which is defined as the difference between the real interest rate of GBs and the expected real return on capital. The premium remains negligible when risk is low: according to consolidation risk, the revenue deficiency is largely filled by tax increases and expenditure reductions. The default rate in the event of consolidation is raised as the debt-to-capital ratio increases, which in turn augments the premium.

To see the effect of deflation and inflation on $\hat{\varepsilon}^{*}_{t+1}$, we compare Scenario 1 (Scenario 4) with Scenarios 2 and 3 (Scenarios 5 and 6). The simulation establishes that the threshold is lowered in the case of deflation and rises in the case of inflation.

<< Insert Figures 4 and 5 about here. >>

We now turn to fiscal inflation, which is calculated in the expected term as in (17). The expected fiscal inflation is lowered as the tax increase in the consolidation grows (i.e., g is high). Figures 6 and 7 show fiscal inflation from the perspective of period t+1.[8] For b_{t+1}/k_{t+1}>0.12, fiscal inflation is extremely high in Scenarios 1 to 3. It becomes 1383% at b_{t+1}/k_{t+1}=0.13. After that, it monotonically increases in b_{t+1}/k_{t+1} as shown in Figure 6. At the near points of b_{t+1}/k_{t+1}=0.45, it reaches over 5000%. This indicates the impact of fiscal inflation (i.e., monetizing all parts of the default using monetary policy).

The expected fiscal inflation also monotonically increases in b_{t+1}/k_{t+1}. The fiscal inflation shown in Figure 7, however, is lower than that in Figure 6 because of the larger tax increase in the consolidation (i.e., higher g). It is zero for b_{t+1}/k_{t+1}<0.43. For b_{t+1}/k_{t+1}>0.44, the disparity becomes prominent, and fiscal inflation in Scenario 5 with $\pi_{t+1} = -0.139$ stays lower than that in Scenario 4 with zero inflation and that in Scenario 6 with $\pi_{t+1} = 0.161$ is higher than in Scenario 4.

It can be thus concluded that initial inflation (π_{t+1}) accelerates the expected fiscal inflation when the default rate in the event of consolidation is raised. The fiscal inflation in Scenario 4 reaches 18.2% at b_{t+1}/k_{t+1}=0.81, that in Scenario 5 23.2% at b_{t+1}/k_{t+1}=0.92, and that in Scenario 6 14.8% at b_{t+1}/k_{t+1}=0.73.

[8] In this paper, although one period in our model is 30 years, we assume that fiscal inflation finishes after three years. Therefore, the expected fiscal inflation in Figures 6 and 7 are calculated as $(E_t[1+\Delta\pi_{t+1}])^{1/3}-1$.

<< Insert Figures 6 and 7 about here. >>

5 Conclusion

In this paper, we analyze the impact of deflation and inflation on the real interest rate of GBs, using an overlapping generations model with the relationship between the real interest rate of GBs and the fiscal consolidation rule. Our key findings are summarized as follows. Deflation may lower the real interest rate of GBs to the same level of public debt to capital, even if the fiscal consolidation rule is same, as opposed to the conventional view that the real interest rate of GBs is determined independent of deflation if the Fisher equation holds. Our results are consistent with how the real interest rate of Japanese GBs reacts in situations of deflation.

This paper also addresses the impact of fiscal inflation (i.e., monetizing all parts of the GB's default using monetary policy). We calculate the expected fiscal inflation when the default rate in the event of fiscal consolidation is raised. The fiscal inflation may be extremely high if the extent of the required tax increase in fiscal consolidation is low. Initial inflation accelerates the expected fiscal inflation, but initial deflation suppresses it.

Our model is highly stylized and highlights certain issues that should be examined in future research. These issues include: (1) the search for the "actual" threshold of regime change and the limitation of using the public debt-to-GDP ratio in the Japanese economy, (2) the effect on our model of the financial crisis, especially the bank runs (Diamond & Dybvig 1983; Diamond & Rajan 2001; Allen & Gale 1998; Uhlig 2010) caused by the default of GBs, and (3) the analysis of the threshold of regime change and the limitation of the public debt-to-GDP ratio in an open economy..

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Table 1: Parameters

Parameters	*Values*
δ	0.5
α	0.3
θ	1.333
A	7.0
$\Omega^0 \equiv \{\tau, \lambda, \zeta\}$	τ=0.1 λ=0.1
$\Omega_{t+1} \equiv \Omega(\Xi_{t+1})$	$\tau(\varepsilon_{t+1}, b_{t+1}/k_{t+1}) = Min\left(g \times \tau, \quad \tau + 3.8\frac{b_{t+1}}{k_{t+1}}\varepsilon_{t+1}\right)$ $\lambda(\varepsilon_{t+1}, b_{t+1}/k_{t+1}) = 0.09$

Table 2: Scenarios

Scenarios	*The extent of the required tax increase*	*Price dynamics*
1	$g = 2$	$\pi_{t+1} = 0$
2	$g = 2$	$\pi_{t+1} = -0.139$
3	$g = 2$	$\pi_{t+1} = 0.161$
4	$g = 5$	$\pi_{t+1} = 0$
5	$g = 5$	$\pi_{t+1} = -0.139$
6	$g = 5$	$\pi_{t+1} = 0.161$

Figure 1: Public debt to GDP, Long-term Real Interest Rate and Deflation

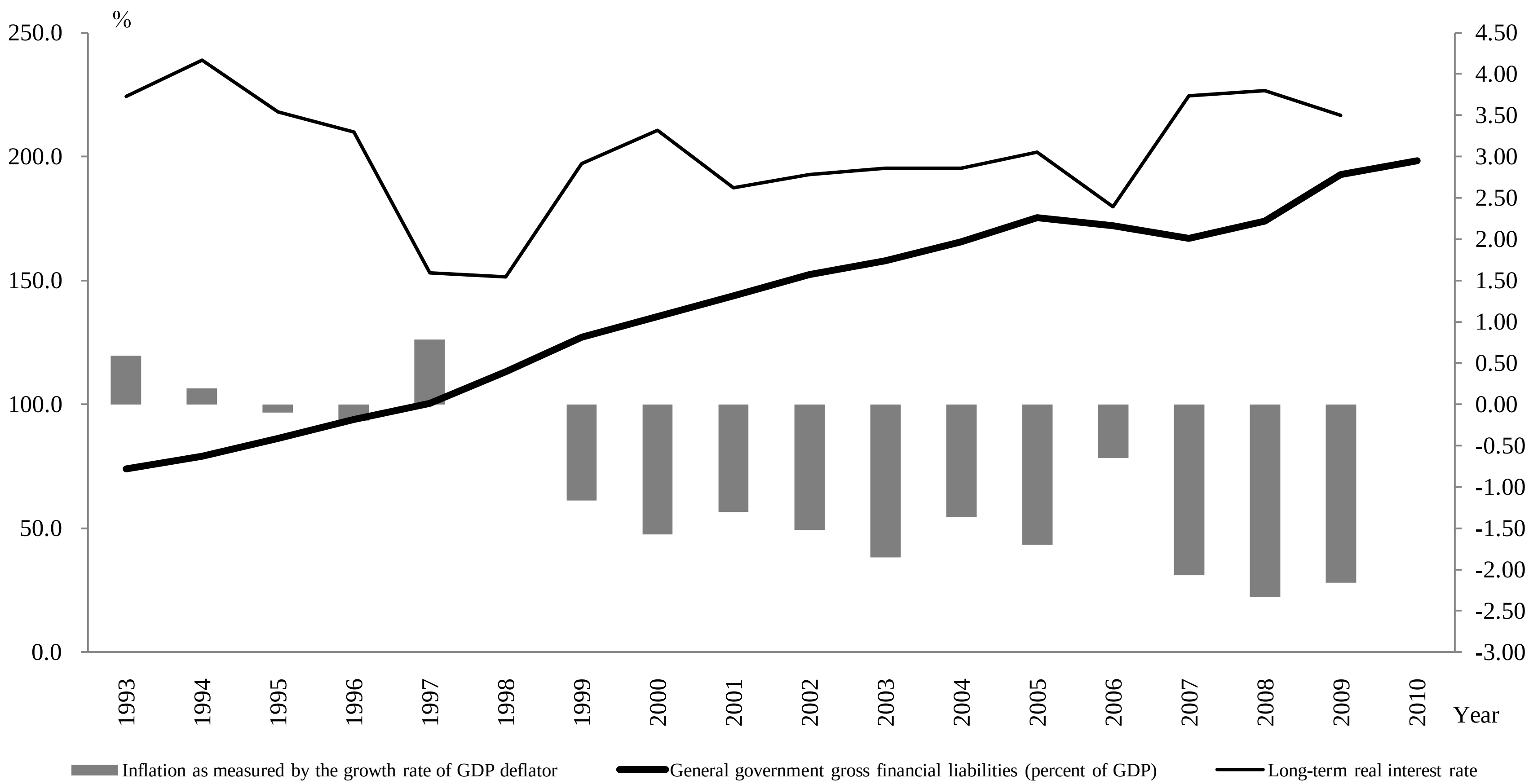

Figure 2: The Real Interest Rate of GBs in Scenarios 1 to 3 ($g = 2$)

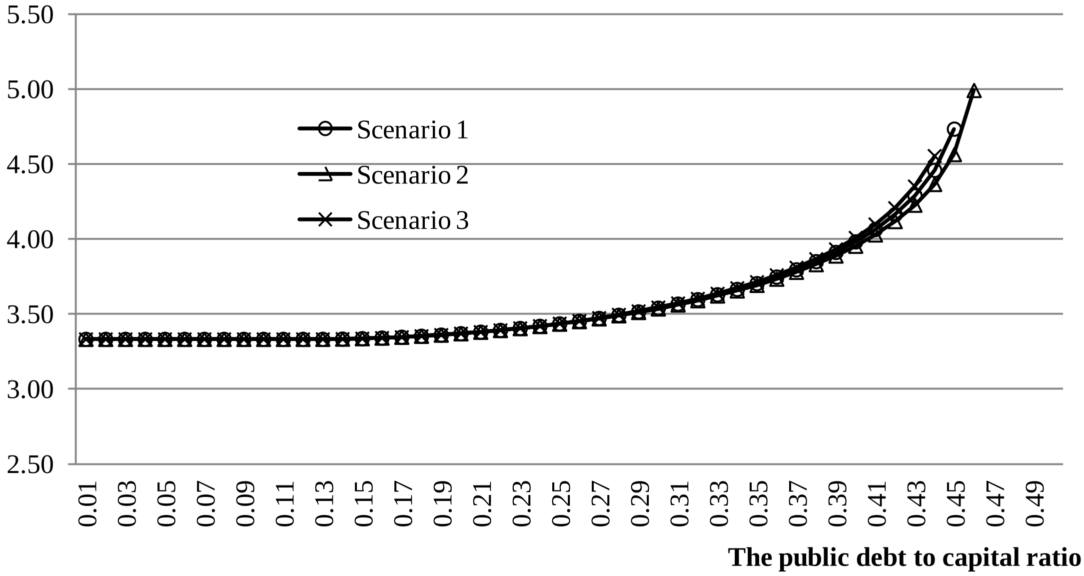

Figure 3: The Real Interest Rate of GBs in Scenarios 4 to 6 ($g = 5$)

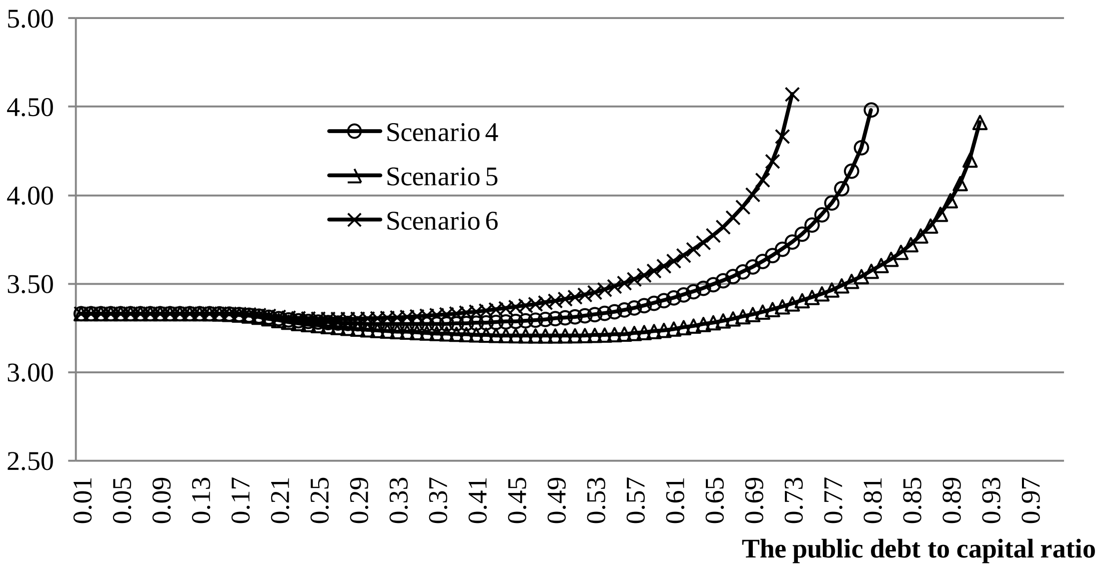

Figure 4: Threshold in Scenarios 1 to 3 ($g = 2$)

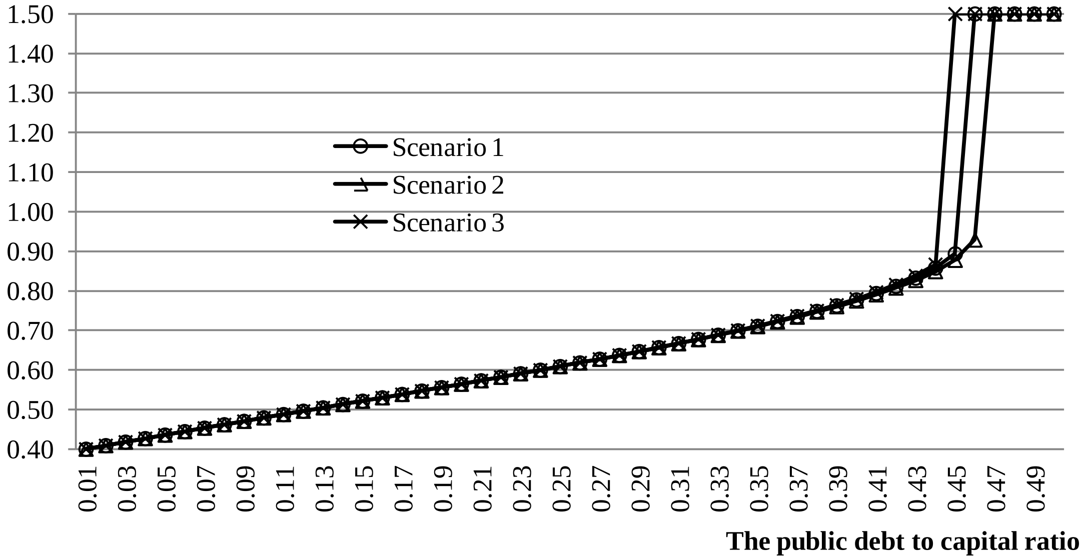

Figure 5: Threshold in Scenarios 4 to 6 ($g = 5$)

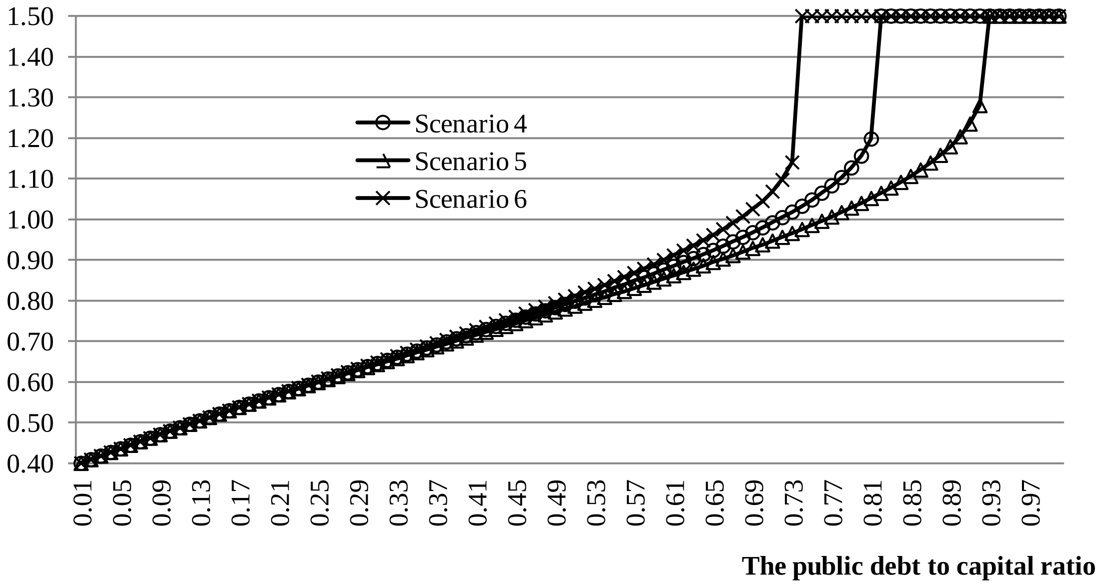

Figure 6: Fiscal inflation in Scenarios 1 to 3 ($g = 2$)

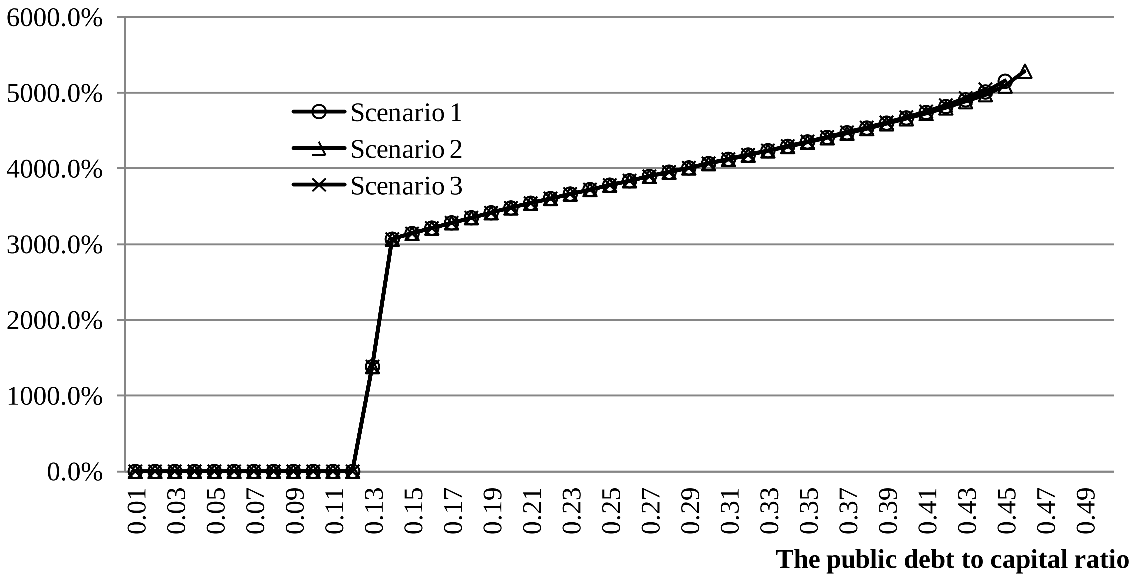

Figure 7: Fiscal inflation in Scenarios 4 to 6 ($g = 5$)