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An empirical analysis of the Nikkei 225 put options using realized GARCH models

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Hi-Stat Discussion Paper

An empirical analysis of the Nikkei 225 put options using realized GARCH models *

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Abstract

This paper analyses whether the realized generalized autoregressive conditional heteroscedasticity (GARCH) model suggested by Hansen et al. [2011] is useful for pricing Nikkei 225 put options. One advantage of this particular model over classic autoregressive conditional heteroscedasticity (ARCH)-type models is that it enables us to estimate simultaneously the dynamics of stock returns using both realized volatility and daily return data. Another advantage is that this model adjusts for the bias in realized volatility caused by the presence of market microstructure noise and non-trading hours, and therefore, it can be apply to any realized measure. The analysis also examines whether realized GARCH models using the realized kernels proposed by Bardorff-Nielsen et al. [2008] improve the performance of option pricing by comparing the results with those obtained using realized volatility as the simple sum of the squares of the intra-day returns. Comparing the estimation results based on the root mean square error indicates that the realized GARCH models perform better than either the exponential GARCH (EGARCH) or the Black–Scholes models in terms of put option pricing. Moreover, the realized GARCH models with the realized kernels without non-trading hour returns perform better than those with realized volatility alone.

Key words: put option pricing; realized GARCH; non-trading hours; microstructure noise; Nikkei 225 stock index

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1 Introduction

One of the most important variables in option pricing is the volatility of the underlying asset, defined as the standard deviation of the returns of financial assets. However, while the well-recognized Black and Scholes [1973] option pricing model assumes that financial asset volatility is constant, it is well known that volatility changes over time. Many alternative time series models are now available to describe the dynamics of volatility. One traditional group of models is the autoregressive conditional heteroscedasticity (ARCH)-type models using daily return data. More recently, realized volatility models using high-frequency data have attracted the attention of financial econometricians as an accurate estimator of volatility. An extension of both ARCH models and time series models of realized volatility is included in the realized generalized ARCH (GARCH) models proposed by Hansen et al. [2011]. This paper analyses whether this particular model is useful for the pricing of Nikkei 225 stock index options. The results indicate that the realized GARCH models in this analysis perform better than either the exponential GARCH (EGARCH) or Black–Scholes (BS) models in terms of put option pricing.

In the mainstream literature, a wide range of traditional ARCH-type models, including the GARCH [generalized ARCH, Bollerslev, 1986] model, GJR [Glosten et al., 1993] model, EGARCH [exponential GARCH, Nelson, 1991] model, APGARCH [asymmetric power GARCH, Ding et al., 1993] model, and FIEGARCH [fractionally integrated EGARCH, Bollerslev and Mikkelsen, 1996] model are commonly analysed. Many of these models have already been applied to option pricing (Bollerslev and Mikkelsen [1999] and Duan [1995]). In strong contrast, realized volatility is merely the sum of the squared intra-day returns using high-frequency data. Ordinarily, to specify the dynamics of realized volatility, time series models are employed, including autoregressive fractionally integrated moving average (ARFIMA) and heterogeneous interval autoregressive (HAR) [Heterogeneous interval autoregressive, Corsi, 2009] models. Bandi et al. [2008], Stentoft [2008], Corsi et al. [2009], Christoffersen et al. [2010] and Ubukata and Watanabe [2011] have applied realized volatilities to option pricing.

Realized GARCH models have a number of advantages over both ARCH-type models and time series models of realized volatility. One advantage is that we can simultaneously estimate the dynamics of stock returns using both realized volatility and daily return data. Another advantage is that we can adjust for the bias in realized volatility caused by the presence of market microstructure noise and non-trading hours. Importantly, to the author's best knowledge, relatively few studies have applied realized GARCH models to option pricing compared with applications to volatility forecasting. Accordingly, this paper applies realized GARCH models to the pricing of Nikkei 225 stock index options traded at the Osaka Securities Exchange, and compares their performance with those using EGARCH and BS models.

As discussed, in actual markets the presence of non-trading hours and market microstructure noise may cause bias in realized volatility. Some methods are available that mitigate the effect of microstructure noise on a realized volatility, such as realized kernels. We use the realized kernels proposed by Bardorff-Nielsen et al. [2008]. For the bias associated with non-trading hours, we employ the bias-adjusted method proposed by Hansen and Lunde [2005]. When using a log-linear specification, realized GARCH models can adjust the bias in realized volatility in the same way as Hansen and Lunde [2005].

To test the effects of this bias adjustment in the realized GARCH models, we estimate realized GARCH models with realized volatilities and kernels adjusted after considering market microstructure noise. Following the estimation of the realized GARCH models, we examine whether the realized kernel methods improve the performance of option pricing by comparing the results with those obtained using realized volatility, which is simply obtained by summing the squares of the returns. If correcting the bias in realized GARCH models were sufficient for adjusting for the total bias in realized volatility, option pricing performance would not improve when using realized kernels.

Our main findings are as follows. First, in terms of option pricing, we find that the realized GARCH models perform better than either the EGARCH or BS models. Second, we also find that the realized GARCH models with realized kernels without the adjustment for non-trading returns also perform better. This suggests that the bias adjustment in realized GARCH models is not sufficient to adjust for the total bias arising from market microstructure noise.

The remainder of the paper is structured as follows. Section 2 describes the realized GARCH models. Section 3 describes the data used in the analysis and discusses integrated and realized volatility. In Section 4, we present the estimation results for the realized GARCH models. Section 5 explains the method of calculating the option prices and compares the performance of the various option pricing models in the analysis. Section 6 concludes.

2 Realized GARCH models

We begin with a brief review of the realized GARCH models proposed by Hansen et al. [2011]. Three equations characterize realized GARCH models, namely, the return equation, the GARCH equation, and the measurement equation. To start with, the return equation is specified as

$$r_t = E(r_t|F_{t-1}) + \epsilon_t, \quad \epsilon_t = \sqrt{h_t} z_t, \quad z_t \sim i.i.d.N(0,1), \tag{2.1}$$

where r_t is the daily return on day t, h_t is the volatility of the daily return r_t, $E(r_t|F_{t-1})$ is the expectation of r_t conditional on the information available up to day $t-1$, and z_t is the standardized error, which follows an independent and identical normal distribution with a mean of zero and a variance of one. In this analysis, the conditional expected return is specified as $E(r_t|F_{t-1}) = r + \nu_r\sqrt{h_t}$, where r is the risk-free rate. We specify this same equation not only for the realized GARCH models, but also for the EGARCH models.

The second equation specified is the GARCH equation. We use the simplest version, the log realized GARCH(1,1) model

$$\ln h_t = \omega + \beta \ln h_{t-1} + \gamma \ln x_{t-1}, \tag{2.2}$$

where x_t is the realized volatility. [1]

The differences between this equation and those found in conventional GARCH models are as follows. First, while GARCH models specify h_t as a function of past values of h_t and error terms (ϵ_t or z_t), realized GARCH models instead specify it as a function of the past value of latent volatility h_t and realized volatility x_t. Second, the persistence of volatility is not summarized by $\beta + \gamma$ in realized GARCH models. Third, the error term for return r_t affects latent volatility h_t through the realized volatility x_{t-1} in realized GARCH models.

[1]Generally, the realized GARCH (p, q) model replaces eq.(2.2) with

$$\ln h_t = \omega + \sum_{i=1}^{p} \beta_i \ln h_{t-i} + \sum_{j=1}^{q} \gamma_j \ln x_{t-j}.$$

We estimate only the realized GARCH(1,1) models.

The third equation specified is the measurement equation, where $\tau(z_t)$ is known as the leverage function. This equation is specified as

$$\ln x_t = \xi + \phi \ln h_t + \tau(z_t) + u_t, \quad u_t \sim N(0, \sigma_u^2), \tag{2.3}$$

$$\tau(z_t) = \tau_1 z_t + \tau_2(z_t^2 - 1). \tag{2.4}$$

Given eq.(2.3) and eq.(2.4), realized volatility (x_t) depends on the current value of z_t. Moreover, the form of eq.(2.4) is convenient because it ensures that $\mathrm{E}\{\tau(z_t)\} = 0$ for any distribution of z_t, so long as $\mathrm{E}(z_t) = 0$ and $\mathrm{Var}(z_t) = 1$. [2]

Although realized volatility includes bias caused by microstructure noise and non-trading hours as discussed below, these biases in realized volatility (x_t) can be corrected with eq.(2.3). For example, if $\xi = 0$ and $\phi = 1$, the realized volatility is an unbiased estimator of the true volatility. Alternatively, if $\xi < 0$ and $\phi < 1$, realized volatility has a downward bias. Therefore, the measurement equation does not require x_t to be an unbiased measure of h_t, and we can estimate the realized GARCH models using a realized volatility that includes bias. We should then expect that $\xi < 0$ and $\phi < 1$.

The leverage function $\tau(z_t)$ expresses the volatility asymmetry. This reflects the well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility. If $\tau_1 < 0$, x_t will be larger when $z_t < 0$ than when $z_t > 0$

$$\ln x_t = \xi + \phi \ln h_t + \tau_1 z_t + \tau_2(z_t^2 - 1) + u_t.$$

[2] Hansen et al. [2011] considered leverage functions that are constructed from Hermite polynomials

$$\tau(z) = \tau_1 z + \tau_2(z^2 - 1) + \tau_3(z^3 - 3z) + \tau_4(z^4 - 6z^2 + 3) + \cdots,$$

and they chose $\tau(z_t) = \tau_1 z_t + \tau_2(z_t^2 - 1)$.

Then, if $\gamma > 0$, h_{t+1} become larger when $z_t < 0$

$$\ln h_{t+1} = \omega + \beta \ln h_t + \gamma \ln x_t.$$

Thus, when $\tau_1 < 0$ and $\gamma > 0$, the volatility asymmetry is observed.

We can derive the volatility persistence from the reduced form. More specifically, a realized GARCH (1,1) model composed of eq.(2.1), eq.(2.2) and eq.(2.3) implies a simple reduced-form model for $\{r_t, h_t\}$

$$\begin{aligned} \ln h_t &= \mu_h + \pi \ln h_{t-1} + \gamma w_{t-1}, \\ \ln x_t &= \mu_x + \pi \ln x_{t-1} + w_t - \beta w_{t-1}, \end{aligned}$$

where $\pi = \beta + \phi\gamma$, $w_t = u_t + \tau(z_t)$, $\mu_h = \omega + \gamma\xi$, $\mu_x = \phi\omega + (1-\beta)\xi$, and w_t is the error term in the measurement equation. The persistence of volatility is summarized by $\pi = \beta + \phi\gamma$. Here, β and ϕ are the parameters reflecting past volatility in the GARCH equation, and γ is the volatility parameter in the measurement equation. Thus, we can calculate the volatility persistence using both the GARCH equation and the measurement equation. In this model, volatility is stationary if $|\pi| < 1$.

Hansen et al. [2011] proposed realized GARCH models with both a linear specification and a log-linear specification. An obvious advantage of a logarithmic specification is that it automatically ensures positive volatility. Moreover, log realized GARCH models can adjust the bias of realized volatility in much the same way as Hansen and Lunde [2005] described below. Thus, we estimate these models with a log-linear specification.

Realized GARCH models can be estimated using quasi-maximum likelihood estimation techniques such that the estimator is distributed asymptotically normal. We adopt Gaussian specifications for the error terms u_t and z_t in the return and measurement equations, respectively, such that the log likelihood function is given by

$$l(r, x; \theta) = -\frac{1}{2} \sum_{t=1}^{n} \left[\ln h_t + \frac{\epsilon_t^2}{h_t} + \ln \sigma_u^2 + \frac{u_t^2}{\sigma_u^2} \right]. \tag{2.5}$$

Here, θ is all of the parameters in the realized GARCH models. [3] Under suitable regularity conditions, they have an asymptotic normal distribution. See Hansen et al. [2011] for details.

While we assume that u_t and z_t follow normal distributions, it is well known that the distribution of stock returns is leptokurtic. If we use Student's t distribution or anything other than normal distributions for the standardized error term z_t in eq.(2.1), we can specify the log likelihood functions and estimate the realized GARCH models using maximum likelihood estimation. In such a case, however, following the estimation of the realized GARCH models, we cannot apply the Duan [1995] method to option pricing (Duan [1999]). To apply the Duan [1995] method to option pricing, we thus adopt a Gaussian specification for z_t.

3 Data

We employ Nikkei NEEDS-TICK data for estimating the realized GARCH models and option pricing simulations. The Japanese certificate of deposit (CD) rate serves as the risk-free rate. We now explain the method of data

[3] We can divide the log likelihood function into two parts by transforming

$$l(r, x; \theta) = -\frac{1}{2} \sum_{t=1}^{n} \left[h(r_t|X_{t-1}) + l(X_t|X_{t-1}) \right]. \tag{2.6}$$

The first part is $h(r_t|X_{t-1}) = \ln h_t + \frac{\epsilon_t^2}{h_t}$, which is the density determined by the normal density of r_t with mean $\mathrm{E}(r_t|F_t)$ and variance h_t. If we have information about the $t-1$ period, we can calculate h_t because there is no t period stochastic variable in eq.(2.2). The second part is $l(X_t|X_{t-1}) = \ln \sigma_u^2 + \frac{u_t^2}{\sigma_u^2}$, which is the normal density with mean $\mu + \phi \ln(\sigma_t^2) + \tau_1 z_t + \tau_2 (z_t^2 - 1)$ and variance σ_u^2.

cleaning following Ubukata and Watanabe [2011], used for the closing prices of the Nikkei 225 stock index and the put option prices.

The dataset comprises the Nikkei 225 stock index price for each minute from 9:01 to 11:00 in the morning session and from 12:31 to 15:00 in the afternoon session. On occasion, the time stamps for the closing prices in the morning and afternoon sessions are slightly after 11:00 and 15:00, because the recorded time appears when the Nikkei 225 stock index is calculated. In such cases, we use all prices up to closing prices. [4]

Nikkei 225 stock index options traded at the Osaka Securities Exchange are European options exercised only on the second Friday of each expiration month. For the most part, put options on the Nikkei 225 stock index trade more heavily than the call options. Further, the put options that have a maturity of 30 days (29 days if the month includes a holiday weekend) trade more heavily than other put options with maturities shorter or longer than 30 days. In what follows, we concentrate on put options with a maturity of 30 days. On such days, we consider put options with different exercise prices whose bid and ask prices are both available at the same time between 14:00 to 15:00. For each option, we use the average of the bid and ask prices instead of the transaction prices because transaction prices are subject to market microstructure noise, such as the bid–ask bounce, as suggested by Campbell et al. [1997]. We also exclude some kinds of put options not priced in the theoretical range from a lower bound at $P_T = \max(0, K\exp(-r\tau))$ to an upper bound at $P_T = K\exp(-r\tau)$.

[4] In their analysis, Hansen and Lunde [2005] use intra-day returns constructed for both bid and ask prices using the previous-tick interpolation method. We define the overnight return as the log difference between the first price (mid quote) of the day and the last price (mid quote) of the preceding day.

3.1 A realized volatility and a realized kernel

We begin with a brief review of integrated volatility and realized volatility using the following continuous price process. We assume that the price process satisfies

$$dp(s) = \mu(s)ds + \sigma(s)dW(s), \tag{3.7}$$

where W is a standard Brownian motion, and μ and σ are smooth time-varying (random) functions that are independent of W. We let integer values of t correspond to the closing time of the afternoon session. The volatility over the interval $(t-1, t)$ is then defined as

$$IV_t = \int_{t-1}^{t} \sigma^2(s)ds. \tag{3.8}$$

We refer to this as the integrated volatility (IV) for day t.

The realized volatility (RV) is an empirical estimate of the IV constructed from intra-day returns. For the special case where intra-day returns are equidistant in calendar time, we define the intra-day returns

$$r(t-1+1/m), r(t-1+2/m), \ldots, r(t)$$

where m is the number of intra-day returns. RV for day t is defined as the sum of squared intra-day returns

$$RV_t = \sum_{i=1}^{m} r(t-1+i/m)^2. \tag{3.9}$$

RV_t will provide a consistent estimator of IV_t

$$\operatorname*{plim}_{m\to\infty} RV_t = IV_t.$$

There are two problems in calculating RV: the first is the presence of non-trading hours, and the second is the presence of microstructure noise. We show that realized GARCH models are able to adjust for the bias associated with non-trading hours. We then detail the method used in Bardorff-Nielsen et al. [2008] for mitigating the effect of microstructure noise. Following this, we examine whether realized GARCH models using the bias-adjusted RV improve the option pricing performance by comparing the results with those obtained using RV.

One problem in calculating RV is the presence of non-trading hours. To calculate a RV that spans a full day, one also requires high-frequency data for the whole day. However, most equities trade for only a fraction of the day. For example, the Tokyo Stock Exchange is only open 9:00–10:00 (morning session) and 12:30–15:00 (afternoon session), so it is impossible to obtain high-frequency returns for the periods 15:00–9:00 (overnight) and 11:00–12:30 (lunchtime). Moreover, in Japan on the first and last trading days of the year, the market is only open 9:00–11:00. In calculating RV using the above data, one may include returns on the non-trading hours, but this can make RV noisy because such returns include much discretization noise. On the other hand, if we calculate the RV as the sum of squared trading hours' returns only, RV may underestimate IV.

In terms of the bias associated with the presence of non-trading hours, Hansen and Lunde [2005] consider a way to extend the RV_t, which is only available for trading hours, to a measure of volatility for the full day. Here, RVN_t indicated RV without non-trading hour returns. Their scaled estimator is

$$RVSC_t \equiv \hat{\delta} RVN_t, \quad \hat{\delta} = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{\sum_{t=1}^{n} RVN_t}, \tag{3.10}$$

where r_t is the daily return, $\bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t$, and $\hat{\delta}$ is a consistent estimator of $\delta \equiv E[\sigma_t^2]/E[RVN_t]$. The mean of the $RVSC_t$ is equal to the volatility of daily returns.

The correcting bias in log realized GARCH models is the same as the method of Hansen and Lunde [2005] in eq.(3.10). When $x_t' = RVSC_t$ and $x_t = RVN_t$, a realized GARCH model using x_t' is

$$\ln h_t = \omega + \beta \ln h_{t-1} + \gamma \ln x_{t-1}', \tag{3.11}$$

$$\ln x_t' = \xi + \phi \ln h_t + \tau(z_t) + u_t. \tag{3.12}$$

From $\ln x' = \ln \hat{\delta} + \ln x_t$,

$$\ln h_t = \omega + \gamma \ln \hat{\delta} + \beta \ln h_{t-1} + \gamma \ln x_{t-1}, \tag{3.13}$$

$$\ln x_t = \xi - \ln \hat{\delta} + \phi \ln h_t + \tau(z_t) + u_t. \tag{3.14}$$

The constant estimates of $RVSC_t$ in eq.(3.11) and eq.(3.12) are different from RVN_t in eq.(3.13) and eq.(3.14), but other estimates of $RVSC_t$ are the same as those of RVN_t. Therefore, when we estimate realized GARCH models, we do not need to calculate $RVSC_t$ and estimate them with RVN_t.

The other problem in the analysis is the presence of microstructure noise, including the bid–ask bounce, non-synchronous trading, rounding errors, and misrecordings (see Campbell et al. [1997], Ch. 3). Without microstructure noise, it would be desirable to use intra-day returns sampled at the highest frequencies. When there is microstructure noise, market microstructure effects cause autocorrelation in intra-day returns, and so RV includes not only the variance of the efficient price but also the variance of microstructure noise. If there is microstructure noise, its variance becomes relatively large in the variance of the true return. That is, the bias caused by microstructure noise increases as the time interval approaches zero.

There are some methods available for mitigating the effect of microstructure noise on RV. The classic approach is to use RV constructed from intraday returns sampled at a moderate frequency. In practice, researchers are necessarily obliged to select a moderate sampling frequency. We calculate realized volatilities using 3- and 5-minute intra-day returns.

To mitigate the effect of a microstructure noise, one of the kernel-based estimators is proposed by Bardorff-Nielsen et al. [2008]. These estimators, called realized kernels (RKs) or flat-top kernels, are specified as

$$RK_t = \hat{\gamma}_0 + \sum_{s=1}^{q} k(x)\left(\hat{\gamma}_s + \hat{\gamma}_{-s}\right), \quad x = \frac{s-1}{H}, \tag{3.15}$$

$$\hat{\gamma}_s = \sum_{j=1}^{m} r(t-1+j/m)r(t-1+(j-s)/m), \quad s = -q, \ldots, q. \tag{3.16}$$

Here, the non-stochastic $k(x)$ for $x \in [0,1]$ is a weight or kernel function, $\hat{\gamma}_0$ represents the RV, and $\hat{\gamma}_s$ represents the $s-$th autocovariance of the intra-day returns. The term of $RK_t - \hat{\gamma}_0 = \sum_{s=1}^{q} k(x)\left(\hat{\gamma}_s + \hat{\gamma}_{-s}\right)$ is the realized kernel correction to RV for market friction. [5]

From Theorem 4 of Bardorff-Nielsen et al. [2008], the asymptotic distribution of this estimator depends on the conditions of $k(x)$ and H. First, they show that if $k(0) = 1$, $k(1) = 0$, and $H = c_0 n^{2/3}$, the resulting estimator is asymptotically mixed Gaussian, converging at rate $n^{1/6}$. Here, c_0 is an estimable constant that can be optimally chosen to minimize the asymptotic variance of this estimator. For example, Bartlett, 2nd order, Epanechnikov kernels are this class of kernels. [6]

When they additionally require that $k'(0)^2 + k'(1)^2 = 0$, then by taking $H = c_0 n^{1/2}$, the resulting estimator is asymptotically mixed Gaussian, and consistent at the rate of convergence $n^{1/4}$ as shown in Bardorff-Nielsen et al.

[5] See Andrews [1991] for the usual kernel estimators and Hansen and Lunde [2006], Bardorff-Nielsen et al. [2008], and Bandi et al. [2008] for other kernel-based estimators of IV.

[6] This special case of a so-called flat-top Bartlett kernel is particularly interesting as its asymptotic distribution is the same as that of the two-scaled estimator.

[2008]. Using this result, it is clear that this estimator converges faster and more efficiently than the previous estimator. Thus, we focus on this estimator. For example, the cubic, 5th to 8th order, Parzen, and Tukey–Hanning kernels are in this class of kernels. [7]

Moreover, with regard to these estimators requiring additional conditions, Bardorff-Nielsen et al. [2008] compared the lower bound of parametric efficiency for some kernels in this class, including the cubic, 5th to 8th order, Parzen, and modified Tukey–Hanning kernels. They concluded that only the modified Tukey–Hanning kernel, as detailed below, approached the lower bound of parametric efficiency. Given this particular kernel is more efficient than other kernels sometimes employed, we focus on the modified Tukey–Hanning kernel estimator.

The flat-top modified Tukey–Hanning$_p$ kernel is defined by

$$k(x) = \sin^2 \left\{ \frac{\pi}{2}(1-x)^p \right\}.$$

This is modified because the case $p = 1$, where $\sin^2\{\pi/2(1-x)\} = \{1 + \cos(\pi x)\}/2$, corresponds to the usual Tukey–Hanning kernel. They focus on the Tukey–Hanning$_2$($p = 2$) kernel in their simulation study because it is near efficient and does not require too many intra-day returns. We employ the flat-top Tukey–Hanning kernel with $p = 2$ to mitigate the effects of microstructure noise

$$\begin{aligned} TH_t &= \hat{\gamma}_0 + \sum_{s=1}^{q} k(x)\left(\hat{\gamma}_s + \hat{\gamma}_{-s}\right), \quad x = \frac{s-1}{H}, \qquad (3.17) \\ & k(x) = \sin^2 \left\{ \frac{\pi}{2}(1-x)^2 \right\}, \\ & \hat{\gamma}_s = \sum_{j=1}^{m} r(t-1+j/m) r(t-1+(j-s)/m), \quad s = -q, \dots, q. \end{aligned}$$

[7] This is a special case, as when $k(x) = 1 - 3x^2 + 2x^3$, this estimator has the same asymptotic distribution as the multiscale estimator.

Moreover, THN_t denotes the flat-top modified Tukey–Hanning kernel with $p = 2$ without non-trading hour returns.

We estimate the asymptotically optimal value of H using 15-minute returns and the highest frequency 1-minute returns. When $H = c\zeta\sqrt{n}$ and $m \to \infty$, the asymptotically optimal value of c that minimizes the asymptotic variance is given by

$$c^* = \sqrt{\rho \frac{k^{1,1}}{k^{0,0}} \left\{ 1 + \sqrt{\frac{3k^{0,0}k^{2,2}}{\rho(k^{1,1})^2}} \right\}}, \tag{3.18}$$

$$\zeta^2 = \sigma_\eta^2/\sqrt{IQ}, \quad \rho = IV/\sqrt{IQ},$$

$$\hat{IQ} = \frac{m^l}{3}\sum_{j=1}^{m^l} r(t-1+j/m^l)^4,$$

$$\hat{\sigma}_\eta^2 = \frac{1}{2m^h}\sum_{i=1}^{m^h} r(t-1+j/m^h)^2,$$

where $(k^{0,0}, k^{1,1}, k^{2,2}) = (0.219, 1.71, 41.7)$. m^l and m^h are the number of low-frequency returns and the highest frequency returns. $\hat{IQ}$ and $\hat{IV}$ are estimated using low-frequency returns, such as 15 minutes, and $\hat{\sigma}_\eta^2$ is estimated using highest frequency returns, such as 1 minute. $\hat{IQ}$ is called realized quarticity. See Bardorff-Nielsen et al. [2008] for details. [8]

Tab.1 summarizes the descriptive statistics for RV and RK. First, as shown, the means of RV_t, RVN_t, TH_t and THN_t become larger as the sampling frequencies increase. This lies contrary to our expectation that RV_t increase as the sampling frequency increases because of microstructure noise. Nonetheless, similar results arise in the volatility signature plots in Hansen and Lunde [2006] and Takahashi et al. [2009]. Therefore, we consider that

[8] From an empirical perspective, Barndorff-Nielsen et al. [2008] point out that end effects can be safely ignored in practice, despite their important theoretical implications for the asymptotic properties of the realized kernel estimators. Thus, we use all samples to calculate the RKs.

this phenomenon is because of the limited frequency available for our data. Second, the standard deviations of RV_t, RVN_t, TH_t and THN_t become larger as the time interval increases, and this confirms that intra-day returns become noisy because of the discretization effect as the interval increases. These results suggest that a more precise estimator of the true volatility may be obtained by correcting the bias associated with non-trading hours and microstructure noise in RV_t. Third, the means of RVN_t and THN_t are relatively lower because RVN_t and THN_t are the RV and RK only for trading hours.

4 Estimation results

We estimate the realized GARCH models using 1,000 daily RVs up to the day before the options trade. The estimated period is from 2001/05 to 2007/09 (77 months). The first options start trading on 9 May 2001. We first estimate the parameters in the realized GARCH models using 1,000 daily RVs, RKs, and returns up to 8 May 2001. We then repeat this procedure up to September 2007.

We first discuss the estimates of the parameters in the measurement equation. The persistence in volatility can be measured by the estimates of $\pi = \beta + \phi\gamma$. We find this is about 0.95, regardless of which realized measure is used. This result exhibits the well-known phenomenon of high persistence in volatility. Next, the asymmetry parameters τ_1 are estimated to be negative for RV_t and RVN_t. This is also consistent with a well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility, such as in Nelson [1991]. However, the estimates of the asymmetry parameters τ_1 are not statistically significant for TH_t and THN_t. Finally, the estimate for ν_r is only significant at the beginning of 2007. The implication is that there is only a risk premium in this period.

For example, Tab.2 provide the estimation results for 2007/09. As shown, the estimates of ξ are negative while those for ϕ are less than one. Consequently, RV_t, RVN_t, TH_t and THN_t exhibit downward biases. While τ_1 are estimated to be negative for RV_t and RVN_t, the estimates of τ_1 are not significant for TH_t and THN_t.

From the results for ν_r, we can assume risk neutrality. If traders are risk neutral, the expected return is r where r is the risk-free rate and r_t is the discrete return, while the expected return is $r - 1/2h_t$ when r_t is the log per cent return. In fact, when we analyse the log per cent return, $r - 1/2h_t$ becomes negative because the risk-free rate r is near zero. This is impossible on theoretical grounds. Thus, we analyse discrete daily close-to-close returns

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}} \times 100.$$

We may represent the expected return under the assumption of risk neutrality by

$$r_t = r + \epsilon_t, \quad \epsilon_t = \sqrt{h_t} z_t, \quad z_t \sim i.i.d.N(0,1). \tag{4.19}$$

We estimate realized GARCH models using the above return equation eq.(4.19).

Under the assumption of risk neutrality, the persistence in volatility $\pi = \beta + \phi\gamma$ is estimated to be about 0.95. The estimates of the asymmetry parameters τ_1 are negative for RV_t and RVN_t, but not significant for TH_t and THN_t. This result is the same as that without the assumption of risk neutrality. Tab.3 provides the estimated results for 2007/09 using the risk-neutral models.

5 Put option pricing

Given the parameter estimates of the realized GARCH models obtained, we now calculate the put option prices. We begin with a brief review of put option pricing using realized GARCH models and calculate put option prices using the risk-neutral return equation in eq.(4.19). Afterwards, we explain the details of Duan [1995] in Section 5.1, and calculate put option prices using eq.(2.1).

The price of a European put option is equal to the discounted present value of the expectation of put option prices on the expiration date. For example, the price of a European put option with the exercise price K and survival period τ is given by

$$P_T = \left(\frac{1}{1+r}\right)^{\tau} E^Q\left[\max\left(K - \tilde{S}_{T+\tau}, 0\right)\right]. \tag{5.20}$$

Here, $\tilde{S}_{T+\tau}$ is the price of the underlying asset on the expiration date $T+\tau$.

We cannot evaluate this expectation analytically if the volatility of the underlying asset follows realized GARCH models. We instead calculate this expectation by simulating $\tilde{S}_{T+\tau}$ from the realized GARCH models. Suppose that $(S^{(1)}_{T+\tau}, \ldots, S^{(m)}_{T+\tau})$ are simulated. Then, eq.(5.20) may be calculated as

$$P_T \approx \left(\frac{1}{1+r}\right)^{\tau} \frac{1}{l}\sum_{i=1}^{l} \max(K - \hat{S}^{(i)}_{T+\tau}, 0). \tag{5.21}$$

For variance reduction, we use the control variate and the negative correlation jointly. We set $m = 10,000$.

For comparison, we also calculate option prices using the EGARCH and BS models. EGARCH(1,0) model (Nelson[1991]) is specified as

$$\ln h_t = \omega + \beta\left\{\ln h_{t-1} - \omega\right\} + \theta z_{t-1} + \gamma\{|z_{t-1}| - E(|z_{t-1}|)\}. \tag{5.22}$$

The well-known BS (Black and Scholes [1973]) formula is specified as

$$
\begin{aligned}
P^{BS} &= -S_T N(-d_1) + K\exp(-r\cdot\tau)N(-d_2), \qquad (5.23)\\
d_1 &= \frac{\ln(S_T/K) + (r+\sigma^2/2)\tau}{\sqrt{\sigma^2\tau}},\\
d_2 &= \frac{\ln(S_T/K) + (r-\sigma^2/2)\tau}{\sqrt{\sigma^2\tau}}.
\end{aligned}
$$

Here, volatility σ is the standard deviation of daily returns over the past 20 days.

To measure the performance of option pricing, we use four loss functions, mean errors (ME), mean percentage errors (MPE), root mean squared errors (RMSE) and root mean squared percentage errors (RMSPE), defined as

$$
ME = \frac{1}{N_P}\sum_{i=1}^{N_P}\left(\tilde{P}_i - P_i\right), \quad MPE = \frac{1}{N_P}\sum_{i=1}^{N_P}\left(\frac{\tilde{P}_i - P_i}{P_i}\right),
$$

$$
RMSE = \sqrt{\frac{1}{N_P}\sum_{i=1}^{N_P}\left(\tilde{P}_i - P_i\right)^2}, \quad RMSPE = \sqrt{\frac{1}{N_P}\sum_{i=1}^{N_P}\left(\frac{\tilde{P}_i - P_i}{P_i}\right)^2},
$$

where N_P is the number of put options used for evaluating performance and $\tilde{P}_i$ is the price of the ith put option calculated by the realized GARCH, EGARCH or BS models. P_i is its market put price calculated as the average of the bid and ask prices at the same time closest to 15:00.

From the results of put option pricing under the assumption of risk neutrality in Tab.4, we can see that the RMSPE of the realized GARCH models are smaller than for any of the EGARCH or BS models, except for that of RVN using 3-minute intra-day returns. Next, the RMSE of the realized GARCH models with TH_t and THN_t are smaller than for any of the EGARCH or BS models. This result is not consistent with that using RMSPE. Because of the functional forms of RMSPE and RMSE, the difference between the results implies that the realized GARCH models perform well when the put option price is low, but not when the put option price is high. Thus, the realized GARCH models perform well with RMSPE. Moreover, the best performing models are the realized GARCH model with THN_t for both RMSE and RMSPE. Accordingly, the flat-top Tukey–Hanning kernel method improves pricing performance in option pricing.

5.1 Duan convert

In the return equation, if the expected return is not equal to the risk-free rate, it implies that risk neutrality is not assumed. Unless traders are risk neutral, we must convert the physical measure P into the risk-neutral measure Q. After converting the models, we evaluate option prices under the risk-neutral measure Q.

Duan [1995] makes the following assumptions on Q, called the local risk-neutral valuation relationship (LRNVR):

- $r_t|F_{t-1}$ follows a normal distribution under the risk-neutral measure Q,
- $E^Q[r_t|F_{t-1}] = r$,
- $Var^Q[r_t|F_{t-1}] = Var^P[r_t|F_{t-1}]$.

For the realized GARCH models, as z_t follows a standard normal distribution, the conditional return under the physical measure P follows

$$r_t|F_{t-1} \sim N(E(r_t|F_{t-1}), h_t|F_{t-1}),$$

where the mean of conditional return $E(r_t|F_{t-1})$ and the variance $h_t|F_{t-1}$ are non-stochastic variables. Thus, the Duan [1995] method can be applied to realized GARCH models.

Daily return under the physical measure P is

$$\begin{aligned} r_t &= E(r_t|F_{t-1}) + \epsilon_t, \\ \epsilon_t &= \sqrt{h_t} z_t, \quad z_t \sim i.i.d.N(0,1). \end{aligned}$$

In this study, $E(r_t|F_{t-1}) = \nu_r \sqrt{h_t}$. Under the assumptions of LRNVR, daily returns under the risk-neutral measure Q must be represented by

$$r_t^Q = r + \eta_t, \tag{5.24}$$

$$\epsilon_t^Q = \eta_t + r - E(r_t|F_{t-1}), \tag{5.25}$$

$$z_t^Q = \frac{\epsilon_t^Q}{\sqrt{h_t^Q}}. \tag{5.26}$$

All we have to do for volatility is to substitute z_t^Q in the realized GARCH models.

From the results using the Duan [1995] method in Tab.5, both RMSPE and RMSE for the realized GARCH models are smaller than for the EGARCH or BS models, and the realized GARCH models with THN_t performs better than RV, RVN_t and TH_t. These results are consistent with the results of RMSPE under the assumption of risk neutrality. Consequently, the realized GARCH models perform well. In addition, we compare the results with the results in Tab.4. Excepting THN_t using 5-minute intra-day returns, RMSE and RMSPE using the Duan [1995] method in Tab.5 are smaller than those under the assumption of risk neutrality in Tab.4. This means that the Duan [1995] method improves pricing performance, even though the estimate of the risk premium parameter is not significant.

6 Conclusions

This paper compares the pricing performance of option prices using realized GARCH and EGARCH models. The main results are as follows. First, from the results assuming risk neutrality, the realized GARCH models with RKs perform better than either the EGARCH or BS models. However, the realized GARCH models with RV improve the performances for RMSPE, but do not improve them for RMSE. Using these results, we can see that realized GARCH models with RV improve pricing performance when the put option price is low. Without the assumptions of risk neutrality, the realized GARCH models with RV and RK perform better than the EGARCH and BS models for both RMSE and RMSPE. Therefore, with the exception of the RMSE of the realized GARCH models with RV under the assumptions of risk neutrality, the realized GARCH models perform better than the EGARCH and BS models when using daily returns.

Irrespective of the risk neutrality assumption, the best performing models are the realized GARCH models with TH_t without the lunch-time and overnight returns. From these results, we can see that the flat-top Tukey–Hanning kernel method improves performance in option pricing. Therefore, correcting for the bias of realized GARCH models is not sufficient; then, the performance of option pricing improves when using accurate estimators of integrated volatility.

Several extensions are possible. First, we assume the risk-neutral volatility dynamics are the same as the physical dynamics. However, Corsi et al. [2009] and Christoffersen et al. [2010] propose option pricing methods when the risk-neutral volatility dynamics differ from the physical volatility dynamics. Barone-Adesi et al. [2008] propose a method for pricing options that allows for different distributions (volatilities) under the physical measure P and the risk-neutral measure Q. We should adapt these methods for realized GARCH models. Second, we did not consider jumps in intra-day returns. Barndorff-Nielsen and Shephard [2004] and Dobrev and Szerszen [2010] have proposed a method to calculate realized volatility taking account of jumps. It will be interesting to see whether the performance of option pricing will improve using these realized measures. Finally, Takahashi et al. [2009], Dobrev and Szerszen [2010], and Koopman and Scharth [2011] propose realized SV models, which have similar advantages as realized GARCH models. We should compare these with the performance of realized GARCH models.

References

Andrews, D. W. K. (1991) "Heteroskedasticity and autocorrelation consistent covariance matrix estimation," *Econometrica*, Vol. 59, pp. 817-858.

Bandi, F. M., J. R. Russell, and C. Yang (2008) "Realized volatility forecasting and option pricing," *Journal of Econometrics*, Vol. 147, pp. 34-46.

Bardorff-Nielsen, O. E., P. Q. R. Hansen, A. Lunde, and N. Shephard (2008) "Desining realized kernels to measure the ex-post variation of equity prices in the presence of noise," *Econometrica*, Vol. 76, pp. 1481-1536.

Barndorff-Nielsen, O. E. and N. Shephard (2004) "Power and bipower variation with stochastic volatility and jumps (with disucuss)," *Journal of Financial Econometrics*, Vol. 2, pp. 1-37.

Barndorff-Nielsen, O. E., P. Q. R. Hansen, A. Lunde, and N. Shephard (2008) "Realized kernels in practice: trades and quotes," *Econometrics Journal*, Vol. 4, pp. 1-32.

Barone-Adesi, G., R. F. Rngle, and L. Mancini (2008) "A GARCH option pricing model with filtered histrorical simulation," *Review of Financial Studies*, Vol. 21, pp. 1223-1258.

Black, F. and M. Scholes (1973) "The pricing of options and corporate liabilities," *Journal of Political Economy*, Vol. 81, pp. 673-659.

Bollerslev, T. (1986) "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, Vol. 31, pp. 673-659.

Bollerslev, T. and H. Mikkelsen (1996) "Modeling and pricing long memory in stock market volatility," *Journal of Econometrics*, Vol. 73, pp. 151-184.

——— (1999) "Long-term equity anticipation securities and stock market volatility dynamics," *Journal of Econometrics*, Vol. 92, pp. 75-99.

Campbell, J.Y., A.W. Lo, and A.C. MacKinlay (1997) *The Econometrics of financial markets*: Princeton University Press.

Christoffersen, P., B. Feunou, K. Jacobs, and N. Meddahi (2010) "The economic value of realized volatility," Working Paper Series.

Corsi, F. (2009) "A simple approximate long-memory model of realized volatility," *Journal of Financial Econometrics*, Vol. 7, pp. 174-196.

Corsi, F., N. Fusari, and D.L. Vecchia (2009) "Realizing smiles: pricing options with realized volatility," Swiss Finance Institute Research Paper No. 10-05.

Ding, Z., C. W. J. Granger, and R. F. Engle (1993) "A long memory property of stock market returns and a new model," *Journal of Empirical Finance*, Vol. 1, pp. 83-106.

Dobrev, D. and P. Szerszen (2010) "The information content of high-frequency data for estimating equity return model and forecasting risk," International Finance Discussion Papers 1005. Board of Governors of the Federal Reserve System (U.S.).

Duan, J. C. (1995) "The GARCH option pricing model," *Mathematical Finance*, Vol. 5, pp. 13-32.

——— (1999) "Conditionally fat-tailed distributions and the volatility smile in options," Working paper. Department of Finance, Hong-Kong University.

Glosten, L. R., R. Jagannathan, and D. Runkle (1993) "On the relation between the expected value and the volatility of nominal excess returns on stocks," *Journal of Finance*, Vol. 48, pp. 1779-1801.

Hansen, P. R. and A. Lunde (2005) "A realized variance for the whole day based on intermittent high-frequency data," *Journal of Financial Econometrics*, Vol. 3, pp. 525-554.

——— (2006) "Realized variance and market microstructure noise," *Journal of Business and Economic Statistics*, Vol. 24, pp. 127-161.

Hansen, P. R., A. Huang, and H. H. Shek (2011) "Realized GARCH: A joint model for returns and realized measures of volatility," *Journal of Applied Econometrics*, in press.

Koopman, S. J. and M. Scharth (2011) "The analysis of stochastic volatility in the presence of daily realised measures," Tinbergen Institute Discussion Paper TI 2011-132/4.

Nelson, D. B. (1991) "Conditional heteroskedasticity in asset returns: a new approach," *Econometrica*, Vol. 59, pp. 347-370.

Stentoft, L. (2008) "Option pricing using realized volatility," CREATES Research Paper 2008-13; EFA 2008 Athens Meetings Paper.

Takahashi, M., Y. Omori, and T Watanabe (2009) "Estimating stochastic volatility model using daily returns and realized volatility simultaneously," *Computational Statistics & Data Analysis*, Vol. 53, pp. 2404-2426.

Ubukata, M. and T. Watanabe (2011) "Pricing Nikkei 225 options using realized volatility," IMES Discussions Paper Series, No.2011-E-18, Institute for Monetary and Economic Studies, Bank of Japan.

Table 1: Descriptive statistics of daily realized measures

		Mean	Std.Dev.	Min	Max
RV	3-minute	1.310	0.965	0.085	9.829
	5-minute	1.352	0.987	0.082	9.078
RVN	3-minute	0.863	0.725	0.056	8.975
	5-minute	0.895	0.735	0.058	8.049
TH	3-minute	1.678	2.110	0.014	36.894
	5-minute	1.697	2.172	0.002	36.146
THN	3-minute	0.995	1.305	0.007	21.160
	5-minute	1.008	1.352	0.003	19.945

RV denotes realized volatility, RVN is RV without non-trading hour returns, TH is the flat-top Tukey–Hanning kernel with $p = 2$ and THN is TH without non-trading hour returns. "3-minute" and "5-minute" are the intraday returns intervals used for calculating the volatilities.

Table 2: Estimation results for 2007/09

		ν_t	ω	β	γ	log likelihood	
3-minute	RV	0.051	0.139*	0.626*	0.441*	−393.822	
		(0.032)	(0.027)	(0.059)	(0.075)		
	RVN	0.042	0.388*	0.541*	0.431*	−181.456	
		(0.032)	(0.047)	(0.046)	(0.045)		
	TH	0.041	0.085*	0.824*	0.162*	−875.565	
		(0.032)	(0.027)	(0.049)	(0.043)		
	THN	0.040	0.163*	0.832*	0.146*	−951.781	
		(0.032)	(0.038)	(0.035)	(0.032)		
5-minute	RV	0.051	0.124*	0.629*	0.440*	−398.610	
		(0.032)	(0.026)	(0.059)	(0.076)		
	RVN	0.043	0.358*	0.558*	0.418*	−222.531	
		(0.032)	(0.047)	(0.046)	(0.047)		
	TH	0.040	0.087*	0.837*	0.147*	−969.193	
		(0.034)	(0.027)	(0.045)	(0.039)		
	THN	0.039	0.159*	0.842*	0.134*	−1047.560	
		(0.033)	(0.038)	(0.033)	(0.030)		
		ξ	ϕ	τ_1	τ_2	σ_u^2	π
5-minute	RV	−0.308*	0.710*	−0.113*	0.083*	0.281*	0.939
		(0.035)	(0.048)	(0.018)	(0.011)	(0.012)	
	RVN	−0.882*	0.947*	−0.110*	0.116*	0.171*	0.949
		(0.044)	(0.064)	(0.016)	(0.011)	(0.008)	
	TH	−0.482*	0.848*	0.089	0.389*	0.661*	0.961
		(0.036)	(0.070)	(0.035)	(0.024)	(0.037)	
	THN	−1.071*	0.883*	−0.042	0.284*	0.785*	0.961
		(0.043)	(0.081)	(0.035)	(0.023)	(0.037)	
5-minute	RV	−0.276*	0.706*	−0.108	0.080*	0.284*	0.940
		(0.035)	(0.048)	(0.018)	(0.011)	(0.013)	
	RVN	−0.842*	0.938*	−0.106*	0.107*	0.187*	0.950
		(0.044)	(0.062)	(0.017)	(0.010)	(0.009)	
	TH	−0.535*	0.825*	0.107	0.429*	0.792*	0.958
		(0.036)	(0.072)	(0.040)	(0.025)	(0.051)	
	THN	−1.135*	0.857*	−0.031	0.323*	0.946*	0.957
		(0.043)	(0.084)	(0.039)	(0.025)	(0.045)	

$*$ indicates significance at the 5% level

Table 3: Estimation results for 2007/09 (risk neutral)

		ν_t	ω	β	γ	log likelihood	
3-minute	RV	$--$	0.140*	0.625*	0.442*	−394.948	
			(0.027)	(0.059)	(0.075)		
	RVN	$--$	0.389*	0.541*	0.431*	−182.333	
			(0.047)	(0.046)	(0.045)		
	TH	$--$	0.086*	0.824*	0.162*	−876.421	
			(0.027)	(0.048)	(0.043)		
	THN	$--$	0.164*	0.831*	0.147*	−952.592	
			(0.038)	(0.035)	(0.032)		
5-minute	RV	$--$	0.125*	0.628*	0.441*	−399.757	
			(0.026)	(0.059)	(0.076)		
	RVN	$--$	0.359*	0.558*	0.418*	−223.461	
			(0.047)	(0.046)	(0.047)		
	TH	$--$	0.087*	0.836*	0.148*	−969.989	
			(0.027)	(0.045)	(0.039)		
	THN	$--$	0.160*	0.842*	0.134*	−1048.332	
			(0.038)	(0.034)	(0.030)		
		ξ	ϕ	τ_1	τ_2	σ_u^2	π
5-minute	RV	−0.304*	0.710*	−0.121*	0.083*	0.281*	0.939
		(0.035)	(0.048)	(0.018)	(0.011)	(0.012)	
	RVN	−0.878*	0.946*	−0.120*	0.116*	0.171*	0.949
		(0.044)	(0.063)	(0.017)	(0.011)	(0.008)	
	TH	−0.485*	0.848*	0.056	0.389*	0.661*	0.961
		(0.037)	(0.070)	(0.035)	(0.024)	(0.037)	
	THN	−1.070*	0.882*	−0.065	0.284*	0.785*	0.960
		(0.043)	(0.080)	(0.032)	(0.023)	(0.037)	
5-minute	RV	−0.272*	0.706*	−0.116*	0.080*	0.285*	0.939
		(0.034)	(0.048)	(0.018)	(0.011)	(0.013)	
	RVN	−0.838*	0.938*	−0.116*	0.107*	0.187*	0.950
		(0.044)	(0.062)	(0.016)	(0.010)	(0.009)	
	TH	−0.539*	0.825*	0.072	0.430*	0.792*	0.958
		(0.038)	(0.073)	(0.108)	(0.025)	(0.051)	
	THN	−1.134*	0.857*	−0.057	0.324*	0.946*	0.957
		(0.043)	(0.084)	(0.031)	(0.025)	(0.045)	

$*$ indicates significance at the 5% level

Table 4: Results for put option pricing (risk neutral)

	ME		RMSE	
BS	16.5834		72.7849	
EG	18.0498		57.1486	
RGAR	3-minute	5-minute	3-minute	5-minute
RV	19.5020	17.9068	60.1766	58.9808
RVN	20.8921	19.1401	63.5518	62.6261
TH	12.4174	11.8082	56.1470	56.1059
THN	6.4320	6.6921	*52.2518	*52.2166
	MPE		RMSPE	
BS	0.3184		1.5708	
EG	0.6139		1.5462	
RGAR	3-minute	5-minute	3-minute	5-minute
RV	0.4795	0.4191	1.4730	1.2315
RVN	0.5111	0.4605	1.6140	1.4037
TH	0.2073	0.1951	1.4506	1.2747
THN	0.0901	0.0809	*0.6801	*0.6203

If the RMSE or RMSPE is smaller than that for the EGARCH models, the values are in blue. $*$ indicates the smallest RMSE or RMSPE.

Table 5: Results for put option pricing

	ME		RMSE	
BS	16.5834		72.7849	
EG	17.0594		56.7497	
RGAR	3-minute	5-minute	3-minute	5-minute
RV	14.3653	12.7962	53.1565	52.8324
RVN	13.5953	12.4255	55.6713	56.1590
TH	11.5681	11.5807	54.7771	55.6656
THN	6.4257	6.5209	*52.0057	*52.0249
	MPE		RMSPE	
BS	0.3184		1.5708	
EG	0.5212		1.3405	
RGAR	3-minute	5-minute	3-minute	5-minute
RV	0.2144	0.2094	0.7562	0.7581
RVN	0.2099	0.2119	0.8492	0.8247
TH	0.1590	0.1757	0.8429	1.1084
THN	0.0792	0.0743	*0.6363	*0.6278

If the RMSE or RMSPE is smaller than that for the EGARCH models, the values are in blue. $*$ indicates the smallest RMSE or RMSPE.