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Appendix:

Business Cycle Implications of Internal Consumption Habit for New Keynesian Models

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Abstract

The appendix discusses computational aspects of the paper "Business Cycle Implications of Internal Consumption Habit for New Keynesian Models." These topics range from solving the baseline new Keynesian dynamic stochastic general equilibrium (NKDSGE) model, estimating the structural infinite-order vector moving averages, checking whether these models recover the fundamental shocks, to computing the permanent and transitory output and consumption growth spectral densities. More evidence about the fit of the NKDSGE models is also reviewed. The NKDSGE models are evaluated using alternative priors, and modification of the VARs generating the posterior distributions, and a different goodness of fit statistic. These evaluations reflect the robustness of the evidence about NKDSG model fit reported in "Business Cycle Implications of Internal Consumption Habit for New Keynesian Models."

[†]The views in this paper represent those of the authors and are not those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

Appendix

The appendix consists of five sections. The sample data is described in section **A0**. Section **A1** enlarges on our discussion in the paper about the internal consumption habit propagation mechanism. We present optimality and equilibrium conditions of the baseline habit new Keynesian dynamic stochastic general equilibrium (NKDSGE) model in section **A2**, along with stochastically detrended, steady state, and linearized versions of these equations. This section also outlines the algorithm applied to solve the linearized NKDSGE models. The next section gives instructions to identify and estimate infinite order structural vector moving averages, SVMA(∞)s. Next, we engage existing literature to show that the SVMA(∞)s retrieve the economic shocks of the NKDSGE models. This is followed by formulas to compute the identified permanent and transitory output and consumption growth spectral densities, $SD_{\Delta Y}$ and $SD_{\Delta C}$. Section **A4** completes the appendix with more information about the Bayesian Monte Carlo experiments discussed in the paper as well as summaries of NKDSGE model fit given the habit parameter h is endowed with a β prior, two different uniform priors, SVMA(∞)s are estimated with VAR(4)s instead of VAR(2)s, and the Kolmogorov-Smirnov goodness of fit statistic is replaced with the Cramer-von Mises goodness of fit statistic.

A0. Data Sources and Construction

This section sketches the $1954Q1$–$2002Q4$ sample data. The source of the data is FRED-II maintained by the Federal Reserve Bank of St. Louis at *http://research.stlouisfed.org/fred2/*. Mnemonics appear in parentheses. The NIPA data are real chained 1996 billion dollars and seasonally adjusted at annual rates. The consumption series equals *Real Personal Consumption Expenditures on Nondurables* (PCNDGC96) plus *Real Personal Consumption Expenditures on Services* (PCESVC96). Investment is constructed by adding together *Real Personal Consumption Expenditures on Durables* (PCDGCC96), *Real Gross Private Domestic Investment* (GPDIC1), *Real National Defense Gross Investment* (DGIC96), and *Real Federal Nondefense Gross Investment*

(NDGIC96). Government spending subtracts *Real National Defense Gross Investment* plus *Real Federal Nondefense Gross Investment* from *Real Government Consumption Expenditures and Gross Investment* (GCEC1). Output equals the sum of consumption, investment and government spending. Aggregate quantities are divided by *Civilian Labor Force* (CLF16OV) to create per capita series. Since the *Civilian Labor Force* is monthly, temporal aggregation produces quarterly observations. Finally, the money stock is equated with the seasonally adjusted, *St. Louis Adjusted Monetary Base* (AMBSL). This monthly series is temporally aggregated to obtain a quarterly series and also made per capita.

A1. Consumption Dynamics under Internal and External Consumption Habits

This sections studies the propagation mechanism of additive internal consumption habit. We also show that subsequent to log linearization additive internal and external consumption habit produce observationally equivalent consumption growth dynamics up to a normalization on the impact shock of the AR(1) real rate.

A1.1 The Internal Consumption Habit Propagation Mechanism

Section 2.2 of the paper presents a calibration exercise that discusses the additive internal consumption habit propagation mechanism. The discussion begins with the Euler equation

$$\lambda_t = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1} R_{t+1}}{1 + \pi_{t+1}} \right\}, \qquad (\mathcal{A}1.1)$$

where the forward-looking marginal utility of consumption is $\lambda_t = \dfrac{1}{c_t - hc_{t-1}} - \mathbf{E}_t \left\{ \dfrac{\beta h}{c_{t+1} - hc_t} \right\}$, h is the habit parameter, c_t is household consumption, β is the household discount factor, the mathematical expectations operator conditional on date t information is $\mathbf{E}_t\{\cdot\}$, R_t is the nominal rate, and $1 + \pi_{t+1}$ $(= P_{t+1}/P_t)$ is date $t + 1$ inflation. Given a random walk (with drift) drives total factor productivity (TFP) A_t, the Euler equation ($\mathcal{A}$1.1) and λ_t are stochastically detrended according to

$$\hat{\lambda}_t = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1} R_{t+1}}{\alpha_{t+1}(1+\pi_{t+1})} \right\}, \tag{$\mathcal{A}$1.2}$$

and

$$\hat{\lambda}_t = \frac{\alpha_t}{\alpha_t \hat{c}_t - h\hat{c}_{t-1}} - \mathbf{E}_t \left\{ \frac{\beta h}{\alpha_{t+1}\hat{c}_{t+1} - h\hat{c}_t} \right\}, \tag{$\mathcal{A}$1.3}$$

where $\hat{\lambda}_t \equiv A_t \lambda_t$ and $\alpha_t = A_t/A_{t-1} = \exp(\alpha + \varepsilon_t)$, $\alpha > 0$, and ε_t is the mean zero, homoskedastic TFP shock innovation. Since household consumption and the marginal utility of consumption are stationary, the Euler equation ($\mathcal{A}$1.2) and marginal utility function ($\mathcal{A}$1.3) can be log linearized around the means (*i.e.* steady state) of $\hat{\lambda}_t$, $\hat{c}_t$, R_t, and π_t. The results are

$$\tilde{\lambda}_t = \mathbf{E}_t \left\{ \tilde{\lambda}_{t+1} - \varepsilon_{t+1} + \tilde{R}_{t+1} - \frac{\pi^*}{1+\pi^*}\tilde{\pi}_{t+1} \right\}, \tag{$\mathcal{A}$1.4}$$

and

$$(\alpha^* - \beta h)(\alpha^* - h)\tilde{\lambda}_t = \alpha^* \beta h \mathbf{E}_t \tilde{c}_{t+1} - (\beta h^2 + \alpha^{*2})\tilde{c}_t + \alpha^* h \tilde{c}_{t-1} - \alpha^* \beta h \mathbf{E}_t \varepsilon_{t+1} + \alpha^* h \varepsilon_t, \tag{$\mathcal{A}$1.5}$$

where, for example, $\tilde{c}_t = \ln \hat{c}_t - \ln c^*$ or $\tilde{R}_t = \ln R_t - \ln R^*$, and $\alpha^* = \exp(\alpha)$ is the deterministic TFP growth rate. We combine equations ($\mathcal{A}$1.4) and ($\mathcal{A}$1.5) to obtain

$$\begin{aligned} &\alpha^* \beta h \mathbf{E}_t \left\{ \Delta\tilde{c}_{t+2} + \varepsilon_{t+2} \right\} - (\beta h^2 + \alpha^{*2}) \mathbf{E}_t \left\{ \Delta\tilde{c}_{t+1} + \varepsilon_{t+1} \right\} + \alpha^* h (\Delta\tilde{c}_t + \varepsilon_t) \\ &\qquad = -(\alpha^* - \beta h)(\alpha^* - h)\mathbf{E}_t \tilde{q}_{t+1}, \end{aligned} \tag{$\mathcal{A}$1.6}$$

where the demeaned real rate is $\tilde{q}_t = \tilde{R}_t - \dfrac{\pi^*}{1+\pi^*}\tilde{\pi}_t$. By exploiting stochastic detrending, the linearized Euler equation ($\mathcal{A}$1.6) can be written as a second-order expectational stochastic difference equation in demeaned household consumption growth

$$\alpha^*\beta h\mathbf{E}_t\widetilde{\Delta c}_{t+2} - (\beta h^2 + \alpha^{*2})\mathbf{E}_t\widetilde{\Delta c}_{t+1} + \alpha^* h\widetilde{\Delta c}_t = -(\alpha^* - \beta h)(\alpha^* - h)\mathbf{E}_t\widetilde{q}_{t+1}, \qquad (\mathcal{A}1.7)$$

where $\widetilde{\Delta c}_t = \Delta \ln \hat{c}_t + \varepsilon_t$ denotes demeaned household consumption growth.

We solve equation ($\mathcal{A}$1.7) to obtain the backward-looking stable root $\varphi_1 = h\alpha^{*-1}$ and forward-looking unstable root $\varphi_2 = \alpha^*/(\beta h)$. These roots are exploited by the lag polynomial $-\mathbf{L}^{-1}(1 - \varphi_1\mathbf{L})(1 - \varphi_2^{-1}\mathbf{L}^{-1})\varphi_2\alpha^*\beta h\widetilde{\Delta c}_t$, which is an alternative to the left side of equation ($\mathcal{A}$1.7). After applying the lag polynomial, we have

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t = \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^{*2}}\sum_{j=0}^{\infty}\left(\frac{\beta h}{\alpha^*}\right)^j \mathbf{E}_t\widetilde{q}_{t+j}, \qquad (\mathcal{A}1.8)$$

which is the unique (*i.e.*, sunspot free) solution of the second-order stochastic difference equation ($\mathcal{A}$1.7). This solution is equation (2) of the paper, where $\Psi = \dfrac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^*\beta h}$. Equation ($\mathcal{A}$1.8) is forward-looking in the expected discounted present value of $\widetilde{q}_t$ and backward-looking in the lag of demeaned consumption growth. Assume $\widetilde{q}_t$ is a AR(1) with persistence parameter ρ_q. In this case, the Wiener-Kolmogorov formulas alter equation ($\mathcal{A}$1.8) to

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t = \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^*(\alpha^* - \rho_q\beta h)}\widetilde{q}_t. \qquad (\mathcal{A}1.9)$$

We employ equation ($\mathcal{A}$1.9), put h on the grid $[0.15\ 0.35\ 0.50\ 0.65\ 0.85]$, calibrate $[\beta\ \ \alpha^*]' = [0.993\ \ \exp(0.004)]'$, and estimate a AR(1) for $\widetilde{q}_t$ to generate the impulse response functions plotted in figure 1.

The real federal funds rate $\widetilde{q}_t$ is measured with the demeaned quarterly nominal federal funds rate and demeaned implicit GDP deflator inflation. The latter is multiplied by the ratio of its mean to one plus its mean and subtracted from the former to create the real federal funds rate $\tilde{q}_t$ on a 1954Q1–2002Q4 sample. Although likelihood ratio tests and the Hannan-Quinn criterion suggest a AR(3), we settle on a AR(1) using the SIC against AR(2) to AR(10) specifications. On the 1954Q1–2002Q4 sample, OLS estimates of the AR(1) of $\tilde{q}_t$ are ρ_q = 0.8687 and the standard error of the regression is 1.2059.

A1.2 An Observational Equivalence Result for Internal and External Consumption Habits

We show in this section that under internal and external additive consumption habit the preferences $\ln[c_t - hc_{t-1}]$ yield log linearized Euler equations that are observationally equivalent up to a normalization of the AR(1) real rate, $\tilde{q}_t$. The consumption habit specification $\ln[c_t - hc_{t-1}]$ is found in the NKDSGE models that Christiano, Eichenbaum, and Evans (2005) and Smets and Wouter (2007) estimate. The latter (former) paper uses internal (external) consumption habit. According to Dennis (2009), the economic content of estimates of linearized NKDSGE models appears to be unaffected by the choice of consumption habit specification. He also shows that the mapping from additive to multiplicative (*i.e.* 'keeping up with the Jones') consumption habit parameters is onto (only in this direction).

Additive external consumption habit restricts the marginal utility of consumption to be purely backward-looking. After stochastic detrending, $\hat{\lambda}_{t,ECH} = \dfrac{\alpha_t}{\alpha_t\hat{c}_t - h\hat{c}_{t-1}}$, where *ECH* denotes external consumption habit. The log linearized Euler equation ($\mathcal{A}$1.4) becomes

$$\alpha^*\tilde{c}_t - h\tilde{c}_{t-1} + h\varepsilon_t = \mathbf{E}_t\Big\{\alpha^*\tilde{c}_{t+1} - h\tilde{c}_t + \alpha^*\varepsilon_{t+1} + (\alpha^* - h)\tilde{q}_{t+1}\Big\}, \qquad (\mathcal{A}1.10)$$

with $(\alpha^* - h)\tilde{\lambda}_{t,ECH} = -\alpha^*\tilde{c}_t + \alpha^*\tilde{c}_{t-1} - h\varepsilon_t$. A bit of rearranging transforms the linearized Euler equation ($\mathcal{A}$1.10) into the first-order expectational stochastic difference equation

$$\mathbf{E}_t\Big\{\Delta\tilde{c}_{t+1} + \varepsilon_{t+1}\Big\} = \frac{h}{\alpha^*}(\Delta\tilde{c}_t + \varepsilon_t) - \frac{\alpha^* - h}{\alpha^*}\mathbf{E}_t\tilde{q}_{t+1}. \qquad (\mathcal{A}1.11)$$

Given a mean zero, homoskedastic expectation error $\vartheta_{\tilde{c},t} = \tilde{c} - \mathbf{E}_{t-1}\tilde{c}$, the first-order stochastic difference equation ($\mathcal{A}$1.11) can be written

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t \;=\; \frac{\alpha^* - h}{\alpha^*}\tilde{q}_t + \vartheta_{\tilde{c},t}, \qquad (\mathcal{A}1.12)$$

which represents reduced-form consumption growth dynamics under additive ECH.

Equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) produce observationally equivalent dynamics in $\widetilde{\Delta c}_t$ up to the impact coefficient on $\tilde{q}_t$ given it is a AR(1). The dynamics are equivalent because equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) share the same leading autoregressive root, which equals h/α^*. Thus across additive internal and external consumption habit, a shock to $\tilde{q}_t$ generates identical responses in $\widetilde{\Delta c}_t$ beyond impact. Only at impact can internal and external consumption habit yield disparate responses in $\widetilde{\Delta c}_t$ to an innovation in $\tilde{q}_t$. As $h \longrightarrow 1$ the impact responses of $\widetilde{\Delta c}_t$ differ by a factor of 12 for internal and external consumption habit at the calibration of section 2.2, but the impact responses converge as $h \longrightarrow 0$.

A2. Solving the Habit NKDSGE Models

This section presents the optimality and equilibrium conditions of the baseline habit NKDSGE models, the stochastically detrended versions of these conditions, the steady state of this economy, the log linearized optimality and equilibrium conditions, and solution method invoked to compute a multivariate linear approximate equilibrium law of motion.

A2.1 Optimality and equilibrium conditions

The baseline habit NKDSGE models have first-order necessary conditions (FONCs) that are restricted by the primitives of preferences, technology, market structure, and monetary policy regime. The FONCs imply optimality and equilibrium conditions that must be satisfied by any candidate equilibrium time series. The optimality and equilibrium conditions are

$$\lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{C_{t+1} - hC_t} \right\}, \tag{$\mathcal{A}$2.1}$$

$$\frac{1 - q_t}{q_t} + S\left(\frac{X_t}{\alpha^* X_{t-1}}\right) + S'\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\frac{X_t}{\alpha^* X_{t-1}} = \frac{\beta}{\alpha^*}\mathbf{E}_t\left\{\frac{\lambda_{t+1} q_{t+1}}{\lambda_t q_t} S'\left(\frac{X_{t+1}}{\alpha^* X_t}\right)\left[\frac{X_{t+1}}{X_t}\right]^2\right\}, \tag{$\mathcal{A}$2.2}$$

$$q_t = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\psi u_{t+1} \phi_{t+1} \frac{Y_{A,t+1}}{K_{t+1}} - a(u_{t+1}) + q_{t+1}(1 - \delta) \right] \right\}, \tag{$\mathcal{A}$2.3}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \frac{\lambda_{t+1}}{P_{t+1}} R_{t+1}, \tag{$\mathcal{A}$2.4}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{P_{t+1}} + \frac{1}{M_{t+1}} \right\}, \tag{$\mathcal{A}$2.5}$$

$$a'(u_t) = \psi \phi_t \frac{Y_{A,t}}{K_t}, \tag{$\mathcal{A}$2.6}$$

$$\frac{W_t}{P_t} = \phi_t (1 - \psi) \frac{Y_{A,t}}{N_t - N_0}, \tag{$\mathcal{A}$2.7}$$

$$\frac{Y_{A,t}}{Y_{D,t}} = \left(\frac{P_{A,t}}{P_t}\right)^{-\xi}, \qquad (\mathcal{A}2.8)$$

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi-1}\right)\frac{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_P\right]^i\lambda_{t+i}\phi_{t+i}Y_{D,t+i}\left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi}}{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_P\right]^i\lambda_{t+i}Y_{D,t+i}\left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi-1}}, \qquad (\mathcal{A}2.9)$$

$$\frac{N_t}{n_t} = \left(\frac{W_{D,t}}{W_t}\right)^{-\xi}, \qquad (\mathcal{A}2.10)$$

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right)\frac{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_W\alpha^{*-\theta(1+1/\gamma)}\right]^i\left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta}N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t\sum_{i=0}^{\infty}\left[\beta\mu_W\alpha^{*(1-\theta)}\right]^i\lambda_{t+i}\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta}\left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1}N_{t+i}}, \qquad (\mathcal{A}2.11)$$

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\right]X_t, \qquad (\mathcal{A}2.12)$$

$$Y_{D,t} = C_t + X_t + a(u_t)K_t, \qquad (\mathcal{A}2.13)$$

$$Y_{A,t} = \left[u_t K_t\right]^{\psi}\left[(N_t - N_0)A_t\right]^{1-\psi}, \qquad (\mathcal{A}2.14)$$

$$P_t^{1-\xi} = \mu_P \left[\frac{P_{t-1}}{P_{t-2}} P_{t-1}\right]^{1-\xi} + (1-\mu_P) P_{c,t}^{1-\xi}, \qquad (\mathcal{A}2.15)$$

$$P_{A,t}^{-\xi} = \mu_P \left[\frac{P_{A,t-1}}{P_{A,t-2}} P_{A,t-1}\right]^{-\xi} + (1-\mu_P) P_{c,t}^{-\xi}, \qquad (\mathcal{A}2.16)$$

$$W_t^{1-\theta} = \mu_W \left(\alpha^* \frac{P_{t-1}}{P_{t-2}} W_{t-1}\right)^{1-\theta} + (1-\mu_W) W_{c,t}^{1-\theta}, \qquad (\mathcal{A}2.17)$$

and

$$W_{D,t}^{-\theta} = \mu_W \left(\alpha^* \frac{P_{t-1}}{P_{t-2}} W_{D,t-1}\right)^{-\theta} + (1-\mu_W) W_{c,t}^{-\theta}, \qquad (\mathcal{A}2.18)$$

where λ_t, C_t, q_t, $S(\cdot)$, X_t, u_t, r_t, δ, $a(u_t)$, P_t, M_t, $Y_{A,t}$, $P_{A,t}$, ξ, $Y_{D,t}$, ϕ_t, ψ, K_t, W_t, $P_{c,t}$, μ_P, θ, μ_W, $W_{c,t}$, $W_{D,t}$, and γ denote the marginal utility of consumption, aggregate consumption, the shadow price of capital (per unit of consumption), the investment growth cost function, the deterministic TFP growth rate, aggregate investment, capital utilization rate, the rental rate of capital, the depreciation rate of capital, the household cost of capital utilization, the aggregate (demand) price level, the aggregate money stock at the end of date $t-1$, aggregate output, the aggregate supply price level, the price elasticity, aggregate demand, real marginal cost, capital's share of output, the aggregate capital stock at the end of date $t-1$, the aggregate nominal wage, the firm's optimal date t price, the fraction of firms forced to update their price at the previous period's inflation rate, the wage elasticity, the fraction of households forced to update their nominal wage at the previous period's inflation rate, the household's optimal date t nominal wage, the aggregate demand nominal wage, and the inverse of the Frisch labor supply elasticity, respectively.

A symmetric equilibrium is imposed on the markets in which final good firms and households have monopolistic power. Along the symmetric equilibrium path, firms i and j choose the same commitment price $P_{c,t} = P_{i,t} = P_{j,t}$. The same restriction is placed on the nominal wages $W_{c,t} = W_{\ell,t} = W_{\wp,t}$ of households ℓ and $\wp$. The optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11) reflect the impact of the symmetric equilibrium assumptions. Rather than $P_{i,t}$ and $W_{\ell,t}$, the symmetric equilibrium impose the final good price $P_{c,t}$ and nominal wage $W_{c,t}$ on the optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11).

The impulse vector consists of TFP and monetary policy shocks. We assume TFP, $\ln A_t$, is a random walk with drift

$$\ln A_t = \alpha + \ln A_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}\left(0,\, \sigma_{\varepsilon}^2\right). \tag{$\mathcal{A}$2.19}$$

The monetary policy shock is either the innovation μ_t of the first order autoregression, AR(1), money growth (MG) supply rule

$$m_{t+1} = (1-\rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0,\, \sigma_{\mu}^2\right), \tag{$\mathcal{A}$2.20}$$

of the NKDSGE-MG model, where m^* is the steady state money growth rate, or the innovation υ_t to the interest rate smoothing Taylor rule (TR)

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(R^* + a_\pi \mathbf{E}_t\left\{\frac{P_{t+1}}{P_t}\right\} + a_{\tilde{Y}}\tilde{Y}_t\right) + \upsilon_t, \ \left|\rho_R\right| < 1, \ \upsilon_t \sim \mathcal{N}\left(0,\, \sigma_{\upsilon}^2\right), \tag{$\mathcal{A}$2.21}$$

of the NKDSGE-TR model, where the steady state nominal rate R^* is the ratio of steady state inflation to the household discount factor, π^*/β, π^* equals the differential of steady state money growth and deterministic TFP growth, $\exp(m^* - \alpha)$, and $\widetilde{Y}_t$ is the output gap (*i.e.*, deviations of output from its trend). The TFP and money growth (or Taylor rule) innovations are assumed to be uncorrelated at leads and lags, $\mathbf{E}\{\varepsilon_{t+i}\,\mu_{t+j}\} = 0$, (or $\mathbf{E}\{\varepsilon_{t+i}\,\upsilon_{t+j}\} = 0$) for all i, j.

Equations ($\mathcal{A}$2.1)–($\mathcal{A}$2.11) are the optimality conditions of the baseline habit NKDSGE model. Internal consumption habit creates the forward-looking marginal utility of current consumption, which is restated by equation ($\mathcal{A}$2.1). Equation ($\mathcal{A}$2.2) sets the cost of adding one unit of aggregate investment, X_t, to its discounted expected benefit. The cost is represented by the ratio of the cost of installing a unit of investment to the market value of extant capital (*i.e.*, the inverse of Tobin's q), $(1 - q_t)/q_t$, plus the total cost of installing a unit of investment and the marginal cost of adding a unit of capital at the investment growth rate, X_t/X_{t-1}, net of steady state growth α^*. The expected benefit equals the foregone marginal cost of future investment valued at the pricing kernel, $\beta\lambda_{t+1}/\lambda_t$, which is weighted by the change in the price of capital. The Euler equation of capital ($\mathcal{A}$2.3) equates the price of increasing the capital stock by one unit to the discounted expected return on the service flow of that unit of capital net of the cost of capital services (or utilization) plus the net value of the unit of capital after production evaluated at the pricing kernel. The riskless bond is priced in the Euler equation ($\mathcal{A}$2.4). The dynamics of the purchasing power of money is described by the Euler equation ($\mathcal{A}$2.5), where money is valued at the marginal utility of consumption. Equations ($\mathcal{A}$2.4) and ($\mathcal{A}$2.5) yield the money demand function of the baseline habit NKDGSE model. Equation ($\mathcal{A}$2.6) is an intratemporal optimality condition that forces the marginal capital utilization rate to match the marginal product of capital, which equals the rental rate of capital. Final good firm labor demand is tied down by the intratemporal optimality condition ($\mathcal{A}$2.7). The ratio of aggregate supply to aggregate demand is connected to the ratio of the alternative aggregate price level to the aggregate price level raised to the negative of the price elasticity by equation

($\mathcal{A}$2.8). Equation ($\mathcal{A}$2.9) specifies optimal pricing of a monopolistically competitive final good firm. This decision is restricted by the Calvo staggered price technology, the firm's discount factor, real marginal cost, aggregate demand, and full indexation to lagged inflation of those firms unable to obtain their optimal price at date t. Aggregate labor demand is equated to aggregate labor supply in equation ($\mathcal{A}$2.10) up to the ratio of the aggregate nominal wage indices raised to the negative of the nominal wage elasticity. The optimal nominal wage decision is characterized by equation ($\mathcal{A}$2.11). The household settles on its optimal nominal wage by balancing the discounted expected disutility of labor supply to the benefits of greater real labor income in marginal utility of consumption units (*i.e.*, the marginal rate of substitution between the expected discounted lifetime disutility of work to the expected discounted value of permanent income). Note that these costs and benefits are affected by the wage and labor supply elasticities, and that those households unable to update their date t nominal wage reset using lagged inflation.

Equilibrium conditions are given by equations ($\mathcal{A}$2.12)–($\mathcal{A}$2.18) for the baseline habit NKDSGE model. Equation ($\mathcal{A}$2.12) is the law of motion of capital with capital adjustment costs. Aggregate demand equals its constituent parts according to equation ($\mathcal{A}$2.13). The constant returns to scale aggregate technology is found in equation ($\mathcal{A}$2.14). Equations ($\mathcal{A}$2.15), ($\mathcal{A}$2.16), ($\mathcal{A}$2.17), and ($\mathcal{A}$2.18) are the laws of motion of the aggregate price levels and nominal wages under Calvo price and nominal wage setting with full indexation.

The laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are added to avoid the curse of dimensionality. Under Calvo staggered price and nominal wage setting, Yun (1996) points out that the price and nominal wage aggregators ($\mathcal{A}$2.15) and ($\mathcal{A}$2.17) place the histories P_t and W_t (from date t = 0) into the state vector of the baseline habit NKDSGE model. The reason is the histories of P_t and W_t drive the process that restrict $P_{C,t}$ and $W_{C,t}$ along any candidate equilibrium path. The aggregate supply price and aggregate demand nominal wage laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are used to replace $P_{C,t}$ and $W_{C,t}$ with $P_{A,t}$ and $W_{D,t}$ in the state vector. This leaves the state vector with P_t, $P_{A,t}$, W_t, and $W_{D,t}$ rather than their histories.

A2.2 Stochastically detrended optimality and equilibrium conditions

The NKDSGE models contain a permanent technology shock A_t. Since this shock is a random walk (with drift), stochastic detrending renders the equilibrium path of state and other endogenous variables stationary. Stochastic detrending consists of $\hat{C}_t \equiv C_t/A_t$, $\hat{X}_t \equiv X_t/A_t$, $\hat{Y}_{j,t} \equiv Y_{j,t}/A_t$, $j = A, D$, $\hat{K}_{t+1} \equiv K_{t+1}/A_t$, $\hat{P}_t \equiv P_t A_t/M_t$, $\hat{P}_{i,t} \equiv P_{i,t} A_t/M_t$, $i = A, c$, $\widehat{W}_t \equiv W_t/M_t$, and $\widehat{W}_{c,t} \equiv W_{\wp,t}/M_t$, $\wp = D, c$. Applying these definitions to equations ($\mathcal{A}$2.1)–($\mathcal{A}$2.18) yields the stochastically detrended optimality and equilibrium conditions

$$\hat{\lambda}_t = \frac{\alpha_t}{\alpha_t \hat{C}_t - h\hat{C}_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{\alpha_{t+1}\hat{C}_{t+1} - h\hat{C}_t} \right\}, \tag{$\mathcal{A}$2.22}$$

$$\frac{1-q_t}{q_t} + S\left(\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}\right) + S'\left(\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}\right)\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}$$

$$= \frac{\beta}{\alpha^*}\mathbf{E}_t \left\{ \alpha_{t+1} \frac{q_{t+1}\hat{\lambda}_{t+1}}{q_t \hat{\lambda}_t} S'\left(\frac{\alpha_{t+1}\hat{X}_{t+1}}{\alpha^* \hat{X}_t}\right) \left[\frac{\hat{X}_{t+1}}{\hat{X}_t}\right]^2 \right\}, \tag{$\mathcal{A}$2.23}$$

$$q_t = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \left[\psi u_{t+1} \phi_{t+1} \frac{\hat{Y}_{t+1}}{\hat{K}_{t+1}} + \frac{q_{t+1}[1-\delta] - a(u_{t+1})}{\alpha_{t+1}} \right] \right\}, \tag{$\mathcal{A}$2.24}$$

$$\frac{\hat{\lambda}_t}{\hat{P}_t} = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1}}{\hat{P}_{t+1}} \frac{R_{t+1}}{\exp(m_{t+1})} \right\}, \tag{$\mathcal{A}$2.25}$$

$$\frac{\hat{\lambda}_t}{\hat{P}_t} = \beta \mathbf{E}_t \left\{ \left[\frac{\hat{\lambda}_{t+1}}{\hat{P}_{t+1}} + 1 \right] \exp(-m_{t+1}) \right\}, \tag{$\mathcal{A}$2.26}$$

$$a'(u_t) = \psi \phi_t \alpha_t \frac{\hat{Y}_t}{\hat{K}_t}, \tag{$\mathcal{A}$2.27}$$

$$\frac{\widehat{W}_t}{\hat{P}_t} = (1-\psi)\phi_t \frac{\hat{Y}_t}{N_t - N_0}, \tag{$\mathcal{A}$2.28}$$

$$\frac{\hat{Y}_{A,t}}{\hat{Y}_{D,t}} = \left(\frac{\hat{P}_{A,t}}{\hat{P}_t} \right)^{-\xi}, \tag{$\mathcal{A}$2.29}$$

$$\exp(m_t - \varepsilon_t)\frac{\hat{P}_{c,t}}{\hat{P}_{t-1}} =$$

$$\left(\frac{\xi}{\xi - 1} \right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \hat{\lambda}_{t+i} \phi_{t+i} \hat{Y}_{D,t+i} \left[\exp(m_{t+i} - \varepsilon_{t+i}) \frac{\hat{P}_{t+i}}{\hat{P}_{t+i-1}} \right]^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \hat{\lambda}_{t+i} \hat{Y}_{D,t+i} \left[\exp(m_{t+i} - \varepsilon_{t+i}) \frac{\hat{P}_{t+i}}{\hat{P}_{t+i-1}} \right]^{\xi - 1}}, \tag{$\mathcal{A}$2.30}$$

$$\frac{N_t}{n_t} = \left(\frac{\widehat{W}_{D,t}}{\widehat{W}_t} \right)^{-\xi}, \tag{$\mathcal{A}$2.31}$$

$$\left[\exp(m_t)\frac{\widehat{W}_{c,t}}{\widehat{P}_{t-1}}\right]^{1+\theta/\gamma} = \frac{\theta}{\theta-1}$$

$$\times \frac{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta\mu_W)^i \exp(\theta(1+1/\gamma)(m_{t+i} + \sum_{j=1}^{i} \varepsilon_{t+j-1}) \left[\left[\frac{\widehat{W}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{\theta} N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta\mu_W)^i \lambda_{t+i} \exp(-(1-\theta)(m_{t+i} + \sum_{j=1}^{i} \varepsilon_{t+j-1})) \left[\frac{\widehat{W}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{\theta} \left[\frac{\widehat{P}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{-1} N_{t+i}}, \quad (\mathcal{A}2.32)$$

$$\hat{K}_{t+1} = \frac{(1-\delta)\hat{K}_t}{\alpha_t} + \left[1 - S\left(\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}\right)\right]\hat{X}_t, \quad (\mathcal{A}2.33)$$

$$\hat{Y}_t = \hat{C}_t + \hat{X}_t + \frac{a(u_t)\hat{K}_t}{\alpha_t}, \quad (\mathcal{A}2.34)$$

$$\hat{Y}_t = \left[u_t \frac{\hat{K}_t}{\alpha_t}\right]^{\psi} \left[N_t - N_O\right]^{1-\psi}, \quad (\mathcal{A}2.35)$$

$$\hat{P}_t^{1-\xi} = \mu_P \left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1})\frac{\hat{P}_{t-1}}{\hat{P}_{t-2}}\hat{P}_{t-1}\right]^{1-\xi} + (1-\mu_P)\hat{P}_{c,t}^{1-\xi}, \quad (\mathcal{A}2.36)$$

$$\hat{P}_{A,t}^{-\xi} = \mu_P \left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1})\frac{\hat{P}_{A,t-1}}{\hat{P}_{A,t-2}}\hat{P}_{A,t-1}\right]^{-\xi} + (1-\mu_P)\hat{P}_{c,t}^{-\xi}, \quad (\mathcal{A}2.37)$$

$$\widehat{W}_t^{1-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{t-1} \right]^{1-\theta} + (1-\mu_W) \widehat{W}_{c,t}^{1-\theta}, \tag{$\mathcal{A}$2.38}$$

and

$$\widehat{W}_{D,t}^{-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{D,t-1} \right]^{-\theta} + (1-\mu_W) \widehat{W}_{c,t}^{-\theta}, \tag{$\mathcal{A}$2.39}$$

where it is understood in equation ($\mathcal{A}$2.32) that at $i = 0$ the sum $\sum_{j=1}^{i} \varepsilon_{t+j-1}$ equals one. Equations ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) constitute the basis of the steady state equilibrium and the first-order linear approximation of the baseline habit NKDSGE models.

A2.3 Deterministic steady state

Let λ^*, C^*, Y^*, X^*, N^*, K^*, q^*, W^*, r^*, P^*, u^*, ϕ^*, and R^* denote deterministic steady state values of the corresponding endogenous variables. The steady state equilibrium rests on $u^* = 1$, $a(1) = 0$, and $S(1) = S'(1) = 0$, which is consistent with Christiano, Eichenbaum, and Evans (2005). Given these assumptions, the following equations characterize the deterministic steady state of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39)

$$C^*\lambda^* = \frac{\alpha^* - \beta h}{\alpha^* - h}, \tag{$\mathcal{A}$2.40}$$

$$q^* = 1, \tag{$\mathcal{A}$2.41}$$

$$\frac{K^*}{Y^*} = \frac{\beta\alpha^*\psi\phi^*}{\alpha^* - \beta(1-\delta)}, \tag{$\mathcal{A}$2.42}$$

$$R^* = \frac{\exp(m^*)}{\beta}, \tag{$\mathcal{A}$2.43}$$

$$\frac{\lambda^*}{P^*} = \frac{\beta}{\exp(m^*) - \beta}, \tag{$\mathcal{A}$2.44}$$

$$a'(1) = \psi\phi^*\alpha^*\frac{Y^*}{K^*}, \tag{$\mathcal{A}$2.45}$$

$$\frac{W^*}{P^*} = (1-\psi)\phi^*\frac{Y^*}{N^* - N_0}, \tag{$\mathcal{A}$2.46}$$

$$\phi^* = \frac{\xi - 1}{\xi}, \tag{$\mathcal{A}$2.47}$$

$$\frac{W^*}{P^*} = \left(\frac{\theta}{\theta - 1}\right)\frac{N^{*1/\gamma}}{\lambda^*}, \tag{$\mathcal{A}$2.48}$$

$$\frac{X^*}{K^*} = 1 - \frac{(1-\delta)}{\alpha^*}, \qquad (\mathcal{A}2.49)$$

$$Y^* = C^* + X^*, \qquad (\mathcal{A}2.50)$$

and

$$Y^* = \left[\frac{K^*}{\alpha^*}\right]^{\psi} \left[N^* - N_O\right]^{1-\psi}. \qquad (\mathcal{A}2.51)$$

Note that equations ($\mathcal{A}$2.46), ($\mathcal{A}$2.48), and ($\mathcal{A}$2.51) imply that the solution for N^* is nonlinear. Also, at the steady state equilibrium, $P^* = P_A^* = P_c^*$ and $W^* = W_D^* = W_c^*$.

A2.4 Log-linearized baseline habit NKDSGE models

We log linearize the optimality and equilibrium conditions of the baseline NKDSGE models in this section. The log linear approximations (*i.e.*, first-order Taylor expansions) of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) are around the deterministic steady state given by equations ($\mathcal{A}$2.40)–($\mathcal{A}$2.51). The approximations exploit, for example, the definitions $\widetilde{C}_t = \ln \widehat{C}_t - \ln C^*$ or $\widetilde{N}_t = \ln N_t - \ln N^*$.

A symmetric equilibrium has several implications for the log linear approximation of the baseline habit NKDSGE models. Subsequent to log linearizing around the steady state, the aggregate price indices are equated $\widetilde{P}_t = \widetilde{P}_{A,t}$, as are the aggregate nominal wages $\widetilde{W}_t = \widetilde{W}_{D,t}$, given $P_0 = P_{A,0}$ and $W_0 = W_{D,0}$. This further reduces the dimension of the state vector.

Log linearizing the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) yields the linear approximate optimality and equilibrium conditions of the baseline habit NKDSGE model. The relevant conditions are

$$(\alpha^* - h)(\alpha^* - \beta h)\widetilde{\lambda}_t = \beta\alpha^* h \mathbf{E}_t \widetilde{C}_{t+1} - (\beta h^2 + \alpha^{*2})\widetilde{C}_t + \alpha^* h(\widetilde{C}_{t-1} - \varepsilon_t), \qquad (\mathcal{A}2.52)$$

$$\beta\varpi \mathbf{E}_t \widetilde{X}_{t+1} - (1+\beta)\varpi\widetilde{X}_t + \varpi\widetilde{X}_{t-1} + \widetilde{q}_t = \varpi\varepsilon_t, \qquad (\mathcal{A}2.53)$$

$$\widetilde{q}_t + \widetilde{\lambda}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} + \beta\psi\phi^* \frac{Y^*}{K^*}\left[\widetilde{\phi}_{t+1} + \widetilde{Y}_{t+1} - \widetilde{K}_{t+1}\right] + \beta\frac{1-\delta}{\alpha^*}\widetilde{q}_{t+1}\right\}, \qquad (\mathcal{A}2.54)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1} + \widetilde{R}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.55)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \frac{\lambda^*}{\lambda^* + P^*}\mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.56)$$

$$\varrho\widetilde{u}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \widetilde{K}_t + \varepsilon_t, \qquad (\mathcal{A}2.57)$$

$$\widetilde{W}_t - \widetilde{P}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \frac{N^*}{N^* - N_0}\widetilde{N}_t, \qquad (\mathcal{A}2.58)$$

$$\mu_P(1+\beta)\widetilde{\pi}_t = \beta\mu_P \mathbf{E}_t \widetilde{\pi}_{t+1} + \mu_P \widetilde{\pi}_{t-1}$$

$$+ \quad (1-\mu_P)(1-\beta\mu_P)\widetilde{\phi}_t + \beta\mu_P \widetilde{m}_{t+1} - \mu_P(1+\beta)(\widetilde{m}_t - \varepsilon_t) + \mu_P(\widetilde{m}_{t-1} - \varepsilon_{t-1}), \quad (\mathcal{A}2.59)$$

$$\left[1 + \beta\mu_W^2 - \frac{\theta(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\widetilde{W}_t = \beta\mu_W \mathbf{E}_t \widetilde{W}_{t+1} + \mu_W \widetilde{W}_{t-1} + \left[\frac{(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\widetilde{N}_t$$

$$- \left[\frac{\gamma(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right](\widetilde{\lambda}_t - \widetilde{P}_t) - \beta\mu_W \widetilde{\pi}_t + \mu_W \widetilde{\pi}_{t-1} + \beta\mu_W \widetilde{m}_{t+1} - (1+\beta)\mu_W \widetilde{m}_t + \mu_W \widetilde{m}_{t-1}$$

$$+ \quad \beta\mu_W \varepsilon_t - \mu_W \varepsilon_{t-1}, \quad (\mathcal{A}2.60)$$

$$\widetilde{K}_{t+1} = \frac{(1-\delta)}{\alpha^*}\left(\widetilde{K}_t - \varepsilon_t\right) + \frac{X^*}{K^*}\widetilde{X}_t, \quad (\mathcal{A}2.61)$$

$$\widetilde{Y}_t = \frac{C^*}{Y^*}\widetilde{C}_t + \frac{X^*}{Y^*}\widetilde{X}_t + \psi\phi^*\widetilde{u}_t, \quad (\mathcal{A}2.62)$$

$$\widetilde{Y}_t = \psi\left(\widetilde{u}_t + \widetilde{K}_t\right) + (1-\psi)\frac{N^*}{N^* - N_0}\widetilde{N}_t - \psi\varepsilon_t, \qquad (\mathcal{A}2.63)$$

and

$$\widetilde{m}_{t+1} \ = \ \rho_m \widetilde{m}_t + \mu_t, \qquad (\mathcal{A}2.64)$$

for NKDSGE-MG rule models, or for NKDSGE-TR models the interest rate rule

$$(1-\rho_R\mathbf{L})\widetilde{R}_t \ = \ (1-\rho_R)\left(a_\pi \mathbf{E}_t\widetilde{\pi}_{t+1} + a_\pi\widetilde{m}_{t+1} + a_y\widetilde{Y}_t\right) \ + \ \upsilon_t, \qquad (\mathcal{A}2.65)$$

where $\varrho \equiv \dfrac{a''(1)}{a'(1)}$ (= 1.174) and $\widetilde{\pi}_t \equiv \widetilde{P}_t - \widetilde{P}_{t-1}$. The linear approximate habit NKDSGE-TR model consists of the linear stochastic difference equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and ($\mathcal{A}$2.65) with the unknowns $\widetilde{\lambda}_t$, $\widetilde{C}_t$, $\widetilde{X}_t$, $\widetilde{q}_t$, $\widetilde{Y}_t$, $\widetilde{K}_{t+1}$, $\widetilde{R}_t$, $\widetilde{u}_t$, $\widetilde{\phi}_t$, $\widetilde{N}_t$, $\widetilde{P}_t$, and $\widetilde{W}_t$. When the AR(1) money growth rule ($\mathcal{A}$2.64) replaces the Taylor rule ($\mathcal{A}$2.65) in the system of linear stochastic difference equations that approximate the baseline habit NKDSGE-MG model, the linearized detrended bond Euler equation ($\mathcal{A}$2.55) can be dropped along with the demeaned nominal rate $\widetilde{R}_t$.

A2.5 Solving the baseline habit NKDSGE models

This section describes the solution method we apply to solve the linear stochastic difference equations that approximate the NKDSGE models. Consider the baseline habit NKDSGE-TR model that employs the monetary policy rule ($\mathcal{A}$2.65). For this model, the vector of endogenous variables is

$$H_t = \left[\widetilde{\lambda}_t \ \ \widetilde{C}_t \ \ \widetilde{X}_t \ \ \widetilde{q}_t \ \ \widetilde{Y}_t \ \ \widetilde{K}_{t+1} \ \ \widetilde{R}_t \ \ \widetilde{u}_t \ \ \widetilde{\phi}_t \ \ \widetilde{N}_t \ \ \widetilde{P}_t \ \ \widetilde{W}_t \ \ \widetilde{m}_{t+1}\right]'.$$

Next define the expectational forecast errors $\vartheta_{\widetilde{\lambda},t+1} = \widetilde{\lambda}_{t+1} - E_t\widetilde{\lambda}_{t+1}$, $\vartheta_{\widetilde{C},t+1} = \widetilde{C}_{t+1} - E_t\widetilde{C}_{t+1}$, $\vartheta_{\widetilde{X},t} = \widetilde{X}_{t+1} - E_t\widetilde{X}_{t+1}$, $\vartheta_{\widetilde{q},t+1} = \widetilde{q}_{t+1} - E_t\widetilde{q}_{t+1}$, $\vartheta_{\widetilde{Y},t+1} = \widetilde{Y}_{t+1} - E_t\widetilde{Y}_{t+1}$, $\vartheta_{\widetilde{u},t+1} = \widetilde{u}_{t+1} - E_t\widetilde{u}_{t+1}$, $\vartheta_{\widetilde{\phi},t+1} = \widetilde{\phi}_{t+1} - E_t\widetilde{\phi}_{t+1}$, $\vartheta_{\widetilde{P},t} = \widetilde{P}_{t+1} - E_t\widetilde{P}_{t+1}$, and $\vartheta_{\widetilde{W},t+1} = \widetilde{W}_{t+1} - E_t\widetilde{W}_{t+1}$. Collect these forecast errors into the vector ϑ_{t+1}. We use H_t and ϑ_t, the linear approximate optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and the Taylor rule ($\mathcal{A}$2.65) to form the multivariate first-order stochastic difference equation system of the baseline habit NKDSGE-TR model

$$\mathbb{G}_0\mathbb{H}_t = \mathbb{G}_1\mathbb{H}_{t-1} + \mathbb{V}\zeta_t + \mathbb{K}\vartheta_t, \qquad (\mathcal{A}2.66)$$

where $\mathbb{H}_t = [H_t \;\; \mathbf{E}_t H_{t+1}]'$ and $\zeta_t = [\varepsilon_t \;\; \upsilon_t]'$ (or when monetary policy is defined by the AR(1) money growth rule ($\mathcal{A}$2.64), $\zeta_t = [\varepsilon_t \;\; \mu_t]'$). It is understood that $\mathbf{E}_t H_{t+1}$ contains only those elements of H_t that enter equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) as one-step ahead expectations. The matrices $\mathbb{G}_0$, $\mathbb{G}_1$, and $\mathbb{V}$ contain cross-equation restriction embedded in the optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63), and the Taylor rule ($\mathcal{A}$2.65).

Sims (2002) studies and solves multivariate linear rational expectations models that match ($\mathcal{A}$2.66). His solution algorithm taps the QZ (or generalized complex Schur) decomposition of matrices $\mathbb{G}_0$ and $\mathbb{G}_1$. The QZ decomposition employs $\mathcal{Q}'\mathcal{F}\mathcal{Z}' = \mathbb{G}_0$ and $\mathcal{Q}'\mathcal{O}\mathcal{Z}' = \mathbb{G}_1$, where $\mathcal{Q}'\mathcal{Q} = \mathcal{Z}'\mathcal{Z} = \mathbf{I}$ and matrices $\mathcal{F}$ and $\mathcal{O}$ are upper triangular. Matrices $\mathcal{Q}$, $\mathcal{Z}$, $\mathcal{F}$ and $\mathcal{O}$ are possibly complex. Let $\mathcal{D}_t = \mathcal{Z}'\mathbb{H}_t$ and premultiply equation ($\mathcal{A}$2.66) by $\mathcal{Q}$ to obtain

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathbf{0} & \mathcal{F}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} \\ \mathbf{0} & \mathcal{O}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \mathcal{Q}_{1\cdot} \\ \mathcal{Q}_{2\cdot} \end{bmatrix} \left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \qquad (\mathcal{A}2.67)$$

where $\mathcal{Q}_{j\cdot}$ denotes the jth block of rows of $\mathcal{Q}$. Although the QZ decomposition of $\mathbb{G}_0$ and $\mathbb{G}_1$ never fails to exist, these decompositions are not unique. Nonetheless, generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ can be unique if infinite values are allowed and zero eigenvalues for $\mathbb{G}_0$ and $\mathbb{G}_1$ are ruled out. Denote the generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ as $f_{ii}^{-1}o_{ii}$. These eigenvalues are ordered to partition the system ($\mathcal{A}$2.67) in such a way to place only explosive elements in $\mathcal{D}_{2,t}$. The 'reduced form' process of $\mathcal{D}_{2,t}$ is the second row of the system ($\mathcal{A}$2.67), which is written

$$\mathcal{D}_{2,t} = \mathcal{M}\mathcal{D}_{2,t-1} + \mathcal{M}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \tag{$\mathcal{A}$2.68}$$

where $\mathcal{M} \equiv \mathcal{F}_{22}^{-1}\mathcal{O}_{22}$. Forward iteration of equation ($\mathcal{A}$2.68) gives

$$\mathcal{D}_{2,t} = -\sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right), \tag{$\mathcal{A}$2.69}$$

where the transversality condition $\mathcal{M}^{-i}\mathcal{D}_{2,t+i}$, $i \longrightarrow \infty$ holds.

Extrinsic or sunspot equilibria are excluded from the solution of the present value ($\mathcal{A}$2.69) of $\mathcal{D}_{2,t}$. The present value invokes a no sunspot result because the expectation error vector ϑ_t has no impact on $\mathcal{D}_{1,t}$ and $\mathcal{D}_{2,t}$. The implications is that $\mathcal{D}_{2,t}$ belongs only to the date t information set (*i.e.*, it includes only the intrinsic shocks of ζ_t), which mean that

$$\mathbf{E}_t \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} = \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right),$$

For an intrinsic equilibrium to exist, Sims (2002) shows that the necessary and sufficient conditions are that the set of equations $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+1} + \mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_{t+1}$ equal a column vector of zeros. A solution is available for the multivariate first order system ($\mathcal{A}$2.66) if (and only if) the column space of $Q_{2\cdot}\mathbb{V}$ is contained in that of $Q_{2\cdot}\mathbb{K}$. Given ζ_t is uncorrelated, the solution follows immediately. This is not true for the NKDSGE-MG models. When the intrinsic shocks are serially correlated, $\mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_t$ is calculated from information in $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_t$.

Suppose that an intrinsic solution exists. When there is no sunspot equilibria, the row space of $\mathcal{Q}_{1\cdot}\mathbb{K}$ is contained in that of $\mathcal{Q}_{2\cdot}\mathbb{K}$. This is a necessary and sufficient condition for uniqueness of the solution of the linear approximate system ($\mathcal{A}$2.66), as Sims (2002) shows. He suggests working with a matrix Φ that yields $\mathcal{Q}_{1\cdot}\mathbb{K} = \Phi Q_{2\cdot}\mathbb{K}$. By premultiplying equation ($\mathcal{A}$2.67) with $[\mathbf{I} \quad -\Phi]$, combining this with equation ($\mathcal{A}$2.68), and noting that this wipes out the expectational forecast errors ϑ_t, we have

$$\mathcal{F}_{11}\mathcal{D}_{1,t} + (\mathcal{F}_{12} - \Phi\mathcal{F}_{22})\mathcal{D}_{2,t} = \mathcal{O}_{11}\mathcal{D}_{1,t-1} + (\mathcal{O}_{12} - \Phi\mathcal{O}_{22})\mathcal{D}_{2,t-1} + (Q_{1\cdot} - \Phi Q_{2\cdot})\mathbb{V}\zeta_t.$$

Stacking these equations on top of the equations of ($\mathcal{A}$2.69) produces

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix} \mathbb{V}\zeta_t + \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_t \sum_{i=0}^{\infty} \mathcal{M}^{-i} \mathcal{O}_{22}^{-1} \mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} \end{bmatrix}.$$

This matrix system maps into the unique intrinsic solution for $\mathbb{H}_t$

$$\mathbb{H}_t = \Theta_{\mathbb{H}}\mathbb{H}_{t-1} + \Theta_\zeta \zeta_t, \qquad (\mathcal{A}2.70)$$

where

$$\Theta_{\mathbb{H}} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathcal{Z}'$$

and

$$\Theta_{\zeta} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix}.$$

We engage the system of first-order stochastic difference equations ($\mathcal{A}$2.70) to produce linear approximate solutions for the NKDSGE models. These solutions generate synthetic data sets that are inputs into our Bayesian simulation experiments.

A3. Estimating SVMAs, Checking Their ABC and Ds, and Spectral Density Computation

This section fills in a few gaps about the methods used to evaluate the NKDSGE models. We review the Blanchard and Quah (1989) decomposition and apply it to vector autoregressions (VARs) of output growth (or consumption growth) and inflation. These VARs are identified with a long-run monetary neutrality (LRMN) restriction that the level of output or consumption is independent of monetary shocks at $t \longrightarrow \infty$. The LRMN restriction yields SVMA(∞), processes of output (or consumption growth) and inflation. We show that it retrieves the TFP and monetary policy shock innovations of the NKDSGE models as in the ABCs and Ds of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). The SVMA(∞) also provides a map to permanent and transitory output and consumption growth spectral densities, $SD_{\Delta Y}$ and $SD_{\Delta C}$. This section ends with a review of several methods available to compute these $SD_{\Delta Y}$ and $SD_{\Delta C}$.

A3.1 VARs and SVMAs

The SVMAs are constructed from a VAR of $\mathcal{X}_t = [\Delta \ln Y_t \;\; \Delta \ln P_t]'$ or $[\Delta \ln C_t \;\; \Delta \ln P_t]'$ and the LRMN restriction using the Blanchard and Quah (1989) decomposition. The unrestricted joint probability distribution of $\mathcal{X}_t$ is approximated by the finite-order VAR

$$\mathcal{X}_t = \mathbb{A}(\mathbf{L})\mathcal{X}_{t-1} + e_t, \quad \mathbb{A}(\mathbf{L}) = \sum_{j=1}^{p} \mathbb{A}_j \mathbf{L}^j, \tag{$\mathcal{A}$3.1}$$

where constants are ignored, the forecast errors $e_t = \mathcal{X}_t - \mathbf{E}\left\{\mathcal{X}_t \middle| \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}\right\}$ are Gaussian, and its covariance matrix is Σ. We set $p = 2$ in sample estimation and for the Bayesian Monte Carlo experiments, but below we report results for $p = 4$.

The unrestricted VAR of ($\mathcal{A}$3.1) is invertible whether estimated or under the NKDSGE models. Inverting this VAR yields the reduced form VMA(∞), $\mathcal{X}_t = \left[\mathbf{I} - \mathbb{A}(\mathbf{L})\right]^{-1} e_t$, or Wold representation of $\mathcal{X}_t$, $\mathbb{C}(\mathbf{L})e_t$, where $\mathbb{C}(\mathbf{L}) = \sum_{i=0}^{\infty} \mathbb{C}_i \mathbf{L}^i$ and the reduced form impact matrix $\mathbb{C}_0 = \mathbf{I}$. The corresponding SVMA(∞) is

$$\mathcal{X}_t = \mathbb{B}(\mathbf{L})\varsigma_t, \quad \varsigma_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right), \tag{$\mathcal{A}$3.2}$$

which summarizes equation (10) of the paper. The NKDSGE models predict that in the long run the levels of output and consumption are independent of monetary policy innovations (*i.e.*, the money growth rule innovation μ_t or Taylor rule innovation υ_t). This is the LRMN restriction, which forces the upper right element of $\mathbb{B}(\mathbf{1})$ to be zero, or $\sum_{j=0}^{\infty} \mathbb{B}_{j,1,2} = \mathbb{B}(\mathbf{1})_{1,2} = 0$. The SVMA ($\mathcal{A}$3.2) and the reduced form VMA(∞) also force $e_t = \mathbb{B}_0\varsigma_t$ and $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$. Note that once estimates of the four unknown elements of the structural impact response matrix $\mathbb{B}_0$ are available, we can compute the SVMA of ($\mathcal{A}$3.2) from the reduced form VMA(∞).

Our goal is to recover the four unknown coefficients of $\mathbb{B}_0$. The map from the structural shocks to the reduced form errors, $e_t = \mathbb{B}_0\varsigma_t$, and the covariances matrices of e_t and ς_t place three restrictions on the four unknowns of $\mathbb{B}_0$. These three restrictions present us with three nonlinear equations that follow from expanding $\Sigma = \mathbb{B}_0\mathbb{B}_0'$ to

$$\begin{aligned}
\Sigma_{1,1} &= \mathbb{B}_{0,1,1}^2 + \mathbb{B}_{0,1,2}^2, \\
\Sigma_{1,2} &= \mathbb{B}_{0,1,1}\mathbb{B}_{0,2,1} + \mathbb{B}_{0,1,2}\mathbb{B}_{0,2,2}, \\
\Sigma_{2,2} &= \mathbb{B}_{0,2,1}^2 + \mathbb{B}_{0,2,2}^2.
\end{aligned} \tag{$\mathcal{A}$3.3}$$

The remaining restriction is found by summing both sides of $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$ from $j \geq 0$, which leads to $\mathbb{B}(\mathbf{1}) = \mathbb{C}(\mathbf{1})\mathbb{B}_0$. The LRMN restriction imposes

$$\mathbb{C}(\mathbf{1})_{1,1}\mathbb{B}_{0,1,2} + \mathbb{C}(\mathbf{1})_{1,2}\mathbb{B}_{0,2,2} = 0, \tag{$\mathcal{A}$3.4}$$

which is a fourth nonlinear equation. We solve the four nonlinear equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4) to calculate estimates of the four unknown coefficients of $\mathbb{B}_0$.

Markov chain Monte Carlo (MCMC) simulations of the SVMA(∞) of equation ($\mathcal{A}$3.2) engage the BACC software of Geweke (1999) and McCausland (2004). The MCMC simulators need priors that are obtained, only in part, from ordinary least squares (OLS) estimates of the reduced form VAR(2) of equation ($\mathcal{A}$3.1). These estimates and related covariance matrices are the prior information used to generate $\mathcal{J}$ (= 5,000) posterior draws of the reduced form VAR(2) coefficients. Next we calculate the reduced form VMA(∞) and apply the BQ decomposition by imposing the LRMN restriction to recover the SVMA(∞) of equation ($\mathcal{A}$3.2). The $\mathcal{J}$ samples of the $\mathbb{B}(\mathbf{L})$s are the basis of the empirical distributions of the permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. The theoretical, $\mathcal{T}$, distributions of these moments are estimated in the same manner, but on synthetic samples generated by NKDSGE models.

We treat synthetic samples generated by MCMC simulations of the unrestricted VAR ($\mathcal{A}$3.1) and the NKDSGE models in the same way, with one caveat. The exception is that although the off diagonal elements of the NKDSGE model structural shock covariance matrix $\Xi = \mathbf{E}\left\{\zeta_t \zeta_t'\right\}$ are zero, its diagonal elements are not unity. The NKDSGE model SVMAs are normalized for the Blanchard and Quah (BQ) decomposition with a correction that relies on the Choleski decomposition of Ξ, $\Xi^{1/2}$. Given $\mathbb{D}(\mathbf{L})$ is the infinite-order lag polynomial matrix of the theoretical SVMA(∞), the normalization is $\mathbb{D}(\mathbf{L})\ \Xi^{1/2}$. This normalization is imposed by the BQ decomposition on the ensemble of $\mathcal{J}$ theoretical SVMAs that are created from synthetic time series of length $\mathcal{M} \times T$ obtained from Bayesian simulations of the NKDSGE models.

A3.2 The ABCs and Ds of the NKDSGE models, LRMN, and SVARs

This section shows how the SVMA(∞) of equation ($\mathcal{A}$3.2) retrieves the economic shocks of a NKDSGE model. This involves restating a result from Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). They study a condition that equates the shocks identified by an econometric model to those of a DSGE model. We exploit their condition to tie the shocks of a structural VAR (SVAR) identified by LRMN to the NKDSGE model shocks ζ_t.

Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (FVRRSW) construct a VAR(∞) driven by DSGE model shocks to expose the condition that links these shocks to those identified by a SVAR(∞). The baseline habit NKDSGE Taylor rule model yields the VAR(∞)

$$\mathcal{X}_t = \Gamma_{\mathbb{H}} \sum_{j=0}^{\infty} \left[\Theta_{\mathbb{H}} - \Theta_{\zeta}\Gamma_{\zeta}^{-1}\Gamma_{\mathbb{H}}\right]^j \Theta_{\zeta}\Gamma_{\zeta}^{-1}\mathcal{X}_{t-j-1} + \Gamma_{\zeta}\zeta_t, \qquad (\mathcal{A}3.5)$$

which combines the equilibrium law of motion ($\mathcal{A}$2.70), the system

$$\mathcal{X}_t = \Gamma_{\mathbb{H}}\mathbb{H}_{t-1} + \Gamma_{\zeta}\zeta_t, \qquad (\mathcal{A}3.6)$$

that relates the observables of $\mathcal{X}_t$ to $\mathbb{H}_t$ and ζ_t, and several steps described by FVRRSW. Note that Γ_ζ is square and its inverse is taken to exist. FVRRSW also examine

$$\mathbb{H}_t = \sum_{j=0}^{\infty} \left[\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}\right]^j \Theta_\zeta \Gamma_\zeta^{-1} \mathcal{X}_{t-j}, \qquad (\mathcal{A}3.7)$$

which results from passing Γ_ζ^{-1} through equation ($\mathcal{A}$3.6), substituting it into the equilibrium law of motion ($\mathcal{A}$2.70), and rearranging terms. Equation ($\mathcal{A}$3.7) recovers the state vector $\mathbb{H}_t$ from the history of $\mathcal{X}_{t-j}$, which consists of observed variables (*i.e.*, there are no latent state variables), if (and only if) the eigenvalues of $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$ are strictly less than one in modulus. This is the condition FVRRSW require to equate shocks identified by a SVAR to the NKDSGE model shocks ζ_t. Given the FVRRSW condition is satisfied by $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$, the coefficients of the lag polynomial implied by $\left[\mathbf{I} - \left(\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}\right) \mathbf{L}\right]$ also fulfill the needs of square summability. By also assuming that $\Gamma_\zeta \zeta_t$ is orthogonal to $\mathcal{X}_{t-j-1}$ (j = 0, 1, ..., ∞), equation ($\mathcal{A}$3.5) can be interpreted as the theoretical VAR(∞) of $\mathcal{X}_t$.

We rely on LRMN for identification of the SVMA(∞) of equation ($\mathcal{A}$3.2). This complicates the problem of using the FVRRSW condition to connect NKDSGE model shock innovations ζ_t to innovations identified by an econometric model. A solution is to exploit an approach of King and Watson (1997) that imposes the LRMN restriction on the SVAR(∞)

$$\begin{bmatrix} 1 & -\Lambda_{\Delta Y,\Delta P,0} \\ -\Lambda_{\Delta P,\Delta Y,0} & 1 \end{bmatrix} \mathcal{X}_t = \begin{bmatrix} \Lambda_{\Delta Y,\Delta Y}(\mathbf{L}) & \Lambda_{\Delta Y,\Delta P}(\mathbf{L}) \\ \Lambda_{\Delta P,\Delta Y}(\mathbf{L}) & \Lambda_{\Delta P,\Delta P}(\mathbf{L}) \end{bmatrix} \mathcal{X}_{t-1} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \qquad (\mathcal{A}3.8)$$

where the impact matrix Λ_0 is nonsingular, $\Lambda(\mathbf{L})$ summarizes the lag polynomial attached to $\mathcal{X}_{t-1}$, $\eta_t = \left[\eta_{1,t} \;\; \eta_{2,t}\right]'$, Ω is the diagonal covariance matrix of $\mathbf{E}\left\{\eta_t \eta_t'\right\}$, $\mathbf{E}\eta_t = \mathbf{0}$, and $\mathbf{E}\left\{\eta_t \eta_{t-i}'\right\}$ $= \mathbf{0}$, for all non-zero i.

King and Watson (1997) are interested in identifying and estimating SVARs with impact and long run restrictions. We focus on the latter type of restriction to identify the SVAR of ($\mathcal{A}$3.8) with LRMN. The identification relies on the response of the level of output to a permanent change in the nominal shock $\eta_{2,t}$, which is

$$\mathcal{L}_{\Delta Y,\Delta P} = \frac{\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})}{1 - \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})}.$$

This ratio is zero when LRMN holds because it measures the long run response of output to a monetary shock. Following King and Watson, the LRMN restriction is imposed on the structural VAR of ($\mathcal{A}$3.8) by rewriting its top equation as

$$\Delta \ln Y_t = [\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})]\Delta \ln P_t + \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1}$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

where, for example, $\Psi_{\Delta Y,\Delta P,i} = -\sum_{s=i+1}^{\infty} \Lambda_{\Delta Y,\Delta P,s}$. Next, multiply and divide the first term after the equality by $\mathcal{L}_{\Delta Y,\Delta P}$ to produce

$$\Delta \ln Y_t - \mathcal{L}_{\Delta Y,\Delta P}\Delta \ln P_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})[\Delta \ln Y_{t-1} - \mathcal{L}_{\Delta Y,\Delta P}\Delta \ln P_t]$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

or under LRMN

$$\Delta \ln Y_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1} + \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t}.$$

The previous equation and the bottom equation of ($\mathcal{A}$3.8) form a just-identified SVAR from which $\eta_{1,t}$ and $\eta_{2,t}$ can be computed. An estimator of these shocks does not rely on identifying either impact coefficient $\Lambda_{\Delta Y,\Delta P,0}$ or $\Lambda_{\Delta P,\Delta Y,0}$. Rather the former coefficient is obtained from $\mathcal{L}_{\Delta Y,\Delta P,0} = 0$ given $\Lambda_{\Delta Y,\Delta Y}(\mathbf{1})$, $\Psi_{\Delta Y,\Delta Y}(\mathbf{L})$ and $\Psi_{\Delta Y,\Delta P}(\mathbf{L})$, while the latter coefficient is obtained from the bottom equation of ($\mathcal{A}$3.8). King and Watson (1997) use an instrumental variable (IV) estimator with $\eta_{1,t}$ serving as the additional instrument. Instead of the IV estimator, we apply the BQ decomposition, equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4), to synthetic samples of $\mathcal{X}_t$, rather than estimate SVAR(∞)s.

The FVRRSW condition enables us to match the shocks of the SVAR of ($\mathcal{A}$3.8) with the NKDSGE shocks ζ_t. This SVAR implies the reduced form VAR(∞)

$$\mathcal{X}_t \;=\; \mathbb{S}(\mathbf{L})\mathcal{X}_{t-1} + \nu_t, \quad \mathbb{S}(\mathbf{L}) = \sum_{j=1}^{\infty} \mathbb{S}_j \mathbf{L}^j, \tag{$\mathcal{A}$3.9}$$

is associated with the SVAR of ($\mathcal{A}$3.8), where $\nu_t = \mathcal{X}_t - \mathbf{E}\Big\{\mathcal{X}_t \Big| \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}, \ldots\Big\}$, $\mathbb{S}(\mathbf{L}) = \Lambda_0^{-1}\,\Lambda(\mathbf{L})$, and $\nu_t = \Lambda_0^{-1}\,\eta_t$. Equation ($\mathcal{A}$3.9) serves to represent the VAR(∞) of $\mathcal{X}_t$ when the sum from $i = 1, \ldots, \infty$ of $\mathbb{S}^2_{\Delta Y,\Delta Y,i} + \mathbb{S}^2_{\Delta Y,\Delta P,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i}$ is finite and the orthogonality condition $\mathbf{E}\Big\{\nu_t \mathcal{X}'_{t-j}\Big\} = \mathbf{0}$, holds for all $j \geq 1$. We can acquire shocks from this reduced form VAR that match those of the baseline habit NKDSGE-TR model when $\Lambda_0 \nu_t = \Gamma_\zeta \zeta_t$. FVRRSW show that the equality links the econometric and NKDSGE model shocks if (and only if) the eigenvalues of $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$ are strictly less than one in modulus.

The FVRRSW restriction is checked at each of the $\mathcal{J} = 5000$ replications of the Bayesian simulations of the 12 NKDSGE models. The simulations reveal that the NKDSGE models satisfy the FVRRSW restriction on the eigenvalues $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$ at all $\mathcal{J}$ replications. Thus, the theoretical SVMA(∞)s estimated on synthetic data always recover the economic shocks of the 12 NKDSGE models.

A3.3 Computing permanent and transitory spectral densities

In Section 3.2, the paper presents the map from the SVMA(∞) of equation ($\mathcal{A}$3.2) to permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$. We reproduce the SD (at frequency ω) that appears at the end of section 3.2 here as

$$SD_{\Delta Y,\iota}(\omega) = \frac{1}{2\pi}\sum_{j=0}^{40}\left|\mathbb{B}_{\Delta Y,\iota,j}e^{-ij\omega}\right|^2, \quad \iota = \varepsilon,\, \upsilon, \qquad (\mathcal{A}3.10)$$

where it is understood that the Bayesian simulations of the NKDSGE models account for the non-unit diagonal elements of Ξ. Although we calculate the permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$ using ($\mathcal{A}$3.10), there are (at least) two other methods available to compute these moments. First, the $SD(\omega)$ can be represented as

$$SD_{\Delta Y,\iota}(\omega) = \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi}\sum_{j=0}^{40}\left|\frac{\mathbb{B}_{\Delta Y,\iota,j}}{\mathbb{B}_{\Delta Y,\iota,0}}e^{-ij\omega}\right|^2,$$

which leads to the factorization

$$1 + \frac{\mathbb{B}_{\Delta Y,\iota,1}}{\mathbb{B}_{\Delta Y,\iota,0}}z + \frac{\mathbb{B}_{\Delta Y,\iota,2}}{\mathbb{B}_{\Delta Y,\iota,0}}z^2 + \ldots + \frac{\mathbb{B}_{\Delta Y,\iota,40}}{\mathbb{B}_{\Delta Y,\iota,0}}z^{40} \;=\; (1-\chi_{\iota,1}z)(1-\chi_{\iota,2}z)\cdots(1-\chi_{\iota,40}z),$$

in terms of the eigenvalues, the $\chi_{\iota,h}$s, of the MA(40) process of output growth with respect to the NKDSGE shocks ε, μ, or υ. The eigenvalue factorization gives

$$SD_{\Delta Y,\iota}(\omega) \;=\; \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi}\prod_{j=1}^{40}\left[1 + \chi^2_{\iota,j} - 2\chi_{\iota,j}\cos(\omega)\right],$$

which provides a third method to compute $SD(\omega)$s.

A4. Additional NKDSGE model evaluation

Our paper grounds its evaluation of 12 NKDSGE models on the minimal econometric interpretation (MEI) of Geweke (2010). The MEI is useful to judge the fit of a NKDSGE model because no pretense is made that it provides a complete description of economic behavior. According to Geweke, a NKDSGE model is incomplete because it fails to produce densities for sample moments and the predictive density of the sample data.

This section presents more information about the NKDSGE model evaluation presented in section 4 of the paper. These are figures 3.A, 3.B, 4.A, 4.B, 5.A, and 5.B. These figures are numbered to match figures 3–5 of the paper. That is, figures 3.A and 3.B contain results for the baseline NKDSGE models, figures 4.A and 3.B contain results for the sticky price (SPrice) NKDSGE models, and figures 5.A and 5.B contain results for the sticky wage (SWage) NKDSGE models. The A denotes results for NKDSGE-MGR models while the NKDSGE-TR models are represented with B.

Figures 3.A–5.B are laid out from the top row to the bottom containing results for permanent $SD_{\Delta Y}$, transitory $SD_{\Delta Y}$, permanent $SD_{\Delta C}$, and transitory $SD_{\Delta C}$. Mean permanent and transitory $SD_{\mathcal{P},\Delta Y}$, $SD_{\mathcal{P},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$ appear in the first column of figures 3.A–5.B. The second (third) column of figures 3.A–5.B contain densities of KS statistics computed using the entire spectrum (constrained to eight to two years per cycle). We denote mean $SD_{\mathcal{P}}$s and $KS_{\mathcal{P}}$ statistic densities with (blue) solid lines, mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities generated by non-habit NKDSGE models with (green) dashed lines, and mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities created by habit NKDSGE models with (red) dot-dash lines in figures 3.A–5.B. Goodness of fit statistic densities appear with associated CIC in figures 3.A–5.B.

We also discuss five more Bayesian Monte Carlo experiments that are grounded on the MEI in this section. To review, we engage the MEI to evaluate 12 NKDSGE models on prior and posterior population moments, permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$, that are functions of actual observable data. By drawing from priors of parameters of a NKDSGE model, its linearized version produces prior population *SD*s from SVMA(∞)s estimated on synthetic samples of length $\mathcal{M}$ (= $T \times \mathcal{W}$). We label these posterior moments theoretical *SD*s, or $SD_{\mathcal{T}}$s. The same SVMA(∞)s are used to build posterior *SD*s, tagged as empirical *SD*s or $SD_{\mathcal{P}}$s, on synthetic samples of length T generated by MCMC simulators. Actual data, unrestricted VAR(2)s, and priors of these models are the conditioning information on which the $SD_{\mathcal{P}}$s are built.

We offer the five new Bayesian Monte Carlo experiments to check the robustness of the evaluation of the 12 NKDSGE models conducted by the paper. The first of these experiments replaces the prior of the habit parameter, $h \sim U(0.05,\ 0.95)$, with a prior drawn from the β distribution, $h \sim \beta(0.65, 0.15)$. Next, we break the prior $h \sim U(0.05,\ 0.95)$ in half to conduct two experiments. One set of simulations condition on the prior $h \sim U(0.50,\ 0.95)$, while the other set relies on the prior $h \sim U(0.05,\ 0.499)$. The fourth experiment retains the original priors, including $h \sim U(0.05,\ 0.95)$, but uses VAR(4)s, rather than VAR(2)s, to construct the SVMA(∞)s. In the final experiment, we return to the structure of the Bayesian Monte Carlo experiments presented in the paper except that the *CIC* are calculated using distributions of Cramer-von Mises (*CvM*) goodness of fit statistics instead of Kolmogorov-Smirnov (*KS*) statistics.

Results of the five additional NKDSGE model evaluation exercises appear in tables A1–A5 and figures A1–A30. Tables A1–A4 contain *CIC* that measure the overlap of $KS_{\mathcal{P}}$ and $KS_{\mathcal{T}}$ distributions generated by Bayesian Monte Carlo experiments employing the prior $h \sim \beta(0.65, 0.15)$, the prior $h \sim U(0.50,\ 0.95)$, the prior $h \sim U(0.050,\ 0.499)$, and switching from VAR(2)s to VAR(4)s, respectively. The source of *CIC* reported in table A5 are densities of *CvM* statistics constructed from distributions of permanent and transitory $SD_{\mathcal{P},\Delta Y}$, $SD_{\mathcal{P},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$.

Figures A1–A30 are laid out in the fashion as figures 3.A–5.B of the paper. From top to bottom, the rows of figures A1-A30 list results for permanent $SD_{\Delta Y}$, transitory $SD_{\Delta Y}$, permanent $SD_{\Delta C}$, and transitory $SD_{\Delta C}$. Mean permanent and transitory $SD_{\mathcal{P},\Delta Y}$, $SD_{\mathcal{P},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$ appear in the first column of figures A1–A30. The second (third) column of figures A1–

A24 contain densities of KS statistics computed using the entire spectrum (constrained to eight to two years per cycle). We denote mean $SD_{\mathcal{P}}$s and $KS_{\mathcal{P}}$ statistic densities with (blue) solid lines, mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities generated by non-habit NKDSGE models with (green) dashed lines, and mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities created by habit NKDSGE models with (red) dot-dash lines in figures A1–A24. Densities of CvM statistics are displayed in the second and third columns of figures A25-A30 using the same scheme. Goodness of fit statistic densities appear with associated CIC in figures A1–A30.

A4.0 Preliminary NKDSGE model fit: Prior predictive analysis

Before reviewing the five Bayesian Monte Carlo experiments, this section presents a prior predictive analysis of the 12 NKDSGE models. Our prior predictive analysis asks if a NKDSGE model can account for sample innovation variances of VAR(2)s estimated on output growth and inflation and consumption growth and inflation data that starts in $1955Q1$ and ends with $2002Q4$, $T = 196$. The data is described in section A0. Draws from priors of NKDSGE model parameters and linearized NKDSGE models generate synthetic samples of output growth, consumption growth, and inflation of length T on which equivalent VARs are estimated to extract $\mathcal{J} = 5000$ pairs of output growth and consumption growth regression forecast innovation variances. These VAR innovation variances form the prior distributions of interest.

Scatter plots of the prior distributions are reported in the three rows and four columns of figure A0. The baseline, SPrice, and SWage versions of these models appear in the rows of figure A0 from top to bottom. From left to right, the columns contain results for non-habit NKDSGE-MGR, habit NKDSGE-MGR, non-habit NKDSGE-TR, and habit NKDSGE-TR models. The 12 scatter plots of figure A0 place the innovation variance of the consumption (output) growth regression on the horizontal (vertical) axes. The symbol "+" in these plots denote the combination of sample innovation variance estimates obtained from output and consumption growth regressions. In each scatter plot of figure A0, clouds of points represent prior distributions of artificial innovation variances.

Figure A0 shows that non-habit NKDSGE models fail to explain sample innovation variances of the output and consumption growth regressions. Neither baseline, SPrice, nor SWage non-habit NKDSGE models produce prior distributions of synthetic innovation variances in the first and third columns of figure A0 that cover the plus sign, "+", that symbolizes the intersection of the sample shock innovations. This is preliminary evidence that non-habit NKDSGE models cannot describe fluctuations in U.S. output and consumption growth data on the $1955Q1$–$2002Q4$ sample.

The habit NKDSGE model are better able to explain the sample innovation variances of the output and consumption growth regression. The second and fourth columns of figure A0 present clouds of prior distributions of shock innovation variances that blanket the sample innovation variances. This result holds for baseline, SPrice, and SWage habit NKDSGE models. Thus, NKDSGE models find it useful to include consumption habit to explain sample output and consumption growth innovation variances.

A4.1 More evidence about NKDSGE model fit

The first column of figures 3.A–5.B include the mean dynamics of the prior and posterior *SD*s that are reported in figures 3–5 of the paper. The second and third columns of figures 3.A–5.B give information about the prior *KS* statistics distributions of the habit and non-habit NKDSGE models compared to the posterior *KS* statistic distributions. The overlap of the prior and posterior distributions of the *KS* statistics confirm the *CIC* of table 2 of the paper. When a $CIC > 0.3$ in table 2, the associated prior *KS* statistic distributions exhibits substantial overlap with the posterior *KS* statistic distributions.

A4.2 NKDSGE model fit under the prior $h \sim \beta(0.65,\ 0.15)$

The uniform prior for the consumption habit parameter, $h \sim U(0.05,\ 0.95)$, only utilizes information about the theoretical restriction that h takes values on the open interval between zero and one. We replace this uninformative prior for h with a β prior informed by evidence from previous DSGE model studies, $h \sim \beta(0.65, 0.15)$. The β prior gives h a mean of 0.65, a standard deviation of 0.15, and a 95 percent coverage interval of [0.3842, 0.8765]. This calibration focuses on estimates of Christiano, Eichenbaum, and Evans (2005) and also covers values of h found in Boldrin, Christiano, and Fisher (2001) and Francis and Ramey (2005), among others. The non-habit NKDSGE models remain defined by the degenerate prior $h = 0$.

Table A1 reports *CIC* generated from Bayesian Monte Carlo experiments of the NKDSGE models given the β prior for h. We include density plots of *KS* statistic distributions based on distributions of permanent and transitory $SD_{\mathcal{P},\Delta Y}$, $SD_{\mathcal{P},\Delta C}$, $SD_{\mathcal{P},\Delta Y}$, and $SD_{\mathcal{P},\Delta C}$ in figures A1–A6. The *CIC* and *KS* statistic densities indicate that the β prior for h produces only minimal changes in NKDSGE model fit compared to *CIC* found in table 2. Figures A1–A6 reinforce this conclusion.

A4.3 NKDSGE model fit under the prior $h \sim U(0.50, 0.95)$

Bounding the prior of h from below at 0.5 yields one important change in the evaluation of the 12 NKDSGE models discussed in the paper. Although the prior $h \sim U(0.50, 0.95)$ is uninformative, it eliminates values of h that suggest weaker consumption habit induced propagation and monetary transmission. With only the prior $h \sim U(0.50, 0.95)$ different, the top half of table A3 shows that habit NKDSGE-MGR models achieve six more $CIC \geq 0.3$ compared to results found in the top half of table 2. These additional matches are mostly made by baseline and SWage habit NKDSGE-MGR models to distributions of transitory $SD_{\mathcal{P},\Delta Y}$ and $SD_{\mathcal{P},\Delta C}$ when fit is restricted to eight to two years per cycle. The SPrice habit NKDSGE-TR model generates an additional $CIC \geq 0.3$ in the bottom half of table A3 when drawing from the prior $h \sim U(0.50, 0.95)$ instead of $h \sim U(0.05, 0.95)$. This match occurs on the transitory $SD_{\mathcal{P},\Delta Y}$ distribution when the evaluation is conducted using the entire spectrum. Visual support for these results are $KS_{\mathcal{P}}$ and $KS_{\mathcal{T}}$ densities displayed in figures A7–A12. The first column of these figures present mean permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that are qualitatively similar to those found in the first column of figures 3.A–5.B.

A4.4 NKDSGE model fit under the prior $h \sim U(0.050, 0.499)$

The reason for replacing the prior $h \sim U(0.50, 0.95)$ with $h \sim U(0.050, 0.499)$ is to generate evidence about the impact of a weaker consumption habit process on NKDSGE model propagation and monetary transmission. Moving to the prior $h \sim U(0.050, 0.499)$ has the unsurprising effect of reducing the number of successful matches, $CIC \geq 0.3$, by five. Compared to the top half of table 2, the top half of table A3 reveals that the SPrice habit NKDSGE-MGR model achieves two fewer matches. SPrice and SWage habit NKDSGE-TR models exhibit three fewer $CIC \geq 0.3$ in the bottom half of table A3 on this dimension when set next to the CIC of table 2 of the paper. These failed matches are to distributions of permanent and transitory $SD_{\mathcal{T},\Delta C}$. The deterioration in the fit of SPrice habit NKDSGE-MGR, SPrice habit NKDSGE-TR, and SWage habit NKDSGE-TR models is reflected in in figures A13–A18 by mean permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that are farther from mean permanent and transitory $SD_{\mathcal{P},\Delta Y}$ and $SD_{\mathcal{P},\Delta C}$ as well as $KS_{\mathcal{P}}$ and $KS_{\mathcal{T}}$ densities that display less overlap.

A4.5 NKDSGE model fit using VAR(4)s to estimate SVMA(∞)s

We estimate unrestricted VARs with longer lags to examine the impact on the construction of SVMA(∞)s, permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$, and our Bayesian evaluation to NKDSGE model. Table A4 includes CIC that indicate switching to VAR(4)s from VAR(2)s has little impact on judging the fit of the 12 NKDSGE models to distributions of $SD_{\mathcal{P},\Delta Y}$ and $SD_{\mathcal{P},\Delta C}$. For the experiments relying on VAR(4)s, there are in net two additional $CIC \geq 0.3$ compared to table 2. The bottom half of table A4 shows the SPrice non-habit NKDSGE-TR model failing to duplicate the distribution of the permanent $SD_{\mathcal{P},\Delta Y}$. The SWage habit NKDSGE-MGR model is responsible for two $CIC \geq 0.3$ as shown in the top half of table A4 given VAR(4)s, instead of VAR(2)s, are engaged by the Bayesian Monte Carlo experiments. These matches occur on distributions of transitory $SD_{\mathcal{P},\Delta Y}$ and $SD_{\mathcal{P},\Delta C}$ when evaluation is constrained to the business cycle frequencies. Another $CIC \geq 0.3$ is provided by the SWage habit NKDSGE-TR model in the bottom half of table A4 that is not observed in table 2. This NKDSGE model replicates the distribution of the transitory $SD_{\mathcal{P},\Delta Y}$ on the entire spectrum.

Figures A19-A24 indicate that the impact of estimating VAR(4)s, rather than VAR(2)s, falls on the mean transitory $SD_{\mathcal{P},\Delta C}$. This SD displays a peak in the business cycle frequencies. Note that figure 2, which is constructed on VAR(2)s, contains a mean transitory $\text{SD}_{\mathcal{P},\Delta C}$ that has a plateau from the growth into the business cycle frequencies. Otherwise, the VAR(4)s have few qualitative implications for mean $SD_{\mathcal{P}}$, mean $SD_{\mathcal{T}}$, $KS_{\mathcal{P}}$ densities, and $KS_{\mathcal{T}}$ densities comparing those in figures A19–A24 to those in figures 3.A–5.B.

A4.5 Gauging NKDSGE model fit with the Cramer-von Mises statistic

Table A5 contains *CIC* for 12 NKDSGE models based on the *CvM* statistic. We ground *CIC* on densities of *CvM* statistics to check the robustness of measures of NKDSGE model fit presented in the paper. The *CvM* statistic is

$$CvM_{\mathcal{D},j} = \int_0^1 \mathcal{B}^2_{\mathcal{D},j}(\kappa)d\kappa,$$

for $\mathcal{D} = \mathcal{P}, \mathcal{T}$ and replication j of the ensemble of $\mathcal{J}$ (= 5000) $\mathcal{P}$ and $\mathcal{T}$ synthetic samples. Section 3.4 provides details about computing $\mathcal{B}_{\mathcal{D},j}(\cdot)$, but to review

$$\mathcal{B}_{\mathcal{D},j}(\kappa) = \frac{\sqrt{2\mathcal{H}}}{2\pi}\left[\mathcal{V}_{\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{\mathcal{D},j}(\pi)\right],$$

where $\kappa \in [0, 1]$ ($[0.064, 0.25]$) when evaluation is conducted on the entire spectrum (on the business cycle frequencies of eight to two years per cycle) and $\mathcal{H} = T$ if $\mathcal{D} = \mathcal{P}$, otherwise $\mathcal{H} = \mathcal{M}$. Also, the partial sum $\mathcal{V}_{\mathcal{D},j}\left(2\pi q/\mathcal{H}\right) = 2\pi \sum_{\ell=1}^{q} \mathcal{R}_{\mathcal{D},j}\left(2\pi\ell/\mathcal{H}\right)/\mathcal{H}$ and the ratio $\mathcal{R}_{\mathcal{D},j}(\omega) = \widehat{I_T}(\omega) \,/\, I_{\mathcal{D},j}(\omega)$, where the numerator (denominator) is the sample (jth $\mathcal{P}$ or $\mathcal{T}$) output or consumption growth SD at frequency ω. Distributions of $CvM_{\mathcal{P}}$ and $CvM_{\mathcal{T}}$ statistics are the basis of *CIC* that quantify the overlap of the ensemble of distributions of permanent and transitory $SD_{\mathcal{P},\Delta Y}$, $SD_{\mathcal{P},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$.

The fit of the NKDSGE models is qualitatively similar across table A5 and table 2 with two exceptions. First, table 2 shows that the SPrice non-habit NKDSGE-MGR model produces one $CIC > 0.30$. Using the CvM statistic allows this NKDSGE model to produce an additional $CIC \geq 0.30$ in the third row of table A5. This row of CIC shows that the SPrice non-habit NKDSGE-MGR model replicates distributions of permanent and transitory $SD_{\mathcal{P},\Delta Y}$s when evaluation is grounded on frequencies between eight to two years per cycle. However, distributions of $CvM_{\mathcal{P}}$ and $CvM_{\mathcal{T}}$ statistics generate CIC limited on the business cycle frequencies that indicate the fit of the SPrice habit NKDSGE-MGR model dominates the fit of the SPrice non-habit NKDSGE model-MGR. The former model also duplicates the distribution of transitory $SD_{\mathcal{P},\Delta C}$ on the entire spectrum using the CvM statistic, which is the other difference between table A5 and table 2 of the paper.

Figures A25–A30 plot mean $SD_{\mathcal{P}}$s and $SD_{\mathcal{T}}$s and densities of $CvM_{\mathcal{P}}$ and $CvM_{\mathcal{T}}$ statistics. A striking feature of these figures is that for $CIC > 0.3$ measured on the entire spectrum, the $CvM_{\mathcal{T}}$ statistic densities often decay smoothly from left to right instead of showing well-defined peaks. For example the middle panel of the bottom row of figure A26 shows that the baseline habit NKDSGE-TR model yields a $CIC = 0.56$ on the distribution of transitory $SD_{\mathcal{P},\Delta C}$, but the relevant CvM statistic density is relatively flat with a long right-hand tail. The explanation is that the quadratic form of the CvM statistic can place weight on large deviations between sample SD, $\widehat{SD_T}(\omega)$, and say, the jth draw from the distribution of $SD_{\mathcal{T}}(\omega)$, $SD_{\mathcal{T},j}(\omega)$. In this case, the density of the $CvM_{\mathcal{T}}$ statistic will be disperse with a long thin right tail. The supremum of the KS statistic, $KS_{\mathcal{T},j} = Max_{\kappa\in[0,1]}\left|\mathcal{B}_{\mathcal{T},j}(\kappa)\right|$, is immune from this dispersion, especially in cases when $CIC > 0.3$. Nonetheless, using the CvM statistic does not alter our evaluation of habit and non-habit NKDSGE models.

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Table A1: CICs of Kolmogorov-Smirnov Statistics

Replace the prior $h \sim U(0.05, 0.95)$ with $h \sim \beta(0.65, 0.15)$

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MGR								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.00	0.03	0.20	0.22	0.01	0.20	0.12	0.22
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.17	0.73	0.15	0.72	0.07	0.62	0.54	0.79
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.01	0.23	0.29	0.01	0.09	0.14	0.29
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.03	0.80	0.46	0.02	0.15	0.53	0.85
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.37	0.83	0.45	0.59	0.14	0.65	0.33	0.76
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.00	0.05	0.62	0.47	0.02	0.14	0.44	0.77

The prior $h \sim \beta(0.65, 0.15)$ implies a 95 percent coverage interval of $[0.3842, 0.8765]$. NKDSGE-MGR and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty:0$ $(8:2)$ indicates that CICs measure the intersection of distributions of $KS_{\mathcal{P}}$ and $KS_{\mathcal{T}}$ statistics computed over the entire spectrum (from eight to two years per cycle).

Table A2: CICs of Kolmogorov-Smirnov Statistics Replace the prior $h \sim U(0.05, 0.95)$ with $h \sim U(0.50, 0.95)$

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MGR								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.00	0.06	0.27	0.32	0.03	0.32	0.26	0.36
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.24	0.80	0.21	0.78	0.17	0.63	0.57	0.77
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.03	0.29	0.39	0.03	0.20	0.26	0.39
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.06	0.78	0.39	0.05	0.28	0.76	0.83
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.44	0.88	0.55	0.56	0.23	0.66	0.61	0.73
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.01	0.10	0.60	0.47	0.05	0.26	0.67	0.70

See the notes to table A1 except that the results of this table rely on $h \sim U(0.50, 0.95)$.

TABLE A3: CICs OF KOLMOGOROV-SMIRNOV STATISTICS
REPLACE THE PRIOR $h \sim U(0.05, 0.95)$ WITH $h \sim U(0.050, 0.499)$

	ΔY w/r/t Trend Sh'k		ΔY w/r/t Transitory Sh'k		ΔC w/r/t Trend Sh'k		ΔC w/r/t Transitory Sh'k	
Model	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$
NKDSGE-MGR								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.01	0.02	0.06	0.02	0.00	0.00	0.00	0.00
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.03	0.48	0.00	0.39	0.00	0.25	0.00	0.20
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.00	0.09	0.08	0.00	0.00	0.00	0.08
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.00	0.50	0.66	0.00	0.00	0.30	0.87
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.41	0.60	0.04	0.74	0.08	0.26	0.04	0.80
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.00	0.00	0.52	0.42	0.00	0.00	0.20	0.84

See the notes to table A1 except that the results of this table rely on $h \sim U(0.050, 0.499)$.

Table A4: *CIC*s of Kolmogorov-Smirnov Statistics Using VAR(4)s to Construct SVMA(∞)s

Model	ΔY w/r/t Trend Sh'k $\infty:0$	$8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	$8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	$8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	$8:2$
NKDSGE-MGR								
Baseline								
Non-Habit	0.01	0.04	0.00	0.03	0.00	0.00	0.00	0.00
Habit	0.02	0.09	0.14	0.19	0.04	0.18	0.14	0.21
SPrice								
Non-Habit	0.14	0.53	0.00	0.36	0.01	0.12	0.00	0.04
Habit	0.29	0.67	0.23	0.75	0.14	0.46	0.28	0.52
SWage								
Non-Habit	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.09
Habit	0.01	0.03	0.23	0.30	0.03	0.11	0.14	0.30
NKDSGE-TR								
Baseline								
Non-Habit	0.00	0.01	0.24	1.00	0.00	0.00	0.03	0.56
Habit	0.02	0.07	0.70	0.98	0.05	0.15	0.44	0.83
SPrice								
Non-Habit	0.13	0.78	0.00	0.92	0.00	0.13	0.00	0.52
Habit	0.40	0.88	0.33	0.89	0.16	0.48	0.34	0.81
SWage								
Non-Habit	0.00	0.00	0.31	1.00	0.00	0.00	0.01	0.65
Habit	0.02	0.06	0.60	0.92	0.04	0.14	0.40	0.84

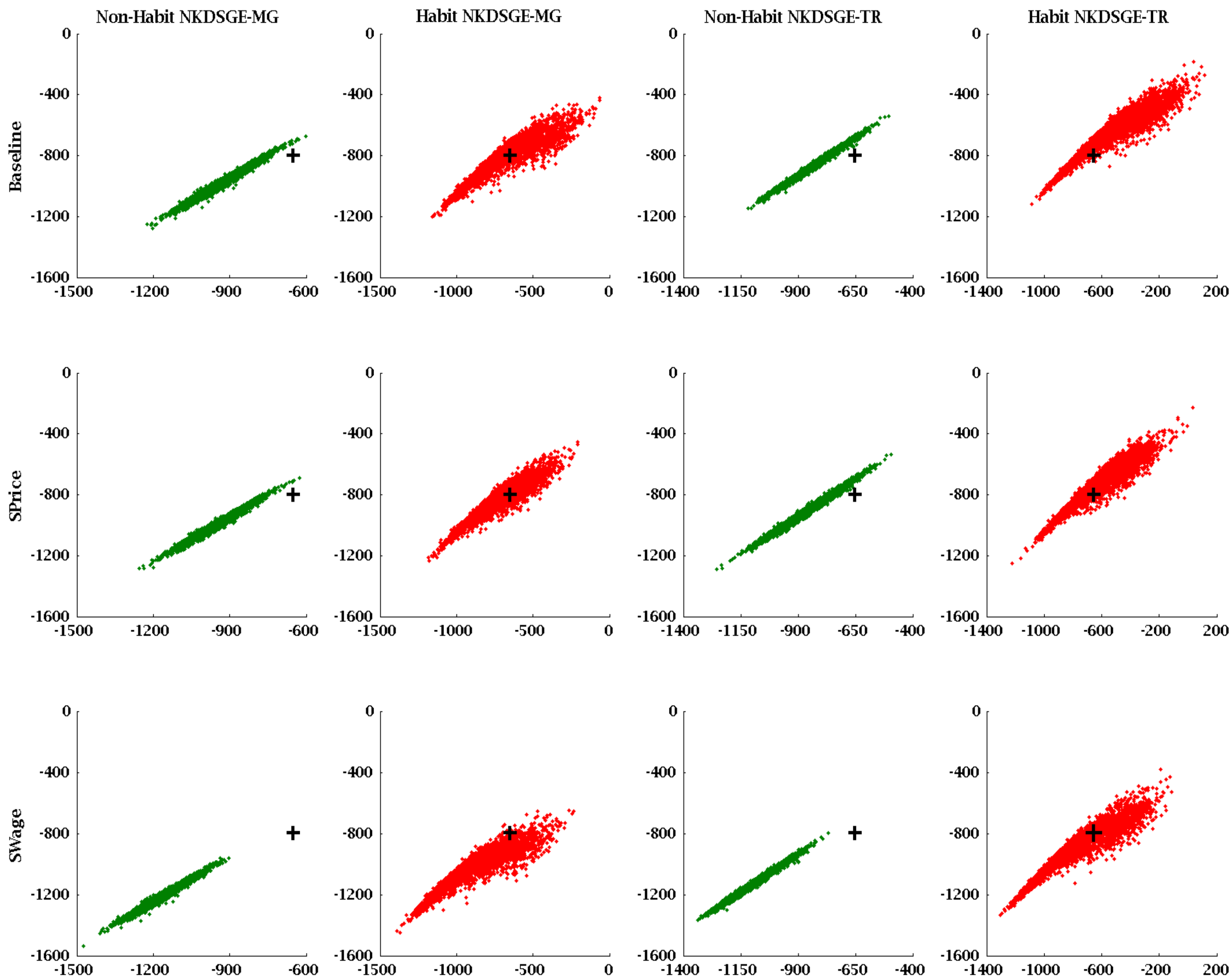
The SVMA(∞) are constructed from unrestricted VAR(4)s estimated on actual and synthetic data. Otherwise, see the notes at the bottom of table A1.

TABLE A5: *CIC*s OF CRAMER-VON MISES STATISTICS

Model	ΔY w/r/t Trend Sh'k ∞ : 0	8 : 2	ΔY w/r/t Transitory Sh'k ∞ : 0	8 : 2	ΔC w/r/t Trend Sh'k ∞ : 0	8 : 2	ΔC w/r/t Transitory Sh'k ∞ : 0	8 : 2
NKDSGE-MGR								
Baseline								
Non-Habit	0.02	0.02	0.00	0.02	0.00	0.00	0.00	0.00
Habit	0.00	0.02	0.20	0.18	0.02	0.12	0.14	0.18
SPrice								
Non-Habit	0.04	0.52	0.00	0.49	0.02	0.21	0.00	0.14
Habit	0.14	0.65	0.09	0.75	0.09	0.46	0.32	0.60
SWage								
Non-Habit	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.01
Habit	0.00	0.01	0.23	0.26	0.01	0.01	0.14	0.19
NKDSGE-TR								
Baseline								
Non-Habit	0.03	0.00	0.12	0.62	0.00	0.00	0.10	0.75
Habit	0.00	0.02	0.63	0.50	0.03	0.14	0.56	0.87
SPrice								
Non-Habit	0.42	0.61	0.00	0.80	0.01	0.21	0.00	0.71
Habit	0.42	0.73	0.27	0.56	0.16	0.50	0.35	0.81
SWage								
Non-Habit	0.00	0.00	0.21	0.33	0.00	0.00	0.03	0.87
Habit	0.00	0.04	0.54	0.43	0.02	0.13	0.46	0.78

The column heading ∞ : 0 (8 : 2) indicates that *CIC*s measure the intersection of distributions of $CvM_{\mathcal{P}}$ and $CvM_{\mathcal{T}}$ statistics computed over the entire spectrum (from eight to two years per cycle). Otherwise, see the notes at the bottom of table A1.

Figure A9: Prior Predictive Analysis: Sample and NKDSDE Prior Estimates of VAR(2) Innovation Variances



The horizontal (vertical) axes contain the innovation variance of the consumption (output) growth regression. The plus symbols denote the intersection of sample innovation variance estimates of the consumption and output growth regressions. The clouds of points are the prior distributions of estimated synthetic innovation variances from the consumption and output growth regressions generated by the NKDSGE models.

Figure 3.A: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the AR(1) Money Growth Rule

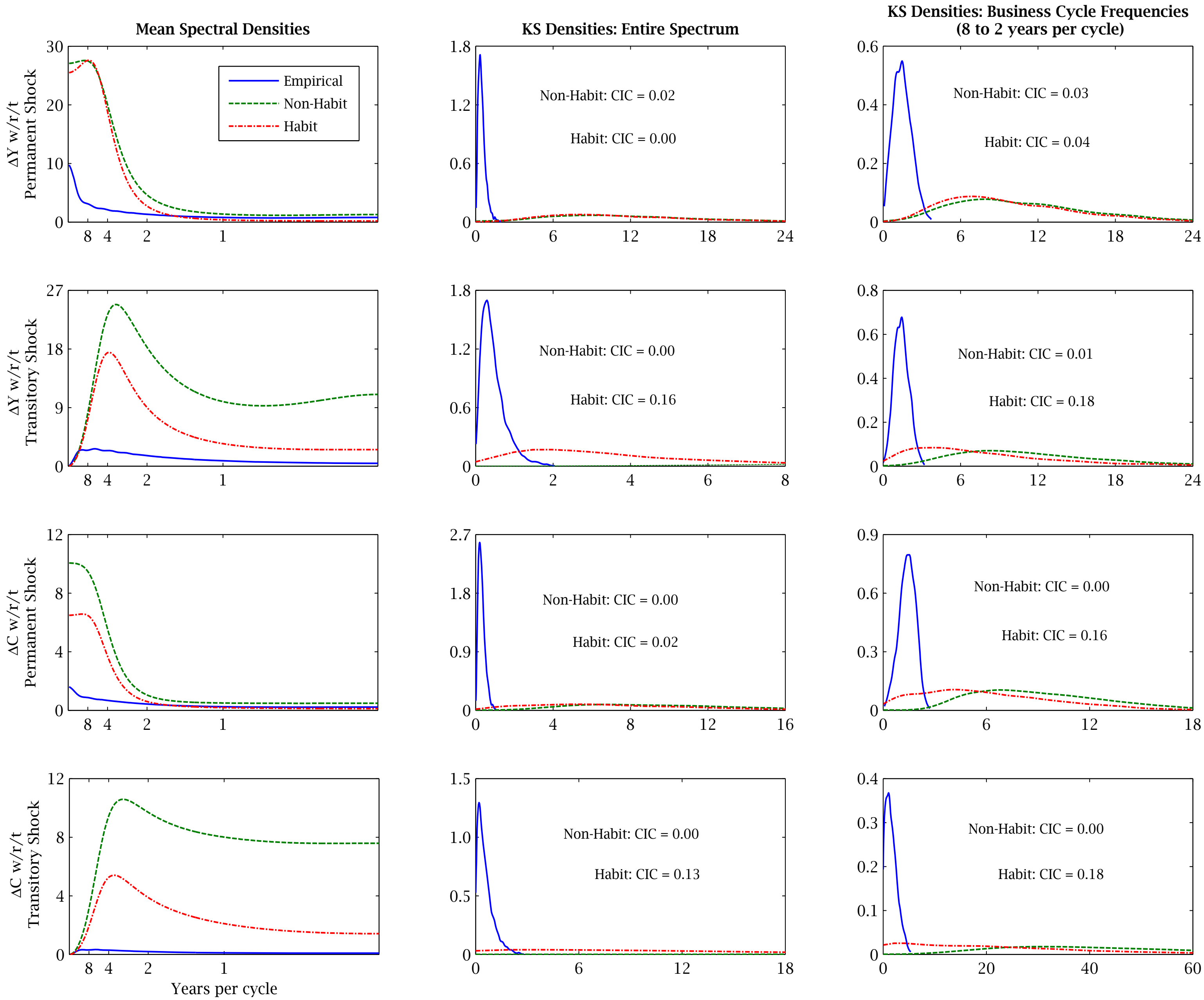


Figure 3.B: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Taylor Rule

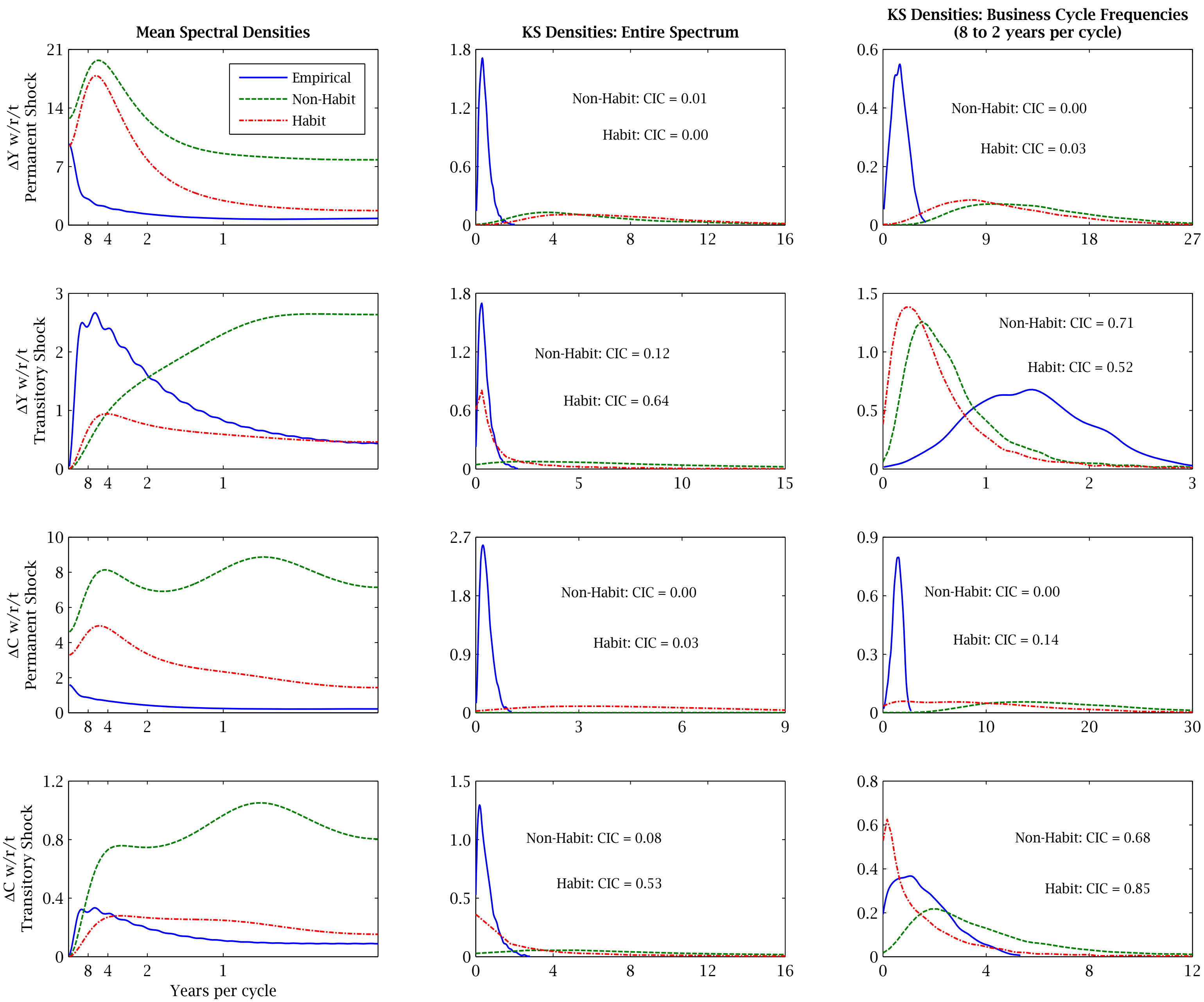


Figure 4.A: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the AR(1) Money Growth Rule and Only Sticky Prices

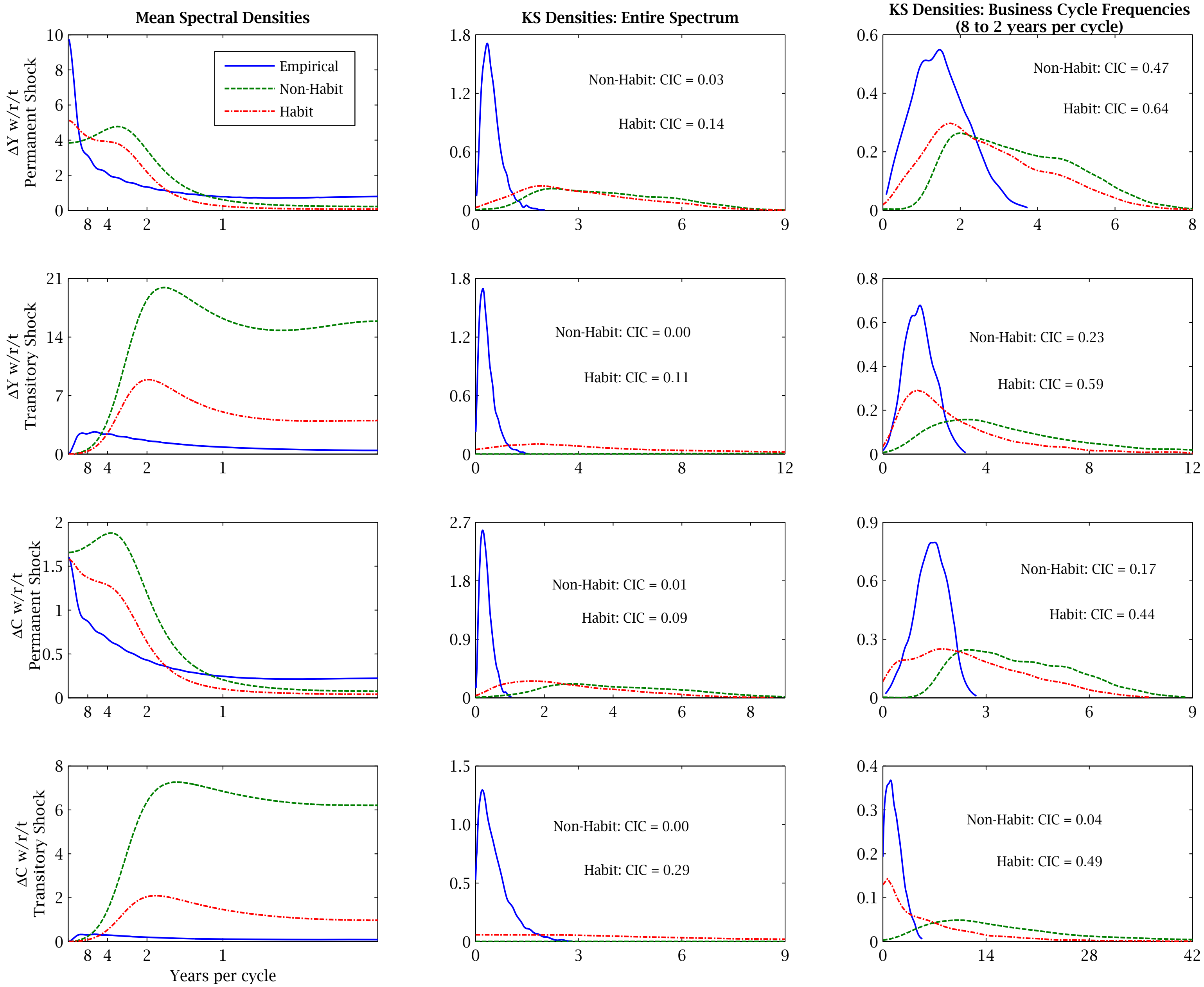


Figure 4.B: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule and only Sticky Prices

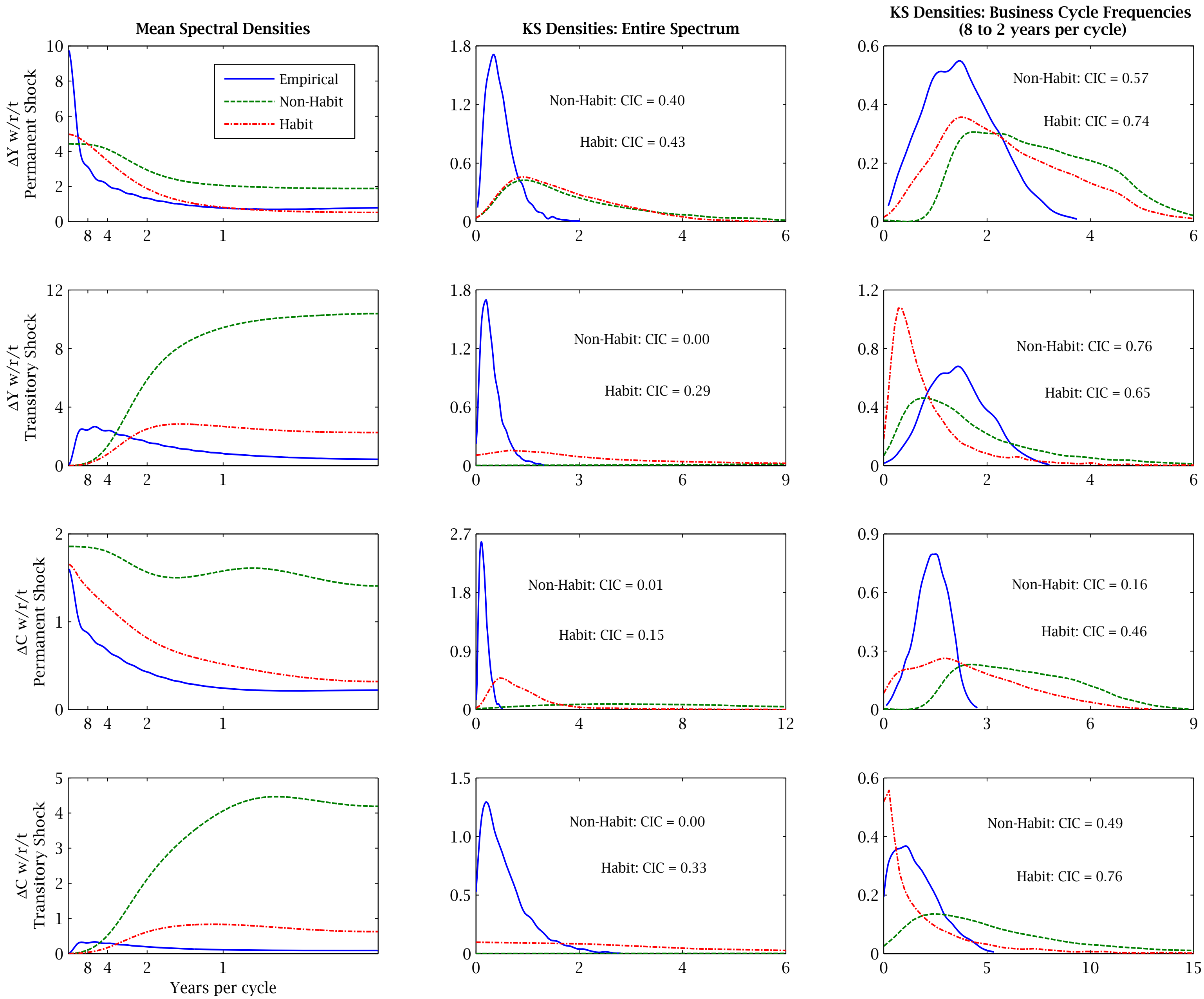


Figure 5.A: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the AR(1) Money Growth Rule and Only Sticky Wages

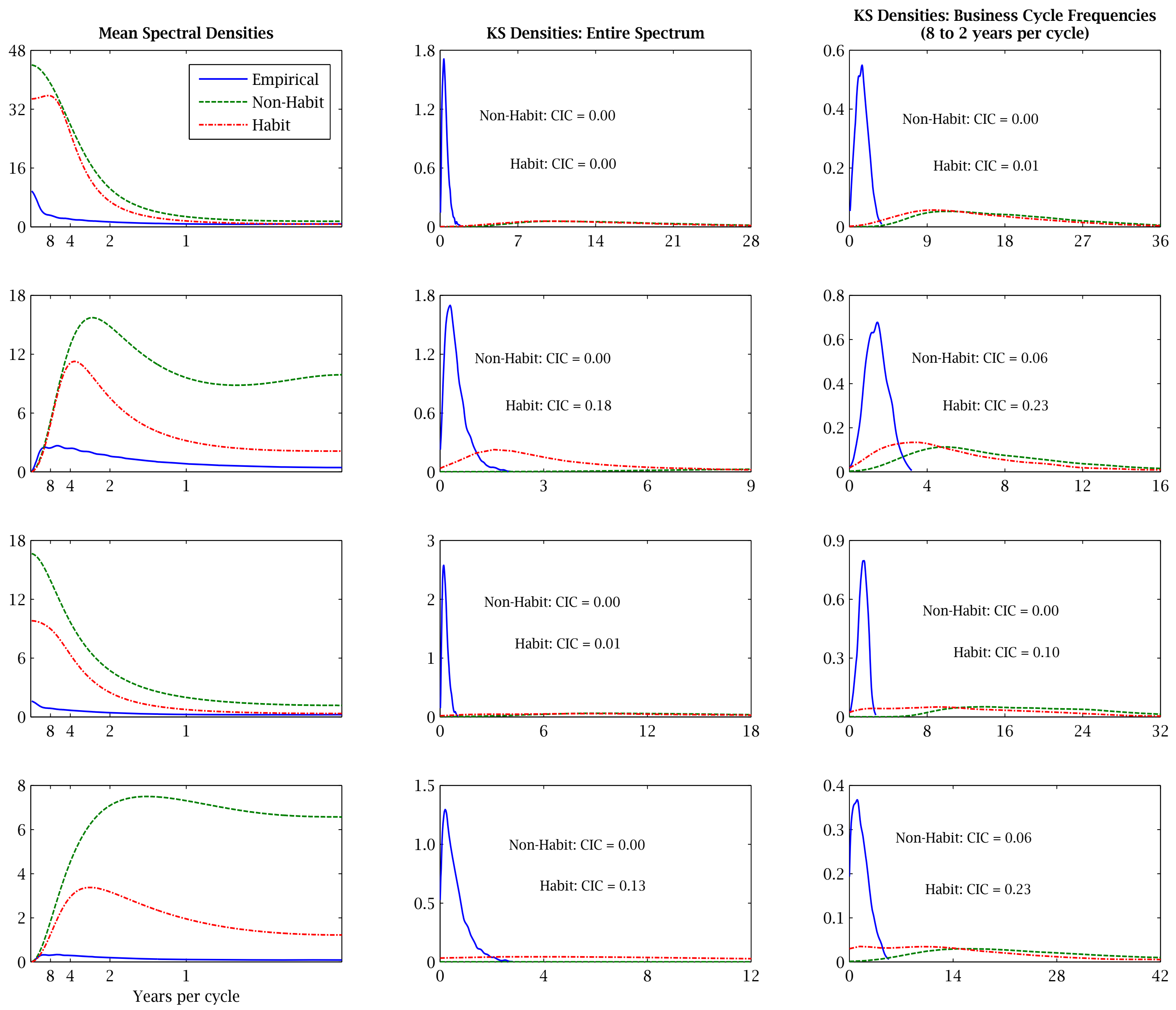


FIGURE 5.B: MEAN STRUCTURAL $\mathcal{P}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR NKDSGE MODELS WITH THE TAYLOR RULE AND ONLY STICKY WAGES

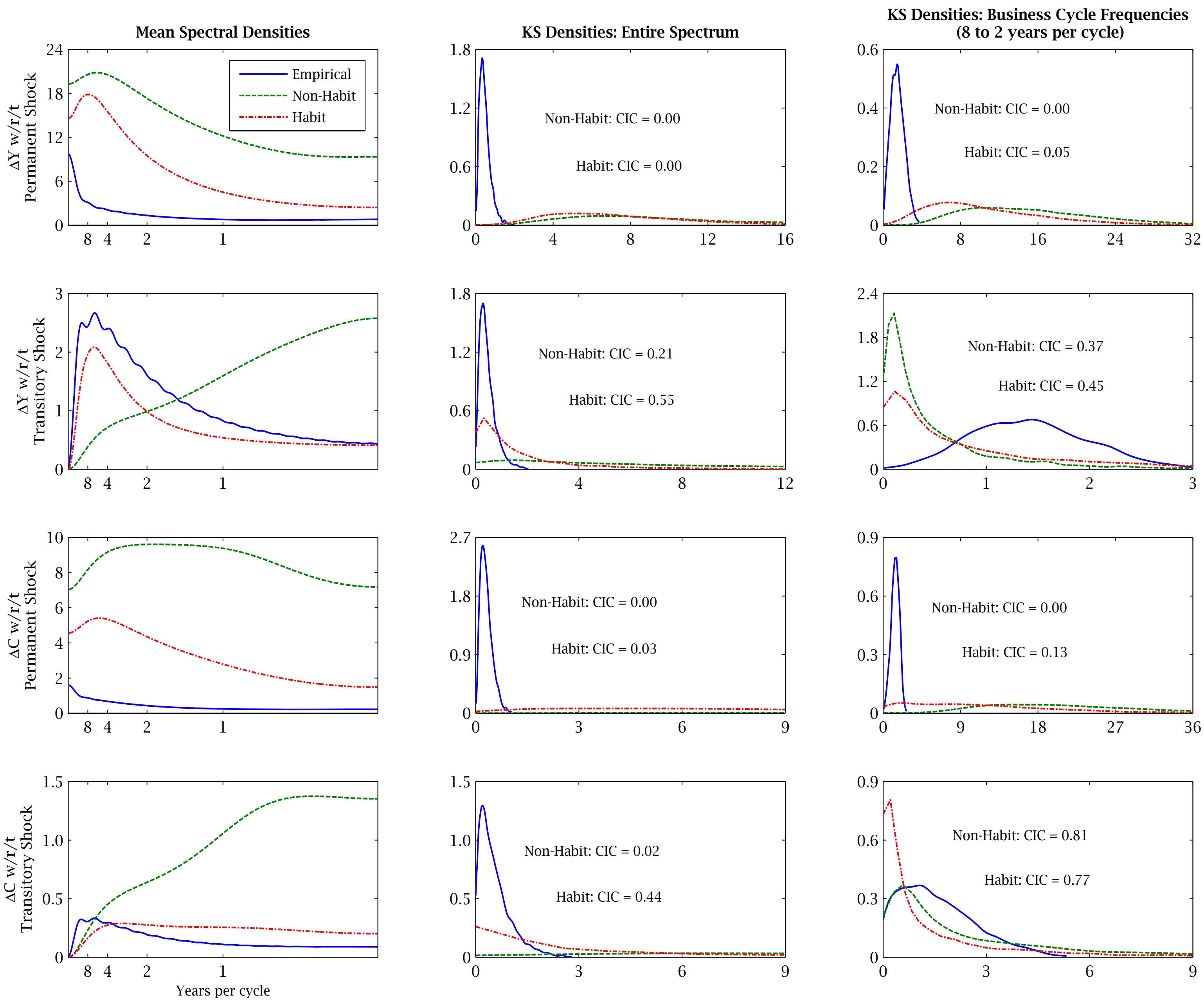


Figure A1: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Money Growth Rule and $h \sim \beta$

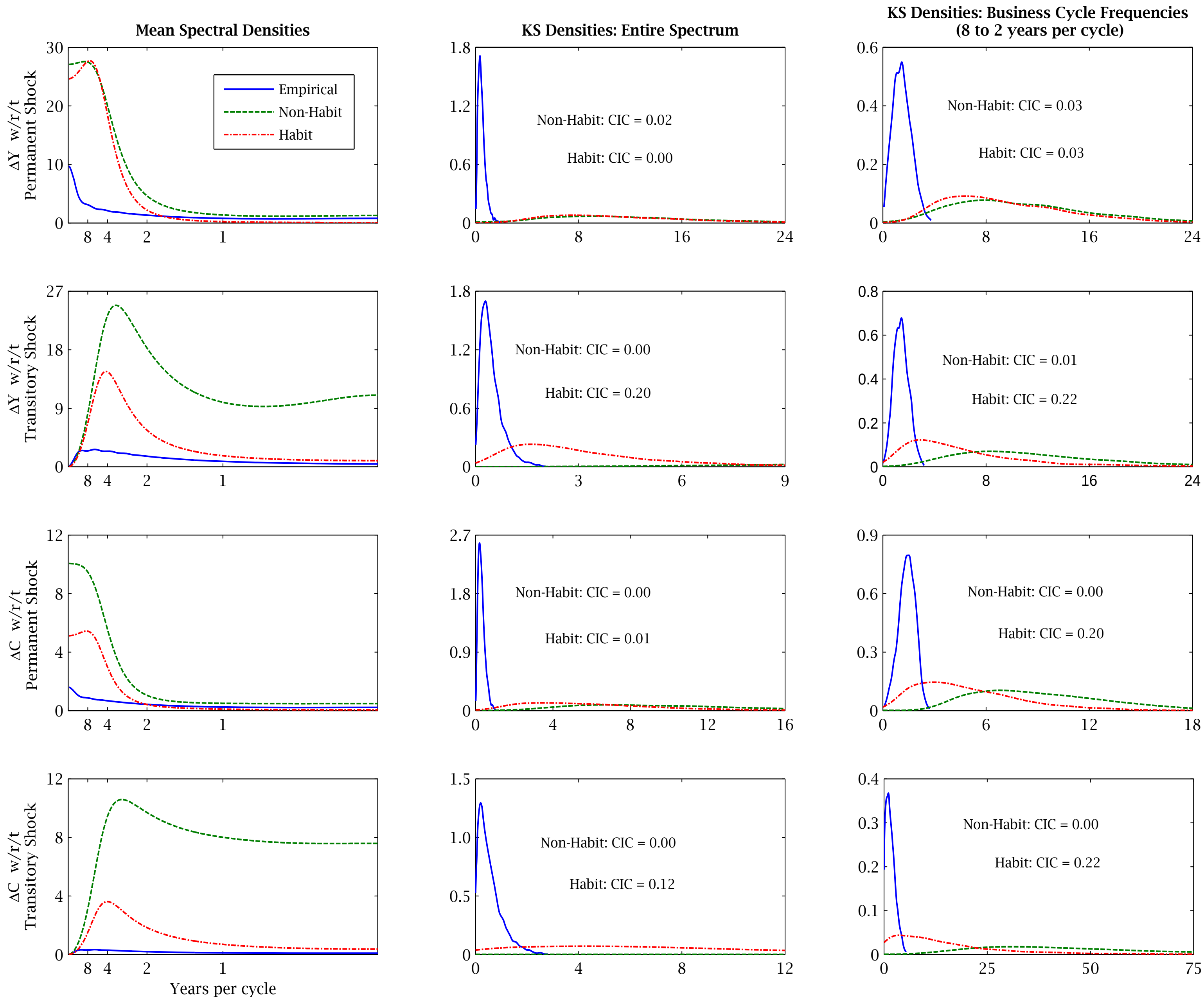


FIGURE A2: MEAN STRUCTURAL $\mathcal{P}$ AND $\mathcal{T}$ SDS AND KS DENSITIES FOR BASELINE NKDSGE MODELS WITH THE TAYLOR RULE AND $h \sim \beta$

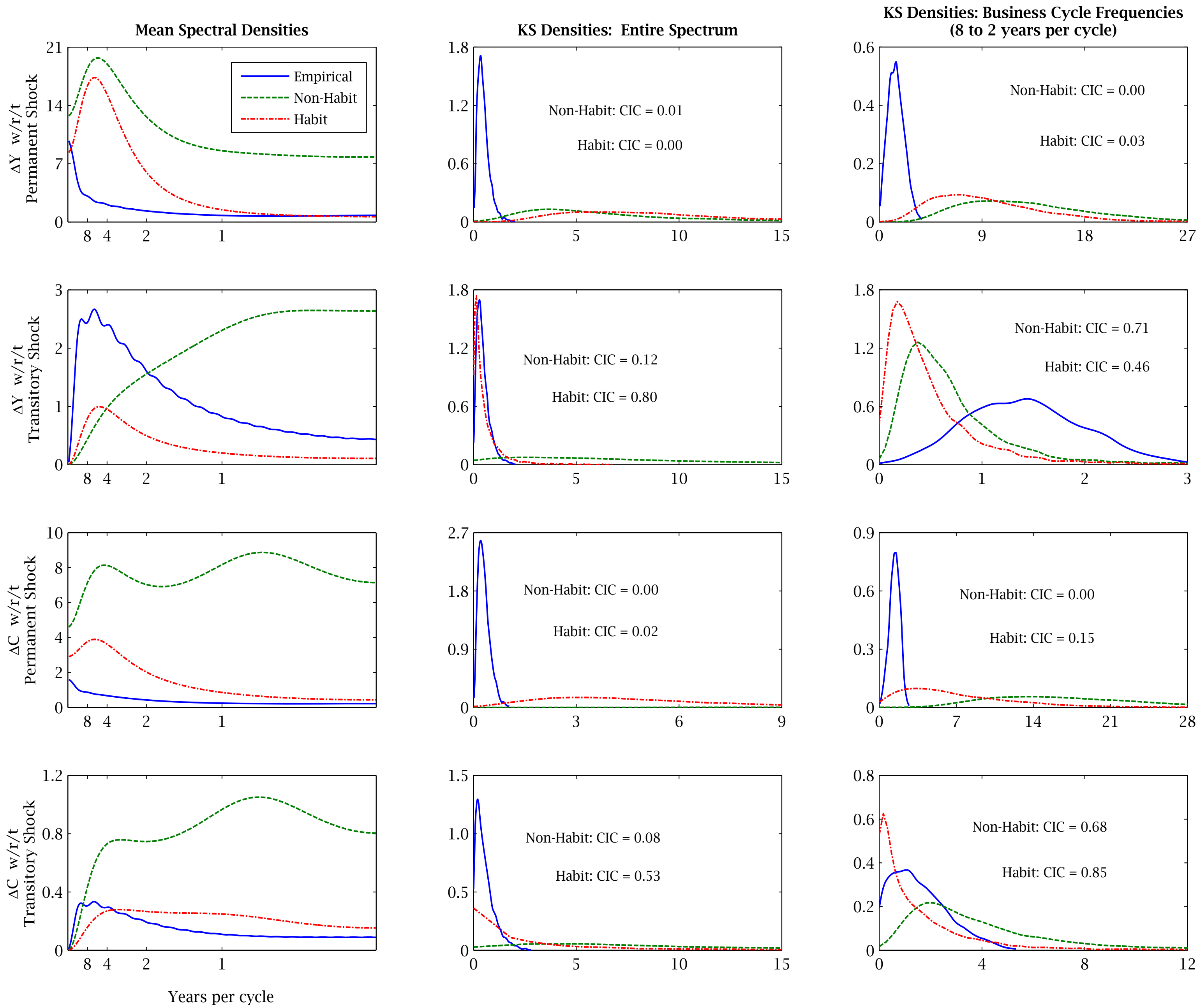


Figure A3: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule, Only Sticky Prices, and $h \sim \beta$

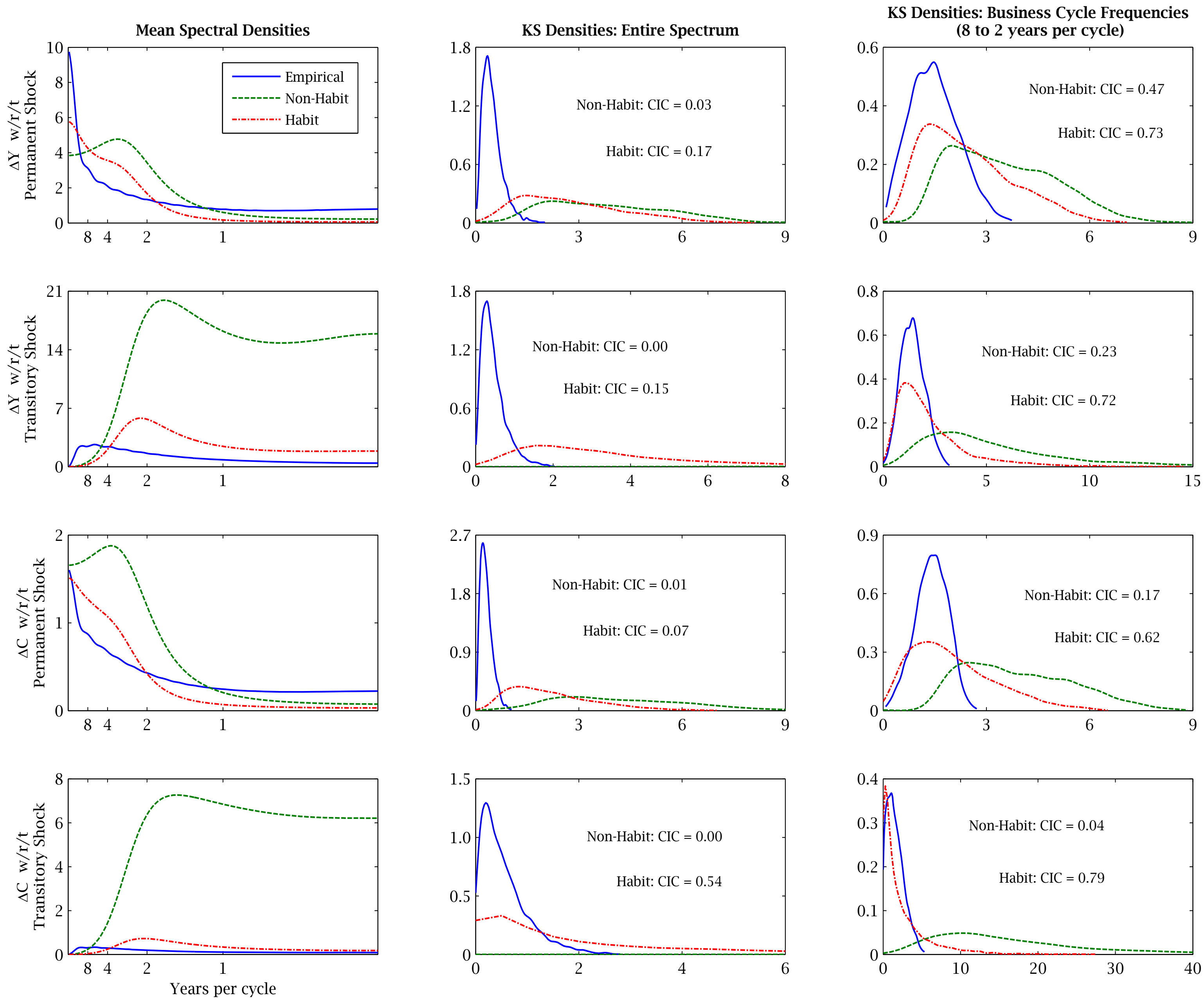


Figure A4: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule, only Sticky Prices, and $h \sim \beta$

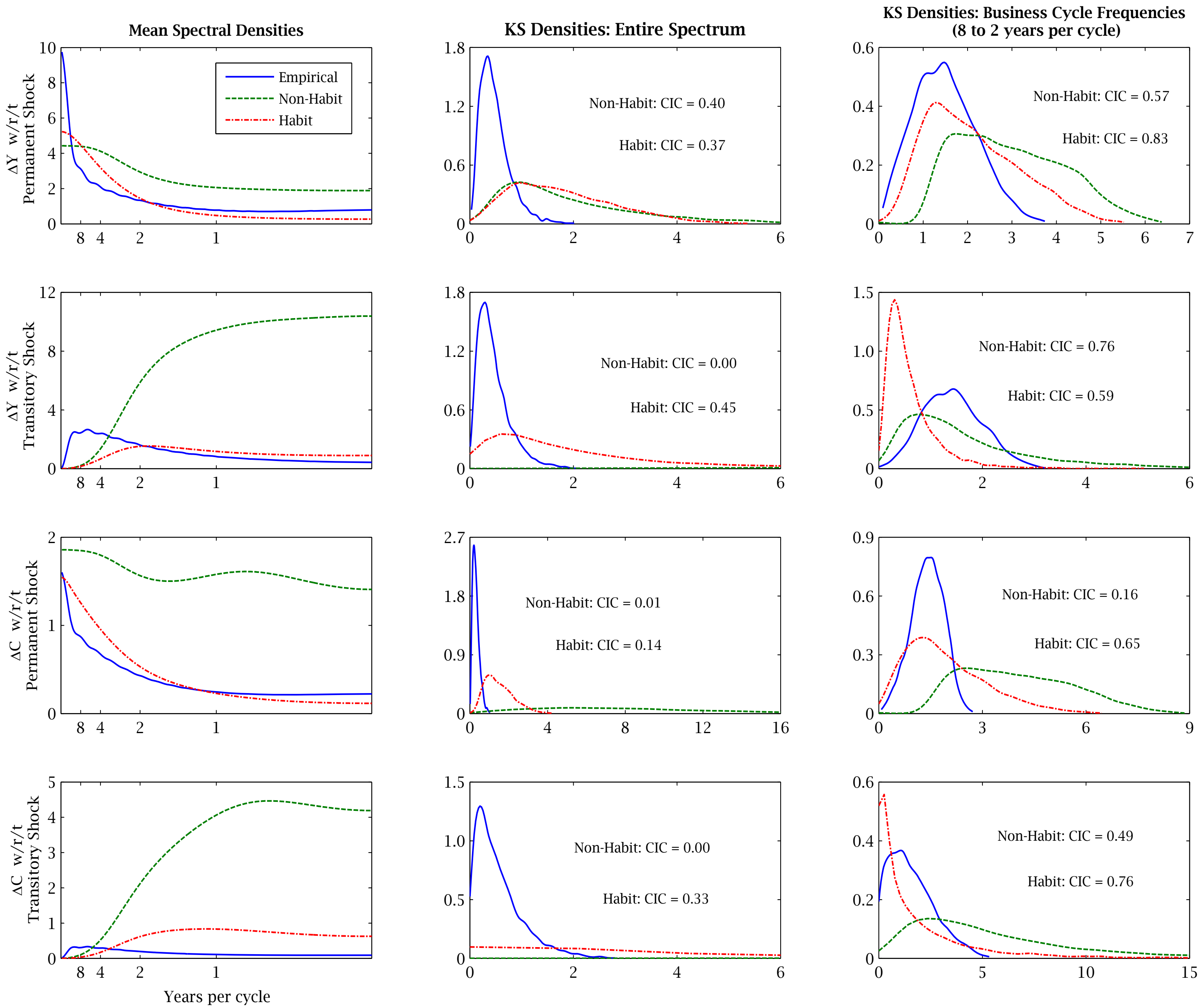


FIGURE A5: MEAN STRUCTURAL $\mathcal{P}$ AND $\mathcal{T}$ SDS AND KS DENSITIES FOR NKDSGE MODELS WITH THE MONEY GROWTH RULE, ONLY STICKY WAGES, AND $h \sim \beta$

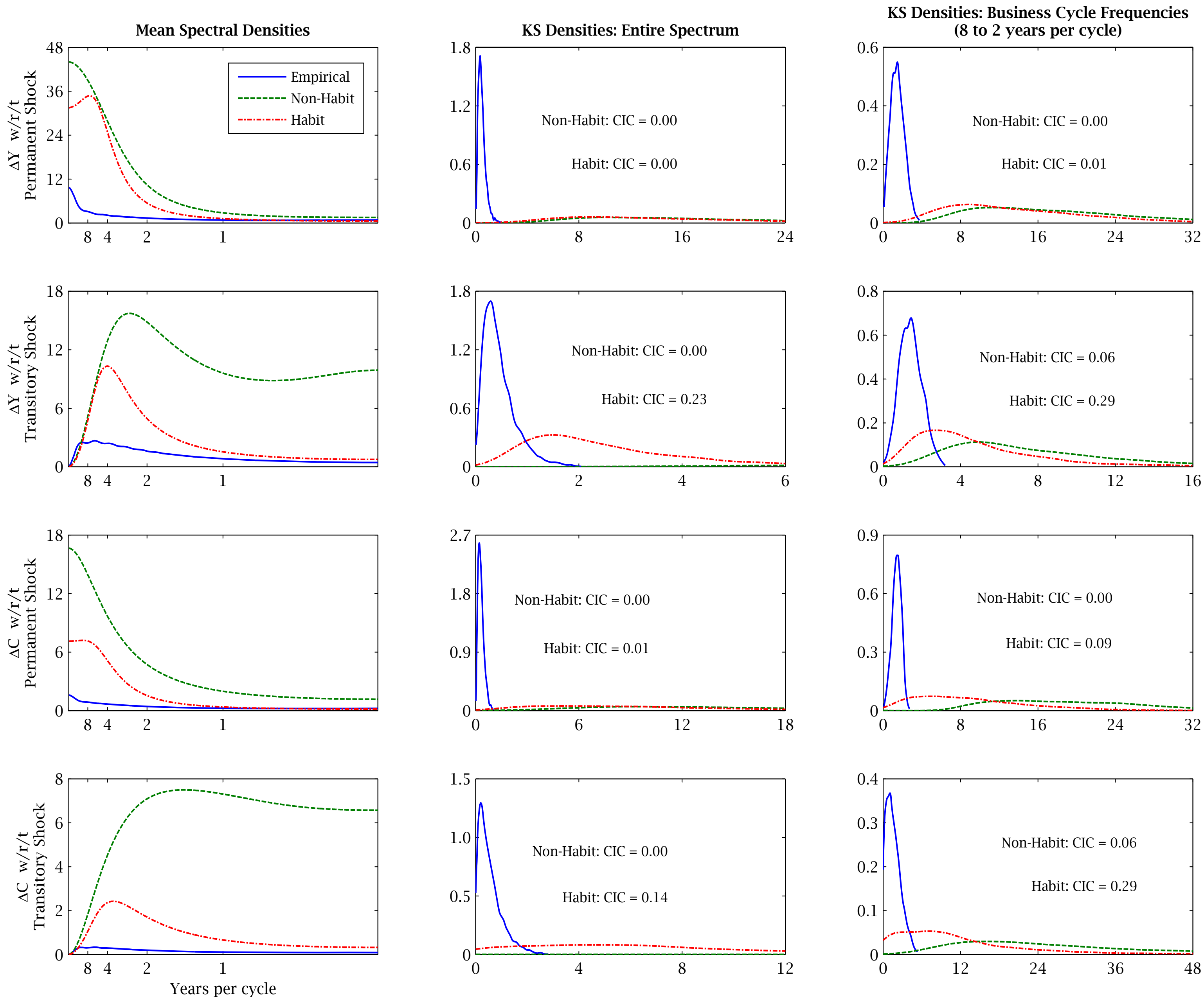


Figure A6: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule, only Sticky Wages, and $h \sim \beta$

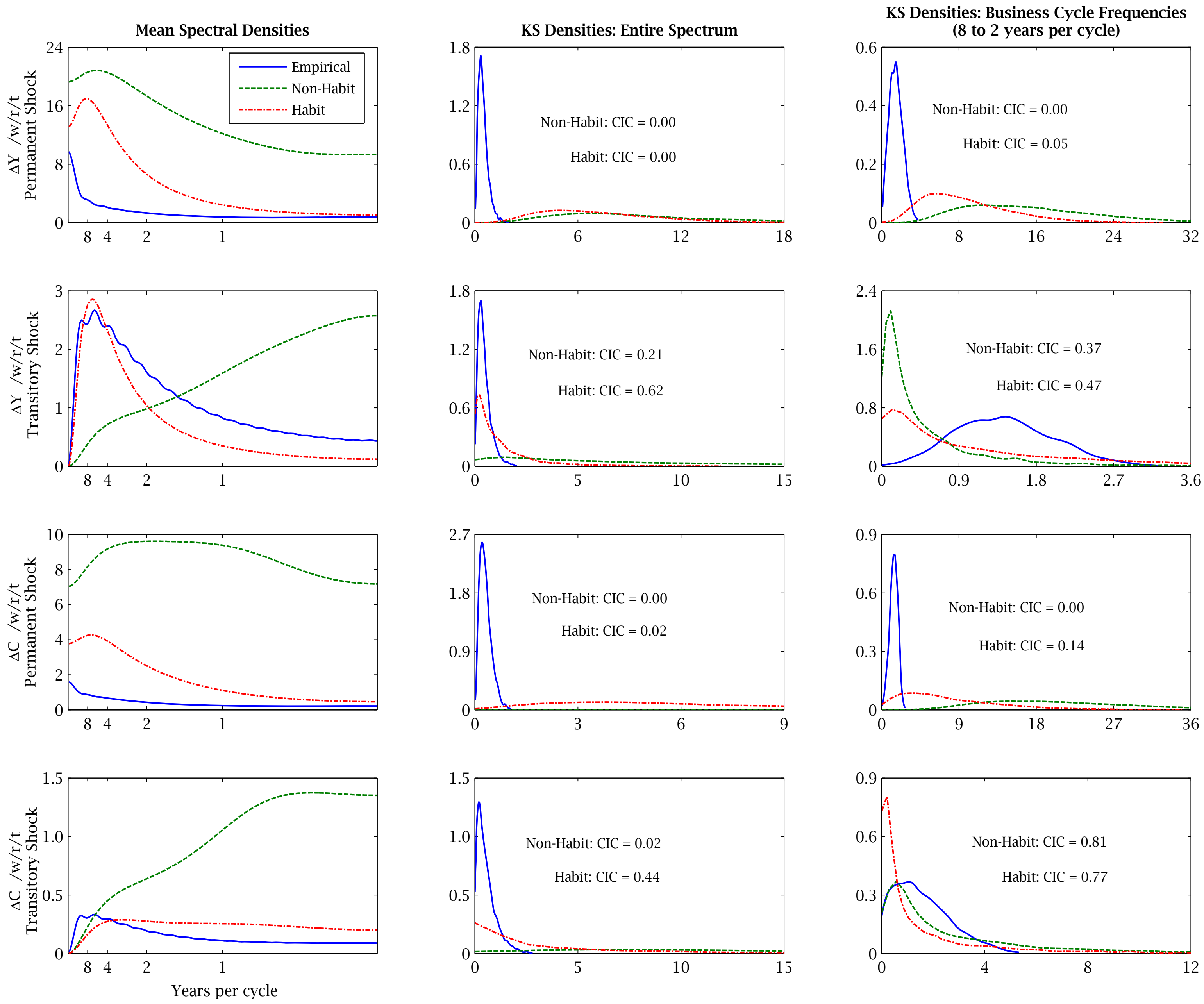


Figure A7: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Money Growth Rule and $h \sim U(0.50, 0.95)$

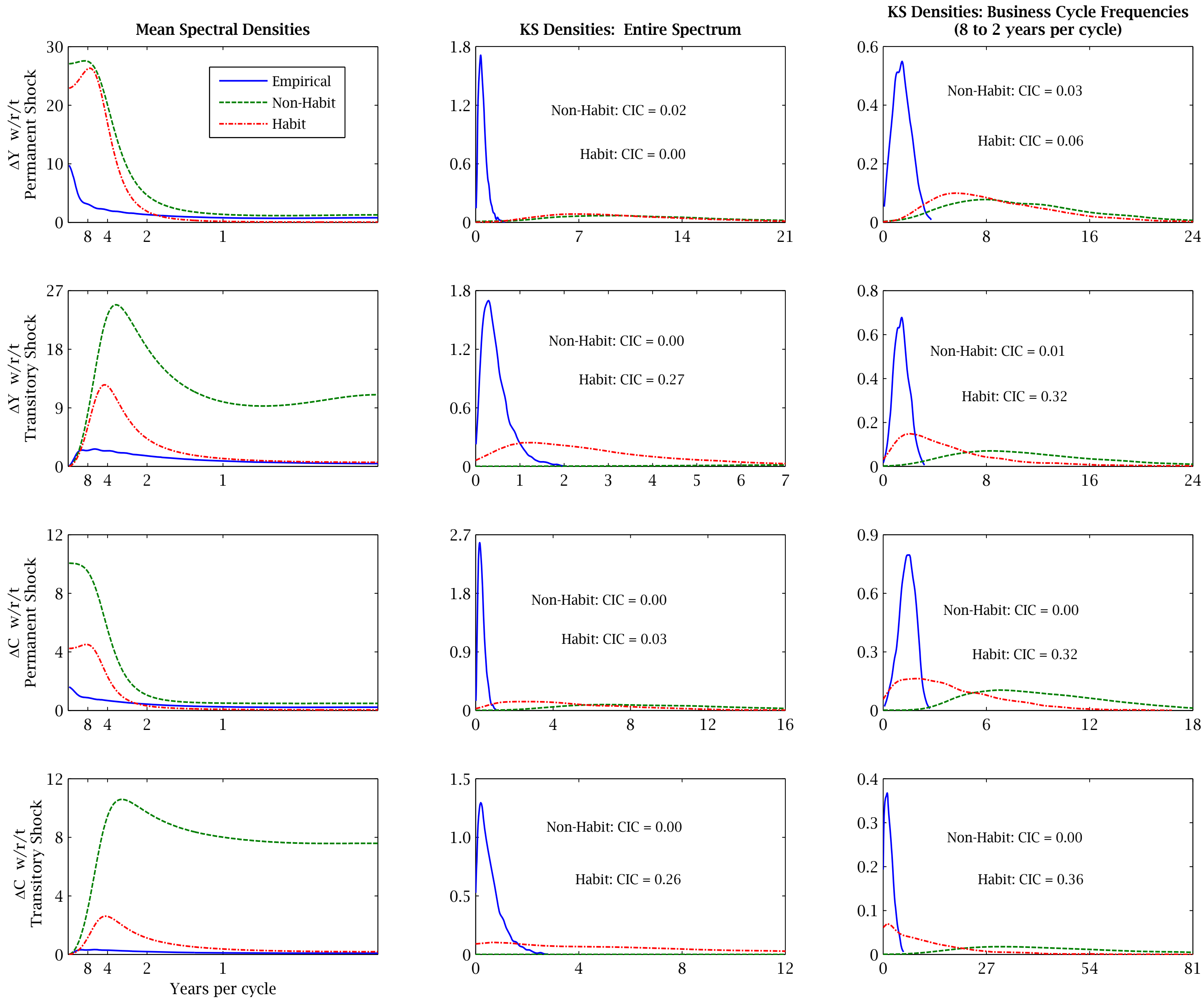


FIGURE A8: MEAN STRUCTURAL $\mathcal{P}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR BASELINE NKDSGE MODEL WITH THE TAYLOR RULE AND $h \sim U(0.50, 0.95)$

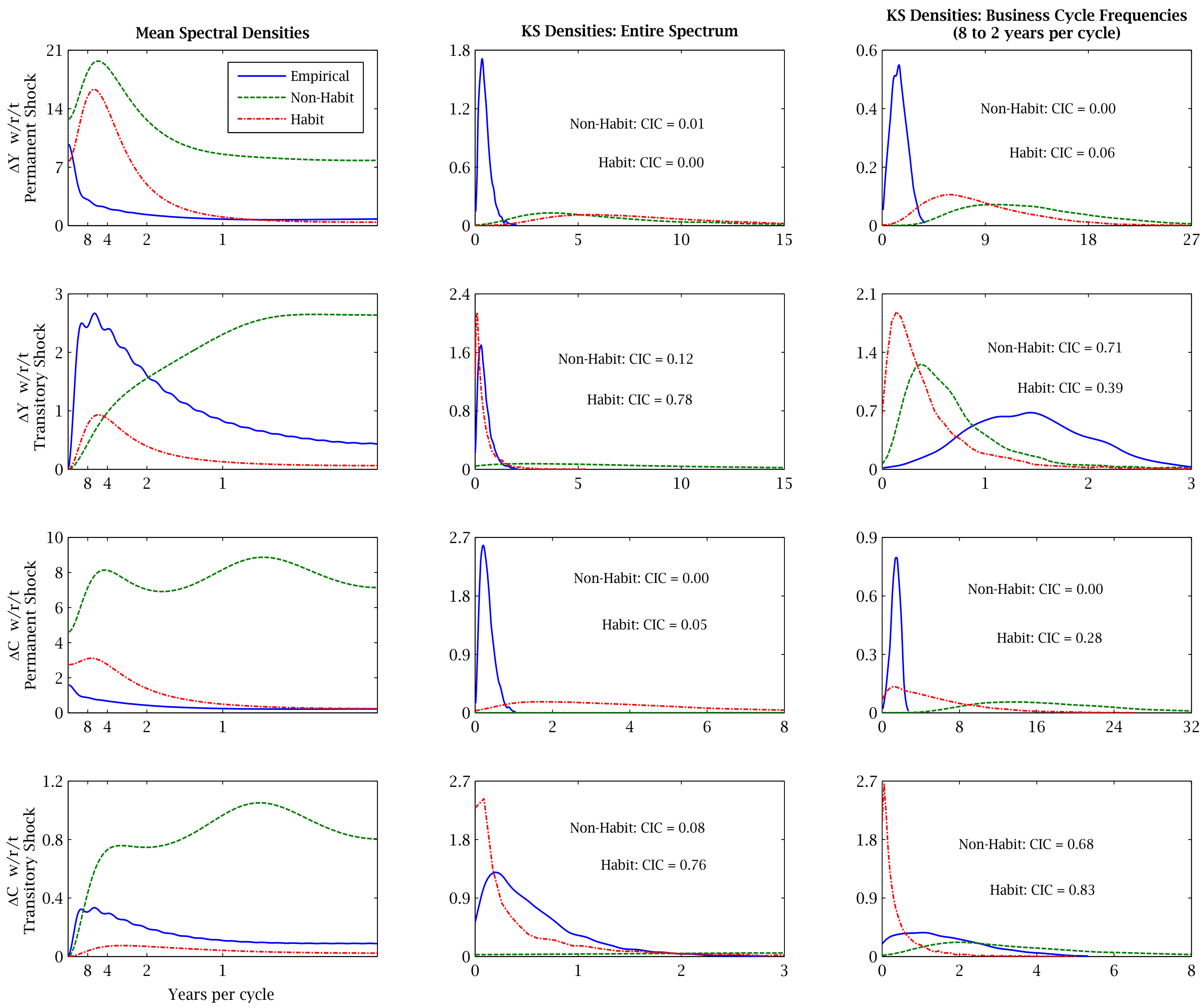


Figure A9: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule, Only Sticky Prices, and $h \sim U(0.50, 0.95)$

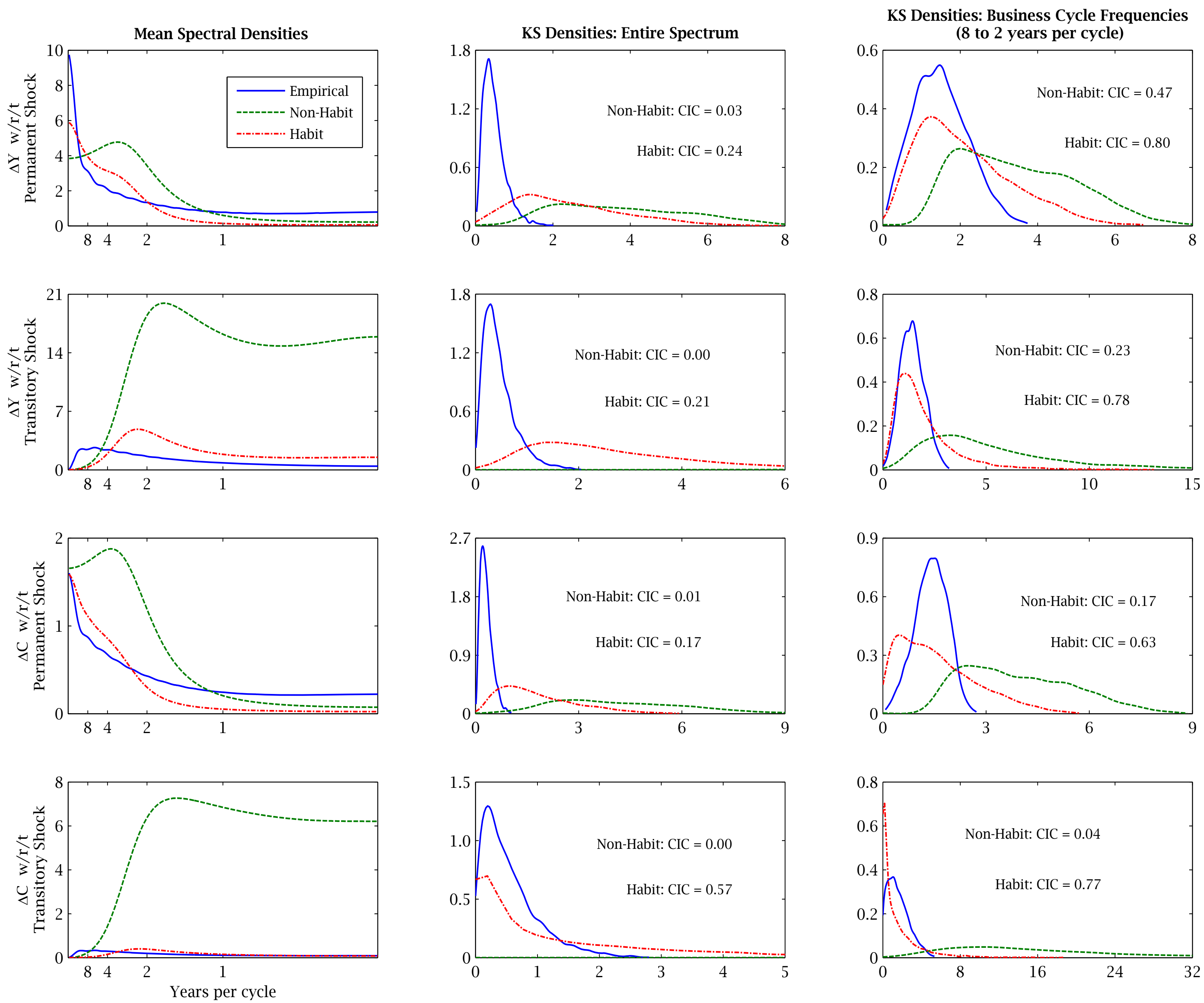


Figure A10: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule, only Sticky Prices, and $h \sim U(0.50, 0.95)$

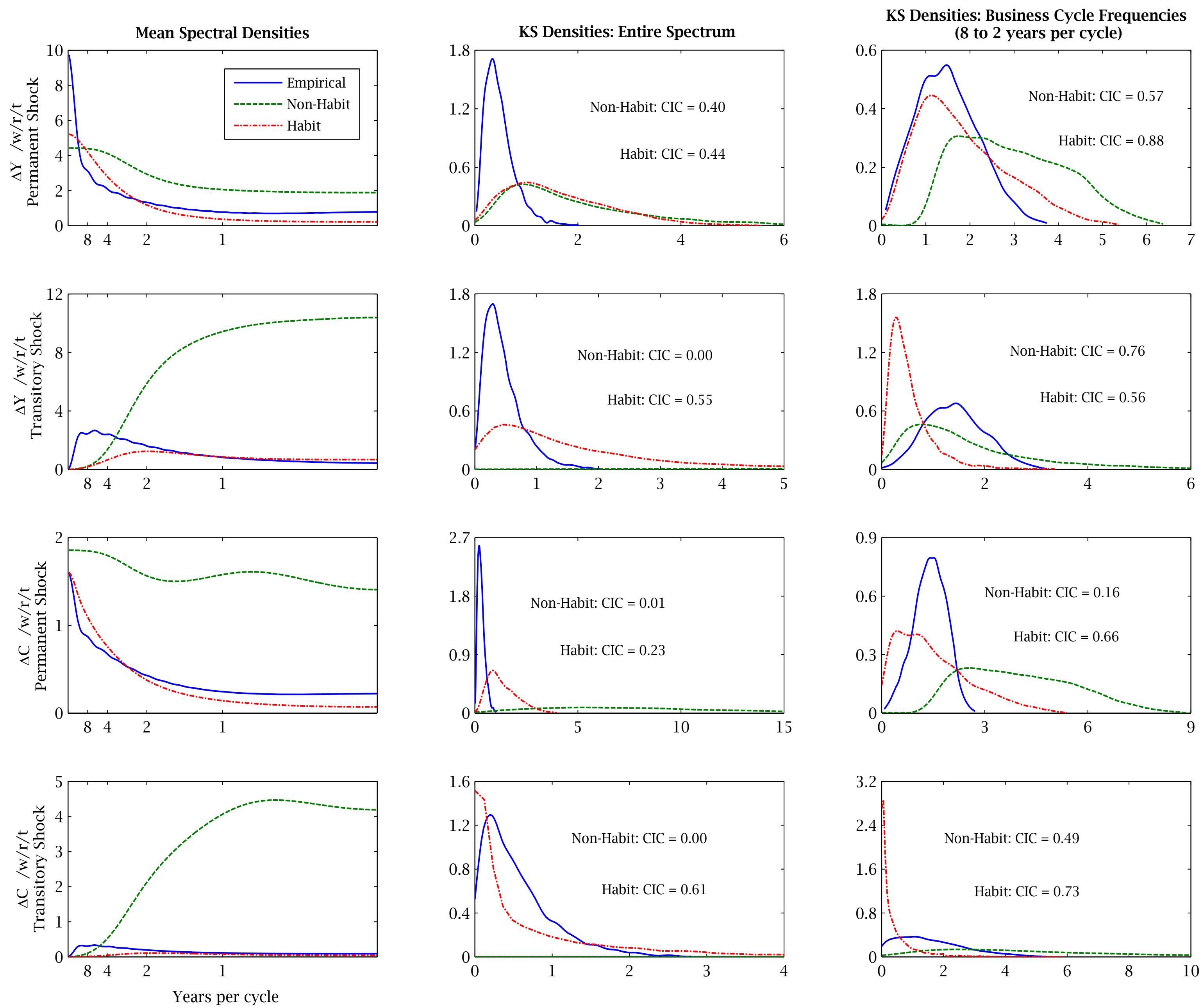


Figure A11: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule, Only Sticky Wages, and $h \sim U(0.50, 0.95)$

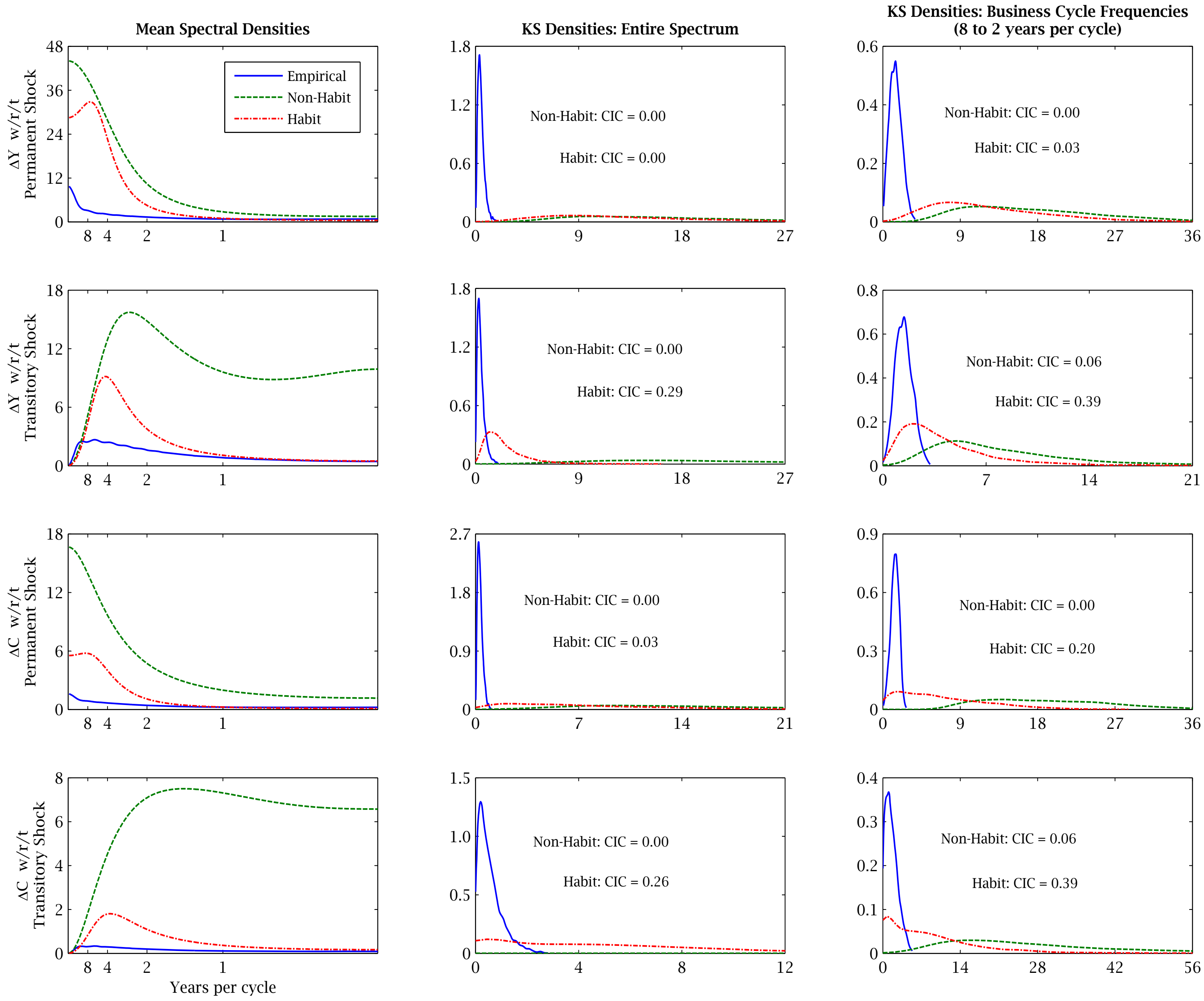


Figure A12: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule, only Sticky Wages, and $h \sim U(0.50, 0.95)$

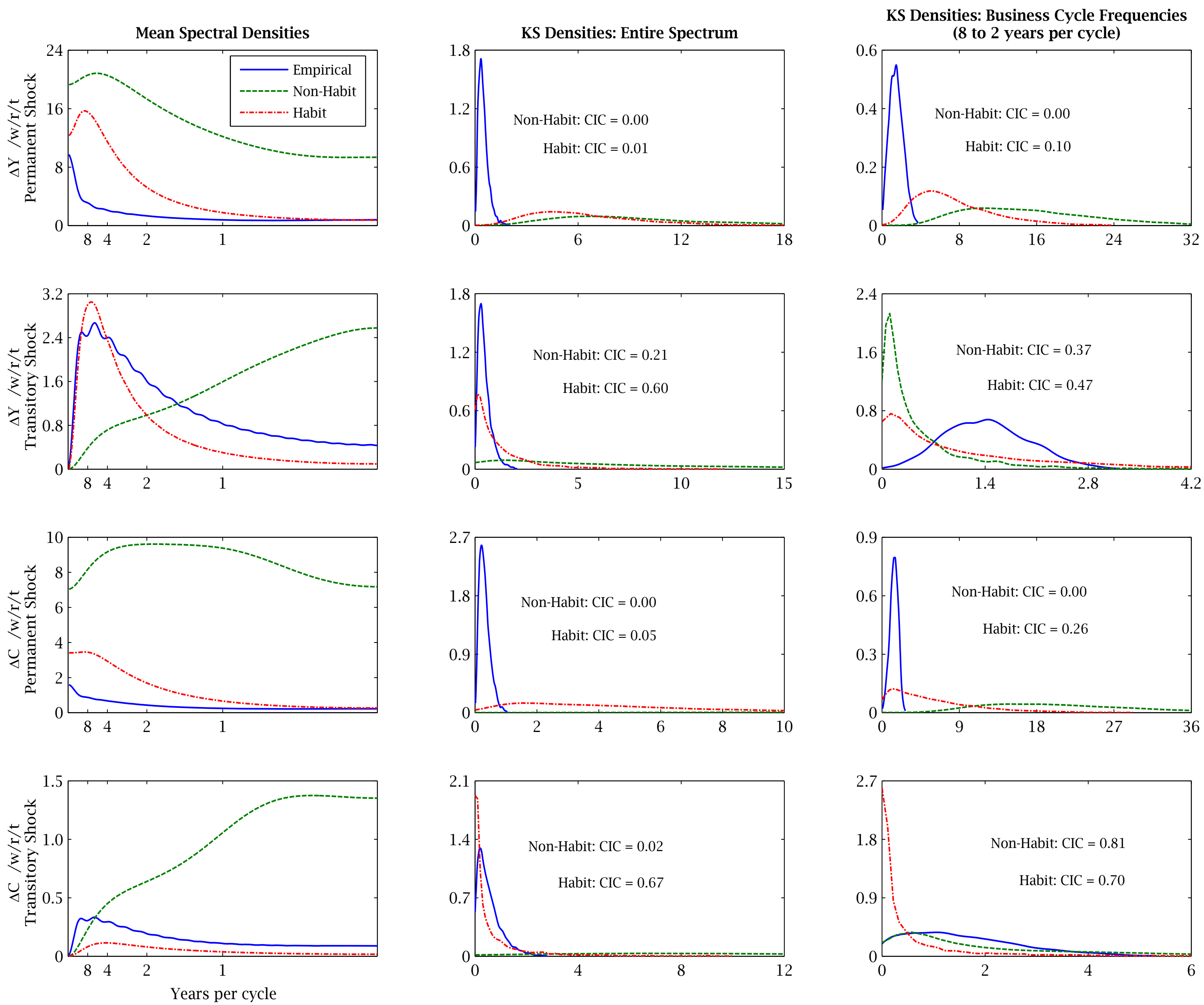


Figure A13: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Money Growth Rule and $h \sim U(0.050, 0.499)$

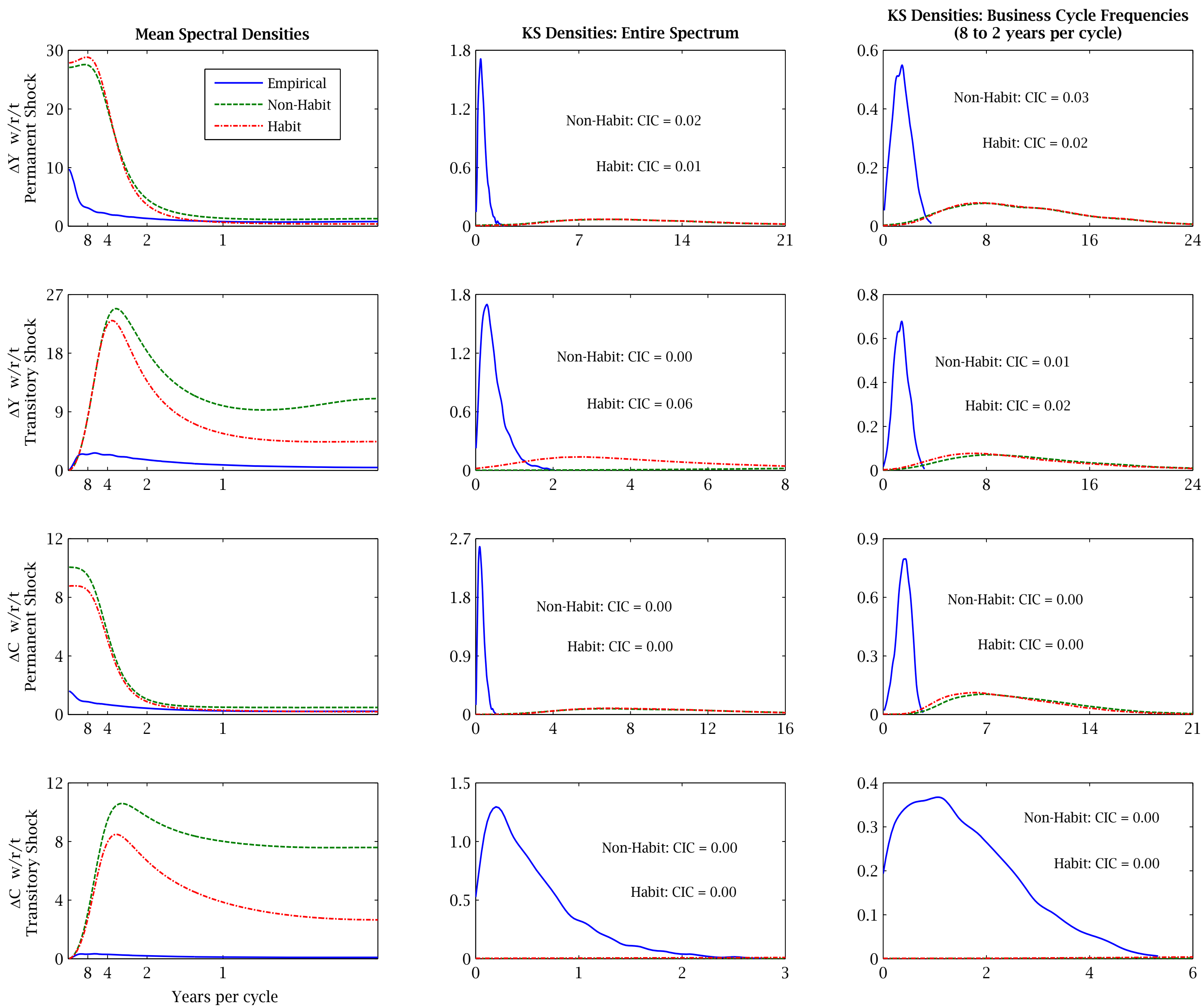


Figure A14: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Taylor Rule and $h \sim U(0.050, 0.499)$

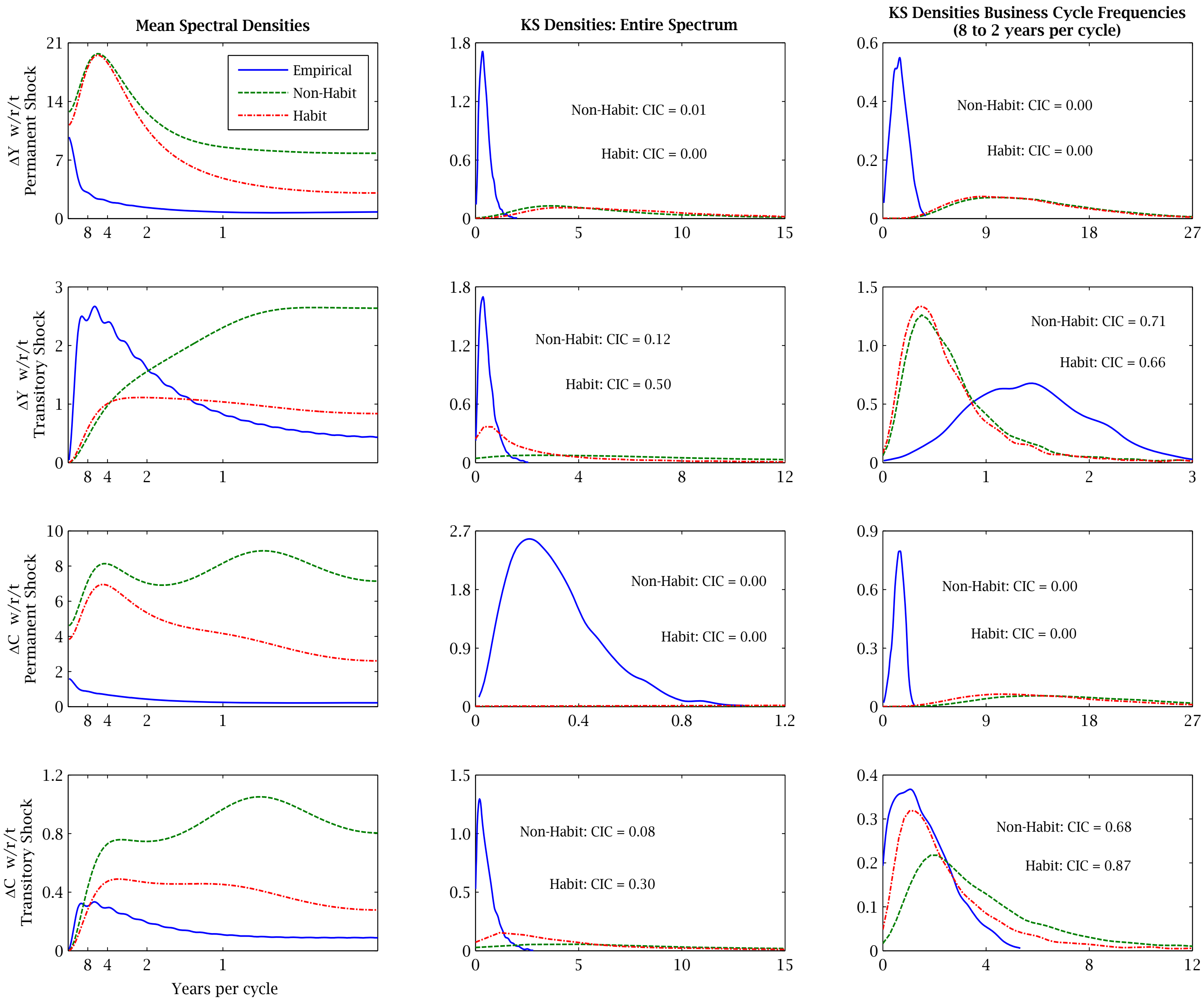


Figure A15: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule, Only Sticky Prices, and $h \sim U(0.050, 0.499)$

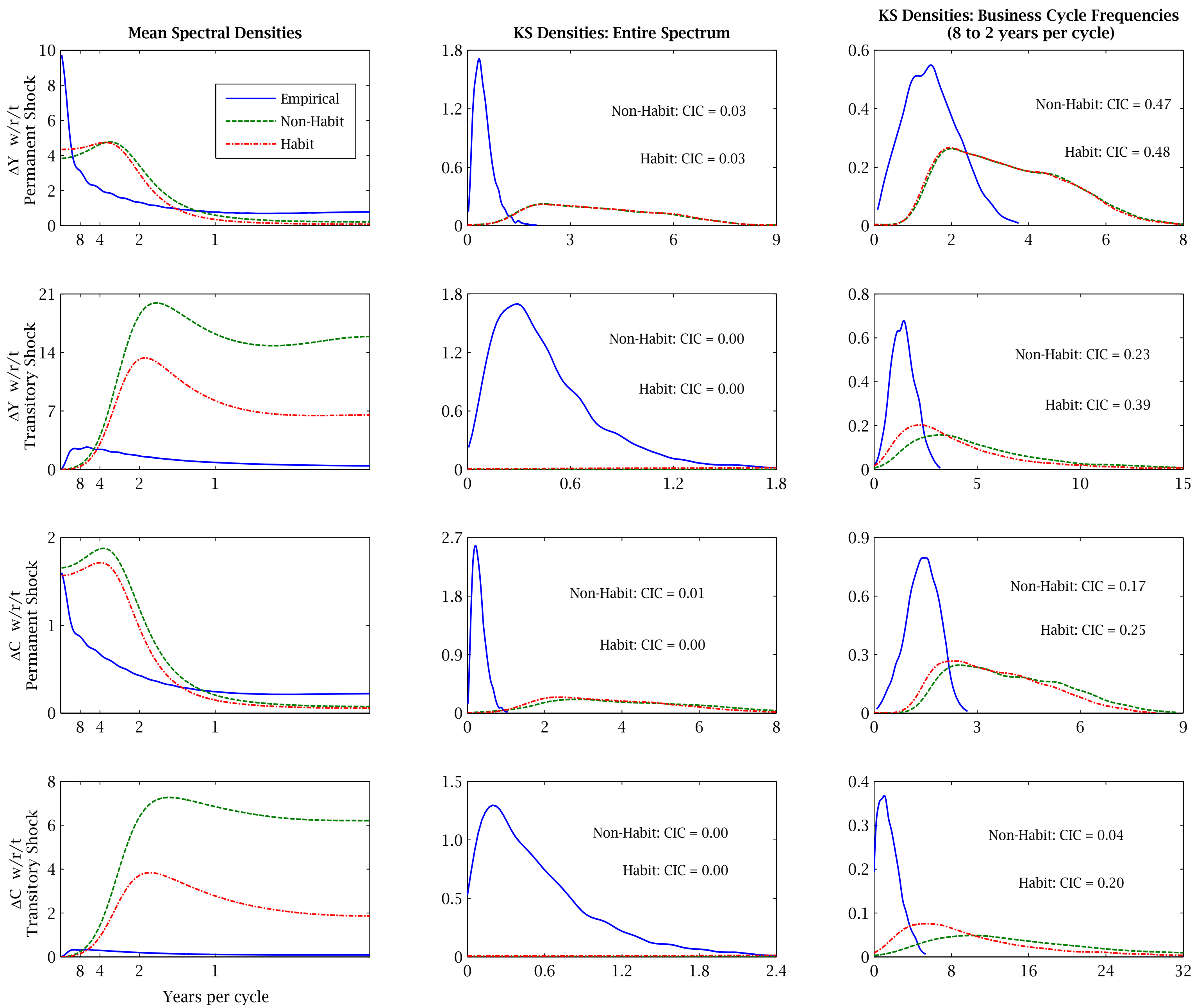


Figure A16: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule, only Sticky Prices, and $h \sim U(0.050, 0.499)$

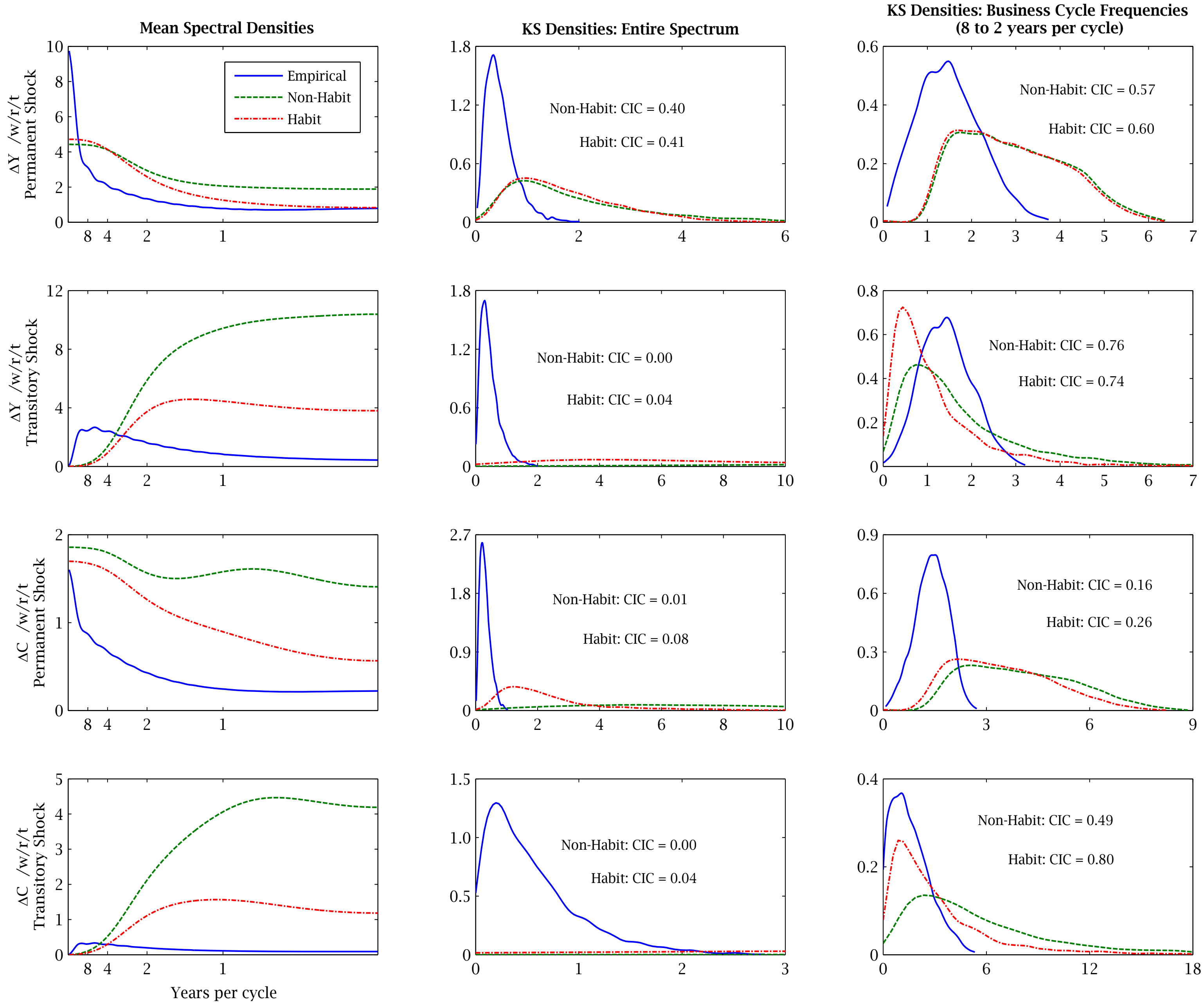


Figure A17: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule, Only Sticky Wages, and $h \sim U(0.050, 0.499)$

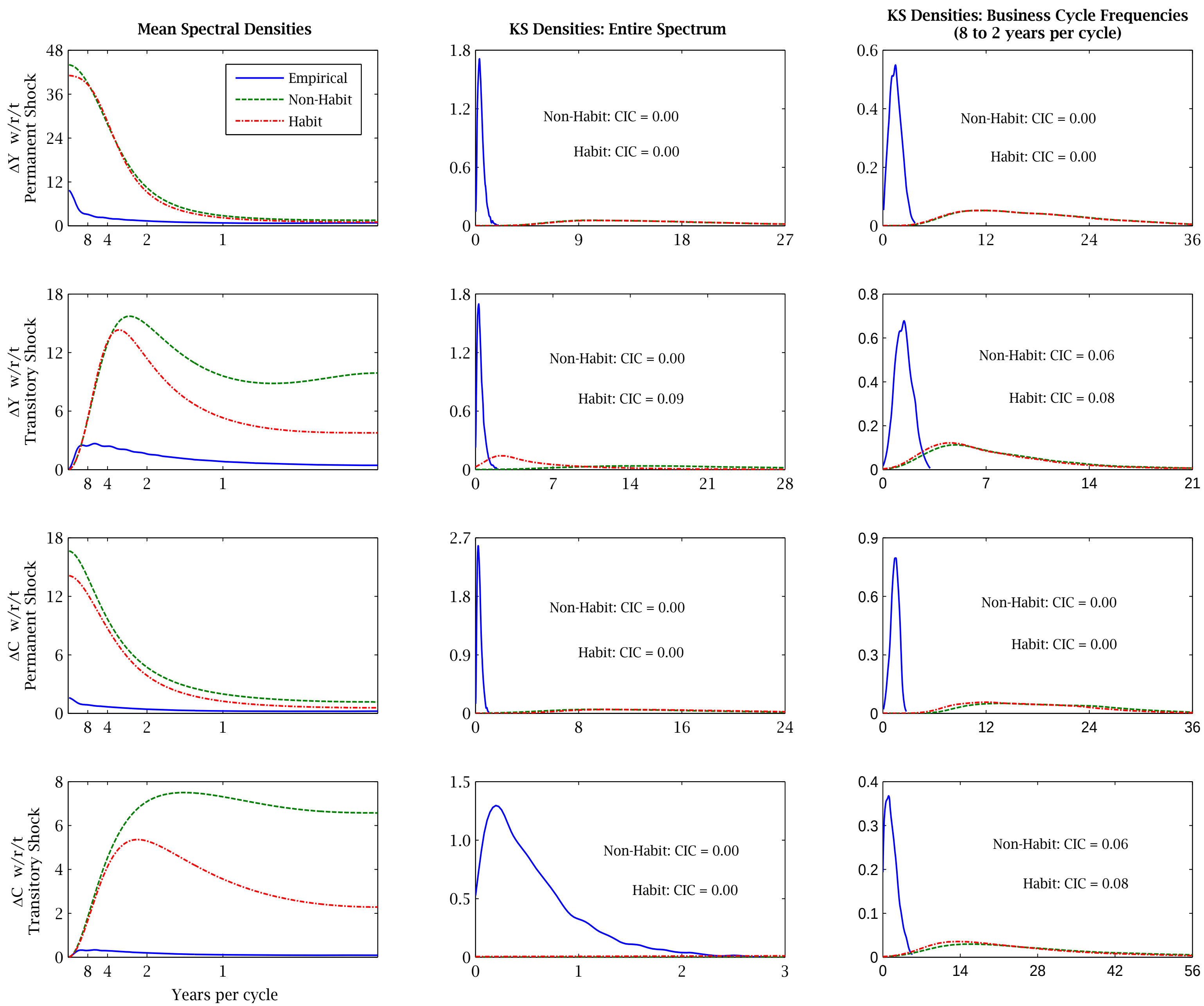


FIGURE A18: MEAN STRUCTURAL $\mathcal{P}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR NKDSGE MODELS WITH THE TAYLOR RULE, ONLY STICKY WAGES, AND $h \sim U(0.050,\ 0.499)$

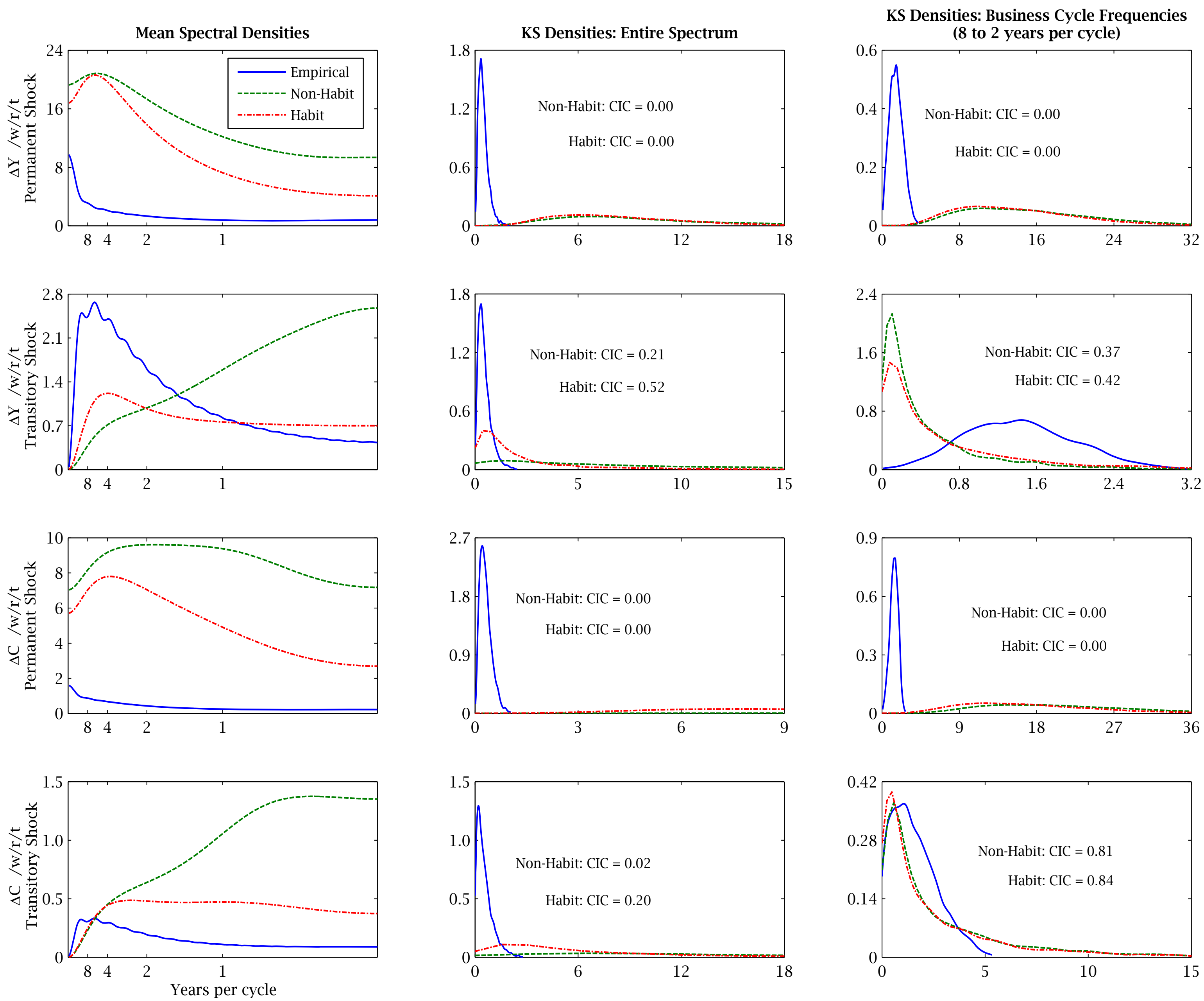


Figure A19: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Money Growth Rule Using a VAR(4)

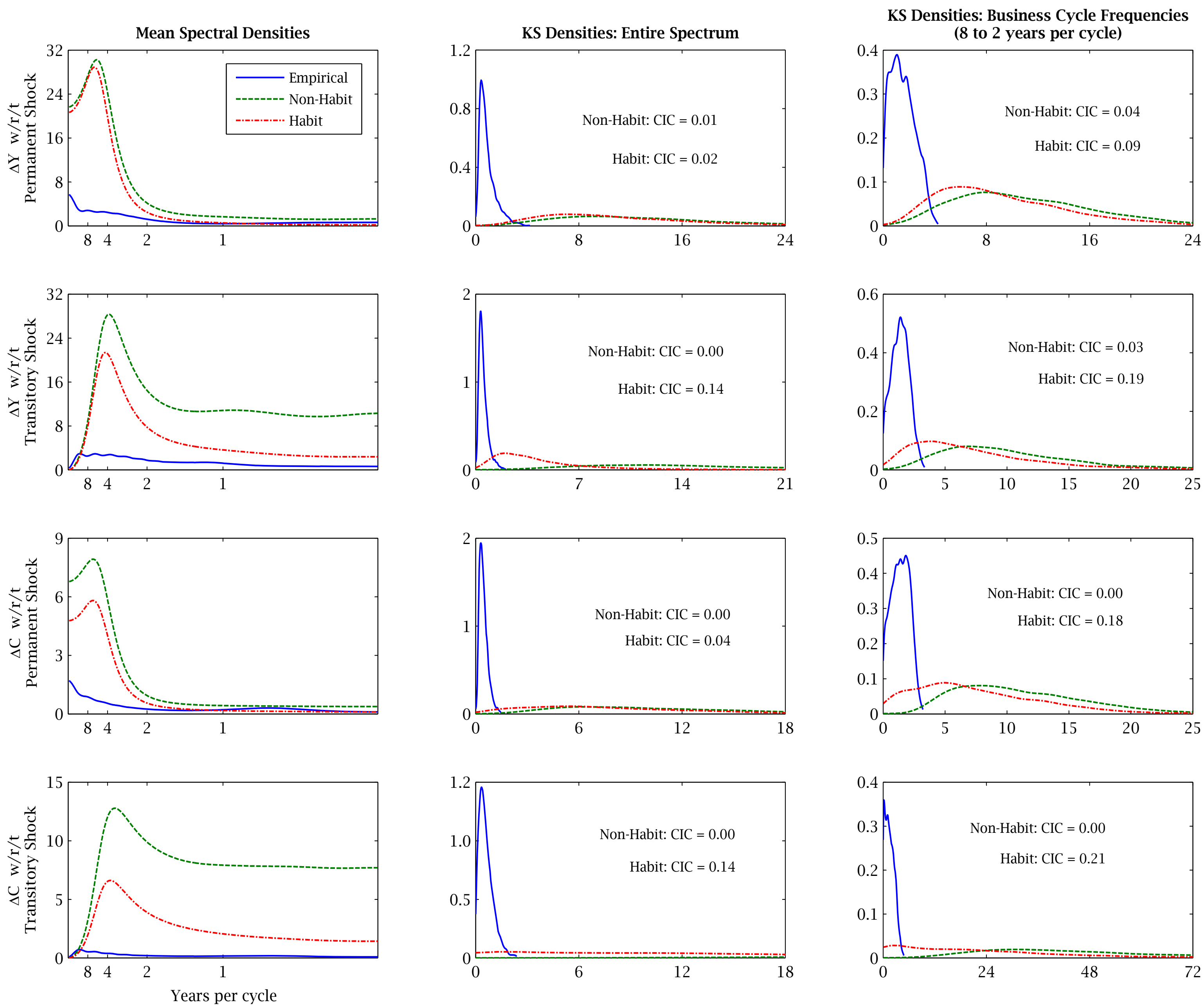


Figure A20: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with the Taylor Rule Using a VAR(4)

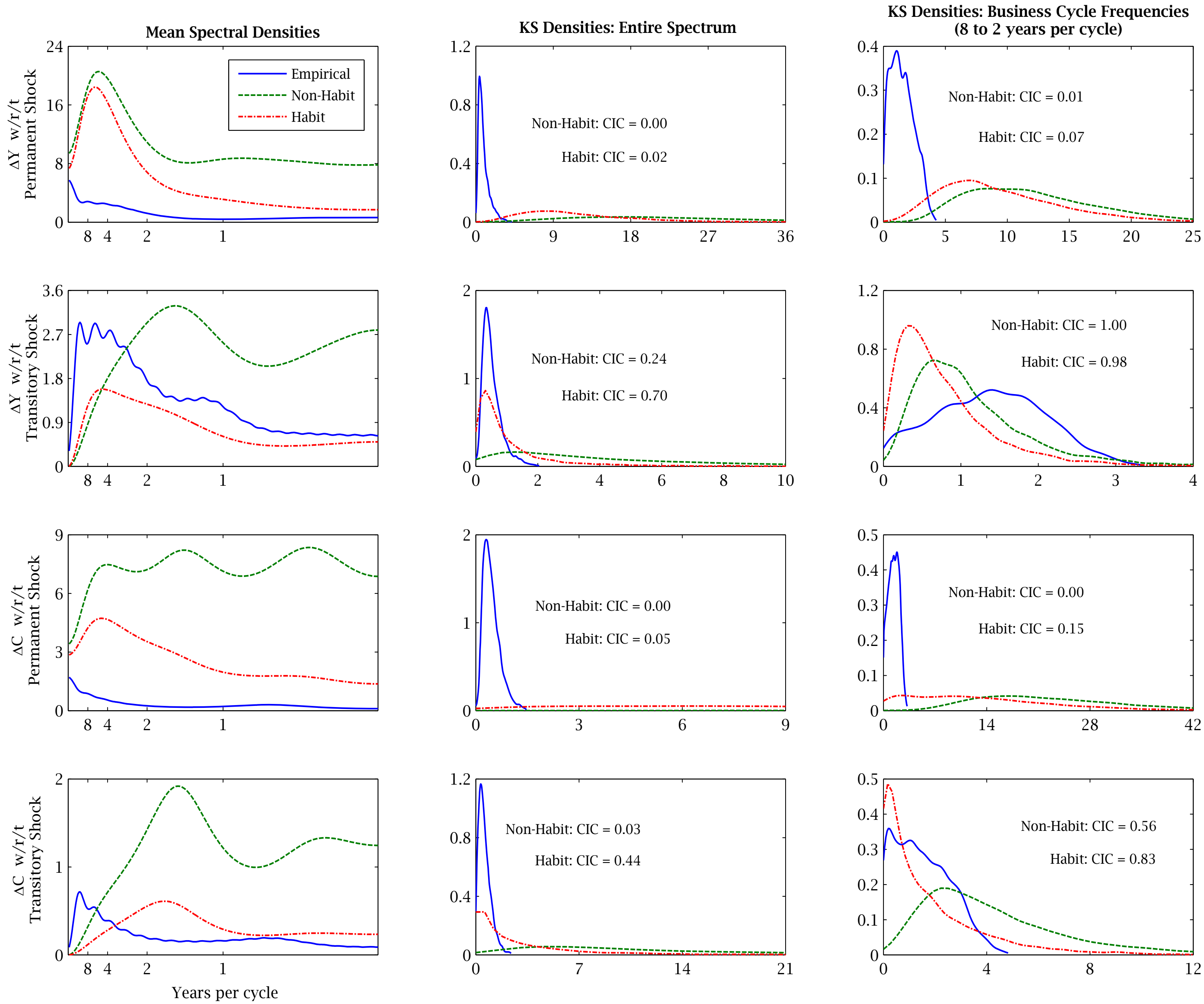


Figure A21: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule and Only Sticky Prices Using a VAR(4)

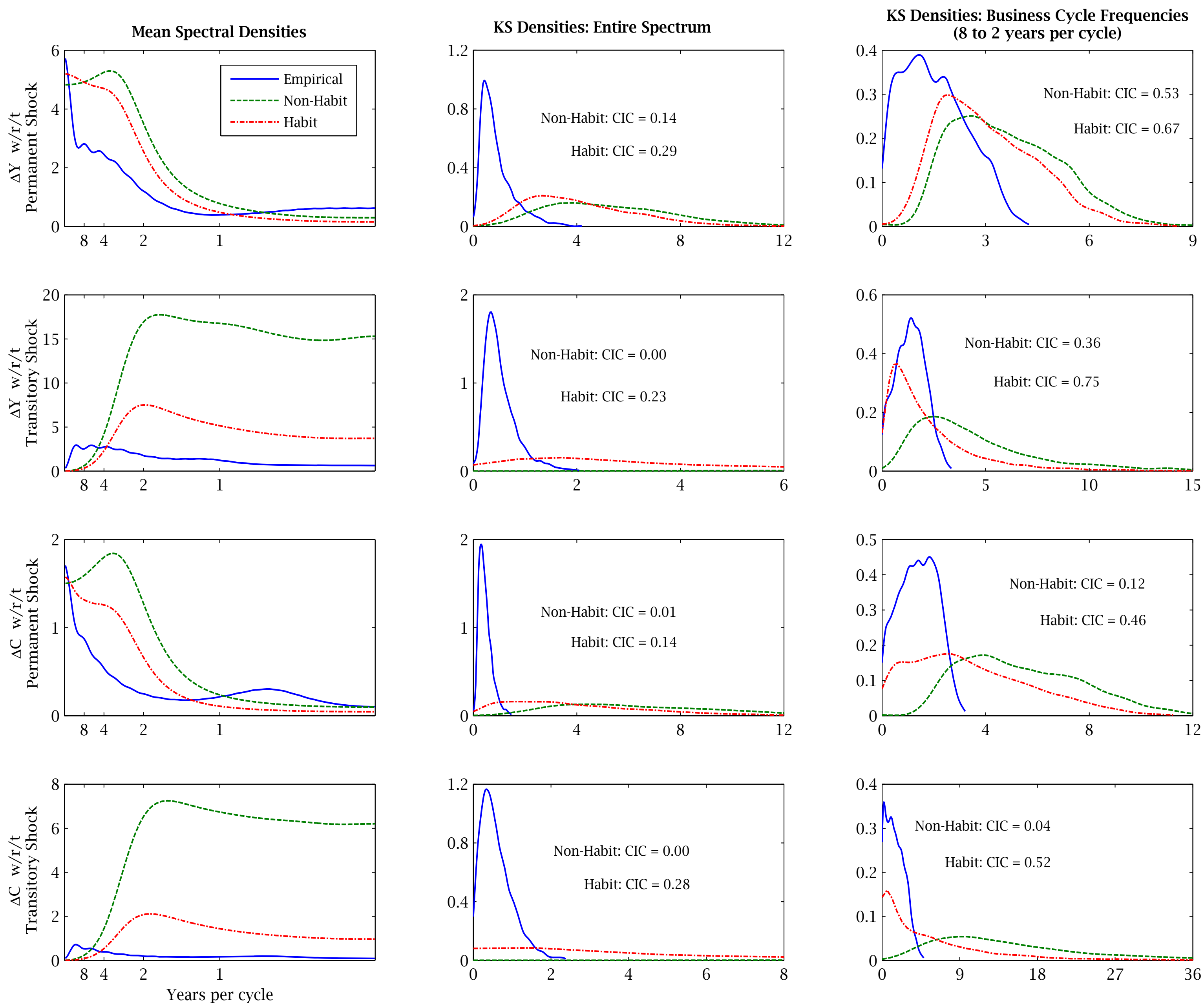


Figure A22: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule and only Sticky Prices Using a VAR(4)

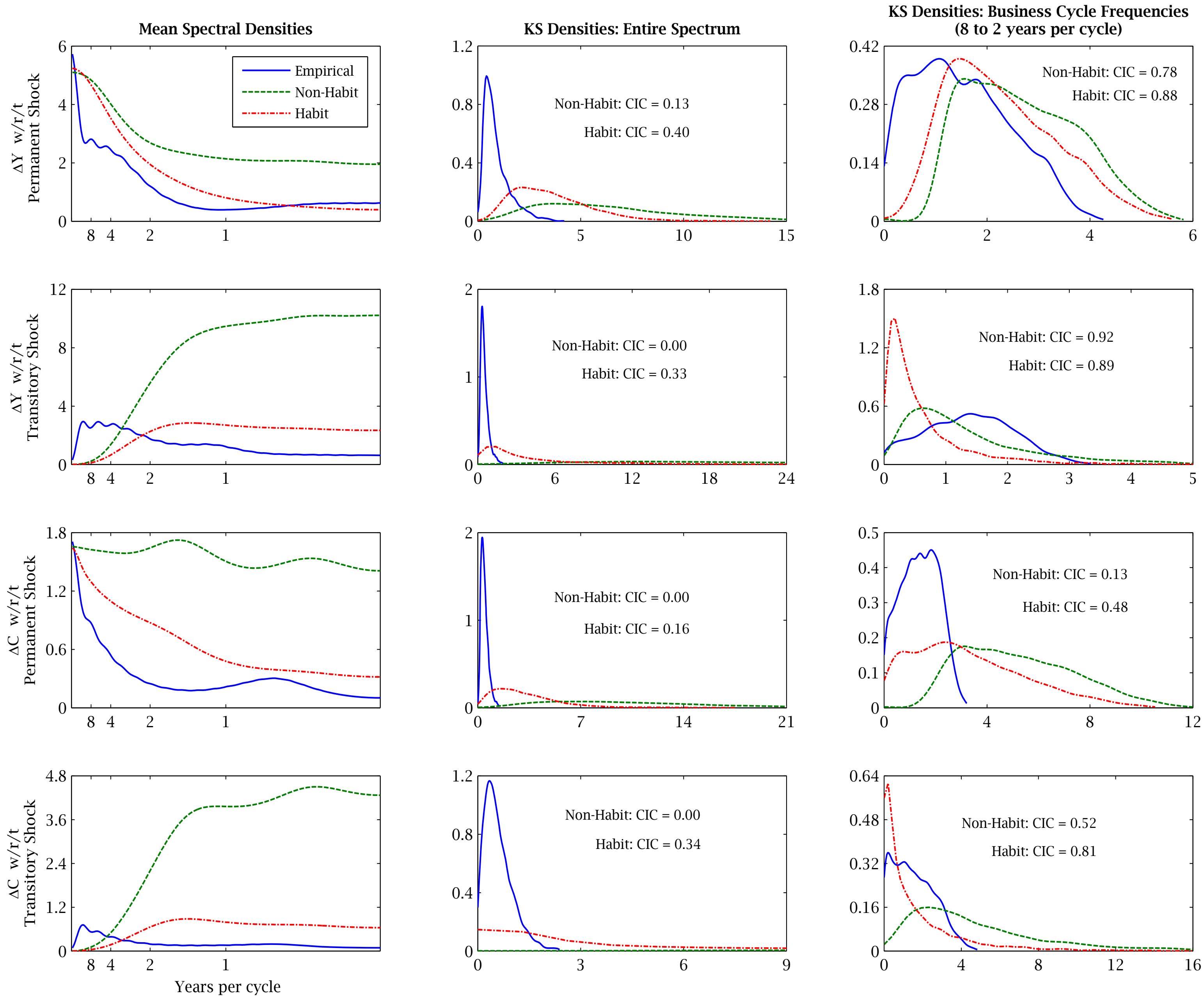


Figure A23: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Money Growth Rule and Only Sticky Wages Using a VAR(4)

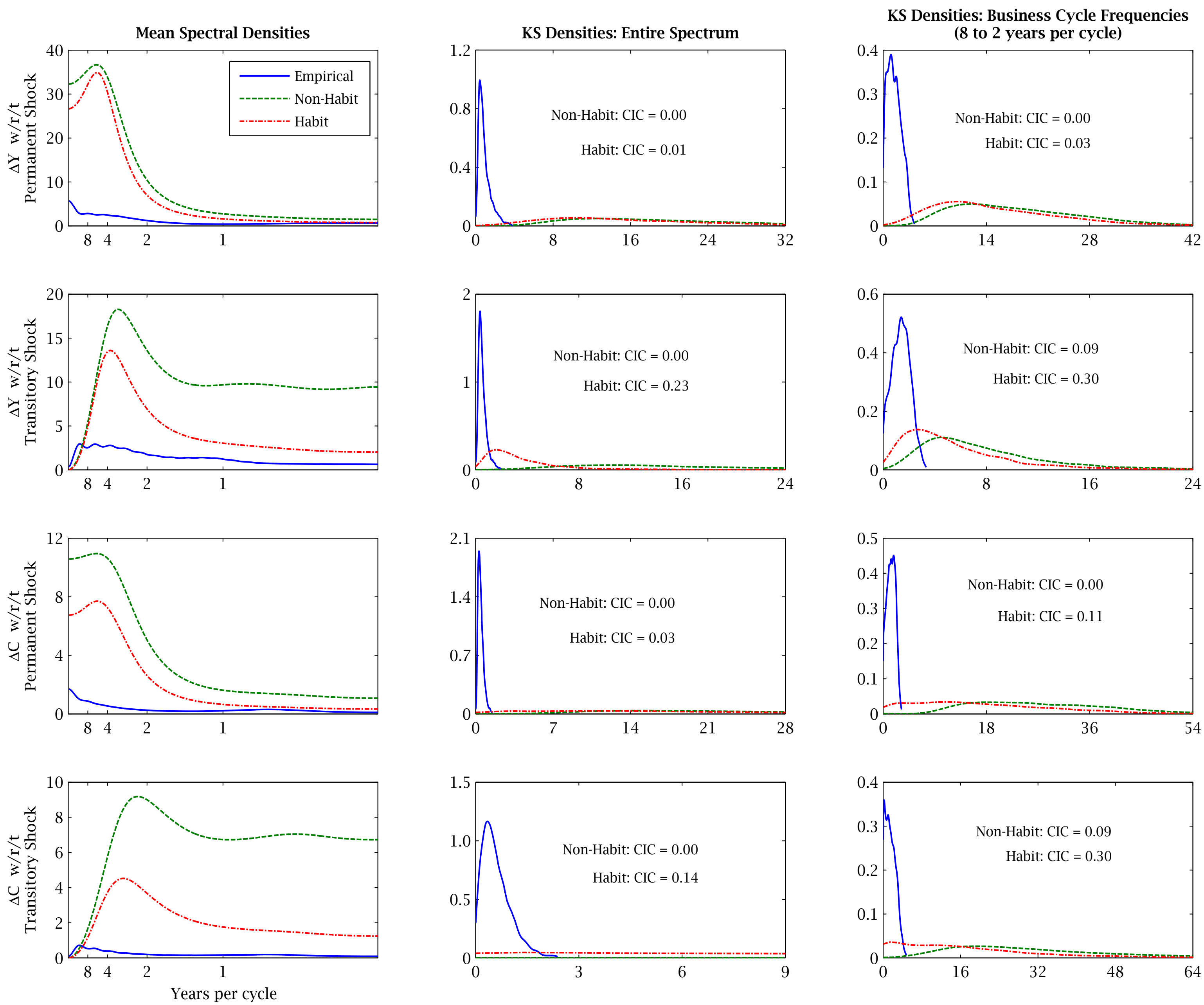


Figure A24: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with the Taylor Rule and only Sticky Wages Using a VAR(4)

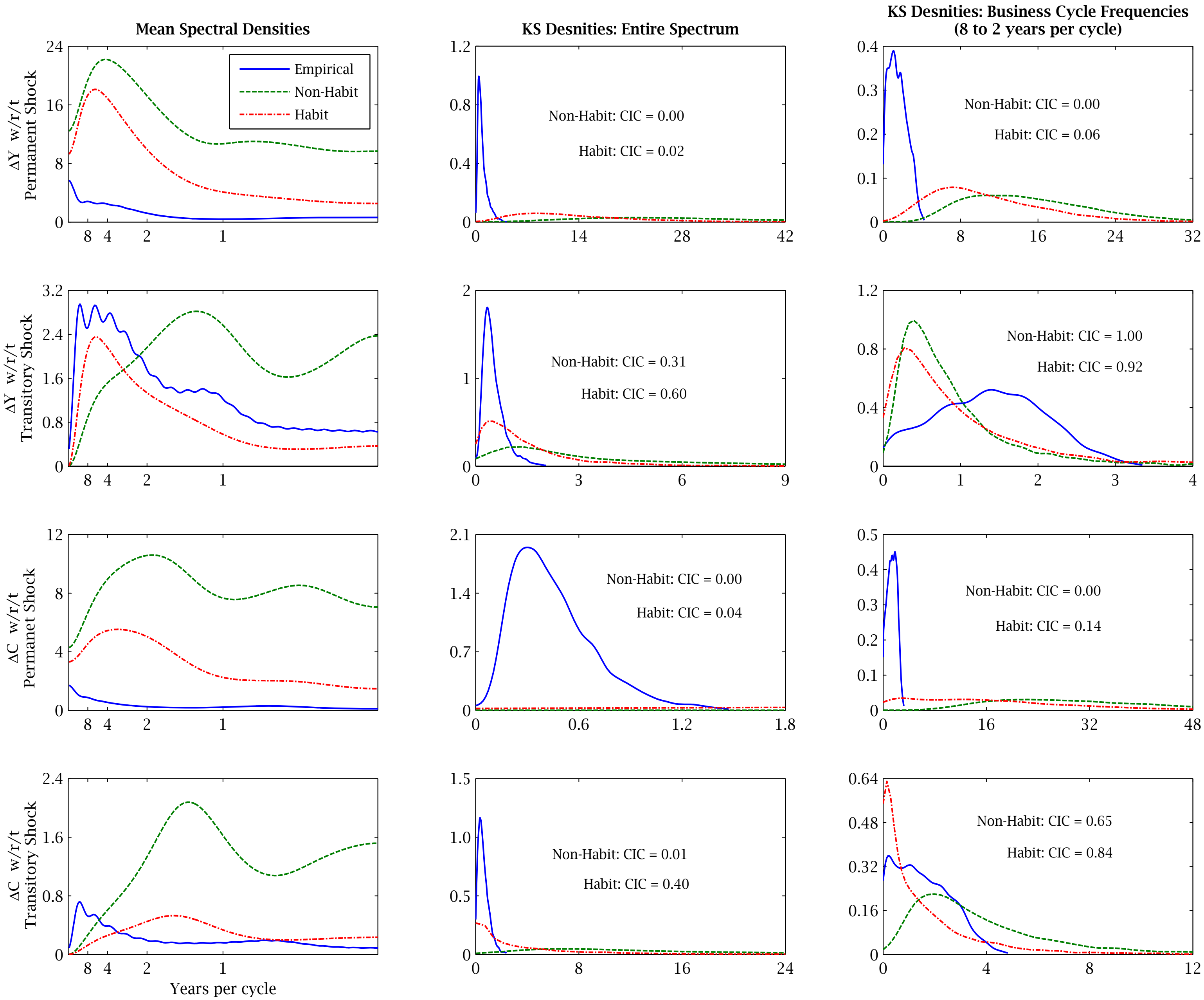


Figure A25: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for Baseline NKDSGE Models with the Money Growth Rule

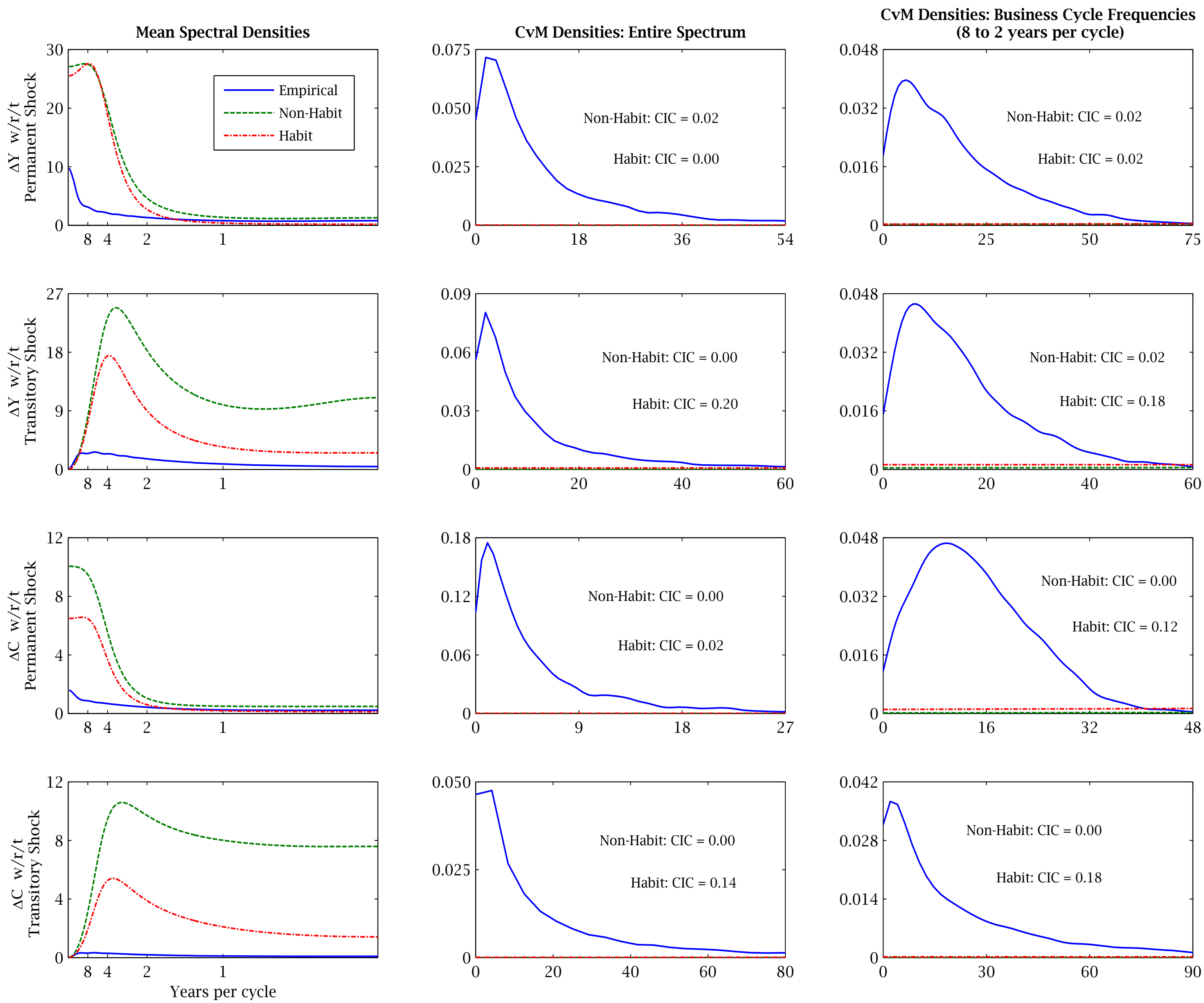


Figure A26: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for Baseline NKDSGE Models with the Taylor Rule

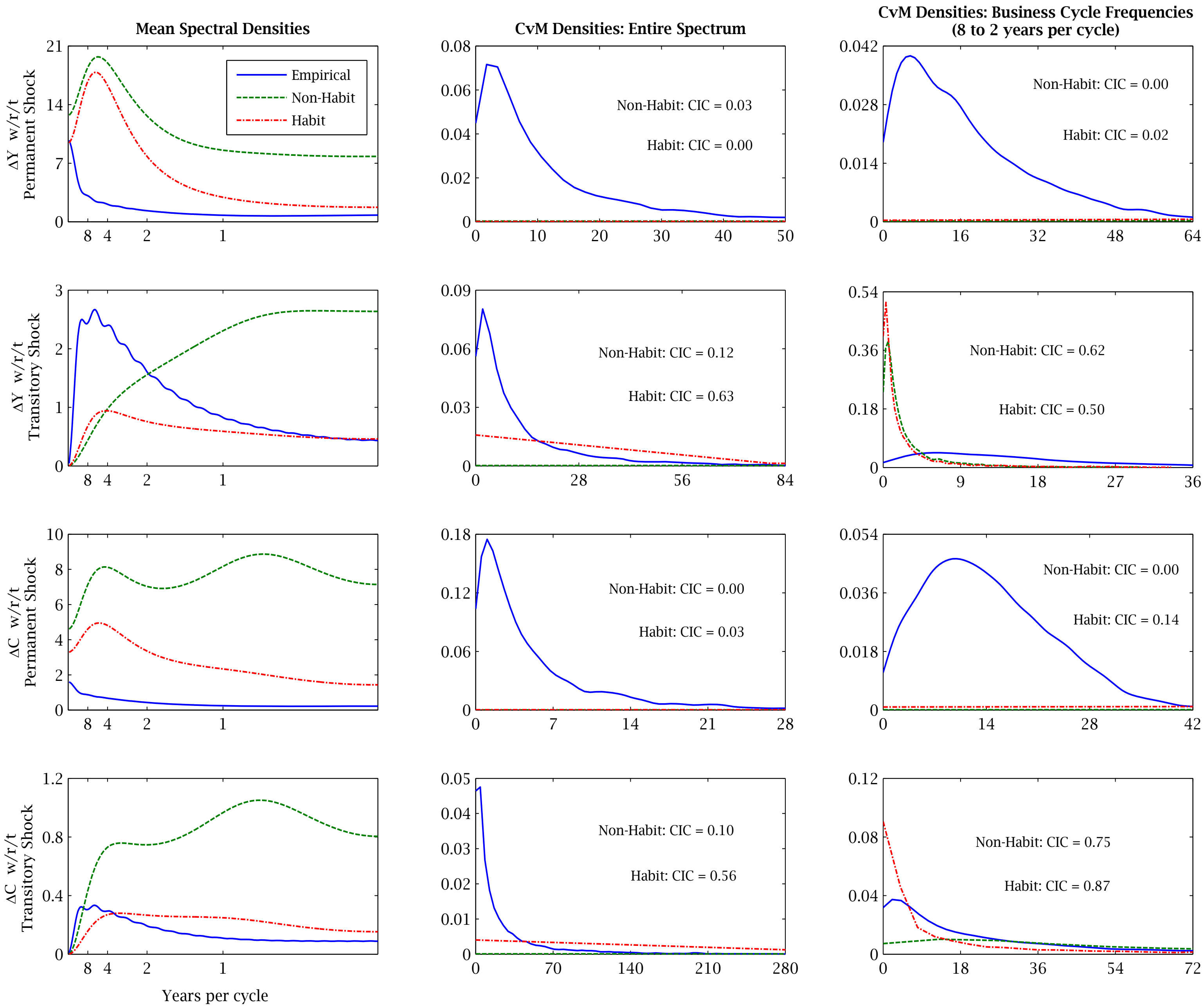


Figure A27: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with the Money Growth Rule and Only Sticky Prices

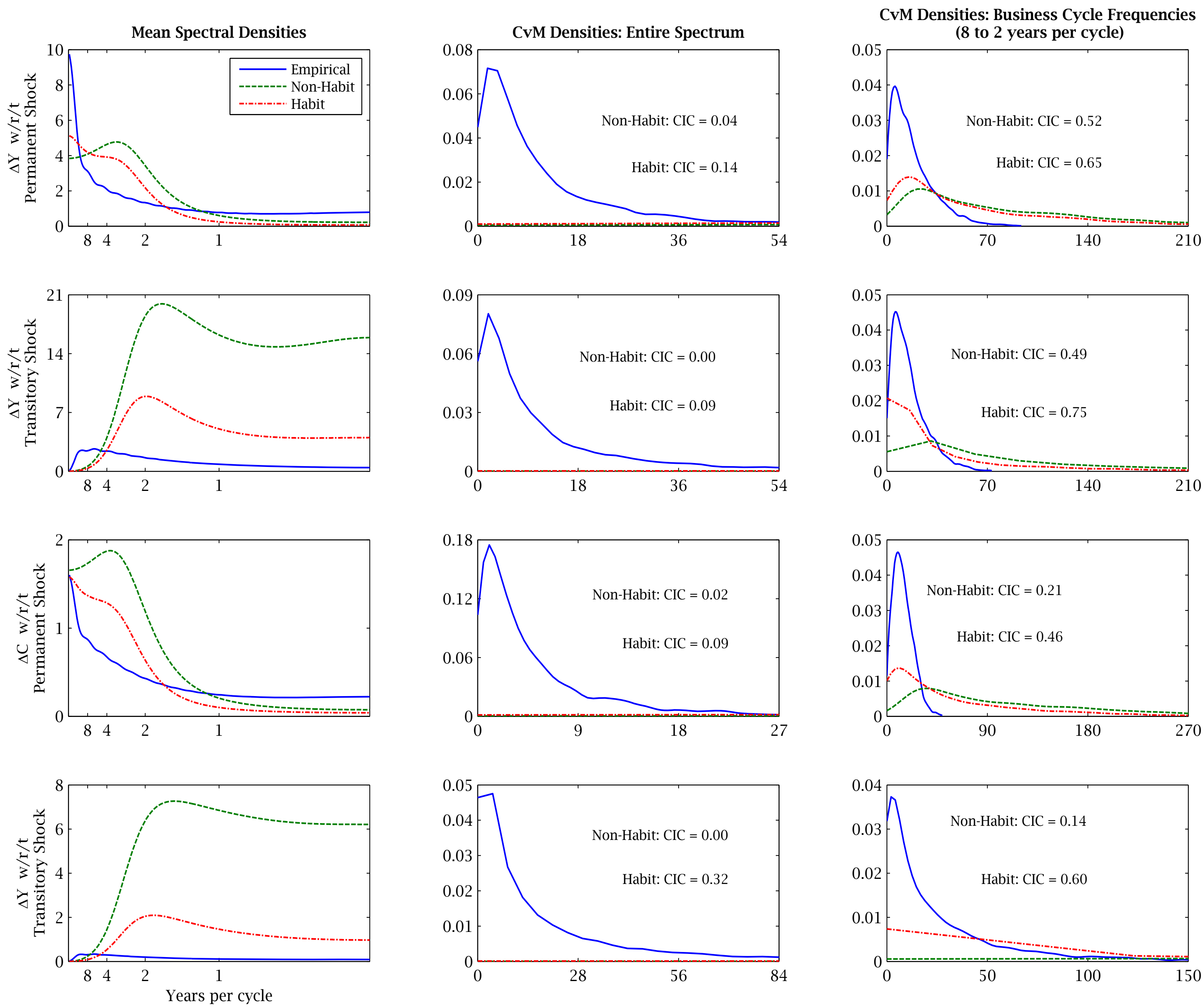


Figure A28: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with the Taylor Rule and only Sticky Prices

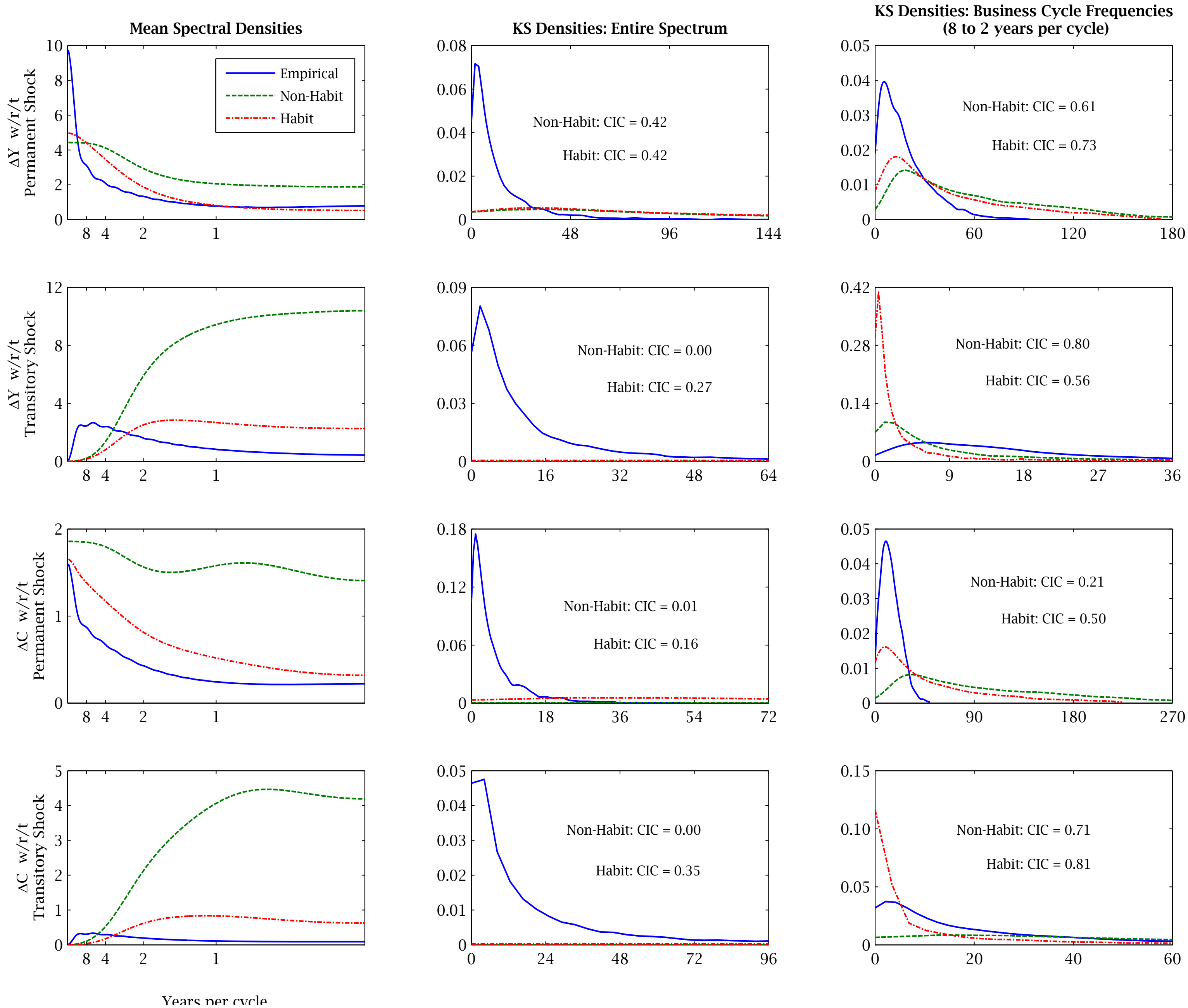


Figure A29: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with the Money Growth Rule and Only Sticky Wages

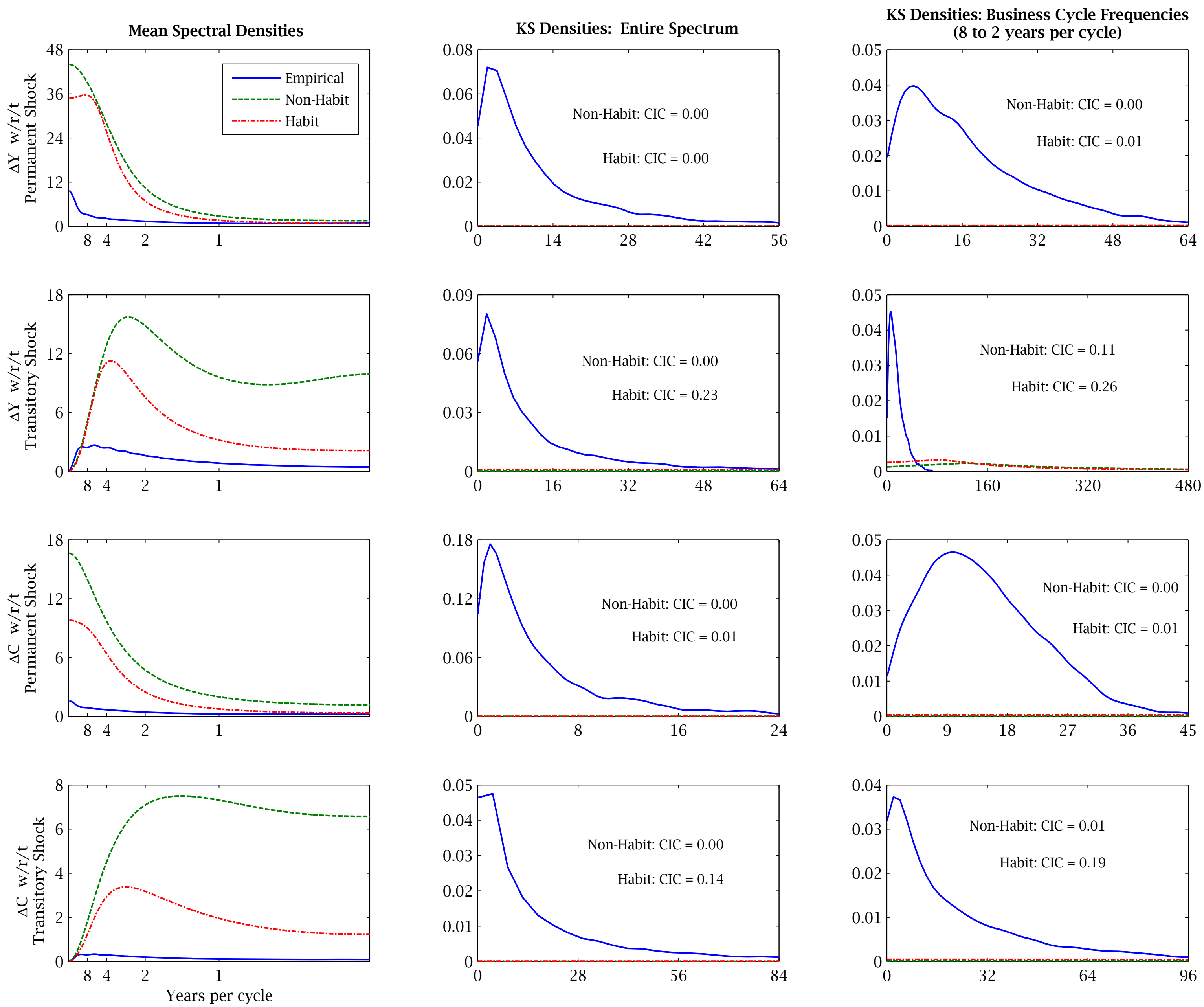


Figure A30: Mean Structural $\mathcal{P}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with the Taylor Rule and only Sticky Wages