Standards and Innovation: Technology vs. Installed Base *

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Abstract

We present a framework to examine how a standard evolves when a standard consortium or a firm (incumbent) innovates either to improve the standard or to strengthen installed base which increases switching cost. Both investments make it more difficult for another firm (entrant) to introduce a standard, also by investing in technology improvement. We show that incumbent's strategy will differ according to if the technology is in infancy or it has matured. The incumbent will deter entry when the technology is in infancy and return from investment is high. In this case ability to raise switching cost is important since entrant also has low cost. If the technology is mature and return to investment is low, then incumbent will choose to allow entry and there is co-existence of two standards. Replacement of standard by the entrant never occurs in equilibrium.

Key Words: standards, innovation, installed base

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1 Introduction

In this paper, we consider a situation where there is a standard in place and examine the incentive of a firm or any other entity with a stake in the current standard to invest in technology to improve the standard or installed base. Having a stake in a standard includes owning patents for the standard or manufacturing products under the standard. In order to maintain the standard, it can either improve the technology and upgrade the standard, or it may invest in installed base to take advantage of inertia (Farrell and Saloner, 1985). Upgrade of a standard will keep it attractive to consumers while investing in installed base increases the consumers' cost of switching to the new standard.

We employ a two stage game where in stage one, incumbent invests in upgrade and installed base and then the entrant invests to improve its potential standard technology. The investments determine the qualities of respective products and switching cost that consumers than buy from the entrant incur. In stage two, firms choose prices simultaneously, i.e., engage in Bertrand competition.

We adopt the Laffont, Rey, and Tirole (1998) approach with differentiated products and heterogeneous consumers that have elastic demand. Thus our model best describes a market such as a smart phone where there are competing platforms, each vendor identified with a platform. Consumers pay a fixed cost and per unit fee and there is cost of switching to a different provider.

Incumbent and entrant also represent patent pools or standard consortium and consumers can be interpreted to be manufactures that pay licensing royalties. It can also be interpreted as game console market if one takes into account the indirect payments consumers make to the console manufacturer through games. Part of price of a game goes to the console manufacturer in licensing fees.[1] The stage two market analysis is a special case of models of non-linear price competition (Calem and Spulber, 1984; Oren, Smith and Wilson, 1983) when restricted to zero switching cost. We provide a more complete characterization of the prices and the welfare implications.

The second stage Bertrand competition will result in one of four outcomes according to configuration of technology and switching costs chosen in stage one: (I) only firm 0 (II) only firm 1 (III) co-existence (unique equilibrium), and (IV) co-existence (multiple-equilibria). "Only firm 0" in regime (I) means the incumbent deters entry through upgrade or more inertia of installed base. "Only firm 1" in regime (II) means the entrant's quality is so good that it drives incumbent out of the market and the standard is replaced.

We characterize the subgame perfect Nash equilibrium of the whole game. The SPNE outcomes are Regimes (I) or (III). Regime (I) occurs when technology improvement is not costly. In this case, incumbent invests in technology improvement or installed base to deter entry. The existing standard will be upgraded. If the technology improvement is costly, the difference in quality

[1] Both console and software are produced by a single firm or at least production is coordinated. We do not model two-sided market.

of the incumbent and entrant will not be too large so that both firms are in the market. Regime (II) never occurs in equilibrium. This is because by investing slightly more in stage one, Firm 0 will avoid being priced out of the market in which case over all payoff will be negative since profit is zero but investment is sunk. Since Regime (II) never occurs, there never will be replacement of a standard in our framework.

Given the decreasing returns to investment in technology, innovation cost will be low when technology is in infancy. In this case incumbent is able to deter entry by upgrading or increasing switching cost. On the other hand, if the technology is mature so that innovation cost is high, then there will be co-existence. Better technology will always increase consumer surplus but higher switching cost decreases consumer surplus, even when both firms are in the market.

Farrell and Saloner (1985) examined a situation where firms can either adopt a technologically superior standard or rely on inertia and show that firms choose not to improve the standard when there is incomplete information. In their framework, technologically superior standard is exogenous to the firms and choice of standards is a coordination problem. Standard is based on the network effect and therefore coordination relevant. We capture the cost of moving from one network to the other as switching cost in a consumer's optimization decision. We then endogenize the technological improvement as investment choice of the firms, as well as the switching cost.

Cabral and Salant (2010) also considers firms that invest in quality improvement of a standard. Their focus is on the effect of unifying a standard (from co-existence of two standards) on incentive to improve the standard. They do not take into account the market interaction of quality improvement and merging and it is assumed that single standard will always improve profit of both firms from network effect. One can interpret that co-existence (incompatible) to single standard (compatible) to be switching cost reduction from infinity to zero. In their framework, technology improvement is a predetermined one step, contrary to choice of size of improvement in our framework. Thus in their analysis, firms are choosing either to reduce switching cost before or after investing in technology. The choice is not "which" but "which" first. We are able to focus on "which" strategy by explicitly modeling consumer behavior.

In the next section, we briefly describe product market and characterize the Bertrand equilibrium, given technology and switching cost. We characterize the equilibrium choices of technology and switching cost in section 3, completing characterization of subgame perfect Nash equilibrium. We discuss consumer and social surplus implications in Section 3.1, and close with closing remarks and possible extensions in Section 4. All proofs are given in the Appendix.

2 Framework

We consider a two stage game of two firms, firms 0 and 1. Firm 0 "owns" the current standard in the sense that it has a stake and controls the current standard. Firm 1 can enter the market if its technology and standard is sufficiently good. In stage one, both firms sequentially invest in technology which determines the level of standard. In stage two, firms engage in Bertrand price competition, given the technology investments made in stage one. In the beginning, Firm 0 is the only firm in the market. So it is possible to interpret Firm 0 as the incumbent and Firm 1 as the entrant. We characterize the subgame perfect Nash equilibrium strategies, technology investment choices, and prices.

We capture the product market as a Hotelling model where consumers are distributed uniformly over the interval [0,1]. Firm 0 is at point 0 and Firm 1 is at 1. Each consumer purchases at most one unit of the good from one of the firms. When a consumer at $x \in [0, 1]$ purchases from Firm 0 at price p_0, his surplus is $v_0 - p_0 - tx$, where t is the per unit transportation cost. If the same consumer purchases from Firm 1, it means he must switch to a new standard and thus he incurs switching cost, S. Consumer's surplus will be $v_1 - p_1 - S - t(1 - x)$.

The intrinsic value of the products, v_i are determined by the technology investments made in stage one. The standard already in place is technology level $\overline{v}$ and we assume,

$$v_i \geq \overline{v} \geq 2t. \tag{M}$$

Any positive investment in stage one by Firm i implies $v_i > \overline{v}$. The second inequality implies that a monopolist will sell to all consumers charging price $v_i - t$. Firm 0 will have sold to all consumers. So all consumers that buy from Firm 1 will incur switching cost S.

2.1 Bertrand Competition Equilibrium

Using the demand derived in the Appendix, we have Firm 0's profit as a function of (p_0, v_0) and (p_1, v_1). Standard Hoteling model analysis (outlined in the Appendix) yields the following Proposition characterizing Bertrand competition. It is summarized in the following proposition and illustrated in Figure 2.

Proposition 1. *There are four regimes of stage two Bertrand price competition equilibrium. Regimes depend on the intrinsic values v_0 and v_1 and transportation cost t. Marginal consumers, $\hat{x}$,$\hat{x}_0$, and $\hat{x}_1$, are defined in the Appendix.*

Regime(I) Only Firm 0 is in the market

If $v_1 - S \leq v_0 + 3t$, the equilibrium prices are

$$p_0^*(v_0, v_1, S) = v_0 - v_1 + S - t, \quad p_1^*(v_0, v_1, S) = S.$$

In this case, all consumers buy from the incumbent. The equilibrium profits are,

$$\pi_0^*(v_0, v_1, S) = v_0 - v_1 - S - t, \quad \pi_1^*(v_0, v_1, S) = 0.$$

Regime(II) Only Firm 1 is in the market

If $v_1 - S \geq v_0 - 3t$, then equilibrium prices are

$$p_0^*(v_0, v_1, S) = 0, \quad p_1^*(v_0, v_1, S) = v_1 - v_0 - t.$$

Now, all consumers buy from the entrant. The equilibrium profits are,

$$\pi_0^*(q_0, q_1, S) = 0, \quad \pi_1^*(q_0, q_1, S) = v_1 - v_0 - S - t.$$

Regime (III) Two firms co-exist in the market (unique equilibrium)

If $v_0 + v_1 - S \geq 3t$ and $v_0 - 3t < v_1 - S < v_0 + 3t$, then equilibrium prices are

$$p_0^*(v_0, v_1, S) = \frac{v_0 - v_1 + S + 3t}{3}, \tag{1}$$

$$p_1^*(v_0, v_1, S) = \frac{v_1 - v_0 - S + 3t}{3}. \tag{2}$$

Both firms make positive sales. The marginal consumer is at $\hat{x}(p_0^*, p_1^*) =$

$\frac{1}{2} + \frac{v_0 - v_1 + S}{6t}$ and has positive surplus $\frac{v_0+v_1-S-3t}{2}$. The equilibrium profits are,

$$\pi_0^*(v_0, v_1, S) = \frac{1}{2t}\left(\frac{v_0 - v_1 + S + 3t}{3}\right)^2,$$
$$\pi_1^*(v_0, v_1, S) = \frac{1}{2t}\left(\frac{v_1 - v_0 - S + 3t}{3}\right)^2.$$

Regime (IV) Two firms co-exist in the market (multiple equilibria)

If $v_0 + v_1 - S < 3t$, then there are continuum of equilibria. The equilibrium prices indexed by $\alpha \in [0,1]$ are,

$$p_0^*(v_0, v_1, S) = \frac{(3-\alpha)v_0}{3} - (1-\alpha)\left(t - \frac{v_1 - S}{3}\right) \qquad (3)$$
$$p_1^*(v_0, v_1, S) = \frac{(2+\alpha)(v_1 - S)}{3} - \alpha\left(t - \frac{v_0}{3}\right). \qquad (4)$$

The marginal consumer is at $\hat{x}(T_0^*, T_1^*) = \hat{x}_0(T_0^*) = \hat{x}_1(T_1^*) = \frac{\alpha v_0}{3t} + (1 - \alpha)\left(1 - \frac{v_1 - S}{3t}\right)$ and has zero surplus. The equilibrium profits are,

$$\pi_0^*(v_0, v_1, S) = \frac{p_0^*(v_0, v_1, S)(v_0 - p_0^*(v_0, v_1, S))}{t},$$
$$\pi_1^*(v_0, v_1, S) = \frac{p_1^*(v_0, v_1, S)(v_1 - p_1^*(v_0, v_1, S) - S)}{t}.$$

Regime (I) occurs when v_0 is large relative to $v_1 - S$ so that equilibrium occurs on the $p_1 = p_0 - t$ segment of R_0. This occurs either when the entrant is significantly less efficient than the incumbent, or when the switching cost is very large, or both. Entry will not result in any consumers actually switching to the new supplier in this regime. However, because of the existence of the entrant, the consumers have higher surplus. In particular, the surplus of the consumer at $x = 1$ increases from 0 when the incumbent was a monopolist to p_1^* after entry. $x = 1$ is the marginal consumer and so is exactly indifferent between switching and not switching.

Regime (II) occurs when v_0 is small relative to $v_1 - S$. In this case, the equilibrium occurs on the $p_1 = p_0 + t$ segment of R_1. This will occur when the entrant is very efficient and the switching cost is low enough so that all consumers switch. Again, the entrant's fixed fee is constrained by the option consumers have of not switching. The consumer at $x = 0$ has positive surplus of p_0^*.

Both firms have positive sales in regimes (III) and (IV), which are depicted in Figure 1. Firms equally split the market when $v_0 = v_1 - S$, which is a subregime of regime (III). However, because of the switching cost, the entrant must be more efficient in order to have the same market share. The entrant will not reduce the final surplus by the whole amount of the switching cost because it takes into account the fact that the incumbent will also reduce its surplus in response. This is direct result of strategic complementarity. For both groups of consumers, equilibrium surplus decreases with the switching cost. However, we can easily show that the equilibrium fee only increases for the incumbent from (1) and (2). The incumbent charges a higher fee and increases its market share with higher switching cost so its profit is increasing in switching cost. The entrant has a lower market share and a lower fee so its profit decreases with the switching cost.

In regime (IV), the intersection of the best-response correspondences is the closed line segment between points $\left(\frac{v_0}{3}, t - \frac{v_0}{3}\right)$ and $\left(t - \frac{v_1-S}{3}, \frac{v_1-S}{3}\right)$. Among these equilibria, the one with largest share for the incumbent, $p_0^*(v_0, v_1, S) = t - \frac{v_1-S}{3}$ is the most profitable for the incumbent. This corresponds to $\alpha = 0$ in the proposition and is at the lower right end of the relevant line segment in Figure 1b.

It is interesting to note that this equilibrium coincides with the subgame perfect equilibrium outcome if prices were determined sequentially and the incumbent chooses first. This is because the best-response correspondence of the entrant (the second mover) is kinked at this point. Prices change from strategic substitutes to strategic complements at this point. The equilibrium reflects the strategic substitute nature of the strategies. When the switching cost increases, the surplus of the entrant's customers increases while that of entrant decreases. From (3) and (4), it is clear that the equilibrium fees for both decrease with higher switching cost. When switching costs increase, the entrant's equilibrium share decreases and the fixed fee is lower. So the entrant's profit unambiguously decreases with the switching cost. Higher switching costs result in lower fees but greater market share for the incumbent. Thus if the fee is relatively large, incumbent profits are increasing in switching cost.

If, in addition to assumption (M), we also assume that the entrant is sufficiently efficient, i.e., $v_1 - S \geq 2t$, then regime (IV) will never occur and the equilibrium will always be unique.

3 Investment Equilibrium

In this section, we try to consider the equilibrium choices of technology improvement and switching cost. Firm 0 can invest either to increase v_0 and improve the current standard, or invest in complementary technology and increase the base of the standard. This will increase the switching cost, S. Firm 1 invests in its technology. The actual investment will be improvement, Δ_i, $i = 0, 1$ over the status quo quality, $\overline{v}$, so that investment realizes quality level $v_i = \overline{v} + \Delta_i$. We assume $\overline{v} \geq 3t$ which is stronger than assumption (M) to simplify the analysis.

Specifically, we assume the indirect utility function takes the following form,

$$v_i = \overline{v} + \Delta_i, \quad i = 0, 1.$$

Cost of investment is,

$$C_0(\Delta_0, S) = \frac{\delta(\Delta_0 + S)^2}{2}, \qquad C_1(\Delta_1) = \frac{\Delta_1^2}{2}.$$

δ means the parameter of investment efficiency. Expected payoffs are,

$$\Pi_0(\Delta_0, \Delta_1, S) = \pi_0^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_0(\Delta_0, S),$$

$$\Pi_1(\Delta_0, \Delta_1, S) = \pi_1^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_1(\Delta_1).$$

$\pi_0(\cdot)$ and $\pi_1(\cdot)$ are defined by Proposition 1 for each Regime.

If there is no investment ($\Delta_0 = \Delta_1 = S = 0$), qualities will be $v_0 = v_1 = \overline{v}$ and regime (III) will transpire. We can consider firm 0 chooses $\Delta \equiv S + \Delta_0$ to maximize his profit since Δ_0 and S are symmetric in this setting. There are two possible regimes after Firm 0 has made its investment choice: $\Delta \equiv \Delta_0 + S > 3t$ (regime (I), Figure 3a) and $\Delta \equiv \Delta_0 + S < 3t$ (regime (III), Figure 3b).

In the regime (I) subgame, depending on firm 1's investment choice, the final outcome will be regime (I), (II), or (III). Next lemma shows the final outcome in the regime (I) .

Lemma 1. *In regime (I) subgame ($\Delta > 3t$), firm 1 does not invest anything and payoff is equal to zero. Then, the final outcome is regime (I).*

Next lemma shows that in regime (III) subgame, final outcome will be (III) or (II).

Lemma 2. *In the regime (III) subgame ($\Delta < 3t$), if $\delta > \frac{1}{3t}$, then Firm 1's optimal investment results in regime (III). Otherwise, firm 1 invests so that final outcome is Regime (II).*

If the final outcome is regime (II), firm 0's payoff will be negative since profit is 0. From the two lemmas, we obtain,

Proposition 2. *In equilibrium (SPNE), if $\delta \leq \frac{1}{3t}$, firm 0's investment is $\Delta^* > 3t$, firm 1 invests $\Delta_1^* = 0$ and final outcome is regime (I) (upgrade and deterrence). If $\delta > \frac{1}{3t}$, firm 0's investment is $\Delta^* < 3t$, firm 1 invests $\Delta_1^* > 0$ and final outcome is regime (III) (co-existence).*

There will be upgrade and no entry if the cost of investment is low, but co-existence when cost is high. Since we have symmetric costs, cost of investment is low for both incumbent and entrant. However, the incumbent is able to invest in switching cost which is more efficient in gaining relative advantage and thus is able to deter entry.

3.1 Welfare Analysis

When the switching cost is reduced, part of R_1 moves upward. However increases in marginal costs of production c_i move part of R_i downward. Within regime (III), a reduction in S will unambiguously increase consumer surplus. However if this results in S being higher than c_i, the total effect might be to reduce consumer surplus. This is because the equilibrium might then change from one regime to another by parameter changes. As a result, it is more useful to analyze welfare in the space of v_0 and $v_1 - S$.

The equilibrium consumer surplus and producer surplus for each of the four regimes (defined in Proposition 1) are summarized below. In regime (IV) where there are multiple equilibria, we choose the one that yields the highest payoff for the incumbent ($\alpha = 0$). The iso-consumer surplus lines are shown in Figure 4.

	Consumer Surplus	Producer Surplus
I	$v_1 - S + \frac{t}{2}$	$v_0 - v_1 + S - t$
II	$v_0 + \frac{t}{2}$	$v_1 - S - v_0 - t$
III	$\frac{(v_0 - v_1 + S)^2}{36t} + \frac{v_1 - S + v_0}{2} - \frac{5}{4}t$	$\frac{(v_0 - v_1 + S)^2}{9t} + t$
IV	$\frac{1}{2t}\left\{(t - \frac{v_1 - S}{3})^2 + (\frac{v_1 - S}{3})^2\right\}$	$v_0 - t - \frac{(v_0 - 2t)(v_1 - S)}{3t} + \frac{(v_1 - S)^2}{9t}$

Table 1: Consumer and Producer Surplus by Regime

In both regimes (I) and (II), the consumers are served by only one of the firms. In regime (I), the incumbent is the sole supplier and thus consumers never actually switch. However the fee they pay reflects the switching cost: the higher the cost, the greater the fee that the incumbent can charge. Thus consumer surplus is decreasing in the switching cost. The switching cost is actually "collected" by the incumbent and thus its profit is increasing in switching cost. The sum of the two surpluses, however, does not depend on switching cost which effectively determines the share of total surplus that accrues to consumers and to the incumbent. Recall that entry (although entrant does not actually sell anything) increases consumer surplus. So high switching costs will syphon some of the benefit to the incumbent.

Since everyone switches in equilibrium in regime (II), switching costs are actually incurred. Thus consumer surplus is increasing in the switching cost but producer surplus is independent of the switching cost because the fee, and thus size of demand in equilibrium, is independent of switching cost.

In regime (III), the switching cost is anti-competitive in the standard sense: consumer surplus decreases and producer surplus increases in the switching cost. This is because the switching cost reduces temptation to cut prices and therefore decreases surplus of consumers. In addition to reducing competition, the switching cost is a cost paid but never collected by anyone within this model. This also contributes to social surplus reductions with higher switching costs.

In regime (IV), the switching cost increases consumer surplus but it is questionable if this is actually pro-competitive. In this regime, higher switching costs increase the surplus for each of the consumers buying from the incumbent and reduce the surplus for those buying from entrant. In addition, the proportion of those buying from the incumbent increases. This increases total consumer surplus. Higher switching costs increase consumer welfare by skewing the surplus distribution so that there are more people in the higher surplus consumer group (which is advantaged) and fewer in the lower surplus consumer group (which is disadvantaged). Producer surplus decreases for a similar reason.

The iso-social surplus curves are presented in Figure 5, which shows the social benefits of equalizing $v_1 - S$ and v_0. Given some level of switching cost reduction, technologies that change the marginal cost may be better. Further, by allocating the costs carefully, it may be possible to realize distributional gain at the same time. For instance, unless S is reduced to zero, allocations that increase c_0 more than c_1 may achieve $v_1 - S = v_0$.

Recall that in regime (III), consumer surplus was always decreasing in S. Social surplus may increase with S in some regions of (III) if the producer surplus gains are large enough. This occurs where $v_1 - S \geq \frac{9}{5}t + v_0$ or $v_1 - S \leq -\frac{9}{5}t + v_0$. In these regions one firm is significantly more efficient, implying significant market power of one of the firms. In this case increasing switching cost does not hurt consumers that much at the margin while producers gain significantly.

4 Concluding Remarks

We have shown that firm or patent pool with a stake in the current standard will invest in upgrade or installed base to deter entry of another standard when the technology is in infancy, When technology is in infancy, cost of innovation is low not only for the incumbent but also for the entrant. Ability to invest in installed base on increase consumer switching cost becomes a valuable strategy in this case to deter entry. As the technology matures, and innovation becomes more costly, the incumbent no longer deters entry and there will be co-existence of standards in the market.

Our analysis suggests that competition and standardization policies should be technology life cycle dependent. While persistence of single standard maybe consistent with high level upgrades, it may also be result of high switching costs. The latter is likely to be consumer surplus reducing. There is argument for anti-trust intervention in this case. However, our analysis also shows that as the technology matures, there is likely to be entry and co-existence of standards even when there is no policy intervention.

When interpreting the incumbent in our analysis to be patent pools or consortium and entrant is firm or consortium (Hotelling model will be that of manufacturers), investment decision is very restrictive. Members of the incumbent entity coordinate investment in our analysis. This maybe problematic in that patent pools or consortiums do not make investment decisions collectively and there may be antitrust concerns regarding such behavior. There also may be more than entrant, although starting with one entrant seems a sensible approach when understanding evolution of standards.

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Appendix

Derivation of Demand with Assumption (M)

We define the bench marks, $\hat{x}_0(p_0)$, $\hat{x}_1(p_1)$, and $\hat{x}(p_0, p_1)$, by

$$v_0 - p_0 - t\hat{x}_0(p_0) = 0, \quad v_1 - p_1 - t(1 - \hat{x}_1(p_1)) = 0, \tag{5}$$

$$v_0 - p_0 - t\hat{x}(p_0, p_1) = v_1 - p_1 - t(1 - \hat{x}(p_0, p_1)). \tag{6}$$

All consumers to left (right) of $\hat{x}_0(p_0)$ ($\hat{x}_1(p_1)$) have positive utility buying from firm 0 (firm 1). All consumers to left (right) of $\hat{x}(p_0, p_1)$ have greater utility from buying from firm 0 (firm 1). By definition, it must be that either (i) $\hat{x}_0(p_0) < \hat{x}(p_0, p_1) < \hat{x}_1(p_1)$, or (ii) $\hat{x}_0(p_0) \geq \hat{x}(p_0, p_1) \geq \hat{x}_1(p_1)$. In case (i), there is an interval of consumers in the middle that do not buy at all. In case (ii), all consumers will buy with three possibilities: all buy from firm 0 if $\hat{x}(p_0, p_1) \leq 0$, all buy from firm 1 if $\hat{x}(p_0, p_1) \geq 1$, and otherwise there is positive sales by both firms.

We use the surplus W_i as a choice variable instead of p_i: $W_0 = v_0 - p_0$ and $W_1 = v_1 - p_1 - S$. Since W_1 already takes into account the switching cost when buying from firm 1, W_1 only needs to cover the transportation cost. From this substitution, we have $\hat{x}_0(p_0) = \frac{W_0}{t}$, $1 - \hat{x}_1(p_1) = \frac{W_1}{t}$, and $\hat{x}(p_0, p_1) = \frac{W_0 - W_1 + t}{2t}$.

Proof of Proposition 1

The problem is to find the W_0 to maximize,

$$\pi_0 = \begin{cases} \pi_0^A = (v_0 - W_0)\frac{W_0}{t} & \text{for } W_0 \leq t - W_1, \\ \pi_0^B = (v_0 - W_0)\frac{W_0 - W_1 + t}{2t} & \text{for } t - W_1 < W_0 \leq t + W_1, \\ \pi_0^C = v_0 - W_0 & \text{for } W_1 + t < W_0. \end{cases}$$

Note that the problem is independent of p_1. Straightforward but tedious calculation yields the following.

Lemma 3. *Firm 0's best response correspondence $W_0 = R_0(W_1)$ is,*

(1) If $t < \frac{v_0}{3}$, then

$$R_0(W_1) = \begin{cases} t + W_1 & \text{for } W_1 \leq v_0 - 3t, \\ \frac{v_0 + W_1 - t}{2} & \text{for } W_1 \geq v_0 - 3t. \end{cases}.$$

(2) If $t > \frac{v_0}{3}$, then

$$R_0(W_1) = \begin{cases} t - W_1 & \text{for } W_1 \leq t - \frac{v_0}{3}, \\ \frac{v_0 + W_1 - t}{2} & \text{for } t - \frac{v_0}{3} \leq W_1. \end{cases}.$$

(3) If $t = \frac{v_0}{3}$, then

$$R_0(W_1) = \frac{v_0 + W_1 - t}{2} \textit{for all } W_1 \geq 0.$$

Firm 1's best response correspondence is obtained similarly, and differs only by the fact that the switching cost must be taken into account in the profit function. Using the same argument as with firm 0, firm 1 chooses W_1 to maximize,

$$\pi_1 = \begin{cases} \pi_1^A = (v_1 - W_1 - S)\frac{W_1}{t} & \text{for } W_1 \leq t - W_0, \\ \pi_1^B = (v_1 - W_1 - S)\frac{t - W_0 + W_1}{2t} & \text{for } t - W_0 < W_1 \leq t + W_0, \\ \pi_1^C = v_1 - W_1 - S & \text{for } t + W_0 < W_1. \end{cases}$$

Lemma 4. *Firm 1's best response correspondence* $W_1 = R_1(W_0)$ *is,*

(1) If $t < \frac{v_1 - S}{3}$*, then*

$$R_1(W_0) = \begin{cases} \frac{v_1 - S}{2} \text{ or } t + W_0 & \text{for } W_0 \leq t - \frac{v_1 - S}{2}, \\ t + W_0 & \text{for } t - \frac{v_1 - S}{2} < W_0 \leq v_1 - S - 3t, \\ \frac{v_1 - S + W_0 - t}{2} & \text{for } v_1 - S - 3t < W_0. \end{cases} \cdot$$

(2) If $t > \frac{v_1 - S}{3}$*, then*

$$R_1(W_0) = \begin{cases} \frac{v_1 - S}{2} & \text{for } W_0 \leq t - \frac{v_1 - S}{2} \\ t - W_0 & \text{for } t - \frac{v_1 - S}{2} < W_0 \leq t - \frac{v_0}{3}, \\ \frac{v_1 - S + W_0 - t}{2} & \text{for } t - \frac{v_1 - S}{3} \leq W_0. \end{cases} \cdot$$

(3) If $t = \frac{v_1 - S}{3}$*, then*

$$R_1(W_0) = \frac{v_1 - S + W_0 - t}{2} \textit{for all } W_0 \geq 0.$$

In case (1), the value of $R_1(W_0)$ for $W_0 \leq t - \frac{v_1 - S}{2}$ is $\frac{v_1 - S}{2}$ if $\pi_1^A(\frac{v_1 - S}{2}) \geq \pi_1^B(t + W_0)$ and the value is $t + W_0$ otherwise. It will always be the case that $R_1(W_0) > W_0$ which guarantees that this segment of the best response function never contains the Nash equilibrium (in pure strategies). Because of the switching cost, firm 1 may not always want to sell to all buyers not buying from firm 0. However, because of the assumption (M), firm 0 will never want to miss making a sale to a buyer who does not buy from firm 1. Using the best-response correspondences, we can characterize the Nash equilibrium prices and allocations.

For both firms, there is a case (case (2) for both) for which strategies can be strategic complements. Competition in fixed fees is effectively competition in prices which are strategic substitutes: when the rival firm lowers its fee, a firm's optimal response is to also lower its fee. That is, when their rival increases demand, each firm finds it profitable to reduce its fee and increase demand (to take back some of the loss in demand due to the rival's fee decrease). In doing so, each firm must forego some surplus it previously collected from its captive consumers. In case (2) however if $W_1 \leq t - \frac{v_0}{3}$, then in response to rival fee decrease, firm 0 finds it optimal to increase its own fee (and further give up demand) to extract more surplus from its captive consumers. For this to be optimal, the reduction in demand due to fee increase must be small relative to surplus, i.e., transportation cost (t) must be sufficiently large, which is the condition for case (2). In addition, the marginal consumer's surplus must be small so that it is not worth retaining ($W_1 \leq t - \frac{v_0}{3}$). A similar argument holds for firm 1's strategic complementarity.

Proof of Lemma 1

First, we consider the firm 1's response when firm 0's investment is large enough ($\Delta_0 + S > 3t$). In this case, firm 1 has to exit from the market unless he improves the quality of products enough. We consider the optimal investments in equilibrium. In order to consider the firm 0's strategy, we have to take into account the firm 1's response.

Firm 1 does not invest ($\Delta_1 = 0$)

When firm 1 does not invest to improve the quality, region 1 is achieved. Then, producers' profits are given by

$$\pi_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2}$$

$$\pi_1 = 0$$

The optimal switching cost S^* and the optimal degree of quality improvement Δ_0^* are the solution to

$$\max_{\Delta_0, S} \pi_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2}$$

$$s.t. \Delta_0 + S \geq 3t$$

We define the Lagrangian

$$L_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2} + \lambda(\Delta_0 + S - 3t)$$

We can consider firm 0 chooses $\Delta \equiv S + \Delta_0$ to maximize his profit since Δ_0 and S are symmetric in this setting. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L_0(\Delta)}{\partial \Delta} = 1 - \delta\Delta + \lambda = 0, \ \Delta \frac{\partial L_0(\Delta)}{\partial \Delta} = 0,$$

$$\frac{\partial L_0(\Delta)}{\partial \lambda} = \Delta - 3t > 0, \ \lambda \geq 0, \ \lambda \frac{\partial L_0(\Delta)}{\partial \lambda} = 0,$$

First, we consider the case $\Delta > 0$, $\lambda = 0$ when $\delta > 1/3t$

$$\Delta^* = 3t,$$

The optimal profits in this region are thus given by

$$\pi_0^* = 2t - \frac{9t^2\delta}{2}, \quad \pi_1^* = 0$$

Second, we consider the case $\Delta > 0$, $\lambda > 0$ when $\delta \leq 1/3t$

$$\Delta^* = \frac{1}{\delta},$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{1}{2\delta} - t, \quad \pi_1^* = 0$$

Firm 1 tries to move to region 3 $(\Delta - 3t < \Delta_1 < \Delta + 3t)$

When firm 1 invests the quality improvement and tries to move to region 3, producers' profits are given by

$$\pi_0 = \frac{(\Delta - \Delta_1 + 3t)^2}{18t} - \frac{\delta\Delta^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$

We need to consider the firm 1's strategy. The optimal Δ_1^* are the solution to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 + 3t \geq \Delta \geq \Delta_1 - 3t$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2} + \lambda_1(\Delta - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta + 3t)$$

The Kuhn-Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta + 3t)}{9t} - \delta\Delta_1 - \lambda_1 + \lambda_2 = 0,\ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda_1} = \Delta - \Delta_1 + 3t > 0,\ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta + 3t > 0,\ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta < 1/9t$

$$\Delta_1^* = \frac{\Delta - 3t}{1 - 9t\delta},$$

The optimal profits in this region are thus given by

$$\pi_1^* = -\frac{\delta(\Delta - 3t)^2}{2(1 - 9t\delta)}$$

When $\delta < 1/9t$, firm 1's equilibrium profit becomes negative. Thus, firm 1 does not choose this strategy.

Firm 1 tries to move to region 2 ($\Delta_1 \geq \Delta + 3t$)

When firm 1 invests the quality improvement enough and tries to move to region 2, producers' profits are given by

$$\pi_0 = -\frac{\delta\Delta^2}{2}, \pi_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2}$$

We need to consider the firm 1's strategy. The optimal Δ_1^* are the solution to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta - t - \frac{\delta \Delta_1^2}{2}$$
$$s.t. \Delta_1 \geq \Delta + 3t$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta - t - \frac{\delta \Delta_1^2}{2} + \lambda(\Delta_1 - \Delta - 3t)$$

The Kuhn-Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta \Delta_1 - \lambda = 0, \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta - 3t > 0, \ \lambda \frac{\partial L_1}{\partial \lambda} = 0,$$

First, we consider the case $\Delta_1 \geq 0$, $\lambda > 0$ when $\max\{3t, \frac{1}{\delta} - 3t\} < \Delta$

$$\Delta_1^* = \Delta^* + 3t,$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta^* + 3t)^2}{2}$$

Second, we consider the case $\Delta_1 \geq 0$, $\lambda = 0$ when $3t < \Delta \leq \frac{1}{\delta} - 3t$

$$\Delta_1^* = \frac{1}{\delta},$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta - t$$

Optimal Investment in this region

We are prepared to consider the firm 0's optimal investment in this region. Firm 1 does not have incentive to move to region 3 since his profit becomes negative. Firm 0 prefers region 1 to region 2. We can easily show that firm 1 does not have incentive to move to region 2 under the firm 0's optimal investment in region 1. Therefore, in this region, firm 0 tries to maximize his profit in region 1 and firm 1 does not invest in equilibrium.

Proof of Lemma 2

This is the case when firm 0's investment is not large ($\Delta_0 + S \leq 3t$), and firm 1 does not have to exit the market unless he improve the quality of products enough. In order to consider the firm 0's strategy, we have to take into account the firm 1's response.

Firm 1 tries to stay in region 3 $(\Delta - 3t < \Delta_1 < \Delta + 3t)$

When firm 1 invests the quality improvement and tries to stay in region 3, producers' profits are given by

$$\pi_0 = \frac{(\Delta - \Delta_1 + 3t)^2}{18t} - \frac{\delta\Delta^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$

We need to consider the firm 1's strategy. The optimal Δ_1^* are the solution to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 + 3t \geq \Delta \geq \Delta_1 - 3t$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2} + \lambda_1(\Delta - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta + 3t)$$

The Kuhn-Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta + 3t)}{9t} - \delta\Delta_1 - \lambda_1 + \lambda_2 = 0, \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda_1} = \Delta - \Delta_1 + 3t > 0, \ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta + 3t > 0, \ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta > 1/9t$

$$\Delta_1^* = \frac{3t - \Delta}{9t\delta - 1},$$

Firm 0 takes into account firm 1's strategy to maximize his profit. Then, the optimal Δ^* are the solution to

$$\begin{aligned} \max_{\Delta} \pi_0 = \frac{\left(\Delta - \frac{3t-\Delta}{9t\delta-1} + 3t\right)^2}{18t} - \frac{\delta\Delta^2}{2}, \\ s.t. \Delta \geq \frac{3t - \Delta}{9t\delta - 1} - 3t, \\ \frac{3t - \Delta}{9t\delta - 1} \geq \Delta - 3t \end{aligned}$$

In this section, we focus on the inner solution. Then, optimal investments in equilibrium are

$$S^* = \Delta_0^* = \frac{3t(9t\delta - 2)}{2(81t^2\delta^2 - 27t\delta + 1)}, \quad \Delta_1^* = \frac{9t(3t\delta - 1)}{81t^2\delta^2 - 27t\delta + 1}$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{t(9t\delta - 2)^2}{2(81t^2\delta^2 - 27t\delta + 1)}, \quad \pi_1^* = \frac{81t^2\delta(3t\delta - 1)^2(9t\delta - 1)}{2(81t^2\delta^2 - 27t\delta + 1)^2}$$

We need to check the conditions are satisfied in equilibrium.

$$\begin{aligned}
\Delta^* < 3t &\iff \frac{9t(9t\delta - 1)(1 - 3t\delta)}{81t^2\delta^2 - 27t\delta + 1} < 0 \\
\Delta^* \geq \frac{3t - \Delta^*}{9t\delta - 1} - 3t &\iff \frac{3t(9t\delta - 1)(9t\delta - 2)}{81t^2\delta^2 - 27t\delta + 1} \geq 0 \\
\frac{3t - \Delta^*}{9t\delta - 1} \geq \Delta^* - 3t &\iff \frac{81t^2\delta(3t\delta - 1)}{81t^2\delta^2 - 27t\delta + 1} \geq 0
\end{aligned}$$

In order to satisfy these conditions, we need to satisfy

$$\text{sign}\left(81t^2\delta^2 - 27t\delta + 1\right) = \text{sign}(9t\delta - 2) = \text{sign}(3t\delta - 1)$$

We consider the case that all signs are positive (when all signs are negative, it is not possible to satisfy all conditions.). We can rewrite the conditions as follows

$$\begin{aligned}
\text{sign}\left(81t^2\delta^2 - 27t\delta + 1\right) > 0 &\iff \frac{9t\delta}{(9t\delta - 1)^2} < 1 \\
\text{sign}(9t\delta - 2) > 0 &\iff \frac{2}{9t} < \delta \\
\text{sign}(3t\delta - 1) > 0 &\iff \delta > \frac{1}{3t}
\end{aligned}$$

$9t\delta/(9t\delta - 1)^2$ is a decreasing function of δ when $\delta > 1/9t$. In addition to that, $9t\delta/(9t\delta - 1)^2$ is smaller than 1 when $\delta = 1/3t$. Therefore, all condition are satisfied when δ is larger than $1/3t$.

Firm 1 tries to move to region 2 $(\Delta_1 > \Delta + 3t)$

When firm 1 invests the quality improvement and tries to move to region 2, producers' profits are given by

$$\pi_0 = -\frac{\delta\Delta^2}{2}, \pi_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2}$$

We need to consider the firm 1's strategy. The optimal Δ_1^* are the solution to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2}$$
$$s.t. \Delta_1 \geq \Delta + 3t$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2} + \lambda(\Delta_1 - \Delta - 3t)$$

The Kuhn-Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta\Delta_1 - \lambda = 0, \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$
$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta - 3t > 0, \ \lambda \frac{\partial L_1}{\partial \lambda} = 0,$$

First, we consider the case $\Delta_1 \geq 0$, $\lambda > 0$ when $\frac{1}{\delta} - 3t < \Delta < 3t$

$$\Delta_1^* = \Delta^* + 3t,$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta + 3t)^2}{2}$$

Second, we consider the case $\Delta_1 \geq 0$, $\lambda = 0$ when $\Delta < \max\{3t, \frac{1}{\delta} - 3t\}$

$$\Delta_1^* = \frac{1}{\delta},$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta - t$$

Optimal Investment in this region

We are prepared to consider the firm 0's optimal investment in this region. Firm 0 prefers region 3 to region 2. Therefore, both firm 0 and firm 1 invests and stay in region 3 when $\delta > 1/3t$. Otherwise, region 2 will be achieved in equilibrium.

Proof of Proposition 2

We are prepared to consider the optimal investment.

If the quality improvement is costly ($\delta > 1/3t$)

When the firm 0's investment is not large enough ($\Delta_0 + S \leq 3t$), region 3 is achieved. When the firm 0's investment is enough ($\Delta_0 + S > 3t$), region 1 is achieved. We can easily show that firm 0's profit under the region 3 is larger than that under the region 1 in this case. Thus, firm 0 tries to stay region 3.

If the quality improvement is not costly ($\delta \leq 1/3t$)

When the firm 0's investment is not large enough ($\Delta_0 + S \leq 3t$), region 2 is achieved. When the firm 0's investment is enough ($\Delta_0 + S > 3t$), region 1 is achieved. Thus, firm 0 tires to invest enough to prevent firm 1's entrant.

Derivation of Consumer Surplus

The consumer surplus for the four regimes in the Mature Industry are,

$$
\begin{aligned}
\text{(I) } CS_I &= \frac{1}{2}(2W_0^* - t), \\
\text{(II) } CS_{II} &= \frac{1}{2}(2W_1^* - t), \\
\text{(III) } CS_{III} &= (W_0^* + W_1^* - t(1 - \hat{x}(p_0^*, p_1^*)))\frac{\hat{x}(p_0^*, p_1^*)}{2} \\
&+ (W_1^* + W_0^* - t\hat{x}(p_0^*, p_1^*))\frac{1 - \hat{x}(p_0^*, p_1^*)}{2} \\
&= \frac{1}{2}(W_0^* + W_1^*) - t\hat{x}(p_0^*, p_1^*)(1 - \hat{x}(p_0^*, p_1^*)), \\
\text{(IV) } CS_{IV} &= \frac{1}{2}\left\{W_0^* \frac{W_0^*}{t} + W_1^*\left(1 - \frac{W_0^*}{t}\right)\right\} = \frac{1}{2t}\{(W_0^*)^2 + (W_1^*)^2\}.
\end{aligned}
$$

Derivation of Iso-social surplus curves

The curves are obtained from,

$$
\begin{aligned}
&(I)\ SS = CS + PS = -\frac{1}{2}t + v_0,\\
&(II)\ SS = -\frac{1}{2}t + v_1 - S.
\end{aligned}
$$

For the remaining regimes, using the following partial derivatives,

$$
\begin{aligned}
(III)\ \frac{\partial SS}{\partial v_0} &= \frac{1}{2}t + \frac{5v_0 - v_1 + S}{18t},\\
\frac{\partial SS}{\partial (v_1 - S)} &= \frac{1}{2}t - \frac{5(v_0 - v_1) + S}{18t},\\
(IV)\frac{\partial SS}{\partial v_0} &= \frac{3t - v_1 + S}{3t},\\
\frac{\partial SS}{\partial (v_1 - S)} &= \frac{4(v_1 - S) - 3v_0 - 3t}{9t}.
\end{aligned}
$$