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The Rejective Core of an Economy with Profit-Making Firms[1]

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In this paper, we consider an economy with producers and introduce a kind of "money" into the economy in order to incorporate producers' behaviors of profit maximization. We define a modified concept of "rejective core" which depends on both consumers' and producers' criterions, and prove the identity of the rejective core and the competitive equilibrium. Namely, the purpose of this paper is to characterize the competitive equilibrium in Arrow-Debreu type economies by applying the concept of "rejective core".

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1. INTRODUCTION

Undeniably, one of the most important results in general equilibrium theory is the coincidence of the core and the set of competitive equilibria. In the studies of core in general equilibrium theory, however, pure exchange economies are mainly considered. As an economic extension, economies with production were considered in several papers such as Scarf [16] in, Vind [20], Hildenbrand [8, 11], Nikaido [14], Champsaur [5], Sondermann [18], Boehm [4], Oddou [15], Shitovitz [17]. However, the definition of core in their arguments is based on only on consumers' criterion of utility maximization, and it does not depend on producers' criterion of profit maximization. Therefore, it is not unreasonable to say that there are essentially no producers, or no firms, in their economies, and that they do not virtually differ from a pure exchange economy. In short, their economies are not economies with producers, but ones with production. Hence, we can say that the problem on the equivalence between the core and the set of competitive equilibria is unsettled in the case of economies with producers.

In the studies of economies containing production, roughly speaking, there are three kinds of approaches. The first one is of the so-called Walras' economy. Walras considered an economy where there are no profits, although he took into account producers, or "entrepreneurs" in his terminology. In his model of economy, the law of "constant returns to scale" prevails, that is, the production possibility set is a cone with vertex at the origin and every producer's profit is zero. In such a zero-profit economy, producers do not play any significant role. Under the

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assumption that every consumer can access to a common production set, the core of the economy, is defined in a similar way and the identity of the core and the set of competitive equilibria is proved. In fact, it was done by Debreu and Scarf [7].

The second approach is of a private ownership economy with a fixed list of firms, such as, in Arrow and Debreu [2] and Debreu [6]. Unlike Walras' economy, each producer's profit can be positive in such a private ownership economy. Therefore, in this case the behavior of producers has significant effects on the economy through the distribution of their profits to consumers. As a result, a difficulty arises in the analysis of the core of such an economy containing producers. In order to show the difficulty, let us quote from Malinvaud [13]:

> The difficulty stems from the fact that profit maximization is no longer suitable as a criterion of choice for producers since they no longer prices as given. The theory must therefore specify how decisions are taken in firms. It is certainly natural to assume that consumers control the firms. But a priori, there are various conceivable ways in which this control and its implications may be specified. The simplest is to assume that each firm is the property of a single consumer who is in full control of it and may use its net output either for his consumption or for the exchanges in which he becomes engaged.
>
> Given this personalisation of firms, the theories of the last two sections can be generalised in a very natural way.

To avoid this difficulty, a third approach was adopted by Hildenbrand [8, 10, 11], that is, a coalition production economy. He proved in [10] that an equilibrium existence theorem for coalition production economies includes that for private ownership economies. Formally, the coalition production economy is a generalization of economies where firms are personalized in the sense of Malinvaud. However, under an assumption that the Radon-Nikodym derivative exists for the set-valued measure that describes the production technologies of coalitions, production technologies are virtually separately possessed by consumers. Therefore, coalition production economies are not essentially different from pure exchange economies in the analysis of the core. Hence, a problem arises, i.e., is the theory of core still adequate to explain the competitive equilibrium in an economy, like private ownership economies, where producers actually exist?

In the present paper, we shall deal with this problem by introducing a kind of "money" into an economy with producers. In our arguments we shall adopt the so-called Arrow-Debreu model as a basic economy. By the word "money", we mean a commodity which has the following two characteristics. The first one is a constancy of its price. In particular, the price of money is assumed to be always unity, so that money becomes a numéraire. Consequently, all economic values can be measured in terms of money. In particular, producers' profits are measured in money. The second is that money is a lawful means of settlement. Namely, every agent can pay off his debt with money, and conversely, no agent can object to payment with money. Thus, producers can use money to pay dividends to consumers who are share holders of their profits. On the other hand, consumers cannot get "utility" by holding money and money has no value for them. However, in the economy we consider, consumers can use money to induce producers to produce commodities for them. Our aim is to analyze the working of a competitive economy with money which has these two properties. To make a long stroy short, the purpose of this paper is to define a new concept of core in an economy with such money and to investigate the relation between the core and the set of competitive equilibria in the economy. In other words, our main aim is to give a new explanation of the competitive equilibrium in a private ownership economy of Arrow-Debreu type in [2, 6].

In an Arrow-Debreu type economy, it is difficult to define a core in a usual way, because each firm may be possessed by several consumers, not necessarily by a single consumer. In such an economy, consumers are indirectly connected with each other through firms, and they cannot act independently if they use production technologies in firms. Also, producers, or firms, cannot determine their production plans independently of consumers who have shares in their profits. To avoid this difficulty, we shall introduce money, which has the above-mentioned two properties, into the economy.

To see the role of money, let us consider the following simple economy: There are only three economic agents who are named s_1, s_2, t. The agent named t is a producer and the other two agents are consumers. Producer t has no obligation to the consumers except for distributing his profit to them in an initially determined way, that is, if his profit is π, $\theta_1\pi$ must be distributed to consumer s_1 and $\theta_2\pi$ to consumer s_2, where $\theta_1 + \theta_2 = 1$ and θ_1, $\theta_2 > 0$. In this economy there are 7 conceivable coalitions, i.e., $\{s_1\}$, $\{s_2\}$, $\{t\}$, $\{s_1, s_2\}$, $\{s_1, t\}$, $\{s_2, t\}$, $\{s_1, s_2, t\}$. Of these, however, coalitions $\{s_1, t\}$ and $\{s_2, t\}$, for example, are not allowable in an economy without money, for producer t has a duty to pay his profit to both consumers s_1 and s_2. But, in this economy with money, such coalitions are admissible. To see this, let us consider, for example, coalition $\{s_1, t\}$. Suppose producer t earns profit π in this coalition. Then he must pay $\theta_2\pi$ to consumer s_2 out of the coalition. Since producer t has no money, he cannot pay it. However, if consumer s_1 has some money, and if he thinks it advantageous to maintain the coalition, he will pay money to consumer s_2 on behalf of producer t. In this way, money makes it possible for producers to form coalitions with consumers, even with consumers who have no shares in the producers' profits. Therefore, producers are independent agents in our economy, and they make a coalition with consumers in order to maximize their profits. On this point, our analysis is essentially different from that in previous works.

As is explained above, by virtue of money, consumers and producers can make coalitions in our economy. The core defined in our economy is a modified one of the so-called "rejective core." The rejective core is a generalization of the usual core and was originally proposed by Konovalov [12]. He defined the rejective core for atomless exchange economies with possibly satiable consumers and extended the classical equivalence result between the core and the competitive equilibrium that was proved by Aumann [1] and Hildenbrand [11]. Our definition of rejective core is essentially the same as that of Konovalov [12]. The purpose of this paper is to characterize the competitive equilibrium in an economy with profit-making firms by applying the concept of rejective core. Our arguments follow the similar mathematical technique of Hildenbrand [8].

2. MODEL AND ASSUMPTIONS

We consider an economy including L commodities and infinitely many agents (continuum of agents). The set of all agents is denoted by a unit interval, $A = [0, 1]$.

For a measurable subset C of $[0,1]$, by $\lambda(C)$ we denote the Lebesgue measure of set C. Following Hildenbrand [9], we distinguish two types of agents, namely, consumers and producers (or firms). We denote the set of consumers by S and the set producers by T, where $\{S, T\}$ is a measurable partition of A.

Each consumer $s \in S$ is characterized by a consumption set X_s, a preference relation $\succ_s$, and an initial endowment $\mathbf{w}(s)$. We assume that $X_s = \mathbf{R}^L_+$, $\mathbf{w}(s) \in \mathbf{R}^L$, and that function $\mathbf{w} : S \to \mathbf{R}^L_+$ is integrable.[2] On the other hand, each producer $t \in T$ is characterized by a production set $Y_t \subset \mathbf{R}^L$.

We assume the following assumptions for consumers and producers.

(A1) For each consumer $s \in S$, the following conditions are satisfied:

- (i) $\mathbf{w}(s)$ is in the interior of $\mathbf{R}^L_+$.
- (ii) $\succ_s$ is irreflexive.
- (iii) For any $x \in \mathbf{R}^L_+$, set $\{y \in \mathbb{R}^L_+ | y \succ_s x\}$ is open in $\mathbf{R}^L_+$.
- (iv) For any $x \in \mathbf{R}^L_+$ and $\epsilon > 0$, there exists $y \in \mathbf{R}^L_+$ such that $y \succ_s x$ and $||y - x|| < \epsilon$.

(A2) For each producer $t \in T$, $\mathbf{0} \in Y_t$.

The profits of producers are distributed to consumers as dividends. The shares of consumers in the profits of producers are described by an integrable function $\theta : S \times T \to \mathbb{R}_+$ such that $\int_S \theta(s,t)d\lambda(s) = 1$ for every $t \in T$. Namely, $\theta(s,t)$ denotes the share of consumer $s \in S$ in the profit of producer $t \in T$ and when each producer $t \in T$ earns profit $\pi(t)$, then each consumer $s \in S$ obtains dividends $\theta(s,t)\pi(t)$ from each producer t and in total $\int_T \theta(s,t)\pi(t)d\lambda(t)$.

Furthermore, we need the following conditions for mathematical treatments.

(A3) For any two measurable functions $\mathbf{f} : S \to \mathbf{R}^L_+$ and $\mathbf{g} : S \to \mathbf{R}^L_+$, set $\{s \in S \mid \mathbf{f}(s) \succ_s \mathbf{g}(s)\}$ is measurable.

(A4) The graph $\{(t,y) \in T \times \mathbf{R}^L \mid y \in Y_t\}$ is measurable.

3. DEFINITION OF CORE

To denote an *allocation* of commodities, we use an integrable function $\mathbf{f} : A \to \mathbf{R}^L$ such that $\mathbf{f}(s) \in X_s$ for almost every (a.e.) $s \in S$ and $\mathbf{f}(t) \in Y_t$ for a.e. $t \in T$. The image $\mathbf{f}(s)$ denotes a consumption bundle allocated to consumer s when $s \in S$, whereas it denotes a production plan of producer t when $t \in T$. An allocation $\mathbf{f} : A \to \mathbf{R}^L$ of commodities is feasible if $\int_S \mathbf{f} = \int_T \mathbf{f} + \int_S \mathbf{w}$.[3]

Producers earn profits and consumers receive dividends from producers. To describe payments of dividends from producers to consumers, we incorporate another commodity, or "fiat money" in the economy. To denote producers' profits and consumers' dividends, we use an integrable function $\mathbf{m} : A \to \mathbf{R}_+$. Namely,

[2] We denote by $\mathbf{R}^L$ an L-dimensional Euclidean space and by $\mathbf{R}^L_+$ its non-negative orthant.

[3] The integral of a measurable function $\mathbf{f}$ on a measurable set U is denoted by $\int_U \mathbf{f}$.

we denote by $\mathbf{m}(t)$ the amount of profits earned by each producer $t \in T$ and by $\mathbf{m}(s)$ the amount of dividends received by each consumer $s \in S$. Since the share of each consumer $s \in$ in each producer $t \in T$ is $\theta(s,t)$, an allocation $\mathbf{m} : A \to \mathbf{R}^L$ of dividends is feasible if $\mathbf{m}(s) = \int_T \theta(s,t)\mathbf{m}(t)d\lambda(t)$ for each $s \in S$.

DEFINITION 1. *A pair* $\{\mathbf{f}, \mathbf{m}\}$ of integrable functions $\mathbf{f} : A \to \mathbb{R}^L$ and $\mathbf{m} : A \to \mathbb{R}_+$ is a *feasible* allocation of commodities and dividends if the following conditions hold:

(i) $\mathbf{f}(s) \in X_s$ for a.e. $s \in S$ and $\mathbf{f}(t) \in Y_t$ for a.e. $t \in T$.

(ii) $\int_S \mathbf{f} = \int_T \mathbf{f} + \int_S \mathbf{w}$.

(iii) $\mathbf{m}(s) = \int_T \theta(s,t)\mathbf{m}(t)d\lambda(t)$ for each $s \in S$.

When the markets of commodities exist, the competitive equilibrium is defined in the following way.

DEFINITION 2. A triplet $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}, \hat{p}\}$ of a feasible allocation $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ and a price vector $\hat{p} \in \mathbf{R}^L$ is a *competitive equilibrium* if the following conditions hold:

(i) For each $s \in S$, $\hat{p} \cdot \hat{\mathbf{f}}(s) \leq \hat{p} \cdot \mathbf{w}(s) + \hat{\mathbf{m}}(s)$ and $x \succ_s \hat{\mathbf{f}}(s)$ implies $\hat{p} \cdot \mathbf{w}(s) + \hat{\mathbf{m}}(s) < \hat{p} \cdot x$.

(ii) For each $t \in T$, $\hat{\mathbf{m}}(t) = \hat{p} \cdot \hat{\mathbf{f}}(t)$ and $\hat{p} \cdot \hat{\mathbf{f}}(t) \geq \hat{p} \cdot y$ for any $y \in Y_t$.

Any measurable subset C of $[0,1]$ is called a *coalition.* The set of consumers in coalition C is $C \cap S$ and the set of producers in C is $C \cap T$. Consumers and producers would collude if they can attain a better allocation in the following way.

DEFINITION 3. A feasible allocation $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ is *rejected* by a coalition C if there is another allocation $\{\mathbf{f}, \mathbf{m}\}$ such that

(i) $\int_{C \cap S} \mathbf{f} = \int_{C \cap T} \mathbf{f} + \int_{C \cap S} \mathbf{w}$,

(ii) $\int_{C \cap T} \mathbf{m} = \int_{C \cap S} \hat{\mathbf{m}}$,

(iii) $\mathbf{f}(s) \succ_s \hat{\mathbf{f}}(s)$ for a.e. $s \in S \cap C$ and $\mathbf{m}(t) > \hat{\mathbf{m}}(t)$ for a.e. $t \in T \cap C$.

In the above definition, when coalition C attains a new allocation $\mathbf{f} : A \to \mathbb{R}^L$ of commodities, condition (i) allows consumers in C to provide their own initial endowments $\int_{C \cap S} \mathbf{w}$ and producers in C to provide their production technologies to produce $\int_{C \cap T} \mathbf{f}$. Condition (ii) allows that consumers in C provide their received dividends $\int_{C \cap S} \hat{\mathbf{m}}$ to bear the amount $\int_{C \cap T} \mathbf{m}$ of profits for producers in C. The condition shows that consumers can induce producers to join in the coalition by bribing them with money.

Condition (iii) means that every agents in coalition C is better off. Note that consumers care only about quantities of commodities whereas producers care only about amounts of profits.

Note that, in allocation $\{\mathbf{f}, \mathbf{m}\}$ in the above definition, we can assume without loss of generality that $\mathbf{f}(s) = \mathbf{w}(s)$ for a.e. $s \in S \setminus C$ and $\mathbf{m}(t) = \hat{\mathbf{m}}(t)$ for a.e. $t \in T \backslash C$. This means that the initial or current situations are ensured for the agents outside coalition C in allocation $\{\mathbf{f}, \mathbf{m}\}$. Namely, the consumers outside coalition C get their initial endowments and the producers outside coalition S get the same amounts of profits. In addition, allocation $\{\mathbf{f}, \mathbf{m}\}$ can be viewed as a feasible one. In fact, if we put $\mathbf{f}(t) = \mathbf{0}$ for each $t \in T \cap C$ and $\mathbf{m}(s) = \int_T \theta(s,t)\mathbf{m}(t)d\lambda(t)$ for each $s \in S$, then allocation $\{\mathbf{f}, \mathbf{m}\}$ become a feasible one. As for such allocation $\{\mathbf{f}, \mathbf{m}\}$, we have the following lemma which shows that consumers inside coalition C do not owe consumers outside the coalition.

LEMMA 1. *As for allocation* $\{\mathbf{f}, \mathbf{m}\}$ *in Definition 3, the following holds:*

$$\int_{S\setminus C}[\int_{T\cap C}\theta(s,t)\mathbf{m}(t)\lambda(dt)]\lambda(ds) \leq \int_{S\cap C}[\int_{T\setminus C}\theta(s,t)\mathbf{m}(t)\lambda(dt)]\lambda(ds).$$

Proof. Since $\int_S \theta(s,t)\lambda(ds) = 1$ for each t, we have

$$\int_{T\cap C}\mathbf{m} = \int_{T\cap C}[\int_S \theta(s,t)\mathbf{m}(t)\lambda(ds)]\lambda(dt).$$

Also, since $\hat{\mathbf{m}}$ is a feasible allocation of money, by Definition 1 we have $\hat{\mathbf{m}}(\mathbf{s}) = \int_T \theta(s,t)\hat{\mathbf{m}}(t)d\lambda(t)$ for each $s \in S$. Thus, (ii) of Definition 3 implies that

$$\int_{T\cap C}[\int_S \theta(s,t)\mathbf{m}(t)\lambda(ds)]\lambda(dt) = \int_{S\cap C}[\int_T \theta(s,t)\hat{\mathbf{m}}(t)\lambda(dt)]\lambda(ds).$$

Therefore, by (iii) of Definition 3, since $\mathbf{m}(t) > \hat{\mathbf{m}}(t)$ for a.e. $t \in T \cap C$ and $\mathbf{m}(t) = \hat{\mathbf{m}}(t)$ for a.e. $t \in T \setminus C$, we have

$$\int_{T\cap C}[\int_S \theta(s,t)\mathbf{m}(t)\lambda(ds)]\lambda(dt) \leq \int_{S\cap C}[\int_T \theta(s,t)\mathbf{m}(t)\lambda(dt)]\lambda(ds).$$

Thus, we can conclude that

$$\int_{T\cap C}[\int_{S\setminus C}\theta(s,t)\mathbf{m}(t)\lambda(ds)]\lambda(dt) \leq \int_{S\cap C}[\int_{T\setminus C}\theta(s,t)\mathbf{m}(t)\lambda(dt)]\lambda(ds). \quad \blacksquare$$

The right hand side of the inequality in the above lemma is the dividends of profits to be paid by producers outside coalition C to consumers inside coalition C, while the left hand side is the dividends of profits to be paid by producers inside coalition C to consumers outside coalition C. Thus, the lemma means that coalition C has no debt to coalition $A \setminus C$ in distributing dividends of profits.

Another interpretation of Lemma 1 is possible. In a static model, the profits of producers can be viewed as their values, i.e., the value of equities published by firms. Therefore, $\mathbf{m}(t)$ stands for a price of equity of firm t. Thus, the inequality of Lemma 1 means that the consumers in coalition C can buy all the equities of firms inside the coalition by selling all their equities of the firms outside the coalition. In other words, the consumers in coalition C are able to take over those firms belonging to the coalition and to utilize their production technologies.

DEFINITION 4. The *rejective core* is the set of all allocations of commodities and dividends that are not rejected by any coalition with positive measure.

The rejective core in this paper is a special case of the rejective core defined by Konovalov [12]. In fact, in the usual definition of rejective core, agents in a coalition can use either their initial endowments or currently assigned allocations in attaining a new allocation. In addition, agents outside the coalition are ensured not to be worse off than in their initial endowments or currently assigned allocations.

On the contrary, in our definition, as for commodities consumers can utilize only their initial endowments and as for money they can utilize their currently assigned money. In this sense the definition of rejection in this paper is more restrictive, and the rejective core becomes larger.

4. EQUIVALENCE BETWEEN CORE AND COMPETITIVE EQUILIBRIUM

Now we are ready to prove the equivalence between the rejective core and the competitive equilibrium. First, by a standard argument, we have the following theorem.

THEOREM 1. *Any allocation of a competitive equilibrium belongs to the rejective core.*

Proof. Let $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}, \hat{p}\}$ be a competitive equilibrium. Suppose that allocation $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}, \hat{p}\}$ does not belong to the rejective core. Then, there is a coalition C with $\lambda(C) > 0$ and an allocation $\{\mathbf{f}, \mathbf{m}\}$ that satisfies conditions (i) - (iii) in Definition 3. By condition (iii) of Definition 3 and the definition of competitive equilibrium, we have the following inequalities:

$$\hat{p} \cdot \mathbf{f}(s) > \hat{p} \cdot \mathbf{w}(s) + \hat{\mathbf{m}}(s) \text{ for a.e. } s \in C \cap S$$

and

$$\mathbf{m}(t) > \hat{\mathbf{m}}(t) = \hat{p} \cdot \hat{\mathbf{f}}(t) \geq \hat{p} \cdot \mathbf{f}(t) \text{ for a.e. } t \in C \cap T.$$

By integrating the above inequalities, we have

$$\hat{p} \cdot \int_{C \cap S} \mathbf{f} > \hat{p} \cdot \int_{C \cap S} \mathbf{w} + \int_{C \cap S} \hat{\mathbf{m}} \text{ and } \int_{C \cap T} \mathbf{m} > \hat{p} \cdot \int_{C \cap T} \mathbf{f}.$$

By (ii) of Definition 3, we have

$$\hat{p} \cdot \int_{C \cap S} \mathbf{f} > \hat{p} \cdot \int_{C \cap S} \mathbf{w} + \hat{p} \cdot \int_{C \cap T} \mathbf{f}$$

This contradicts (i) of Definition 3. ∎

The following is the converse of Theorem 1, which implies the equivalence between the rejective core and the competitive equilibrium.

THEOREM 2. *For any allocation $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ belonging to the rejective core, there is a price vector $\hat{p}$ such that $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}, \hat{p}\}$ is a competitive equilibrium, or such that $\{\hat{\mathbf{f}}, \mathbf{0}, \hat{p}\}$ is a competitive equilibrium where every producer's maximum profit at price $\hat{p}$ is zero.*

To prove the above theorem, let $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ be an allocation belonging to the rejective core. Define a mapping $H : A \to \mathbf{R}^{L+1}$ as follows:

$$H(s) := \{(x - \mathbf{w}(s), \alpha) \mid x \succ_s \hat{\mathbf{f}}(s) \text{ and } \alpha = \hat{\mathbf{m}}(s)\} \text{ for each } s \in S \text{ and}$$

$$H(t) := \{(-y, -\alpha) \mid y \in Y_t, \alpha > \hat{\mathbf{m}}(t)\} \text{ for each } t \in T.$$

Define a subset of $\mathbf{R}^{L+1}$ by

$$\Delta := \{\int_C \mathbf{h} \mid \mathbf{h} \in \mathcal{L}(H),\ C \text{ is a measurable subset of } A \text{ with } \lambda(C) > 0.\},$$

where $\mathcal{L}(H) = \{\mathbf{h} \mid \mathbf{h}$ is an integrable function of A into $\mathbf{R}^{L+1}$ such that $h(a) \in H(a)$ a.e. in A.$\}$.

LEMMA 2. *Δ is a non-empty convex subset of $\mathbf{R}^{L+1}$ such that $\mathbf{0} \notin \Delta$.*

Proof. The non-emptiness of set Δ can be proved by using conditions (iii) and (iv) of (A1), (A2), and (A4). The convexity of set Δ follows from Liapunov's theorem (Cor.2 in Hildenbrand [8], p.451, due to Vind [19]).

Now, suppose that $\mathbf{0} \in \Delta$. Then, there exist an integrable function $\mathbf{h} \in \mathcal{L}(H)$ and a measurable subset C of A with $\lambda(C) > 0$ such that $\int_C \mathbf{h} = \mathbf{0}$. Let $\mathbf{f} : A \to \mathbb{R}^L$ and $\mathbf{m} : A \to \mathbb{R}$ be functions such that

$(\mathbf{f}(a), \mathbf{m}(a)) = \mathbf{h}(a) + (\mathbf{w}(a), 0)$ for each $a \in C \cap S$

and

$(\mathbf{f}(a), \mathbf{m}(a)) = -\mathbf{h}(a)$ for each $a \in C \cap T$.

Then, by definition of $\mathfrak{L}(H)$,

$\mathbf{f}(s) \succ_s \hat{\mathbf{f}}(s)$ and $\mathbf{m}(s) = \hat{\mathbf{m}}(s)$ for each $s \in C \cap S$

and

$\mathbf{f}(t) \in Y_t$ and $\mathbf{m}(t) > \hat{\mathbf{m}}(t)$ for each $t \in C \cap T$.

In addition, since $\int_C \mathbf{h} = \mathbf{0}$, we have

$\int_{C\cap S} \mathbf{f} = \int_{C\cap T} \mathbf{f} + \int_{C\cap S} \mathbf{w}$ and $\int_{C\cap T} \mathbf{m} = \int_{C\cap S} \hat{\mathbf{m}}$.

Thus, coalition C rejects $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$. This contradicts that $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ belongs to the rejective core. ∎

By a separation theorem, there is a non-zero vector $(\hat{p}, -\hat{\gamma}) \in \mathbf{R}^L \times \mathbf{R}$ such that $\hat{p} \cdot z - \hat{\gamma}\alpha \geq 0$ for all $(z, \alpha) \in \Delta$. From the shape of set $H(t)$ of each $t \in T$, it follows that $\hat{\gamma} \geq 0$. Thus, by a standard argument we can prove that for a.e. $a \in A$, $\hat{p} \cdot z - \hat{\gamma}\alpha \geq 0$ for all $(z, \alpha) \in H(a)$.

Therefore, for each $s \in S$, if $x \succ_s \hat{\mathbf{f}}(s)$, then $\hat{p} \cdot x \geq \hat{p} \cdot \mathbf{w}(s) + \hat{\gamma}\hat{\mathbf{m}}(s)$. In addition, by condition (iv) of (A1), we have

$$\hat{p} \cdot \hat{\mathbf{f}}(s) \geq \hat{p} \cdot \mathbf{w}(s) + \hat{\gamma}\hat{\mathbf{m}}(s) \quad \text{for a.e. } s \in S. \tag{1}$$

On the other hand, for each $t \in T$, if $y \in Y_t$ and $\alpha > \hat{\mathbf{m}}(t)$, then $\hat{p} \cdot y \leq \hat{\gamma}\alpha$. Therefore,

$$\hat{\gamma}\hat{\mathbf{m}}(t) \geq \hat{p} \cdot \hat{\mathbf{f}}(t) \quad \text{for a.e. } t \in T. \tag{2}$$

If strict inequality holds in the above two inequalities for some agent, then, by integrating them we have $\hat{\gamma} \int_T \hat{\mathbf{m}} + \hat{p} \cdot \int_S \hat{\mathbf{f}} > \hat{p} \cdot \int_T \hat{\mathbf{f}} + \hat{p} \cdot \int_S \mathbf{w} + \hat{\gamma} \int_S \hat{\mathbf{m}}$, which contradicts that $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}\}$ is a feasible allocation. Thus, equality holds in both (1) and (2). Thus, when $\hat{\gamma} > 0$, without loss of generality, we can put $\hat{\gamma} = 1$, and $\{\hat{\mathbf{f}}, \hat{\mathbf{m}}, \hat{p}\}$ is a competitive equilibrium.

On the other hand, when $\hat{\gamma} = 0$, by assumption of (A2), we have that $\hat{p} \cdot \hat{\mathbf{f}}(t) = 0$ for every $t \in T$, i.e., every producer's maximum profit is zero. Thus, under $\hat{p}$, $\{\hat{\mathbf{f}}, \mathbf{0}, \hat{p}\}$ is a competitive equilibrium. This completes the proof of Theorem 2.

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