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The Social Value of Public Information with Convex Costs of Information Acquisition*

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Abstract

In a beauty contest framework, welfare can decrease with public information if the precision of private information is exogenous, whereas welfare necessarily increases with public information if the precision is endogenous with linear costs of information acquisition. The purpose of this paper is to reconcile these results by considering nonlinear costs of information acquisition. The main result of this paper is a necessary and sufficient condition for welfare to increase with public information. Using it, we show that costs of information acquisition are linear if and only if welfare necessarily increases with public information. Thus, welfare can decrease with public information for any strictly convex costs. This is because convex costs mitigate the so-called crowding-out effect of public information on private information, thereby making the social value of public information with endogenous precision closer to that with exogenous precision.

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1 Introduction

In multi-agent situations, more information is not necessarily valuable. A notable example is the anti-transparency result in a beauty contest game of Morris and Shin (2002). They demonstrate that welfare can decrease with public information if agents have access to sufficiently precise private information.

The anti-transparency result of Morris and Shin (2002) has prompted considerable debates.[1] In particular, Colombo and Femminis (2008) reach the opposite conclusion by allowing agents to choose the precision of private information given the precision of public information. They show that welfare necessarily increases with public information if the cost of the precision is linear. In their result, the so-called crowding-out effect of public information on private information has an essential role: an increase in the precision of public information reduces the incentives for acquisition of private information and thus delivers substantial cost savings enough to compensate any decrease in the expected payoffs.

The purpose of this paper is to reconcile the anti-transparency result with exogenous precision and the pro-transparency result with endogenous precision by incorporating general nonlinear costs in the model of Colombo and Femminis (2008).

The main result of this paper is a necessary and sufficient condition for welfare to increase with public information. Using it, we show that costs of information acquisition are linear if and only if welfare necessarily increases with public information. Thus, welfare can decrease with public information for any strictly convex costs. This is because convex costs mitigate the crowding-out effect of public information, thereby making the social value of public information with endogenous precision closer to that with exogenous precision.

Colombo et al. (2014) also compare the social value of public information with endogenous precision and that with exogenous precision in a general model with nonlinear costs. They give a sufficient condition guaranteeing that the former is positive whenever the latter is positive and show that the model of Colombo and Femminis (2008) satisfies the condition. Their condition is based upon a comparison between the equilibrium and the socially optimal strategy profile. Such a comparison is useful in understanding the social value of public information, which is the focus of Colombo et al. (2014). In contrast, our focus is the property of convex costs mitigating the crowding-out effect and a full characterization of the resulting social value of public information with endogenous precision.[2]

[1] See Angeletos and Pavan (2004, 2007), Hellwig (2005), Svensson (2006), Cornand and Heinemann (2008), and James and Lawler (2011), among others.

[2] Information acquisition in a beauty contest framework is also studied by Hellwig and Veldkamp (2009) and Myatt and Wallace (2012), but their focus is not the social value of public information.

2 The model

There is a continuum of agents indexed by $i \in [0, 1]$. Agent i chooses an action $a_i \in \mathbb{R}$, and we write a for the action profile. The payoff function of agent i is

$$u_i(a, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}), \tag{1}$$

where θ is the state, r is a constant with $0 < r < 1$, and

$$L_i = \int_0^1 (a_j - a_i)^2 dj, \ \bar{L} = \int_0^1 L_j dj.$$

Agent i aims to minimize the weighted mean of the squared error of his action from the state and that from an opponent's action. This payoff function is characterized by two conditions: (i) the best response is the weighted mean of the state and the average action of the opponents; (ii) the total ex ante expected payoff is proportional to $-\int E[(a_i - \theta)^2]di$.

Agent i observes a public signal $y = \theta + \eta$ and a private signal $x_i = \theta + \varepsilon_i$, where θ, η and ε_i are independently and normally distributed with

$$E[\theta] = \bar{\theta}, \ E[\eta] = E[\varepsilon_i] = 0, \ \mathrm{var}[\theta] = 1/\alpha_\theta, \ \mathrm{var}[\eta] = 1/\alpha_y, \ \mathrm{var}[\varepsilon_i] = 1/\beta_i,$$

and ε_i and ε_j are independent for $i \neq j$. We refer to α_y and β_i as the precision of public information and that of private information, respectively. We write $\alpha = \alpha_\theta + \alpha_y$.

Following Colombo and Femminis (2008),[3] we consider a two-stage game with information acquisition. In the first stage, agent i chooses the precision of private information β_i. In the second stage, he observes public and private signals and chooses an action a_i. The second-stage game when $\beta_i = \beta_j$ for all $i, j \in [0, 1]$ is the beauty contest game of Morris and Shin (2002).

The cost of choosing β_i in the first stage is $C(\beta_i) \geq 0$, where $C : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and convex; that is, $C'(\beta_i) > 0$ and $C''(\beta_i) \geq 0$ for all $\beta_i \geq 0$. The net payoff function of agent i is

$$u_i(a, \theta) - C(\beta_i).$$

Colombo and Femminis (2008) restrict attention to a linear cost function, but we consider all convex cost functions. Our model is a special case of the model of Colombo et al. (2014) who study information acquisition in symmetric linear quadratic Gaussian games of Angeletos and Pavan (2007).[4]

[3]Colombo and Femminis (2008) also consider a stage in which a public authority endogenously determines the precision of public information. We do not consider such a stage because we focus on the effects of public information on agents' acquisition of private information.

[4]The earliest papers on information acquisition in linear quadratic Gaussian games are Li et al. (1987) and Vives (1988), who study Cournot games.

3 The equilibrium

We consider a symmetric equilibrium. To characterize it and obtain the expected payoff, we utilize two known results. The following result on the second stage is due to Morris and Shin (2002).

Lemma 1. *In the unique equilibrium of the second-stage game when $\beta_i = \beta$ for all $i \in [0, 1]$, agent i's action is*

$$a_i = \lambda E[\theta|y] + (1 - \lambda)x_i,$$

where $\lambda = \alpha/(\alpha + (1 - r)\beta)$ and $E[\theta|y] = (\alpha_\theta\bar{\theta}+\alpha_y y)/(\alpha_\theta+\alpha_y)$. The total ex ante expected payoff in this equilibrium is

$$F(\alpha, \beta) \equiv -(1 - r) \int E[(a_i - \theta)^2]di = -(1 - r)(\lambda^2/\alpha + (1 - \lambda)^2/\beta). \tag{2}$$

Morris and Shin (2002) show that $dF(\alpha,\beta)/d\alpha_y = \partial F(\alpha,\beta)/\partial\alpha < 0$ if and only if $r > 1/2$ and $\alpha < (2r - 1)(1 - r)\beta$; that is, welfare can decrease with public information if $r > 1/2$ when the precision of private information is exogenous. As explained by Morris and Shin (2002), a strategic complementarity induces players' overreaction to public information relative to the efficiency benchmark, which results in the detrimental effects of public information.

The following result on the first stage is due to Colombo and Femminis (2008).

Lemma 2. *When all the opponents follow the equilibrium strategy in the second-stage game given in Lemma 1, player i's marginal benefit of the precision β_i in the first stage evaluated at $\beta_i = \beta$ is*

$$\frac{d}{d\beta_i}E[u_i(a,\theta)]\bigg|_{\beta_i=\beta} = (1 - \lambda)^2/\beta^2 = (1 - r)^2/(\alpha + (1 - r)\beta)^2.$$

By this lemma, the first order condition for the equilibrium precision is

$$(1 - r)^2/(\alpha + (1 - r)\beta)^2 = C'(\beta). \tag{3}$$

Note that the marginal benefit is strictly decreasing in α and β, whereas the marginal cost is increasing in β (see Figure 1). The equilibrium precision is the unique value of β solving (3) if $C'(0) \le (1 - r)^2/\alpha^2$ and it is zero if $C'(0) > (1 - r)^2/\alpha^2$. We denote the equilibrium precision by $\phi(\alpha)$ as a function of $\alpha = \alpha_\theta + \alpha_y$. Let $\overline{\beta} \equiv \phi(0) = \sup_{\alpha>0}\phi(\alpha)$ be the supremum of the equilibrium precision, which equals the unique solution of $\beta\sqrt{C'(\beta)} = 1$.

For example, suppose that $C(\beta) = c\beta$ with $c > 0$. Then, $\phi(\alpha) = 1/\sqrt{c} - \alpha/(1 - r)$ if $c \le (1 - r)^2/\alpha^2$, as shown by Colombo and Femminis (2008). Only in the linear case, we can obtain $\phi(\alpha)$ in such a simple form.

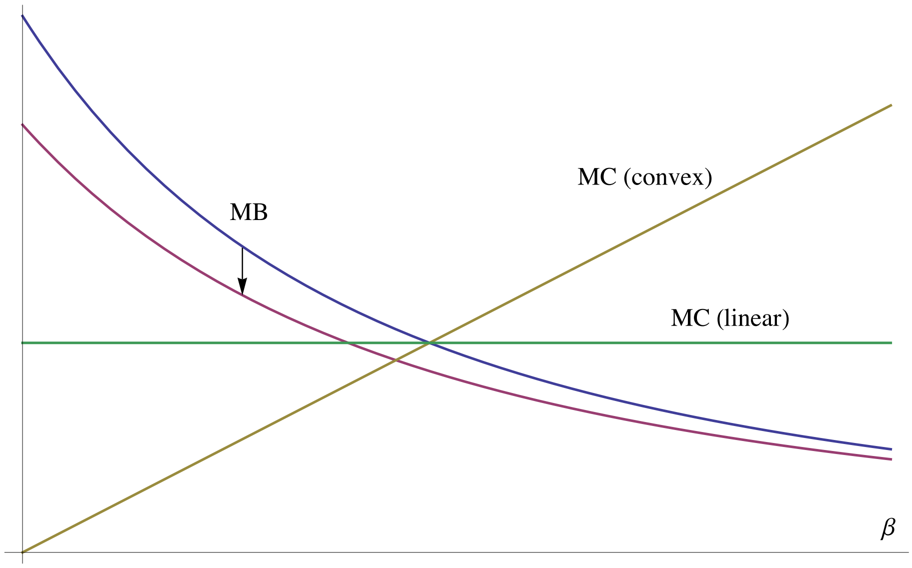

Figure 1: The marginal benefit curve and the marginal cost curve. An increase in α shifts the marginal benefit curve down, by which the equilibrium precision decreases.

It should be noted that an increase in the precision of public information results in a decrease in the precision of private information by (3) (see Figure 1), i.e., $\phi'(\alpha) < 0$. Colombo and Femminis (2008) refer to this effect as the crowding-out effect of public information on private information. The next section will go into detail.

4 Main results

We write the total ex ante expected net payoff as a function of the precision of public information

$$W(\alpha_y) \equiv F(\alpha_\theta + \alpha_y, \phi(\alpha_\theta + \alpha_y)) - C(\phi(\alpha_\theta + \alpha_y)),$$

which is our measure of welfare. Colombo and Femminis (2008) show that if $C(\beta) = c\beta$ with $c > 0$ then $W'(\alpha_y) = c$; that is, welfare necessarily increases with public information if the cost is linear.

In the nonlinear case, it is difficult to directly calculate $W(\alpha_y)$ because we do not have $\phi(\alpha)$ in a closed form. However, by using the inverse function

$$\psi(\beta) \equiv \phi^{-1}(\beta) = (1 - r)(1/\sqrt{C'(\beta)} - \beta), \tag{4}$$

we can represent $W(\alpha_y)$ as a function of β by (2):

$$F(\psi(\beta), \beta) - C(\beta) = r\beta C'(\beta) - \sqrt{C'(\beta)} - C(\beta). \tag{5}$$

This representation leads us to the following main result of this paper.

Proposition 1. *Let $\underline{\beta}$ be the unique solution of $\beta\sqrt{C'(\beta)} = 1/2$. If $\alpha_\theta + \alpha_y \geq \psi(\underline{\beta})$, then $W'(\alpha_y) > 0$. If $\alpha_\theta + \alpha_y < \psi(\underline{\beta})$, then, for $\beta = \phi(\alpha_\theta + \alpha_y)$,*

$$W'(\alpha_y) \gtreqless 0 \Leftrightarrow r \lesseqgtr R(\beta) \equiv \frac{C'(\beta) + C''(\beta)/(2\sqrt{C'(\beta)})}{C'(\beta) + C''(\beta)\beta}. \tag{6}$$

In the latter case, $R(\beta) \leq 1$, where the equality holds if and only if $C''(\beta) = 0$.

Proof. Note that $\underline{\beta} < \overline{\beta}$ because $\overline{\beta}$ is the unique solution of $\beta\sqrt{C'(\beta)} = 1$. Let $\beta = \phi(\alpha_\theta + \alpha_y)$. Because

$$W'(\alpha_y) = \frac{\partial F}{\partial \alpha} + \left(\frac{\partial F}{\partial \beta} - C'\right)\phi' = \frac{1}{\psi'}\left(\frac{\partial F}{\partial \alpha}\psi' + \frac{\partial F}{\partial \beta} - C'\right) = \frac{1}{\psi'}\frac{d}{d\beta}(F(\psi(\beta), \beta) - C(\beta)),$$

the sign of $W'(\alpha_y)$ is opposite to that of $d(F(\psi(\beta), \beta) - C(\beta))/d\beta$. By (5), we have

$$\frac{d}{d\beta}(F(\psi(\beta), \beta) - C(\beta)) = -(1-r)C'(\beta) + C''(\beta)\left(r\beta - \frac{1}{2\sqrt{C'(\beta)}}\right).$$

By rearranging the above, we obtain (6) for all $\beta < \overline{\beta}$. If $\alpha_\theta + \alpha_y \geq \psi(\underline{\beta})$, then $\beta \leq \underline{\beta}$ and thus $\beta \leq 1/(2\sqrt{C'(\beta)})$, which implies that $R(\beta) \geq 1$ and thus $W'(\alpha_y) > 0$ because $r < 1$. If $\alpha_\theta + \alpha_y < \psi(\underline{\beta})$, then $\beta > \underline{\beta}$ and thus $\beta > 1/(2\sqrt{C'(\beta)})$, which implies the last statement. □

Proposition 1 says that welfare increases with public information if $\alpha \geq \psi(\underline{\beta})$ or if $\alpha < \psi(\underline{\beta})$ and $r < R(\beta)$ but decreases if $\alpha < \psi(\underline{\beta})$ and $r > R(\beta)$. In particular, if $C''(\beta) = 0$ for all β, then $r < R(\beta) = 1$ and thus $W'(\alpha_y) > 0$ for all $\alpha > 0$. That is, if the cost is linear, then welfare necessarily increases with public information for all $\alpha > 0$ and $r \in (0, 1)$, as shown by Colombo and Femminis (2008). We can say more than this: welfare increases with public information for all $\alpha > 0$ and $r \in (0, 1)$ if and only if the cost is linear on $(\underline{\beta}, \overline{\beta})$.

Corollary 2. $W'(\alpha_y) > 0$ *for all* $\alpha > 0$ *and* $r \in (0, 1)$ *if and only if* $C''(\beta) = 0$ *for all* $\beta \in (\underline{\beta}, \overline{\beta})$.

Proof. If $C''(\beta) = 0$ for all $\beta \in (\underline{\beta}, \overline{\beta})$, then $r < R(\beta) = 1$ and thus $W'(\alpha_y) > 0$ for all $\alpha > 0$ and $r \in (0, 1)$ by Proposition 1. If $W'(\alpha_y) > 0$ for all $\alpha > 0$ and $r \in (0, 1)$, then $r < R(\beta) \leq 1$ for all $\beta \in (\underline{\beta}, \overline{\beta})$ and $r \in (0, 1)$ by Proposition 1, and thus $R(\beta) = 1$. This implies that $C''(\beta) = 0$. □

The "only if" part implies that, for any strictly convex costs, welfare can decrease with public information. To illustrate it, suppose that $C''(\overline{\beta}) > 0$ and

$$1 > r > R(\overline{\beta}) = \left(C''(\overline{\beta})/(2C'(\overline{\beta})) + \sqrt{C'(\overline{\beta})}\right) / \left(C''(\overline{\beta})/C'(\overline{\beta}) + \sqrt{C'(\overline{\beta})}\right). \tag{7}$$

Then, welfare decreases with public information if α is sufficiently small.

Corollary 3. *Suppose that $C''(\overline{\beta}) > 0$ and $r > R(\overline{\beta})$. Let $\beta^* < \overline{\beta}$ be the maximum value of β solving $r = R(\beta)$. If $\alpha < \psi(\beta^*)$ then $W'(\alpha_y) < 0$. Suppose, in addition, that β^* is the unique solution of $r = R(\beta)$. Then, $W'(\alpha_y) < 0$ if and only if $\alpha < \psi(\beta^*)$.*

Proof. If $\psi(\overline{\beta}) = 0 < \alpha = \psi(\beta) < \psi(\beta^*)$, then $\beta^* < \beta < \overline{\beta}$ and thus $r > R(\beta)$ by the definition of β^*. Therefore, $W'(\alpha_y) < 0$ by Proposition 1. Suppose that β^* is the unique solution of $r = R(\beta)$. Then, $r > R(\beta)$ if and only if $\beta > \beta^*$ because $r < R(\underline{\beta}) = 1$. Therefore, if $W'(\alpha_y) < 0$ then $r > R(\phi(\alpha))$ by Proposition 1 and thus $\phi(\alpha) > \beta^*$, i.e., $\alpha < \psi(\beta^*)$. □

By the latter half of the corollary, when $R(\beta)$ is strictly decreasing in β, welfare decreases with public information if and only if α is sufficiently small. For example, consider an isoelastic cost function $C(\beta) = c\beta^\rho/\rho$ with $\rho > 0$, where

$$R(\beta) = \left(1 + (\rho - 1)\beta^{-(\rho+1)/2}/(2\sqrt{c})\right)/\rho$$

is strictly decreasing in β. By calculating $R(\overline{\beta})$ and $\psi(\beta^*)$, we obtain the following result.

Corollary 4. *Suppose that $C(\beta) = c\beta^\rho/\rho$ with $\rho > 1$ and $r > (\rho + 1)/(2\rho)$. Then, $W'(\alpha_y) < 0$ if and only if*

$$\alpha < \frac{(1 - r)(2\rho r - (\rho + 1))}{c^{\frac{1}{\rho+1}}(\rho - 1)^{\frac{\rho-1}{\rho+1}}(2\rho r - 2)^{\frac{2}{\rho+1}}}.$$

This corollary says that welfare can decrease with public information if $r > (\rho + 1)/(2\rho)$. Moreover, the threshold $(\rho + 1)/(2\rho)$ decreases to $1/2$ as ρ goes to infinity. This is because $R(\overline{\beta})$ can be arbitrarily close to $1/2$ if $C''(\overline{\beta})/C'(\overline{\beta})$ is large enough by (7), where $C''(\overline{\beta})/C'(\overline{\beta})$ is analogous to the Arrow-Prat measure of risk aversion. Recall that, when the precision of private information is exogenous, welfare can decrease with public information if $r > 1/2$, as shown by Morris and Shin (2002). Therefore, convex costs make the social value of public information with endogenous precision arbitrarily close to that with exogenous precision.

To explain the role of convex costs in the above result, we show that convex costs mitigate the crowding-out effect of public information measured by $\phi'(\alpha)$.

Proposition 5. *Let $\phi_C(\alpha)$ be the equilibrium precision with strictly convex costs and let $\phi_L(\alpha)$ be the equilibrium precision with linear costs. If $\phi_C(\alpha) = \phi_L(\alpha) > 0$, then $\phi'_L(\alpha) < \phi'_C(\alpha) < 0$.*

Proof. Let ψ_C and ψ_L be the inverse functions of ϕ_C and ϕ_L, respectively. For $\beta = \phi_C(\alpha) = \phi_L(\alpha)$, $1/\phi'_C(\alpha) = \psi'_C(\beta) = -(1 - r)(1 + C''(\beta)(C'(\beta))^{-3/2}/2) < -(1 - r) = \psi'_L(\beta) = 1/\phi'_L(\beta) < 0$. Thus, $\phi'_L(\alpha) < \phi'_C(\alpha) < 0$. □

Recall that the equilibrium precision is determined by the intersection of the downward sloping marginal benefit curve and the upward sloping marginal cost curve (See Figure 1). An increase in the precision of public information shifts the marginal benefit curve down, by which the equilibrium precision decreases. Clearly, the equilibrium precision decreases more when the marginal cost curve is flat, i.e., the cost is linear.

Proposition 5 explains the following role of convex costs. Because

$$W'(\alpha_y) = \frac{\partial F}{\partial \alpha} + \left(\frac{\partial F}{\partial \beta} - C'\right)\phi',$$

the welfare effects of public information consist of a direct effect $\partial F/\partial\alpha$ and an indirect effect $(\partial F/\partial\beta - C')\phi'$ through the crowding-out effect ϕ'. With convex costs, the crowding-out effect is small, thus making the direct effect dominant. In this case, the total effect can be negative because the direct effect can be negative as shown by Morris and Shin (2002). With linear costs, the crowding-out effect is large, thus making the indirect effect dominant. In this case, the total effect is positive because, whenever the direct effect is negative, the indirect effect is positive due to substantial cost savings, as shown by Colombo and Femminis (2008).

Remark 1. The above discussion on the role of convex costs does not depend upon the specific setting of the beauty contest model. Thus, our finding on the property of convex costs in Proposition 5 may be relevant in other models with information acquisition, such as the general model of Colombo et al. (2014). To explore this issue, the use of the inverse function (4) may also be useful, which is a methodological difference between this paper and Colombo et al. (2014).

Remark 2. Colombo et al. (2014) also compare the social value of public information with endogenous precision and that with exogenous precision. They give a sufficient condition guaranteeing that the total effect $W'(\alpha_y)$ is positive whenever the direct effect $\partial F/\partial\alpha$ is positive and show that the beauty contest model satisfies the condition. Their condition is based upon a comparison between the equilibrium and the socially optimal strategy profile. Such a comparison is useful in understanding the social value of public information with endogenous precision, which is the focus of Colombo et al. (2014). Note that, using (5), we can also confirm that $\partial F/\partial\alpha > 0$ implies $W'(\alpha_y) > 0$ because

$$\left.\frac{\partial F}{\partial \alpha}\right|_{\alpha=\psi(\beta)} = \frac{2(C'(\beta))^{3/2}}{1-r}\left(\frac{1}{2\sqrt{C'(\beta)}} - r\beta\right),$$

$$W'(\psi(\beta) - \alpha_\theta) = \left((1-r)C'(\beta) + C''(\beta)\left(\frac{1}{2\sqrt{C'(\beta)}} - r\beta\right)\right)\left(-\frac{1}{\psi'(\beta)}\right).$$

References

Angeletos, G.-M., Pavan, A., 2004. Transparency of information and coordination in economies with investment complementarities. Amer. Econ. Rev. 94, 91–98.

Angeletos, G.-M., Pavan, A., 2007. Efficient use of information and social value of information. Econometrica 75, 1103–1142.

Colombo, L., Femminis, G., 2008. The social value of public information with costly information acquisition. Econ. Letters 100, 196–199.

Colombo, L., Femminis, G., Pavan, A., 2014. Information acquisition and welfare. Working paper, Northwestern University.

Cornand, C., Heinemann, F., 2008. Optimal degree of public information dissemination. Econ. J. 118, 718–742.

Hellwig, C., 2005. Heterogeneous information and the welfare effects of public information disclosures. Working paper, UCLA.

Hellwig, C., Veldkamp, L., 2009. Knowing what others know: coordination motives in information acquisition. Rev. Econ. Stud. 76, 223–251.

James, J. G., Lawler, P., 2011. Optimal policy intervention and the social value of public information. Amer. Econ. Rev. 101, 1561–1574.

Li, L., McKelvey, R. D., Page, T., 1987. Optimal research for Cournot oligopolists. J. Econ. Theory 42, 140–166.

Morris, S., Shin, H. S., 2002. Social value of public information. Amer. Econ. Rev. 92, 1521–1534.

Myatt, D. P., Wallace, C., 2012. Endogenous information acquisition in coordination games. Rev. Econ. Stud. 79, 340–374.

Svensson, L. E. O., 2006. Social value of public information: comment: Morris and Shin (2002) is actually pro-transparency, not con. Amer. Econ. Rev. 96, 448–452.

Vives, X., 1988. Aggregation of information in large Cournot markets. Econometrica 56, 851–876.