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Abstract

Financial and macroeconomic time-series data often exhibit infrequent but large jumps. Such jumps may be considered as outliers that are independent of the underlying data-generating processes and contaminate inferences on their model. In this study, we investigate the effects of such jumps on asymptotic inference for large-dimensional common factor models. We first derive the upper bound of jump magnitudes with which the standard asymptotic inference goes through. Second, we propose a jump-correction method based on a series-by-series outlier detection algorithm without accounting for the factor structure. This method gains standard asymptotic normality for the factor model unless outliers occur at common dates. Finally, we propose a test to investigate whether the jumps at a common date are independent outliers or are of factors. A Monte Carlo experiment confirms that the proposed jump-correction method retrieves good finite sample properties. The proposed test shows good size and power. Two small empirical applications illustrate usefulness of the proposed methods.

JEL Classification Number: C12, C38

Keywords: outliers, large-dimensional common factor models, principal components, jumps

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1 Introduction

The common factor model is found to be a useful and effective tool for statistical inference with financial or economic high-dimensional data sets. Major applications are found in the empirical asset pricing literature of the well-known Arbitrage Pricing Theory (Ross, 1976). For classical examples, Lehmann and Modest (1988) and Connor and Korajczyk (1988) apply a multifactor model to cross sections of stock returns. Recently, Ando and Bai (2014) develop a multifactor model with group structure and apply it to Chinese stock returns. The list of studies pertaining to fixed-income assets such as government and corporate bonds includes Litterman and Scheinkman (1991), Elton et al. (1995), Ang and Piazzesi (2003), and Ludvigson and Ng (2009). Lustig et al. (2011) provide an application to currency returns. There is also a strand of research investigating macroeconomic time-series data using dynamic factor models following, as far as the author knows, Geweke (1977), Sargent and Sims (1977), and Stock and Watson (2002ab). This list is by no means comprehensive.

One remarkable features of such data sets is that they often exhibit infrequent but large jumps. While the source and dates of these jumps are sometimes of interest by themselves, we may simply consider the jumps nuisance outliers that are independent of the underlying data-generating processes. In the latter case, it is well-recognized that such outliers can easily contaminate inferences based on the underlying jump-free model. Therefore, a large amount of research has gone into identifying and correcting such outlier effects. The most popular issue was to detect outliers in the stationary autoregressive moving average (ARMA) models, for which methods have been proposed by Fox (1972), Box and Tiao (1975), Tsay (1986), and Chen and Liu (1993), among others. For examples of unit root and cointegration tests, see Franses and Haldrup (1994), Vogelsang (1999), and Perron and Rodríguez (2003) and for examples of inference for conditionally heteroskedastic models with outliers, see Franses and Ghijsels (1999) and Charles and Darné (2005).[1]

Following the aforementioned outlier detection/correction literature, we investigate the effects of outliers on the recently developed asymptotic inference for large-dimensional common factor models using the principal component approach (e.g., Bai and Ng, 2002; Bai, 2003; Amengual and Watson, 2007; and Bates et al., 2012). To make this attempt feasible and attractive, we extend the standard large-dimensional common factor model as follows.

[1] In this perspective, a strand of literature uses high-frequency data to asymptotically infer jump-free processes or the jump itself. See Barndorff-Nielsen and Shephard (2007), Aït-Sahalia and Jacod (2014), and the references therein. Aït-Sahalia and Xiu (2015) apply principal component analysis using high-frequency financial data.

First, we model the infrequent jumps of each response variable as increments of a mixture of Poisson processes, with the intensity parameter p/T, where p is a small constant value and T is the time dimension of data. This is a popular strategy to model infrequent events in financial time series. The jumps are infrequent because the probability of a jump at a given time goes to zero as $T \to \infty$. Second, the magnitudes of jumps are modeled as a function of data dimension. This device provides useful asymptotic approximations of the effects of jumps on inferences. Third, we consider jumps that occur at dates specific to one response variable (idiosyncratic jumps) and those that occur at the same date in other response variables (common jumps). Finally, we consider the possibility of the underlying factors exhibiting large jumps. This is in contrast to the case where jumps are independent from the factors and thus they are regarded as outliers.

Under this setting, we first derive the upper bounds of jump magnitudes with which the standard asymptotic inference goes through. Furthermore, we provide two useful applications of this result. The first application pertains to a method to correct the effects of outliers on inferences. This is a simple application of a series-by-series outlier detection algorithm without considering the factor structure in the data. This method enables us to apply standard asymptotic normality of common factor models unless common jumps occur. Even when they do, the consistency of factor estimates is obtained. The second application pertains to the factor jump test—a test to investigate whether jumps at a common date are independent outliers or are of factors. This test is important because outliers may spuriously induce jumps in factor estimates even if the true factors have no jumps.

A Monte Carlo experiment confirms the following results in finite samples. First, independent large outliers easily contaminate the standard asymptotic inference in large-dimensional factor models. They significantly deteriorate the coverage rates of asymptotic confidence intervals, reduce the correlation between the true and estimated factors, and induce over- and under-estimation of a number of factors. However, the proposed jump-correction method retrieves good finite sample properties unless T is too small. Finally, the factor jump test shows good size when the outliers are sufficiently large. The test also exhibits good power. We then apply these methods to daily log-returns data of 25 currencies against the U.S. dollar for the recent financial crisis period. We observe infrequent large jumps in many currencies and identify a few common ones. From the common jumps on May 6–7 and September 30, 2008, a factor closely related to currencies such as the Hungarian forint, Norwegian krone, and Polish zloty shows strong evidence of jumps. On the other hand, a factor related to currencies such as the Swiss franc and Japanese yen exhibits no jump. This factor exhibits very weak evidence of jumps during that period. We also apply the method to Japanese prefectural new car registration data for the period January 1985 to December 2014. Note that there were two large earthquakes, in 1995 and 2011. We find that the jumps following the 2011 earthquake represent a jump in a common factor, whereas the jumps following the 1995 earthquake do not represent a jump in factors.

The rest of this paper is structured as follows. Section 2 presents our model and assumptions. Section 3 provides the upper bounds with which the standard asymptotic inference results go through. Section 4 discusses two useful applications: the jump-correction method and the factor jump tests. Section 5 investigates their finite sample properties via Monte Carlo simulations. Section 6 serves as two small empirical applications, and section 7 concludes the paper. We use the following notations throughout the paper. The Euclidean norm of vector x is denoted by $\|x\|$. For matrices, we use the vector-induced norm. Symbols $O(\cdot)$ and $o(\cdot)$ denote the standard asymptotic order of sequences; symbol $\xrightarrow{p}$ represents the convergence in probability under probability measure P, and symbol $\Rightarrow$ denotes the convergence in distribution. Symbols $O_p(\cdot)$ and $o_p(\cdot)$ are the orders of convergence in probability under P. We let $c_{NT} = \min\left\{\sqrt{N}, \sqrt{T}\right\}$.

2 Model and assumptions

2.1 Model

We consider the common factor model with cross-sectional dimension N and time-dimension T where N and T are both large:

$$x_{it}^* = \lambda_i' F_t + u_{it}, \quad \text{for } i = 1, ..., N \text{ and } t = 1, ..., T, \tag{1}$$

where x_{it}^* is the ith response variable at time t, F_t is an $r \times 1$ vector of common factors, λ_i is an $r \times 1$ vector of factor loadings, and u_{it} is an idiosyncratic error. Without loss of generality, we use demeaned data so that intercepts are omitted from the model. In matrix form, model (1) can be written as

$$X^* = F\Lambda + u, \tag{2}$$

where $X^* = [x_1^*, ..., x_N^*]$ is a $T \times N$ matrix with $x_i^* = [x_{i1}^*, ..., x_{iT}^*]'$ being a $T \times 1$ vector of response variables, $F = [F_1, .., F_T]'$ is a $T \times r$ matrix of common factors, $\Lambda = [\lambda_1, ..., \lambda_N]'$ is an $N \times r$ matrix of factor loadings, and $u = [u_1, ..., u_N]$ is a $T \times N$ matrix of idiosyncratic errors with $u_i = [u_{i1}, ..., u_{iT}]'$ being a $T \times 1$ vector.

In this study, we consider the model in which response variable x^*_{it} is not observed but x_{it} is, so that

$$x_{it} = x^*_{it} + z_{it}, \tag{3}$$

where z_{it} consists of infrequently occurring jumps. Specifically, we consider the following increments of a mixture of Poisson processes:

$$z_{it} = \eta^c_t \delta^c_{it} + \eta_{it}\delta_{it}. \tag{4}$$

In the two terms on the right-hand side of (4), η^c_t and η_{it} are *i.i.d.* Bernoulli random variables with probabilities p^c/T and p/T, respectively, where p^c and p are (typically small) positive constants. Furthermore, δ^c_{it} and δ_{it} are random variables associated with jump magnitudes. Note that if the first term shows a jump ($\eta^c_t = 1$), every response variable (x_{it} for every i) also jumps on the same date t. Therefore, we call them common jumps. On the other hand, the second term consists of jumps occurring on idiosyncratic dates, and so we call them idiosyncratic jumps. We make two observations with regard to this model. First, jumps are infrequent in the sense that the probabilities of jumps p^c/T and p/T diminish to zero as T increases. This is a popular modeling of rare events such as jumps and level shifts in financial returns and volatilities. Second, we assume that both common and idiosyncratic jumps are independent of the underlying factors and so are regarded as nuisance outliers in the factor model.

2.2 Assumptions

This section introduces our assumptions. Assumptions 1 to 5 apply to model (1), following the standard literature of Bai (2003) and Bates et al. (2013).

Assumption 1. $E\|F_t\|^4 < \infty$ and $T^{-1}\sum_{t=1}^{T} F_t F_t' \xrightarrow{p} \Sigma_F$, as $T \to \infty$, for some positive definite matrix Σ_F.

Assumption 2. $E\|\lambda_i\| \leq \lambda < \infty$ and $\Lambda'\Lambda/N \xrightarrow{p} \Sigma_\Lambda$, as $N \to \infty$, for some positive definite matrix Σ_Λ.

Assumption 3. The following conditions hold for all N and T, where M is a generic constant.

(a) $E(u_{it}) = 0$, $E|u_{it}|^8 \leq M$.

(b) $\gamma_N(s,t) = E(u_s' u_t / N)$ for all (s,t),

$$|\gamma_N(s,t)| \leq M \text{ for all } s,$$

and

$$T^{-1} \sum_{s=1}^{T} \sum_{t=1}^{T} |\gamma_N(s,t)| \leq M.$$

(c) $\kappa_{ij,ts} = E(u_{it} u_{js})$ for all (i,j,s,t). $|\kappa_{ij,tt}| \leq |\kappa_{ij}|$ for some κ_{ij} and for all t, while

$$N^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} |\kappa_{ij}| \leq M,$$

and

$$(NT)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{s=1}^{T} \sum_{t=1}^{T} |\kappa_{ij,ts}| \leq M.$$

(d) For every (s,t),

$$E \left| N^{-1/2} \sum_{i=1}^{N} [u_{is} u_{it} - E(u_{is} u_{it})] \right|^4 \leq M.$$

Assumption 4. For all (i,j,s,t), F_t, u_{is}, and λ_j are mutually independent.

Assumption 5. The eigenvalues of $\Sigma_F \Sigma_\Lambda$ are distinct.

Assumptions 6 and 7 specify the jump process (4) regarded as outliers.

Assumption 6. The followings hold for all (i,j,s,t)
(a) z_{it} and x_{js}^* are mutually independent.
(b) η_t^c, δ_{it}^c, η_{js}, and δ_{js} are mutually independent.
(c) δ_{it}^c and δ_{js} follow $i.i.d.N(0, \sigma_{NT}^2)$.

Assumption 7. With k_{NT} as an arbitrary function of N and T, the standard deviation of jumps is $\sigma_{NT} = k_{NT}\sigma$, where $0 < \sigma < \infty$ is a fixed constant.

Assumption 6 (a) ensures that jumps are independent outliers in the factor model. Furthermore, Assumption 6 (c) assumes that jump magnitudes follow a normal distribution with zero mean. However, normality is not essential and solely for derivational simplicity. The zero-mean assumption is not without loss of generality, however, since it solves an identification problem and greatly simplifies theoretical results, we keep this assumption within this paper.[2] Assumption 7 assumes that the standard deviation of jumps is asymptotically large and represented by scale factor k_{NT}. As shown later, this enables us to obtain meaningful asymptotic results pertaining to jump magnitudes.

Throughout the paper, factors are estimated using the principal component method, that is,

$$(\hat{\Lambda}, \hat{F}) = \arg\min_{\Lambda, F} \textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \lambda_i' F_t)^2, \tag{5}$$

imposing a normalization $\hat{F}'\hat{F}/T = I_r$, where I_r is the r-dimensional identity matrix. This yields $\hat{F}$, that is, $\sqrt{T}$ times r eigenvectors of XX' corresponding to its r largest eigenvalues and $\hat{\Lambda} = X'\hat{F}(\hat{F}'\hat{F})^{-1}$.

3 Asymptotic results

This section presents the asymptotic inference results for large-dimensional factor models as established by the literature in the presence of jumps. Again, jumps in this section are regarded as outliers independent of the underlying factor model. We examine the conditions for the scale of jump magnitudes k_{NT} under which standard results are unaffected. To this end, we study the insight of Bates et al. (2013), who discuss the conditions for magnitudes of factor loading instabilities with which standard asymptotic results go through.

We first consider the asymptotic normality originally obtained by Bai (2003) in the following theorem.

Theorem 1 *(Asymptotic normality of factors and factor loadings) Suppose Assumptions 1–7 hold and u_{it} follows i.i.d. with mean zero and variance σ_u^2. (i) If $k_{NT} < \sqrt{T}$ and $\eta_t^c = 0$, then*

$$N^{1/2}(\hat{F}_t - H'F_t) \Rightarrow N(0, \Omega_{\hat{F}}), \tag{6}$$

as $N, T \to \infty$ under $\sqrt{N}/T \to 0$, where $H = V_{NT}^{-1}(\hat{F}'F/T)(\Lambda'\Lambda/N)$ and $\Omega_{\hat{F}} = \sigma_u^2 V^{-1} Q \Sigma_\Lambda Q' V^{-1}$ with $Q = V^{1/2}\Phi'\Sigma_\Lambda^{-1/2}$. Matrices V_{NT} and V are diagonal, the main diagonals being the r largest eigenvalues of $XX'/(NT)$ and $\Sigma_\Lambda^{1/2}\Sigma_F\Sigma_\Lambda^{1/2}$, respectively, and Φ is the eigenvector matrix corresponding to the latter.

[2]Suppose $E(\delta_{it}) = \delta < \infty$. Then, under the following additional conditions on the original factor model, a model with non zero mean jumps can be regarded as a model with zero mean jumps. When common jumps occur ($\eta_t^c = 1$), the condition is $E(\lambda_i) = \lambda \neq 0$ for all i. Then, the new factor at t is defined as $F_t + \mu/\lambda$ in the case of $r = 1$ so that the new jumps have zero mean. When a idiosyncratic jump occurs ($\eta_{it} = 1$), the condition is $E(F_t) = \mu_F \neq 0$ for all t. Then the new loading is defined as $\lambda_i + \mu/\mu_F$ to be compatible with the model with zero mean jumps.

(ii) If $k_{NT} \leq T/\sqrt{N}$, then

$$T^{1/2}(\hat{\lambda}_i - H^{-1}\lambda_i) \Rightarrow N(0, \Omega_{\hat{\Lambda}}), \tag{7}$$

where $\Omega_{\hat{\Lambda}} = \sigma_u^2 Q'^{-1}\Sigma_F Q^{-1}$, as $N, T \to \infty$ under $\sqrt{T}/N \to 0$.

This theorem implies that the upper bounds of jump magnitudes are given by $\sqrt{T}$ for factor estimates and $T/\sqrt{N}$ for factor loading estimates to obtain standard asymptotic normality. We interpret these upper bounds as a larger T extending the bound so that it helps obtain asymptotic inferences for both estimators in the presence of outliers. On the other hand, a larger N lowers the bound for factor loadings and hence may harm the inference for factor loadings. This is intuitive because jumps are infrequent and so the total number of jumps in a data set does not increase as T increases, but increases as N does.

The theorem also implies that the asymptotic normality of $\hat{F}_t$ is available only when common jumps do not occur at t ($\eta_t^c = 0$). To deal with this problem, the following corollary guarantees its consistency with the timing of common jumps.

Corollary 1 *(Consistency of factors under common jumps) Suppose Assumptions 1–7 hold and $\eta_t^c = 1$. If $k_{NT} < \sqrt{N}$, then*

$$\left\| \hat{F}_t - H'F_t \right\| = o_p(1). \tag{8}$$

We next consider the upper bound of jump magnitudes with which the information criteria of Bai and Ng (2002) give consistent estimates for the number of factors r. The information criteria are defined as

$$\hat{r} = \arg \max_{0 \leq l \leq l_{\max}} \log V(l) + l \times g(N, T), \tag{9}$$

where $V(l) \equiv \sum_{i=1}^N \sum_{t=1}^T (x_{it} - \hat{\lambda}_i^{l\prime} \hat{F}_t^l)^2$ and $\hat{F}_t^l$ is the principal component factor estimate, assuming l factors and $\hat{\lambda}_i^l = (\sum_{t=1}^T \hat{F}_t^l \hat{F}_t^{l\prime})^{-1} (\sum_{t=1}^T \hat{F}_t^l x_{it})$. We obtain the following theorem as a direct consequence of Amengual and Watson (2007).

Theorem 2 *(Information criteria) Suppose Assumptions A1–A9 of Amengual and Watson (2007) hold. If $k_{NT} \leq \max\left\{1, T^{1/4}N^{-1/4}\right\}$, then $\hat{r} \xrightarrow{p} r$ as $N, T \to \infty$.*

For this theorem, we need Amengual and Watson's (2007) set of assumptions for the underlying jump-free factor model; however, they are very similar to our Assumptions 1 to 5.

4 Two useful applications

4.1 Series-by-series jump-correction algorithm

This section discusses two useful applications of the results presented in the previous section. The first pertains to the correction of jump effects. We consider the algorithms developed for univariate time-series data. For this, we apply them series-by-series without considering their common factor structure. The idea is that if jumps are outliers, removing their effects will not change their factor structure. We then identify and estimate the common factors with the set of individually jump-corrected response variables. Here, we consider an example in which x_{it} follows a stationary ARMA process $\theta_i(L)x_{it} = v_{it}$, where $\theta_i(L)$ is a polynomial of the standard lag operator L for every i and all jumps represent the so-called additive outliers.[3][4] Then, we have the following algorithm.

Algorithm: Implement the following steps for $i = 1, ..., N$.

Step 1. Compute $\tau_i(t) = \hat{v}^*_{it}/\hat{\sigma}_{it}$, where $\hat{v}^*_{it}$ is the residual from maximum-likelihood estimation, using x_{it} without considering its common factor structure. Estimate the standard error $\hat{\sigma}_{it}$ which is not affected by the jumps present in $\{x_{it}\}_{t=1}^{T}$.[5].

Step 2. If $\max_{1\leq t\leq T} |\tau_i(t)| \geq \xi$, where ξ is a predetermined critical value,

$$\hat{T}_i = \arg\max_{1\leq t\leq T} |\tau_i(t)|$$

is considered a possible jump location. Now, go to Step 3. If $\max_{1\leq t\leq T} |\tau_i(t)| < \xi$, the ith series exhibits no (more) jumps. Assume that $\hat{x}^*_{it} = x_{it}$, and go back to Step 1 to proceed with the $(i+1)$th series.

Step 3. Estimate the realized jump magnitude with least squares estimation of coefficient ω_i in the regression

$$\hat{v}^*_{it} = \omega_i w_{it} + \epsilon_{it}, \quad \text{for } t = 1, ..., T,$$

[3] A sufficient condition for this to be directly applicable is that factors and idiosyncratic errors follow stationary ARMA processes, because the sum of two ARMA processes is an ARMA process.

[4] For an extension to the autoregressive integrated moving average (ARIMA) model with additive and innovational outliers, see Chen and Liu (1993). Franses and Ghijsels (1999) and Charles and Darné (2005) provide methods using conditionally heteroskedastic models.

[5] For example, Chen and Liu (1993) propose the following three methods: (1) the median absolute deviation method, (2) the α% trimmed method, and (3) the omit-one method.

where $w_{it} = 0$ for $t < \hat{T}_i$, $w_{it} = 1$ for $t = \hat{T}_i$, and $w_{it} = -\theta_{il}$ for $t = \hat{T}_i + l$. Compute $\hat{x}^*_{it} = x_{it} - \hat{\omega}_i w_{it}$. Go back to Step 1 and use $\hat{x}^*_{it}$ as a new x_{it}.

We next provide an asymptotic justification for this algorithm. To this end, we first confirm that statistic $|\tau_i(t)|$ is informative with respect to jump locations as long as it explodes as $N, T \to \infty$.

Proposition 1 *If Assumptions 1–7 hold, $p\lim_{N,T\to\infty} \hat{\sigma}^2_{it}$ is a finite constant and $k_{NT} < N^{1/2}T$. Then, the jump component z_{it} becomes the dominating term in $|\tau_i(t)|$ if $k_{NT} \to \infty$, as $N, T \to \infty$.*

It may not be straightforward to require condition $k_{NT} < N^{1/2}T$ in Proposition 1, because $|\tau_i(t)|$ could be considered informative for jumps larger than $N^{1/2}T$. When the jumps are too large, factor estimation errors may also explode as fast as the jumps in theory. However, this is an extreme situation and may not materialize in practice. This is because the algorithm is sequential and large jumps must be removed first and $|\tau_i(t)|$ functions better as large jumps get removed. In contrast, and more importantly, the algorithm may fail to detect non-explosive jumps in the data. For example, assume that the jump magnitude is $k_{NT} = T/\sqrt{N}$ and asymptotic inference requires $\sqrt{T}/N \to c < \infty$, as shown in Theorem 2. Then, these jumps may remain in the data and affect the inference results. We address this concern with the following theorem:

Theorem 3 *Suppose that factors (F) and factor loadings (Λ) are estimated by (5) and we estimate the number of factors (r) by (9) using $\hat{x}^*_{it}$. From Assumptions 1–7 and $\hat{\omega}_i - \omega_i = O_p(1)$, for every jump detected by the algorithm, the following conditions hold:*

(i-a) If $\eta^c_t = 0$, then (6) holds under $\sqrt{N}/T \to 0$, as $N, T \to \infty$.

(i-b) If $\eta^c_t = 1$, then (8) holds, as $N, T \to \infty$.

(ii) (7) holds under $\sqrt{T}/N \to 0$ and $\sqrt{N}/T \to c$ ($0 \le c < \infty$), as $N, T \to \infty$.

(iii) $\hat{r} \xrightarrow{p} r$, as $N, T \to \infty$.

Several useful implications follow. Part (i-a) states that unless common jumps occur at t, we can have standard asymptotic inferences for the factors in Bai (2003) without any additional condition (we already have condition $\sqrt{N}/T \to 0$ in the standard result). In other words, Theorem 1 states that if the jumps are not larger than $\sqrt{T}$, we obtain asymptotic results, although they can be asymptotically identified with the algorithm because they are explosive as long as $T \to \infty$. Therefore, what we require is only the existing condition $\sqrt{N}/T \to 0$. Part (i-b) suggests that if we have common jumps at t, we cannot have asymptotic normality for F_t, although the factor space can still be consistently estimated. Part (ii) means that the inference for factor loading requires condition $\sqrt{N}/T \to c$ ($0 \leq c < \infty$) in addition to the existing condition $\sqrt{T}/N \to 0$. If this is not satisfied, jumps smaller than or equal to $T/\sqrt{N}$ may not be detected in theory because $T/\sqrt{N} \to c^{-1} < \infty$. This means again that jumps remain in the data and may contaminate the inference results. However, this condition is not more restrictive than that required in part (i-a). Finally, part (iii) simply ensures that after correcting the jumps, Bai and Ng's (2002) information criteria can consistently estimate the number of factors.

4.2 Factor jump tests

So far, jumps follow Assumption 6 and are independent of factor structure. Moreover, from Assumption 1 ($E\|F_t\| < \infty$), underlying factors should not show large jumps. However, if we allow for the underlying factors to jump, the response variables also exhibit common jumps, so that they must be identified as factor jumps. From an empirical perspective, whether factors show jumps or not is an important question but very often not a priori known to researchers.

To illustrate this, we present two dissimilar models exhibiting common jumps at time t. If the jumps are outliers independent of factors, the model is the same as (3) and (4),

$$x_{it} = \lambda_i' F_t + z_{it} + u_{it}. \tag{10}$$

On the other hand, if the jumps are of factors, by denoting them by J_t, an $r \times 1$ vector, the model becomes

$$\begin{aligned} x_{it} &= \lambda_i'(F_t + J_t) + u_{it}, \\ &= \lambda_i' F_t + \lambda_i' J_t + u_{it}. \end{aligned} \tag{11}$$

The two models have very different implications, but the difference is not trivial by observing x_{it}. To this end, we propose a factor jump test for the null hypothesis of model (10) against the alternative hypothesis (11) as follows.

Factor jump test

Step 1. Estimate the jump-free factors $\hat{F}_t$ and factor loadings $\hat{\lambda}_i$ using the jump-correction procedure proposed in the previous subsection.

Step 2. Obtain residuals from cross-sectional regression: $\hat{u}_{it} = x_{it} - \hat{\lambda}_i'\hat{F}_t$ at t for $i = 1, ..., N$.

Step 3. Let a factor jump be suspected at $t = T^c$. Implement an F test for the null hypothesis $H_0 : \gamma_1 = 0_{r\times 1}$ against the alternative hypothesis $H_1 : \gamma_1 \neq 0_{r\times 1}$ in the following cross-sectional regression:

$$\hat{u}_{iT^c} = \gamma_0 + \hat{\lambda}_i'\gamma_1 + \varepsilon_i, \quad i = 1, ..., N, \tag{12}$$

that is,

$$F^J = \frac{(SSR_r - SSR_u)/r}{SSR_u/(N-r)},$$

where SSR_r and SSR_u are the restricted and unrestricted sums of squared regression residuals (12).

If the test rejects the null hypothesis, we conclude that the common jumps at time T^c are of factors. If not, the jumps are outliers independent of factors. We formally present this test property in the following theorem.

Theorem 4 *Let Assumptions 1–7 hold. (i) Under model (10) of the null hypothesis that jumps are independent of common factors, $rF^J \Rightarrow \chi_r^2$ as $N, T \to \infty$. (ii) Under model (11) of the alternative hypothesis that jumps are part of common factors, $F^J \to \infty$ as $N, T \to \infty$.*

Remark 1 *We can also consider a t test in regression (12) for individual factors to investigate whether an individual factor jumps or not. This version is especially useful if the estimated individual factors can be identified and interpreted.*

5 Monte Carlo simulation

In this section, we study the finite sample properties of asymptotic inference for common factor models in the presence of jumps via Monte Carlo simulations. We examine how independent jumps contaminate the standard inference and how the proposed jump-correction method improves performance. We also investigate the finite sample size and power of the proposed test.

We generate the data by

$$x_{it}^* = \lambda_i' f_t + u_{it}, \tag{13}$$

$$x_{it} = x_{it}^* + z_{it}, \tag{14}$$

$$z_{it} = \eta_t^c \delta_{it}^c + \eta_{it}\delta_{it}, \tag{15}$$

where $f_t \sim i.i.d.N(0, I_r)$, $\lambda_i \sim i.i.d.N(0, I_r)$, and $u_{it} \sim i.i.d.N(0, 1)$. Jump process z_{it} has a common component, where $\eta_t^c \sim i.i.d.B(p_c/T)$ and $\delta_{it}^c \sim i.i.d.N(0, \sigma^2)$, and an idiosyncratic component, where $\eta_{it} \sim i.i.d.B(p/T)$ and $\delta_{it} \sim i.i.d.N(0, \sigma^2)$. Importantly, jumps are independent of factor structure in this model. Throughout this experiment, the jump-correction method assumes a white noise for every series and we use the critical value $\xi = 5$ for $|\tau_{it}|$. We consider a case in which jumps are not corrected (denoted by "no correction" in the tables) and one in which jumps are corrected using the proposed method (denoted by "correction" in the tables). In total, we run 3,000 replications.

We first investigate the distributional properties of the factor and factor loading estimates. For this, we set $r = 1$ and compute the coverage rate and average length of the confidence intervals of (rotation-adjusted) factor Hf_t, factor loading $\lambda_i H^{-1}$, and common component $\lambda_i f_t$, where

$$H = v^{-1}(T^{-1} \textstyle\sum_{t=1}^T \hat{f}_t f_t)(N^{-1} \sum_{i=1}^N \lambda_i^2),$$

and v is the largest eigenvalue of $XX'/(NT)$.[6] The asymptotic confidence intervals are constructed by Bai (2003) such that

$$[\hat{f}_t - z_{\alpha/2}\sqrt{\widehat{Var}(\hat{f}_t)},\ \hat{f}_t + z_{\alpha/2}\sqrt{\widehat{Var}(\hat{f}_t)}],$$
$$[\hat{\lambda}_i - z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\lambda}_i)},\ \hat{\lambda}_i + z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\lambda}_i)}],$$
$$[\hat{f}_t\hat{\lambda}_i - z_{\alpha/2}\sqrt{\widehat{Var}(\hat{f}_t\hat{\lambda}_i)},\ \hat{f}_t\hat{\lambda}_i + z_{\alpha/2}\sqrt{\widehat{Var}(\hat{f}_t\hat{\lambda}_i)}],$$

where, respectively,

$$\begin{aligned}
\widehat{Var}(\hat{f}_t) &= (N^{-1} \textstyle\sum_{j=1}^N \hat{u}_{jt}^2)(\sum_{j=1}^N \hat{\lambda}_j^2)^{-1}, \\
\widehat{Var}(\hat{\lambda}_i) &= (T^{-1} \textstyle\sum_{s=1}^T \hat{u}_{is}^2)(\sum_{s=1}^T \hat{f}_s^2)^{-1}, \\
\widehat{Var}(\hat{f}_t\hat{\lambda}_i) &= [(N^{-1} \textstyle\sum_{j=1}^N \hat{u}_{jt}^2)(N^{-1} \sum_{j=1}^N \hat{\lambda}_j^2)^{-1}\hat{\lambda}_i^2 + (T^{-1} \sum_{s=1}^T \hat{u}_{is}^2)(T^{-1} \sum_{s=1}^T \hat{f}_s^2)^{-1}\hat{f}_t^2,
\end{aligned}$$

and $z_{\alpha/2}$ is the $100 \times (1 - \alpha/2)\%$ quantile of the standard normal distribution. We consider the set of parameter values associated with jump magnitudes $\sigma = [0, 5, 10, 50, 100]$, that are in turn associated with jump frequencies $(p_c, p) = [(1, 0),\ (5, 0),\ (0, 1),\ (0, 5),\ (1, 1)]$, and the set of sample sizes $(N, T) = [(20, 500), (50, 200), (100, 100), (200, 50), (500, 20)]$. We also consider the 90% confidence intervals for, without loss of generality, f_T, λ_1, and $\lambda_1 f_T$. Since Theorem 1 requires no common jumps, we set $\eta_T^c = 0$. The results are reported in Tables

[6] Since we set $r = 1$ in this experiment, H is a scalar. Still, it is important to incorporate it because it is not necessarily 1.

1 to 3. Tables 1(a) and 1(b) give the coverage rate and average length of the confidence interval of Hf_T. They show that even when jumps are not corrected, the coverage rate goes close to 0.9 except for the case of $(N,T) = (500, 20)$; however, the average length inflates as σ increases. On the other hand, when jumps are corrected, the coverage rate is again close to 0.9, except except for the case of $(N,T) = (500, 20)$, and the average length does not inflate. This shows that the proposed jump-correction method works well for factor estimation as long as $\sqrt{N}/T \to 0$ is relevant, as discussed in Theorem 3 (i-a). We now examine the results of factor loading in Tables 2(a) and 2(b). When jumps are not corrected, the coverage rate significantly deteriorates and the average length shortens as σ increases. On the other hand, when jumps are corrected, we observe significant improvement except for the case of $(N,T) = (500, 20)$, where $\sqrt{N}/T \to c < \infty$ could be irrelevant, as Theorem 3(ii) predicts. We also observe that the coverage rate deteriorates when $(N,T) = (20, 500)$, because condition $\sqrt{T}/N \to 0$ as required in Theorem 3(ii) may not be relevant; however, the errors are minor in this case. Finally, Tables 3(a) and 3(b) show the confidence interval results for the common component. Again, the coverage rate moves away from the nominal level 0.9 as σ increases when jumps are not corrected; however, jump correction significantly improves performance except for the case of $(N,T) = (500, 20)$. We again observe some errors in coverage rate for the case of $(N,T) = (20, 500)$, but they are minor.

The above results are direct consequences of Theorems 1 and 3. However, a good coverage ratio of $\hat{f}_T$ without jump correction may be questionable. We here show that the observed coverage rate is pointwise and does not reflect a good estimate for $\left\{\hat{f}_t\right\}_{t=1}^T$ as a series. To this end, we compute the correlation coefficient between the estimated factor $\left\{\hat{f}_t\right\}_{t=1}^T$ and the (rotated) true factor $\{Hf_t\}_{t=1}^T$. Table 4 gives the average correlation coefficient over simulation. When jumps are not corrected, it moves significantly away from 1 as σ becomes larger. This is the case even if all the jumps are idiosyncratic ($p_c = 0$). The average correlation coefficient moves very close to 1 when jumps are corrected in almost all cases. Furthermore, Figure 1 gives a sample path of a true factor and factor estimates without jump correction when the data show (a) common jumps at $t = \lfloor 0.5T \rfloor$ with $\sigma = 10$ and (b) an idiosyncratic jump in x_{1t} at $t = \lfloor 0.5T \rfloor$ with $\sigma = 100$. The factor estimate exhibits a jump in response to these outliers at $t = \lfloor 0.5T \rfloor$. Thus, we do not obtain a good estimate for the series as a whole. More importantly, this occurs even if the outlier is idiosyncratic as long as the magnitude is sufficiently large.

Table 5 investigates Theorems 2 and 3(iii) and reports the average estimated number of factors in Bai and Ng's (2002) information criteria. We here set the true number of factors to $r = 4$ and consider the three suggested information criteria (IC_p1, IC_p2, and IC_p3). In every case of the sample size and jump frequency, the number without jump correction moves away from 4 as σ increases. One must be careful because our theory does not determine the direction of either under- or over-estimation. For example, it tends to over-estimate when common jumps occur; we also observe significant over-estimation when only one idiosyncratic jump occurs, that is, $(p_c, p) = (0, 1)$. However, we observe under-estimation when idiosyncratic jumps are more frequent, $(p_c, p) = (0, 5)$. Again, after jumps are corrected, it recovers the true number 4 in most cases as suggested by Theorem 3 (iii).

Finally, we investigate the size and power of the factor jump test. Figure 1 shows that even if the true factor does not jump, independent outliers in the response variables (even if it occurs in one response variable) could induce a spurious jump in factor estimates, showing the importance of this test. We first examine the size of the test. The data in models (13) and (14), that is, under the null hypothesis of no factor jumps, are generated with $r = 2$. We also simplify the model by assuming that no idiosyncratic jumps occur. Thus, we generate process (15) with

$$z_{it} = I(t = \lfloor 0.5T \rfloor) \times \delta_{it}^c,$$

where $\delta_{it}^c \sim i.i.d.N(0, \sigma^2)$. On the other hand, to investigate power of the test, we assume that $z_{it} = 0$ for all i and t, so that although no independent outliers are present, the factors jump such that

$$f_t = f_t^* + f_t^J,$$

where $f_t^* \sim i.i.d.N(0, I_r)$ represents jump-free factors and $f_t^J = [I(t = \lfloor 0.5T \rfloor) \times \delta,\ 0]$ with $\delta \sim N(0, \sigma^2)$ corresponds to a jump of the first factor. Since the jump-free factor estimates used in Steps 1 and 2 can affect the performance of the test, it is instructive to compare the results for the following two cases. Case 1 considers an unfeasible test that assumes the presence of true jump-free observations x_{it}^*. The test is constructed from the factors and factor loading estimated using them. Case 2 pertains to a feasible test that uses jump-corrected factor and factor loading estimates to construct the test.

Table 6 reports the size of the factor jump test at the nominal 5% level with the set of jump magnitudes and sample sizes. Case 1 illustrates a very good size; however, the feasible test in Case 2 suffers some size distortions when σ is small. This is consistent with the theory, because, as elaborated in the proof of Theorem 4 in the appendix, the pseudo-true coefficients attached to factor loading estimates in the cross-section regression of Step 2 have random quantity in finite samples. However, since they shrink to zero at the rate of

$o_p(k_{NT}^{-1}N^{-1/2})$, the size improves remarkably as σ becomes large. The size is also distorted when T is small, because the jump-correction algorithm does not work well in such cases, as shown in Tables 1 and 2. However, the size improves as T increases. Finally, Table 7 illustrates the power as a rejection frequency of the test at the nominal 5% level. It shows that the test has good power against factor jumps.

6 Empirical examples

6.1 Daily currency returns against the U.S. dollar

Much attention has been paid to comovements of currency returns. Especially, recent empirical evidence of deviation from the theory of uncovered interest parity has motivated many authors and policy makers to identify the risk factors in currency markets besides interest rate differentials. For example, Lustig et al. (2011) apply a common factor model to monthly returns on 35 currencies against the U.S. dollar (minus the interest differential). Using the estimates of principal component factors, they identify the global risk factor as the series closely related to the world's stock market volatility and find it consists of an important element of exchange rate dynamics. While they use monthly data, it is well-known that large jumps are likely to occur if daily currency returns data are used.

We provide a small empirical example related to such data. To this end, we use the daily log-returns on 25 major foreign currencies with relatively stable volatilities against the U.S. dollar for the recent financial crisis period. The sample period is from August 1, 2007, to September 30, 2008, totaling 305 business days. The currency returns are computed as $r_{i,t} = \log(e_{i,t}/e_{i,t-1})$, where e_{it} is the daily spot exchange rate of currency i against the U.S. dollar at day t. Table 8 gives the list of currencies. The data, e_{it}, are quoted at 15:00 EST by Bankers Trust Co., and are downloaded from the Datastream database. Figure 2 plots the 25 individual currency returns, clearly showing a few large jumps in some currencies.

We first identify the jump dates using the proposed method. For this, we fit a white noise model to individual series and use the critical value of 5 for $|\tau_i(t)|$. Table 8 gives the number of jumps identified using this method. Jumps are relatively scarce, but 13 out of 25 currencies exhibit them. Figure 3 provides information on how many series exhibit a jump each day, with no jump or only a few jumps occurring on most days considered as individual jumps. However, nine jumps are identified on May 6 and 7 and four jumps on September 29, 2008.

Turning to factor estimation, Figures 4-1 and 4-2 present the first and second estimated factors, respectively. For each set of figures, panel (a) shows the factor estimates with and without jump correction and panel (b) gives their difference. A visual inspection shows that the first factor estimate may include three jumps, on May 6 and 7 and September 29, 2008. The second factor estimate may also exhibit jumps on these days. To examine whether these jumps are due to the independent outliers or jumps in the factors, we present the results of the test for factor jumps in Table 9: an F test for a jump of the two factors jointly and t tests for a jump of each factor. The table shows that the null hypothesis of independent outliers is rejected at the 5% level for the jumps on May 6, 2008, suggesting that they are of factors. We also find that the t test for the first factor is significant at the 5 % level but insignificant for the second factor. Finally, we try to interpret the factor estimates. The first factor is related to some European currencies (the Hungarian forint, Norwegian krone, Polish zloty, etc.), the Australian dollar, and the New Zealand dollar. In contrast, the second factor is related to the Swiss franc and Japanese yen. Given that the latter two currencies exhibit much more market liquidity, we may conclude that a factor jump is found in the common risk factor related to currencies with less liquidity, that is, the first factor.

6.2 Japanese prefectural data following earthquake shocks

The second example involves the new car registrations data for 47 Japanese prefectures. The data consist of monthly spans from January 1985 to December 2014 (seasonally adjusted) and are taken from the Nikkei CIDIc database. We consider a monthly growth rate computed by the first difference of its natural logarithms so that the time dimension of the data is $T = 12 \times 30 - 1 = 359$. Instead of presenting all 47 series, Figure 5 gives the individual series of four selected prefectures illustrating the features of the data well. The top two panels present Tokyo and Osaka, the two largest prefectures in Japan, while the two figures at the bottom two panels represent Hyogo and Miyagi prefectures. Hyogo prefecture clearly exhibits a large jump in January 1995, because it was the epicenter of the Great Hanshin earthquake. On the other hand, Miyagi prefecture also exhibits a large jump in 2011 following the Great East Japan earthquake in March 2011. Tokyo and Osaka may only be indirectly affected by these events. The question we examine is whether these large jumps affect our factor estimation.

To this end, we first follow the series-by-series jump-correction procedure. We fit a white noise model for individual series and set the critical value of $|\tau_i(t)|$ at 5. Table 10 shows that only one prefecture exhibits a jump following the 1995 earthquake, whereas 23 prefectures experienced a jump after the 2011 earthquake. From Bai and Ng's (2002) information criteria

(IC_p2), the number of factors estimated with the original data is four, but this becomes two with jump-corrected data. Hence, the number may be contaminated by these jumps. Finally, Figure 6 gives the first four non-corrected estimates (in the top four panels) and the two jump-corrected factor estimates (in the bottom two panels). As expected, the non-corrected estimates exhibit jumps. In particular, the second and third non-corrected estimates exhibit jumps in March 2011. To examine whether these jumps are of factors, we implement factor jump tests in Table 11. We observe strong evidence of factor jumps in March 2011, with p-value 0.00 for the F-test. The t-tests indicate that the jump is associated with the first factor with p-value 0.03, while the p-value for the second factor is 0.40. Finally, the fourth non-corrected factor estimate shows a large jump in January 1995 following the Great Hanshin earthquake, although only Hyogo prefecture exhibits a jump. Table 11 shows no evidence of factor jumps in January 1995. Therefore, we conclude that the jump in factor estimate in January 1995 was spuriously caused by an individual outlier in Hyogo prefecture and that the factors did not jump.

7 Conclusion

Financial and economic time-series data often exhibit infrequent but large jumps. This paper explored the problems pertaining to such jumps in recently developed large-dimensional common factor models. To make this attempt feasible and attractive, we introduce the following extensions of the standard model. First, jumps are modeled as increments of a mixture of Poisson processes independent of the underlying factor structure. Second, the jump magnitudes are modeled as a function of data dimension to derive meaningful asymptotic results. Third, we consider idiosyncratic jumps and common jumps. Under this setting, we primarily derive the upper bounds of jump magnitudes with which the standard asymptotic inference goes. Furthermore, this result is followed by two useful applications: the series-by-series jump-correction method and the factor jump test. A Monte Carlo experiment confirms that independent large outliers easily contaminate standard asymptotic inference. However, the proposed jump-correction method retrieves good finite sample properties unless T is very small. The factor jump test shows good size when outliers are sufficiently large and exhibit good power. The usefulness of the proposed method is highlighted in a small empirical example using daily log-returns data of 25 currencies against the U.S. dollar as well as Japanese prefectural new car registration data following the two large earthquakes.

Appendix : Proof of Theorems

For notational simplicity, we assume that $E\left\|F_t\right\|^2 = \sigma_F^2$ for all t, $E\left\|\lambda_i\right\|^2 = \lambda^2$ for all i, and $E(u_{it}^2) = \sigma_u^2$ in the following proofs. This simplification does not qualitatively affect our final results.

Lemma 1: Let $b_t = \sum_{i=1}^N \sum_{j=1}^N E(z_{it}z_{jt})$ and $d_t = \sum_{s=1}^T \sum_{i=1}^N \sum_{j=1}^N E(z_{is}z_{it}z_{js}z_{jt})$. From Assumptions 6 and 7, we have

$$b_t = \begin{cases} O(k_{NT}^2 N), & \text{if } \eta_t^c = 1 \\ O(k_{NT}^2 NT^{-1}), & \text{if } \eta_t^c = 0 \end{cases},$$

and

$$\bar{b} = T^{-1}\sum_{t=1}^T b_t = O_p(k_{NT}^2 NT^{-1}).$$

We also have

$$d_t = \begin{cases} O(k_{NT}^4 N^2), & \text{if } \eta_t^c = 1 \\ O(k_{NT}^4 \max\left\{NT^{-1}, N^2T^{-2}\right\}), & \text{if } \eta_t^c = 0 \end{cases},$$

and

$$\bar{d} = T^{-1}\sum_{t=1}^T d_t = \begin{cases} O_p(k_{NT}^4 N^2 T^{-1}), & \text{if } p_c = 0 \\ O_p(k_{NT}^4 \max\left\{NT^{-1}, N^2T^{-2}\right\}), & \text{if } p_c \neq 0 \end{cases}.$$

.

Proof of Lemma 1: For all i and t, $E(z_{it}^2) = k_{NT}^2\sigma^2$ if $\eta_t^c = 1$ and $E(z_{it}^2) = \frac{p}{T}\sigma^2$ if $\eta_t^c = 0$. Because $E(z_{it}z_{jt}) = 0$ for $i \neq j$, by Assumption 6,

$$\begin{aligned} b_t &= \sum_{i=1}^N E(z_{it}^2) + \sum_{\substack{i=1 \\ i\neq j}}^N \sum_{j=1}^N E(z_{it}z_{jt}) \\ &= \sum_{i=1}^N E(z_{it}^2) = \begin{cases} k_{NT}^2 N\sigma^2 + k_{NT}^2 N(p/T)\sigma^2, & \text{if } \eta_t^c = 1 \\ k_{NT}^2 N\frac{p}{T}\sigma^2, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

and the result for b_t follows. For $\bar{b}$,

$$\begin{aligned} \bar{b} &= T^{-1}p_c k_{NT}^2 N\sigma^2 + T^{-1}\sum_{t=1}^T k_{NT}^2 N(p/T)\sigma^2, \\ &= T^{-1}p_c k_{NT}^2 N\sigma^2 + k_{NT}^2 N(p/T)\sigma^2, \\ &= O(k_{NT}^2 NT^{-1}), \end{aligned}$$

and the result follows.

We turn to the bound of d_t. From Assumption 6(c), $E(z_{it}^4) = k_{NT}^4 3\sigma^4$ if $\eta_t^c = 1$ and $E(z_{it}^4) = (p/T)k_{NT}^4 3\sigma^4$ if $\eta_t^c = 0$ for all i and t, and so

$$\begin{aligned}
d_t &= \textstyle\sum_{s=1}^T \sum_{i=1}^N \sum_{j=1}^N E(z_{is}z_{it}z_{js}z_{jt}), \\
&= \textstyle\sum_{i=1}^N E(z_{it}^4) + \sum_{\substack{i=1 \\ i\neq j}}^N \sum_{j=1}^N E(z_{it}^2)E(z_{jt}^2) \\
&\quad + \textstyle\sum_{\substack{s=1 \\ s\neq t}}^T \sum_{i=1}^N E(z_{is}^2)E(z_{it}^2) + \sum_{\substack{s=1 \\ s\neq t}}^T \sum_{\substack{i=1 \\ i\neq j}}^N \sum_{j=1}^N E(z_{is}z_{it}z_{js}z_{jt}), \\
&= I + II + III + IV.
\end{aligned}$$

If $\eta_t^c = 1$, then

$$\begin{aligned}
I &= Nk_{NT}^4 3\sigma^4 + N(p/T)k_{NT}^4 3\sigma^4, \\
II &= (N^2 - N)k_{NT}^4\sigma^4 + (N^2 - N)(p/T)^2 k_{NT}^4\sigma^4, \\
III &= N(T-1)(p/T)k_{NT}^4\sigma^4 + N(T-1)(p/T)^2 k_{NT}^4\sigma^4,
\end{aligned}$$

and $IV = 0$. Therefore, term II dominates and $d_t = O(k_{NT}^4 N^2)$. If $\eta_t^c = 0$, then

$$\begin{aligned}
I &= N(p/T)k_{NT}^4 3\sigma^4, \\
II &= (N^2 - N)(p/T)^2 k_{NT}^4\sigma^4, \\
III &= N(T-1)(p/T)^2 k_{NT}^4\sigma^4,
\end{aligned}$$

and $IV = 0$. Therefore, $d_t = O(k_{NT}^4 \max\{NT^{-1}, N^2T^{-2}\})$. For $\bar{d}$,

$$\begin{aligned}
\bar{d} &= T^{-1}p_c N k_{NT}^4 3\sigma^4 + N(p/T)k_{NT}^4 3\sigma^4, \\
&\quad + T^{-1}p_c(N^2 - N)k_{NT}^4\sigma^4 + (N^2 - N)(p/T)^2 k_{NT}^4\sigma^4, \\
&\quad + T^{-1}p_c N(T-1)(p/T)k_{NT}^4\sigma^4 + N(T-1)(p/T)^2 k_{NT}^4\sigma^4, \\
&= I + II + III + IV + V + VI.
\end{aligned}$$

If $p_c \neq 0$, then term III dominates and $\bar{d} = O(k_{NT}^4 N^2 T^{-1})$. If $p_c = 0$, then terms I, III, and V are zero. Then, $\bar{d} = O(k_{NT}^4 \max\{NT^{-1}, N^2T^{-2}\})$. ■

Lemma 2: From Assumptions 1–7,

$$T^{-1}\textstyle\sum_{t=1}^T \left\| \hat{F}_t - H'F_t \right\|^2 = O_p(J_{NT}),$$

where

$$J_{NT} = \begin{cases} \max\{k_{NT}^4 T^{-2}, k_{NT}^2 N^{-1}T^{-1}\} & \text{if } p_c \neq 0 \\ \max\left\{k_{NT}^4 c_{NT}^{-2} T^{-2}, k_{NT}^2 N^{-1}T^{-1}\right\} & \text{if } p_c = 0 \end{cases},$$

as $N, T \to \infty$.

Proof of Lemma 2: Using steps very similar to those applied in the proof of Theorem 1 of Bates et al. (2013), we start with the results of the proof of Theorem 1 of Bai and Ng (2002):

$$\begin{aligned} \hat{F}_t - H'F_t &= (NT)^{-1}\Big\{\hat{F}'F\Lambda'u_t + \hat{F}'u\Lambda F_t + \hat{F}'uu_t + \hat{F}'F\Lambda'z_t \\ &\quad +\hat{F}'Z\Lambda F_t + \hat{F}'Zz_t + \hat{F}'uz_t + \hat{F}'Zu_t\Big\} \\ &\equiv \textstyle\sum_{h=1}^{8} d_{ht}, \end{aligned} \tag{A.1}$$

where $d_{1t} = (NT)^{-1}\hat{F}'F\Lambda'u_t$, etc. Since

$$T^{-1}\textstyle\sum_{t=1}^{T}\left\|\hat{F}_t - H'F_t\right\|^2 \leq 8\sum_{h=1}^{8}\left(T^{-1}\sum_{t=1}^{T}\|d_{ht}\|^2\right).$$

and we know from Bai and Ng (2002) that the terms for d_{1t}, d_{2t}, and, d_{3t} are $O_p(c_{NT}^{-2})$, we consider the bounds for the remaining terms. For $h = 4$,

$$\|d_{4t}\|^2 \leq N^{-2}\underbrace{(T^{-1}\textstyle\sum_{s=1}^{T}\left\|\hat{F}_s\right\|^2)}_{=tr(I_r)=r}\underbrace{(T^{-1}\textstyle\sum_{s=1}^{T}\|F_s\|^2)}_{\xrightarrow{p}\sigma_F^2}\|\Lambda'z_t\|^2.$$

where

$$\begin{aligned} E\|\Lambda'z_t\|^2 &= \textstyle\sum_{i=1}^{N}\sum_{j=1}^{N}E(\lambda_i'\lambda_j)E(z_{it}z_{jt}), \\ &\leq \lambda^2 b_t. \end{aligned}$$

Therefore,

$$T^{-1}\textstyle\sum_{t=1}^{T}E\|d_{4t}\|^2 \leq N^{-2}r\sigma_F^2\lambda^2\bar{b},$$

and so from the result of $\bar{b}$ in Lemma 1, we obtain

$$T^{-1}\textstyle\sum_{t=1}^{T}\|d_{4t}\|^2 = O_p(k_{NT}^2N^{-1}T^{-1}).$$

For $h = 5$,

$$\|d_{5t}\|^2 \leq N^{-2}T^{-1}(T^{-1}\textstyle\sum_{s=1}^{T}\left\|\hat{F}_s\right\|^2)\|Z\Lambda F_t\|^2,$$

where

$$\begin{aligned} E\|Z\Lambda F_t\|^2 &= \textstyle\sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N}|E(z_{is}z_{js})|E\left\|\lambda_i'F_t\lambda_j'F_t\right\|, \\ &\leq T\lambda^2\sigma_F^2\bar{b}. \end{aligned}$$

Therefore,

$$E\|d_{5t}\|^2 \leq N^{-2}r\lambda^2\sigma_F^2\bar{b},$$

so that

$$T^{-1}\textstyle\sum_{t=1}^{T}\|d_{5t}\|^2 = O_p(k_{NT}^2N^{-1}T^{-1}).$$

For $h = 6$,

$$\|d_{6t}\|^2 = N^{-2}T^{-1}(T^{-1}\sum_{s=1}^{T}\left\|\hat{F}_s\right\|^2)\|Z'z_t\|^2,$$

where

$$E\|Zz_t\|^2 = \sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N}|E(z_{is}z_{it}z_{js}z_{jt})| = d_t.$$

Therefore,

$$E\|d_{6t}\|^2 \leq N^{-2}T^{-1}rd_t,$$

so that

$$T^{-1}\sum_{t=1}^{T}E\|d_{6t}\|^2 \leq N^{-2}T^{-1}r\bar{d},$$

and

$$T^{-1}\sum_{t=1}^{T}\|d_{6t}\|^2 = \begin{cases} O_p(k_{NT}^4T^{-2}), & \text{if } p_c \neq 0 \\ O_p(k_{NT}^4\max\{N^{-1}T^{-2}, T^{-3}\}), & \text{if } p_c = 0 \end{cases},$$

or by using symbol $c_{NT} = \min\left\{\sqrt{N}, \sqrt{T}\right\}$,

$$T^{-1}\sum_{t=1}^{T}\|d_{6t}\|^2 = \begin{cases} O_p(k_{NT}^4T^{-2}), & \text{if } p_c \neq 0 \\ O_p(k_{NT}^4c_{NT}^{-2}T^{-2}), & \text{if } p_c = 0 \end{cases}.$$

For $h = 7$,

$$\|d_{7t}\|^2 = N^{-2}T^{-1}(T^{-1}\sum_{s=1}^{T}\left\|\hat{F}_s\right\|^2)\|uz_t\|^2,$$

where

$$\begin{aligned} E\|uz_t\|^2 &= \sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N}E(u_{is}u_{is})E(z_{it}z_{jt}), \\ &\leq T\sigma_u^2b_t. \end{aligned}$$

Therefore,

$$E\|d_{7t}\|^2 \leq N^{-2}r\sigma_u^2b_t,$$

so that

$$T^{-1}\sum_{t=1}^{T}E\|d_{7t}\|^2 \leq N^{-2}r\sigma_u^2\bar{b},$$

and

$$T^{-1}\sum_{t=1}^{T}\|d_{7t}\|^2 = O_p(k_{NT}^2N^{-1}T^{-1}).$$

For $h = 8$,

$$\|d_{8t}\|^2 = N^{-2}T^{-1}(T^{-1}\sum_{s=1}^{T}\left\|\hat{F}_s\right\|^2)\|Zu_t\|^2,$$

where

$$\begin{aligned} E\|Zu_t\|^2 &= \sum_{s=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N}E(u_{it}u_{it})E(z_{is}z_{js}), \\ &\leq T\sigma_u^2\bar{b}. \end{aligned}$$

Therefore,

$$E\left\|d_{8t}\right\|^2 \leq N^{-2} r \sigma_u^2 \bar{b},$$

so that

$$T^{-1}\sum_{t=1}^{T}\left\|d_{8t}\right\|^2 = O_p(k_{NT}^2 N^{-1}T^{-1}).$$

Therefore, the bounds of the five terms are $O_p(k_{NT}^2 N^{-1}T^{-1})$, $O_p(k_{NT}^2 N^{-1}T^{-1})$, $O_p(k_{NT}^4 T^{-2})$, $O_p(k_{NT}^2 N^{-1}T^{-1})$, and $O_p(k_{NT}^2 N^{-1}T^{-1})$ if $p_c \neq 0$. The third term becomes $O_p(k_{NT}^4 c_{NT}^{-2} T^{-2})$ if $p_c = 0$. The result follows.■

Lemma 3: From Assumptions 1–7, the following hold:

$$\begin{aligned}
&\text{(a)} \quad \left\|T^{-1}\textstyle\sum_{t=1}^{T}(\hat{F}_t - H'F_t)F_t'\right\| &=& \quad O_p(c_{NT}^{-2}) + O_p(k_{NT}N^{-1/2}T^{-1}) + O_p(k_{NT}^2 N^{-1/2}T^{-2}),\\
&\text{(b)} \quad \left\|T^{-1}\textstyle\sum_{t=1}^{T}(\hat{F}_t - H'F_t)\hat{F}_t'\right\| &=& \quad O_p(c_{NT}^{-2}) + O_p(k_{NT}N^{-1/2}T^{-1}) + O_p(k_{NT}^2 N^{-1/2}T^{-2}),\\
&\text{(c)} \quad \left\|T^{-1}\textstyle\sum_{t=1}^{T}(\hat{F}_t - H'F_t)u_{it}\right\| &=& \quad O_p(c_{NT}^{-2}) + O_p(k_{NT}N^{-1/2}T^{-1}) + O_p(k_{NT}^2 N^{-1/2}T^{-2}),\\
&\text{(d)} \quad \left\|T^{-1}\textstyle\sum_{t=1}^{T}(\hat{F}_t - H'F_t)e_t\right\| &=& \quad O_p(c_{NT}^{-2}) + O_p(k_{NT}N^{-1/2}T^{-1}) + O_p(k_{NT}^2 N^{-1/2}T^{-2}).
\end{aligned}$$

Proof of Lemma 3: (a) We start with Bai and Ng's (2002) expression:

$$\begin{aligned}
T^{-1}\textstyle\sum_{t=1}^{T}(\hat{F}_t - H'F_t)F_t' &= N^{-1}T^{-2}(\hat{F}'F\Lambda u'F + \hat{F}'u\Lambda F'F + \hat{F}'uu'F + \hat{F}'F\Lambda' Z'F\\
&\quad + \hat{F}'Z\Lambda F'F + \hat{F}'ZZ'F + \hat{F}'uZ'F + \hat{F}'Zu'F),\\
&= \textstyle\sum_{h=1}^{8} D_h.
\end{aligned}$$

Terms $D_1 + D_2 + D_3$ do not involve jumps and their sum is $O_p(c_{NT}^{-2})$. In the following, we compute the stochastic bounds for terms D_4 to D_8.

For D_4,

$$\begin{aligned}
N^{-1}T^{-2}\left\|\hat{F}'F\Lambda' Z'F\right\| &\leq N^{-1}T^{-1}\left\|T^{-1/2}\hat{F}\right\|\left\|T^{-1/2}F\right\|\left\|\Lambda' Z'F\right\|,\\
&\leq N^{-1}T^{-1}\left\|T^{-1/2}\hat{F}\right\|\left\|T^{-1/2}F\right\|\sqrt{\textstyle\sum_{t=1}^{T}\sum_{i=1}^{N}\left\|\lambda_i\right\|^2 z_{it}^2\left\|F_t\right\|^2},
\end{aligned}$$

although from independent assumptions (Assumptions 4 and 6(a)), we obtain

$$\begin{aligned}
E(\left\|\lambda_i\right\|^2 z_{it}^2\left\|F_t\right\|^2) &= E\left\|\lambda_i\right\|^2 E(z_{it}^2)E\left\|F_t\right\|^2,\\
&= \begin{cases} k_{NT}^2\sigma^2\lambda^2\sigma_F^2 + k_{NT}^2(p/T)\sigma^2\lambda^2\sigma_F^2, & \text{if } \eta_t^c = 1\\ k_{NT}^2(p/T)\sigma^2\lambda^2\sigma_F^2, & \text{if } \eta_t^c = 0\end{cases}.
\end{aligned}$$

Therefore,

$$E\textstyle\sum_{t=1}^{T}\sum_{i=1}^{N}\left\|\lambda_i\right\|^2 z_{it}^2\left\|F_t\right\|^2 = Nk_{NT}^2(p_c + p)\sigma^2\lambda^2\sigma_F^2,$$

so that

$$\|\Lambda' Z' F\| = O_p(k_{NT} N^{1/2}).$$

This results in

$$D_4 = O_p(k_{NT} N^{-1/2} T^{-1}).$$

For D_5,

$$N^{-1}T^{-2} \left\| \hat{F}' Z \Lambda F' F \right\| \leq N^{-1}T^{-1} \left\| \hat{F}' Z \Lambda \right\| \left\| T^{-1/2} F \right\| \left\| T^{-1/2} F \right\|,$$

where

$$\left\| \hat{F}' Z \Lambda \right\| \leq \sqrt{\textstyle\sum_{t=1}^{T} \sum_{i=1}^{N} \left\| \hat{F}_t \right\|^2 z_{it}^2 \left\| \lambda_i \right\|^2}.$$

However,

$$E \left\| \hat{F}_t \right\|^2 z_{it}^2 \left\| \lambda_i \right\|^2 = \begin{cases} k_{NT}^2 \sigma^2 \lambda^2 r + (p/T) k_{NT}^2 \sigma^2 \lambda^2 r, & \text{if } \eta_t^c = 1 \\ (p/T) k_{NT}^2 \sigma^2 \lambda^2 r, & \text{if } \eta_t^c = 0 \end{cases}$$

$$E \textstyle\sum_{t=1}^{T} \sum_{i=1}^{N} \left\| \lambda_i \right\|^2 z_{it}^2 \left\| \hat{F}_t \right\|^2 = N k_{NT}^2 (p_c + p) \sigma^2 \lambda^2 r,$$

so that $\left\| \hat{F}' Z \Lambda \right\| = O_p(k_{NT}^2 N^{1/2})$ and $D_5 = O_p(k_{NT} N^{-1/2} T^{-1})$.

For D_6,

$$N^{-1}T^{-2} \left\| \hat{F}' Z Z' F \right\| \leq N^{-1}T^{-2} \sqrt{\textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} \left\| \hat{F}_t z_{it} z_{is} F_s' \right\|^2}.$$

However,

$$\begin{aligned} E \left\| \hat{F}_t z_{it} z_{is} F_s' \right\|^2 &= E \left\| \hat{F}_t \right\|^2 E(z_{it}^2) E(z_{is}^2) E \left\| F_s \right\|^2 \\ &= \begin{cases} k_{NT}^2 \sigma^2 E(z_{it}^2) \sigma_F^2 r + (p/T) k_{NT}^2 \sigma^2 E(z_{it}^2) \sigma_F^2 r, & \text{if } \eta_s^c = 1 \\ (p/T) k_{NT}^2 \sigma^2 E(z_{it}^2) \sigma_F^2 r, & \text{if } \eta_s^c = 0 \end{cases}, \end{aligned}$$

so that

$$\begin{aligned} \textstyle\sum_{s=1}^{T} E \left\| \hat{F}_t z_{it} z_{is} F_s' \right\|^2 &\leq k_{NT}^2 \sigma^2 (p_c + p) E(z_{it}^2) \sigma_F^2 r, \\ &= \begin{cases} [k_{NT}^2 \sigma^2 (p_c + p) \sigma_F^2 r] k_{NT}^2 (1 + p/T) \sigma^2, & \text{if } \eta_t^c = 1 \\ [k_{NT}^2 \sigma^2 (p_c + p) \sigma_F^2 r] k_{NT}^2 (p/T) \sigma^2, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

so that

$$\textstyle\sum_{t=1}^{T} \sum_{s=1}^{T} E \left\| \hat{F}_t z_{it} z_{is} F_s' \right\|^2 \leq [k_{NT}^2 \sigma^2 (p_c + p)]^2 \sigma_F^2 r,$$

and

$$\textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} E \left\| \hat{F}_t z_{it} z_{is} F_s' \right\|^2 \leq N [k_{NT}^2 \sigma^2 (p_c + p)]^2 \sigma_F^2 r.$$

Therefore, $D_6 = O_p(k_{NT}^2 N^{-1/2} T^{-2})$.

For D_7,

$$N^{-1}T^{-2}\left\|\hat{F}'uZ'F\right\| \le N^{-1}T^{-2}\sqrt{\sum_{i=1}^N \sum_{t=1}^T \sum_{s=1}^T \left\|\hat{F}_t u_{it} z_{is} F_s'\right\|^2}.$$

However,

$$\begin{aligned} E\left\|\hat{F}_t u_{it} z_{is} F_s'\right\|^2 &= E\left\|\hat{F}_t\right\|^2 E(u_{it}^2)E(z_{is}^2)E\left\|F_s\right\|^2, \\ &= \begin{cases} k_{NT}^2\sigma^2\sigma_u^2 r\sigma_F^2 + (p/T)k_{NT}^2\sigma^2\sigma_u^2 r\sigma_F^2, & \text{if } \eta_s^c = 1 \\ (p/T)k_{NT}^2\sigma^2\sigma_u^2 r\sigma_F^2, & \text{if } \eta_s^c = 0 \end{cases} \end{aligned}$$

so that

$$\sum_{s=1}^T E\left\|\hat{F}_t u_{it} z_{is} F_s'\right\|^2 = k_{NT}^2(p_c + p)\sigma^2\sigma_u^2 r\sigma_F^2,$$

and

$$\sum_{i=1}^N \sum_{t=1}^T \sum_{s=1}^T E\left\|\hat{F}_t u_{it} z_{is} F_s'\right\|^2 = NTk_{NT}^2(p_c + p)\sigma^2\sigma_u^2 r\sigma_F^2.$$

Therefore, $D_7 = O_p(k_{NT} N^{-1/2} T^{-3/2})$. For D_8, we use a similar computation as D_7 to obtain $D_8 = O_p(k_{NT} N^{-1/2} T^{-3/2})$. Therefore, terms D_4 and D_5 consist of the second component and terms D_7 and D_8 correspond to the third component of the final result. Term D_6 is dominated by terms D_4 and D_5, and we obtain the final result.

(b) We essentially follow the same computation as (a).

(c) We start with

$$\begin{aligned} T^{-1}\sum_{t=1}^T(\hat{F}_t - H'F_t)u_{it} &= N^{-1}T^{-2}(\hat{F}'F\Lambda u'u_i + \hat{F}'u\Lambda F'u_i + \hat{F}'uu'u_i + \hat{F}'F\Lambda' Z'u_i \\ &\quad + \hat{F}'Z\Lambda F'u_i + \hat{F}'ZZ'u_i + \hat{F}'uZ'u_i + \hat{F}'Zu'u_i), \\ &= \sum_{h=1}^8 D_h. \end{aligned}$$

Terms $D_1 + D_2 + D_3$ do not involve jumps and their sum is $O_p(c_{NT}^{-2})$. In the following, we compute the stochastic bounds for D_4 to D_8. For D_4,

$$\begin{aligned} N^{-1}T^{-2}\left\|\hat{F}'F\Lambda' Z'u_i\right\| &\le N^{-1}T^{-1}\left\|T^{-1/2}\hat{F}\right\|\left\|T^{-1/2}F\right\|\left\|\Lambda' Z'u_i\right\|, \\ &\le N^{-1}T^{-1}\left\|T^{-1/2}\hat{F}\right\|\left\|T^{-1/2}F\right\|\sqrt{\sum_{t=1}^T \sum_{j=1}^N \left\|\lambda_j\right\|^2 z_{jt}^2 u_{it}^2}, \end{aligned}$$

and

$$\begin{aligned} E(\left\|\lambda_j\right\|^2 z_{jt}^2 u_{it}^2) &= E\left\|\lambda_j\right\|^2 E(z_{jt})^2 E(u_{it}^2), \\ &= \begin{cases} k_{NT}^2\sigma^2\lambda^2\sigma_u^2 + k_{NT}^2(p/T)\sigma^2\lambda^2\sigma_u^2, & \text{if } \eta_t^c = 1 \\ k_{NT}^2(p/T)\sigma^2\lambda^2\sigma_u^2, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

so that

$$D_4 = O_p(k_{NT} N^{-1/2} T^{-1}).$$

For D_5,

$$\begin{aligned} N^{-1}T^{-2}\left\|\hat{F}'Z\Lambda F'u_i\right\| &\leq N^{-1/2}T^{-3/2} \underbrace{\left\|N^{-1/2}\hat{F}Z\Lambda\right\|}_{=O_p(k_{NT}) \text{ by } D_5 \text{ in (a)}} \left\|T^{-1/2}Fu_i\right\|, \\ &= O_p(k_{NT}N^{-1/2}T^{-3/2}). \end{aligned}$$

For D_6,

$$N^{-1}T^{-2}\left\|\hat{F}'ZZ'u_i\right\| \leq N^{-1}T^{-2}\sqrt{\textstyle\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\left\|\hat{F}_t z_{jt} z_{js} u_{is}\right\|^2}.$$

However,

$$\begin{aligned} E\left\|\hat{F}_t z_{jt} z_{js} u_{is}\right\|^2 &= E\left\|\hat{F}_t\right\|^2 E(z_{jt}^2)E(z_{js}^2)E(u_{is}^2), \\ &= \begin{cases} k_{NT}^2\sigma^2 E(z_{jt}^2)\sigma_u^2 r + (p/T)k_{NT}^2\sigma^2 E(z_{jt}^2)\sigma_u^2 r, & \text{if } \eta_s^c = 1 \\ (p/T)k_{NT}^2\sigma^2 E(z_{jt}^2)\sigma_u^2 r, & \text{if } \eta_s^c = 0 \end{cases}, \end{aligned}$$

so that

$$\begin{aligned} \textstyle\sum_{s=1}^{T} E\left\|\hat{F}_t z_{jt} z_{js} u_{is}\right\|^2 &\leq p_c k_{NT}^2\sigma^2 E(z_{jt}^2)\sigma_u^2 r + p k_{NT}^2\sigma^2 E(z_{jt}^2)\sigma_u^2 r, \\ &= \begin{cases} [k_{NT}^2\sigma^2(p_c+p)\sigma_u^2 r]k_{NT}^2(1+p/T)\sigma^2, & \text{if } \eta_t^c = 1 \\ [k_{NT}^2\sigma^2(p_c+p)\sigma_u^2 r]k_{NT}^2(p/T)\sigma^2, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

so that

$$\textstyle\sum_{t=1}^{T}\sum_{s=1}^{T} E\left\|\hat{F}_t z_{it} z_{is} u_{is}\right\|^2 \leq [k_{NT}^2\sigma^2(p_c+p)\sigma_u^2]^2 r,$$

and

$$\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} E\left\|\hat{F}_t z_{it} z_{is} u_{is}\right\|^2 \leq N[k_{NT}^2\sigma^2(p_c+p)\sigma_u^2]^2 r.$$

Therefore,

$$D_6 = O_p(k_{NT}^2 N^{-1/2} T^{-2}).$$

For D_7,

$$N^{-1}T^{-2}\left\|\hat{F}'uZ'u_i\right\| \leq N^{-1}T^{-2}\sqrt{\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\left\|\hat{F}_t u_{it} z_{is} u_{is}\right\|^2}.$$

However,

$$\begin{aligned} E\left\|\hat{F}_t u_{it} z_{is} u_{is}\right\|^2 &= E\left\|\hat{F}_t\right\|^2 E(u_{it}^2)E(z_{is}^2)E(u_{is}^2), \\ &= \begin{cases} k_{NT}^2\sigma^2\sigma_u^4 r + (p/T)k_{NT}^2\sigma^2\sigma_u^4 r, & \text{if } \eta_s^c = 1 \\ (p/T)k_{NT}^2\sigma^2\sigma_u^4 r, & \text{if } \eta_s^c = 0 \end{cases} \end{aligned}$$

so that

$$\sum_{s=1}^{T} E\left\|\hat{F}_t u_{it} z_{is} u_{is}\right\|^2 = k_{NT}^2 (p_c + p)\sigma^2 \sigma_u^4 r,$$

and

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} E\left\|\hat{F}_t u_{it} z_{is} u_{is}\right\|^2 = NT k_{NT}^2 (p_c + p)\sigma^2 \sigma_u^4 r.$$

Therefore, $D_7 = O_p(k_{NT} N^{-1/2} T^{-3/2})$.

For D_8,

$$N^{-1}T^{-2}\left\|\hat{F}' Z u' u_i\right\| \le N^{-1}T^{-2}\sqrt{\sum_{j=1}^{N}\sum_{s=1}^{T}\sum_{t=1}^{T}\left\|\hat{F}_t z_{jt} u_{js} u_{is}\right\|^2}.$$

However,

$$\begin{aligned} E\left\|\hat{F}_t z_{jt} u_{js} u_{is}\right\|^2 &= E\left\|\hat{F}_t\right\|^2 E(z_{jt}^2)E(u_{js}^2)E(u_{is}^2), \\ &= \begin{cases} k_{NT}^2\sigma^2\sigma_u^4 r + (p/T)k_{NT}^2\sigma^2\sigma_u^4 r, & \text{if } \eta_t^c = 1 \\ (p/T)k_{NT}^2\sigma^2\sigma_u^4 r, & \text{if } \eta_t^c = 0 \end{cases} \end{aligned}$$

so that

$$\sum_{s=1}^{T} E\left\|\hat{F}_t z_{jt} u_{js} u_{is}\right\|^2 = k_{NT}^2 (p_c + p)\sigma^2 \sigma_u^4 r,$$

and

$$\sum_{i=1}^{N}\sum_{s=1}^{T}\sum_{t=1}^{T} E\left\|\hat{F}_t z_{jt} u_{js} u_{is}\right\|^2 = NT k_{NT}^2 (p_c + p)\sigma^2 \sigma_u^4 r,$$

so that $D_8 = O_p(k_{NT} N^{-1/2} T^{-3/2})$. Therefore, term D_4 corresponds to the second component and terms D_7 and D_8 consist of the third component of the final result. Terms D_5 and D_6 are dominated by term D_4. We obtain the final result.

(d) We essentially follow the same computation as (c).■

Proof of Theorem 1: Part (i): We start (A.1). Terms d_{1t} to d_{3t} have nothing to do with jumps, and we know that from Theorem 1 of Bai (2003), if there is no jump,

$$\sqrt{N}(\hat{F}_t - H'F_t) = V^{-1}(F'F/T)N^{-1/2}\sum_{i=1}^{N}\lambda_i u_{it} + O_p(N^{1/2}T^{-1/2}c_{NT}^{-1}) + O_p(c_{NT}^{-1}).$$

Therefore, the stochastic bound of terms d_{1t} to d_{3t} (multiplied by $\sqrt{N}$) is given as above. In the rest of this proof, we compute the stochastic bound for terms d_{4t} to d_{8t} (multiplied by $\sqrt{N}$).

For d_{4t},

$$\begin{aligned} N^{-1/2}T^{-1}\left\|\hat{F}' F \Lambda' z_t\right\| &= N^{-1/2}\left\|\hat{F}' F/T\right\|\left\|\Lambda' z_t\right\|, \\ &= N^{-1/2}\left\|\hat{F}' F/T\right\|\sqrt{\sum_{i=1}^{N}\left\|\lambda_i z_{it}\right\|^2}, \end{aligned}$$

where

$$E\left\|\lambda_i z_{it}\right\|^2 = E\left\|\lambda_i\right\|^2 E(z_{it}^2) = \begin{cases} k_{NT}^2\sigma^2\lambda^2 + (p/T)k_{NT}^2\sigma^2\lambda^2, & \text{if } \eta_t^c = 1 \\ (p/T)k_{NT}^2\sigma^2\lambda^2, & \text{if } \eta_t^c = 0 \end{cases},$$

so that

$$\textstyle\sum_{i=1}^N \left\|\lambda_i z_{it}\right\|^2 = \begin{cases} O_p(k_{NT}^2 N), & \text{if } \eta_t^c = 1 \\ O_p(k_{NT}^2 N T^{-1}), & \text{if } \eta_t^c = 0 \end{cases}.$$

Therefore,

$$N^{-1/2}T^{-1}\left\|\hat{F}'F\Lambda' z_t\right\| = \begin{cases} O_p(k_{NT}), & \text{if } \eta_t^c = 1 \\ O_p(k_{NT}T^{-1/2}), & \text{if } \eta_t^c = 0 \end{cases}.$$

For d_{5t},

$$\begin{aligned} N^{-1/2}T^{-1}\left\|\hat{F}'Z\Lambda F_t\right\| &\leq T^{-1}\underbrace{\left\|N^{-1/2}\hat{F}'Z\Lambda\right\|}_{=O_p(k_{NT})}\left\|F_t\right\|, \\ &= O_p(k_{NT}T^{-1}). \end{aligned}$$

For d_{6t},

$$N^{-1/2}T^{-1}\left\|\hat{F}'Zz_t\right\| \leq N^{-1/2}T^{-1}\sqrt{\textstyle\sum_{i=1}^N\sum_{s=1}^T\left\|\hat{F}_s z_{is} z_{it}\right\|^2},$$

where

$$\begin{aligned} E\left\|\hat{F}_s z_{is} z_{it}\right\|^2 &= E\left\|\hat{F}_s\right\|^2 E(z_{is}^2)E(z_{it}^2) \\ &= \begin{cases} k_{NT}^2\sigma^2 E(z_{it}^2)r + (p/T)k_{NT}^2\sigma^2 E(z_{it}^2)r, & \text{if } \eta_t^c = 1 \\ (p/T)k_{NT}^2\sigma^2 E(z_{it}^2)r, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

so that

$$\begin{aligned} \textstyle\sum_{s=1}^T E\left\|\hat{F}_s z_{is} z_{it}\right\|^2 &= k_{NT}^2(p_c + p)\sigma^2 E(z_{it}^2)r, \\ &= \begin{cases} [k_{NT}^2\sigma^2(p_c+p)r]k_{NT}^2(1+p/T)\sigma^2, & \text{if } \eta_t^c = 1 \\ [k_{NT}^2\sigma^2(p_c+p)r]k_{NT}^2(p/T)\sigma^2, & \text{if } \eta_t^c = 0 \end{cases}, \end{aligned}$$

and

$$N^{-1/2}T^{-1}\left\|\hat{F}'Zz_t\right\| = \begin{cases} O_p(k_{NT}^2 N^{-1/2}T^{-1}), & \text{if } \eta_t^c = 1 \\ O_p(k_{NT}^2 N^{-1/2}T^{-3/2}), & \text{if } \eta_t^c = 0 \end{cases}.$$

For d_{7t},

$$N^{-1/2}T^{-1}\left\|\hat{F}'uz_t\right\| \leq N^{-1/2}T^{-1}\sqrt{\sum_{i=1}^{N}\sum_{s=1}^{T}\left\|\hat{F}_s u_{is} z_{it}\right\|^2},$$

where

$$\begin{aligned} E\left\|\hat{F}_s u_{is} z_{it}\right\|^2 &= E\left\|\hat{F}_s\right\|^2 E(u_{is}^2)E(z_{it}^2), \\ &= \left\{\begin{array}{ll} k_{NT}^2\sigma^2\sigma_u^2 r + k_{NT}^2(p/T)\sigma^2\sigma_u^2 r, & \text{if } \eta_t^c = 1 \\ k_{NT}^2(p/T)\sigma^2\sigma_u^2 r, & \text{if } \eta_t^c = 0 \end{array}\right., \end{aligned}$$

so that

$$\sum_{i=1}^{N}\sum_{s=1}^{T} E\left\|\hat{F}_s u_{is} z_{it}\right\|^2 = \left\{\begin{array}{ll} NTk_{NT}^2\sigma^2\sigma_u^2 r + Nk_{NT}^2 p\sigma^2\sigma_u^2 r, & \text{if } \eta_t^c = 1 \\ Nk_{NT}^2 p\sigma^2\sigma_u^2 r, & \text{if } \eta_t^c = 0 \end{array}\right..$$

Hence,

$$N^{-1/2}T^{-1}\left\|\hat{F}'uz_t\right\| = \left\{\begin{array}{ll} O_p(k_{NT}T^{-1/2}), & \text{if } \eta_t^c = 1 \\ O_p(k_{NT}T^{-1}), & \text{if } \eta_t^c = 0 \end{array}\right..$$

For d_{8t},

$$N^{-1/2}T^{-1}\left\|\hat{F}'Zu_t\right\| \leq N^{-1/2}T^{-1}\sqrt{\sum_{i=1}^{N}\sum_{s=1}^{T}\left\|\hat{F}_s z_{is} u_{it}\right\|^2},$$

where

$$\begin{aligned} E\left\|\hat{F}_s z_{is} u_{it}\right\|^2 &= E\left\|\hat{F}_s\right\|^2 E(z_{is}^2)E(u_{it}^2), \\ &= \left\{\begin{array}{ll} k_{NT}^2\sigma^2\sigma_u^2 r + k_{NT}^2\sigma^2(p/T)\sigma_u^2 r, & \text{if } \eta_s^c = 1 \\ k_{NT}^2\sigma^2(p/T)\sigma_u^2 r, & \text{if } \eta_s^c = 0 \end{array}\right., \end{aligned}$$

so that

$$\sum_{s=1}^{T} E\left\|\hat{F}_s z_{is} u_{it}\right\|^2 = k_{NT}^2\sigma^2(p_c + p)\sigma_u^2 r,$$

and

$$\sum_{i=1}^{N}\sum_{s=1}^{T} E\left\|\hat{F}_s u_{is} z_{it}\right\|^2 = Nk_{NT}^2\sigma^2(p_c + p)\sigma_u^2 r.$$

Therefore,

$$N^{-1/2}T^{-1}\left\|\hat{F}'Zu_t\right\| = O_p(k_{NT}T^{-1}).$$

This computation gives

$$\sqrt{N}(d_{4t} + d_{5t} + d_{6t} + d_{7t} + d_{8t}) = \left\{\begin{array}{ll} O_p(k_{NT}), & \text{if } \eta_t^c = 1 \\ O_p(k_{NT}T^{-1/2}), & \text{if } \eta_t^c = 0 \end{array}\right.,$$

to complete the proof.

Part (ii): We extend the factor loading estimate:

$$\begin{aligned}\hat{\lambda}_i &= (\hat{F}'\hat{F})^{-1}\hat{F}'X_i, \\ &= (\hat{F}'\hat{F})^{-1}\hat{F}'F\lambda_i + (\hat{F}'\hat{F})^{-1}\hat{F}'u_i, \\ &= (\hat{F}'\hat{F})^{-1}\hat{F}'\hat{F}H^{-1}\lambda_i - (\hat{F}'\hat{F})^{-1}\hat{F}'(\hat{F}-FH)H^{-1}\lambda_i \\ &\quad +(\hat{F}'\hat{F})^{-1}H'F'u_i + (\hat{F}'\hat{F})^{-1}(\hat{F}-FH)'u_i,\end{aligned}$$

so that

$$\begin{aligned}T^{1/2}(\hat{\lambda}_i - H^{-1}\lambda_i) &= T^{-1/2}H'F'u_i + T^{-1/2}(\hat{F}-FH)'u_i - T^{-1/2}\hat{F}'(\hat{F}-FH)H^{-1}\lambda_i, \\ &= O_p(1) + O_p(T^{1/2}c_{NT}^{-2}) + O_p(k_{NT}N^{-1/2}T^{-1/2}) + O_p(k_{NT}^2 N^{-1/2}T^{-3/2}), \\ &= I + II + III + IV,\end{aligned}$$

from Lemma 3 (b) and 3 (c). The condition for term II to diminish is

$$T^{1/2}c_{NT}^{-2} = \max\left\{\frac{T^{1/2}}{N}, \frac{T^{1/2}}{T}\right\} \to 0,$$

or $\sqrt{T}/N \to 0$. Further, the condition for term III to diminish is

$$k_{NT}N^{-1/2}T^{-1/2} = k_{NT}(\sqrt{T}/N)(\sqrt{N}/T) \to 0,$$

which is implied when $k_{NT}(\sqrt{N}/T)$ is bounded or

$$k_{NT} \leq T/\sqrt{N}.$$

The condition for term IV to diminish is

$$k_{NT}^2 N^{-1/2}T^{-3/2} \leq k_{NT}(\sqrt{T}/N)(\sqrt{N}/T^2),$$

or

$$k_{NT} \leq T^2/\sqrt{N},$$

which is satisfied when $k_{NT} \leq T/\sqrt{N}$. Hence, the additional condition is $k_{NT} \leq T/\sqrt{N}$.■

Proof of Corollary 1: This immediately holds because from Theorem 1 (a), if $\eta_t^c = 1$, then

$$\begin{aligned}\left\|\hat{F}_t - H'F_t\right\| &= \textstyle\sum_{h=1}^{3} d_{ht} + \sum_{h=4}^{8} d_{ht}, \\ &= o_p(1) + O_p(k_{NT}N^{-1/2}).\end{aligned}$$

Therefore, we have $o_p(1)$ if $k_{NT} < N^{-1/2}$.■

Proof of Theorem 2: We use Observation 1 of Bates et al. (2013) and require

$$k_{NT}^2 \sum_{i=1}^N \sum_{t=1}^T E\left\|z_{it}F_t\right\|^2 = O(\max\{N,T\}),$$

or

$$k_{NT}^2 \sigma_F^2 \sum_{i=1}^N \sum_{t=1}^T E(z_{it}^2) = O(\max\{N,T\}), \tag{A.2}$$

under our simplification assumptions. However,

$$\sum_{t=1}^T E(z_{it}^2) = (p_c + p)k_{NT}^2\sigma^2,$$

and

$$\sum_{i=1}^N \sum_{t=1}^T E(z_{it}^2) = N(p_c + p)k_{NT}^2\sigma^2.$$

Hence, (A.2) becomes

$$k_{NT}^2 \sigma_F^2 N(p_c + p)k_{NT}^2\sigma^2 = O(\max\{N,T\}),$$

or

$$k_{NT}^4 = O(\max\{1, T/N\}),$$

or

$$k_{NT} \leq \max\left\{1, T^{1/4}N^{-1/4}\right\}.$$

This completes the proof.■

Proof of Proposition 1: We consider the numerator only. Because

$$\begin{aligned}
\hat{v}_{it}^* &= z_{it} + u_{it} + \lambda_i' F_t - \hat{\lambda}_i' \hat{F}_t, \\
&= z_{it} + u_{it} + \lambda_i' H'^{-1}(H'F_t - \hat{F}_t) - (\hat{\lambda}_i' - \lambda_i' H'^{-1})H'F_t - (\hat{\lambda}_i' - \lambda_i' H'^{-1})(H'F_t - \hat{F}_t), \\
&= z_{it} + u_{it} + I + II + III.
\end{aligned}$$

and $E\left|u_{it}\right|$ is bounded, we consider only terms I, II, and III. Theorem 1 implies that

$$\begin{aligned}
I &= O_p(k_{NT}N^{-1/2}), \\
II &= O_p(k_{NT}N^{-1/2}T^{-1}) + O_p(k_{NT}^2 N^{-1/2}T^{-2}), \\
III &= O_p(k_{NT}^2 N^{-1}T^{-1}) + O_p(k_{NT}^3 N^{-1}T^{-2}).
\end{aligned}$$

For all the terms in I, II, and III to be smaller than $|z_{it}| = O_p(k_{NT})$, we require that $k_{NT} < N^{1/2}T$.■

Proof of Theorem 3: Part (i-a) holds because the jumps satisfying $k_{NT} > \sqrt{T}$ are always corrected by Proposition 1.

Part (i-b) holds because the jumps satisfying $k_{NT} > \sqrt{N}$ are always corrected by Proposition 1.

Part (ii) is a straightforward consequence of Theorem 1 (ii).

Part (iii) holds because the jumps satisfying $k_{NT} > \max\left\{1, T^{1/4}N^{-1/4}\right\}$ are always corrected by Proposition 1.■

Proof of Theorem 4: We consider the cross-sectional regression

$$\hat{u}_{it} = \gamma_0 + \hat{\lambda}_i'\gamma_1 + \varepsilon_i, \quad \text{for } i = 1, ..., N, \tag{A.3}$$

where ε_i is the error term. We know that the residuals $\hat{u}_{it}$ become

$$\hat{u}_{it} = u_{it} + z_{it} + (\lambda_i' F_t - \hat{\lambda}_i' \hat{F}_t), \tag{A.4}$$

under H_0 and

$$\hat{u}_{it} = u_{it} + \lambda_i' J_t + (\lambda_i' F_t - \hat{\lambda}_i' \hat{F}_t), \tag{A.5}$$

under H_1, where $\hat{\lambda}_i$ and $\hat{F}_t$ are jump-corrected estimates. We show that the regression model (A.3) induced by true process (A.4) has a pseudo-true coefficient $\gamma_1 = 0$ and the model (A.3) induced by true process (A.5) has a pseudo-true coefficient $\gamma_1 \neq 0$ with error term ε_i showing a finite variance. First, note that under H_0, (A.4) becomes

$$\hat{u}_{it} = \hat{\lambda}_i'(HF_t - \hat{F}_t) + (\lambda_i H^{-1} - \hat{\lambda}_i)' HF_t + u_{it} + z_{it},$$

so that

$$\begin{aligned}
\gamma_0 &= 0, \\
\gamma_1 &= p \lim_{N,T\to\infty} (\underbrace{HF_t - \hat{F}_t}_{=O_p(N^{-1/2})}) = 0, \\
\varepsilon_i &= (\underbrace{\lambda_i H^{-1} - \hat{\lambda}_i}_{=O_p(T^{-1/2})})' HF_t + u_{it} + z_{it} + o_p(1), \\
&= I + II + III + o_p(1),
\end{aligned}$$

from Theorem 3 (i-b) and 3 (ii). Therefore, error ε_i consists of three terms, I, II, and III. Term I has a variance shrinking to zero, term II a finite variance σ_u^2, and term III variance $k_{NT}^2\sigma^2$. Since the F test is invariant to model scaling, it is the same as that applied to regression model (A.3) with

$$\begin{aligned}
\gamma_0 &= 0, \\
\gamma_1 &= p \lim_{N,T\to\infty} k_{NT}^{-1}(HF_t - \hat{F}_t) = 0, \\
\varepsilon_i &= k_{NT}^{-1} z_{it} + o_p(1).
\end{aligned}$$

Under this model, error ε_i has a finite variance σ^2 and pseudo-true coefficients γ_1 are zero at rate $o_p(N^{-1/2})$ so that the F test multiplied by the numerator's degree of freedom has the standard Chi square limit distribution. Under H_1, (A.5) becomes

$$\hat{u}_{it} = \hat{\lambda}_i' HJ_t + (\hat{\lambda}_i - \lambda_i H^{-1})' HJ_t + \hat{\lambda}_i'(HF_t - \hat{F}_t) - (\lambda_i H^{-1} - \hat{\lambda}_i)' HF_t + u_{it}, \tag{A.6}$$

so that we obtain the regression model (A.3) with

$$
\begin{aligned}
\gamma_0 &= 0, \\
\gamma_1 &= p \lim_{N,T\to\infty} HJ_t \neq 0, \\
\varepsilon_i &= (\underbrace{\hat{\lambda}_i - \lambda_i H^{-1}}_{=O_p(T^{-1/2})})' HJ_t + \hat{\lambda}_i'(\underbrace{HF_t - \hat{F}_t}_{=O_p(N^{-1/2})}) - (\underbrace{\lambda_i H^{-1} - \hat{\lambda}_i}_{=O_p(T^{-1/2})})' HF_t + u_{it} + o_p(1), \\
&= I + II + III + u_{it} + o_p(1).
\end{aligned}
$$

Terms II and III diminish as $N, T \to \infty$ regardless of k_{NT}. Hence, we separately consider the cases where I diminishes and does not diminish. We first suppose that $k_{NT} < T^{1/2}$. Then, terms I, II,and III show a variance that shrinks to zero and u_{it} has a finite variance σ_u^2. We next suppose that $k_{NT} \geq T^{1/2}$. Now, scaling by $k_{NT}^{-1} T^{1/2}$ makes the regression model (A.3) have

$$
\begin{aligned}
\gamma_0 &= 0, \\
\gamma_1 &= p \lim_{N,T\to\infty} T^{1/2} k_{NT}^{-1} HJ_t \neq 0, \\
\varepsilon_i &= \underbrace{\sqrt{T}(\hat{\lambda}_i - \lambda_i H^{-1})'}_{\Rightarrow N(0,\Omega_{\hat{\Lambda},i})} \underbrace{k_{NT}^{-1} HJ_t}_{\sim N(0,\sigma^2)} + o_p(1),
\end{aligned}
$$

so that the error term ε_i has a zero mean and finite variance. The final result follows.■

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Table 1(a). Coverage ratio of the confidence interval for the factor at the 90% nominal level

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.87	0.87	0.88	0.88	0.88	0.88	0.85	0.85	0.71	0.75
5	0.85	0.85	0.88	0.88	0.88	0.88	0.85	0.85	0.80	0.75
10	0.85	0.85	0.87	0.88	0.88	0.88	0.88	0.86	0.82	0.75
50	0.88	0.86	0.88	0.87	0.89	0.88	0.89	0.84	0.82	0.73
100	0.85	0.86	0.87	0.88	0.89	0.88	0.88	0.85	0.82	0.74
	pc=5, p=0									
0	0.87	0.87	0.88	0.88	0.88	0.88	0.85	0.86	0.71	0.75
5	0.85	0.85	0.87	0.88	0.87	0.88	0.84	0.85	0.85	0.67
10	0.83	0.85	0.83	0.87	0.85	0.88	0.88	0.85	0.90	0.63
50	0.87	0.85	0.89	0.87	0.90	0.88	0.90	0.83	0.91	0.67
100	0.87	0.86	0.89	0.88	0.89	0.88	0.89	0.84	0.90	0.71
	pc=0, p=1									
0	0.87	0.87	0.88	0.88	0.88	0.88	0.85	0.85	0.71	0.75
5	0.85	0.85	0.88	0.88	0.88	0.88	0.85	0.86	0.73	0.80
10	0.85	0.86	0.87	0.87	0.87	0.87	0.84	0.86	0.77	0.80
50	0.82	0.85	0.82	0.88	0.85	0.89	0.87	0.87	0.89	0.76
100	0.86	0.86	0.87	0.87	0.90	0.89	0.89	0.87	0.90	0.78
	pc=0, p=5									
0	0.87	0.87	0.88	0.88	0.88	0.88	0.85	0.85	0.71	0.75
5	0.84	0.85	0.87	0.88	0.87	0.88	0.85	0.87	0.79	0.72
10	0.84	0.86	0.84	0.88	0.82	0.88	0.80	0.81	0.84	0.35
50	0.80	0.86	0.84	0.87	0.87	0.87	0.89	0.78	0.90	0.40
100	0.86	0.86	0.89	0.89	0.89	0.87	0.89	0.89	0.90	0.80
	pc=1, p=1									
0	0.87	0.87	0.88	0.88	0.88	0.88	0.85	0.86	0.71	0.75
5	0.85	0.85	0.88	0.88	0.88	0.88	0.84	0.85	0.81	0.77
10	0.85	0.86	0.87	0.88	0.88	0.87	0.88	0.87	0.85	0.76
50	0.86	0.86	0.87	0.88	0.88	0.89	0.89	0.86	0.90	0.72
100	0.84	0.86	0.88	0.88	0.89	0.89	0.90	0.87	0.89	0.73

Table 1(b). Average length of the confidence interval for the factor at the 90% nominal level

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.78	0.78	0.47	0.47	0.33	0.33	0.23	0.23	0.15	0.15
5	0.78	0.78	0.48	0.48	0.34	0.33	0.26	0.23	0.86	0.15
10	0.80	0.78	0.52	0.48	2.37	0.33	7.71	0.23	5.12	0.14
50	87.24	0.78	213.93	0.48	82.57	0.33	51.74	0.23	47.66	0.14
100	438.52	0.78	335.73	0.48	168.89	0.33	143.65	0.23	89.02	0.14
	pc=5, p=0									
0	0.78	0.78	0.47	0.47	0.33	0.33	0.23	0.23	0.15	0.15
5	0.79	0.79	0.49	0.48	0.35	0.33	0.29	0.23	0.74	0.16
10	0.88	0.79	1.32	0.48	3.21	0.33	33.82	0.23	12.95	0.22
50	191.32	0.78	472.57	0.48	134.00	0.33	93.95	16.67	75.27	1.07
100	666.97	0.78	440.37	0.48	370.32	0.34	197.63	5.20	229.42	1.99
	pc=0, p=1									
0	0.78	0.78	0.47	0.47	0.33	0.33	0.23	0.23	0.15	0.15
5	0.78	0.78	0.48	0.48	0.33	0.33	0.24	0.23	0.16	0.14
10	0.79	0.78	0.48	0.47	0.34	0.33	0.25	0.23	0.19	0.14
50	13.83	0.78	26.70	0.47	15.84	0.33	16.58	0.23	16.17	0.14
100	243.78	0.78	134.88	0.47	86.35	0.33	80.17	0.23	37.30	0.14
	pc=0, p=5									
0	0.78	0.78	0.47	0.47	0.33	0.33	0.23	0.23	0.15	0.15
5	0.79	0.78	0.48	0.48	0.34	0.33	0.25	0.23	0.19	0.15
10	0.83	0.78	0.51	0.47	0.37	0.32	0.30	0.22	0.43	0.14
50	108.84	0.78	87.48	0.47	2634.54	0.32	47.27	0.38	54.86	0.93
100	421.01	0.78	233.91	0.47	184.07	0.36	480.69	13.42	79.54	7.05
	pc=1, p=1									
0	0.78	0.78	0.47	0.47	0.33	0.33	0.23	0.23	0.15	0.15
5	0.79	0.78	0.48	0.48	0.34	0.33	0.31	0.23	1.42	0.15
10	0.81	0.78	0.55	0.48	3.82	0.33	7.64	0.23	6.25	0.14
50	168.26	0.78	118.59	0.47	100.12	0.33	109.33	0.23	70.39	0.14
100	402.83	0.78	279.14	0.47	222.35	0.33	193.49	0.24	87.11	0.16

Table 2(a). Coverage ratio of the confidence interval for the factor loading at the 90% nominal level

σ	N=20, T=500 no correction	correction	N=50, T=200 no correction	correction	N=100, T=100 no correction	correction	N=200, T=50 no correction	correction	N=500, T=20 no correction	correction
	pc=1, p=0									
0	0.67	0.67	0.87	0.87	0.88	0.88	0.87	0.86	0.86	0.85
5	0.66	0.67	0.84	0.87	0.84	0.89	0.79	0.89	0.45	0.85
10	0.57	0.66	0.66	0.86	0.43	0.89	0.35	0.89	0.34	0.86
50	0.23	0.67	0.32	0.85	0.33	0.89	0.34	0.89	0.33	0.85
100	0.26	0.68	0.31	0.86	0.34	0.88	0.34	0.88	0.33	0.83
	pc=5, p=0									
0	0.67	0.67	0.87	0.87	0.88	0.88	0.87	0.86	0.86	0.85
5	0.60	0.65	0.79	0.87	0.77	0.89	0.64	0.89	0.33	0.73
10	0.40	0.64	0.35	0.87	0.13	0.89	0.06	0.91	0.06	0.78
50	0.01	0.66	0.01	0.85	0.01	0.89	0.01	0.77	0.01	0.67
100	0.01	0.68	0.00	0.85	0.01	0.88	0.01	0.69	0.01	0.65
	pc=0, p=1									
0	0.67	0.67	0.87	0.87	0.88	0.88	0.87	0.86	0.86	0.85
5	0.66	0.67	0.86	0.86	0.87	0.88	0.87	0.88	0.83	0.84
10	0.61	0.67	0.82	0.87	0.86	0.88	0.84	0.88	0.70	0.85
50	0.09	0.68	0.06	0.86	0.06	0.88	0.05	0.89	0.05	0.83
100	0.01	0.69	0.01	0.86	0.01	0.88	0.02	0.87	0.02	0.83
	pc=0, p=5									
0	0.67	0.67	0.87	0.87	0.88	0.88	0.87	0.86	0.86	0.85
5	0.63	0.66	0.84	0.86	0.86	0.87	0.85	0.87	0.75	0.79
10	0.54	0.66	0.74	0.87	0.78	0.88	0.74	0.86	0.52	0.79
50	0.02	0.70	0.02	0.86	0.03	0.87	0.03	0.79	0.03	0.50
100	0.00	0.71	0.00	0.88	0.01	0.86	0.01	0.12	0.01	0.06
	pc=1, p=1									
0	0.67	0.67	0.87	0.87	0.88	0.88	0.87	0.86	0.86	0.85
5	0.65	0.67	0.83	0.86	0.84	0.88	0.78	0.88	0.44	0.82
10	0.53	0.66	0.65	0.87	0.45	0.88	0.35	0.88	0.28	0.84
50	0.03	0.68	0.02	0.86	0.02	0.88	0.02	0.89	0.02	0.82
100	0.00	0.68	0.00	0.86	0.00	0.88	0.01	0.88	0.01	0.80

Table 2(b). Average length of the confidence interval for the factor loading at the 90% nominal level

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.13	0.13	0.22	0.22	0.32	0.32	0.45	0.45	0.69	0.70
5	0.13	0.13	0.23	0.23	0.35	0.33	0.51	0.49	0.51	0.81
10	0.14	0.13	0.25	0.23	0.27	0.33	0.22	0.48	0.30	0.80
50	0.05	0.13	0.08	0.22	0.12	0.32	0.18	0.46	0.27	0.73
100	0.05	0.13	0.08	0.22	0.12	0.32	0.17	0.45	0.27	0.73
	pc=5, p=0									
0	0.13	0.13	0.22	0.22	0.32	0.32	0.45	0.45	0.69	0.70
5	0.14	0.14	0.27	0.24	0.44	0.38	0.68	0.62	0.67	1.12
10	0.16	0.14	0.31	0.24	0.25	0.37	0.14	0.60	0.15	1.12
50	0.01	0.13	0.01	0.23	0.01	0.33	0.01	0.46	0.02	0.72
100	0.00	0.13	0.00	0.22	0.01	0.33	0.01	0.38	0.01	0.67
	pc=0, p=1									
0	0.13	0.13	0.22	0.22	0.32	0.32	0.45	0.45	0.69	0.70
5	0.13	0.13	0.23	0.23	0.35	0.33	0.53	0.49	0.91	0.81
10	0.14	0.13	0.26	0.23	0.42	0.33	0.67	0.49	1.06	0.83
50	0.10	0.13	0.12	0.22	0.13	0.33	0.14	0.48	0.12	0.78
100	0.01	0.13	0.02	0.22	0.02	0.33	0.04	0.47	0.04	0.79
	pc=0, p=5									
0	0.13	0.13	0.22	0.22	0.32	0.32	0.45	0.45	0.69	0.70
5	0.14	0.14	0.27	0.25	0.45	0.39	0.77	0.63	1.30	1.21
10	0.17	0.14	0.37	0.25	0.66	0.39	1.08	0.66	1.18	1.36
50	0.05	0.13	0.05	0.24	0.07	0.36	0.07	0.66	0.07	0.90
100	0.01	0.13	0.01	0.23	0.02	0.36	0.03	0.23	0.03	0.18
	pc=1, p=1									
0	0.13	0.13	0.22	0.22	0.32	0.32	0.45	0.45	0.69	0.70
5	0.14	0.13	0.24	0.23	0.37	0.35	0.58	0.52	0.63	0.91
10	0.15	0.13	0.28	0.23	0.33	0.34	0.32	0.52	0.45	0.93
50	0.04	0.13	0.05	0.23	0.05	0.33	0.06	0.49	0.05	0.82
100	0.01	0.13	0.01	0.22	0.01	0.33	0.02	0.48	0.02	0.80

Table 3(a). Coverage ratio of the confidence interval for the common component at the 90% nominal level

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.88	0.88	0.89	0.89	0.89	0.89	0.88	0.88	0.87	0.86
5	0.87	0.87	0.89	0.90	0.88	0.89	0.83	0.90	0.51	0.86
10	0.84	0.87	0.80	0.89	0.58	0.89	0.48	0.90	0.42	0.86
50	0.58	0.88	0.54	0.88	0.50	0.90	0.47	0.90	0.42	0.86
100	0.58	0.88	0.52	0.89	0.50	0.90	0.45	0.89	0.43	0.85
	pc=5, p=0									
0	0.88	0.88	0.89	0.89	0.89	0.89	0.89	0.89	0.87	0.86
5	0.86	0.87	0.87	0.90	0.83	0.89	0.71	0.89	0.42	0.73
10	0.78	0.87	0.62	0.89	0.35	0.90	0.25	0.91	0.20	0.76
50	0.40	0.88	0.33	0.89	0.28	0.90	0.22	0.81	0.16	0.70
100	0.43	0.88	0.31	0.88	0.28	0.89	0.21	0.72	0.18	0.69
	pc=0, p=1									
0	0.88	0.88	0.89	0.89	0.90	0.90	0.88	0.88	0.87	0.86
5	0.87	0.87	0.90	0.90	0.89	0.89	0.89	0.88	0.87	0.85
10	0.87	0.87	0.87	0.88	0.89	0.89	0.87	0.88	0.76	0.85
50	0.40	0.87	0.23	0.89	0.18	0.90	0.16	0.89	0.14	0.84
100	0.15	0.88	0.11	0.88	0.11	0.89	0.11	0.88	0.11	0.85
	pc=0, p=5									
0	0.88	0.88	0.89	0.90	0.90	0.90	0.88	0.88	0.87	0.86
5	0.87	0.87	0.89	0.88	0.89	0.89	0.87	0.88	0.79	0.78
10	0.87	0.87	0.88	0.89	0.87	0.88	0.81	0.87	0.58	0.76
50	0.25	0.88	0.22	0.88	0.23	0.88	0.22	0.79	0.17	0.43
100	0.18	0.87	0.17	0.89	0.20	0.89	0.22	0.19	0.18	0.11
	pc=1, p=1									
0	0.88	0.88	0.89	0.89	0.90	0.90	0.88	0.88	0.87	0.86
5	0.86	0.87	0.89	0.89	0.88	0.89	0.83	0.89	0.52	0.83
10	0.84	0.87	0.79	0.89	0.59	0.89	0.48	0.89	0.39	0.85
50	0.41	0.88	0.29	0.89	0.23	0.89	0.19	0.89	0.14	0.83
100	0.31	0.88	0.25	0.88	0.19	0.89	0.17	0.88	0.13	0.83

Table 3(b). Average length of the confidence interval for the common component at the 90% nominal level

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.60	0.60	0.44	0.44	0.41	0.41	0.44	0.44	0.59	0.59
5	0.60	0.60	0.45	0.45	0.43	0.41	0.50	0.47	0.47	0.66
10	0.60	0.60	0.47	0.45	0.43	0.42	0.34	0.45	0.34	0.67
50	0.72	0.60	0.48	0.44	0.38	0.41	0.33	0.46	0.34	0.62
100	0.71	0.60	0.48	0.45	0.38	0.41	0.32	0.45	0.33	0.60
	pc=5, p=0									
0	0.60	0.60	0.44	0.44	0.41	0.41	0.44	0.44	0.59	0.59
5	0.61	0.60	0.48	0.46	0.50	0.44	0.65	0.57	0.62	0.85
10	0.62	0.60	0.53	0.45	0.47	0.44	0.33	0.54	0.25	0.85
50	0.78	0.61	0.50	0.44	0.37	0.42	0.27	0.46	0.19	0.61
100	0.77	0.60	0.51	0.45	0.36	0.42	0.27	0.40	0.18	0.54
	pc=0, p=1									
0	0.60	0.60	0.44	0.44	0.41	0.41	0.44	0.44	0.59	0.59
5	0.61	0.60	0.45	0.45	0.43	0.42	0.51	0.47	0.77	0.66
10	0.60	0.59	0.46	0.44	0.46	0.40	0.61	0.47	0.91	0.67
50	0.54	0.61	0.33	0.45	0.29	0.42	0.24	0.46	0.19	0.64
100	0.36	0.60	0.18	0.45	0.16	0.41	0.16	0.45	0.13	0.62
	pc=0, p=5									
0	0.60	0.60	0.44	0.44	0.41	0.41	0.44	0.44	0.59	0.59
5	0.61	0.60	0.48	0.45	0.51	0.45	0.70	0.56	1.11	0.91
10	0.61	0.60	0.55	0.46	0.65	0.43	0.94	0.55	1.05	0.93
50	0.48	0.61	0.35	0.44	0.34	0.43	0.29	0.57	0.22	0.62
100	0.36	0.60	0.29	0.45	0.28	0.42	0.27	0.25	0.20	0.16
	pc=1, p=1									
0	0.61	0.61	0.44	0.44	0.41	0.41	0.44	0.44	0.59	0.59
5	0.60	0.60	0.46	0.45	0.45	0.43	0.55	0.50	0.57	0.72
10	0.61	0.60	0.49	0.45	0.45	0.41	0.42	0.48	0.47	0.74
50	0.68	0.60	0.42	0.44	0.34	0.42	0.25	0.46	0.18	0.65
100	0.62	0.61	0.41	0.45	0.28	0.41	0.22	0.46	0.15	0.62

Table 4. Average correlation coefficient between the estimated and true factors

σ	N=20, T=500 no correction	N=20, T=500 correction	N=50, T=200 no correction	N=50, T=200 correction	N=100, T=100 no correction	N=100, T=100 correction	N=200, T=50 no correction	N=200, T=50 correction	N=500, T=20 no correction	N=500, T=20 correction
	pc=1, p=0									
0	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
5	0.97	0.97	0.99	0.98	0.99	0.99	0.98	1.00	0.71	0.98
10	0.96	0.97	0.96	0.98	0.75	0.99	0.52	0.99	0.51	0.97
50	0.43	0.97	0.37	0.98	0.43	0.99	0.45	0.99	0.48	0.96
100	0.41	0.97	0.37	0.98	0.42	0.99	0.45	0.99	0.47	0.96
	pc=5, p=0									
0	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
5	0.97	0.97	0.98	0.97	0.98	0.99	0.95	0.99	0.69	0.94
10	0.93	0.97	0.86	0.97	0.49	0.97	0.27	0.96	0.30	0.87
50	0.09	0.97	0.06	0.97	0.09	0.97	0.12	0.85	0.19	0.73
100	0.06	0.97	0.06	0.97	0.09	0.97	0.12	0.78	0.18	0.72
	pc=0, p=1									
0	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
5	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
10	0.97	0.97	0.98	0.98	0.99	0.99	0.99	1.00	0.99	0.99
50	0.51	0.97	0.37	0.98	0.26	0.99	0.23	1.00	0.25	0.99
100	0.13	0.97	0.10	0.98	0.11	0.99	0.13	1.00	0.20	0.99
	pc=0, p=5									
0	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
5	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99
10	0.95	0.97	0.96	0.98	0.97	0.99	0.96	0.99	0.89	0.99
50	0.23	0.97	0.15	0.98	0.13	0.99	0.15	0.93	0.20	0.62
100	0.08	0.97	0.07	0.98	0.09	0.99	0.12	0.36	0.19	0.24
	pc=1, p=1									
0	0.97	0.97	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99
5	0.97	0.97	0.99	0.98	0.99	0.99	0.98	0.99	0.72	0.98
10	0.96	0.97	0.95	0.98	0.74	0.99	0.52	0.99	0.50	0.97
50	0.26	0.97	0.17	0.98	0.15	0.99	0.16	0.99	0.20	0.96
100	0.08	0.97	0.07	0.98	0.09	0.99	0.12	0.98	0.19	0.95

Table 5. Estimated number of factors by Bai and Ng's (2002) information criteria

	N=50, T=200						N=100, T=100						N=200, T=50					
	no correction			correction			no correction			correction			no correction			correction		
σ	ICp1	ICp2	ICp3	ICp1	ICp2	ICp3	ICp1	ICp2	ICp3	ICp1	ICp2	ICp3	ICp1	ICp2	ICp3	ICp1	ICp2	ICp3
	pc=1, p=0																	
0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	5.34	4.00	4.00	4.00	4.05	4.03	4.18
5	5.02	5.01	5.04	4.00	4.00	4.07	5.02	5.02	5.02	4.72	4.30	6.39	5.06	5.06	5.06	5.10	5.08	5.22
10	5.00	5.00	5.00	4.00	4.00	4.04	4.98	4.98	4.99	4.37	4.11	6.41	5.01	5.01	5.01	5.01	4.95	5.19
50	4.99	4.99	4.99	4.00	4.00	4.00	4.97	4.97	4.97	4.01	4.00	5.46	5.01	5.01	5.01	4.04	4.02	4.17
100	5.01	5.01	5.01	4.00	4.00	4.00	5.01	5.01	5.02	4.00	4.00	5.43	4.98	4.98	4.98	4.04	4.02	4.17
	pc=5, p=0																	
0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.01	4.00	5.34	4.00	4.00	4.00	4.05	4.03	4.18
5	8.76	8.56	8.98	4.04	4.01	4.44	9.05	9.05	9.15	6.83	4.80	10.92	9.06	9.06	9.06	9.11	9.08	9.26
10	8.97	8.97	8.97	4.02	4.01	4.25	8.98	8.98	9.07	5.22	4.31	10.88	9.02	9.02	9.02	8.58	8.19	9.20
50	9.02	9.02	9.02	4.00	4.00	4.00	9.01	9.01	9.08	4.00	4.00	6.10	8.97	8.97	8.97	4.03	4.01	4.21
100	9.02	9.02	9.02	4.00	4.00	4.00	9.01	9.01	9.11	4.00	4.00	6.08	8.99	8.99	8.99	4.05	4.03	4.27
	pc=0, p=1																	
0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.01	4.00	5.33	4.00	4.00	4.00	4.05	4.03	4.18
5	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.83	4.00	4.00	5.25	4.00	4.00	4.00	4.02	4.01	4.09
10	4.21	4.14	4.64	4.00	4.00	4.00	4.25	4.07	19.25	4.00	4.00	5.51	4.05	4.02	4.28	4.02	4.01	4.11
50	19.93	19.68	20.00	4.00	4.00	4.00	12.82	3.56	20.00	4.00	4.00	5.48	1.11	1.04	4.59	4.03	4.01	4.14
100	19.95	19.88	20.00	4.00	4.00	4.00	14.29	5.21	20.00	4.00	4.00	5.48	1.25	1.09	5.20	4.03	4.02	4.14
	pc=0, p=5																	
0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.01	4.00	5.33	4.00	4.00	4.00	4.05	4.03	4.18
5	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	5.30	4.00	4.00	5.27	4.00	4.00	4.00	4.00	4.00	4.00
10	4.03	4.01	4.18	4.00	4.00	4.00	4.00	3.99	15.24	4.00	4.00	6.64	2.99	2.57	3.76	4.00	4.00	4.02
50	1.03	1.01	1.66	4.00	4.00	4.00	1.00	1.00	16.96	4.00	4.00	6.29	1.00	1.00	1.00	4.00	4.00	4.07
100	1.06	1.02	2.13	4.00	4.00	4.00	1.00	1.00	16.76	4.00	4.00	6.22	1.00	1.00	1.00	4.00	4.00	4.07
	pc=1, p=1																	
0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.01	4.00	5.33	4.00	4.00	4.00	4.05	4.03	4.18
5	4.98	4.96	5.02	4.00	4.00	4.05	5.00	5.00	6.00	4.58	4.17	6.33	5.06	5.06	5.06	5.07	5.05	5.14
10	5.23	5.15	5.73	4.00	4.00	4.04	5.25	5.05	19.50	4.27	4.06	6.57	5.07	5.04	5.34	4.90	4.78	5.10
50	19.93	19.75	20.00	4.00	4.00	4.00	13.51	4.57	20.00	4.00	4.00	5.59	1.72	1.56	6.18	4.02	4.01	4.14
100	19.95	19.89	20.00	4.00	4.00	4.00	14.80	6.14	20.00	4.00	4.00	5.60	1.93	1.62	6.59	4.02	4.01	4.15

Figure 1. Sample path of factor and factor estimate in the presence of outlier

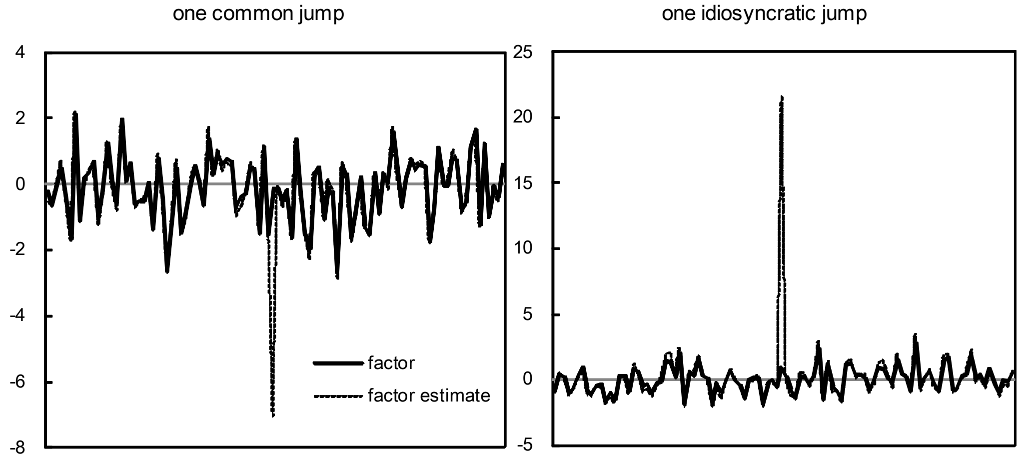


Table 6. Size of factor jump test

σ	N=20 T=500	N=50 T=200	N=100 T=100	N=200 T=50	N=500 T=20
	Case 1: X* is available				
5	0.05	0.06	0.05	0.04	0.04
10	0.05	0.07	0.06	0.05	0.05
50	0.05	0.07	0.06	0.06	0.04
100	0.05	0.07	0.07	0.06	0.05
	Case 2: X* is estimated				
5	0.07	0.01	0.03	0.03	0.65
10	0.06	0.08	0.12	0.20	0.56
50	0.05	0.07	0.07	0.07	0.08
100	0.05	0.07	0.07	0.06	0.06

Table 7. Power of factor jump test

σ	N=20 T=500	N=50 T=200	N=100 T=100	N=200 T=50	N=500 T=20
	Case 1: X* is available				
5	0.90	0.94	0.96	0.97	0.98
10	0.95	0.97	0.98	0.99	0.99
50	0.99	1.00	1.00	1.00	1.00
100	1.00	1.00	1.00	1.00	1.00
	Case 2: X* is estimated				
5	0.58	0.34	0.35	0.39	0.56
10	0.76	0.62	0.65	0.65	0.76
50	0.95	0.92	0.93	0.92	0.95
100	0.98	0.96	0.96	0.96	0.98

Figure 2. Log-returns on currencies against the U.S. dollar

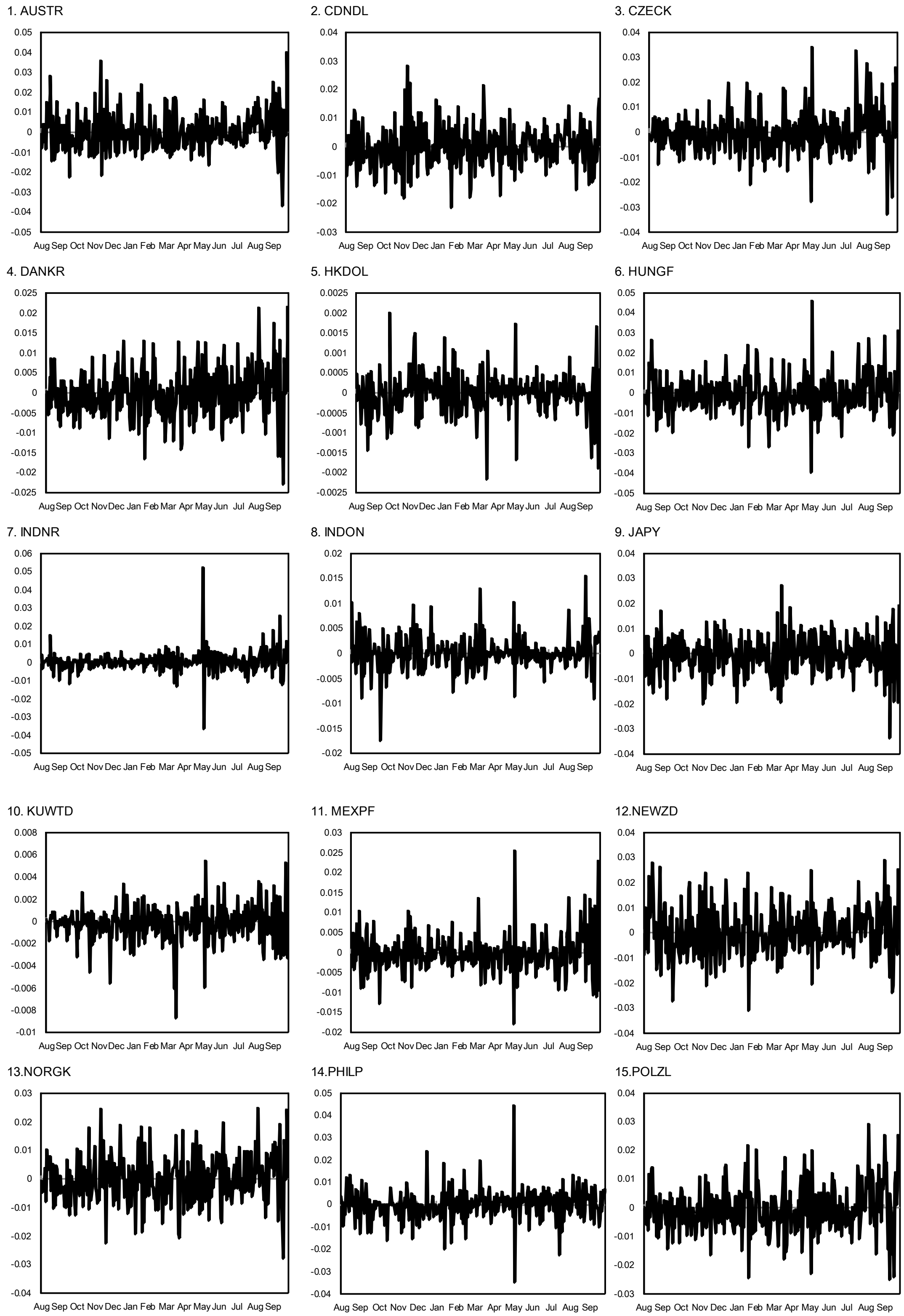


Figure 2. Log-returns on currencies against the U.S. dollar (continued)

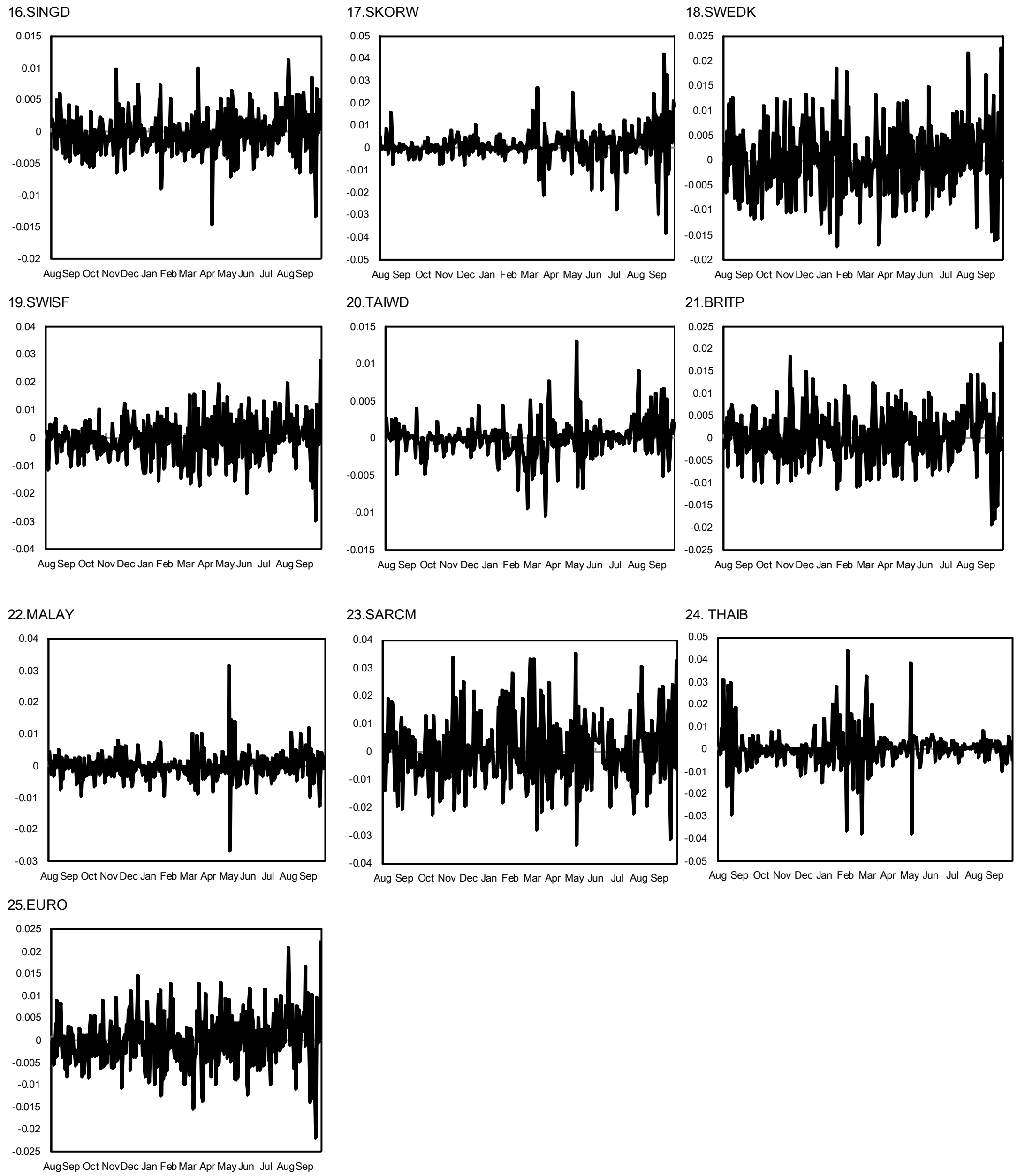


Table 8. List of currencies, number of jumps, and common jumps

			# of jumps	common jump dates		
				06 May	07 May	29 Sep
1	Australian Dollar	AUSTR	1			X
2	Canadian Dollar	CDNDL	0			
3	Czech Republic Koruna	CZECK	0			
4	Danish Krone	DANKR	0			
5	Hong Kong Dollar	HKDOL	7	X	X	X
6	Hungrian Forint	HUNGF	1		X	
7	Indian Rupee	INDNR	4	X	X	
8	Indonesian Rupiah	INDON	11	X	X	
9	Japanese Yen	JAPYN	0			
10	Kuwaiti Dinar	KUWTD	6	X	X	
11	Mexican Peso	MEXPF	3	X	X	X
12	New Zealand Dollar	NEWZD	0			
13	Norwegian Krone	NORGK	0			
14	Philippines Peso	PHILP	3	X	X	
15	Polish Zloty	POLZL	0			
16	Singaporean Dollar	SINGD	1			
17	South Korean Won	SKORW	21	X		X
18	Swedish Krona	SWEDK	0			
19	Swiss Franc	SWISF	0			
20	UK Pound	BRITP	11			
21	Malaysian Ringgit	MALAY	0			
22	Taiwan Dollar	TAIWD	2	X	X	
23	South African Rand	SARCM	0			
24	Thai Baht	THAIB	20	X	X	
25	Euro	EURO	0			

Notes : 1. "# of jumps" indicates how many jumps are detected by the proposed method between 1 Aug 2008 and 30 Sep. 2008.

2. The common jumps dates are those on which more than 3 currencies have a jump. These currencies have a mark "X".

Figure 3. Number of jumps in a day

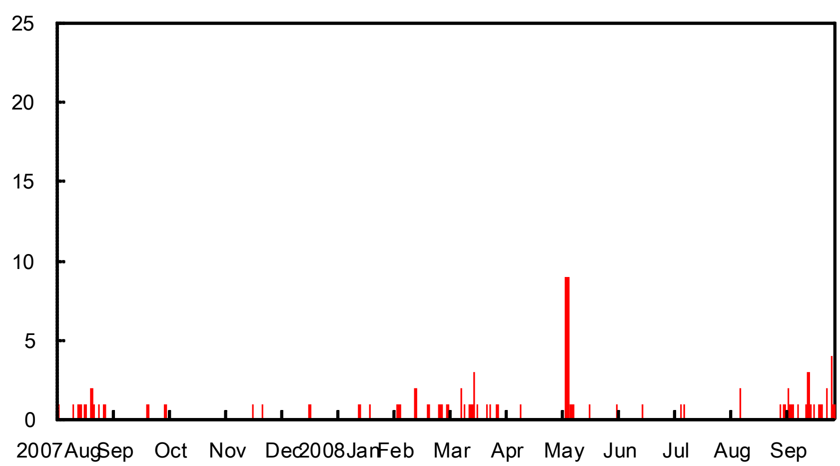
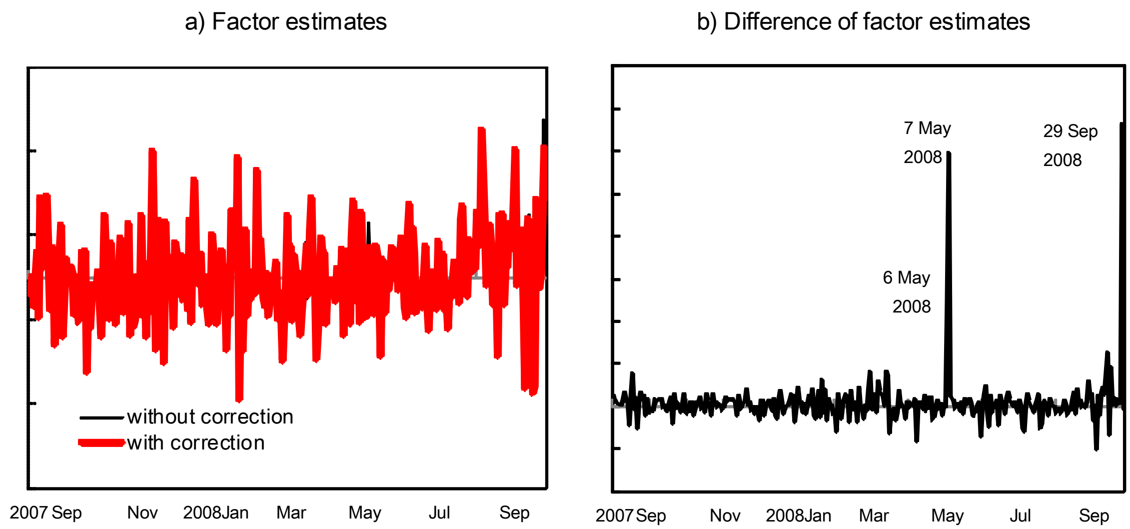


Figure 4. Factor estimates with and without jump correction

1) First factor



2) Second factor

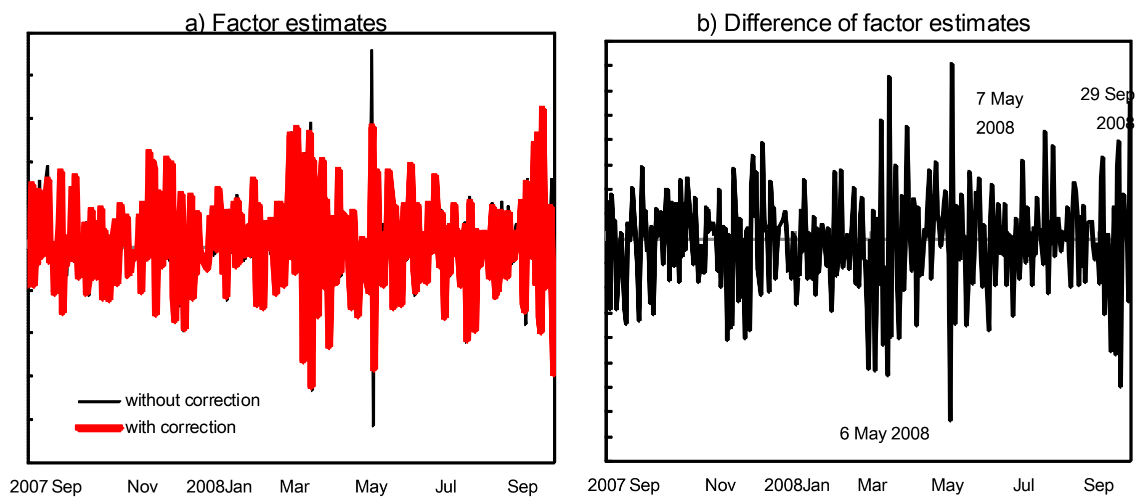


Table 9. Test for jump of factors: Currency return data

	F	p-value	t (1st factor)	p-value	t (2nd factor)	p-value
2008/5/6	3.58**	(0.04)	-2.65**	(0.01)	-0.73	(0.47)
2008/5/7	3.09*	(0.06)	2.48**	(0.02)	0.47	(0.64)
2008/9/29	2.90*	(0.07)	1.45	(0.16)	-1.71	(0.10)

Note: ** and * indicate significance at the 5% and 10% levels, respectively.

Figure 5. Monthly growth rates of new car registrations in selected Japanese prefectures

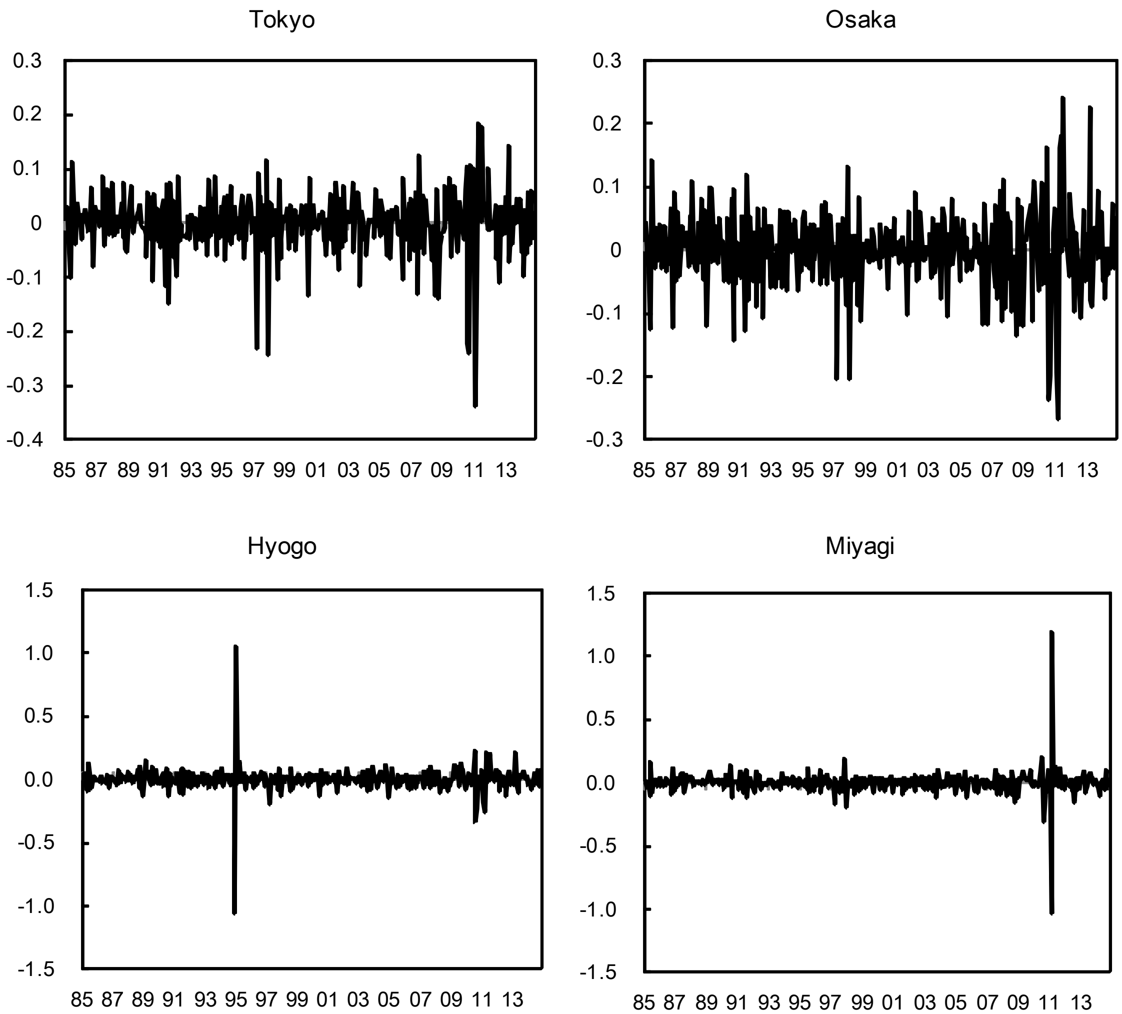
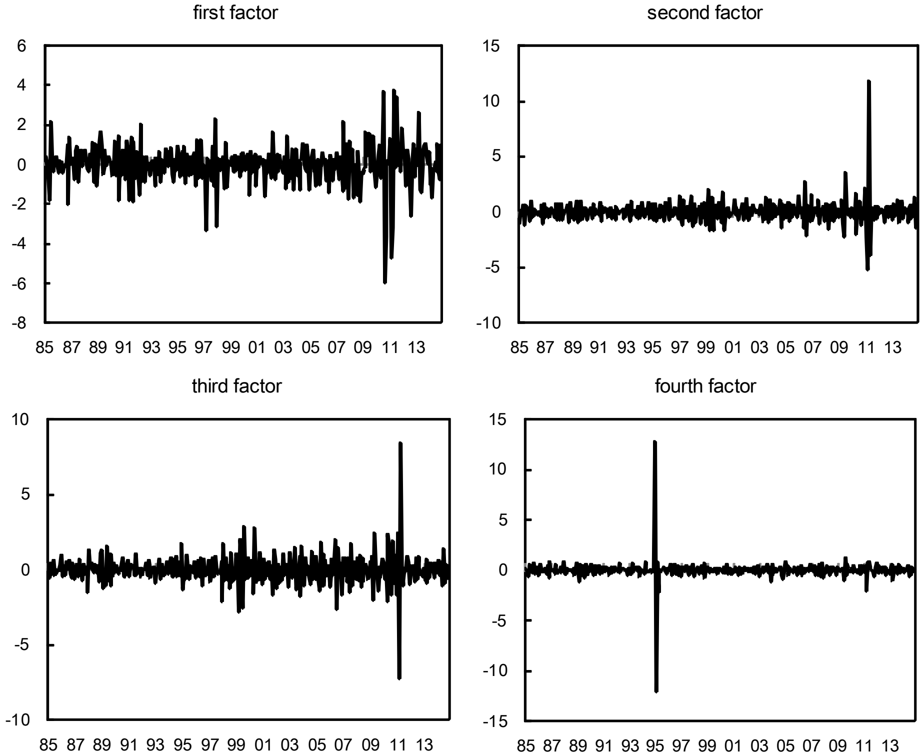


Table 10. Prefectures showing a jump in earthquake periods

	# of pref.	Prefectures that have a jump
Jan 1995	1	Hyogo
May 2011	23	Hokkaido, Aomori, Iwate, Miyagi, Akita, Yamagata Fukushima, Ibaragi, Tochigi, Gunma, Saitama Chiba, Tokyo, Kanagawa, Yamanashi, Gifu, Nagano, Shizuoka, Aichi, Shimane, Okayama, Hiroshima, Fukuoka

Figure 6. Japanese prefectural new car registration factor estimates

non-corrected



jump-corrected

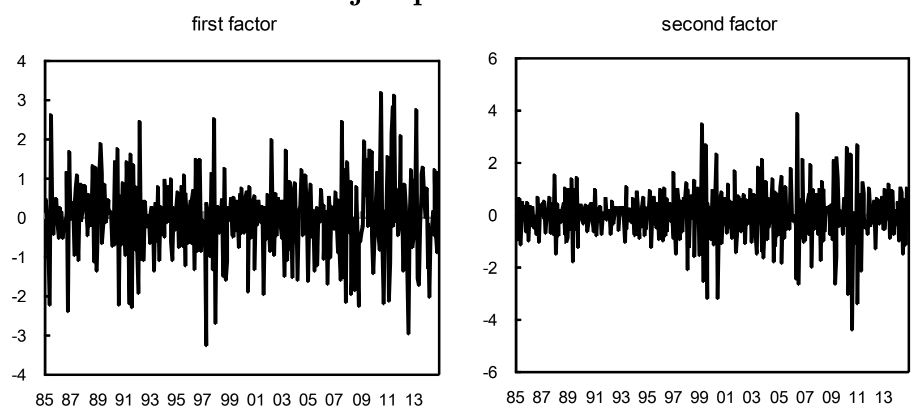


Table 11: Tests for factor jump: Japanese prefectural new car registration data

	F	p-value	*t* (1st factor)	p-value	*t* (2nd factor)	p-value
Jan 1995	0.04	(0.96)	0.06	(0.95)	0.17	(0.87)
Mar 2011	6.61***	(0.00)	2.21**	(0.03)	0.84	(0.40)

Note: ** and * indicate significance at the 5% and 10% levels, respectively.