Public Education, Pension and Debt Policy*

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Abstract

Our paper sets the model with public education investment, pension benefit and public debt stock and examines how tax burden and expenditure share between education policy and pension policy affect the public debt stock ratio to Gross Domestic Product (GDP). Moreover, our paper considers the target policy to be constant public debt ratio to GDP over time.

Based on Domar condition, our paper examines fiscal sustainability and how tax and expenditure policy affect on the public debt stock ratio to GDP in the long run. The change of expenditure share between public education investment and pension benefit can decrease the public debt ratio to GDP. Moreover, our paper derives two positive income tax rate to hold constant public debt ratio to GDP. Thanks to low tax rate, physical capital accumulation increases and then both income growth and income level increase.

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1 Introduction

Our paper examines how the intergenerational policy affects on the public debt: one for the public education investment for younger people and the other for pension benefit for older people. Under a constant tax revenue, an increase in expenditure for eaxh policies can be considered to bring about an increase in public debt. However, even if an public education investment increases fiscal burden, an increase in human capital carried by public education investment increases income level and then increases tax revenue. Finally, this effect contains a decrease in public debt.

Greiner (2008a) examines economic growth with public education investment and public debt and derived that high primaly balance surplus can bring about high level of public debt stock. Some research examines how the public debt stock is determined in the model with public capital that is productive government expenditure, which increases the labor productivity. Greiner (2007) examines the condition that the fiscal deficit finance is sustainable, and it is determined by the primary surplus. Public capital increases Gross Domestic Product (GDP) and then contains the effects that increases tax revenue and decreases public debt GDP ratio, as shown by Yakita (2008). Futagami, Iwaisako and Ohdoi (2008) sets the model with productive government spending and derives that constant public debt stock policy generates two equilibrium: one for low income level and the other for high income level. This same public debt policy is examined by Minea and Villieu (2013), too. Minea and Villieu (2013) considers debt policy that public debt stock per GDP is constant. Then, only a equilibrium exists. Teles and Mussolini (2014) considers dynamics of public debt in the model with productive government expenditure. Chalk (2000) and Moraga and Vidal (2008) also examine public debt in terms of fiscal sustainability. Moraga and Vidal (2008) derives how an aging society affects the public debt and the condition to stay constant public debt level.

Some studies concentrate on the analysis of public debt in the model with productive government expenditure. On the other hand, Ono (2003) examines how the dynamics of public debt stock is determined in the model with social security policy. Ono (2003) obtains the result that an aging society may increase public debt stock. Kunze (2014), which does not consider the social security policy but public education investment, investigates the effect of an aging society on income growth.

An endogenous growth with human capital is investigated by Greiner (2008b) and McDonald and

Zhang (2012). These studies examines not only income growth but also income distribution. Greiner (2008b) sets the model that public education spending increases the human capital accumulation and derives that if large level of human capital is inputed in an educational sector, human capital accumulation is promoted.

Cremer, Gahavari and Pestieau (2011) also consider the same policy in the model without fiscal deficit and derives an optimal policy to bring about first best allocations. Our paper considers the share policy between education investment for younger people and pension benefit for older people and investigates how the dynamics of public debt stock ratio to GDP can be depicted. As shown by Fig.1, the expenditure share of between public education investment and social security expenditure is changed in Japan, as we can see, and it is due to an aging society. Moreover, public debt stock ratio to GDP in Japan increases steeply compared with other countries, as shown by Fig.2.

[Insert Fig.1 around here.]

[Insert Fig.2 around here.]

It is important to increase public education investment as similar with productive government expenditure because an increase in human capital stock raises the labor productivity and then brings about a high level of GDP and income per capita. This positive effect brings about fiscal conslidation since the tax revenue increases and decreases public debt stock per GDP. However, because of an aging society, the government can not raise the share of public education investment and must raise the share of social security for older people. Our paper examines how the share of expenditure between public education investment and pension benefit affects on the public debt stock ratio to GDP.

In Euro countries, the public debt stock to GDP ratio and the fiscal deficit to GDP ratio must be less than 60% and 3%, respectively. The target policy helps fiscal sustainability and the govenrment commitment for fiscal management. Our paper assumes that the government adopts the target policy that public debt stock is constant over time and examines how the tax rate to hold the public debt policy is determined.

Our paper sets the model with public education investment, pension benefit and public debt stock and examines how tax burden and expenditure share between education policy and pension policy affect the public debt stock ratio to Gross Domestic Product (GDP). Moreover, our paper considers the target policy to be constant public debt ratio to GDP over time.

Based on Domar condition, our paper examines fiscal sustainability and how tax and expenditure policy affect on the public debt stock ratio to GDP in the long run. The change of expenditure share between public education investment and pension benefit can decrease the public debt ratio to GDP. Moreover, our paper derives two positive income tax rate to hold constant public debt ratio to GDP. Thanks to low tax rate, physical capital accumulation increases and then both income growth and income level increase.

The reminder in this paper consists of the following sections. Section 2 sets the model with public debt, public education investment and pension benefit. Section 3 derives the equilibrium in the model economy and the steady state equilibrium. Section 4 considers the some policies and how the public debt stock ratio to GDP in the steady state is affected by the policies. Moreover, our paper considers the target policy of public debt stock ratio to GDP and examines how the tax rate to hold the target policy is determined. Section 5 concludes our analyses.

2 Model

In this economy model, three type agents exist: household, firm and government. This section explains the model settings.

2.1 Household

The individuals live in two periods, young and old period. Our paper sets the overlapping generations model, that is, there exists young generation and old generation in each period. It is no population growth and population size is assumed as unity over time. The household's utility function is assumed by the following log utility function as

$$u_t = \alpha \ln c_{1t} + (1-\alpha) \ln c_{2t+1},\ 0 < \alpha < 1, \tag{1}$$

where c_{1t} and c_{2t+1} denote the consumption in young and old, respectively. In young period, the younger people work inelastically to gain wage income and the wage income is allocated into the consumption in young period and the saving to consume in old period. Moreover, the government levies labor income tax to provide the pension and public education investment. Then, the household's lifetime budget constraint is shown as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau)w_t H_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{2}$$

$1+r_{t+1}$ and w_t denotes an interest rate and wage rate per human capital H_t. p_{t+1} denotes the pension benefit. τ denotes the tax rate to provide public education investment and pension benefit $(0 < \tau < 1)$.

It is assumed that the human capital is accumulated by only public education investment E_t as follows,[1]

$$H_{t+1} = \beta E_t, 0 < \beta. \tag{3}$$

2.2 Firm

Firms produces the final goods with capital stock and labor input in perfectly competitive market. The product function is assumed as

$$Y_t = K_t^{\gamma} H_t^{1-\gamma},\ 0 < \gamma < 1. \tag{4}$$

Y_t denotes the final goods and K_t denotes the capital stock in t period. Then, maximizing firm's profit in competitive market, the demand for the physical capital stock and labor input are shown as follows,

$$w_t = (1-\gamma)k_t^{\gamma}, \tag{5}$$

$$1 + r_t = \gamma k_t^{\gamma-1}, \tag{6}$$

where $k_t = \frac{K_t}{H_t}$. It is assumed that physical capital stock is fully depreciated in one period.

2.3 Government

Our paper considers two policies: one for younger people and the other for older people. The government provides the pension benefit for older people and public education investment for younger people. It is assumed that the government is allowed to issue the bond to collect the revenue, that is, debt finance or fiscal deficit, in addition to taxation for labor income. Then, the government budget constraint is shown as follows,

$$B_{t+1} = E_t + p_t - \tau w_t H_t + (1+r_t)B_t. \tag{7}$$

[1] Glomm and Ravikumar (1992) assumes that the growth rate of human capital is described by the schooling time, public education investment and parental human capital. In Greiner (2008), the growth rate of human capital is determined by public education investment and human capital stock. Our paper assumes simply human capital accumulation.

B_t denotes the public debt stock. $E_t + p_t - \tau w_t H_t$ denotes the primary deficit. If the primary deficit is equal to zero, the public debt stock grows at the rate of $1 + r_t$. It is assumed that the public education investment E_t and pension benefit p_t are proportionally provided as $E_t = \epsilon w_t H_t$ and $p_t = \eta w_t H_t$ ($0 < \epsilon < 1$, $0 < \eta < 1$), the government budget constraint is shown by

$$B_{t+1} = (\epsilon + \eta - \tau) w_t H_t + (1 + r_t) B_t. \tag{8}$$

Then, $\epsilon + \eta - \tau$ shows the primary deficit. Recursively substituting, we obtain the government budget constraint written by the another form as follows,

$$\lim_{s\to\infty} \frac{B_{t+s}}{\prod_{j=0}^{s}(1+r_{t+j})} = (\epsilon + \eta - \tau) \frac{\sum_{i=0}^{s-1} \prod_{i=j}^{s-1}(1 + r_{t+i}) w_{t+i} H_{t+i}}{\prod_{j=0}^{s}(1+r_{t+j})} + B_t. \tag{9}$$

Considering Non-Ponzi condition, which shows that the government can not borrow forever or the public debt stock B_t grows at more than an interest rate,

$$\lim_{s\to\infty} \frac{B_{t+s}}{\prod_{j=0}^{s-1}(1+r_{t+j})} = 0 \tag{10}$$

must be held. Then, the government budget constraint is

$$B_t = (\tau - \epsilon - \eta) \lim_{s\to\infty} \sum_{j=0}^{s-1} \frac{w_{t+j} H_{t+j}}{\prod_{i=0}^{j}(1+r_{t+i})}. \tag{11}$$

An initian debt B_t equals to discounted primary surplus described by the right hand side of this equation. Then, given positive B_t, $\tau - \epsilon - \eta$ must be positive.

3 Equilibrium

This section derives the equilibrium. Now, considering human capital accumulation, the growth rate of human capital $1 + g_t$ is obtained by $E_t = \epsilon w_t H_t$ and (5).

$$1 + g_t = \frac{H_{t+1}}{H_t} = \beta\epsilon(1-\gamma)k_t^{\gamma}. \tag{12}$$

Defining $b_t = \frac{B_t}{H_t}$, the budget constraint (8) is reduces to

$$b_{t+1} = \frac{1}{\beta\epsilon}\left(\epsilon + \eta - \tau + \frac{\gamma}{1-\gamma}\frac{b_t}{k_t}\right). \tag{13}$$

Physical capital market clearing condition is $B_{t+1} + K_{t+1} = s_t$, which s_t denotes the household's saving given by $s_t = (1-\tau)w_t H_t - c_{1t}$ Then, we obtain the dynamics of k_t as follows,[2]

$$k_{t+1} = \frac{\gamma}{\gamma + \alpha\eta(1-\gamma)} \left(\frac{(1-\alpha)(1-\tau)}{\beta\epsilon} - b_{t+1} \right). \tag{14}$$

Rewriting (14) at t period and substituting this equation into (13) derives the dynamics equation of b_t. The equilibrium in this model economy is specified by an initial condition b_0 and the following dynamics equation of b_t,

$$b_{t+1} = \frac{\epsilon + \eta - \tau}{\beta\epsilon} + \frac{\gamma + \alpha\eta(1-\gamma)}{1-\gamma} \frac{1}{\frac{(1-\alpha)(1-\tau)}{b_t} - \beta\epsilon}. \tag{15}$$

Then, the dynamics are depicted as Fig.3.

[Insert Fig.3-1 around here.]

[Insert Fig.3-2 around here.]

Fig.3-1 shows the dynamics of b_t with primary surplus and No-Ponzi condition holds.[3] The steady state equilibrium E_0 is unstable steady state. If an initial public debt b_0 is less than the public debt at E_0, the public debt stock continues decreasing and fiscal sustainability can be brought about. On the other hand, if an initial public debt stock b_0 is more than the public debt at E_0, b_t increases steeply and diverges. Fig.3-2 shows the dynamics of b_t with primary deficit. We consider an another condition to examines fiscal sustainability, Domar condition. Domar condition implies that if income growth rate is larger than the interest rate, the public debt stock to GDP ratio converges to certain level. Considering the government budget constraint (8), we obtain the following equation,

$$b_{t+1} = \frac{\epsilon + \eta - \tau}{\beta\epsilon} + \frac{1 + r_t}{1+g} b_t. \tag{16}$$

Moreover, we obtain the following equation,[4]

$$b_{t+s} = \left(1 + \frac{1+r}{1+g} + \cdots + \left(\frac{1+r}{1+g}\right)^{t+s-1} \right) \frac{\epsilon + \eta - \tau}{\beta\epsilon} + \left(\frac{1+r}{1+g}\right)^{s} b_t. \tag{17}$$

[2] We obtain $s_t = (1-\alpha)(1-\tau)w_t H_t - \frac{\alpha p_{t+1}}{1+r_{t+1}}$. Considering (6) and $p_{t+1} = \eta w_{t+1} H_{t+1}$, (14) can be derived.

[3] See Appendix for a detail proof.

[4] Because of briefly calculating, the interest rate and income growth rate are constant over time. We can consider this situation at the steady state.

With $s \to \infty$ and $g > r$, We obtain the following constant public debt ratio to GDP,

$$\bar{b} = \frac{1+g}{g-r}\frac{\epsilon+\eta-\tau}{\beta\epsilon}, \tag{18}$$

where $\bar{b}$ denotes the public debt stock b_t at the steady state. Fig.3-2 shows the two steady state equilibrium, E_1 and E_2. E_1 is stable steady state and E_2 is unstable one. At the steady state, b_t and k_t are constant over time and then K_t, B_t, Y_t and H_t grow at the rate $1+g$ given by (12). Moreover the condition of $g > r$ needs the following condition

$$\bar{k} > \frac{\gamma}{(1-\gamma)\beta\epsilon}, \tag{19}$$

where $\bar{k}$ denotes the physical capital ratio to human capital at the steady state. Moreover, considering (14), the condition of $\bar{b}$ to have $g > r$ is

$$\bar{b} < \frac{(1-\tau-\alpha\eta-\alpha)(1-\gamma)(1-\tau)-\gamma}{(1-\gamma)\beta\epsilon}. \tag{20}$$

The public debt $\bar{b}$ at the steady state are given by

$$\bar{b} = \frac{-X \pm \sqrt{X^2 - 4(\epsilon+\eta-\tau)(1-\alpha)(1-\tau)}}{2\beta\epsilon}, \tag{21}$$

where $X = \frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma} - (\epsilon+\eta-\tau) - (1-\alpha)(1-\tau)$. With $X^2 - 4(\epsilon+\eta-\tau)(1-\alpha)(1-\tau) \geq 0$, steady state equilibrium exists, Otherwise, no steady state equilibrium exists, as shown by the dashed line in Fig.3-2.

4 Public Debt and Policies

Our paper consideres some policies. First, we examine whether an increase in public education investment ϵ, pension benefit η and tax rate τ increase public debt $\bar{b}$ at the steady state E_1 or decrease. Second, we consider how the government should decrease the primary deficit. Third, we consider the target level of $\bar{b}$ and derives the policy parameter to sustain the target debt level.

4.1 Comparative Statics

This subsection derives how an increase in ϵ, η and τ affects on $\bar{b}$ with coparative statics. Comparative statics derives the following results,

$$\frac{d\bar{b}}{d\epsilon} = \frac{\frac{\tau-\eta}{\beta\epsilon^2} + \frac{\beta(\gamma+\alpha\eta(1-\gamma))\bar{b}^2}{(1-\gamma)\left((1-\alpha)(1-\tau)-\beta\epsilon\bar{b}\right)^2}}{Y}, \tag{22}$$

$$\frac{d\bar{b}}{d\eta} = \frac{\frac{1}{\beta\epsilon} + \frac{\alpha\bar{b}}{(1-\alpha)(1-\tau)-\beta\epsilon\bar{b}}}{Y} > 0, \tag{23}$$

$$\frac{d\bar{b}}{d\tau} = \frac{-\frac{1}{\beta\epsilon} + \frac{(1-\alpha)(\gamma+\alpha\eta(1-\gamma))(1-\alpha)\bar{b}}{(1-\gamma)\left((1-\alpha)(1-\tau)-\beta\epsilon\bar{b}\right)^2}}{Y}, \tag{24}$$

where $Y \equiv 1 - \frac{(\gamma+\alpha\eta(1-\gamma))(1-\alpha)(1-\tau)}{(1-\gamma)\left((1-\alpha)(1-\tau)-\beta\epsilon\bar{b}\right)^2} > 0$.[5]

An increase in pension benefit η always increases the public debt $\bar{b}$. However, an increases in public education investment ϵ does not always increases the public debt $\bar{b}$. Although an increase in ϵ brings about the fiscal burden, an increase in an income growth rate $1+g$ implies to alleviate fiscal burden thanks to an increase in human capital accumulation. We define $\hat{b}_\epsilon$ to hold $\frac{\tau-\eta}{\beta\epsilon^2} + \frac{\beta(\gamma+\alpha\eta(1-\gamma))}{(1-\gamma)\left(\frac{(1-\alpha)(1-\tau)}{\hat{b}_\epsilon}-\beta\epsilon\right)^2} = 0$. Then, we can obtain $\frac{d\bar{b}}{d\epsilon}$ if $\bar{b} > \hat{b}_\epsilon$. Similarly, the sign of $\frac{d\bar{b}}{d\tau}$ is ambiguous. Defining $\hat{b}_\tau$ to hold $-\frac{1}{\beta\epsilon} + \frac{(\gamma+\alpha\eta(1-\gamma))(1-\alpha)\hat{b}_\tau}{(1-\gamma)\left((1-\alpha)(1-\tau)-\beta\epsilon\hat{b}_\tau\right)^2}$, we obtain $\frac{d\bar{b}}{d\tau} > 0$ if $\bar{b} > \hat{b}_\tau$. Then, we can establish the following proposition.

Proposition 1 An increase in η always raises $\bar{b}$ at the stable steady state. An increase in ϵ and τ raise $\bar{b}$ with $\bar{b} > \hat{b}_\epsilon$ and $\bar{b} > \hat{b}_\tau$, respectively.

Although we consider that an increase in tax rate decreases the public debt because of an increase in tax revenue, an increase in τ can not always decreases $\bar{b}$. An increase in tax rate decreases the houdhold's savings and then physical capital stock decreases. Then, the growth rate $1+g$ decreases and the interest rate $1+r$ increases. Therefore, the fiscal burden relatively become large.

4.2 Primary Balance Policy

Now, we consider the following policies,

$$\epsilon = m\tau\theta,$$

$$\eta = m\tau(1-\theta).$$

With $m = 1$, the primary balance is not deficit and not surplus. Then, the public debt increases at the pace of $1+r$ in any period. $m > 1$ denotes primary deficit. θ denotes the share of expenditure between two policies. Considering the fiscal situation in Japan, we analyze the stable steady state equilibrium E_1

[5] Totally differentiating (15) by b_t and b_{t+1} at the steady state, we obtain $\frac{db_{t+1}}{db_t} = \frac{(\gamma+\alpha\eta(1-\gamma))(1-\alpha)(1-\tau)}{(1-\gamma)((1-\alpha)(1-\tau)-\beta\epsilon\bar{b})^2} > 0$. Locally stable condition at the steady state is $\frac{db_{t+1}}{db_t} < 1$. Then, we obtain $Y > 0$.

with primary deficit $m > 1$. We examines how the public debt in the steady state $\bar{b}$ is affected by m and θ as follows,

$$\frac{d\bar{b}}{dm} = \frac{\frac{1}{\beta\theta m^2} + \frac{\alpha\tau(1-\theta)\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}} - \beta m\tau\theta\right) + \frac{\beta\tau\theta(\gamma+\alpha m\tau(1-\theta)(1-\gamma))}{1-\gamma}}{\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}} - \beta m\tau\theta\right)^2}}{Y} > 0, \tag{25}$$

$$\frac{d\bar{b}}{d\theta} = \frac{-\frac{m-1}{\beta m\theta^2} - \frac{\alpha m\tau}{\frac{(1-\alpha)(1-\tau)}{\bar{b}} - \beta m\tau\theta} + \frac{\beta m\tau(\gamma+\alpha m\tau(1-\theta)(1-\gamma))}{(1-\gamma)\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}} - \beta m\tau\theta\right)^2}}{Y}. \tag{26}$$

We can obtain positive sign of $\frac{d\bar{b}}{d\theta}$ if β is sufficiently small. Then, the following proposition is established.

Proposition 2 The sign of $\frac{d\bar{b}}{dm}$ is always positive. On the other hand, with sufficiently small β, the sign of $\frac{d\bar{b}}{d\theta}$ is negative.

An increase in m means an increase in expenditure under a constant tax rate. Even if an increase in m raises the public education investment, income growth alleviates fiscal burden and public debt may decrease. However, in addition to an increase in public education investment, pension benefit also raises. Finally, these two effects always make public debt $\bar{b}$ increase.

On the other hand, an increase in θ can decrease the public debt $\bar{b}$ if β is small. This term means fiscal burden given by a decrease in physical capital stock. An increase in public education investment raises income growth and decreases the household's saving ratio to human capital and then physical capital stock k decreases. Finally, a decrease in k increases payment of interest rate and fiscal burden occurs.

4.3 Debt Target Policy

This subsection considers the debt target policy. In Futagami, Iwaisako and Ohdoi (2008), Minea and Villieu (2013) sets the debt target policy in the model with productive government expenditure and derives two equilibrium with high income and low income. Our paper also considers the debt target policy that the public debt b_t is fixed by $b_t = \bar{b}$ and derives the tax rate τ to hold $b_t = \bar{b}$. Considering (15) and $b_t = \bar{b}$, income tax rate is detemined to equalize the following equation,

$$\bar{b} + \frac{\tau - (\epsilon + \eta)}{\beta\epsilon} = \frac{\gamma + \alpha\eta(1-\gamma)}{1-\gamma} \frac{1}{\frac{(1-\alpha)(1-\tau)}{\bar{b}} - \beta\epsilon}. \tag{27}$$

Then, if the condition to have τ is held, we obtain the following Fig.4.[6]

[6] See Appendix for a detail proof.

[Insert Fig.4 around here.]

R and L denote $R \equiv \frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma} \frac{1}{\frac{(1-\alpha)(1-\tau)}{\bar{b}}-\beta\epsilon}$ and $L \equiv \bar{b} + \frac{\tau-(\epsilon+\eta)}{\beta\epsilon}$. Then, the following proposition is established.

Proposition 3 If the government adopts the public debt target policy $b_t = \bar{b}$, two positive income tax rate τ exist, given some conditions.

Both τ_{low} and τ_{high} can maintain the public debt target $\bar{b}$. Why can τ_{low} maintain $\bar{b}$? If the tax rate is low, we consider that tax revenue is small. However, τ_{low} increases the household's saving and physical capital stock become large. Then, the public education investment is also large and the high income growth rate and high income level are brought about and the government can receive sufficient amount of tax revenue to hold $\bar{b}$.

Now, we examine the condition that a decrease in target level of $\bar{b}$ reduces the tax rate τ as determined by (27). Totally differentiating (27) by $\bar{b}$ and τ, we obtain the follows,

$$\frac{d\tau}{d\bar{b}} = \frac{1 - \frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma} \frac{\frac{(1-\alpha)(1-\tau)}{\bar{b}^2}}{\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}}-\beta\epsilon\right)^2}}{\frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma} \frac{\frac{1-\alpha}{\bar{b}}}{\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}}-\beta\epsilon\right)^2} - \frac{1}{\beta\epsilon}}. \tag{28}$$

The numerator of (28) is a positive sign if we consider stable steady state equilibrium.[7] Then, if the denominator of (28) is a negative sign as following inequality, we obtain $\frac{d\tau}{d\bar{b}} > 0$, that is, a decrease in $\bar{b}$ reduces τ.

$$\frac{(1-\alpha)\bar{b} + \frac{(1-\alpha)(\tau-(\epsilon+\eta))}{\beta\epsilon}}{(1-\alpha)(1-\tau)-\beta\epsilon\bar{b}} > \frac{1}{\beta\epsilon}. \tag{29}$$

The left hand side of (29) increases with $\bar{b}$. Then, even if the inequality does not hold in small $\bar{b}$, an increase in $\bar{b}$ can hold the inequality (29). Then, considering (12) and (14), an income growth rate $1+g$ always rises. An increase in income growth rate and a wage rate and a decrease in tax rate raises the household disposable income and this effect pulls up the utility. However, an increase in physical capital-human capital ratio k reduces an interest rate $1+r$, which has the negative effect on consumption in

[7] Totally differentiating (15) by db_{t+1} and db_t at the steady state, we obtain $\frac{db_{t+1}}{db_t} = \frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma} \frac{\frac{(1-\alpha)(1-\tau)}{\bar{b}^2}}{\left(\frac{(1-\alpha)(1-\tau)}{\bar{b}}-\beta\epsilon\right)^2}$. The condition to have stable steady state is $-1 < \frac{db_{t+1}}{db_t} < 1$. Then, we find that the numerator of (28) is a positive sign.

old period c_{2t} because of $c_{2t} = (1-\alpha)(1+r_t)\left((1-\tau)w_t H_t + \frac{\eta w_{t+1} H_{t+1}}{1+r_{t+1}}\right)$. Therefore, the effects on the utility is ambiguous.

5 Conclusions

Our paper sets the model with public education investment, pension benefit and public debt stock and examines how tax burden and expenditure share between education policy and pension policy affect the public debt stock in the steady state. Moreover, our paper considers the target policy about public debt that the public debt stock is constant over time. Fiscal sustainability is considered by No Ponzi condition, Domar condition, and so on. No Ponzi condition shows that the growth rate of public debt stock is less than the interest rate and needs that the sum of primary surplus discounted by the interest rate in future is equal to the public debt in present period. Domar condition is condition not to diverge public debt stock ratio to GDP and needs that income growth rate is more than the interest rate.

Based on Domar condition, our paper examines fiscal sustainability and how tax and expenditure policy affect on the public debt stock in the long run. Moreover, if the government adopts the target of public debt stock, our paper derives two positive tax rate: one for low tax rate and the other for high tax rate. Even if tax rate is low, the target policy to be constant public debt stock ratio to GDP can be held because low tax rate increases physical capital accumulation and then increases income growth rate thanks to an increase in public education investment.

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Appendix

Fiscal Rule with No Ponzi Condition

We consider the steady state with No Ponzi condition. Defining $\tilde{b}$ as the public debt with no ponzi condition at the steady state and considering (11), we can show the following equation.

$$\tilde{b} = (\tau - \epsilon - \eta) \lim_{s \to \infty} \sum_{j=0}^{s-1} w \prod_{i=0}^{j} \left(\frac{1+g}{1+r} \right). \tag{30}$$

Then, consideting (5), (6) and (12), we obtain the following equation,

$$\begin{aligned}\tilde{b} &= \frac{(\tau - \epsilon - \eta) w (1+r)}{r-g} \\ &= \frac{(\tau - \epsilon - \eta)\gamma(1-\gamma)k^{\gamma}}{\gamma - \beta\epsilon(1-\gamma)k}.\end{aligned} \tag{31}$$

$\tilde{b}$ and $\tilde{k}$ to hold No Ponzi condition are given by the following equations.

$$\tilde{b} = \frac{(\tau - \epsilon - \eta)\gamma(1-\gamma)k^{\gamma}}{\gamma - \beta\epsilon(1-\gamma)k}, \tag{32}$$

$$\tilde{k} = \frac{\gamma}{\gamma + \alpha\eta(1-\gamma)} \left(\frac{(1-\alpha)(1-\tau)}{\beta\epsilon} - \tilde{b} \right). \tag{33}$$

Then, we obtain $\tilde{b}$ to have an intersection point given by these equations. With $\bar{b} < \tilde{b}$, $\bar{b}$ at Fig.3-1 holds No Ponzi condition.

[Insert Fig.5 around here.]

The proof of Proposition 3

Considering (27), we obtain the following quadratic equation,

$$\begin{aligned}F(\tau) &= (1-\alpha)\tau^2 - ((1-\alpha)(1+\epsilon+\eta) - \beta\epsilon\bar{b}(1+(1-\alpha)\bar{b}))\tau \\ &\quad - \left(\beta\epsilon\bar{b} \left(1 + \epsilon + \eta - \alpha - \frac{\gamma + \alpha\eta(1-\gamma)}{1-\gamma} - \beta\epsilon\bar{b} \right) - (1-\alpha)(\epsilon+\eta) \right) = 0.\end{aligned} \tag{34}$$

The following conditions are held, we obtain the two positive tax rate τ_{low} and τ_{high}.

$$F(0) = -\left(\beta\epsilon\bar{b} \left(1 + \epsilon + \eta - \alpha - \frac{\gamma + \alpha\eta(1-\gamma)}{1-\gamma} - \beta\epsilon\bar{b} \right) - (1-\alpha)(\epsilon+\eta) \right) > 0, \tag{35}$$

$$F'(\tau)|_{\tau=0} = -Z < 0, \tag{36}$$

$$Z^2 + 4(1-\alpha) \left(\beta\epsilon\bar{b} \left(1 + \epsilon + \eta - \alpha - \frac{\gamma + \alpha\eta(1-\gamma)}{1-\gamma} - \beta\epsilon\bar{b} \right) - (1-\alpha)(\epsilon+\eta) \right) > 0, \tag{37}$$

$$\tau < 1 - \frac{\beta\epsilon\bar{b}}{1-\alpha}, \tag{38}$$

where $Z = (1-\alpha)(1+\epsilon+\eta) - \beta\epsilon\bar{b}(1+(1-\alpha)\bar{b})$. Then, the tax rate τ is given by

$$\tau = \frac{Z \pm \sqrt{Z^2 + 4(1-\alpha)\left(\beta\epsilon\bar{b}\left(1+\epsilon+\eta-\alpha-\frac{\gamma+\alpha\eta(1-\gamma)}{1-\gamma}-\beta\epsilon\bar{b}\right)-(1-\alpha)(\epsilon+\eta)\right)}}{2(1-\alpha)}. \tag{39}$$

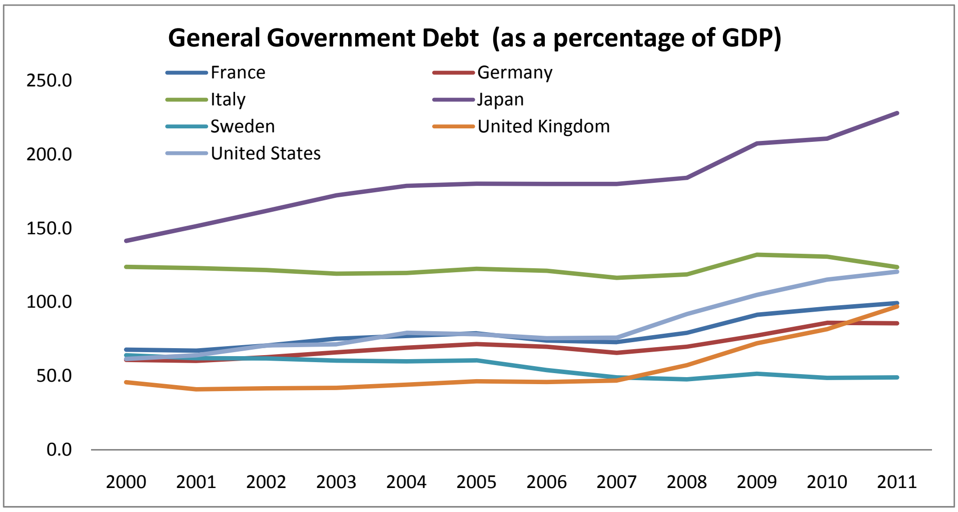

Fig.1 General Government Debt (as a percentage of GDP)

(Data: OECD Factbook 2014 Economic, Environmental and Social Statistics)

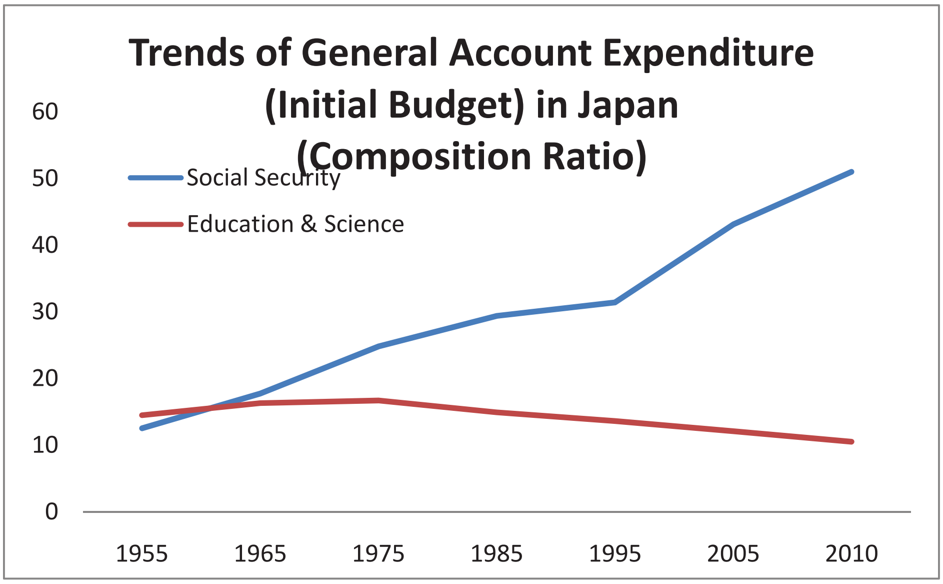

Fig.2 Trends of General Account Expenditure (Initial Budget in Japan) (Composition ratio)

(Data: Ministry of Finance Japan 'Trends of General Account Expenditure')

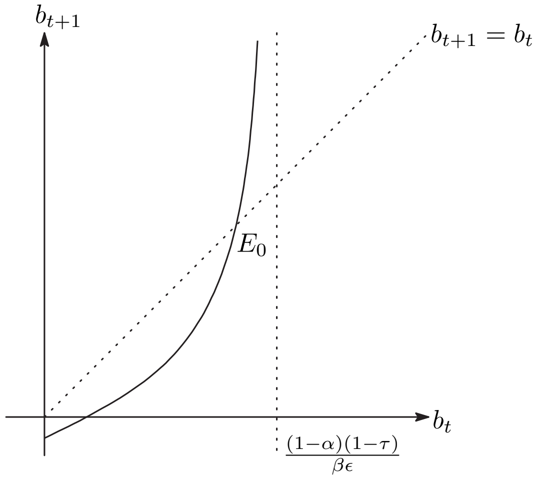

Fig.3-1: A Positive $\bar{b}$ Steady State Equilibrium (Unstable)

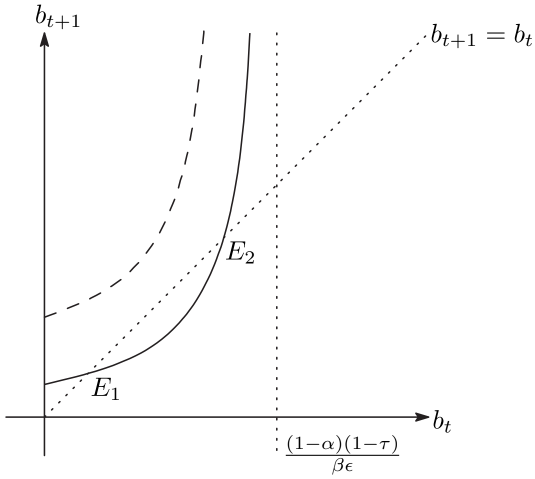

Fig.3-2: Two Positive Steady State Equilibrium

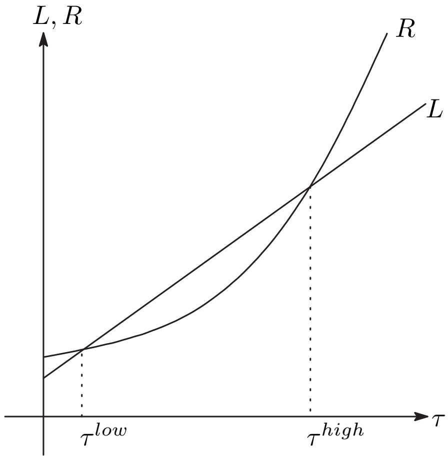

Fig.4: Debt Target Policy and Tax Rate

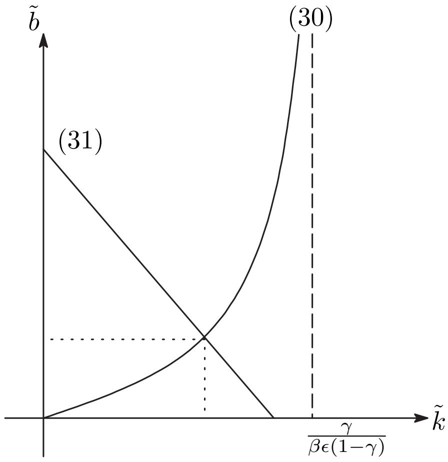

Fig.5 $\tilde{b}$ with No Ponzi condition