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**Evolution of Fairness and Group Formation
in Multi-Player Ultimatum Games**

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Evolution of Fairness and Group Formation in Multi-Player Ultimatum Games*

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Abstract

Group formation is a fundamental activity in human society. Humans often exclude others from a group and divide the group benefit in a fair way only among group members. Such an allocation is called *in-group fair*. Does natural selection favor an in-group fair allocation? We investigate the evolution of fairness and group formation in a three-person Ultimatum Game (UG) in which the group value depends on its size. In a stochastic model of the frequency-dependent Moran process, natural selection favors the formation of a two-person subgroup in the low mutation limit if its group value exceeds a high proportion (0.7) of that of the largest group. Stochastic evolutionary game theory provides theoretical support to explain the behavior of human subjects in economic experiments of a three-person UG.

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1 Introduction

Group formation is a fundamental activity in all human societies. Humans formed various groups to cooperate for their survival. They jointly hunted animals and shared foods in hunter-gatherers societies. Cooperation and distributions were major activities in groups of hunters. As the society has developed beyond the primitive stage, the roles of human groups have been significantly expanded. In an agricultural society, people cooperated for growing crops, raising live stocks and constructing irrigation systems. In modern societies, people, firms, and states form a variety of social, economic, and political groups such as community, club, labor union, political party, coalition, joint venture, cartel, alliance, international organizations, and so on. Group formation greatly affects efficiency and fairness of resource allocation.

We consider the evolution of group formation, focusing on distributional efficiency and fairness. When people form groups, there are two kinds of fairness in payoff allocations. One may be termed as *universal fairness*, in which all people cooperate and distribute the largest payoffs equally. The other may be termed as *in-group fairness*, in which a fair allocation is realized only within a sub-group and non-members of the group are excluded from the payoff allocation. The universal fairness implies that the largest group forms. In contrast, in-group fairness implies that a smaller group forms and non-members are excluded. In-group fairness leads to economic inefficiency and social exclusion.

The related concept of in-group favoritism has been studied in social psychology and biology (1–5); researchers have been concerned as to whether and why humans behave towards members of their own group in a favorable way, given that group members share some attributes such as identity and tastes. For each group member, there is no economically significant difference between members and non-members. A social group characterized by non-economic conditions is exogenously given. Group-forming behavior is not within the scope of the existing literature on in-group favoritism.

In contrast, we consider the evolution of strategically formed groups. Players can produce

a positive value only when they cooperate with other group members. Players cannot receive positive payoffs if they do not join any group. While players need to cooperate with group members, they compete with each other in the process of group formation. There is a variety of possible groups ranging from the largest to the smallest. The largest group may not be formed due to a conflict of allocations, although it can produce the largest value. Using evolutionary game theory (6–9), we consider the following questions: Which group is evolutionary stable, and what kinds of allocations prevail within a group? Specifically, under what conditions does natural selection favor in-group fairness?

A two-person Ultimatum Game (UG) has been employed by a great number of works that investigate theoretically and experimentally how people allocate their resources in bilateral contexts (10–15). While traditional game theory assuming selfish and rational players predicts that proposers offer the smallest possible share and responders accept it, many experimental studies observe that human subjects tend to offer fair allocations. Modal and median offers are 40 to 50 percent, and offers below 20 percent are rejected by the majority of responders (10, 12–14). Deterministic evolutionary game models agree with traditional game theory (7, 8). In the one-shot anonymous UG, natural selection favors low offers and low demands. To explain fairness in an evolutionary context in the two-person UG, different approaches have been introduced, including inequality aversion (16), reputation formation (17, 18), assortative matching (19), asymmetric mutation (20), and stochastic reproduction (21).

2 The Model

While almost all literature on UG considers two player games, we study a multi-player UG to investigate the problem of group formation. Consider the following three-person UG. Players, indexed by 1, 2, and 3, are assumed to be symmetric.

If players cooperate, they can receive some joint payoff, depending on the number of cooperators. The three-person group has the largest payoff value, normalized to be one. Every

two-person group has a smaller value v ($0 < v < 1$). The situation describes a classic problem in game theory, namely a three-person cooperative (symmetric) game.

The game proceeds as follows. One randomly chosen player, say 1, chooses one of three groups including himself: $\{1, 2, 3\}$, $\{1, 2\}$, or $\{1, 3\}$. If a proposer chooses the three-person group $\{1, 2, 3\}$, then he further proposes a payoff allocation of the group value, one. Thereafter, two other members respond to the proposal sequentially according to a fixed order.¹ If all responders accept it, then the three players receive their own payoffs in the proposal. Otherwise, the game stops, and no players receive positive payoffs. A similar rule is applied when the proposer chooses a two-person group. If a proposer chooses a two-person group, e.g. $\{1, 2\}$, the proposer proposes a payoff allocation of v . A single group member responds to it. If he accepts, then the proposal is agreed and the two players receive their own payoffs in the proposal. A single player outside the group receives nothing. If the responder rejects, then the three players receive nothing. Like the two-person UG, the traditional game theory assuming selfish and rational players predicts that a proposer chooses the largest group $\{1, 2, 3\}$ and exploits almost all of the group value, one. Two responders receive the smallest possible shares.

We examine whether either universal fairness or in-group fairness emerges in a three-person UG in an evolutionary context. A strategy is defined by a triplet (m, p, q) , where $m = 2, 3$ is the size of a group, p is a payoff offered to all other members in the group, and q is the smallest payoff that a player accepts when acting as a responder, in other words, an acceptance threshold. When $m = 2$, it is assumed that the proposer chooses each of the two other players randomly as his group partner. In the formulation of a strategy, we assume that a proposer treats two members (if any) of his group equally.²

Let S be a finite set of strategies (m, p, q) , where $m = 2, 3$, $0 \leq p \leq 1$ (when $m = 3$),

¹ The order of responders does not affect the result in any critical manner.

² This assumption is supported by experimental data from a three-person UG (22).

$0 \leq p \leq v$ (when $m = 2$), and $0 \leq q \leq 1$.³ Special attention is paid to the following three strategies: (i) Selfish strategy: $(3, 0, 0)$; (ii) Universally Fair strategy: $(3, 1/3, 1/3)$; and (iii) In-Group Fair strategy: $(2, v/2, v/2)$. In Selfish strategy, a proposer chooses the largest group $\{1, 2, 3\}$ and he exploits the total payoff, one. In Universally Fair strategy, a proposer chooses the largest group $\{1, 2, 3\}$ and splits the total payoff equally among all players. In In-Group Fair strategy, a proposer chooses one of two other players randomly and splits the group value v equally with his partner. Like the two-person UG, traditional game theory predicts that rational and selfish players employ Selfish strategies.

When the strategy set S consists only of three strategies, $s_1 = \text{Selfish}$, $s_2 = \text{Universally Fair}$, and $s_3 = \text{In-Group Fair}$, the payoff matrices of a three-person UG with the three strategies are given in Table 1, depending on whether the two-person group value v is larger than the threshold $2/3$ or not. The numbers in the payoff matrices represent payoffs of the row-player (who selects a row). For example, suppose that all three players employ In-Group Fair strategies s_3 . Then, every player joins a two-person group with probability $2/3$, whereby he receives payoff $v/2$. Thus, the expected payoff for each player is $v/3$. Suppose now that one player, say 1, employs Universally Fair strategy s_2 . Then, player 1 offers payoff $1/3$ to players 2 and 3 with probability $1/3$, and he is offered payoff $v/2$ by them with probability $1/3$. If $0 < v/2 < 1/3$, i.e. $0 < v < 2/3$, then player 1's offer $1/3$ is accepted, and he rejects the offer $v/2$ from players 2 and 3. Thus, player 1 receives expected payoff $1/9$. If $2/3 < v < 1$, player 1's offer $1/3$ is rejected, and he accepts the offer $v/2$ from players 2 and 3. Thus, player 1 receives expected payoff $v/6$. In Table 1, Selfish and Universally Fair strategies are strict Nash equilibria⁴, independent of the two-person group value v . In-Group Fair strategy is a strict Nash equilibrium if $v > 1/3$.

³ In this strategy formulation, a responder's acceptance threshold q is not conditioned by the group size m .

⁴A Nash equilibrium is called strict if every player has a unique best response.

	s_1	s_2	s_3
s_1	1/3	1/9	$v/12$
s_2	1/9	2/9	1/9
s_3	$v/6$	$\frac{1}{9} + \frac{v}{12}$	$v/4$

	s_1	s_2	s_3
s_1	1/9	2/9	$\frac{1}{9} + \frac{v}{12}$
s_2	2/9	1/3	2/9
s_3	$\frac{1}{9} + \frac{v}{12}$	2/9	$\frac{1}{9} + \frac{v}{6}$

	s_1	s_2	s_3
s_1	$v/12$	$\frac{1}{9} + \frac{v}{12}$	$v/6$
s_2	1/9	2/9	1/9
s_3	$v/4$	$\frac{1}{9} + \frac{v}{6}$	$v/3$

	s_1	s_2	s_3
s_1	1/3	1/9	$v/12$
s_2	1/9	2/9	$v/12$
s_3	$v/6$	$v/6$	$v/4$

	s_1	s_2	s_3
s_1	1/9	2/9	$v/12$
s_2	2/9	1/3	$v/12$
s_3	$v/6$	$v/6$	$v/4$

	s_1	s_2	s_3
s_1	$v/12$	$v/12$	$v/6$
s_2	$v/12$	$v/12$	$v/6$
s_3	$v/4$	$v/4$	$v/3$

Table 1. Three left and right tables represent payoff matrices of the three-person UG when strategies are restricted to Selfish s_1 , Universally Fair s_2 and In-Group Fair s_3 in the cases of $0 < v < 2/3$ and $2/3 < v < 1$, respectively.

3 Results

We first consider the deterministic replicator dynamics for the evolution of strategies in an infinite population (6–8). The strategy set S consists of three strategies: Selfish, Universally Fair, and In-Group Fair. Each player in the population may be matched randomly with two other players, and they play a three-person UG. A player adopting strategy s_i receives payoff (or fitness) $A(s_i, s_j, s_k)$ when two other players adopt strategies s_j and s_k . Let $x = (x_j)_{j=1,2,3}$ be a strategy distribution in the population, where x_j is the proportion of strategy s_j . Each strategy s_i receives the average payoff $A(s_i, x) = \sum_{j,k=1,2,3} A(s_i, s_j, s_k)x_jx_k$. The average payoff of the population is given by $A(x) = \sum_{i=1,2,3} x_iA(s_i, x)$. The replicator dynamics is given by $\dot{x}_i = x_i(A(s_i, x) - A(x))$ for $i = 1, 2, 3$. The growth rate of each strategy s_i is equal to its average payoff $A(s_i, x)$ minus the average payoff $A(x)$ of the population. The replicator dynamics describes biological and cultural reproduction process where a more successful strategy than the average payoff in the population reproduces more offspring.

We present the dynamic flow of the replicator dynamics for a three-person UG where only Selfish, Universally Fair and In-Group Fair exist in the population. Figure 1 illustrates the evolution of the three strategies in two cases of $\nu = 1/6, 5/6$.⁵ Selfish and Universally Fair strategies are asymptotically stable in both cases, but In-Group Fair is so only when $\nu = 5/6$. It can be verified that Selfish and Universally Fair strategies are asymptotically stable for every value of $\nu(0 < \nu < 1)$, and that In-Group Fair strategy is so only when $\nu > 1/3$ (see Appendix 1).

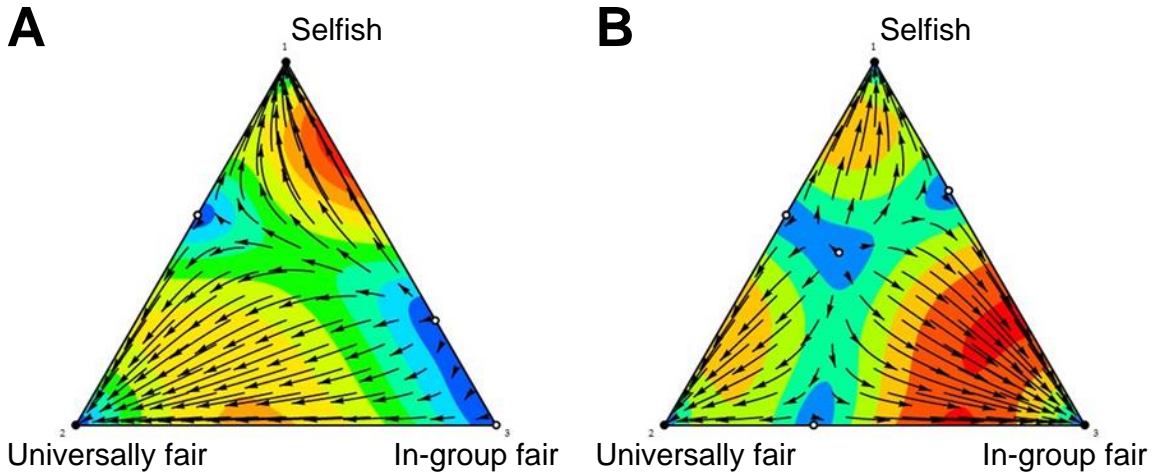


Fig. 1. The phase diagram of the replicator dynamics for a three-person UG when the two-person group value is $\nu = 1/6$ (A) and $\nu = 5/6$ (B). The value of the three-person group is one. The population consists of three strategies, Selfish, Universally Fair, and In-group Fair. A black circle \bullet and a white circle \circ represent the asymptotically stable rest state and the asymptotically unstable rest state, respectively. In (A), Selfish and Universally Fair are asymptotically stable and in (B), all three strategies are asymptotically stable.

We next examine the robustness of evolutionary stability in Figure 1 when the strategy set S is expanded. A strategy (m, p, q) is called consistent if $p = q$. In words, a player offers to group members the same amount of payoffs as he demands when he is a responder. Selfish, Universally Fair, and In-Group Fair strategies are all consistent. When a strategy set S composed of consistent strategies is expanded, the following results hold: (1) Selfish strategy is asymptotically stable for every two-person group value ν ($0 < \nu < 1$). (2) Universally Fair

⁵ Figure 1 is drawn by the software in (23).

strategy is asymptotically stable if and only if $0 < v < 2/3$. (3) In-Group Fair strategy is asymptotically stable if and only if $2/3 < v < 1$ (see Appendix 1). In the two-person UG, it is well-known that the rational strategy with selfish motivations is asymptotically stable (7, 8, 17, 20). The first result extends this to a three-person case. The last two results are new findings in the multi-person UG. They show that there exists a threshold value of a two-person group that determines the evolutionary stability of Universally Fair and In-Group Fair strategies. Universally Fair strategy is asymptotically stable if the two-person group value is less than the threshold value, i.e. $v < 2/3$. Otherwise ($v > 2/3$), In-Group Fair strategy is asymptotically stable (see Appendix 1).

When there exist only Selfish, Universally Fair, and In-Group Fair strategies in a population, Universally Fair strategy is always asymptotically stable. It, however, is fragile against a mutant when the group value v of two persons is high. The instability of Universally Fair strategy for $v > 2/3$ can be explained as follows. Consider the fourth strategy $s_4 = (2, 1/3, 1/3)$, called Strategic Fair, where a player chooses randomly one of two other players, offers $1/3$ to his partner, and demands the same amount of payoffs as a responder. In Strategically Fair strategy, a proposer exploits the residual payoff $v - 1/3$ by forming a two-person group. If Strategic Fair strategy appears in the population of Universally Fair strategy, then it receives an average payoff of $1/3(v - 1/3) + 2/3(1/3)$, which is strictly higher than the average payoff $1/3$ of the population if $v > 2/3$. Thus, Strategically Fair strategy can invade the population. As a result, Universally Fair strategy is not asymptotically stable when $v > 2/3$. On the other hand, when $v < 2/3$, Universally Fair strategy is asymptotically stable by the following reason. When a mutant $s = (3, p, p)$ ($p \neq 1/3$) appears in the population of Universally Fair strategies, the average payoff of the mutant is less than the average payoff $1/3$ of the population. Thus, the mutant does not survive. When a mutant $t = (2, q, q)$ appears in the population, the average payoff of the mutant is less than or equal to $1/3(v - 1/3) + 2/9$ (the equality holds if $q = 1/3$), which is less than the average payoff $1/3$ of the population when $v < 2/3$. Thus, the mutant

does not survive in this case, either.

When $v > 2/3$, In-Group Fair strategy is asymptotically stable. If a mutant $s = (3, p, p)$ appears in the population of In-Group Fair strategies, then the average payoff of the mutant is $v/6$ for $0 < p < v/2$, $(2 - v)/6$ for $p = v/2$, and $1/3(1 - 2p)$ for $v/2 < p < 1/2$, which is less than the average payoff $v/3$ of the population for every p if $v > 2/3$. Thus, the mutant does not survive. If another mutant $t = (2, q, q)$ ($q \neq v/2$) appears in the population, then the average payoff of the mutant is less than the average payoff $v/3$ in the population. Thus, In-Group Fair strategy is asymptotically stable if $v > 2/3$ (for details, see Appendix 1).

So far, the analysis has been restricted to the evolutionary stability of three strategies, Selfish, Universally Fair, and In-Group Fair in the population of consistent strategies. How is the evolutionary stability of these strategies changed if the population includes inconsistent strategies? It can be shown that none of Selfish, Universally Fair, or In-Group Fair is asymptotically stable. For example, consider the population where all players employ Selfish strategies $s = (3, 0, 0)$. Suppose that a mutant $t = (3, 0, 1)$ invades. Since this mutant rejects any offer, it receives the same average fitness $1/3$ as the rest of the population. Random drift determines the evolution of the population, and the Selfish strategy is not asymptotically stable. Thus, the deterministic replicator dynamics is not suitable to study the evolutionary stability of group formation and fairness when the strategy set includes inconsistent strategies. By this reason, we consider the Moran process (24, 25) as a stochastic evolutionary model.

Now, we consider a three-person UG with inconsistent strategies. We first characterize all Nash equilibria when the strategy set S contains all consistent and inconsistent strategies.⁶ The following results hold (see Appendix 1). Let $0 \leq p \leq \frac{1}{2}$ and $0 \leq q \leq v$. (1) Every consistent strategy $(3, p, p)$ is a Nash equilibrium if and only if $p \leq 1 - v$. (2) Every consistent strategy $(2, p, p)$ is a Nash equilibrium if and only if $1 - v \leq p$. (3) Inconsistent strategies $(3, p, q)$ and

⁶The strategy set S is given by the union of two rectangles, $\{(3, p, q) \mid 0 \leq p \leq \frac{1}{2} \text{ and } 0 \leq q \leq 1\}$ and $\{(2, p, q) \mid 0 \leq p \leq v \text{ and } 0 \leq q \leq 1\}$.

$(2, 0, q)$ are Nash equilibria if $q \geq \max\{1/2, v\}$.⁷

An intuition of the result is as follows. Consider the population where all players employ strategy $s = (3, p, p)$. The average fitness of the population is $(1 - 2p)/3 + 2p/3 = 1/3$. Assume that a mutant $t = (2, p, p)$ appears in the population. The average fitness of mutant t is $(v - p)/3 + 2p/3 = (v + p)/3$. In order for s to be a Nash equilibrium, it must hold that $1/3 \geq (v + p)/3$, i.e. $p \leq 1 - v$. By a similar reason, every consistent strategy $(2, p, p)$ can be a Nash equilibrium if and only if $1 - v \leq p$.

Figure 2 illustrates the range of Nash equilibria for each value v of a two-person group. The set of Nash equilibria is a continuum one. When inconsistent strategies are feasible, every Nash equilibrium is not strict since there exists alternative best reply to other players' strategies. The deterministic model in the classical evolutionary game theory does not give us a further insight to which strategy is observed more frequently.

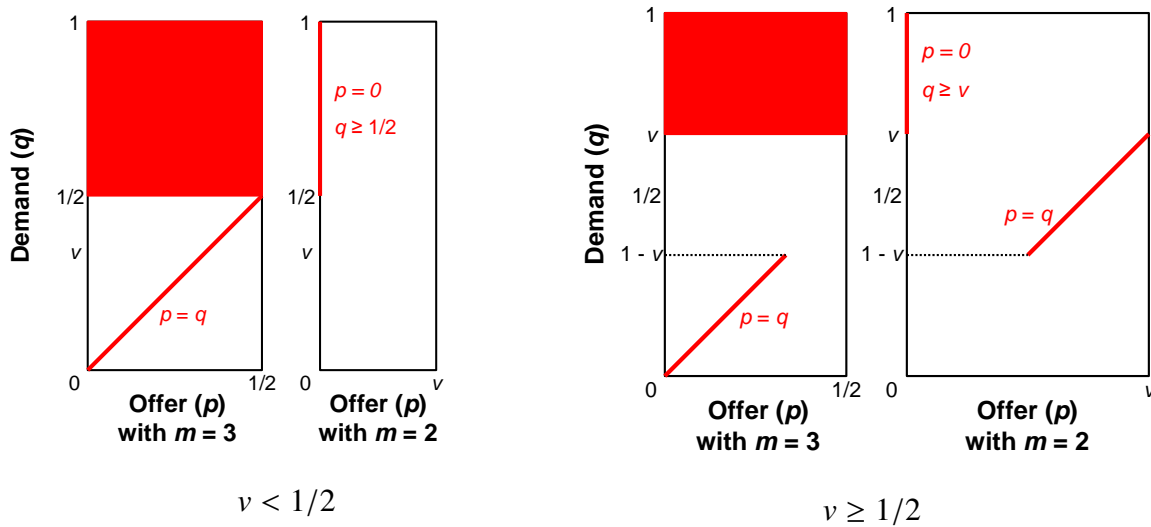


Fig. 2. The set of Nash equilibria of the three-person UG when all consistent and inconsistent strategies are feasible. Two left and right rectangles represent the strategy set for $v < 1/2$ and $v \geq 1/2$, respectively. Each point on red lines and red rectangles represents a Nash equilibrium.

⁷In these strategies, all responders are greedy. They demand excessively, i.e. more than $1/2$ in the three-person group, and v in a two-person group. All offers are rejected, and thus all players receive zero payoffs. In these Nash equilibria, the average fitness of the population is zero. Any mutant has the same average fitness if it invades into the population. These Nash equilibria are not evolutionary stable. Their existence is caused by an asymmetric property of a feasible strategy where an offer is subject to a group value, but a demand is not.

Stochastic evolutionary game theory (26, 27) offers us an effective tool to study the long-run frequency of strategies in finite populations in which randomness plays a key role in the course of evolution. Human behavior is affected by randomness in many ways. Players may make errors in social learning due to their bounded rationality. They may make incorrect estimations about their own payoffs. Payoffs are subject to random fluctuations. We consider the frequency-dependent Moran process (24, 28, 29) in a population of size $N < \infty$. Each player i plays a three-person UG with each pair of two players from the other $N - 1$ players, and receives an average payoff π_i . Player i 's effective payoff is defined as $\exp w\pi_i$, where w is called the intensity of selection. In the Moran process, one player of each generation is randomly selected to change strategy (death and birth). Let $u > 0$ be the mutation rate. With probability u , a mutation occurs and a random strategy is employed. With probability $1 - u$, another strategy is selected proportional to the effective payoffs to be imitated. When the selection intensity w goes to infinity, only the highest payoff strategy is imitated (strong selection). When w goes to zero, a random strategy is imitated so that natural drift determines the evolution (weak selection).

We report agent-based simulation results⁸ with parameters $N = 50$, $w = 10^{-0.5}, 10^2$, $u = 10^{-3}$, and $v = 1/6, 5/6$ where N is the number of players, w the intensity of selection, u a mutation rate, and v a two-person group value. Figure 3 shows the frequencies of strategies averaged over 10^8 generations for different values of parameters v and w . The strategy set is colored according to the frequency of each strategy. Red color indicates high frequency, and yellow color does low frequency. The most abundant strategy is indicated with a black \times . It is $(3, 5/12, 1/3)$ for $v = 1/6$ and $w = 10^{-0.5}$, $(2, 1/4, 1/4)$ for $v = 5/6$ and $w = 10^{-0.5}$, and $(3, 1/6, 1/6)$ for $v = 1/6, 5/6$ and $w = 10^2$. Strong selection ($w = 10^2$) drives the population to a consistent strategy $(3, 1/6, 1/6)$, close to the Selfish strategy. On the other hand, when selection is weak ($w = 10^{-0.5}$), the most abundant strategy is away from the Selfish strategy.

⁸Theoretical results on the most abundant strategy are given in Appendix 2.

The outcome depends critically on the two-person group value. When the group value is low ($v = 1/6$), the most abundant strategy chooses the three-person group and is closest to the Universally Fair strategy; a proposer offers payoff $5/12$ and responders accept offers of more than $1/3$. When the group value is high ($v = 5/6$), the most abundant strategy chooses a two-person group; a proposer offers payoff $1/4$ and receives $1/4$ for himself.

We also analyze theoretically the most abundant strategy in the low mutation limit in which u converges to 0 (Appendix 2). It can be proved that the most abundant strategy is consistent ($p = q$). Figure 4 shows how it varies with changes in the two-person group value v . For $0 < v \leq 0.7$, the most abundant strategy chooses the three-person group. The offer p value is not monotone with respect to v , and it ranges approximately from 0.2 to 0.23. For $0.75 < v < 1$, the most abundant strategy chooses a two-person group. The offer p value is monotonically increasing in v , and it ranges approximately from 0.25 to 0.32.

From numerical simulations and theoretical investigations of the Moran process for the three-person UG, we obtain the following observations: (i) Unlike the results of the deterministic replicator dynamics for the two-person UG (7, 8, 17, 20), the most abundant strategy produces a fair allocation that gives a positive payoff to responders, and (ii) unlike the result of the Moran process for the two-person UG (21), the most abundant strategy produces an economically inefficient and in-group favorable allocation with the exclusion of non-members.

4 Discussion

We discuss whether the stochastic evolutionary model can explain experimental observations of human behavior in a three-person UG (22).⁹ In the experiment, the value of the three-person group is fixed to be 3000 points. The value of a two-person group varies over 2800, 2500, 2100,

⁹ The experiment was conducted at the CREED laboratory at the Faculty of Economics and Econometrics of the University of Amsterdam in March 2003. Subjects played eight rounds of a three-person UG with random matching in each round with a fixed parameter for the two-person group value.

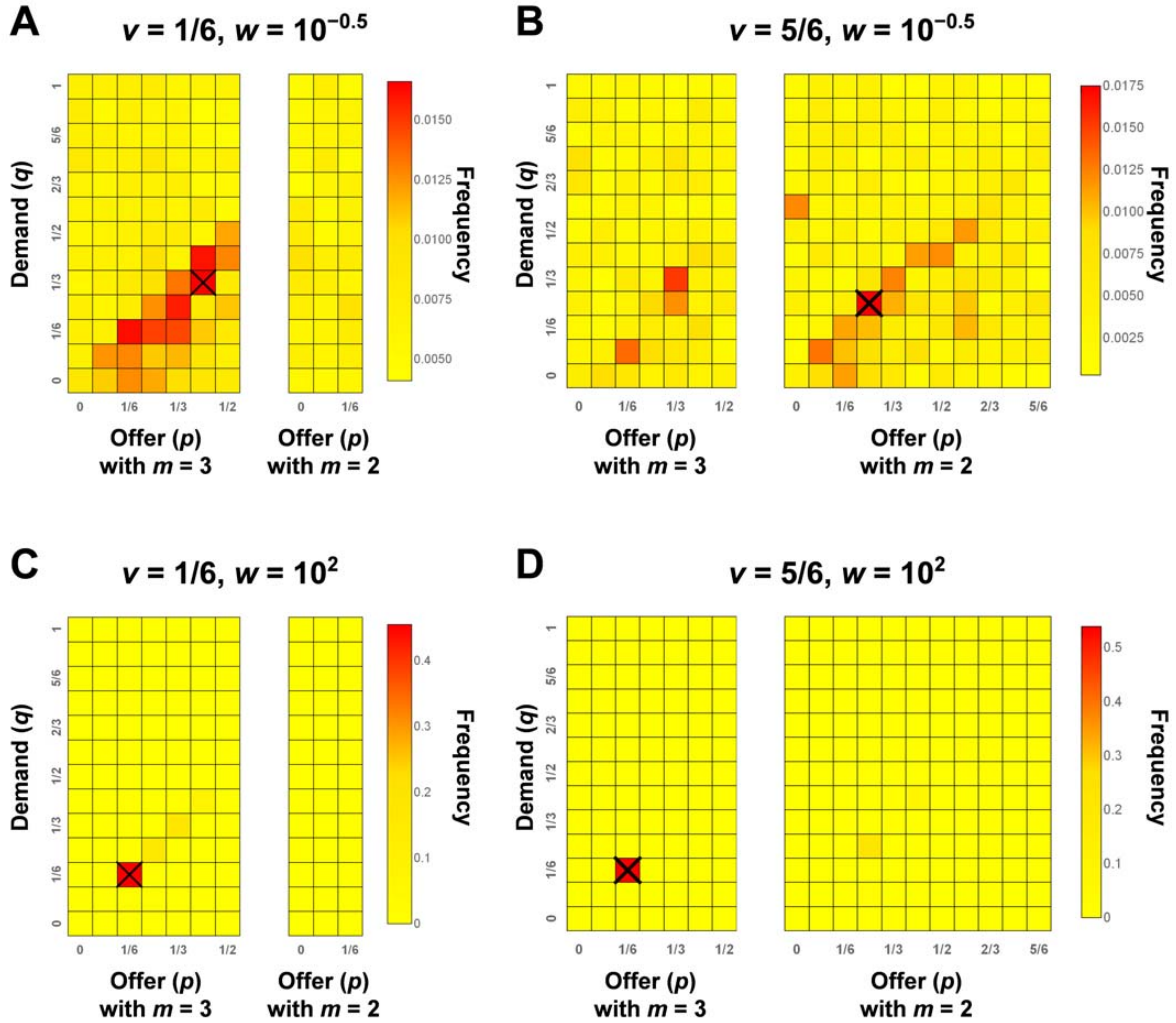


Fig. 3. Simulation results of the frequency-dependent Moran process where the number of players is $N = 50$ and the mutation rate is $u = 10^3$. Parameters of the two-person group value (v) and the selection intensity (w) are: (A) $v = 1/6, w = 10^{-0.5}$; (B) $v = 5/6, w = 10^{-0.5}$; (C) $v = 1/6, w = 10^2$; and (D) $v = 5/6, w = 10^2$. The strategy parameters, p (offer) and q (acceptance threshold), are discretized in increments of $1/12$. The strategy set is colored according to the average frequency of each strategy over 10^8 generations. The most abundant strategy indicated with a black \times is $(3, 5/12, 1/3)$ in (A), $(2, 1/4, 1/4)$ in (B), and $(3, 1/6, 1/6)$ in (C) and (D). In the case of weak selection ($w = 10^{-0.5}$), the most abundant strategy chooses a two-person group when the group value (v) is high. In the case of strong selection ($w = 10^2$), the most abundant strategy is close to the Selfish strategy in choosing the three-person group.

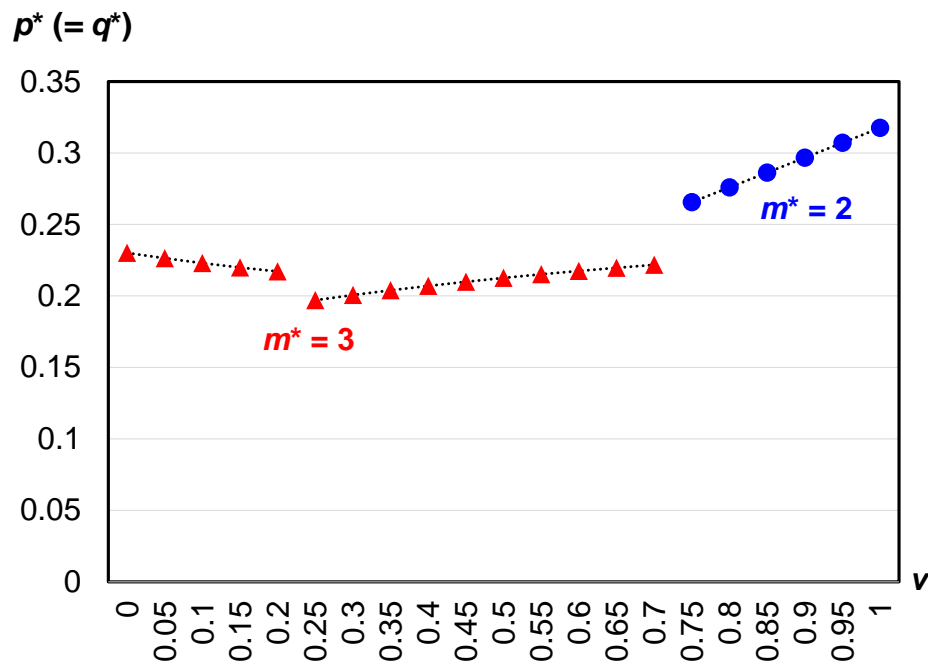


Fig. 4. Numerical results of the group size chosen by the most abundant strategy (m, p, q) in the Moran process in the limit where the mutation rate (u) converges to zero for each two-person group value (v) in increments of 0.05. For $0 < v \leq 0.7$, the most abundant strategy chooses the three-person group and the offer value p ranges approximately from 0.2 to 0.23. For $0.75 < v < 1$, it chooses a two-person group and the offer ranges from 0.25 to 0.32.

and 1200 points.¹⁰ Normalizing 3000 to be one, these parameters correspond to four cases that a two-person group value v is 0.93, 0.83, 0.7 and 0.4, respectively. Human subjects played eight rounds of a three-person UG with random matching in each round with a fixed parameter for the two-person group value v .

Experimental observations are as follows:¹¹ (1) When $v = 0.93$, 82.5 percent of proposers choose two-person groups. In a two-person group, an average offer is 30.9, and any offer above 37.5 percent is accepted for sure. (2) When $v = 0.83$, 90.0 percent of proposers choose two-person groups. In a two-person group, an average offer is 32.7 percent, and any offer above 38 percent is accepted for sure. (3) When $v = 0.7$, 67.5 percent of proposers choose a three-person group. In the group, an average offer is 23.0 percent, and any offer above 33.3 percent is accepted for almost surely. (4) When $v = 0.4$, almost all proposers (97 percent) choose the three-person group. Proposers treat two group members equally. In the three-person group, the average offer is 25.0 percent, and any offer above 33.3 percent is accepted almost surely. (5) Like a large number of two-person UG experiments, the extreme allocation implemented by the rational strategy with selfish motivations is not observed in the three-person UG experiment.

The theoretical results of the stochastic evolutionary model in Figure 4 are surprisingly consistent with the experimental observations. Regarding group size, the most abundant strategy in the Moran process predicts the formation of a two-person group in the cases of $v = 0.93, 0.83$. In the experiment, more than 80 percent of proposers choose a two-person group. In the case of $v = 0.7$, which is the theoretical threshold for the group size, both the three-person group and a two-person group are chosen by more than 30 percent of proposers in the experiment. In the case of $v = 0.4$, the most abundant strategy predicts the formation of the three-person group, which is consistent with experimental observations. Regarding

¹⁰The exchange rate from points to EURO was 250 points = 1 EURO. Subjects' average earning was about 17 EURO (including a show-up fee of 5 EURO).

¹¹ The frequencies of two-person groups except for $v = 0.4$ are the data in the first round. They do not decrease over time.

the offer, the most abundant strategy in the Moran process is close to the average offer in the experiment: 0.303 (theory) vs. 0.309 (experiment) for $\nu = 0.93$; 0.282 vs. 0.327 for $\nu = 0.83$; 0.222 vs. 0.23 for $\nu = 0.7$; and 0.207 vs. 0.250 for $\nu = 0.4$.¹²

5 Conclusion

In this paper, we have investigated the evolution of group formation in the three-person UG, focusing on distributional efficiency and fairness. Although the deterministic replicator dynamics shows that the magnitude of a two-person group value is critical in the evolution of group formation, its result is indeterminate in that the set of Nash equilibria is a continuum one. The frequency-dependent Moran process as a stochastic evolutionary model shows that natural selection favors a two-person group with an in-group fair allocation if its group value is high. The formation of a subgroup leads to economic inefficiency and social exclusion. The result of the stochastic evolutionary game theory fits with experimental evidences on group formation in the three-person UG surprisingly well.

¹² The offers by the most abundant strategy are calculated in Appendix 2. The observed offers are reported in (22).

Appendix

1 Replicator Dynamics

Consider an infinite population involving only three strategies, $s_1 = \text{Selfish}$, $s_2 = \text{Universally Fair}$, and $s_3 = \text{In-Group Fair}$. Let x_i denote a proportion of players who adopt strategy s_i . A state of the population is represented by $x = (x_1, x_2, x_3)$. A state where all players employ Selfish strategies is represented by $x = (1, 0, 0)$. For simplicity, we call the state $x = (1, 0, 0)$ Selfish. Similarly, we call the state $x = (0, 1, 0)$ Universally Fair, and the state $x = (0, 0, 1)$ In-Group Fair.

The replicator dynamics is given by

$$\dot{x}_i = x_i(A(s_i, x) - A(x)), \quad (1)$$

for $i = 1, 2, 3$, where $A(s_i, x) = \sum_{j,k=1,2,3} A(s_i, s_j, s_k)x_jx_k$ is the average fitness of strategy s_i , and $A(x) = \sum_{i=1,2,3} x_iA(s_i, x)$ is that of the population. From the payoff matrices, the functions $A(s_i, x)$ ($i = 1, 2, 3$) and $A(x)$ are given as follows: For $v \in (0, 2/3)$,

$$\begin{aligned} A(s_1, x) &= \frac{1}{3}x_1^2 + \frac{2}{9}x_2^2 + \frac{v}{6}x_3^2 + \frac{2}{9}x_1x_2 + \frac{v}{6}x_1x_3 + \left(\frac{2}{9} + \frac{v}{6}\right)x_2x_3 \\ A(s_2, x) &= \frac{1}{9}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{9}x_3^2 + \frac{4}{9}x_1x_2 + \frac{2}{9}x_1x_3 + \frac{4}{9}x_2x_3 \\ A(s_3, x) &= \frac{v}{6}x_1^2 + \frac{2}{9}x_2^2 + \frac{v}{3}x_3^2 + \left(\frac{2}{9} + \frac{v}{6}\right)x_1x_2 + \frac{v}{2}x_1x_3 + \left(\frac{2}{9} + \frac{v}{3}\right)x_2x_3 \\ A(x) &= \frac{1}{3}x_1^3 + \frac{1}{3}x_2^3 + \frac{v}{3}x_3^3 + \frac{1}{3}x_1^2x_2 + \frac{v}{3}x_1^2x_3 + \frac{2}{3}x_1x_2^2 + \frac{2v}{3}x_1x_3^2 \\ &\quad + \frac{2}{3}x_2^2x_3 + \left(\frac{1}{3} + \frac{v}{3}\right)x_2x_3^2 + \left(\frac{2}{3} + \frac{v}{3}\right)x_1x_2x_3, \end{aligned}$$

and for $v \in (2/3, 1)$,

$$A(s_1, x) = \frac{1}{3}x_1^2 + \frac{2}{9}x_2^2 + \frac{v}{6}x_3^2 + \frac{2}{9}x_1x_2 + \frac{v}{6}x_1x_3 + \frac{v}{6}x_2x_3$$

$$A(s_2, x) = \frac{1}{9}x_1^2 + \frac{1}{3}x_2^2 + \frac{v}{6}x_3^2 + \frac{4}{9}x_1x_2 + \frac{v}{6}x_1x_3 + \frac{v}{6}x_2x_3$$

$$A(s_3, x) = \frac{v}{6}x_1^2 + \frac{v}{6}x_2^2 + \frac{v}{3}x_3^2 + \frac{v}{3}x_1x_2 + \frac{v}{2}x_1x_3 + \frac{v}{2}x_2x_3$$

$$A(x) = \frac{1}{3}x_1^3 + \frac{1}{3}x_2^3 + \frac{v}{3}x_3^3 + \frac{1}{3}x_1^2x_2 + \frac{v}{3}x_1^2x_3 + \frac{2}{3}x_1x_2^2 + \frac{2v}{3}x_1x_3^2 + \frac{v}{3}x_2^2x_3 + \frac{2v}{3}x_2x_3^2 + \frac{2v}{3}x_1x_2x_3.$$

The following notions are standard in the theory of dynamic systems.

Definition 1.1.

- A state \bar{x} is a *rest point* if it satisfies $\dot{x}_i = 0$ for every i .
- A state x^* is a *Nash equilibrium point* if (i) $A(s_i, x^*) = A(s_j, x^*)$ for any pair i, j with $x_i, x_j > 0$, and (ii) $A(s_i, x^*) \geq A(s_j, x^*)$ for any pair i, j with $x_i > 0$ and $x_j = 0$. A state x^* is a *strict Nash equilibrium point* if $A(s_i, x^*) > A(s_j, x^*)$ for any i with $x_i > 0$ and any $j \neq i$.
- A state x^* is an *asymptotically stable point* if the following two conditions hold:
 - (i) (Liapunov stability) Given any neighborhood U_1 of x^* , there exists some neighborhood U_2 of x^* such that for any path $x = x(t)$, $x(0) \in U_2$ implies $x(t) \in U_1$ for all $t > 0$.
 - (ii) There exists some neighborhood V of x^* such that for any path $x = x(t)$, $x(0) \in V$ implies $\lim_{t \rightarrow \infty} x(t) = x^*$.

Table A1 summarizes all rest points of the replicator dynamics (1) with Selfish, Universally Fair and In-Group Fair strategies. In Table A1, $\hat{v} \in (1/3, 2/3)$ satisfies $1 - \frac{1}{3v} - \frac{3v + \sqrt{9v^2 + 96}}{48} = 0$. Table A2 summarizes all Nash equilibrium points.

	Rest points $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$
$v \in (0, 1/3)$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}})$
$v \in [1/3, \hat{v})$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}}),$ $(0, 1 - \frac{1}{3v}, \frac{1}{3v})$
$v \in [\hat{v}, 2/3)$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}}),$ $(0, 1 - \frac{1}{3v}, \frac{1}{3v}), (\frac{3v + \sqrt{9v^2 + 96}}{24}, 1 - \frac{1}{3v} - \frac{3v + \sqrt{9v^2 + 96}}{48}, \frac{1}{3v} - \frac{3v + \sqrt{9v^2 + 96}}{48})$
$v \in (2/3, 1)$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}}),$ $(0, \sqrt{\frac{v}{2}}, 1 - \sqrt{\frac{v}{2}}), (\sqrt{\frac{(9+2\sqrt{3})v}{46}}, \sqrt{\frac{(12-5\sqrt{3})v}{23}}, 1 - \sqrt{\frac{(9+2\sqrt{3})v}{46}} - \sqrt{\frac{(12-5\sqrt{3})v}{23}})$

Table. A1. The list of rest points in the replicator dynamics with Selfish, Universally Fair and In-Group Fair strategies.

	Nash equilibrium points (x_1^*, x_2^*, x_3^*)
$v \in (0, 1/3)$	$(1, 0, 0), (0, 1, 0), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0)$
$v \in [1/3, \hat{v})$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0),$ $(0, 1 - \frac{1}{3v}, \frac{1}{3v})$
$v \in [\hat{v}, 2/3)$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}}),$ $(0, 1 - \frac{1}{3v}, \frac{1}{3v}), (\frac{3v + \sqrt{9v^2 + 96}}{24}, 1 - \frac{1}{3v} - \frac{3v + \sqrt{9v^2 + 96}}{48}, \frac{1}{3v} - \frac{3v + \sqrt{9v^2 + 96}}{48})$
$v \in (2/3, 1)$	$(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0), (\sqrt{\frac{v}{2}}, 0, 1 - \sqrt{\frac{v}{2}}),$ $(0, \sqrt{\frac{v}{2}}, 1 - \sqrt{\frac{v}{2}}), (\sqrt{\frac{(9+2\sqrt{3})v}{46}}, \sqrt{\frac{(12-5\sqrt{3})v}{23}}, 1 - \sqrt{\frac{(9+2\sqrt{3})v}{46}} - \sqrt{\frac{(12-5\sqrt{3})v}{23}})$

Table. A2. The list of Nash equilibrium points in the replicator dynamics with Selfish, Universally Fair, and In-Group Fair strategies.

The following theorem characterizes asymptotically stable points, depending on the two-person group value v .

Theorem 1.1. *The replicator dynamics (1) has the following asymptotically stable points.*

- (1) *When $v \in (0, 1/3]$, Selfish and Universally Fair states are asymptotically stable.*
- (2) *When $v \in (1/3, 1)$ (except $v = 2/3$), Selfish, Universally Fair, and In-Group Fair states are asymptotically stable.*

Proof. The theorem can be proved from the well-known result that a strict Nash equilibrium point is an asymptotically stable point, which is a Nash equilibrium point (7). If every eigenvalue of the Jacobian matrix $(\frac{\partial g}{\partial x}(x^*)_{ij})_{i,j=1,2,3}$ of functions $g_i(x) = x_i(A(s_i, x) - A(x))$ at a rest point x^* has a negative real-part, then x^* is asymptotically stable (see e.g. (30)). Also, if the Jacobian matrix at x^* has at least one eigenvalue whose real-part is positive, then x^* is not stable, thus not asymptotically stable.

Case 1. $v \in (0, 1/3]$. It is easy to see from Table 1 that two states $(1, 0, 0)$, $(0, 1, 0)$ are strict Nash equilibrium points. We examine asymptotic stability of other Nash equilibrium points. Consider the Nash equilibrium point $x = (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0)$. The eigenvalues of the Jacobian matrix of g at x are $-\frac{1}{27}(9 - 2\sqrt{3})$, $-\frac{1}{18}(2 - \sqrt{3}v)$, and $\frac{2}{27}(3 - \sqrt{3})$. Since the Jacobian matrix has a positive eigenvalue $\frac{2}{27}(3 - \sqrt{3})$, x is not asymptotically stable. Similarly, for every other Nash equilibrium point x^* and every $v \in (0, 1/3]$, it can be shown that the Jacobian matrix of x^* has at least one positive eigenvalue. Thus, asymptotically stable points are only $(1, 0, 0)$ and $(0, 1, 0)$ for $v \in (0, 1/3]$.

Case 2. $v \in (1/3, 2/3)$. Similarly to Case 1, two states $(1, 0, 0)$, $(0, 1, 0)$ are strict Nash equilibrium points. Consider the Nash equilibrium state $x = (0, 0, 1)$. When $v \in (1/3, 2/3)$, the eigenvalues of the Jacobian matrix of g at x are $-\frac{v}{3}$, $-\frac{v}{6}$, and $-\frac{v}{6}$. Since all eigenvalues are negative, x is asymptotically stable for $v \in (1/3, 2/3)$.

Next, consider the Nash equilibrium point $y = (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0)$. For every $v \in (1/3, 2/3)$, the eigenvalues of the Jacobian matrix of g at y are $-\frac{1}{27}(9 - 2\sqrt{3})$, $-\frac{1}{18}(2 - \sqrt{3}v)$, and $\frac{2}{27}(3 - \sqrt{3})$. Since the Jacobian matrix has a positive eigenvalue $\frac{2}{27}(3 - \sqrt{3})$, y is not asymptotically stable for $v \in (1/3, 2/3)$. Similarly, it can be shown that every other Nash equilibrium point has at least one positive eigenvalue for every $v \in (1/3, 2/3)$. Hence, only three states $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ are asymptotically stable.

Case 3. $v \in (2/3, 1)$. It can be seen from Table 1 that three states $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ are strict Nash equilibrium points, and thus asymptotically stable. Next, consider the Nash equilibrium point $y = (\frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0)$. For every $v \in (2/3, 1)$, the same calculation as in Case 2 shows that the eigenvalues of the Jacobian matrix of g at y are $-\frac{1}{27}(9 - 2\sqrt{3})$, $-\frac{1}{54}(18 + 4\sqrt{3} + 9v)$, and $\frac{2}{27}(3 - \sqrt{3})$. Since the Jacobian matrix has a positive eigenvalue $\frac{2}{27}(3 - \sqrt{3})$, y is not asymptotically stable. Similarly, it can be shown that every other Nash equilibrium point has at least one positive eigenvalue for every $v \in (2/3, 1)$. Hence, only three states $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ are asymptotically stable. \square

In the following, we expand the strategy set S . We consider the case that the strategy set is composed by any finite number of consistent strategies including Selfish, Universally Fair and In-Group Fair strategies. We also assume that the strategy set contains consistent strategies $(2, \frac{1}{3}, \frac{1}{3})$ and $(3, \frac{v}{2}, \frac{v}{2})$.

Theorem 1.2. *When all strategies are consistent, the following hold:*

- (1) *Selfish state is asymptotically stable for any $v \in (0, 1)$.*
- (2) *Universally Fair state is asymptotically stable if and only if $v \in (0, 2/3)$.*
- (3) *In-Group Fair state is asymptotically stable if and only if $v \in (2/3, 1)$.*

Proof. (1): Consider Selfish state x . The average fitness of the population at x is given by $A(x) = \frac{1}{3} \times 1 + \frac{2}{3} \times 0 = \frac{1}{3}$. Suppose that a mutant $s = (3, p, p)$ with $p > 0$ invades into the

population. The average fitness of s at x is $A(s, x) = \frac{1}{3} \times (1 - 2p) + \frac{2}{3} \times 0$, which is lower than $A(x)$ since $p > 0$. Thus, mutant s cannot survive at x . Next, consider a mutant $s = (2, p, p)$. The average fitness of s at x is $A(s, x) = \frac{1}{3} \times (v - p) + \frac{2}{3} \times 0$, which is lower than $A(x)$ since $v < 1$. Again, mutant s cannot survive at x . By these arguments, Selfish state x is a strict Nash equilibrium point.

(2): Consider Universally Fair state y . The average fitness of the population at y is given by $A(y) = \frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{1}{3}$. Assume that $v < \frac{2}{3}$. Suppose that a mutant $s = (3, p, p)$ with $p \neq 1/3$ invades into the population. The average fitness $A(s, y)$ of s at y is lower than $A(y)$ by $p \neq 1/3$. Thus, mutant s cannot survive at y . Next, consider a mutant $s = (2, p, p)$. The average fitness $A(s, y)$ of s at y is lower than or equal to $\frac{1}{3} \times (v - \frac{1}{3}) + \frac{2}{3} \times \frac{1}{3}$. Since $v < \frac{2}{3}$, it holds that $A(s, y) < \frac{1}{3} = A(y)$. Again, mutant s cannot survive at y . By these arguments, Universally Fair state y is a strict Nash equilibrium point if $v < \frac{2}{3}$.

Assume that $v > \frac{2}{3}$. Suppose that a mutant $s = (2, \frac{1}{3}, \frac{1}{3})$ invades into the population at y . The average fitness $A(s, y)$ of s at y is equal to $\frac{1}{3} \times (v - \frac{1}{3}) + \frac{2}{3} \times \frac{1}{3}$, which is higher than $A(y) = \frac{1}{3}$ since $v > \frac{2}{3}$. Thus, mutant s can invade at y . In other words, Universally Fair state y is not a Nash equilibrium point, and thus not asymptotically stable, if $v > \frac{2}{3}$.

Assume that $v = \frac{2}{3}$. For any sufficiently small $\epsilon > 0$, take a state y^ϵ where the proportion of Universally Fair strategy s_2 is $1 - \epsilon$ and that of strategy $s = (2, \frac{1}{3}, \frac{1}{3})$ is ϵ . The average fitness of s_2 at y^ϵ is equal to $\frac{1}{3}(1 - \epsilon)^2 + \frac{5}{18} \times 2(1 - \epsilon)\epsilon + \frac{2}{9}\epsilon^2 = \frac{1}{3} - \frac{1}{9}\epsilon$. Similarly, it can be shown that the average fitness of s at y^ϵ is the same as that of s_2 . Since for any $\epsilon > 0$, the trajectory starting from y^ϵ does not converge to state s_2 , y is not asymptotically stable.

(3): Consider In-Group Fair state z . The average fitness of the population at z is given by $A(z) = \frac{1}{3} \times \frac{v}{2} + \frac{1}{3} \times \frac{v}{2} = \frac{v}{3}$. Assume that $v > \frac{2}{3}$. Suppose that a mutant $s = (3, p, p)$ invades into the population. The average fitness $A(s, z)$ of s at z is lower than or equal to $\frac{1}{3} \times (1 - v) + \frac{1}{3} \times \frac{v}{2}$, which is lower than $A(z) = \frac{v}{3}$ by $v > \frac{2}{3}$. Thus, mutant s cannot survive at z . Next, consider a mutant $s = (2, p, p)$ with $p \neq v/2$. The average fitness $A(s, z)$ of s at z is

lower than $\frac{1}{3} \times \frac{v}{2} + \frac{1}{3} \times \frac{v}{2} = \frac{v}{3}$ since a proposer employing s receives either payoff $v - p < \frac{v}{2}$ or zero. Again, mutant s cannot survive at z . By these arguments, In-Group Fair state z is a strict Nash equilibrium point if $v > \frac{2}{3}$.

Assume that $v < \frac{2}{3}$. Suppose that a mutant $s = (3, \frac{v}{2}, \frac{v}{2})$ invades into the population at z . The average fitness $A(s, z)$ of s at z is equal to $\frac{1}{3} \times (1 - v) + \frac{1}{3} \times \frac{v}{2}$, which is higher than $A(z) = \frac{v}{3}$ since $v < \frac{2}{3}$. Thus, mutant s can invade at z . In other words, In-Group Fair state z is not a Nash equilibrium point, and thus not asymptotically stable, if $v < \frac{2}{3}$.

Assume that $v = \frac{2}{3}$. For any sufficiently small $\epsilon > 0$, take a state z^ϵ where the proportion of In-Group Fair strategy s_3 is $1 - \epsilon$ and that of strategy $s = (3, \frac{v}{2}, \frac{v}{2})$ is ϵ . The average fitness of s_3 at z^ϵ is equal to $\frac{v}{3}(1 - \epsilon)^2 + \frac{5v}{12} \times 2(1 - \epsilon)\epsilon + \frac{v}{2}\epsilon^2 = \frac{2}{9} + \frac{1}{9}\epsilon$. Similarly, it can be shown that the average fitness of s at z^ϵ is the same as that of s_3 . Since for any $\epsilon > 0$, the trajectory starting from z^ϵ does not converge to state s_3 , z is not asymptotically stable. \square

Finally, we consider a general case that the strategy set S includes inconsistent strategies. In this case, Theorem 1.2 does not hold. None of Selfish, Universally Fair, or In-Group Fair is asymptotically stable due to random drift.

Theorem 1.3. *When the strategy set S contains all consistent and all inconsistent strategies, a three-person UG has the following Nash equilibrium points:*

- (1) *A consistent strategy $(3, p, p)$ with $0 \leq p \leq \frac{1}{2}$ is a Nash equilibrium if and only if $p \leq 1 - v$.*
- (2) *A consistent strategy $(2, p, p)$ with $0 \leq p \leq v$ is a Nash equilibrium if and only if $1 - v \leq p$.*
- (3) *An inconsistent strategy $(3, p, q)$ is a Nash equilibrium if and only if $q \geq \max\{1/2, v\}$.*
- (4) *An inconsistent strategy $(2, p, q)$ is a Nash equilibrium if and only if $p = 0$ and $q \geq \max\{1/2, v\}$.*

Proof. (1): Let $s = (3, p, p)$ with $0 \leq p \leq \frac{1}{2}$. Suppose that all players in the population employ s . The average fitness of the population s is $(1 - 2p)/3 + 2p/3 = 1/3$. Note that any mutant

proposing the three-person group cannot obtain a higher average fitness than s in the population. When $p > v$, the same thing holds for a mutant proposing a two-person group since a proposal is rejected. Thus, assume that $p \leq v$ without any loss of generality. Among all mutants proposing two-person groups, a mutant $t = (2, p, p)$ can obtain the highest average fitness in the population $s = (3, p, p)$. The average fitness of $t = (2, p, p)$ is $(v - p)/3 + 2p/3 = (v + p)/3$. Thus, state s is a Nash equilibrium point if and only if $(v + p)/3 \leq 1/3$, i.e. $p \leq 1 - v$.

(2): Let $s = (2, p, p)$ with $0 \leq p \leq v$. The average fitness of the population s is $(v - p)/3 + p/3 = v/3$. Parallel to (1), notice that any mutant proposing a two-person group cannot obtain a higher average fitness than strategy s in the population. When $p > 1/2$, the same thing holds for a mutant proposing the three-person group since a proposal is rejected. Thus, assume that $p \leq 1/2$, without any loss of generality. Among all mutants proposing the three-person group, mutant $t = (3, p, p)$ can obtain the highest average fitness in the population $s = (2, p, p)$. The average fitness of t in population s is $(1 - 2p)/3 + p/3 = (1 - p)/3$. Thus, state s is a Nash equilibrium point if and only if $(1 - p)/3 \leq v/3$, i.e. $1 - p \leq v$.

(3): Let $s = (3, p, q)$ with $p \neq q$ and $q \geq \max\{1/2, v\}$. Since $p \leq 1/2$, it holds that $p < q$. The average fitness of population s is 0. Since $q \geq \max\{1/2, v\}$ and the strategy s with $p < q$ proposes the three-person group, any mutant has the average fitness 0. Hence, s is a Nash equilibrium point.

We will show that any other inconsistent strategy $s = (3, p, q)$ with $p \neq q$ is not a Nash equilibrium point if $q < \max\{1/2, v\}$. Assume first that $p < q$. The average fitness of population s is 0. If $q < 1/2$, then the average fitness of mutant $t = (3, q, q)$ in population s is $(1 - 2q)/3 > 0$. If $q < v$, then the average fitness of mutant $t = (2, q, q)$ in population s is $(v - q)/3 > 0$. In both cases, population s is invaded by some mutant, and thus it is not a Nash equilibrium point. Assume next that $p > q$. The average fitness of population s is $1/3$. Consider a mutant $t = (3, q, q)$. The average fitness of t in population s is $(1 - 2q)/3 + 2p/3$, which is higher than $1/3$. Mutant t invades into population s , and thus s is not a Nash

equilibrium point.

(4): Let $s = (2, 0, q)$ with $q \geq \max\{1/2, v\}$. The average fitness of population s is 0. Since $q \geq \max\{1/2, v\}$, any mutant has the average fitness 0. Trivially, s is a Nash equilibrium point.

We will show that any other inconsistent strategy $s = (2, p, q)$ with $p \neq q$ is not a Nash equilibrium point if either $p > 0$ or $q < \max\{1/2, v\}$ holds. Assume first that $p < q$. The average fitness of population s is 0. If $p > 0$, then the average fitness of mutant $t = (3, p, p)$ in population s is $\frac{1}{3} \times 0 + \frac{2}{3} \times \frac{1}{2}p = p/3 > 0$. If $q < 1/2$, then the average fitness of mutant $t = (3, q, q)$ in population s is $(1 - 2q)/3 > 0$. If $q < v$, then the average fitness of mutant $t = (2, q, q)$ in population s is $(v - q)/3 > 0$. In all cases, population s is invaded by some mutant, and thus it is not a Nash equilibrium point. Assume next that $p > q$. The average fitness of population s is $v/3$. The average fitness $t = (2, q, q)$ in population s is $(v - q)/3 + p/3$, which is higher than $v/3$. Mutant t invades into population s , and thus s is not a Nash equilibrium point. \square

2 Moran Process

We analyze the Moran process in the main text (see [13, 17, 19] for technical details). Let us consider a population of N players. Let $S = \{s_1, \dots, s_n\}$ be the set of n strategies in a three-person UG. A state of the population is described by $x = (x_1, \dots, x_n) = X/N = (X_1, \dots, X_n)/N$, where X_i is the number of players using strategy i . The effective payoff of a player using strategy i is given by $\exp[wA(s_i, x)]$, which is approximately equal to $1 + wA(s_i, x)$ for w close to zero. Here, w is the intensity of selection. We assume weak selection so that $wN \ll 1$.

In state x , the average number of offspring of an l -player due to selection is $\omega_l = 1 - 1/N + [1 + wA(s_l, x)]/N[1 + wA(x)]$. For $w \rightarrow 0$, the number can be written as

$$\omega_l = 1 + wN^{-1}[A(s_l, x) - A(x)] + \mathcal{O}(w^2).$$

Due to selection, the frequency change of l -players on average is given by

$$\Delta x_l^{\text{sel}} = x_l \omega_l - x_l = w \Delta x_l^{(1)} [1 + \mathcal{O}(w)], \quad (2)$$

where the first derivative with respect to w is

$$\Delta x_l^{(1)} = N^{-1} x_l [A(s_l, x) - A(x)]. \quad (3)$$

In the stationary state of the Moran process, the system is in state X with probability $P_w(X)$.

The stationary probabilities are continuous at $w = 0$, and we can approximate them as

$$P_w(X) = P_{w=0}(X) [1 + \mathcal{O}(w)] \text{ for any state } X \text{ near } w = 0.$$

Hence by averaging Δx_l^{sel} in (2) in the stationary state, we obtain

$$\langle \Delta x_l^{\text{sel}} \rangle_w \equiv \sum_X \Delta x_l^{\text{sel}} P_w(X) = w \sum_X \Delta x_l^{(1)} P_{w=0}(X) \times [1 + \mathcal{O}(w)].$$

Define $a_{ijk} = A(s_i, s_j, s_k)$ for each i, j, k . Using expression (3) for $\Delta x_l^{(1)}$, we can write the average change due to selection in the first order in w as

$$\begin{aligned} \langle \Delta x_l^{\text{sel}} \rangle_w &= w N^{-1} \langle x_l [A(s_l, x) - A(x)] \rangle \\ &= w N^{-1} \left(\sum_{j,k} a_{ljk} \langle x_l x_j x_k \rangle - \sum_{i,j,k} a_{ijk} \langle x_l x_i x_j x_k \rangle \right), \end{aligned} \quad (4)$$

where $\langle \cdot \rangle$ denotes the average in the neutral stationary state ($w = 0$).

By taking into account mutation as well as selection, the expected total change of frequency in state X can be written as

$$\Delta x_l^{\text{tot}} = \Delta x_l^{\text{sel}} (1 - u) + \frac{u}{N} \left(\frac{1}{n} - x_l \right), \quad (5)$$

where u is the mutation rate. In the stationary state the average total change of frequency is zero, $\langle \Delta x_l^{\text{tot}} \rangle_w = 0$. Hence by averaging Δx_l^{tot} in (5), we obtain the abundance (average frequency) in the stationary state expressed by the average change due to selection as

$$\langle x_l \rangle_w = \frac{1}{n} + N \frac{1-u}{u} \langle \Delta x_l^{\text{sel}} \rangle_w. \quad (6)$$

It follows from (6) that the condition $\langle x_l \rangle_w > \frac{1}{n}$ is equivalent to

$$\langle \Delta x_l^{\text{sel}} \rangle_w > 0.$$

In what follows, we will evaluate $\langle \Delta x_l^{\text{sel}} \rangle_w$ in (4). To do so, we need to calculate averages of the form $\langle x_l x_j x_k \rangle$ and $\langle x_l x_i x_j x_k \rangle$. We remark that exchanging indexes does not affect the averages since all players are symmetric in the neutral stationary state. For example, $\langle x_1 x_1 x_1 \rangle = \langle x_3 x_3 x_3 \rangle$ and $\langle x_1 x_2 x_2 x_2 \rangle = \langle x_1 x_3 x_3 x_3 \rangle$. By taking into account these symmetries, only eight different averages appear in (4):

$$\langle x_1 x_1 x_1 \rangle = \langle x_i x_i x_i \rangle,$$

$$\langle x_1 x_2 x_2 \rangle = \langle x_i x_j x_j \rangle,$$

$$\langle x_1 x_2 x_3 \rangle = \langle x_i x_j x_k \rangle,$$

$$\langle x_1 x_1 x_1 x_1 \rangle = \langle x_i x_i x_i x_i \rangle,$$

$$\langle x_1 x_1 x_2 x_2 \rangle = \langle x_i x_i x_j x_j \rangle,$$

$$\langle x_1 x_2 x_2 x_2 \rangle = \langle x_i x_j x_j x_j \rangle,$$

$$\langle x_1 x_1 x_2 x_3 \rangle = \langle x_i x_i x_j x_k \rangle,$$

$$\langle x_1 x_2 x_3 x_4 \rangle = \langle x_l x_i x_j x_k \rangle$$

for all i, j, k, l which are different each other. Then, assuming $n \geq 4$, we can from (4) that

$$\begin{aligned}
N_W^{-1} \langle \Delta x_l^{\text{sel}} \rangle_w &= \langle x_1 x_1 x_1 \rangle a_{lll} + \langle x_1 x_2 x_2 \rangle \sum_{i \neq l} (a_{lii} + a_{lli} + a_{lil}) + \langle x_1 x_2 x_3 \rangle \sum_{\substack{i,j: \\ l \neq i \neq j \neq l}} a_{lij} \\
&\quad - \langle x_1 x_1 x_1 x_1 \rangle a_{llll} - \langle x_1 x_2 x_2 x_2 \rangle \sum_{i \neq l} (a_{lli} + a_{lil} + a_{ill} + a_{iii}) \\
&\quad - \langle x_1 x_1 x_2 x_2 \rangle \sum_{i \neq l} (a_{lii} + a_{ili} + a_{iil}) \\
&\quad - \langle x_1 x_1 x_2 x_3 \rangle \sum_{\substack{i,j: \\ l \neq i \neq j \neq l}} (a_{lij} + a_{ilj} + a_{ijl} + a_{iij} + a_{iji} + a_{jii}) \\
&\quad - \langle x_1 x_2 x_3 x_4 \rangle \sum_{\substack{i,j,k: \\ \text{different each other,} \\ i \neq l, j \neq l, k \neq l}} a_{ijk}. \tag{7}
\end{aligned}$$

We can rearrange (7) as

$$\begin{aligned}
N_W^{-1} \langle \Delta x_l^{\text{sel}} \rangle_w &= a_{lll} (\langle x_1 x_1 x_1 \rangle - 3 \langle x_1 x_2 x_2 \rangle + 2 \langle x_1 x_2 x_3 \rangle - \langle x_1 x_1 x_1 x_1 \rangle \\
&\quad + 4 \langle x_1 x_2 x_2 x_2 \rangle + 3 \langle x_1 x_1 x_2 x_2 \rangle - 12 \langle x_1 x_1 x_2 x_3 \rangle + 6 \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_i a_{lii} (\langle x_1 x_2 x_2 \rangle - \langle x_1 x_2 x_3 \rangle - \langle x_1 x_1 x_2 x_2 \rangle + 2 \langle x_1 x_1 x_2 x_3 \rangle - \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_i (a_{lli} + a_{lil}) (\langle x_1 x_2 x_2 \rangle - \langle x_1 x_2 x_3 \rangle - \langle x_1 x_2 x_2 x_2 \rangle + 3 \langle x_1 x_1 x_2 x_3 \rangle - 2 \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_i (a_{ill} + a_{iii}) (-\langle x_1 x_2 x_2 x_2 \rangle + 3 \langle x_1 x_1 x_2 x_3 \rangle - 2 \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_i (a_{ili} + a_{iil}) (-\langle x_1 x_1 x_2 x_2 \rangle + 2 \langle x_1 x_1 x_2 x_3 \rangle - \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_{i,j} a_{lij} (\langle x_1 x_2 x_3 \rangle - \langle x_1 x_1 x_2 x_3 \rangle + \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_{i,j} (a_{ilj} + a_{ijl} + a_{iij} + a_{iji} + a_{jii}) (-\langle x_1 x_1 x_2 x_3 \rangle + \langle x_1 x_2 x_3 x_4 \rangle) \\
&\quad + \sum_{i,j,k} a_{ijk} (-\langle x_1 x_2 x_3 x_4 \rangle). \tag{8}
\end{aligned}$$

As is known from the literature on population genetics (25), when N is large, the frequencies $x = (x_1, \dots, x_n)$ in the stationary state follow a Dirichlet distribution with a probability density function

$$f(x_1, \dots, x_{n-1}; \alpha_1, \dots, \alpha_n) = \frac{\Gamma(\sum_{i=1}^n \alpha_i)}{\prod_{i=1}^n \Gamma(\alpha_i)} \prod_{i=1}^n x_i^{\alpha_i-1},$$

where Γ is the gamma function. It is well known (31) that moments of Dirichlet-distributed random variables can be expressed as

$$E \left[\prod_{i=1}^n x_i^{\gamma_i} \right] = \frac{\Gamma(\sum_{i=1}^n \alpha_i)}{\Gamma(\sum_{i=1}^n (\alpha_i + \gamma_i))} \times \prod_{i=1}^n \frac{\Gamma(\alpha_i + \gamma_i)}{\Gamma(\alpha_i)}.$$

A simple computation also shows that $\alpha_1 = \dots = \alpha_n \equiv \alpha = \mu/n$, where $\mu = Nu$ is the rescaled mutation rate. These facts imply that the eight averages are given by

$$\begin{aligned} \langle x_1 x_1 x_1 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 3)} \frac{\Gamma(\alpha + 3)}{\Gamma(\alpha)} = \frac{(n + \mu)(2n + \mu)}{n^3(1 + \mu)(2 + \mu)} = (3 + \mu)(n + \mu)(2n + \mu)nC, \\ \langle x_1 x_2 x_2 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 3)} \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)} \frac{\Gamma(\alpha + 2)}{\Gamma(\alpha)} = \frac{\mu(n + \mu)}{n^3(1 + \mu)(2 + \mu)} = \mu(3 + \mu)(n + \mu)nC, \\ \langle x_1 x_2 x_3 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 3)} \left\{ \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)} \right\}^3 = \frac{\mu^2}{n^3(1 + \mu)(2 + \mu)} = \mu^2(3 + \mu)nC, \\ \langle x_1 x_1 x_1 x_1 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 4)} \frac{\Gamma(\alpha + 4)}{\Gamma(\alpha)} = \frac{(n + \mu)(2n + \mu)(3n + \mu)}{n^4(1 + \mu)(2 + \mu)(3 + \mu)} = (n + \mu)(2n + \mu)(3n + \mu)C, \\ \langle x_1 x_1 x_2 x_2 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 4)} \left\{ \frac{\Gamma(\alpha + 2)}{\Gamma(\alpha)} \right\}^2 = \frac{\mu(n + \mu)^2}{n^4(1 + \mu)(2 + \mu)(3 + \mu)} = \mu(n + \mu)^2 C, \\ \langle x_1 x_2 x_2 x_2 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 4)} \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)} \frac{\Gamma(\alpha + 3)}{\Gamma(\alpha)} = \frac{\mu(n + \mu)(2n + \mu)}{n^4(1 + \mu)(2 + \mu)(3 + \mu)} = \mu(n + \mu)(2n + \mu)C, \\ \langle x_1 x_1 x_2 x_3 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 4)} \left\{ \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)} \right\}^2 \frac{\Gamma(\alpha + 2)}{\Gamma(\alpha)} = \frac{\mu^2(n + \mu)}{n^4(1 + \mu)(2 + \mu)(3 + \mu)} = \mu^2(n + \mu)C, \\ \langle x_1 x_2 x_3 x_4 \rangle &= \frac{\Gamma(n\alpha)}{\Gamma(n\alpha + 4)} \left\{ \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)} \right\}^4 = \frac{\mu^3}{n^4(1 + \mu)(2 + \mu)(3 + \mu)} = \mu^3 C, \end{aligned}$$

where $C = [n^4(1 + \mu)(2 + \mu)(3 + \mu)]^{-1}$. With these correlations, Eq. (8) takes the form

$$\begin{aligned} \frac{Nw^{-1}\langle\Delta x_l^{\text{sel}}\rangle_w}{C} &= \mu n^2 \sum_i (2a_{lll} + 2a_{lil} + a_{lli} + a_{lil} - 2a_{ill} - 2a_{iii} - a_{ili} - a_{iil}) \\ &\quad + \mu^2 n \sum_{i,j} (a_{lil} + a_{lli} + a_{lil} + 2a_{lij} - a_{ilj} - a_{ijl} - a_{iij} - a_{iji} - a_{jii}) \\ &\quad + \mu^3 \sum_{i,j,k} (a_{lij} - a_{ijk}). \end{aligned} \quad (9)$$

By defining

$$\begin{aligned} L_l &= \frac{1}{n} \sum_i [2(a_{lll} - a_{ill}) + 2(a_{lil} - a_{iii}) + (a_{lli} - a_{ili}) + (a_{lil} - a_{iil})], \\ M_l &= \frac{1}{n^2} \sum_{i,j} [(a_{lij} - a_{iij}) + (a_{lij} - a_{jij}) + (a_{lil} - a_{jil}) + (a_{lli} - a_{jli}) + (a_{lil} - a_{jil})], \\ H_l &= \frac{1}{n^3} \sum_{i,j,k} (a_{lij} - a_{kij}), \end{aligned} \quad (10)$$

we finally arrive at the formula

$$\langle\Delta x_l^{\text{sel}}\rangle_w = \frac{w\mu(L_l + \mu M_l + \mu^2 H_l)}{nN(1 + \mu)(2 + \mu)(3 + \mu)}.$$

Strategy s_l is more abundant than the average if it satisfies $\langle\Delta x_l^{\text{sel}}\rangle_w > 0$. In the case of the low mutation rate, $\mu \ll 1$, the condition for abundance is given by $L_l > 0$. In what follows, we will characterize the most abundant strategy that maximizes L_l in (10).

In a three-person UG, a strategy is represented by three parameters as $s_i = (m_i, p_i, q_i)$ where $m_i = 2, 3$ is a group size, p_i is an offer and q_i is a minimum acceptance payoff. For three strategies s_i, s_j, s_k , payoff $a_{ijk} = A(s_i, s_j, s_k)$ for strategy s_i is determined. To compute a_{ijk} , we introduce the 0-1 function $I(E)$ for a condition E such that $I(E) = 1$ if E is true, and otherwise, $I(E) = 0$. In particular, we define $I_{ij} = I(p_i \geq q_j)$ for any i, j . Similar notations are used under the same convention. Then, payoff a_{ijk} is given as follows (ignoring a $1/3$ factor,

the probability with which each player has the role of a proposer)

$$a_{ijk} = \begin{cases} I_{ij}I_{ik}(1 - 2p_i) + I_{ji}I_{jk}p_j + I_{ki}I_{kj}p_k & \text{if } m_i = m_j = m_k = 3 \\ I_{ij}I_{ik}(1 - 2p_i) + I_{ji}I_{jk}p_j + \frac{1}{2}I_{ki}p_k & \text{if } m_i = m_j = 3 \text{ and } m_k = 2 \\ I_{ij}I_{ik}(1 - 2p_i) + \frac{1}{2}I_{ji}p_j + \frac{1}{2}I_{ki}p_k & \text{if } m_i = 3 \text{ and } m_j = m_k = 2 \\ \frac{1}{2}(I_{ij} + I_{ik})(v - p_i) + \frac{1}{2}I_{ji}p_j + \frac{1}{2}I_{ki}p_k & \text{if } m_i = m_j = m_k = 2 \\ \frac{1}{2}(I_{ij} + I_{ik})(v - p_i) + \frac{1}{2}I_{ji}p_j + I_{ki}I_{kj}p_k & \text{if } m_i = m_j = 2 \text{ and } m_k = 3 \\ \frac{1}{2}(I_{ij} + I_{ik})(v - p_i) + I_{ji}I_{jk}p_j + I_{ki}I_{kj}p_k & \text{if } m_i = 2 \text{ and } m_j = m_k = 3. \end{cases} \quad (11)$$

Noting that $a_{lli} = a_{lil}$ and $a_{ili} = a_{iil}$, we can divide the summation of Eq. (10) into two parts as

$$L_l = \frac{2}{n} \left(\sum_{i:m_i=3} + \sum_{i:m_i=2} \right) [(a_{lll} - a_{ill}) + (a_{lil} - a_{ili}) + (a_{lli} - a_{lil})]$$

In the game, the strategy s_i for each player takes finitely many values in the strategy space $\{(p_i, q_i) \mid 0 \leq p_i \leq 1/2, 0 \leq q_i \leq 1\}$ if $m_i = 3$, and does so in the strategy space $\{(p_i, q_i) \mid 0 \leq p_i \leq v, 0 \leq q_i \leq 1\}$ if $m_i = 2$. As the total number n of strategies grows infinitely large, it converges to the integrals

$$L(m_l, p_l, q_l) = \frac{2}{1/2 + v} \left(\int_0^{1/2} \int_0^1 + \int_0^v \int_0^1 \right) [(a_{lll} - a_{ill}) + (a_{lil} - a_{ili}) + (a_{lli} - a_{lil})] dq_i dp_i. \quad (12)$$

Notice that the sum of the areas of the two strategy spaces for $m_i = 3$ and $m_i = 2$ is $1/2 + v$.

To compute the most abundant strategy $s_l = (m_l, p_l, q_l)$ that maximizes $L(m_l, p_l, q_l)$ in (12), we first show the explicit formula of the objective function L .

Case 1. $m_l = 3$. Let $s_l = (3, p, q)$. Using (11), we obtain the following equations:

$$\int_0^{\frac{1}{2}} \int_0^1 a_{lll} dq_i dp_i + \int_0^v \int_0^1 a_{lll} dq_i dp_i = \int_0^{\frac{1}{2}} \int_0^1 I_{ll} dq_i dp_i + \int_0^v \int_0^1 I_{ll} dq_i dp_i = I(p \geq q) \left(\frac{1}{2} + v \right).$$

$$\begin{aligned} & \int_0^{\frac{1}{2}} \int_0^1 a_{ill} dq_i dp_i + \int_0^v \int_0^1 a_{ill} dq_i dp_i \\ &= \int_0^{\frac{1}{2}} \int_0^1 [I_{il}(1 - 2p_i) + 2I_{il}I_{ll}p] dq_i dp_i + \int_0^v \int_0^1 [I_{il}(v - p_i) + 2I_{il}I_{ll}p] dq_i dp_i \\ &= I(q \leq 1/2) \int_q^{\frac{1}{2}} (1 - 2p_i) dp_i + \int_0^p p I_{ll} dq_i + I(q \leq v) \int_q^v (v - p_i) dp_i + \int_0^p 2vp I_{ll} dq_i \\ &= I(p \geq q) 2p^2 \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{4} - q + q^2 \right) + I(q \leq v) \left(\frac{v^2}{2} - vq + \frac{q^2}{2} \right) \end{aligned}$$

$$\begin{aligned} & \int_0^{\frac{1}{2}} \int_0^1 a_{lii} dq_i dp_i + \int_0^v \int_0^1 a_{lii} dq_i dp_i \\ &= \int_0^{\frac{1}{2}} \int_0^1 [I_{li}(1 - 2p) + 2I_{li}I_{ii}p_i] dq_i dp_i + \int_0^v \int_0^1 [I_{li}(1 - 2p) + I_{li}p_i] dq_i dp_i \\ &= \int_0^p \frac{1}{2}(1 - 2p) dq_i + I(q \leq 1/2) \int_q^{\frac{1}{2}} (2p_i^2) dp_i + \int_0^p v(1 - 2p) dq_i + I(q \leq v) \int_q^v p_i dp_i \\ &= (p - 2p^2) \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{12} - \frac{2}{3}q^3 \right) + I(q \leq v) \left(\frac{v^2}{2} - \frac{q^2}{2} \right) \end{aligned}$$

$$\begin{aligned} & \int_0^{\frac{1}{2}} \int_0^1 a_{iii} dq_i dp_i + \int_0^v \int_0^1 a_{iii} dq_i dp_i \\ &= \int_0^{\frac{1}{2}} \int_0^1 [I_{ii}(1 - 2p_i) + 2I_{ii}p_i] dq_i dp_i + \int_0^v \int_0^1 [I_{ii}(v - p_i) + I_{ii}p_i] dq_i dp_i \\ &= \int_0^{\frac{1}{2}} p_i(1 - 2p_i) dp_i + \int_0^{\frac{1}{2}} (2p_i^2) dp_i + \int_0^v p_i(v - p_i) dp_i + \int_0^v p_i^2 dp_i \\ &= \frac{1}{8} + \frac{v^3}{2} \end{aligned}$$

$$\int_0^{\frac{1}{2}} \int_0^1 a_{lll} dq_i dp_i + \int_0^v \int_0^1 a_{lll} dq_i dp_i$$

$$\begin{aligned}
&= \int_0^{\frac{1}{2}} \int_0^1 [I_{ll}I_{li}(1-2p) + I_{ll}I_{li}p + I_{li}p_i] dq_i dp_i + \int_0^v \int_0^1 \left[I_{ll}I_{li}(1-2p) + I_{ll}I_{li}p + \frac{I_{li}}{2}p_i \right] dq_i dp_i \\
&= I(p \geq q)p(1-p)\left(\frac{1}{2} + v\right) + I(q \leq 1/2) \int_q^{\frac{1}{2}} p_i dp_i + I(q \leq v) \int_q^v \frac{p_i}{2} dp_i \\
&= I(p \geq q)(p-p^2)\left(\frac{1}{2} + v\right) + I(q \leq 1/2)\left(\frac{1}{8} - \frac{q^2}{2}\right) + I(q \leq v)\left(\frac{v^2}{4} - \frac{q^2}{4}\right)
\end{aligned}$$

$$\begin{aligned}
&\int_0^{\frac{1}{2}} \int_0^1 a_{iii} dq_i dp_i + \int_0^v \int_0^1 a_{iii} dq_i dp_i \\
&= \int_0^{\frac{1}{2}} \int_0^1 [I_{ii}I_{il}(1-2p_i) + I_{ii}I_{il}p_i + I_{il}p] dq_i dp_i + \int_0^v \int_0^1 \left[\frac{(I_{ii} + I_{il})}{2}(v-p_i) + \frac{I_{ii}}{2}p_i + I_{il}p \right] dq_i dp_i \\
&= I(q \leq 1/2) \int_q^{\frac{1}{2}} p_i(1-p_i) dp_i + p^2\left(\frac{1}{2} + v\right) + \int_0^v \frac{vp_i}{2} dp_i + I(q \leq v) \int_q^v \frac{v-p_i}{2} dp_i \\
&= p^2\left(\frac{1}{2} + v\right) + I(q \leq 1/2)\left(\frac{1}{12} - \frac{q^2}{2} + \frac{q^3}{3}\right) + I(q \leq v)\left(\frac{v^2}{4} - \frac{vq}{2} + \frac{q^2}{4}\right) + \frac{v^3}{4}
\end{aligned}$$

Substituting these equations into (12) yields

$$\begin{aligned}
L(3, p, q) &= I(p \geq q)(-3p^2 + p + 1)\left(\frac{1}{2} + v\right) + (-3p^2 + p)\left(\frac{1}{2} + v\right) - \frac{3}{4}v^3 - \frac{1}{8} \\
&\quad + I(q \leq 1/2)\left(-q^3 - q^2 + q - \frac{1}{8}\right) + I(q \leq v)\left(-\frac{3}{2}q^2 + \frac{3}{2}vq\right) \tag{13}
\end{aligned}$$

(ignoring the factor $\frac{2}{1/2+v}$). The function $L(3, \cdot, \cdot)$ is continuous in (p, q) on each region of $\{(p, q) \mid 0 \leq p \leq 1/2, 0 \leq q \leq p\}$ and $\{(p, q) \mid 0 \leq p \leq 1/2, p < q \leq 1\}$, and has downward jumps in q on the line $p = q$.

Case 2. $m_l = 2$. Let $s_l = (2, p, q)$. Using (11), we obtain the following equations:

$$\int_0^{\frac{1}{2}} \int_0^1 a_{lll} dq_i dp_i + \int_0^v \int_0^1 a_{lll} dq_i dp_i = \int_0^{\frac{1}{2}} \int_0^1 I_{ll}v dq_i dp_i + \int_0^v \int_0^1 I_{ll}v dq_i dp_i = I(p \geq q)v\left(\frac{1}{2} + v\right).$$

$$\int_0^{\frac{1}{2}} \int_0^1 a_{iii} dq_i dp_i + \int_0^v \int_0^1 a_{iii} dq_i dp_i$$

$$\begin{aligned}
&= \int_0^{\frac{1}{2}} \int_0^1 [I_{il}(1-2p_i) + I_{li}p] dq_i dp_i + \int_0^v \int_0^1 [I_{il}(v-p_i) + I_{li}p] dq_i dp_i \\
&= I(q \leq 1/2) \int_q^{\frac{1}{2}} (1-2p_i) dp_i + \int_0^p \frac{p}{2} dq_i + I(q \leq v) \int_q^v (v-p_i) dp_i + \int_0^p v p dq_i \\
&= p^2 \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{4} - q + q^2 \right) + I(q \leq v) \left(\frac{v^2}{2} - vq + \frac{q^2}{2} \right)
\end{aligned}$$

$$\begin{aligned}
&\int_0^{\frac{1}{2}} \int_0^1 a_{lii} dq_i dp_i + \int_0^v \int_0^1 a_{lii} dq_i dp_i \\
&= \int_0^{\frac{1}{2}} \int_0^1 [I_{li}(v-p) + 2I_{il}I_{ii}p_i] dq_i dp_i + \int_0^v \int_0^1 [I_{li}(v-p) + I_{il}p_i] dq_i dp_i \\
&= \int_0^p \frac{1}{2}(v-p) dq_i + I(q \leq 1/2) \int_q^{\frac{1}{2}} (2p_i^2) dp_i + \int_0^p v(v-p) dq_i + I(q \leq v) \int_q^v p_i dp_i \\
&= (vp - p^2) \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{12} - \frac{2}{3}q^3 \right) + I(q \leq v) \left(\frac{v^2}{2} - \frac{q^2}{2} \right)
\end{aligned}$$

$$\int_0^{\frac{1}{2}} \int_0^1 a_{iii} dq_i dp_i + \int_0^v \int_0^1 a_{iii} dq_i dp_i = \frac{1}{8} + \frac{v^3}{2}$$

$$\begin{aligned}
&\int_0^{\frac{1}{2}} \int_0^1 a_{lii} dq_i dp_i + \int_0^v \int_0^1 a_{lii} dq_i dp_i \\
&= \int_0^{\frac{1}{2}} \int_0^1 \left[\frac{I_{ll} + I_{li}}{2}(v-p) + \frac{I_{ll}}{2}p + I_{il}p_i \right] dq_i dp_i + \int_0^v \int_0^1 \left[\frac{I_{ll} + I_{li}}{2}(v-p) + \frac{I_{ll}}{2}p + \frac{I_{il}}{2}p_i \right] dq_i dp_i \\
&= I(p \geq q) \frac{v}{2} \left(\frac{1}{2} + v \right) + \frac{p}{2}(v-p) \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \int_q^{\frac{1}{2}} p_i dp_i + I(q \leq v) \int_q^v \frac{p_i}{2} dp_i \\
&= I(p \geq q) \frac{v}{2} \left(\frac{1}{2} + v \right) + \left(\frac{vp}{2} - \frac{p^2}{2} \right) \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{8} - \frac{q^2}{2} \right) + I(q \leq v) \left(\frac{v^2}{4} - \frac{q^2}{4} \right)
\end{aligned}$$

$$\begin{aligned}
&\int_0^{\frac{1}{2}} \int_0^1 a_{iil} dq_i dp_i + \int_0^v \int_0^1 a_{iil} dq_i dp_i \\
&= \int_0^{\frac{1}{2}} \int_0^1 \left[I_{ii}I_{il}(1-2p_i) + I_{ii}I_{il}p_i + \frac{I_{li}}{2}p \right] dq_i dp_i + \int_0^v \int_0^1 \left[\frac{(I_{ii} + I_{il})}{2}(v-p_i) + \frac{I_{ii}}{2}p_i + \frac{I_{li}}{2}p \right] dq_i dp_i
\end{aligned}$$

$$\begin{aligned}
&= I(q \leq 1/2) \int_q^{\frac{1}{2}} p_i(1-p_i)dp_i + \frac{p^2}{2} \left(\frac{1}{2} + v \right) + \int_0^v \frac{vp_i}{2} dp_i + I(q \leq v) \int_q^v \frac{v-p_i}{2} dp_i \\
&= \frac{p^2}{2} \left(\frac{1}{2} + v \right) + I(q \leq 1/2) \left(\frac{1}{12} - \frac{q^2}{2} + \frac{q^3}{3} \right) + I(q \leq v) \left(\frac{v^2}{4} - \frac{vq}{2} + \frac{q^2}{4} \right) + \frac{v^3}{4}
\end{aligned}$$

Substituting these equations into (12) yields

$$\begin{aligned}
L(2, p, q) &= I(p \geq q) \left(\frac{3}{4}v + \frac{3}{2}v^2 \right) - \frac{3}{4}v^3 - \frac{1}{8} + \left(-3p^2 + \frac{3}{2}vp \right) \left(\frac{1}{2} + v \right) \\
&\quad + I(q \leq 1/2) \left(-q^3 - q^2 + q - \frac{1}{8} \right) + I(q \leq v) \left(-\frac{3}{2}q^2 + \frac{3}{2}vq \right) \quad (14)
\end{aligned}$$

(ignoring the factor $\frac{2}{1/2+v}$). The function $L(2, \cdot, \cdot)$ is continuous in (p, q) on each region of $\{(p, q) \mid 0 \leq p \leq v, 0 \leq q \leq p\}$ and $\{(p, q) \mid 0 \leq p \leq v, p < q \leq 1\}$, and has downward jumps in q on the line $p = q$.

The most abundant strategy in the population is a solution (m^*, p^*, q^*) to the following maximization problem:

$$\max_{m \in \{2,3\}, p, q} L(m, p, q),$$

where the objective functions $L(3, p, q)$ and $L(2, p, q)$ are given by (13) and (14), respectively.

By tedious calculations, the optimal solution is characterized as follows:

$$(m^*, p^*, q^*) = \begin{cases} (3, p^1(v), p^1(v)) & \text{if } 0 < v \leq v^1, \\ (3, p^2(v), p^2(v)) & \text{if } v^1 < v \leq v^2, \\ (2, p^3(v), p^3(v)) & \text{if } v^2 < v < 1, \end{cases}$$

where

$$p^1(v) = \frac{1}{3}(-4 - 6v + \sqrt{22 + 54v + 36v^2})$$

$$p^2(v) = \frac{1}{6}(-11 - 12v + \sqrt{145 + 306v + 144v^2})$$

$$p^3(v) = \frac{1}{6}(-8 - 6v + \sqrt{76 + 123v + 54v^2})$$

and v^1 and v^2 satisfy $L(3, p^1(v^1), p^1(v^1)) = L(3, p^2(v^1), p^2(v^1))$ and $L(3, p^2(v^2), p^2(v^2)) = L(2, p^3(v^2), p^3(v^2))$ respectively. Table A4 gives numerically the optimal solution (m^*, p^*, q^*) for every value of $v \in [0, 1]$ in increments of 0.05.

v	m^*	$p^*(=q^*)$	v	m^*	$p^*(=q^*)$	v	m^*	$p^*(=q^*)$
0	3	0.230	0.35	3	0.204	0.7	3	0.222
0.05	3	0.226	0.4	3	0.207	0.75	2	0.266
0.1	3	0.223	0.45	3	0.210	0.8	2	0.276
0.15	3	0.220	0.5	3	0.213	0.85	2	0.286
0.2	3	0.217	0.55	3	0.215	0.9	2	0.297
0.25	3	0.197	0.6	3	0.217	0.95	2	0.307
0.3	3	0.201	0.65	3	0.220	1	2	0.318

Table A4. The most abundant strategy (m^*, p^*, q^*) for every value of $v \in [0, 1]$ in increments of 0.05.

3 Simulation Results

We present simulation results of the Moran process with parameters $N = 50$, $w = 10^{-0.5}$, $u = 10^{-2.5}, 10^{-3.5}$ and $v = 1/6, 5/6$. Figure A1 illustrates the frequencies of strategies averaged over 10^8 generations in the case of weak selection ($w = 10^{-0.5}$). Regarding the group size choice, the most abundant strategy is not affected when the mutation rate u changes from $u = 10^{-3}$ to $u = 10^{-2.5}, 10^{-3.5}$. The values of p and q are mildly changed except the p -value when $v = 1/6$: p takes values $1/4, 5/12$ and $1/3$ for $u = 10^{-2.5}, 10^{-3}, 10^{-3.5}$, respectively.

Figure A2 shows the mean frequencies of two-person groups over 10^8 generations for different values of u and w in the simulations when the group value $v = 5/6$. For comparison, the mean frequency of two-person groups in the experiments (22) is also given. In the

experiment, subjects played eight rounds of a three-person UG with random matching in each round. The data show the mean frequency of two-person groups chosen by proposers in the first round. The mean frequency was stable over rounds.

In the simulations, the mean frequencies of two-person groups are stable for a change of the mutation rate u in the case of weak selection ($w = 10^{-0.5}$). When the intensity of selection increases from $w = 10^{-0.5}$ to $w = 10^2$, the mean frequency of two-person groups drops from 63.8% to 43.2% when $u = 10^{-3}$. The mean frequency of two-person groups in the experiment is higher than those of simulations in Figure A2.

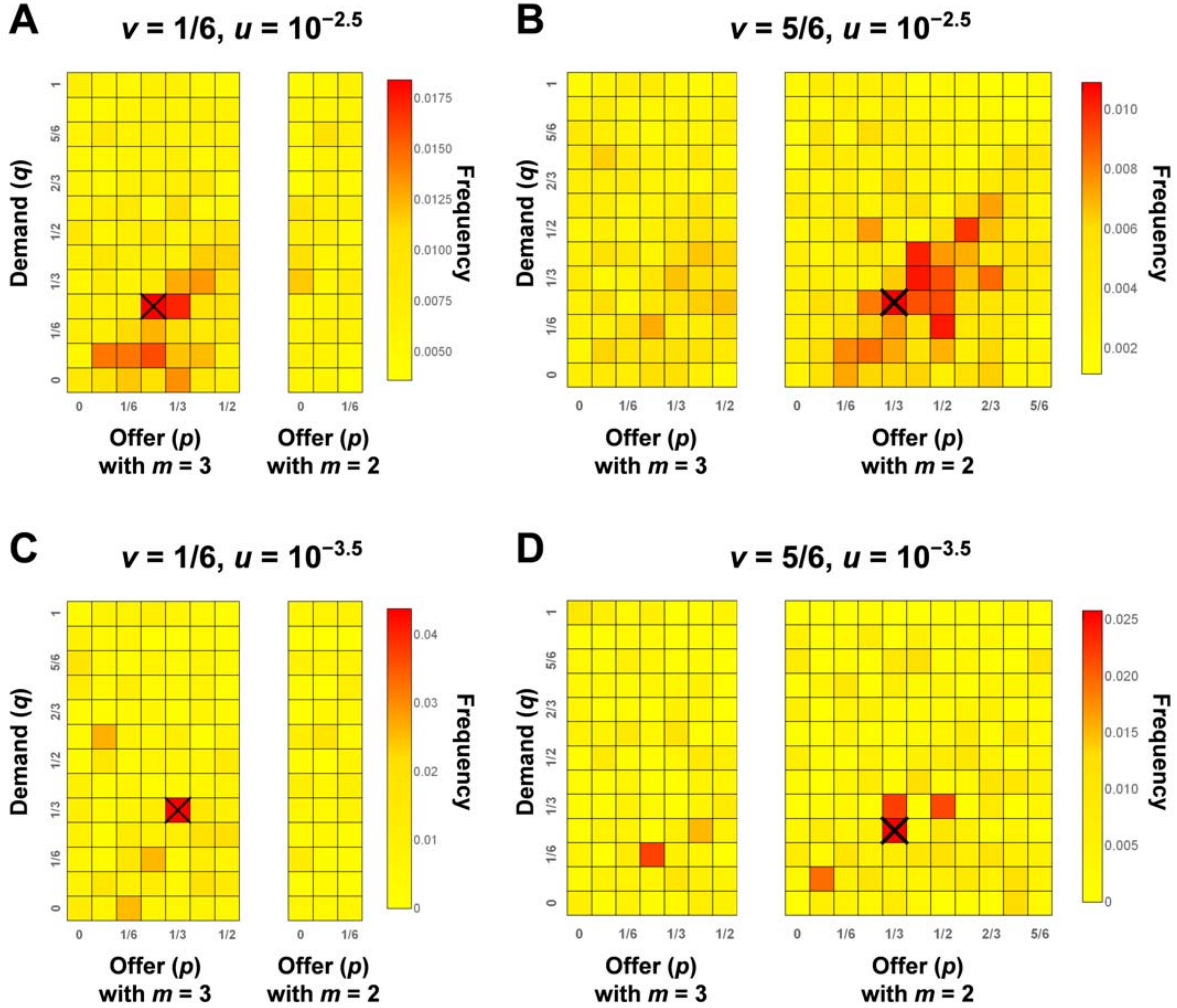


Fig. A1. Simulation results of the frequency-dependent Moran process where the number of players is $N = 50$ and the selection intensity is $w = 10^{-0.5}$. Parameters of the two-person group value (v) and the mutation rate (u) are: (A) $v = 1/6, u = 10^{-2.5}$; (B) $v = 5/6, u = 10^{-2.5}$; (C) $v = 1/6, u = 10^{-3.5}$; and (D) $v = 5/6, u = 10^{-3.5}$. The strategy parameters, p (offer) and q (acceptance threshold), are discretized in increments of $1/12$. The strategy set is colored according to the average frequency of each strategy over 10^8 generations. The most abundant strategy indicated with a black \times is $(3, 1/4, 1/4)$ in (A), $(2, 1/3, 1/4)$ in (B) and (D), and $(3, 1/3, 1/3)$ in (C).

Experimental and simulation results ($\nu = 5/6$)

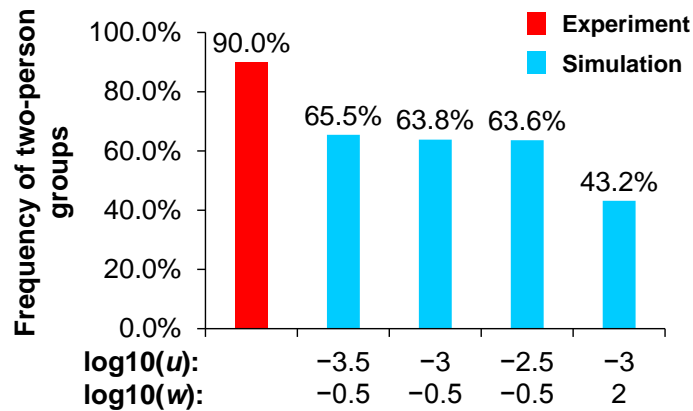


Fig. A2. Mean frequencies of two-person groups when $\nu = 5/6$. Red bar shows the mean frequency of two-person groups chosen in the first round of the experiment (22). Blue bars show the mean frequencies of two-person groups over 10^8 generations for different values of u and w in the simulations.

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