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Is Growth Declining in the Service Economy?*

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Abstract

This study extends Baumol's (1967) two-sector (manufacturing and services) unbalanced growth model to analyze a situation in which, first, services are used for both final consumption and intermediate inputs in manufacturing production, and second, the productivity of the manufacturing and services sectors endogenously evolves. Using this model, we investigate how the employment share of services and economic growth rate evolve through time. Our results are summarized as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, the economic growth rate decreases at first, begins to increase after some time, decreases again, and finally, approaches zero.

Keywords: service economy; economic growth; endogenous productivity growth; business services

JEL Classification: J21; J24; O11; O14; O30; O41

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1 Introduction

Why does the share of services tend to increase over time? What is the relationship between the tendency toward services and economic growth? This study is an attempt to answer these questions.

Here, we define the tendency toward services as an increase in the employment share of the service sector. This tendency toward services is observed broadly in developed economies.[1)] Figure 1 shows the time series of the employment shares of the service sectors in Japan and the US during 1980–2010. From this, the tendency toward services progresses in both Japan and the US.

[Figure 1 around here]

Baumol (1967) is a pioneering work that examines the relationship between the tendency toward services and economic growth.[2)] He builds a two-sector (manufacturing and services) unbalanced growth model to investigate why the employment share of services increases and the relationship between the tendency toward services and economic growth.[3)] He shows that if the productivity growth of manufacturing is higher than that of services (Baumol's first assumption) and, in addition, if there is a constant ratio of demand for services and demand for manufacturing (Baumol's second assumption), then the employment share of services increases over time, and the economic growth rate continues to decline before finally approaching the rate of productivity growth of services.

After Baumol's seminal work, many studies have been produced on the relationship between the tendency toward services and economic growth.[4)] The present study integrates the elements of Sasaki (2007) and Sasaki (2012), both of which develop Baumol's (1967) argument.[5)]

1) In addition to studies that consider structural change from manufacturing toward services, many studies consider structural change from agriculture toward manufacturing. For example, Imrohoroglu *et al.* (2014) present a model that considers structural change from agriculture toward manufacturing, and empirically investigate factors of structural change in the Turkish economy.

2) For increasing costs of labor-intensive services (e.g., healthcare and education), which are called "cost disease," see Baumol (2012).

3) Kapur (2012) builds a model and investigates a situation in which the service sector is divided into two subsectors: a service sector in which productivity stagnates and a service sector in which productivity grows. Akbulut (2011) investigates the growth of the service sector as an explanation for the increase in women's employment. She develops an economic model that can account for the increase in women's employment and the growth of the service sector at the same time.

4) Duarte and Restuccia (2010) investigate the role of sectoral labor productivity in explaining the process of structural transformation—the secular reallocation of labor across sectors—and the time path of aggregate productivity across countries. Rogerson (2008) presents a three-sector model that captures time allocation between manufacturing, market services, and nonmarket production.

5) For a survey of the literature on the relationship between the tendency toward services and economic growth, see studies cited in Sasaki (2007, 2012).

To begin with, Sasaki (2007) introduces intermediate service inputs into the Baumol model and investigates how Baumol's results change. In Baumol (1967), services are used entirely for final consumption. In this respect, Oulton (2001) observes that in reality, services are used for intermediate inputs as well as final consumption, and models such a situation. Schettkat and Yocarini (2003) provide data for intermediate services in manufacturing. Using input–output tables, they calculate the share of intermediate services to total manufacturing output. Table 1 shows that intermediate services increase through time in France, Germany, the UK, and the US.[6] Therefore, considering intermediate business services in manufacturing is important for the analysis of the service economy.[7]

[Table 1 around here]

Oulton (2001) concludes that as the employment share of services increases, the rate of economic growth also increases. However, in Oulton's model, services are devoted entirely to intermediate inputs into manufacturing, and hence, no services are used for final consumption. Based on this argument, Sasaki (2007) builds a model to capture a situation in which services are used for both intermediate inputs into manufacturing and final consumption. He reaches a conclusion similar to Baumol (1967): if enough time passes, the rate of economic growth declines with the tendency toward services.

Next, Sasaki (2012) introduces endogenous technological progress into the Baumol model and investigates the relationship between the tendency toward services and economic growth. His study is influenced by the work of Pugno (2006), who considers that the consumption of services augments human capital à la Lucas (1988). The consumption of healthcare and education services would lead to human capital accumulation. Accordingly, the consumption of services increases the productivity of workers, thereby resulting in an increase in the productivity of both manufacturing and services. Pugno (2006) incorporates this human capital accumulation effect into Baumol's model and shows that if this effect is relatively strong, the employment shift toward services increases, not decreases, the rate of economic growth. Sasaki (2012) considers a learning-by-doing effect in manufacturing as well as Pugno's (2006) human capital accumulation effect. He shows that the employment shift toward services and the rate of economic growth have a U-shaped relationship. That is, if the employment share of services begins to increase from a value of zero, the economic

6) Schettkat and Yocarini (2003) also provide data for manufacturing intermediate inputs in service sectors. They show that manufacturing intermediate inputs in France, Germany, and the US roughly stay constant whereas they increase in the UK. These empirical facts suggest that manufacturing intermediate inputs do not necessarily increase in all countries. Accordingly, our model abstracts manufacturing intermediate inputs in services.

7) Services as intermediate inputs are related to the outsourcing of services. For the relationship between the outsourcing of services and economic growth, see Fixler and Siegel (1999).

growth rate begins to decline with an increase in the employment share of services, but after some time, it begins to increase with the increase in the employment share of services.

The present study, by integrating the elements of Sasaki (2007) and Sasaki (2012), builds a more general model and investigates the relationship between the employment shift toward services and the economic growth rate.[8] Specifically, services are used for both final consumption and intermediate inputs, human capital is accumulated through the consumption of services, and the productivity of manufacturing increases through learning by doing. Moreover, productivity specific to the services sector increases through time whereas it is constant in Sasaki (2012).

The main results are as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, the economic growth rate decreases first, begins to increase after some time, decreases again, and finally, approaches zero.

The remainder of the paper is organized as follows. Section 2 builds our model. Section 3 derives the instantaneous equilibrium. Section 4 obtains long-run growth rates. Section 5 investigates the equilibrium path by using both an analytical method and numerical simulations. Section 6 concludes the paper.

2 Model

Consider a closed economy that consists of the manufacturing and service sectors. In the manufacturing sector, manufactured goods are produced with labor inputs and intermediate service inputs. In the service sector, services are produced with only labor inputs. Consumers consume both manufactured goods and services. Intermediate service inputs can be regarded as outsourcing by manufacturing firms. Accordingly, service firms supply both services for consumers and services for manufacturing firms (i.e., outsourcing).

8) In our model, only labor is the primary factor of production. For an analysis of a service-oriented economy using models that consider capital accumulation, see Kongsamut *et al*. (2001), Klyuev (2005), Bonatti and Felice (2008), and Batabyal and Nijkamp (2013).

2.1 Firms

We specify the production functions of both sectors as follows

$$Q_m = A_m \left[\beta^{\frac{1}{\psi}} (hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}} S^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad \beta \in (0,1),\ \psi > 0,\ \psi \neq 1, \tag{1}$$

$$Q_s = A_s(hL_s), \tag{2}$$

where Q_m denotes the output of manufacturing; Q_s the output of services; L_m the employment of manufacturing; L_s the employment of services; S the intermediate service inputs in manufacturing; A_m the productivity specific to manufacturing; h the level of human capital; β a positive parameter; and ψ the elasticity of substitution between labor inputs and intermediate service inputs. Oulton (2001), who considers intermediate business services in manufacturing, assumes that $\psi > 1$. In Section 3, we refer to the size of ψ. A_s denotes the productivity specific to services. Human capital is accumulated through workers themselves consuming services, and accordingly, both L_m and L_s are multiplied by h.

Profits of manufacturing firms π_m and service firms π_s are provided as follows

$$\pi_m = p_m Q_m - (wL_m + p_s S), \tag{3}$$

$$\pi_s = p_s Q_s - wL_s, \tag{4}$$

where p_m denotes the price of manufactured goods and p_s the price of services. The wage rate is denoted by w. Suppose that labor is perfectly free to move between the two sectors. Then, the nominal wages in both sectors are equalized.

2.2 Consumers

We specify the problem of utility maximization of consumers. Suppose that the representative consumer solves the following optimization problem

$$\max_{c_m, c_s} u = \left[\alpha^{\frac{1}{\sigma}} c_m^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (c_s + \gamma)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

$$\alpha \in (0,1),\ \sigma > 0,\ \sigma \neq 1,\ \gamma > 0,$$

$$\text{s.t. } p_m c_m + p_s c_s = w, \tag{6}$$

where c_i denotes per capita consumption ($c_i = C_i/L$); and σ the elasticity of substitution between the two types of consumption. Following many previous studies on the service economy, we assume $\sigma < 1$, which implies that the demand for services is inelastic to price. The parameter α denotes a positive parameter governing the weight of expenditure for manufacturing; and γ a positive parameter. Such a nonhomothetic preference is also adopted by Iscan (2010). When $\gamma = 0$, the preference is homothetic, and hence, the income elasticities of manufacturing consumption and service consumption are unity. When $\gamma > 0$, the preference is nonhomothetic, and hence, the income elasticity of manufacturing demand is less than unity and that of service demand is greater than unity. Introducing a positive γ does not affect the dynamics of the employment share very much. However, it does affect the dynamics of the consumption ratio C_s/C_m. Baumol (1967) assumes that the consumption ratio remains constant over time. The assumption $\gamma > 0$ together with $\sigma < 1$ corresponds to Baumol's Assumption 2.

2.3 Labor and goods markets

Suppose that total labor supply L is constant. Then, the labor market-clearing condition is given by

$$L_m + L_s = L. \tag{7}$$

The goods market-clearing condition is given by

$$Q_m = C_m, \tag{8}$$

$$Q_s = C_s + S. \tag{9}$$

All of the manufactured goods are used for final consumption. On the other hand, services are used for both final consumption and intermediate inputs.

2.4 Productivity growth

Here, we define economic growth used in this study. In our model, there is no capital accumulation and population growth, and hence, productivity growth is the engine of growth. In what follows, we refer to an increase in the total factor productivity (TFP) of the whole economy g_{TFP} as economic growth.[9)]

9) The expression "TFP" is used to describe the case in which there are some factors of production. In contrast, in our model, labor is the sole primary factor of production, and so, the expression "TFP" may not be appropriate. In this case, the expression "total labor productivity (TLP)" is more appropriate. However, we refer to TFP because it is a more general expression than TLP.

The growth rate of the TFP of the manufacturing sector $g_{\text{TFP},m}$ is given by

$$g_{\text{TFP},m} = g_{A_m} + \frac{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}}}{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}}S^{\frac{\psi-1}{\psi}}}\, g_h, \tag{10}$$

where $g_x \equiv \dot{x}/x$ denotes the growth rate of a variable x.

On the other hand, the growth rate of the TFP of the service sector $g_{\text{TFP},s}$ is given by

$$g_{\text{TFP},s} = g_{A_s} + g_h. \tag{11}$$

Let us specify g_{A_m}, g_{A_s}, and g_h that appear in Equations (10) and (11).

First, the productivity specific to A_s increases exogenously, which is specified as follows

$$A_s(t) = A_{s,0}(1+\mu\theta t)^{\frac{1}{\theta}}, \quad \theta \geq 0,\ \mu > 0, \tag{12}$$

where $A_{s,0}$ denotes the initial level of A_s; and μ and θ are positive parameters. In Sasaki (2012), A_s is assumed to be constant. As explained later, whether A_s increases considerably affects the results.

When specifying an exogenous and continuous increase in a variable, we usually assume that the variable increases at a constant rate, and hence, we use an exponential function. Instead, based on the study of Groth *et al.* (2010), we use a more flexible specification than an exponential function. From Equation (12), the growth rate of A_s is given by

$$g_{A_s} = \frac{\mu}{1+\mu\theta t}. \tag{13}$$

From this, we know that if $\theta > 0$, the growth rate of A_s declines over time and $\lim_{t\to+\infty} g_{A_s} = 0$.

Equation (12) includes some special cases.

$$A_s = \begin{cases} A_{s,0}e^{\mu t} & \text{if } \theta = 0, \\ A_{s,0}(1+\mu t) & \text{if } \theta = 1, \\ A_{s,0} & \text{if } \theta \to +\infty. \end{cases} \tag{14}$$

That is, A_s becomes an exponential function if $\theta \to 0$, a linear function of time if $\theta = 1$, and constant if $\theta \to +\infty$.

Next, we specify the productivity specific to the manufacturing sector, A_m. We assume that A_m is an increasing function of the knowledge stock K_m.

$$A_m = K_m^{\phi}, \quad \phi > 0, \tag{15}$$

where ϕ denotes the elasticity of A_m with respect to K_m. We assume that the knowledge stock depends on the production experience that is accumulated until now; we specify the knowledge stock as follows

$$K_m = \exp\left[\int_{-\infty}^{t} \frac{L_m(\tau)}{L(\tau)}\, d\tau\right]. \tag{16}$$

Note that the production experience K_m is measured by L_m/L. We use the manufacturing employment share, and not the level of manufacturing employment, to ascertain that the dynamics of the model hold even when the labor force grows. In addition, we use manufacturing employment, and not manufacturing output, for the following reason. In our model, labor is the sole factor of production; then, an increase in output has a one-to-one relationship with an increase in employment. Therefore, for simplicity, we measure production experience by employment of manufacturing, not by output of manufacturing.

Substituting Equation (16) into Equation (15) and differentiating the resultant expression with respect to time, we obtain

$$\dot{A}_m = \left(\phi \frac{L_m}{L}\right) A_m = \phi\left(1 - \frac{L_s}{L}\right) A_m. \tag{17}$$

That is, A_m becomes an increasing (decreasing) function of the employment share of manufacturing (services).[10] Therefore, if the employment share of the service sector increases, the productivity specific to manufacturing decreases.

Then, we specify the accumulation of human capital h. According to Pugno (2006), human capital is accumulated through consumption of services.[11]

$$\dot{h} = \delta c_s^{\lambda}, \quad \delta > 0,\ 0 < \lambda \leq 1, \tag{18}$$

where δ denotes a positive parameter and captures the efficiency of human capital accumulation. Note that Equation (18) is different from the specifications of Pugno (2006) and Sasaki (2012). They assume that $\lambda = 1$ whereas we assume that $0 < \lambda \leq 1$. The reason is that we investigate a broader situation, which includes a knife-edge case $\lambda = 1$. As will be shown

10) De Vincenti (2007) adopts a specification in which the growth rate of manufacturing productivity is a decreasing function of the employment share of manufacturing.

11) The relationship between health services and economic growth is analyzed in Van Zon and Muysken (2001).

later, if we assume that $0 < \lambda < 1$, economic growth is not sustainable in the very long run. Instead, we introduce productivity specific to services, that is, A_s and we consider the case in which A_s increases through time, which can offset the diminishing effect of $\dot{h}$.

We derive the growth rate of TFP of the whole economy. Note that in our model, services are used for intermediate inputs in manufacturing. In this case, in line with Oulton (2001) and Sasaki (2007), it is appropriate to use Domar aggregation presented by Domar (1961).[12)] Using Domar aggregation, we obtain the growth rate of TFP as follows

$$g_{\mathrm{TFP}} = \frac{p_m Q_m}{\mathrm{GDP}} g_{\mathrm{TFP},m} + \frac{p_s Q_s}{\mathrm{GDP}} g_{\mathrm{TFP},s}. \tag{19}$$

Gross domestic product (GDP) is given by $p_m C_m + p_s C_s = wL$. Equation (19) considers that services are used for intermediate inputs, and hence, the sum of the weights exceeds unity, that is, $p_m Q_m + p_s Q_s > \mathrm{GDP}$.

3 Instantaneous equilibrium

At some point in time, A_m, A_s, and h are given. We obtain the instantaneous equilibrium as follows

1. By solving the utility-maximization problem of consumers, we obtain demand functions for manufacturing and services, which depend on p_m, p_s, and w.

2. By solving the profit-maximization problem of firms, we obtain the optimal ratio of L_m and S, which depends on p_s and w.

3. By solving the zero-profit conditions of firms, we obtain p_m and p_s, which depend on A_m, A_s, h, and w.

4. By solving the goods market-clearing condition, we obtain L_m and L_s.

From the utility-maximization problem, we obtain demand functions for manufacturing and services.

$$C_m = \frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}\gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right] L, \tag{20}$$

12) In addition, for Domar aggregation, see Hulten (1978) and Ten Raa and Schettkat (2001).

$$C_s = \frac{w - \dfrac{\alpha}{1-\alpha} p_m \left(\dfrac{p_s}{p_m}\right)^{\sigma} \gamma}{p_s + \dfrac{\alpha}{1-\alpha} p_m \left(\dfrac{p_s}{p_m}\right)^{\sigma}} \cdot L. \tag{21}$$

From the profit-maximization problem of manufacturing and service firms, that is, the equalization between the marginal rate of substitution and the relative price, we obtain

$$S = \frac{1-\beta}{\beta} A_s^{\psi} (hL_m). \tag{22}$$

That is, intermediate service inputs are linear with respect to effective labor hL_m.

From the zero-profit conditions, we obtain

$$p_m = \frac{w}{\beta^{\frac{1}{\psi-1}} A_m h \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{1}{\psi-1}}}, \tag{23}$$

$$p_s = \frac{w}{A_s h}. \tag{24}$$

Hence, the relative price is given by

$$\frac{p_s}{p_m} = \beta^{\frac{1}{\psi-1}} \frac{A_m}{A_s} \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{1}{\psi-1}}. \tag{25}$$

With Equation (22), we can express Q_m as a function of L_m.

$$Q_m = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}. \tag{26}$$

That is, manufacturing output is linear with respect to manufacturing employment.

The share of intermediate services in manufacturing output is given by

$$\frac{p_s S}{p_m Q_m} = \frac{1}{1 + \frac{\beta}{1-\beta} A_s^{1-\psi}}. \tag{27}$$

When A_s is constant trough time, the share of intermediate services in manufacturing is also constant. When A_s increases through time, the share of intermediate services in manufacturing increases if $\psi > 1$ but decreases if $\psi < 1$. As shown in Table 1, this share increases through time in reality, and hence, we assume $\psi > 1$ in the following analysis.

From the goods market-clearing condition $C_m = Q_m$, we obtain

$$\frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha}\, p_m \left(\frac{p_s}{p_m}\right)^{\sigma} \gamma}{p_s + \frac{\alpha}{1-\alpha}\, p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right] L = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}. \tag{28}$$

Solving equation (28) for L_m/L, we obtain

$$\frac{L_m}{L} = \frac{\frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha}\, p_m \left(\frac{p_s}{p_m}\right)^{\sigma} \gamma}{p_s + \frac{\alpha}{1-\alpha}\, p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right]}{\beta^{\frac{1}{\psi-1}} A_m h \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}}. \tag{29}$$

Consumption of services is given by $C_s = Q_s - S$, which is rewritten in per capita terms as

$$c_s = A_s h \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi} h \left(1 - \frac{L_s}{L}\right). \tag{30}$$

From this, the rate of human capital accumulation is given by

$$\frac{\dot{h}}{h} = \delta A_s^{\lambda} h^{\lambda-1}\left[\frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi-1}\left(1 - \frac{L_s}{L}\right)\right]^{\lambda}. \tag{31}$$

For the rate of human capital accumulation to be positive, that is, $\dot{h}/h > 0$, we need

$$\frac{L_s}{L} > \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{1-\psi}}. \tag{32}$$

The ratio of manufacturing output to GDP and the ratio of service output to GDP are given by the following equations

$$\frac{p_m Q_m}{\text{GDP}} = \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)\left(1 - \frac{L_s}{L}\right), \tag{33}$$

$$\frac{p_s Q_s}{\text{GDP}} = \frac{L_s}{L}, \tag{34}$$

respectively.

The growth rate of the TFP of manufacturing is given by

$$g_{\text{TFP},m} = g_{A_m} + \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{\psi-1}}\, g_h. \tag{35}$$

Therefore, the growth rate of the TFP for the whole economy is given by

$$g_{\mathrm{TFP}} = \left(1 - \frac{L_s}{L}\right)\left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) g_{\mathrm{TFP},m} + \frac{L_s}{L} g_{\mathrm{TFP},s}. \tag{36}$$

4 Long-run growth rates

We investigate the growth rate of each sector when the tendency toward services progresses and finally, $L_s/L = 1$. In the following analysis, we consider two cases: $\lambda = 1$ and $0 < \lambda < 1$.

4.1 Constant returns to scale in human capital accumulation

When the human capital accumulation function exhibits constant returns to scale with respect to consumption of services, that is, $\lambda = 1$, each growth rate when $L_s/L = 1$ is given by

$$\lim_{L_s/L \to 1} g_{A_m} = 0, \tag{37}$$

$$\lim_{L_s/L \to 1} g_h = \delta A_s, \tag{38}$$

$$\lim_{L_s/L \to 1} g_{\mathrm{TFP},m} = \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{\psi-1}} \delta A_s, \tag{39}$$

$$\lim_{L_s/L \to 1} g_{\mathrm{TFP},s} = g_{A_s} + \delta A_s, \tag{40}$$

$$\lim_{L_s/L \to 1} g_{\mathrm{TFP}} = g_{\mathrm{TFP},s} = g_{A_s} + \delta A_s. \tag{41}$$

Equations (40) and (41) shows that the growth rate of the TFP for the whole economy is equal to the growth rate of the TFP of the service sector.

These equations show that long-run growth rates depend significantly on the growth of A_s. When $\theta \geq 0$, and hence, A_s increases through time, g_{TFP} continues to increase through time. This suggests that the level of TFP continues to increase indefinitely and diverges to infinity within finite time. Accordingly, in the case of $\lambda = 1$, as long as A_s continues to increase through time, irrespective of whether exponentially, the level of TFP becomes infinity, and hence, we cannot permit the growth of A_s. In contrast, when $\theta \to +\infty$, and hence, A_s stays constant through time, g_{TFP} converges to a positive constant value $g_{\mathrm{TFP}} = \delta A_{s,0}$. That is, the economy grows at a positive and constant rate in the long run.

Summarizing these results, we obtain the following proposition.

Proposition 1. *When the human capital accumulation function exhibits constant returns to scale with respect to consumption of services, the growth rate of the productivity specific to* *the service sector has to be zero. In this case, if the tendency toward services completely finishes, that is, the employment share of the service sector approaches unity, the growth rate of TFP for the whole economy approaches $\delta A_{s,0}$ in the long run.*

4.2 Decreasing returns to scale in human capital accumulation

When the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services, that is, $0 < \lambda < 1$, the growth rate of each sector when $L_s/L = 1$ is given by

$$\lim_{L_s/L \to 1} g_{A_m} = 0, \tag{42}$$

$$\lim_{L_s/L \to 1} g_h = \delta A_s^{\lambda} h^{\lambda-1}, \tag{43}$$

$$\lim_{L_s/L \to 1} g_{\text{TFP},m} = \frac{1}{1+\frac{1-\beta}{\beta}} \delta A_s^{\lambda} h^{\lambda-1}, \tag{44}$$

$$\lim_{L_s/L \to 1} g_{\text{TFP},s} = g_{A_s} + \delta A_s^{\lambda} h^{\lambda-1}, \tag{45}$$

$$\lim_{L_s/L \to 1} g_{\text{TFP}} = g_{\text{TFP},s} = g_{A_s} + \delta A_s^{\lambda} h^{\lambda-1}. \tag{46}$$

Suppose that A_s is constant or increases less than exponentially. Since $g_h \geq 0$ from Equation (43), h is nondecreasing with respect to time. Moreover, since $\partial g_h/\partial h \leq 0$ from Equation (43), the growth rate of human capital g_h converges to zero after enough time passes. This means that $g_{\text{TFP},m}$, $g_{\text{TFP},s}$, and g_{TFP} converge to zero. Therefore, in this case, the growth rate of the TFP for the whole economy converges to zero in the long run.

In contrast, suppose that A_s increases exponentially. In this case, even when the rate of human capital accumulation is zero, g_{TFP} converges to μ because A_s increases at a positive and constant rate μ. That is, the economy grows at a positive and constant rate in the long run.

Summarizing these results, we obtain the following proposition.

Proposition 2. *Suppose that the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services. When the exogenous productivity specific to the service sector stays constant through time or increases less than exponentially, the growth rate of the TFP for the whole economy converges to zero in the long run if the tendency toward services completely finishes. In contrast, when the exogenous productivity specific to the service sector increases at a constant rate μ, the growth rate of the TFP for the whole economy converges to μ in the long run if the tendency toward services completely finishes.*

5 Equilibrium path and numerical simulations

The analysis in Section 4 assumes that the tendency toward services progresses (i.e., $L_s/L \to 1$) and that enough time passes (i.e., $t \to +\infty$). However, the analysis does not reveal whether the employment share of the service sector increases; moreover, it does not reveal each growth rate along the transitional dynamics path. Accordingly, we investigate the employment share of the service sector and each growth rate along the transitional dynamics path by using both analytical and numerical methods.

5.1 Constant returns to scale in human capital accumulation

In this case, from Equation (36), the growth rate of the TFP for the whole economy leads to

$$g_{\text{TFP}} = \beta^{-1} \frac{L_m}{L}(g_{A_m} + \beta g_h) + \left(1 - \frac{L_m}{L}\right) g_h. \tag{47}$$

From this, we see that the elasticity of substitution between labor inputs and intermediate service inputs in manufacturing, that is, ψ, never affects the result.

The value of $g_{\text{TFP}}(0)$, that is, the growth rate of the TFP for the whole economy when the employment share of services is zero, must be positive.

$$g_{\text{TFP}}(0) = \beta^{-1}[\phi - \delta(1 - \beta)]. \tag{48}$$

From this, we require the following condition

$$\phi - \delta(1 - \beta) > 0. \tag{49}$$

Hereafter, we assume that this condition holds in the case of $\lambda = 1$.

Comparing $g_{\text{TFP}}(0)$ with $g_{\text{TFP}}(1)$, which is obtained when $L_s/L = 1$, we can see whether an increase in the employment share of services increases or decreases the rate of economic growth in the long run. If $g_{\text{TFP}}(0) < g_{\text{TFP}}(1)$, the rate of economic growth increases in the long run. In contrast, if $g_{\text{TFP}}(0) > g_{\text{TFP}}(1)$, the rate of economic growth decreases in the long run. From this, we obtain

$$g_{\text{TFP}}(0) \lesseqgtr g_{\text{TFP}}(1) \iff \phi \lesseqgtr \delta. \tag{50}$$

With the calculation of g_{TFP}, we find that it is a quadratic function of L_s/L, which is given by

$$
\begin{aligned}
g_{\mathrm{TFP}} &= \frac{\phi}{\beta}\left(\frac{L_s}{L}\right)^2 + \frac{\delta - 2\phi}{\beta}\frac{L_s}{L} + \frac{\phi - \delta(1-\beta)}{\beta} \\
&= \frac{\phi}{\beta}\left(\frac{L_s}{L} - \frac{2\phi - \delta}{2\phi}\right)^2 + \delta\left(\frac{4\beta\phi - \delta}{4\beta\phi}\right). \qquad (51)
\end{aligned}
$$

This result is almost the same as that of Sasaki (2012). With $\beta = 1$, Equation (51) is the same as that of Sasaki (2012). Hence, we obtain the following proposition.

Proposition 3. *If* $2\phi < \delta$, *the growth rate of* TFP *increases with the employment shift toward services and converges to* δ. *If* $2\phi > \delta$, *the growth rate of* TFP *decreases until the employment share of services reaches* $(L_s/L)^* = (2\phi - \delta)/2\phi$, *and from then onward, it increases with the employment shift toward services, finally converging to* δ.

This proposition is shown in Figures 2–4.

[Figures 2–4 around here]

When deriving Proposition 3, we assume that the employment share of services increases over time. In our model, productivity growth and employment share are dependent on each other. Therefore, we must analyze the dynamics of employment share and productivity growth simultaneously.

Each sector's employment share, each sector's productivity growth, and the productivity growth of the whole economy depend on A_m and h. Moreover, $\dot{A}_m$ and $\dot{h}$ depend on A_m and h. As a result, if we provide the initial values of A_m and h, and if we examine the system of differential equations of A_m and h, we can determine the time paths of A_m and h. In addition, by using this result, we can obtain the time paths of all variables. However, the differential equations of our model are nonlinear, and hence, analytical solutions are difficult to obtain. Accordingly, we use numerical simulations.

Here, we investigate the following three cases.

$$\text{Case 1} : \phi < \delta,\ 2\phi < \delta, \qquad (52)$$

$$\text{Case 2} : \phi < \delta,\ 2\phi > \delta, \qquad (53)$$

$$\text{Case 3} : \phi > \delta. \qquad (54)$$

In Cases 1 and 2, we have $g(0) < g(1)$, and in Case 3, we have $g(0) > g(1)$.

We set the parameters as in Table 1 and the initial values as $A_m(0) = h(0) = 1$. Then, we change the elasticity of substitution σ from 0.1 to 0.9 in intervals of 0.1.

[Table 2 around here]

Figures 5–10 show the results of the numerical simulation. Figures 5, 7, and 9 show the time paths of the employment share of services. Figures 6, 8, and 10 show the time paths of the growth rate of the TFP for the whole economy. In all cases, the employment share of services increases over time. In Cases 1 and 2, the growth rate of TFP first declines, then increases, and finally, converges to δ. In Case 3, the growth rate of TFP increases constantly over time, which is similar to the result of Oulton (2001). These results are obtained assuming that $\theta \to +\infty$.

[Figures 5–10 around here]

5.2 Decreasing returns to scale in human capital accumulation

Let $0 < \lambda < 1$ in Equation (36). Then, we obtain the growth rate of TFP as follows

$$g_{\mathrm{TFP}} = \phi \frac{1-\beta}{\beta} A_s^{\psi-1} \left(1 - \frac{L_s}{L}\right)^2 + \delta A_s^{\lambda} h^{\lambda-1} \left[\left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi-1}\right]^{\lambda} + g_{A_s} \frac{L_s}{L}. \tag{55}$$

Note that A_s and g_{A_s} depend on time, and h depends on L_s/L.

In this case, analytical solutions are difficult to obtain, and thus, we resort to numerical simulations.

We set the parameters as in Table 3 and the initial values as $A_m(0) = h(0) = 1$. Then, we change the elasticity of substitution σ from 0.1 to 0.9 in intervals of 0.1.

[Table 3 around here]

As stated in Equation (41), as long as enough time passes, the growth rate of the TFP for the whole economy approaches zero. However, along with transitional dynamics, the employment share of services and the growth rate of TFP display interesting behavior. In Case 4, the employment share of services monotonically increases whereas the growth rate of TFP first declines, then increases, and finally, declines after some time.

[Figures 11 and 12 around here]

The time paths of the growth rate of TFP in Case 4 can be explained as follows. First, as the employment share of services increases, the learning-by-doing effect of manufacturing weakens, which lowers the economic growth rate. However, as the employment share of services increases further, human capital accumulation proceeds further, and productivity specific to services increases with the passage of time. These contribute to an increase in the growth rate of TFP. Nevertheless, as enough time passes, there is a diminishing positive effect of an increasing service employment share on the growth rate of TFP. In the long run, the growth rate of TFP approaches zero. The phase of the decreasing to increasing effect is similar to the time path obtained in Sasaki (2012) while the phase of the increasing to decreasing effect is similar to the time path obtained in Sasaki (2007).

Our model introduces intermediate services into Sasaki (2012) and considers decreasing returns to scale in the human capital accumulation function whereas the constant returns to scale function is used in Sasaki (2012). In addition, productivity specific to the service sector A_s is introduced in our model. We explain which one of these three extensions is crucial to our results.

Table 4 summarizes our results. When $\lambda = 1$, that is, the human capital accumulation function exhibits constant returns to scale and A_s is constant, TFP growth decreases as the employment share of services increases but at some point in time, it begins to increase and converges to a constant value, irrespective of whether services are used for intermediate inputs. This result is the same as that of Sasaki (2012). When $\lambda = 1$ and A_s increases through time, the long-run growth rate of TFP continues to increase and the level of TFP diverges to infinity within finite time.

[Table 4 around here]

Next, we investigate the case in which the human capital accumulation function exhibits decreasing returns to scale. This case is more realistic than the constant returns to scale case.

First, when A_s is constant, TFP growth continues to decrease and converges to zero as the employment share of services increases, irrespective of whether services are used for intermediate inputs. In the case of decreasing returns to scale, a shift in employment share toward services decreases the growth rate of human capital.

Second, when A_s increases less than exponentially, the growth rate of TFP first declines, then increases, declines after some time, and converges to zero in the long run, irrespective of whether services are used for intermediate inputs.

Third, when A_s increases exponentially, TFP growth decreases as the employment share of services increases but at some point in time, it begins to increase and converges to a constant value, irrespective of whether services are used for intermediate inputs. This result is the same as that obtained in the case in which $\lambda = 1$ and A_s are constant.

Summarizing these results, we obtain the following proposition.

Proposition 4. *Suppose that the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services. In addition, suppose that the growth rate of the exogenous productivity specific to the service sector diminishes with time. Then, as the employment share of the service sector increases, the growth rate of the TFP for the whole economy decreases first, begins to increase after some time, decreases again, and finally, approaches zero.*

These results show that the results are not affected by whether services are used for intermediate inputs. In addition, since the result of the case in which $\lambda = 1$ and A_s is constant is the same as that of the case in which $\lambda < 1$ and A_s increases exponentially, it is crucial for the results that either A_s or h increase exponentially. When the human capital accumulation function exhibits decreasing returns to scale, a shift in employment share toward services decreases the growth rate of human capital. This implies that the long-run growth rate of TFP converges to zero unless an increase in A_s exceeds the decrease of human capital accumulation.

5.3 Interpretations

We explain in detail the reason why we obtain the above results. We restrict ourselves to the case in which $\lambda = 1$ and $\theta \to +\infty$ and the case in which $0 < \lambda < 1$ and $0 < \theta \ll +\infty$.

The growth rate of the TFP for the whole economy comprises four components.

$$g_{\text{TFP}} = \underbrace{\frac{p_m Q_m}{\text{GDP}}}_{\text{(A)}} \underbrace{g_{\text{TFP},m}}_{\text{(B)}} + \underbrace{\frac{p_s Q_s}{\text{GDP}}}_{\text{(C)}} \underbrace{g_{\text{TFP},s}}_{\text{(D)}} \tag{56}$$

$$= \underbrace{\left(1 - \frac{L_s}{L}\right)\left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)}_{\text{(A)}} \underbrace{g_{\text{TFP},m}}_{\text{(B)}} + \underbrace{\frac{L_s}{L}}_{\text{(C)}} \underbrace{g_{\text{TFP},s}}_{\text{(D)}}. \tag{57}$$

When $\lambda = 1$ and $\theta \to +\infty$, (A) and (B) decrease while (C) and (D) increase through time. When the employment share of the service sector is small, the negative effect of decreasing (A) and (B) is large, whereas the positive effect of increasing (C) and (D) is small. Accordingly, the growth rate of TFP decreases. However, as the employment share of the service sector increases, the positive effect of (C) and (D) exceeds the negative effect of (A) and (B), and finally, converges to a positive and constant rate.

The case in which $0 < \lambda < 1$ and $0 < \theta \ll +\infty$ is more complicated. Part (A) continues to decrease with time when $\psi > 1$.[13] Part (B) continues to decrease with time when $\psi > 1$.[14]

13) Even when $\psi < 1$, Part (A) continues to decrease with time.

14) When $\psi < 1$, Part (B) decreases first, then increases, and finally, decreases.

Part (C) continues to increase through time and converges to unity. Part (D) decreases first, then increases, and finally, decreases. Combined with these effects, g_{TFP} decreases first, begins to increase after some time, decreases again, and finally, approaches zero.

Figure 13 decomposes the growth rate of the TFP for the whole economy into Parts (A) and (B) and Parts (C) and (D) when $\sigma = 0.5$. Parts (C) and (D) turn from increasing to decreasing, which is caused by the evolution of the rate of human capital accumulation g_h.

$$g_h = \underbrace{\delta A_s^{\lambda} h^{\lambda-1}}_{\text{(a)}} \left[\left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1} \right) \underbrace{\frac{L_s}{L}}_{\text{(b)}} - \frac{1-\beta}{\beta} A_s^{\psi-1} \right]^{\lambda} . \tag{31'}$$

Part (a) decreases and finally approaches zero, which has a negative effect on g_h, whereas Part (b) increases with a rise in L_s/L, which has a positive effect on g_h. Therefore, as L_s/L increases, g_h increases, begins to decrease after some time, and finally, converges to zero.

[Figure 13 around here]

In Figure 13, the negative effects of (A) and (B) are dominant in Phase I, and hence, the growth rate of TFP decreases. In Phase II, the positive effects of (C) and (D) are dominant, and hence, the growth rate of TFP increases. Finally, in Phase III, the effect of decreasing returns to scale in human capital accumulation is dominant, and the growth rate of TFP decreases, and finally, approaches zero.

We mention the implications of outsourcing. We can regard $p_s S/(p_m Q_m)$, that is, the share of intermediate services in manufacturing output, as a measure of outsourcing. In our model, outsourcing increases through time, and this corresponds to the actual data (see Table 1). Then, in our model, as outsourcing increases, $g_{\text{TFP},m}$ as well as g_{TFP} decrease and converge to zero in the long run if human capital accumulation exhibits decreasing returns to scale. This implies that even if manufacturing firms conduct outsourcing to raise their productivity, their productivity growth decreases. Outsourcing expands the service sector (i.e., an increase in L_s/L), which decreases the g_{A_m} that constitutes $g_{\text{TFP},m}$. Moreover, continuous increases in L_s/L make g_h converge to zero in the long run. Therefore, as outsourcing increases, manufacturing TFP growth decreases and the TFP growth for the whole economy decreases.

6 Conclusion

This study extended Baumol's unbalanced growth model to investigate the relationship between the tendency toward services and economic growth. In our model, the productivity growth of both manufacturing and services were determined endogenously, while services were used for intermediate inputs in manufacturing as well as for final consumption.

We showed that the results are affected by the specifications of human capital accumulation and productivity specific to the service sector. If the human capital accumulation function has constant returns to scale with respect to per capita consumption of services, the result is similar to that of Sasaki (2012), who does not consider intermediate service inputs. In this case, we obtain a U-shaped relationship between the employment share of services and economic growth. In contrast, if the human capital accumulation function has decreasing returns to scale with respect to per capita consumption of services, we obtain the combination of a U-shaped and an inverted U-shaped relationship between the employment share of services and economic growth.

Our model considers both the case of services playing the role of intermediate inputs and the case of consumption of educational and health services leading to human capital accumulation. Hence, the model is more realistic than the previous literature. The main result is that the relationship between the tendency toward services and economic growth is not monotonous. This result has implications for empirical analysis regarding the relationship between the tendency toward services and economic growth.

Hartwig (2012) empirically tests the three hypotheses about the relationship between the tendency toward services and economic growth: Baumol's (1967) hypothesis—a shift in employment share toward the services sector decreases the rate of economic growth; Kaldor's (1957) hypothesis—the growth rate of per capita real GDP is almost constant despite the presence of structural changes; and Pugno's (2006) hypothesis—a shift in the employment share toward services increases the economic growth rate. Hartwig (2012) concludes that no evidence is found to support Pugno's hypothesis and it is not evident whether structural change is compatible with balanced growth or whether it leads to long-term stagnation.

Our model is able to explain why such ambiguous empirical results are obtained. The relationship between the employment shift toward services and economic growth is not monotonous. Consequently, if we perform a cross-country analysis for countries whose stages of development are diversified, we are likely to obtain ambiguous results. Moreover, if we perform a time-series analysis for a specific country, we are likely to obtain different results depending on the period of data used.

Finally, our results imply that irrespective of whether services are used for intermediate inputs, to achieve sustainable economic growth when the employment share of services continues to increase, it is necessary for the productivity of services to increase sustainably.

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Tables and Figures

Table 1: Percentage of intermediate services in manufacturing output (Source: Schettkat and Yocarini, 2003)

	France	Germany	UK	US
1968			9.16	
1972	10.82			11.91
1977	11.11			12.65
1978		12.98		
1979			10.22	
1990	16.77	16.73	13.14	13.19

Table 2: List of parameters ($\lambda = 1$)

	α	β	γ	δ	ϕ
Case 1	0.5	0.5	1	0.02	0.015
Case 2	0.5	0.9	1	0.03	0.02
Case 3	0.5	0.5	1	0.02	0.04

Table 3: List of parameters ($0 < \lambda < 1$)

	α	β	γ	δ	ϕ	ψ	λ	μ	θ
Case 4	0.5	0.5	1	0.005	0.02	2	0.9	0.01	0.9

Table 4: Classifications of the time series of TFP growth

	$\lambda = 1$		$0 < \lambda < 1$	
	$\beta = 1$	$0 < \beta < 1$	$\beta = 1$	$0 < \beta < 1$
A_s: constant	$\searrow \nearrow \rightarrow$	$\searrow \nearrow \rightarrow$	$\searrow \rightarrow$	$\searrow \rightarrow$
A_s: less than exponential	explosive	explosive	$\searrow \nearrow \searrow \rightarrow$	$\searrow \nearrow \searrow \rightarrow$
A_s: exponential	explosive	explosive	$\searrow \nearrow \rightarrow$	$\searrow \nearrow \rightarrow$

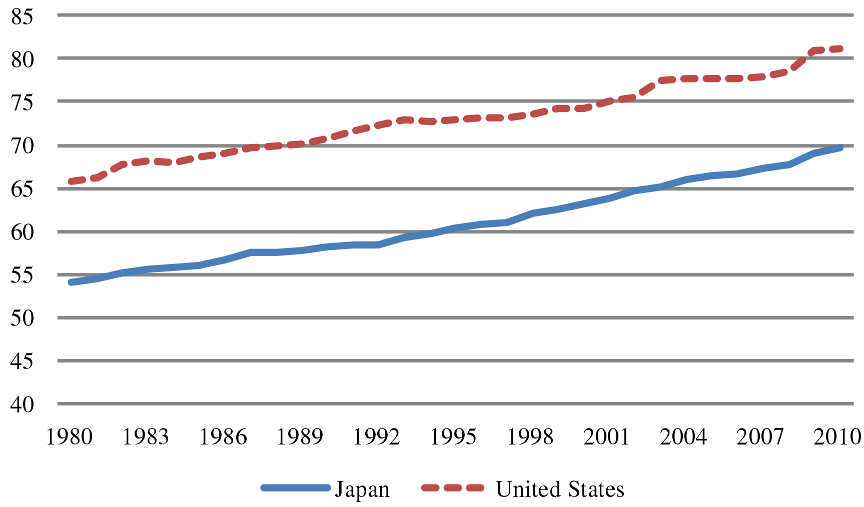

Figure 1: Employment share of services in Japan and the US between 1980 and 2010 (Source: World Bank, *World Development Indicators*)

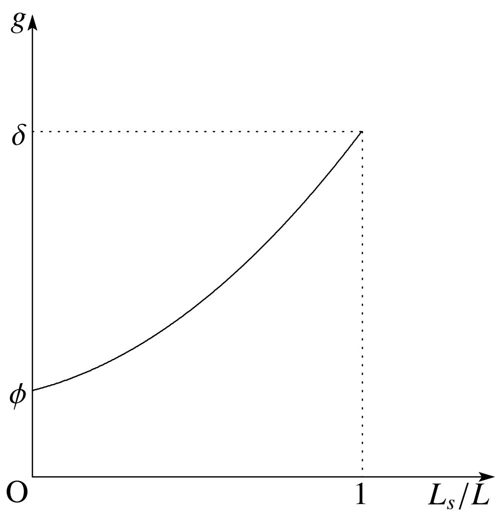

Figure 2: The relationship between L_s/L and g_{TFP} ($\phi < \delta$ and $2\phi < \delta$)

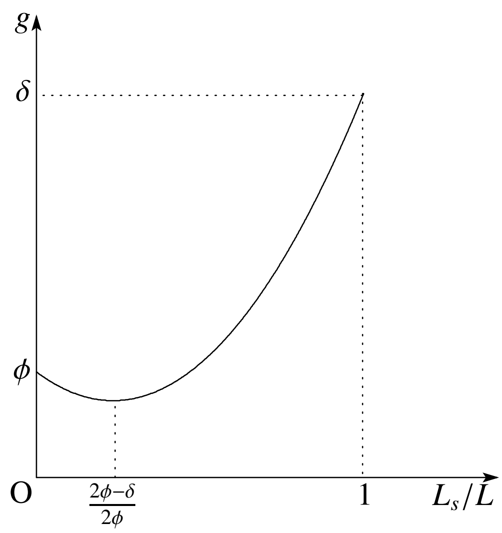

Figure 3: The relationship between L_s/L and g_{TFP} ($\phi < \delta$ and $2\phi > \delta$)

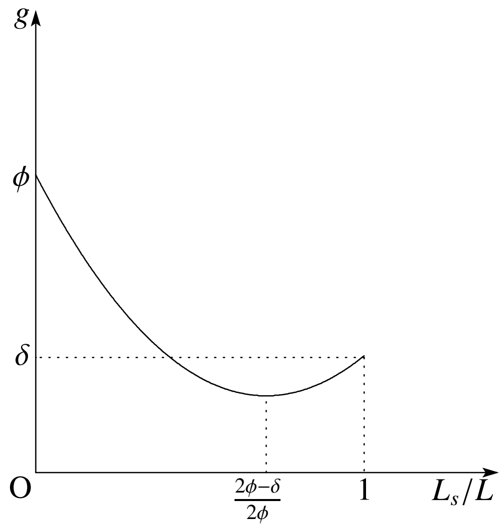

Figure 4: The relationship between L_s/L and g_{TFP} ($\phi > \delta$)

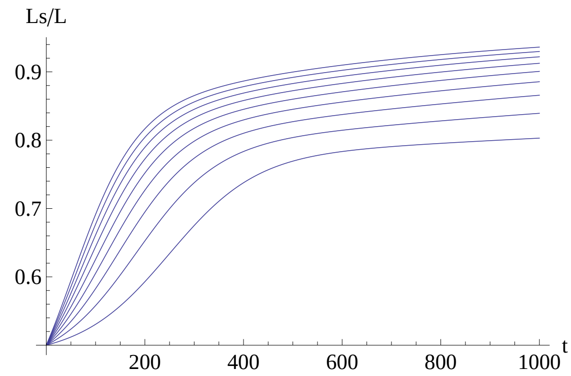

Figure 5: Employment share of services in Case 1

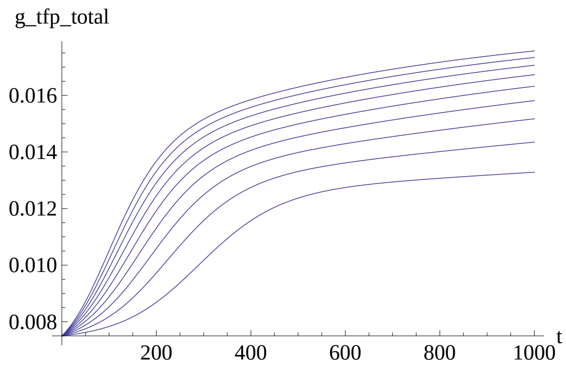

Figure 6: TFP growth rate in Case 1

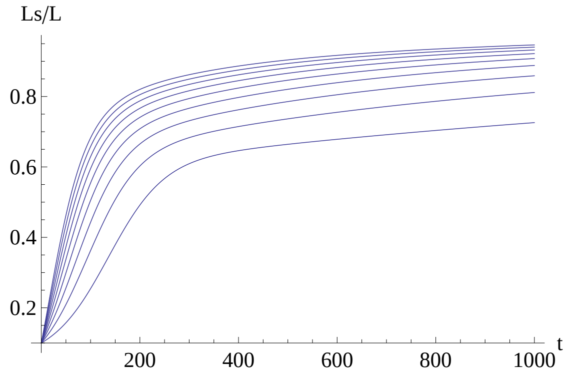

Figure 7: Employment share of services in Case 2

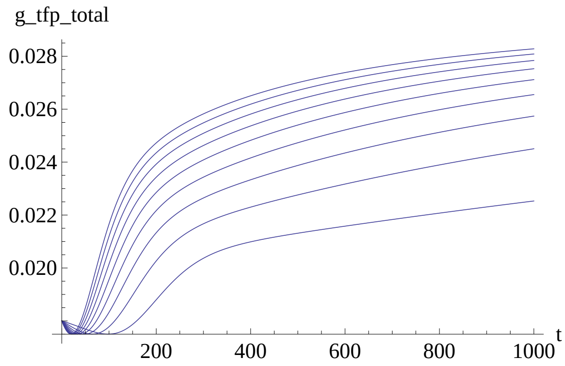

Figure 8: TFP growth rate in Case 2

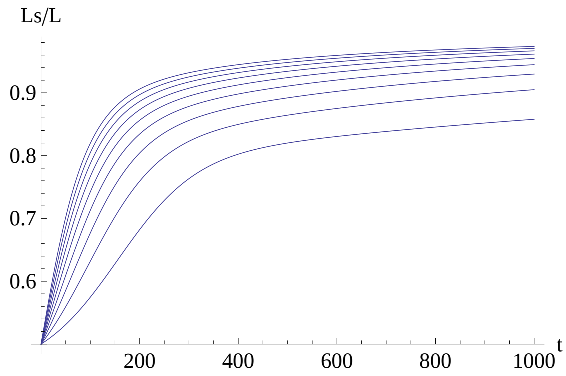

Figure 9: Employment share of services in Case 3

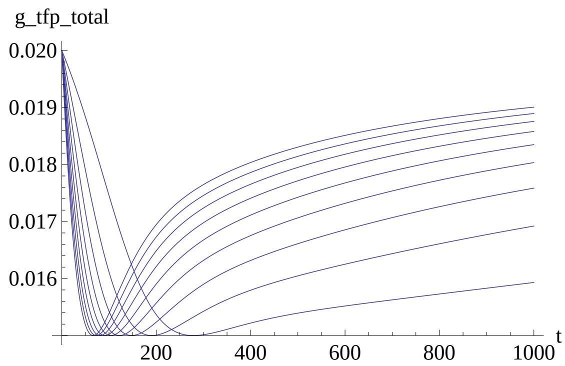

Figure 10: TFP growth rate in Case 3

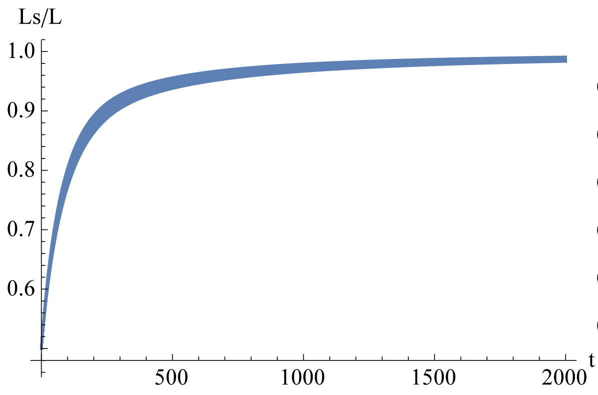

Figure 11: Employment share of services in Case 4

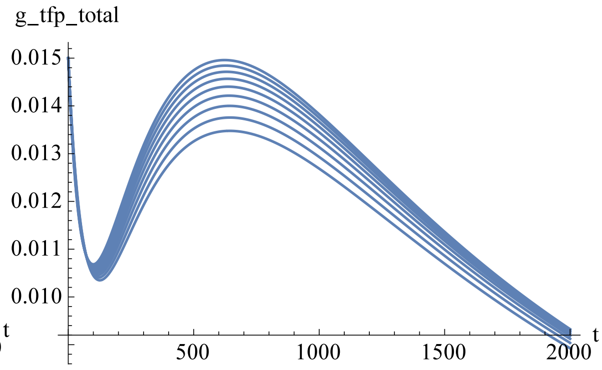

Figure 12: TFP growth rate in Case 4

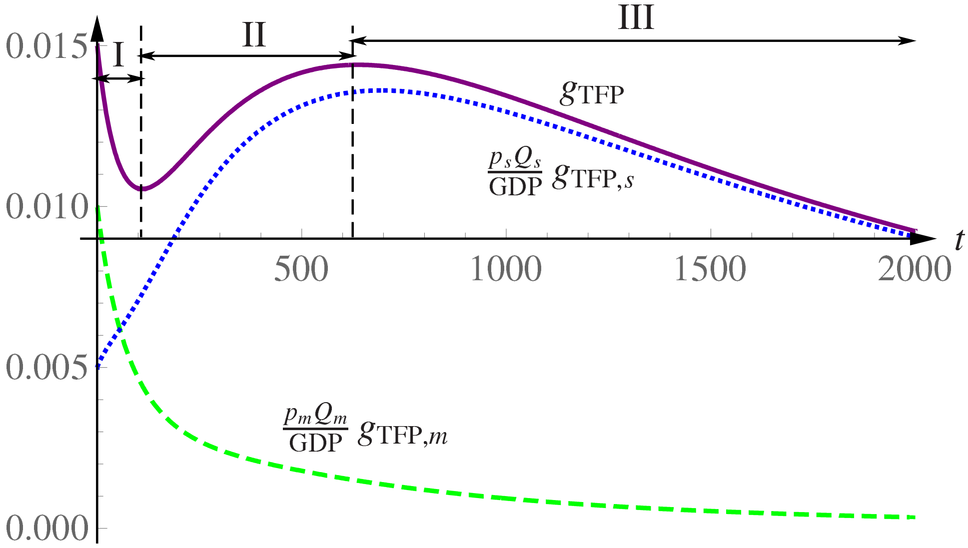

Figure 13: Decomposition of TFP growth in Case 4