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Abstract

This paper develops a model of patent trolls to understand various litigation strategies employed by nonpracticing entities (NPE). We show that when a NPE faces multiple potential infringers who use related technologies, it can gain a credible threat to litigate even when it has no such credibility vis-à-vis any single potential infringer in isolation. This is due to an information externality generated by an early litigation outcome for subsequent litigation. Successful litigation creates an option value against future potential infringers through Bayesian updating. This renders a credible litigation threat against the initial defendant and allows the NPE to extract more rents. We discuss policy implications including the adoption of the British system of "loser-pays" fee shifting and the use of injunctive relief.

Keywords: patent portfolios, patent litigation, non-practicing entities, patent troll
JEL: D43, L13, O3

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1 Introduction

The patent system is designed to protect and promote innovation by granting innovators exclusive rights to commercially exploit their inventions for a limited period of time. However, patent law does not require that only the inventor enforce the patent. Patents can be transferred to other parties and be enforced by whoever owns them (Lemley and Melamed 2013). Recently, the emergence of non-practicing entities (NPEs) as a major driver of patent litigation has spawned a heated debate on their role in the overall patent system and their impacts on innovation. NPEs, also derisively called "patent trolls," are a new organizational form whose sole purpose is to use patents primarily to obtain license fees rather than to support the development of technology.[1] They amass patents not for the purpose of commercializing a new product, but to litigate and demand licensing fees.

The proponents of NPEs emphasize potential positive roles of NPEs. They argue that NPEs help small independent inventors to monetize their intellectual property (IP) rights against potential misappropriation by established companies, thereby inducing more innovation by small inventors. In contrast, the opponents are concerned that NPEs simply raise the costs of innovation and can drag the innovation process. Due to their business models, they seek patents to pursue "freedom to litigate" rather than "freedom to operate." The value of a patent thus can be based on the "exclusion value" rather than the "intrinsic value" when it is held by NPEs (Chien 2010). More importantly, the recent surge in the number of lawsuits initiated by patent trolls became a cause for concern for businesses and policy-makers alike.[2] One recent statistic shows that patent trolls are responsible for 67 percent of all patent lawsuits (Morton and Shapiro 2014). Bessen, Ford, and Meurer (2011) estimate that trolls cost the economy $500 billion over the last twenty years, mostly in the IT industry.

This paper develops a model of patent trolls to understand various litigation strategies employed by nonpracticing entities. We show that when a NPE faces multiple potential infringers who use related technologies, it can gain a credible threat to litigate even when it has no such credibility vis-a-vis any single potential infringer in isolation. This is due to an

[1] NPEs are also called patent asserting entities (PAEs).

[2] Even President Obama expressed his concern about the harmful effects of patent trolls in his speech. See Reuters, "Obama Says Patent Reform Needs to Go Farther," February 14, 2013.

information externality generated by an early court outcome for subsequent litigation. Successful litigation creates an option value against future potential infringers through Bayesian updating. This renders a credible litigation threat against the initial defendant and allows the NPE to extract more rents.

Lemley and Melamed (2013) point out that patent trolls do not employ a unitary business model and there are at least three different troll business models. A "lottery-ticket" troll is an owner of a patent that reads on a significant technology area. They target big established firms with an uncertain shot at a big payout. It is particularly important to these trolls that the perceived probability of infringement is high when they face their big practicing entity (PE) target. By contrast, "bottom-feeder" trolls are not particularly concerned as to whether the patent is infringed or not. They rely on the high cost of patent litigation, and aim to settle for small amounts of money. Finally, "patent aggregator" trolls acquire huge patent portfolios to convince target company to pay royalties for the portfolio license.

Our paper formalizes how the exclusion value is created by the credible threat to litigate and explores its implications for NPEs' litigation strategies. For instance, consider bottom-feeder trolls who search for "quick, low-value settlements for a variety of patents."[3] The logic is that the defendants prefer to settle for small amounts of money rather than pay the high cost of patent litigation that could easily run into millions. However, high litigation costs cut both ways and the logic begs the question of why defendants would consider the litigation threat by bottom feeders seriously. Given the considerable litigation costs relative to a meager expected payout, why don't they ignore the threat? We do not rely on a reputational mechanism, but rather provide a theory of litigation credibility based on information externalities. We show that a NPE may have a patent portfolio that is not strong enough to make its litigation threat credible in isolation, but in the presence of multiple defendants, the litigation threat becomes credible due to its option value for other future defendants. As pointed out by Lemley and Melamed (2013), "the universe of technology users against which a troll might assert patents is ... potentially much larger than the group of competitors against which a practicing entity is likely to assert its patents."

To understand the role of information externalities in patent litigation, consider the following simple numerical example. First, consider a situation in which an NPE faces only one PE that uses its patented technology. Let the PE's profit be 20 and if the NPE is successful

[3] See Lemley and Melamed (2013, p.2126).

against the PE in its litigation, it can extract half of the PE's profit via Nash bargaining with the threat of injunction. The probability that the PE's patent is valid and infringed by the PE is given by 1/4, and the legal costs for each party is 4. Then the expected payoff from litigation for the NPE is given by $(1/4) * 10 - 4 = -1.5 < 0$, and the NPE's litigation threat is not credible. Now suppose that there are two PEs with the same profit level of 20. For simplicity, these two PEs are not competing each other, but assumed to use the same technology. This implies that the infringement by the two PEs is perfectly correlated.[4] In this case, litigation against one PE reveals perfect information about the infringement by the other PE. Thus, if the NPE wins against one PE, it has a credible threat to litigate against the remaining PE and can extract 10 for sure. This implies that the NPE's threat against the first PE is credible because $(1/4) * (10 + 10) - 4 = 1 > 0$. The NPE and the targeted PE will settle out of court to save litigation costs. With Nash bargaining, the NPE will be able to receive a licensing fee of 5 from the targeted PE due to the presence of another PE that offers an additional option value for the initial litigation. We thus show how the presence of other potential infringers enhances the credibility of the patent holder's litigation threats and enables him to "double dip." However, note that the NPE no longer has any credible threat against the remaining PE once it extracts the licensing fee from the first PE. This simple example also suggests that the NPE may have higher incentives to acquire patent portfolios for the purpose of litigation vis-a-vis PEs. Suppose that the target firm is randomly selected because the two firms are symmetric. Then, each firm's expected licensing cost is 2.5. As a result, each PE will have incentives to bid up to 2.5 if the patent is up for sale whereas the NPE has incentives to bid up to 5. Acquiring the patent in this example is like providing a public good between the PEs because if one PE acquires the patent, the other PE benefits as much because the acquiring PE will not have any credible threat against the other PE. This type of provision of public good problem can also explain the emergence of defensive aggregators.

Hovenkamp (2013) is related to our paper in that he considers the NPE's incentives to litigate and the credibility of litigation threat. However, the mechanism by which the NPE gains credibility with weak patents is very different. He develops a dynamic model of predatory litigation that relies on the NPE's litigious reputation and behavioral type of "im-

[4] According to the legal principle of "res judicata," a matter that has been judged on the merits may not, generally, be relitigated.

pressionable" PEs which are easily intimidated by the NPE's predatory litigation behavior. In contrast, we do not assume any asymmetric information about firms' types and our main results are driven by information externalities across litigation suits. Lemus and Temnyalov (2014) analyze the role of patent asserting entities (PAEs) on litigation and innovation incentives. To address this issue, they consider a model in which PAE is allowed to acquire patents from practicing entities, and compare the equilibrium in such a set-up to a situation in the absence of the PAE. They identify two effects created by the PAE that are immune to counter-litigation: enhanced patent monetization effect and loss of the value of defensive patent portfolios. They show that when the former effect dominates the latter, PAEs can enhance innovation incentives and social welfare. The main focus of their paper, however, is different from ours and can be complementary to ours in understanding the tactics and roles of NPEs/PAEs in the overall patent system. They are concerned with the price of patent acquisition by the PAE and how this in turn changes the returns to R&D. We are more interested in the litigation strategies of NPEs and focus on litigation externalities, which are absent in their model.

Choi (1998) considers the implications of information externality in patent litigation, but in a different context. He considers a setting in which a patent holder is the incumbent facing multiple potential entrants. Launching a patent suit in face entry can be a risky proposition for the incumbent because of potentially harmful information that would invite further entry if its patent is invalidated. He explores the implications of such information revelation on entry dynamics and show that the nature of the entry game can be one of either waiting or preemption depending on the strength of the patent. However, the nature of information revelation in Choi (1998) is different from ours because the patent holder is a practicing entity and the issue is entry dynamics rather than extraction of rents by NPEs.

Che and Yi (1993) and Daughety and Reinganum (1999) are also closely related to our paper in that they consider strategic implications of information that comes out of the initial litigation for the subsequent litigation and settlement outcomes. Che and Yi (1993) consider a situation in which a single defendant faces multiple plaintiffs and once a precedent is set, it can have a lasting effect on successive trial outcomes. Daughety and Reinganum (1999) consider an incomplete information model in which an initially uninformed plaintiff makes a menu of settlement demands of the informed defendant who faces other potential plaintiffs. They consider implications of information revelation to outsiders from the existing negotiation, and

analyze incentives that one of both participants may have to limit the transmission of that information. They show that the possibility that there are other plaintiffs the defendant might face improves the current plaintiff's bargaining position as the outcome of the current case may invite further follow-on suits. As a result, the defendant may be willing to pay "hush money" to keep the negotiation outcome confidential. In our model, we consider a symmetric information structure and the fact that the patent holder has many defendants enhances the bargaining position. Our work thus differs from theirs both in important features of the model and in the questions analyzed.

The remainder of the paper is organized in the following way. In Section 2, we set up a simple model of patent litigation with information externalites. To illustrate the main idea, we consider one NPE that can assert its patent portfolio against multiple NPEs in a pre-determined order, and analyze the implications of multiple PEs on the credibility of litigation threat. In section 3, we endogenize the NPE's target choice and derive the optimal sequence of litigation targets. Section 4 extends our analysis to an environment in which PEs compete with each other and we explore the role of injunctive relief. Section 5 considers the implications of the British cost shifting rule under which the loser pays all legal expenses. Section 6 extends the basic model in several directions and checks the robustness of the main results. Section 7 closes the paper with concluding remarks. Longer proofs for lemmas and propositions are relegated to the Appendix.

2 Benchmark Model

We consider a situation in which one NPE or patent troll intends to assert its patent portfolio against multiple PEs. The NPE has a patent portfolio of size S, which translates into an infringement probability of $\theta \in [0,1]$ for any PE. This infringement parameter can be interpreted as the strength of the NPE's patent portfolio.[5] For simplicity, let us assume that there are two PEs, firm 1 (PE_1) and firm 2 (PE_2), and the NPE is negotiating sequentially with each of them. To illustrate the nature of the information externality across litigation cases, we first assume that the sequence is pre-determined in the benchmark model. This

[5] Suppose that the probability that the PE's product infringes a particular patent is q and this probability is the same and independent of each other across patents. Then, the probability that the PE's product will infringe at least one patent is given by $\theta = 1 - (1-q)^S$, where S is the number of patents held by the NPE. More generally, the probability of infringing will depend not only on the NPE's patent portfolio size, but also on the patent quality.

would be the case if PEs are entering the market sequentially over time.[6] By contrast, in the next section, we consider a scenario, in which both PEs are already in the market and the NPE can endogenously choose whether to approach the PEs simultaneously or sequentially, and if sequentially, which firm to target first when the PEs are asymmetric. The PEs are not competing with each other, but they use related technologies. This means that the litigation outcome for one firm does not affect the other firm's profitability through competitive effects.[7] Nonetheless, the litigation outcome for one PE may have implications for the likelihood of the other PE's infringement on the NPE's patent portfolio when they use related technologies. For instance, many industries have evolved by integrating technologies from a variety of different scientific disciplines. The interdisciplinary approach and convergence of technologies have made it commonplace for the same type of related technologies to be adopted in previously separate industries, blurring the boundaries of traditional industries and creating new ones. Consider the convergence of broadcasting and telephone industries. Traditionally, they represented very different forms of communications in many dimensions, including the mode of transmission and the nature of communication. As a result, they were considered separate industries. Digital convergence now enables both person-to-person communication services and broadcast content with similar technologies. We represent the technological overlap between the two firms with a parameter $\rho \in [0, 1]$.

More specifically, there are four possible litigation outcomes if there are patent suits against both PEs: $(I, I), (I, NI), (NI, I)$, and (NI, NI), where I and NI respectively denote infringement and no infringement. The probabilities of each event are given by:

$$\Pr(I, I) = \theta^2 + \rho\theta(1-\theta), \;\; \Pr(I, NI) = \Pr(NI, I) = (1-\rho)\theta(1-\theta)$$
$$\Pr(NI, NI) = (1-\theta)^2 + \rho\theta(1-\theta).$$

We can interpret ρ as a correlation coefficient in litigation outcomes across the PEs. If $\rho = 1$, there is perfect correlation between the litigation outcomes. At the other extreme, if $\rho = 0$, the litigation outcomes are independent. As a result, the litigation outcome for one party does not reveal any information about the likelihood of litigation outcomes for the other party. More generally, the updated beliefs about one firm's infringement probability given

[6] The order would be inconsequential if the two PEs are symmetric.

[7] We consider the case of product market competition among the PEs in Section 4.

the litigation outcome for the other firm is given by

$$\Pr(I|I \text{ for the other firm}) = \frac{\Pr(I,I)}{\Pr(I)} = \theta + \rho(1-\theta) \equiv \overline{\theta},$$
$$\Pr(I|NI \text{ for the other firm}) = \frac{\Pr(I,NI)}{\Pr(NI)} = (1-\rho)\theta \equiv \underline{\theta}.$$

Figure 1 below illustrates how the infringement probability can be updated depending on the outcome of litigation for the other party. The gap between the two lines, $\overline{\theta} - \underline{\theta}$, represent the updating in beliefs depending on the outcome of litigation, and is given by ρ. As expected, a higher ρ leads to more information revelation from litigation on the infringing probability of other firms.

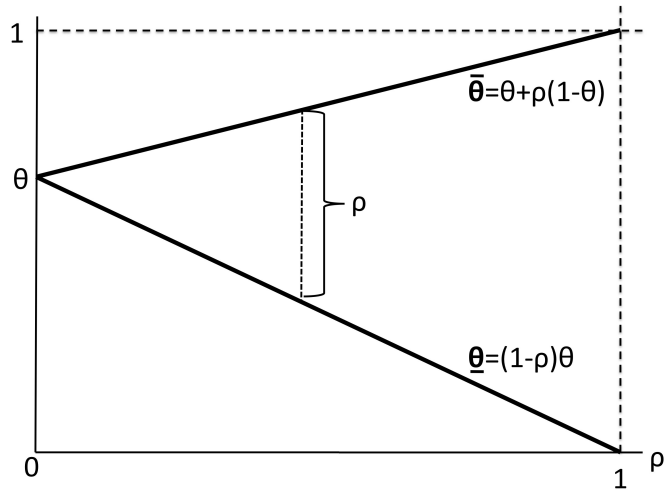

Figure 1: *Updating of infringement probability*

Now let us analyze the NPE's incentives to litigate against the PEs. Let D_i denote the expected damage payment or prospective licensing revenue the NPE expects to receive from firm i if the NPE litigates and firm i is found to infringe on the NPE's patent portfolio. We can imagine various scenarios in which D_i is determined. Until *eBay v. MercExchange*, an injunction order was issued more or less automatically in the absence of exceptional circumstances if a patent was found valid and infringed. Suppose, for instance, that injunctive relief is granted to the NPE when it wins in the litigation case. Then, the NPE can threaten to shut down the business of the PE and extract licensing revenues. If we assume Nash bargaining between the NPE and the infringing firm, the expected payment from PE_i would

be $D_i = \pi_i/2$, where π_i, $i = 1, 2$, denotes firm i's operating profit without litigation.

However, in the landmark case of eBay, the Supreme Court unanimously ruled that the decision to grant an injunction should be based on traditional principles of equity. In particular, it can be denied if legal damages are "sufficient to compensate for the infringement and an injunction may not serve the public interest."[8] Justice Kennedy's concurring opinion, in which NPEs were characterized as firms using patents "not as a basis for producing and selling goods but, instead, primarily for obtaining licensing fees," was often interpreted as advocating denial of injunctive relief to NPEs.[9] In case an injunction is not available, the legal statute stipulates that the claimant be awarded lost profits "adequate to compensate for the infringement, but in no event less than a reasonable royalty."[10] For an NPE, the appropriate damage is a reasonable royalty rate because the NPE does not produce any products or services. *Georgia-Pacific* established 15 factors that can be considered in determining the reasonable royalty rate, with the essence being considered as a "hypothetical license" approach that defines the reasonable royalty rate as "[t]he amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time of the infringement began) if both had been reasonably and voluntarily trying to reach an agreement."[11]

To accommodate various scenarios in which the patent holder is compensated, we adopt a general approach with the assumption that $D_i = D(\pi_i)$ with $0 < D_i < \pi_i$ and $\partial D_i/\partial \pi_i \geq 0$. If injunctive relief is available to the NPE, $D_i = \pi_i/2$. In the reasonable royalty rate case, our approach simply assumes that the damage payment is increasing in firm profits. Litigation incurs a cost of $L > 0$ for each involved party.

To highlight the importance of information externality, we first analyze the NPE's incentives to litigate against the PEs when they use unrelated technologies ($\rho = 0$) and consequently the litigation outcome against one PE has no implications for the other PE. In this

[8] See *eBay, Inc. v. MercExchange*, L.L.C., 547 U.S. 388 (2006).

[9] See, however, Denicolo et al. (2008) who interpret the *eBay* ruling as more of a call for a balancing test from a nearly automatic granting of injunctive relief in the past.

[10] 35 U.S.C. 284 (2006). The claimant can also be compensated for treble damages if a willfull infringement can be demonstrated.

[11] Georgia-Pacific Corp. v. United States Plywood Corp., 318 FSupp 1116, 6 USPQ 235 (SD NY 1970). See Choi (2009) for a discussion on logical consistency issues associated with the concept of "reasonable" royalty rates and alternative damage rules when intellectual property rights are probabilistic.

case, the NPE has a credible incentive to litigate against firm i if and only if

$$\theta D_i \geq L. \tag{1-i}$$

The NPE wins the infringement case against firm i with probability θ. In this case, the NPE receives a damage award payment D_i from firm i.[12] If the PE wins, it retains its entire market profit as it is not infringing on the NPE's patent portfolio.

Now assume that the technologies of the PEs are related. The NPE first negotiates with firm 1 and then with firm 2 in a predetermined order. We consider the case with reversed roles and the optimal choice of litigation targets in the next section. The NPE's interaction with firm 1 can end up in three scenarios: They litigate and the NPE wins in court, they litigate and the PE wins, and finally, they do not litigate and settle out of court. Hence, the posterior belief that firm 2 infringes based on the outcome of the NPE's interaction with firm 1 is given by $\hat{\theta} \in \{\overline{\theta}, \underline{\theta}, \theta\}$. The NPE will have a credible threat to litigate against firm 2 if and only if

$$\hat{\theta} D_2 \geq L \tag{1}$$

and the value of the NPE's patent portfolio with respect to firm 2 is given by

$$V_2(\hat{\theta}) = \begin{cases} \hat{\theta} D_2 & \text{if } L \leq \hat{\theta} D_2, \\ 0 & \text{otherwise.} \end{cases}$$

In equilibrium, the NPE and firm 2 never litigate and the extent to which the NPE can extract rents from the PE depends on the threat of litigation, that is, the belief that firm 2 infringes.

Now consider the litigation incentives between the NPE and firm 1. To do so, let Ψ_2 define the information externality of litigating the first firm in terms of expected profits with the second firm. If the NPE successfully litigates against firm 1, the probability of infringement of the second firm is revised upwards. On the other hand, upon losing litigation, the probability of the second firm infringing decreases relative to the case where the firms settle. Hence, we

[12] In section 6.3, we consider PEs with asymmetric infringement probabilities and derive qualitatively similar results.

have

$$\Psi_2 = \theta V_2(\overline{\theta}) + (1-\theta)V_2(\underline{\theta}) - V_2(\theta).$$

It follows that the NPE has a credible incentive to litigate the first firm if

$$\theta D_1 - L + \Psi_2 \geq 0. \tag{2}$$

If negotiations with the first firm fail, the NPE litigates if the sum of litigation profits with firm 1 and the information externality with firm 2 is positive. The next lemma determines the sign of the information externality.

Lemma 1 *If $\underline{\theta}D_2 < L < \theta D_2$, the information externality Ψ_2 is negative. If $\theta D_2 < L < \overline{\theta}D_2$, then the information externality Ψ_2 is positive. Otherwise, $\Psi_2 = 0$.*

PROOF:

$$\begin{aligned}
\Psi_2 =& \theta \begin{cases} \overline{\theta}D_2 & \text{if } L \leq \overline{\theta}D_2, \\ 0 & \text{otherwise} \end{cases} + (1-\theta) \begin{cases} \underline{\theta}D_2 & \text{if } L \leq \underline{\theta}D_2, \\ 0 & \text{otherwise} \end{cases} - \begin{cases} \theta D_2 & \text{if } L \leq \theta D_2, \\ 0 & \text{otherwise} \end{cases} \\
=& \begin{cases} 0 & \text{if } L \leq \underline{\theta}D_2, \\ -\theta(1-\overline{\theta})D_2 & \text{if } \underline{\theta}D_2 < L \leq \theta D_2, \\ \theta\overline{\theta}D_2 & \text{if } \theta D_2 < L \leq \overline{\theta}D_2, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}.$$

The information externality can be negative or positive as a function of the amount of expected damage payments relative to the cost of litigation. If the expected damage payments from the second firm are high relative to the cost of litigation, the externality can be negative. If the NPE settles with the first firm, no information is revealed to the second firm and the NPE still has a credible incentive to litigate and extract rents from that firm. By contrast, if the NPE litigates and loses, the expected probability of infringement of the second firm decreases and makes the threat of litigation against firm 2 non-credible. Hence, the presence of the second firm exerts a negative information externality on the NPE as an unsuccessful litigation against firm 1 would eliminate future licensing revenues with firm 2.

If the expected payments from the second firm are small relative to the litigation cost, the externality can be positive. In the absence of litigation against the first firm, the NPE would

not have a credible threat to sue the second firm. However, a positive litigation outcome could increase the perceived probability of infringement such that litigating the second firm would become credible. In this case, the presence of firm 2 has a positive externality on the NPE as a successful litigation outcome could also raise licensing revenues with the other firm.

If condition (2) holds, the NPE has a credible threat to litigate against the first firm. Firm 1 and the NPE will settle rather than going to court if their joint profits of settlement exceed their joint profits from litigation, that is, if

$$\pi_1 + V_2(\theta) \geq \pi_1 - 2L + \theta V_2(\overline{\theta}) + (1-\theta)V_2(\underline{\theta}) \quad \text{or} \quad \Psi_2 \leq 2L.$$

The NPE settles with firm 1 if the information externality from litigation does not exceed the total cost of litigation. This holds a priori when the externality is negative or zero. In the presence of a positive externality, the most the NPE can extract, that is, the maximum value of $\Psi_2 = \theta\overline{\theta}D_2$ is the expected profits with the second firm, θD_2. Litigation would be optimal if this gain would exceed both parties' litigation cost. However, a positive information externality requires that the expected profit with firm 2 is more than the litigation cost of the NPE. Hence, litigation never takes place in the benchmark model.[13]

It follows that when condition (2) is satisfied, firms Nash bargain. The NPE receives a total expected profit of

$$\Pi^{NPE} = \begin{cases} \theta D_1 + \Psi_2/2 & \text{if } L \leq \theta D_1 + \Psi_2, \\ 0 & \text{otherwise.} \end{cases} + V_2(\theta).$$

We can thus characterize the outcome of the benchmark model as follows.

Proposition 1 *Consider the equilibrium of the benchmark model with exogenously ordered sequential litigation.*

(i) There is no litigation in equilibrium.

(ii) When the information externality is negative and $D_1 \leq (2-\theta)D_2$, there exist parameter values such that the NPE has no credible litigation threat with respect to the first firm although litigation would be credible if it would deal with this firm in isolation.

[13] The model should thus be viewed as informing the terms of settlement in licensing contracts rather than offering predictions about the conditions under which litigation takes place. However, see the analysis in section 6.1, where litigation can take place in equilibrium.

(iii) When the information externality is positive, there exist parameter values such that the NPE has a credible threat to litigate against firm 1 although it would not be credible to sue that firm in isolation.
(iv) Compared to the case with unrelated technologies, the NPE may be able to extract higher (lower) total licensing fees when the information externality is positive (negative).

PROOF: *(ii)* Assume $\underline{\theta}D_2 < L \leq \theta D_2$. Condition (2) holds if

$$L \leq \theta(D_1 - (1 - \overline{\theta})D_2).$$

The RHS is increasing in ρ and takes value θD_1 at $\rho = 1$. There always exist parameter values to satisfy this condition if the RHS at $\rho = 0$ is less than θD_2 or

$$\theta(D_1 - (1 - \theta)D_2) \leq \theta D_2.$$

This gives the condition in the text.
(iii) If $\theta D_2 < L \leq \overline{\theta} D_2$, then (2) holds if

$$L \leq \theta(D_1 + \overline{\theta} D_2).$$

The RHS is increasing in ρ and takes value $\theta(D_1 + D_2) > \theta D_1$ at $\rho = 1$.
(iv) Suppose $\Psi_2 < 0$. In this case, the NPE makes profits of $\theta D_1 + \theta D_2$ when the PEs technologies are unrelated. Consider correlated technologies. If litigation against PE_1 is credible, the NPE earns $\theta D_1 + \Psi_2/2 + \theta D_2$ while if litigation is not credible it gets θD_2. Suppose $\Psi_2 > 0$. If the NPE faces unrelated PEs he obtains $V_1(\theta)$. With correlated technologies, the NPE $\theta D_1 + \Psi_2/2$ if litigation against PE_1 is credible. If litigation is not credible, the NPE makes zero profits with and without correlated technologies. The proposition follows.

In equilibrium, the NPE never sues the first firm for infringement and no information is revealed in the process. The NPE then interacts with the second firm as in the case of isolation. The information externality affects the NPE through its effect on the credibility of litigation incentives and on the Nash bargaining settlement with firm 1 via the negotiation threat points.

The presence of information externalities can explain different types of troll business models. For instance, consider a case where the NPE would have an incentive to litigate

against firm 1 in isolation, that is, $L \leq \theta D_1$. However, due to the fact that the NPE might lose licensing revenues with firm 2 if it loses litigation with firm 1, the NPE will not enforce its property rights with the first firm and wait for the other, more lucrative target. This equilibrium outcome can explain the behavior of "lottery ticket" patent trolls that aim at and wait for opportunities for a big payout rather than pursuing every licensing opportunity in the presence of a negative information externality. In contrast, the "bottom feeder" business model of patent trolls can be explained by the presence of a positive information externality. Consider a case where $L > \theta D_i$, $i = 1, 2$. In this case, both PEs' profits are too low relative to the litigation cost to make litigation profitable for the NPE when dealing with them in isolation. However, the possibility of a positive information externality from a successful litigation outcome increases the threat of litigation against firm 1 and allows the NPE to extract additional rents in negotiations. Despite the relatively high cost of litigation, information externalities allow the NPE to create a litigation threat and make profit.

Figure 2 below shows a diagram in the (L, ρ) space that illustrates the credibility of the litigation threat against firm 1 for the symmetric case of $\pi_1 = \pi_2 = \pi$ and $D_1 = D_2 = D(\pi) = D$. The grey shaded area depicts all parameter values for which litigation credibility against PE_1 is affected due to the presence of information externality. Area A in the graph refers to point *(ii)* of the Proposition. As $L \leq \theta D$, the NPE would have an incentive to litigate against firm 1 in isolation. Nonetheless, it will not enforce its property rights with the first firm due to the possibility of a negative information externality in dealing with the other PE. Area B refers to point *(iii)* of the Proposition. As $L > \theta D$, both PEs' profits are too low to make litigation profitable for the NPE when dealing with them in isolation. However, the presence of a positive information externality allows the NPE to retain litigation credibility and extract licensing revenues from firm 1.

How does the information externality affect total licensing revenues of the NPE relative to a situation with unrelated technologies? In regions A and C the NPE is worse off, whereas in region B profits are higher. In region A, due to the negative information externality, the NPE is only able to extract rents from firm 2 whereas it would be able to extract rents from both firms in isolation. In region C, the NPE is able to sell a license to firm 1 but negotiated license fees are lower due to the lower threat point of litigation. Finally, in region B, the NPE would not receive any license income with uncorrelated technologies. However, due to the positive information externality, litigation becomes credible and the NPE can extract rents

from firm 1.

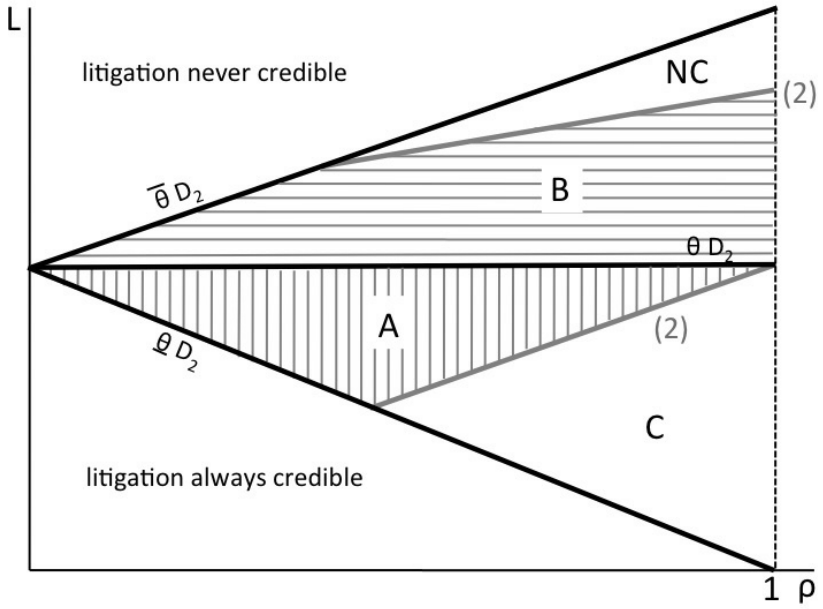

Figure 2: *Credible litigation threat with informational externality for symmetric PEs.*

3 Strategic Litigation Target Choice

In the previous section we have assumed that the PEs arrive in a predetermined order. Suppose now that the PEs are both operating in their respective market. The NPE can thus choose whether to approach the PEs simultaneously or sequentially, and which firm to target first. Suppose, without loss of generality, that PE_2 is the more profitable target, that is $\pi_1 < \pi_2$ and $D_1 \leq D_2$. In this section we investigate the optimal negotiation strategy and target choice for the NPE in the presence of information externalities.

In analyzing strategic litigation target choice, we assume that the NPE negotiates with the PEs only once, either in sequence or simultaneously. For instance, when the NPE approaches one PE and the first target refuses to pay, the NPE has two choices: either litigate against the first target or just move on to the other target; it cannot come back to the first target again later after it strikes a deal or engages in litigation with the remaining PE. We justify this assumption on two grounds. First, the assumption is made for analytical simplicity. Even if we allow the NPE to come back later to the first target who refuses to pay, we can derive

qualitatively the same results. Second, when the first target refuses to pay and the NPE decides not to act on its threat, the NPE's inaction may be interpreted as a tacit withdrawal of patent claims and the NPE may be barred to bring an infringement suit against the first target based on equitable estoppel.[14]

Similar to the benchmark model in the previous section, define V_1 and Ψ_1 as the continuation value and information externality when the NPE approaches firm 2 first. Furthermore, let us write the litigation credibility constraint of the NPE when he approaches the PEs in the order of firm i first and firm j second, where $j \neq i$, as

$$\theta D_i - L + \Psi_j \geq 0. \tag{2-i}$$

Moreover, let Π_{ij}^{NPE} denote the NPE's payoff from sequentially approaching the PEs in the order of firm i first and firm j second. Then it follows from our analysis above that

$$\Pi_{ij}^{NPE} = \begin{cases} \theta D_i + \Psi_j/2 & \text{if (2-i) holds,} \\ 0 & \text{otherwise.} \end{cases} + V_j(\theta). \tag{3}$$

Now consider the situation where the NPE approaches both firms simultaneously. In this case, there are no information externalities and the NPE settles with both firms at the same terms as if the technologies were unrelated. Hence, with simultaneous negotiations, the NPE makes a profit of $V_1(\theta) + V_2(\theta)$.

The NPE chooses the negotiation strategy, sequential or simultaneous, that maximizes his expected profit. When solving for the optimal litigation strategy, we adopt the following tie-breaking rule. If the NPE achieves the same expected profits with simultaneous negotiations and with the best sequential negotiation strategy, he chooses the former. The NPE's optimal target choice can then be summarized as follows.

Proposition 2 *Consider the NPE's optimal target choice with two PEs and $D_2 \geq D_1$.*
(i) If $\Psi_1 = \Psi_2 = 0$, that is, in the absence of information externalities, simultaneous and sequential negotiations yield the same expected profit.
(ii) If $\Psi_1 > 0$ and (2-2) is satisfied, the NPE chooses sequential negotiations with the more

[14] See, for example, *Aspex Eyewear Inc. v. Clariti Eyewear, Inc.*, Nos. 09-1147, -1162 (Fed. Cir. May 24, 2010).

profitable firm 2 as first target.

(iii) If $\Psi_2 > \Psi_1 = 0$ and (2-1) is satisfied, the NPE chooses sequential negotiations with the less profitable firm 1 as first target.

(iv) Otherwise, the NPE weakly prefers simultaneous negotiations.

The timing of negotiations is irrelevant when there are no information externalities. In this case, it follows from Eq. (3) that the NPE's profits with sequential negotiations are the same as with simultaneous negotiations and equal to $V_1(\theta) + V_2(\theta)$. Figure 3 below illustrates in the (L, ρ) space under which conditions this irrelevance result holds.

In the presence of an information externality, that is, when at least one of Ψ_i, $i = 1, 2$, is non-zero, the optimal strategy of the NPE depends on the sign and strength of the externality. A positive information externality allows the NPE to increase the credibility of his litigation threat and extract more rents from the optimally chosen first target in sequential negotiations. Hence, sequential negotiations are optimal if and only if at least one sequence generates a positive externality.

What is the optimal first target for the NPE in sequential negotiations? There are two distinct forces. The NPE might choose its first target to maximize the positive information externality as he partially internalizes this externality through the Nash bargaining procedure with the first target (if the litigation is credible). For instance, if $\Psi_2 \leq 0$ and $\Psi_1 > 0$, targeting firm 2 first is the optimal strategy because it can create a positive information externality of Ψ_1 when litigation takes place, whereas targeting firm 1 first will generate no or a negative externality. This case is included in point *(ii)* of Proposition 2 and arises when the NPE has no threat to litigate against the less profitable in isolation while it would have a credible threat against the more profitable target. Similarly, if $\Psi_1 = 0$ and $\Psi_2 > 0$, targeting firm 2 first yields no information externality whereas targeting firm 1 first generates a positive information externality. This is the case in point *(iii)* of the proposition. This scenario arises when the NPE has no credible threat against any of the targets in isolation.

There is, however, also the possibility that both externalities are strictly positive. Here, with the assumption of $D_1 \leq D_2$, it holds that $\Psi_2 \geq \Psi_1 > 0$ but the NPE prefers to approach firm 2 first although it generates a weaker information externality than approaching firm 1 first. The reason is as follows. This case may arise when the NPE has credible threats against neither firm in isolation. In particular, the NPE does not have any credible threat against the second target once it settles with the first target. Thus, the only source of revenue is with the

first target, the credibility against which is achieved with the presence of the other practicing entity. Even though the magnitude of the information externality is larger when the less profitable firm is the first target, the direct effect of extracting licensing income from the more profitable firm outweighs the indirect effect of the positive information externality from targeting the less profitable firm. Moreover, for the same reason, the credibility constraint is weakly easier to satisfy with firm 2 as the first target. Hence, whenever targeting 2 is credible for $\Psi_2 \geq \Psi_1 > 0$, it arises as the optimal strategy for the NPE. This completes point *(ii)* in the above proposition.

Finally, point *(iv)* implies that if at least one of the externalities is negative while the other is not strictly positive, the NPE is at least as well off with simultaneous negotiations as with the best sequential strategy. When the NPE has a credible threat against either PE in isolation, both information externalities can be strictly negative. In this case, simultaneous negotiations are strictly preferred by the NPE.

The light shaded area in Figure 3 gives a summary of the parameter values, for which targeting the less profitable firm 1 first strictly dominates. The darker shaded area depicts the parameter values such that targeting the more profitable PE is optimal. Sequential negotiations tend to yield higher profits when the cost of litigation is of intermediate size relative to profits and when technologies are sufficiently closely related.

Our results on strategic sequencing of litigation targets are related to the literature on optimal negotiation sequence. Krasteva and Yildirim (2012), for instance, consider the sequencing choice of a buyer who negotiates with the sellers of two complementary objects with uncertain payoffs. They show that the buyer's optimal sequencing is to negotiate with the weak seller if the sellers have diverse bargaining powers. However, they show that if all buyer's valuations were common knowledge, the buyer would be indifferent to the sequence. Our model, however, analyzes very different issues and assume *symmetric* information between the NPE and PEs. Nonetheless, sequencing matters for the NPE. In addition, we allow simultaneous bargaining whereas most papers in the literature do not allow such a possibility.[15]

[15] Krasteva and Yildirim (2014) extend their analysis to allow for endogenous information acquisition. They show that for moderate complements, the value of information is negative and the buyer would optimally commit to be uninformed even with costless information. In our model, the NPE's choice of simultaneous bargaining can be interpreted as a way of limiting information externalities.

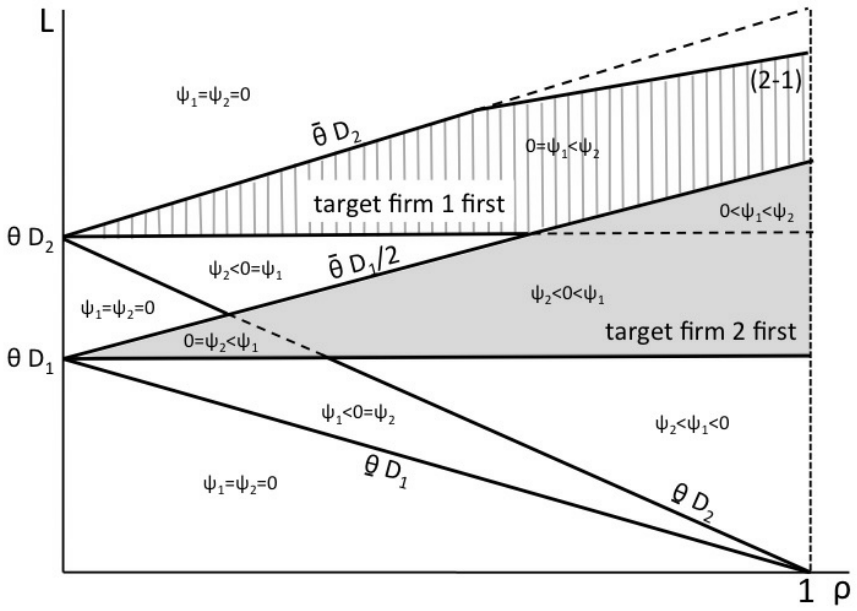

Figure 3: *Strategic Litigation Targets for NPE for* $\theta\pi_2 \leq \pi_1$.

4 Downstream Competition and Injunctive Relief

We have analyzed the NPE's litigation strategies in the framework that can encompass both enforcement by injunction and enforcement by liability for damages.[16] One policy question in relation to NPEs has been the availability of injunctive relief. More specifically, people expressed concerns that injunctive relief confers NPEs the ability to "hold up" PEs with the threat to shut down their businesses once they have made sunk investments, which can lead to licensing royalties far in excess of the true value of the patents involved (Lemley and Shapiro, 2007; Shapiro 2010). This hold-up concern led to the landmark case of *eBay* in which the US Supreme Court established four equitable factors that should be considered in determining whether an injunction should issue.[17] In particular, Justice Kennedy recognized NPE business models in which "firms use patents not as a basis for producing and selling

[16] According to legal theory, injunction is a property rule of entitling the claimant in that patent owners have a right to completely prevent all uses of the patented technology. In contrast, under a liability rule, someone might use the patent with adequate compensation to the patent holder. See Calabresi and Melamed (1972) for a distinction between property rules and liability rules.

[17] For injunctive relief, a plaintiff need "to demonstrate: (1) that it has suffered an irreparable injury; (2) that remedies available at law are inadequate to compensate for that injury; (3) that considering the balance of hardships between the plaintiff and defendant, a remedy in equity is warranted; and (4) that the public interest would not be disserved by a permanent injunction." See *eBay Inc. v. MercExchange, L.L.C.*, 547 U.S. 388 (2006)

goods but, instead, primarily for obtaining licensing fees." This concurring opinion in the case has been interpreted as denial of injunctive relief to NPEs, as injunctions can be used "as a bargaining tool to charge exorbitant fees."

When a NPE litigates against competing PEs, we point out that the availability of injunctive relief raises a new set of issues as the NPE acquires the ability to monopolize the downstream market to increase its profits. Consider our benchmark and suppose that the two PEs are product market competitors. Negotiations are again sequential; the NPE first bargains with firm 1 and then with firm 2. Let π^d denote the duopoly profits each PE is making when they both either own a license for the NPE's technology or are not found infringing or are not challenged by the NPE. If exactly one firm gets a license or exactly one firm is not infringing, then this firm earns monopoly profits $\pi^m (\geq 2\pi^d)$.

Consider the bargaining with the second firm when firm 1 is active in the product market. This could be either due to the fact that firm 1 settled (out of court or after an infringement verdict) or the court found firm 1's technology not to be infringing on the NPE's patent. Let $\hat{\theta}$ again be the updated belief that firm 2's product is infringing. Given the product market presence of firm 1, the NPE and the second firm always prefer to settle to avoid the cost of litigation. With the availability of injunction as a remedy, the NPE's profits with the second firm are thus

$$V^d(\hat{\theta}) = \begin{cases} \hat{\theta}\pi^d/2 & \text{if } L \leq \hat{\theta}\pi^d/2, \\ 0 & \text{otherwise.} \end{cases}$$

Now consider the second case in which the first firm was found infringing and the NPE did not sell a license to firm 1. With the updated belief of $\hat{\theta} = \overline{\theta}$, the NPE again settles with the second firm and receives

$$V^m(\overline{\theta}) = \begin{cases} \overline{\theta}\pi^m/2 & \text{if } L \leq \overline{\theta}\pi^m/2, \\ 0 & \text{otherwise.} \end{cases}$$

Let us turn to the NPE's negotiations with the first firm. Suppose the NPE decides to litigate against infringement and wins the court case. The NPE now has the choice to either license the technology or use the injunction to exclude PE_1 from the market. Exclusion occurs when the expected profits of the NPE when selling a single license to PE_2 exceed the

joint profits of the NPE with PE_1, that is,

$$V^m(\bar{\theta}) \geq \pi^d + V^d(\bar{\theta}). \tag{4}$$

The next lemma gives a condition under which this equation is satisfied.

Lemma 2 *Suppose the first PE has been found infringing on the NPE's patent. If product market competition is sufficiently intense, the NPE uses the injunction to exclude the infringing PE.*

An injunction allows the NPE to exclude the first PE from the market and reach a license agreement with the second PE as the monopolistic supplier in the market. By contrast, if the NPE sells a license to the first PE, the available rent he can extract from the second PE depends on the degree of product market competition. The lower the duopoly profits π^d, the lower the joint profits of PE_1 and NPE, and the more profitable is exclusion. Note that exclusion can be optimal in situations where the NPE has a credible threat against PE_2 and, a fortiori, when there is no such threat.

If exclusion is not optimal, PE_1 and NPE Nash bargain and share their joint surplus. Let $V^I$and J^I denote the NPE profits and the joint profits of PE_1 and NPE, respectively, when the court finds that PE_1 infringes on the patent and injunctive relief is available. In its initial negotiations with the first PE, the NPE has a credible threat to litigate if

$$[\theta V^I(\bar{\theta}) + (1-\theta)V^d(\underline{\theta})] - L \geq V^d(\theta). \tag{5}$$

Litigation arises in equilibrium if the joint profits from licensing for NPE and firm 1 are less than the profits from litigation, that is

$$\theta J^I(\bar{\theta}) + (1-\theta)[\pi^d + V^d(\underline{\theta})] - 2L \geq \pi^d + V^d(\theta). \tag{6}$$

The following proposition characterizes the equilibrium with product market competition and explores the role of injunctions on litigation incentives.

Proposition 3 *Suppose the PEs are competitors and injunctive relief is available.*
(i) If product market competition is sufficiently intense and the cost of litigation relatively low, the NPE litigates against the first PE and, if successful, uses the injunction to exclude

the firm while selling a license to the second PE.
(ii) When the technologies are perfectly related, the NPE has a credible litigation threat with the first PE, which is at least as strong as if the NPE would face a monopolist in the product market.

In the presence of injunctive relief, the NPE is able to exclude a PE, which has been found infringing on its patent. Such exclusionary licensing is profitable if downstream competition reduces the amount of rents that the NPE could extract in negotiations with competing PEs. Hence, if product market competition is intense and litigation cost is relatively low, the NPE is suing the first PE for infringement in an attempt to increase downstream profits. Even when the NPE prefers to settle with the first PE, the NPE is still able to use injunctive relief as the threat point of his bargaining with PE_1. This is particularly effective when the technologies of the firms are closely related. Consider the case where the NPE has no litigation incentive against PE_2 once he sells a license to PE_1. If ρ approaches 1, condition (4) holds and the NPE always has an incentive to exclude PE_1 should he prevail in court. Hence, the threat point of the NPE with the first NPE is $\theta\pi^m/2$, which is the same as if the NPE faces a monopolist in the product market.

5 Cost Shifting and Litigation Incentives of NPEs

In the US, the default rule for patent litigation is that each party bears its own attorneys' fees. In patent cases, under 35 U.S.C. §285, attorneys' fees are only shifted in exceptional cases, which have been very rare. The Congress is currently considering different pieces of legislation that all aim to reduce NPE patent litigation by adopting "loser-pays" fee shifting, also called the British rule of legal fee allocation. The idea is that if NPEs face the possibility of paying the target firm's attorneys' fees, they would not initiate litigation unless the case has sufficient merit. In this section, we analyze whether the British rule of legal fee allocation reduces the NPE's incentives to litigate and the profitability of their business model relative to the American rule.

Consider the set-up of our benchmark model under the British rule with symmetric firms such that $\pi_1 = \pi_2$ and $D_1 = D_2$. Suppose the NPE is facing the second PE. Under the British rule, the NPE only pays attorneys' fees when losing the court case but then it also has to cover the defendant's fees. Hence, for a given belief $\hat{\theta}$, the NPE has an incentive to

litigate the second PE if and only if

$$\hat{\theta}D_2 - (1-\hat{\theta})2L \geq 0 \quad \text{or} \quad L \leq L(\hat{\theta}) \equiv \frac{\hat{\theta}}{1-\hat{\theta}}\frac{D_2}{2}. \tag{7}$$

For $\hat{\theta} = \theta$ this condition coincides with the incentive constraint for litigation when the NPE faces each PE in isolation. Compare this condition with the credibility of litigation under the American rule, $\hat{\theta}D_2 - L \geq 0$. Under the British rule the expected cost of litigation is $2L$ times the probability of losing. Hence, the threat of litigation against the second PE is lower under the British (American) rule if and only if the perceived infringement probability $\hat{\theta}$ is less (greater) than 1/2. It follows that after a win against PE_1, the British rule gives more credibility with the second PE if $\overline{\theta} > 1/2$, that is, when the initial merit and the correlation are sufficiently high. After a loss, the British rule yields more credibility if $\underline{\theta} > 1/2$ which holds for a high initial merit and a low correlation. In particular, if $\rho \geq 1/2$, then $\overline{\theta} > 1/2$ and $\underline{\theta} < 1/2$, that is, the British rule gives more (less) credibility against PE_2 after a win (loss) against PE_1.

When the NPE has a credible threat, Nash bargaining ensues and the value of the NPE's patent portfolio with respect to firm 2 is given by

$$V^B(\hat{\theta}) = \begin{cases} \hat{\theta}D_2 + 2L(\hat{\theta} - 1/2) & \text{if } L \leq L(\hat{\theta}), \\ 0 & \text{otherwise.} \end{cases}$$

With the British cost allocation, the perceived infringement probability does not only affect the expected damage payment but also the expected litigation expenses. The party, that is more likely to win litigation, can extract more rents as it is more likely not to pay the overall litigation expenditures.

We can now analyze the decision to litigate against the first firm. The NPE has a credible threat of litigation if

$$\theta D_1 - (1-\theta)2L + \Psi^B \geq 0 \tag{8}$$

where the information externality under the British rule is defined as

$$\Psi^B = \theta V^B(\overline{\theta}) + (1-\theta)V^B(\underline{\theta}) - V^B(\theta).$$

The next lemma establishes the sign of the information externality under the British cost allocation.

Lemma 3 *If $L(\underline{\theta}) \leq L < L(\theta)$, then the information externality may be positive or negative. If $L(\theta) \leq L \leq L(\overline{\theta})$, then the information externality is positive. Otherwise, it is zero.*

With the British cost allocation, the information externality may be positive when litigating PE_2 is credible in isolation but not after a loss against PE_1. The reason is that under the British rule a win against PE_1 can imply a higher profit gain for the NPE as it additionally reduces the probability that the NPE has to pay the litigation cost against PE_2. This effect is stronger when the correlation ρ and the actual cost of litigation L are high, which is also when $\theta V^B(\overline{\theta})$ dominates $V^B(\theta)$.

The next proposition characterizes the incentives to litigate against the first PE under the British cost allocation rule.

Proposition 4 *Consider litigation incentives under the British fee allocation.*
(i) When the information externality is negative and $\rho < 1/2$, there exist parameter values such that the NPE has no incentive to litigate against firm 1 whereas it would have a credible threat against each PE in isolation.
(ii) When the information externality is positive, the NPE has weakly stronger incentives to litigate relative to a situation where it faces each PE in isolation.
(iii) If the merit of the case is sufficiently strong, the NPE has an incentive to litigate the first PE for any level of litigation cost.

Points *(i)* and *(ii)* state that the information externalities can have a negative or positive effect on the NPE's litigation credibility relative to facing the PEs in isolation. However, as discussed in the context of Lemma 3, the British rule reduces the extent of the negative externality. As a consequence, if the correlation parameter is greater than 1/2, the NPE always has at least as strong incentives to litigate in the presence of two related PEs as if it would facing the PEs in isolation. For the last point of the proposition, note that it follows from (7) that if $\hat{\theta} \in \{\theta, \overline{\theta}\}$ and θ approaches 1, litigation against the second PE is always credible. However, as θ approaches 1, the externality goes to zero. That is, the NPE always has a credible threat against the first (and second) PE independent of the cost of litigation.

Let us now compare litigation incentives under the American and the British rule.

Proposition 5 *Compare the litigation credibility under the American and British cost allocation.*

(i) If $\theta \geq \sqrt{2}-1$ *and* $\rho \geq 1/2$*, then the NPE has weakly more credibility to litigate under the British cost allocation compared to the American rule.*

(ii) Otherwise, the incentives to litigate might be weaker or stronger under the British rule.

(iii) There exist parameter values where the merit of the case θ *is arbitrarily close to zero and the NPE has a credible threat to litigate under the British but not under the American cost allocation.*

The first point is the main result of this section. For the parameter values, for which information externalities are most relevant, that is, for high correlation and high merit of the case, the British cost allocation provides more litigation credibility to the NPE. As discussed above, under the British rule, the information externality is positive for any $\rho \geq 1/2$. This guarantees that litigation is credible for any $L \leq L(\theta)$ while there might be no litigation threat under the American rule due to a negative information externality. Furthermore, consider situations in which litigation against firm 2 is only credible if the NPE wins against firm 1 under both the American and British rule. In this case, the British rule allows for a stronger information externality since winning against the first firm increases the probability that the NPE is successful against firm 2 and pays no litigation cost. This, in turn, increases the NPE's outside option and credibility vis-a-vis the first firm.

Point *(ii)* emphasizes the fact that if the correlation and merit are low, the comparison between American and British rule yields ambiguous results. In situations where negative information externalities can arise under both rules, the litigation threat against the first PE depends on the credibility constraints against PE_2. However, the strength of litigation incentives against the second PE is a function of the parameters and no clear ranking of the cost allocation rules can be established. Finally, the last point of the proposition states that no matter how weak the NPE's case might be, as long as the correlation is sufficiently high, the British rule can still provide a more credible threat of litigation against the first PE. When dealing with PEs in isolation, the British rule is less favorable to the NPE when the merit of the case is low. This standard result can, however, be reversed with multiple PEs as the American cost allocation is more likely to generate negative information externalities.

6 Extensions

In this section, we extend the model in several directions and checks the robustness of the main results.

6.1 Litigation Incentives with an Infinite Horizon

In this section we extend our benchmark model to situations where PEs arrive over an infinite horizon. In particular, we consider an infinite horizon model with discrete periods, in which in each period one PE is entering and potentially infringing on the NPE's patent. The expected damage payment with any PE satisfies $0 < D < \pi$. For simplicity, assume that the technologies of all PEs are perfectly correlated and that the NPE discounts future profits with discount factor $0 \leq \delta < 1$. We first characterize the equilibrium of the game with an infinite horizon. Then we analyze the effect of the merit of the case θ on the incentives to litigate and show that the NPE might be better off in situations where the case for infringement is weaker.

As a benchmark, suppose the NPE faces a single PE. The NPE has a credible threat to litigate if the cost of litigation is less than the expected profit in the event that the PE infringing, that is,

$$L \leq \theta D. \tag{9}$$

Now consider a stream of sequentially arriving PEs and assume for the moment that the NPE has a credible threat to litigate. The value of settling with the current PE, $v(\theta)$, in Nash bargaining is recursively given by

$$v(\theta) = \frac{1}{2}[\pi + \delta v(\theta)] + \frac{1}{2}[\theta(D + \frac{\delta}{1-\delta}D) - L] - \frac{1}{2}[\pi - \theta D - L]$$

or

$$v(\theta) = \frac{\theta}{1-\delta}D.$$

This is the maximum continuation profit the NPE can achieve without litigation. However, this value is equal to $\theta v(1)$, which is the expected, discounted value from litigation. Hence, the information externality is always weakly positive. This implies, that when (9) is satisfied and the current expected profits exceed the cost, litigation is credible against all PEs. For low values of the litigation cost and a sufficiently strong merit of the case, litigation is credible

independent of the value of the discount factor. The fact that litigation is credible against all PEs also implies that the joint profits from litigation never exceed the value of settlement. Hence, litigation does not occur in equilibrium.[18]

Now consider the incentives to litigate if condition (9) is not satisfied. The NPE has a credible threat to litigate if

$$\theta D - L + \theta \frac{\delta}{1-\delta} D \geq 0 \qquad \text{or} \qquad L \leq \frac{\theta}{1-\delta} D. \tag{10}$$

In this case the information externality of litigation is strictly positive as the NPE is unable to extract any profits in the absence of litigation that validates the infringement claim. This makes current litigation relatively more profitable and increases the credibility of the NPE's litigation incentives. Finally, consider the condition under which litigation maximizes the joint profits of the NPE and the current PE. Litigation is optimal if

$$\pi + \theta \frac{\delta}{1-\delta} D - 2L \geq \pi \qquad \text{or} \qquad L \leq \frac{\delta\theta}{2(1-\delta)} D. \tag{11}$$

Comparing the above incentive constraints yields our first result.

Proposition 6 *Suppose PEs arrive over an infinite horizon. If the discount factor is sufficiently high, the NPE always has a credible threat to litigate the PEs independent of the cost of litigation. If condition (9) fails to hold and the discount factor is sufficiently close to 1, the NPE litigates against the first PE in equilibrium.*

If the cost of litigation is low and condition (9) holds, there is no information externality as there is always a credible threat to litigate. In this case, litigation never arises as the parties try to avoid the cost of litigation. For higher values of L, there is a positive information externality of value $\theta\delta v(1)$. This value increases as the PEs arrive more frequently and δ goes up. As the discount factor approaches one, current profits and litigation cost are negligible and litigation is optimal from the point of view of the NPE and the PE-NPE pair jointly. Moreover, the NPE always has a stronger incentive to litigate compared to the PE-NPE pair jointly as the NPE ignores the cost it imposes on the PE. It follows that for high discount factors, litigation occurs in equilibrium, while for intermediate values, the NPE has a credible

[18] This is the same situation as in the benchmark model where for perfectly correlated technologies, the NPE always has an incentive to litigate against the first PE if there is a credible threat against a single PE.

threat of litigation and settles with all PEs. These results are illustrated below in the (L, δ) space where the black lines correspond to the incentive constraints with a patent that is infringed upon with certainty (θ=1).

What is the effect of the merit of the case on the incentives to litigate and the profits of the NPE? We compare a situation where the patent is infringed upon with certainty with a situation where the infringement probability is $\theta < 1$. Figure 4 illustrates this comparison. The gray lines correspond to the constraints with the uncertain infringement case. As the case becomes weaker all three conditions above become harder to satisfy. This directly implies that there are situations where the PE-NPE would have litigated with certain infringement, whereas with a lower merit, the NPE still has a credible threat but settles with all PEs (region A). There is a second effect of weakening the merit of a case. A weaker case reduces the sustainability of future license extraction if there is no litigation. This creates a positive information externality and increases the incentives to litigate in the current period. Hence, in region B, a weak patent induces litigation where the NPE and PE would have settled with certain infringement. Finally, note that a weaker case reduces the NPE's credible threat of litigation in region C.

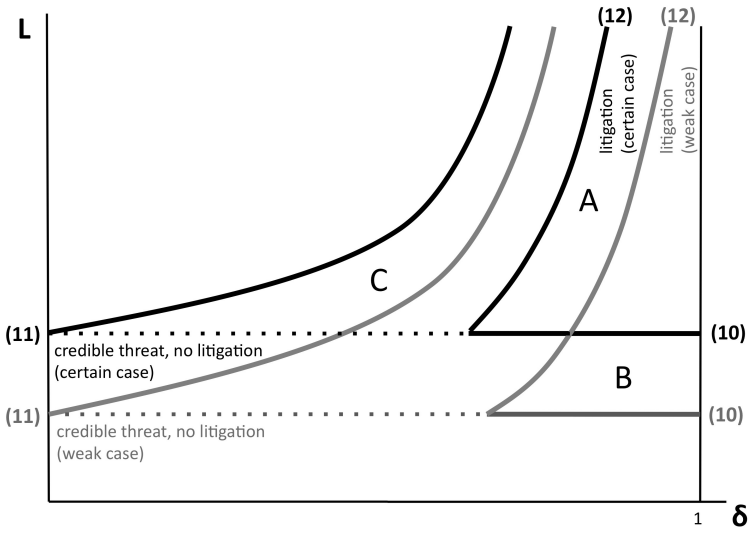

Figure 4: *Litigation incentives with a certain (black lines) and a weak infringement claim (gray lines).*

It is easy to check that if a weaker case induces litigation (region B) or jeopardizes the litigation credibility (region C), the NPE is always worse off. However, consider the case

where a weaker case induces settlement while certain infringement would lead to litigation. In region A, the NPE prefers a stream of settlement profits with a weak case over litigation profits with certain infringement if and only if

$$v(\theta) \geq D - L + \frac{\delta}{1-\delta} D \qquad \text{or} \qquad L \geq \frac{1-\theta}{1-\delta} D. \tag{12}$$

If litigation costs are relatively high and the merit of the case not too low, the NPE prefers settlement with a weak claim over litigating a certain claim to judgement. The next proposition gives a condition under which (12) is satisfied in region A and summarizes the results of the comparative statics with respect to the merit of the case.

Proposition 7 *A weaker infringement claim may lead to more or less litigation in equilibrium. If $\theta \geq 1/2$, that is, the claim is not too weak and if it avoids litigation relative to a case with certain infringement, then the NPE may make higher profits with a weaker, uncertain infringement claim.*

6.2 Endogenous Litigation Effort

We have assumed that the probability that the patent holder prevails in litigation is exogenously given by the relationship between patent claims and the technologies of the PEs. The probability of winning in the court may also depend on the litigation efforts by both parties. In this section we show that when endogenous litigation efforts are considered, two additional effects arise, which might increase or decrease the credibility of the NPE.

Consider the same set-up as the benchmark model. In each period one PE enters. The market profit of both PEs is given by π and the associated damage payment is D. To focus on the effects from endogenous litigation effort, suppose the fixed cost of litigation is zero, that is, $L = 0$. However, we assume that there are discretionary legal expenses that each party can spend to influence the court outcome. Let e_N and e_P be the variable legal expenses incurred by the NPE and a PE, respectively. We consider a Tullock type contest to model the strategic litigation effort of the parties. In this contest, the effectiveness of legal expenses depends on the perceived strength of the infringement case. This could, for example, reflect the fact that producing convincing evidence is harder, the worse the case is stuck against a party. Hence, assume that the expected probability of the NPE winning the court case is

given by

$$p(e_N, e_P) = \frac{\theta e_N}{\theta e_N + (1-\theta)e_P}.$$

At equal levels of expenditure, the winning probability is equal to the expected merit of the case θ.

Consider the NPE facing the second PE when the expected probability of infringement is $\hat{\theta}$. Suppose the NPE and PE go to court. The NPE maximizes his expected profit net of his legal expenses,

$$\max_{e_N} \quad p(e_N, e_P)D - e_N.$$

The PE chooses the litigation effort that minimizes his expected loss and legal expenses or

$$\max_{e_P} \quad \pi - p(e_N, e_P)D - e_P.$$

It is straightforward to show that there exists a Nash equilibrium with positive effort from both parties. The following lemma summarizes this equilibrium.

Lemma 4 *The NPE and the second PE exert the same amount of effort in court. The equilibrium winning probability of the NPE is $\hat{\theta}$. Both parties make positive profits in equilibrium and the NPE's litigation threat is always credible.*

PROOF: The first-order condition for party $i \in \{N, P\}$ facing party $j \in \{N, P\}$, $j \neq i$ is

$$\frac{\hat{\theta}(1-\hat{\theta})e_j}{(\hat{\theta}e_N + (1-\hat{\theta})e_P)^2} D = 1.$$

It follows that $e_N^{**} = e_P^{**} = \hat{\theta}(1-\hat{\theta})D$ and $p^{**} = \hat{\theta}$. The parties' profits are

$$\Pi_N^{**} = \hat{\theta}D - \hat{\theta}(1-\hat{\theta})D > 0, \quad \Pi_P^{**} = \pi - \hat{\theta}D - \hat{\theta}(1-\hat{\theta})D > 0.$$

The lemma follows.

In the absence of fixed cost, the NPE's threat of litigation is always credible. However, both parties are better off avoiding costly litigation in court. In the Nash bargaining settlement, the NPE gets

$$V_2(\hat{\theta}) = D + \frac{1}{2}[p^*D - e_N^*] - \frac{1}{2}[\pi - p^*D - e_P^*] = \hat{\theta}D.$$

In other words, the NPE can expect the same value from the second PE as in the benchmark model while always maintaining a credible litigation threat.

Let us turn to the NPE's interaction with the first PE. Suppose the NPE initiates litigation. The PE's litigation effort solves the same maximization problem as before. In contrast, the NPE takes into account the effect of the litigation outcome on the infringement belief and the profits with the second PE. The NPE solves

$$\max_{e_N} \quad \Pi_N = p(e_N, e_P)[D + V_2(\overline{\theta})] + (1 - p(e_N, e_P))V_2(\underline{\theta}) - e_N$$

which gives

$$\frac{\partial p(e_N, e_P)}{\partial e_N}[D + V_2(\overline{\theta}) - V_2(\underline{\theta})] = 1.$$

The LHS is the marginal benefit of increasing effort. More expenses result in a higher probability of winning and extracting rents from the first and the second PE. The following lemma considers the nature of strategic interaction between NPE and the first PE.

Lemma 5 *If $e_N/e_F \leq (1-\theta)/\theta$, the best response of the NPE (first PE) is decreasing (increasing). Otherwise, the best response of the NPE (first PE) is increasing (decreasing).*

PROOF: From totally differentiating the first-order condition for each party we have

$$\frac{\partial^2 \Pi_N}{\partial e_N \partial e_P} = -(1+\rho)\frac{\partial^2 \Pi_N}{\partial e_N \partial e_P} = \frac{\theta(1-\theta)[\theta e_N - (1-\theta)e_P]}{(\theta e_N + (1-\theta)e_P)^3} D.$$

The lemma follows.

The strategic interaction of legal expenses depends on whether the merit of the case is high or low compared to the relative effort of NPE and PE. In the former case, the NPE responds to more effort from its rival by also increasing effort, while the PE reduces his effort when the NPE invests more. Vice versa, it the merit of the case low, the NPE reduces his effort and the PE increases his effort as a response to more expenses from the other party.

We can now explicitly solve for the equilibrium values. In the unique Nash equilibrium with positive effort levels, the PE and NPE invest, respectively,

$$e_P^* = p^*(1-p^*)D, \quad e_N^* = (1+\rho)e_P^*$$

where the equilibrium winning probability of the NPE is given by

$$p^* = \theta + \frac{\rho\theta(1-\theta)}{1+\rho\theta} \geq \theta.$$

The comparative statics of these equilibrium values depend on the type of strategic interaction as defined in Lemma 5. The condition in Lemma 5 is satisfied in equilibrium if

$$\frac{e_N^*}{e_P^*} = (1+\rho) \leq \frac{1-\theta}{\theta} \qquad \text{or} \qquad \theta \leq \frac{1}{2+\rho}. \tag{13}$$

and we have the following lemma.

Lemma 6 *The equilibrium comparative statics for $i \in \{P, N\}$ can be summarized as follows*

$$\frac{\partial e_N^*}{\partial \rho} > 0, \ \frac{\partial e_P^*}{\partial \rho} \begin{cases} \geq 0 & \textit{if (13) holds,} \\ < 0 & \textit{otherwise} \end{cases}, \ \frac{\partial e_i^*}{\partial \theta} \begin{cases} \geq 0 & \textit{if (13) holds,} \\ < 0 & \textit{otherwise} \end{cases}, \ \frac{\partial p^*}{\partial \rho} > 0, \ \frac{\partial p^*}{\partial \theta} > 0$$

When the merit of the case and the technology correlation are high, the NPE's effort is a strategic complement while the PE's effort is a strategic substitute. Otherwise, the NPE is more accommodating while the PE is more aggressive with respect to litigation expenses. What is the effect of increasing θ and ρ in these regimes? Increasing the correlation parameter raises the future gains with the second PE and shifts out the reaction function of the NPE. The PE's reaction function is not affected. It follows that the expenses of the NPE always increase in ρ. Meanwhile, the equilibrium effort of the PE increases in ρ if its expenses are a strategic complement, that is, if condition (13) is satisfied. In this case, it is clear that the presence of a second PE induces both parties to invest more in legal expenses than they would if there was just one PE. The effect of a higher θ on the reaction functions are the same for both parties. If condition (13) does not hold, the reaction functions shift inwards and both exert less effort. Otherwise, they shift outwards and both exert more effort as θ increases. Finally, the NPE's equilibrium probability of winning increases both in θ and ρ.

Now consider the individual litigation incentive constraint for the NPE. Litigation is credible if the current and future expected gains from litigation outweigh the future profits from settlement when there is no additional information revealed via litigation, that is, if

$$p^* D - e_N^* + \Omega^L \geq 0 \tag{14}$$

where the externality term is defined as

$$\Omega^L = p^* V_2(\overline{\theta}) + (1 - p^*) V_2(\underline{\theta}) - V_2(\theta) = (p^* - \theta)\rho D \geq 0.$$

Unlike in the benchmark model, the information externality is not driven by the litigation incentive against the second PE as the NPE's threat is always credible with endogenous litigation costs. Litigating against the first PE has the benefit of endogenously raising the success probability above the initial merit of the case. This makes it more likely to face the second PE with a strong case for infringement and raises the expected future settlement payoff. Hence, like in the benchmark model, there is a positive externality of litigation.

However, there might also be a negative effect due to the presence of a second PE. When condition (13) is satisfied, the PE and NPE invest more in legal expenses relative to what they would if there was an isolated PE. This means that the contest dissipates more rents and litigation is more costly for the NPE. In particular, it can be shown that if θ is sufficiently small, the incentive constraint (14) is not satisfied and the NPE has no incentive to litigate against the first PE. To see this, consider a marginal increase in θ at $\theta = 0$. This change has a first-order effect on future profits in the incentive constraint but a negligible effect on the endogenous litigation efforts with the first PE. The next proposition gives the main result of this analysis.

Proposition 8 *Consider the model with endogenous litigation effort. If the initial merit of the case θ is sufficiently low, the NPE has no credible threat of litigation against the first PE and is worse off relative to facing the PEs in isolation. Otherwise, if the threat against the first PE is credible, the NPE always extracts more rents relative to facing two isolated PEs. Litigation never occurs in equilibrium.*

With endogenous litigation efforts, two cases arises. If the NPE has a credible threat to litigate against the first PE, then the positive externality allows the NPE to extract more rents relative to facing unrelated, isolated PEs. However, if litigating the first PE is too costly and condition (14) fails to hold, the NPE is worse off in the presence of a second PE.

6.3 Asymmetric Infringement Probabilities across Firms

So far we have assumed that all PEs have the same probability of infringement of NPE's patent portfolios and the inference process from one firm's litigation outcome for other firms'

infringement probability was symmetric even though we allowed PEs' profits to be different. In this section, we consider the case where PEs can have different infringement probabilities and the inference process can differ depending on which firm is litigated first. To address this issue, we consider the case where the set of technologies used by one PE is a subset of the other PE's. For instance, we can imagine that one PE's product has strictly more features than the other PE's or incorporates more sophisticated technologies. To focus on this issue, we assume that expected damage payments in case of infringement are the same across firms i.e., $D_1 = D_2 = D$ and $\pi_1 = \pi_2 = \pi$. Without any loss of generality, assume that PE2's product has more features. Let $k \in [0, 1]$ parametrize the proportion of technologies used by PE_1 compared to PE_2. This is reflected by differences in infringement probabilities. More specifically, we assume that firm 1's infringement probability is given by $\theta_1 = k\theta$ whereas PE_2's infringement probability is given by $\theta_2 = \theta$. This implies that the updating process from litigation outcomes is different depending on the identity of the defendant firm. For instance, if firm 1 is litigated and found to infringe on the NPE's patents, this is a sure signal that firm 2 also infringes on the NPE's patents because firm 2's set of technologies used is a superset of firm 1's, while firm 2's infringement does not necessarily mean that firm 1 also infringed. Likewise, if firm 2 is found not to have infringed on NPE's patent, this is a sure sign that firm 1 has not infringed, either, while the converse is not necessarily true. Let I_i and NI_i denote infringement and no infringement by PEi,respectively, $i = 1, 2$. The application of Bayes' rule implies the following inference process

$$\Pr(I_1,I_2) = k\theta,\ \ \Pr(I_1, NI_2) = 0,\ \ \Pr(NI_1, I_2) = (1-k)\theta,\ \ \Pr(NI_1,\ NI_2) = (1-\theta)$$

and it follows

$$\begin{aligned}
&\overline{\theta}_1 = \Pr(I_1|I_2) = k, \quad \underline{\theta}_1 = \Pr(I_1|NI_2) = 0, \\
&\overline{\theta}_2 = \Pr(I_2|I_1) = 1, \quad \underline{\theta}_2 = \Pr(I_2|NI_1) = \frac{(1-k)\theta}{1-k\theta}.
\end{aligned}$$

We can derive parallel results to the ones in section 3 where we analyzed strategic sequence of litigation targets when firms are asymmetric in their profits, but symmetric in terms of the information updating process. To be more specific, we define Ψ_i as the information externality for firm i when litigation takes place against firm j, as in section 3. Then, in the

current context with asymmetric infringement probabilities, we have

$$\Psi_i = \theta_j V_i(\overline{\theta}_i) + (1-\theta_j)V_i(\underline{\theta}_i) - V_i(\theta_i), \text{ where } i = 1, 2 \text{ and } j \neq i$$

Once again, we can easily verify that the order of sequence is irrelevant in the absence of any information externality, i.e., if $\Psi_1 = \Psi_2 = 0$. In addition, we have the following proposition, which parallels Proposition 2.

Proposition 9 *Consider the NPE's optimal target choice with two PEs when PE_2's product incorporates strictly more features than PE_1's, which results in a higher infringement probability for PE_2.*
(i) If $\Psi_1 > 0$ and litigation threat against firm 2 is credible, the NPE chooses sequential negotiations with the more profitable firm 2 as first target.
(ii) If $\Psi_2 > \Psi_1 = 0$ and litigation threat against firm 1 is credible, the NPE chooses sequential negotiations with the less profitable firm 1 as first target.
(iii) Otherwise, the NPE weakly prefers simultaneous negotiations.

The intuition for the result is similar to the one for Proposition 2. The NPE chooses its first target to maximize the positive information externality (or minimize the negative information externality). When the sign of information externality is the same, the NPE targets a more lucrative target first. In the current context where both firms have the same operating profits, PE_2 is a more lucrative target because its infringement probability is higher.

6.4 Patent Portfolio Acquisition

Our analysis so far has assumed that a non-practicing entity has a patent portfolio of certain strength. We now analyze the NPE's incentives to acquire patent portfolio vis-a-vis PEs'. Suppose that a patent portfolio of strength $\theta > 0$ has been put up for sale. We ask which type of entities is more likely to acquire the patent portfolio. To illustrate the implications of litigation externalities for patent portfolio acquisition incentives, we consider the simplest setting of one NPE and two PEs bidding for the available patent packet. To simplify the analysis, we analyze a setting in which all parties have no existing patent portfolios.[19] As

[19] We can easily extend the analysis of the game to a setting in which firms have existing patent portfolios. See Choi and Gerlach (2014) for more details of such an analysis.

a benchmark case, we first establish that all firms have the same willingness to pay for the patent portfolio in the *absence* of any litigation externalities.

Lemma 7 *If there are no litigation externalities (i.e., $\rho = 0$), all firms bidding for the patent portfolio have the same willingness to pay.*

The intuition for this result is simple. The acquisition incentives for the NPE are determined by the amount of licensing revenues it can extract from the two PEs. Let R_1 and R_2 be the amount of revenues the NPE can extract from each firm with the acquisition of the patent portfolio. Then, the NPE's maximum willingness to pay is (R_1+R_2). For PE_1, its payoff from the acquisition of the patent portfolio is R_2, which is the licensing income it can generate from PE_2.[20] If PE_1 does not acquire the patent portfolio, its payoff will be $-R_1$. As a result, PE_1's maximum willingness to pay for the patent portfolio is the same as the NPE's and given by (R_1+R_2). The same logic applies to PE_2.

However, if we allow for the possibility of litigation externalities, we can show that the NPE has higher incentives to acquire patent portfolios than any PEs due to a free rider problem between PEs. To see this, consider the case of two symmetric PEs with positive information externalities, that is, $D_1 = D_2 = D$ and $\theta D < L < \overline{\theta} D$ with $\Psi = \theta\overline{\theta} D$. In this case, the NPE can only extract rents from the first PE it approaches while PEs have no credible threat against each other if they acquire the patent. If the NPE gets the patent, it randomly chooses one of the PEs and extracts $\theta D + \Psi/2$. From the PEs' perspectives, they will be chosen as the first target with probability $1/2$. Acquiring the patent only serves a defensive purpose as each PE is willing to bid up to the expected losses from the NPE's purchase of the patent packet, that is up to $[\theta D + \Psi/2]/2$. Hence, the NPE has a higher willingness to pay.

For this type of result to hold, some uncertainty about the identity of the NPE's first litigation target is required at the time of patent portfolio acquisition. This could be due to the fact that the NPE considers the PEs as equally valuable targets like above. This argument also applies to situations where the expected damage payment of each PE is unknown to the NPE and only revealed in litigation. If the ex ante expected damage payment from each PE

[20] Here the assumption that PE2 does not have any existing patent portfolio it can use against PE1 as a countermeasure is important. If there is any existing patent portfolio for PE2, the licensing income PE1 can generate will depend on the relative patent portfolio strength and can be different from what the NPE can extract from PE2.

is the same, the NPE will be indifferent about which firm to target first. A second possibility is that, at the moment of acquisition, the PEs are equally valuable but at the moment of choosing the first litigation target, the NPE strictly prefers one PE.

Let us briefly sketch this argument. If we consider positive information externalities with $D_1 < D_2$, the NPE will choose PE_2 as the first target and once the NPE settles with PE_2, it does not have any litigation credibility against PE_1. In this case of complete information, the NPE and PE_2 will have the same willingness to acquire the patent portfolio. Now suppose that acquisition takes place before the market matures and the values of the products are uncertain until they are actually introduced. We thus introduce uncertainty about D_i at the time of patent portfolio acquisition. Consider the following timing. When a patent portfolio is up for sale, the potential damage payments D_i are not known but they are distributed according to $F(.)$. Once the patent portfolio acquisition takes place, the values of D_i are revealed in the market and the patent holder makes a litigation decision.[21] The following lemma considers this set-up.

Lemma 8 *Assume that D_1 and D_2 are distributed according to a joint distribution $F(.,.)$ on R^2_+ and revealed after acquisition but before any litigation decision. Then, the NPE has a strictly higher willingness to pay for the patent portfolio for sale.*

Suppose the distributions were restricted to values of D_i such that there are only positive information externalities for both firms. In this case, the firm with the higher realized D_i is the first target. As the ranking is uncertain at the time of acquisition, our result follows. For other realizations of D_i, the NPE has at least as high a willingness to pay for the patent portfolio as the maximum of the PEs' willingness to pay. This is due to the fact that the NPE can avoid negative information externalities by litigating simultaneously. Thus, the NPE will have a strictly higher willingness to pay and we can summarize the findings as follows.

Proposition 10 *In the presence of a positive information externality and if there is some uncertainty about the identity of the first litigation target at the time of acquisition, the NPE has a higher willingness to pay for the patent portfolio than the PEs.*

[21] Given the two PEs are active and known at the time of acquisition, we assume that the NPE can choose to litigate simultaneously or sequentially. If the NPE does not have the option to litigate both PEs simultaneously, the PEs might have a higher willingness to pay due to negative information externalities.

7 Concluding Remarks

We have developed a model of patent trolls to understand NPEs' business models and litigation tactics used to maximize their licensing revenues. We have considered a setting in which the technologies covered by NPEs' patent portfolios can be potentially infringed by multiple PEs who use related technologies. The main driver of our analysis is information that can be revealed in litigation for future licensing negotiations with other potential infringers. If the credibility of litigation threats is not an issue, such potential information externalities are in expectation neutral and licensing bargaining with each potential infringer is not affected by the presence of other infringers and can be analyzed in isolation. However, if the outcome of current litigation affects the credibility of future litigation threat, information externalities arise and a rich set of predictions can be derived depending on the signs and magnitudes of such information externality.

Our analysis has focused on NPEs' litigation credibility and tactics in the presence of information externalities, with the entry of PEs and the level of innovative activities exogenously given. One natural and important extension would be to explore implications of NPEs' litigation for entry and innovation incentives for PEs. For instance, Bessen *et al.* (2011) estimate that NPE lawsuits are associated with half a trillion dollars of lost wealth to defendants from 1990 to 2010 and reduced innovation incentives. In a different vein, Cohen *et al.* (2015) empirically document that increased litigation risk by NPEs has driven innovators to shield themselves by shifting innovation away from public and private firms into universities. To mitigate negative impacts of NPEs, they suggest "cost-shifting" in legal fees to limit the power of NPEs. However, our analysis indicates that the effect of cost-shifting on litigation incentives can be subtle in the presence of multiple PEs, reversing the standard result that the British rule is less favorable to the NPE when the merit of the case is low. Our analysis of litigation externalities also suggests that there could be unmeasured costs of reduced entry of PEs when additional entry restores litigation credibility for NPEs.

Appendix

PROOF of Proposition 2: For notational convenience, let L_{ij} be the threshold value for which condition (2-i) holds with equality when firm i is the first target. With the assumption of $D_1 \leq D_2$, it can easily be shown that parameters such that $\Psi_1 < 0$ and $\Psi_2 > 0$ do not exist. For point (ii), there are two distinct cases to consider with $\Psi_1 > 0$. First consider $\Psi_2 \leq 0 < \Psi_1$, which corresponds to parameter values such that $\theta D_1 < L \leq \min\{\overline{\theta} D_1, \theta D_2\}$. Here the 21-sequence allows credibility against the first target as (2-2) is satisfied since $L < \theta D_2 + \Psi_1$. However, there is no credibility against the second target after settlement with the first one. By contrast, since $L > \theta D_1 + \Psi_2$, the 12-sequence is not credible against the first target. Finally, the 21-sequence strictly dominates simultaneous negotiations since

$$\theta D_2 + \frac{1}{2}\Psi_1 > \theta D_2.$$

The second case is where $\Psi_2 > \Psi_1 > 0$ which holds if $\theta D_2 < L \leq \overline{\theta} D_1$. Such values only exist if $D_1 > \theta D_2$. In this case, litigation against the second target is not credible for either sequence. If litigation with the first target is credible, profits are weakly higher in the 21-sequence since

$$\theta D_2 + \frac{1}{2}\Psi_1 = \theta(1-\overline{\theta}) D_2 \geq \theta D_1 + \frac{1}{2}\Psi_2 = \theta(1-\overline{\theta}) D_1.$$

Moreover, we have

$$L_{21} = \theta D_2 + \theta\overline{\theta} D_1 \geq L_{12} = \theta D_1 + \theta\overline{\theta} D_2.$$

The 21-sequence is always credible while the 12-sequence is credible and there exist values where the 21-sequence is the only sustainable strategy. Finally, since simultaneous negotiations yield no returns, the NPE strictly prefers the 21-sequence as long as (2-2) is satisfied.
For point (iii): The case $\Psi_2 > \Psi_1 = 0$ corresponds to parameter values such that $L > \overline{\theta} D_1$ and $\theta D_2 < L \leq \overline{\theta} D_2$. Litigation is not credible in the 21-sequence and with simultaneous negotiations. However, if $L \leq L_{21}$, the 12-sequence has a credible threat with the first target. As $L_{21}(\rho = 0) = \theta D_2 + \theta^2 D_1 > \theta D_2$ such parameter values always exist.
For point (iv), three more cases are possible. The case $\Psi_1 < \Psi_2 = 0$ corresponds to parameter values such that $\underline{\theta} D_1 < L \leq \theta D_1$ and $L < \underline{\theta} D_2$. Litigation is always credible against the second target in both sequences. Litigation is also always credible with the first target in the 12-sequence. If $L \leq L_{21}$, then litigation is credible against the first target in the 21-sequence.

However, the 12-sequence is more profitable since

$$\theta D_1 + \frac{1}{2}\Psi_2 + \theta D_2 > \theta D_2 + \frac{1}{2}\Psi_1 + \theta D_1$$

for $\Psi_1 < \Psi_2 = 0$. However, the best sequential strategy cannot improve on simultaneous litigation which yields $\theta(D_1+D_2)$. The second case to consider is $\Psi_2 < 0 = \Psi_1$ corresponding to parameter values such that $L > \overline{\theta}D_1$ and $\underline{\theta}D_2 < L \leq \theta D_2$. The 12-sequence has no credibility. The 21-sequence has a credible threat against the first firm but as $\Psi_1 = 0$, the NPE makes the same profit, θD_2, as with simultaneous litigation. The last case is $\Psi_2 < \Psi_1 < 0$ which holds if $\underline{\theta}\pi_2/2 < L \leq \theta\pi_1/2$. If sustainable the equilibrium profit for the NPE with firm i as first target is always less than what it can earn with simultaneous negotiations,

$$\theta D_i + \frac{1}{2}\Psi_j + \theta D_j < \theta D_i + \theta D_j$$

since $\Psi_j < 0$. The proposition follows.

PROOF of Lemma 2 and Proposition 3: Consider $L \leq \bar{\theta}\pi^d/2$. Condition (4) holds if and only if

$$\pi^d \leq \frac{\bar{\theta}}{2+\bar{\theta}}\pi^m.$$

Let Γ_1 be the set of all parameter values (L, π^d) such that for $L \leq \bar{\theta}\pi^d/2$, the NPE prefers licensing. Then consider $\bar{\theta}\pi^d/2 < L \leq \bar{\theta}\pi^m/2$. Condition (4) holds if and only if

$$\pi^d \leq \frac{\bar{\theta}}{2}\pi^m.$$

Let Γ_2 be the set of all parameter values (L, π^d) such that $\bar{\theta}\pi^d/2 < L \leq \bar{\theta}\pi^m/2$ and the NPE prefers licensing. The statement in the lemma follows.

The NPE's profits from a successful litigation outcome with PE_1 are

$$V^I(\bar{\theta}) = \frac{\pi^d}{2} + \begin{cases} \bar{\theta}(\pi^d + \pi^m)/4 & \text{if } (L, \pi^d) \in \Gamma_1, \\ \bar{\theta}(\pi^m)/4 & \text{if } (L, \pi^d) \in \Gamma_2, \\ (\bar{\theta}\pi^m - \pi^d)/2 & \text{otherwise,} \end{cases}$$

while the joint profits are

$$J^I(\bar{\theta}) = \pi^d + \begin{cases} \bar{\theta}\pi^d/2 & \text{if } (L, \pi^d) \in \Gamma_1, \\ 0 & \text{if } (L, \pi^d) \in \Gamma_2, \\ \bar{\theta}\pi^m/2 - \pi^d & \text{otherwise.} \end{cases}$$

For point *(i)* of the proposition, consider condition (6). Upon inspection, if $(L, \pi^d) \in \Gamma_1$ or if $(L, \pi^d) \in \Gamma_2$, the condition cannot be satisfied. For the remaining parameter values, successful litigation leads to the exclusion of PE_1. Three cases need to be distinguished. First, if additionally $L \leq \underline{\theta}\pi^d/2$, the litigation condition holds if

$$L \leq \frac{1}{4}\theta\bar{\theta}\pi^m - [3\theta - (1-\theta)\underline{\theta}]\frac{1}{4}\pi^d.$$

The RHS is decreasing in π^d and positive if and only if $\pi^d \leq \bar{\theta}\pi^m/(2+\bar{\theta})$. Hence, the condition is satisfied if L and π^d sufficiently small. Second, if $\underline{\theta}\pi^d/2 < L \leq \theta\pi^d/2$, condition (6) is satisfied if

$$L \leq \frac{1}{4}\theta\bar{\theta}\pi^m - \frac{3\theta}{4}\pi^d,$$

which always holds for L and π^d sufficiently small. Finally, if $L > \theta\pi^d/2$, the condition is

$$L \leq \frac{1}{4}\theta\bar{\theta}\pi^m - \theta\pi^d$$

and point *(i)* of the proposition follows.

For point *(ii),* note that the set Γ_2 is empty at $\bar{\theta} = 1$. Consider $(L, \pi^d) \in \Gamma_1$. If $0 < L \leq \theta\pi^d/2$, condition (5) is satisfied for $\rho = 1$ if

$$L \leq \frac{1}{4}\theta(\pi^d + \pi^m).$$

The RHS is always larger than $\theta\pi^d/2$. Hence, the condition is satisfied. If $\theta\pi^d/2 < L < \pi^d/2$, the condition is

$$L \leq \frac{1}{4}\theta(\pi^m + 3\pi^d).$$

The RHS is larger than $\theta\pi^m/2$ if $\pi^d \geq \pi^m/3$, that is for any admissible value of this parameter region. Finally consider (L, π^d) such that the NPE would exclude the first PE after winning

the court case. If $0 < L \leq \theta\pi^d/2$, condition (5) is satisfied for $\rho = 1$ if

$$L \leq \frac{1}{2}\theta(\pi^m - \pi^d).$$

The RHS is always larger than $\theta\pi^d/2$ and the condition holds. If $L > \theta\pi^d/2$, the condition is $L \leq \theta\pi^m/2$ and the point follows.

PROOF of Lemma 3:

$$\Psi^B = \theta \begin{cases} \overline{\theta}D_2 + 2L(\overline{\theta} - 1/2) & \text{if } L \leq L(\overline{\theta}), \\ 0 & \text{otherwise.} \end{cases} + (1-\theta) \begin{cases} \underline{\theta}D_2 + 2L(\underline{\theta} - 1/2) & \text{if } L \leq L(\underline{\theta}), \\ 0 & \text{otherwise.} \end{cases}$$

$$- \begin{cases} \theta D_2 + 2L(\theta - 1/2) & \text{if } L \leq L(\theta), \\ 0 & \text{otherwise.} \end{cases}$$

$$= \begin{cases} 0 & \text{if } L \leq L(\underline{\theta}), \\ -(1-\overline{\theta})\theta D_2 + 2L(1-\theta)(1/2 - \underline{\theta}) & \text{if } L(\underline{\theta}) < L \leq L(\theta), \\ \overline{\theta}\theta D_2 + 2L\theta(\overline{\theta} - 1/2) & \text{if } L(\theta) < L \leq L(\overline{\theta}), \\ 0 & \text{otherwise.} \end{cases}$$

Note that Ψ^B is linear and continuous in L for $L(\underline{\theta}) < L \leq L(\theta)$. At L=$L(\theta)$ we have $\Psi^B = \theta(\rho - 1/2)D_2$ which can be positive or negative. Next, as L approaches $L(\theta)$ from above, Ψ^B goes to $\theta D_2[\theta + (1-\theta)2\rho]/[2(1-\theta)] > 0$. Ψ^B is linear in L, takes value $\theta D_2[\underline{\theta} + \rho]/[2(1-\theta)(1-\rho)] > 0$ at $L(\overline{\theta})$ and, hence, the externality is strictly positive for $L(\theta) < L \leq L(\overline{\theta})$.

PROOF of Proposition 4: For point *(i)*, note that for $L(\underline{\theta}) < L \leq L(\theta)$ and $D = D_1 = D_2$ condition (8) is

$$L \leq \frac{\theta\overline{\theta}}{1 + \theta - 2\theta\overline{\theta}} D. \tag{App-1}$$

The RHS is increasing in ρ and is less or equal than $L(\theta)$ if and only if

$$\frac{\theta D(1/2 - \rho)}{(1-\theta)(1 + \theta - 2\theta\overline{\theta})} \geq 0$$

or $\rho \leq 1/2$. For $\rho \geq 1/2$, there is always an incentive to litigate for $L(\underline{\theta}) < L \leq L(\theta)$.

Point *(ii)* follows directly from Lemma *3*. For point *(iii)* note that $L(\underline{\theta}, \theta = 1) < \infty$. However, $L(\theta)$ goes to ∞ and the RHS of *(App-1)* goes to ∞ as θ goes to 1. Hence the litigation threat

is always credible as θ goes to 1.

PROOF of Proposition 5: Consider point *(i)*. From section 2 follows that under the American rule, the highest L such that the NPE has an incentive to litigate is given by

$$L = \min\{\theta(1+\overline{\theta})D, \overline{\theta}D\}. \tag{15}$$

It is easy to check that the first term is binding if $\theta \leq \theta'$ with $\theta' \in[\sqrt{2}-1, 1/2]$ for $\rho \geq 1/2$. Note that there exist values below this upper bound and $\underline{\theta}D < L < \theta D$ where litigation is not credible due to the negative externality. From the previous proposition follows that if $\rho \geq 1/2$, then litigation is always credible for $L \leq L(\theta)$ under the British rule, and more generally if

$$L \leq \min\{\frac{\theta(1+\overline{\theta})}{2(1-\theta/2-\theta\overline{\theta})}D, L(\overline{\theta})\}. \tag{16}$$

The binding constraint is the first term if and only if $\theta \leq \theta''$ with $\theta'' > 1/2$ for $\rho \geq 1/2$. Finally, the first term in (15) is smaller than the first term (16) if and only if $\theta \geq \theta^*$ with $\theta^* \in[1/3, \sqrt{2}-1]$ for $\rho \geq 1/2$. Hence, from $\theta^* \leq \theta' \leq \theta''$ and $L(\overline{\theta}) \geq \overline{\theta}D$ for $\rho \geq 1/2$ follows that litigation is more credible under the British rule if $\theta \geq \theta^*$. As θ^* decreases in ρ, the lowest value such that this result holds for all $\rho \geq 1/2$ is $\theta \geq \theta^*(\rho = 1/2) = \sqrt{2}-1$.

For point *(ii)* consider $\rho < 1/2$. In this case, under the British rule there exist parameter values such that if L is sufficiently close but below $L(\theta)$, litigation is not credible due to the negative information externality. Similarly, if L is sufficiently close but below θD, litigation is not credible under the American rule. Now check that $L(\theta) < \underline{\theta}D$ if and only if $\rho \leq 1/2 - \theta/[2(1-\theta)]$. For this condition, there exist parameter values such that litigation is credible under the American but not under the British rule. Vice versa, if $\theta D < L(\underline{\theta})$ or $\rho \leq 1/3 - 4(1-\theta)/[3(1+2\theta)]$, there exist values such that litigation is credible under the British but not the American rule.

For point *(iii)*, consider $\rho \geq 1/2$ such that litigation is credible under the British rule if and only if (16) is satisfied. As shown above, for small values of θ, the first term in (16) is binding. Under the American rule, if $\theta\overline{\theta}D < L < \theta D$, litigation is not credible due to the negative information externality. Hence, to show point *(iii)*, it suffices to prove that

$$z(\theta) = \frac{\theta(1+\overline{\theta})}{2(1-\theta/2-\theta\overline{\theta})}D - \theta\overline{\theta}D > 0$$

for θ sufficiently small. This follows directly from $z(0) = 0$ and $z'(0) = (1-\rho)D/2 > 0$.

PROOF of Proposition 7: There exist parameter values such that conditions (11) and (12) for certain infringement hold if the RHS of (11) is larger than the RHS of (12) which is satisfied for $\delta \geq 2(1-\theta)$. Thus, if $\theta \geq 1/2$, such values always exist. Condition (11) for a certain claim implies (10) for a weak claim if the RHS of (10) is larger than the RHS of (11) which holds if $\delta \leq 2\theta$. This condition is always satisfied if $\theta \geq 1/2$. This establishes the second part of the proposition.

PROOF of Lemma 6: The comparative statics follow from

$$\frac{\partial p(e_N, e_P)}{\partial e_i} = \frac{\theta(1-\theta)e_j}{(\theta e_N + (1-\theta)e_P)^2} = \frac{p^*(1-p^*)}{e_i}.$$

and $V_2(\overline{\theta}) - V_2(\underline{\theta}) = \rho D$. We then have

$$\begin{aligned}
&\frac{\partial e_N^*}{\partial \rho} = \frac{(1+\rho)2\theta(1-\theta)^2}{(1+\rho\theta)^3}D, \frac{\partial e_P^*}{\partial \rho} = \frac{\theta(1-\theta)[1-\theta(2+\rho)]}{(1+\rho\theta)^3}D, \\
&\frac{\partial e_P^*}{\partial \theta} = (1+\rho)\frac{\partial e_N^*}{\partial \theta} = \frac{(1+\rho)[1-\theta(2+\rho)]}{(1+\rho\theta)^3}D, \\
&\frac{\partial p^*}{\partial \rho} = \frac{\theta(1-\theta)}{(1+\rho\theta)^2}, \frac{\partial p^*}{\partial \theta} = \frac{1+\rho}{(1+\rho\theta)^2}.
\end{aligned}$$

The lemma follows.

PROOF of Proposition 8: Condition (14) can be rewritten as

$$\begin{aligned}
\varphi &= p^*[D + V_2(\overline{\theta}) - V_2(\underline{\theta})] - e_N^* - [V_2(\theta) - V_2(\underline{\theta})] \geq 0 \\
&= (1+\rho)(p^*)^2 D - [\rho\theta D] \geq 0 \\
&= [\frac{(1+\rho)^3\theta^2}{(1+\rho\theta)^2} - \rho\theta]D \geq 0
\end{aligned}$$

Check that

$$\frac{\partial \varphi}{\partial \theta} = \frac{2(1+\rho)^2\theta}{(1+\rho\theta)^3} - \rho \text{ and } \frac{\partial^2 \varphi}{(\partial\theta)^2} = \frac{2(1+\rho)^2(1-2\rho\theta)}{(1+\rho\theta)^4}.$$

The second derivative is positive if $\theta < 1/(2\rho)$ and negative otherwise. Hence, φ takes value 0 at $\theta = 0$, has a negative slope there and is convex for low values of θ and concave for higher values. Moreover, since $\varphi(\theta = 1)$=1, there exists a unique θ' such that if $\theta \geq \theta'$, the condition is satisfied whereas if $\theta < \theta'$, the condition does not hold. Litigation does not maximize the

joint profits of the first PE and NPE since

$$e_N^* + e_P^* - \Omega^L = \frac{\theta(1-\theta)[2+3\rho-\theta\rho^3]}{(1+\rho\theta)^2}\frac{\pi}{2} > 0$$

Now consider the difference between the NPE profits from bargaining with the first PE and the profits when facing two unrelated PEs, that is,

$$\begin{aligned} &p^*D + V_2(\theta) + \frac{1}{2}[\Omega^L - (e_N^* - e_P^*)] - 2V_2(\theta) \\ &= \frac{\theta(1-\theta)\rho[1+\theta(2+\rho)\rho]}{2(1+\rho\theta)^2}D \geq 0. \end{aligned}$$

PROOF of Proposition 9: Since the proof parallels the one for Proposition 2, we only provide a sketch.

(i) There are two distinct cases to consider with $\Psi_1 > 0$. First consider $\Psi_2 \leq 0 < \Psi_1$. In this case, the 21-sequence allows credibility against the first target whereas the 12-sequence is not credible against the first target. In addition, the 21-sequence strictly dominates simultaneous negotiations since

$$\theta D + \frac{1}{2}\Psi_1 > \theta D.$$

The second case is where $\Psi_2 = \Psi_1 = k\theta D > 0$ which holds if $max\{\theta, \underline{\theta}_2\}D < L \leq kD$. Such values only exist if $k > \theta$. In this case, litigation against the second target is not credible for either sequence. If litigation with the first target is credible, profits are weakly higher in the 21-sequence since

$$\theta D + \frac{1}{2}\Psi_1 = \theta(1+\frac{k}{2})D \geq k\theta D + \frac{1}{2}\Psi_2 = \theta(k+\frac{k}{2})D.$$

Moreover, the 21-sequence is always credible while the 12-sequence is credible and there exist values where the 21-sequence is the only sustainable strategy. Finally, since simultaneous negotiations yield no returns, the NPE strictly prefers the 21-sequence as long as the credibility constraint for the 21-sequence is satisfied.

(ii) The case $\Psi_2 > \Psi_1 = 0$ corresponds to parameter values such that $\max\{\theta, k\}D < L \leq D$. Litigation is not credible in the 21-sequence and with simultaneous negotiations. However, if $L \leq 2k\theta$, the 12-sequence has a credible threat with the first target.

(iii) Three more cases are possible. The case $\Psi_1 < \Psi_2 = 0$ corresponds to parameter values

such that $L < \min\{k\theta, \underline{\theta}_2\}D$. Litigation is always credible against the second target in both sequences. Litigation is also credible with the first target in the 12-sequence. Since $L \leq \theta D + \Psi_1 = \theta D - (1-k)\theta D = k\theta D$, litigation is also credible against the first target in the 21-sequence. However, the 12-sequence is more profitable since

$$\theta D + \frac{1}{2}\Psi_2 + k\theta D = \theta(1+k)D > \theta D + \frac{1}{2}\Psi_1 + k\theta D = \theta(1+k-(\frac{1-k}{2})]D$$

for $\Psi_1 < \Psi_2 = 0$. However, the best sequential strategy cannot improve on simultaneous litigation which yields $\theta(1+k)D$. The second case to consider is $\Psi_2 < 0 = \Psi_1$ corresponding to parameter values such that $L > \overline{\theta}D_1$ and $\max\{\underline{\theta}_2, k\}D < L \leq \theta D$. The 12-sequence has no credibility. The 21-sequence has a credible threat against the first firm but as $\Psi_1 = 0$, the NPE makes the same profit, θD, as with simultaneous litigation. The last case is $\Psi_2 = \Psi_1 = (1-k)\theta D < 0$ which holds if $\underline{\theta}_2 D < L < k\theta D$. If sustainable the equilibrium profit for the NPE with firm i as first target is always less than what it can earn with simultaneous negotiations,

$$\theta(1+k)D + \frac{1}{2}\Psi_i = \theta(1+k-(\frac{1-k}{2})]D < \theta(1+k)D.$$

PROOF of Lemma 8: Assume that D_1 and D_2 are distributed according to a joint distribution $F(.,.)$ on an interval $[\underline{D}\ , \overline{D}]^2 \subset [L/\overline{\theta}, L/\theta]^2$, that is, the sign of the information externality is always positive for all realizations of D_1 and D_2. Let $D_{(1)}$ and $D_{(2)}$ denote the first and second order statistic, respectively, i.e., $D_{(1)} = \min\{D_1, D_2\}$ and $D_{(2)} = \max\{D_1, D_2\}$. Then, given the parameter restrictions, the NPE's willingness to pay for the patent portfolio is given by

$$E_F[\theta D_{(2)} + \Psi_{(1)}/2], \text{ where } \Psi_{(1)} = \theta\overline{\theta}D_{(1)}.$$

In contrast, each PE's willingness to pay is $E_F[\theta D_i + \Psi_j/2 | D_i > D_j] \Pr[D_i > D_j], i = 1, 2$ and $j \neq i$. Since $E_F[\theta D_i + \Psi_j/2 | D_i > D_j] \Pr[D_i > D_j] > 0$ and $\sum_{i=1}^{2} E_F[\theta D_i + \Psi_j/2 | D_i > D_j] \Pr[D_i > D_j] = E_F[\theta D_{(2)} + \Psi_{(1)}/2]$, the NPE has a higher willingness to pay for the patent portfolio than PEs. This result extends to distributions on R_+^2 as argued in the main text.

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