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Kaz Miyagiwa
Jiangyun(Yunyun) Wan

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Kaz Miyagiwa* and Yunyun Wan†

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Abstract

The merger paradox is revisited in the presence of cost-reducing R&D in Cournot oligopoly. Two cases are found, in which merger is profitable without satisfying the 80-percent threshold requirement of Salant et al (1983).

JOE codes:
Key words: merger paradox, R&D, Cournot oligopoly

Corresponding author: Kaz Miyagiwa, Department of Economics, Florida International University, 11200 SW 8th St, Miami, FL33199, U.S.A., E-mail: kmiyagiwa@gmail.com

* Florida International University, HIAS (Hitotsubashi University) and ISER (Osaka University)
† Institute of Innovation Research, Hitotsubashi University

1. Introduction

The merger paradox refers to the fact that it is difficult to explain merger with simple oligopoly models. More specifically, Salant, Switzer and Reynolds (1983) showed that horizontal merger in standard Cournot oligopoly is unprofitable unless it comprises at least 80 percent of all the firms in the industry. Since real-world merger cases never clear this 80-percent threshold, a number of authors have sought ways out of this paradox, some introducing asymmetries into Cournot oligopoly; e.g., Daughety (1990) and Perry and Porter (1985), and others switching to price competition; e. g., Davidson and Deneckere (1986) and Compte, Jenny and Rey (2002). More recent work has examined increasingly complicated settings; e.g., Pesendorfer (2005) and Nocke and Whinston (2013).

In this paper we revisit the merger paradox in Cournot oligopoly. We depart from the literature in that we endow a firm or firms with the opportunities to invest in cost-reducing R&D. We thus ask the question whether R&D opportunities make merger more likely. We give the answer in the affirmative in two scenarios. In the first scenario, we endow only one firm with an R&D capability, and find that, over a certain range of R&D technology, merger by this R&D firm and any number of non-R&D firms is profitable, regardless of the size of the industry. The second scenario supposes that there are multiple firms capable of R&D in the industry, and asks when they have the incentive to merge into a single firm. Here, too, a range of R&D technology can be found over which merger by all R&D firms is profitable, regardless of the number of R&D firms and the total number of firms in the industry.

2. A single R&D firm

Consider an n-firm industry, where $n \geq 3$. Initial marginal cost is identical across firms and fixed at $c > 0$. Demand is linear and takes the familiar form $p = a – Q$, where $a > c$ is demand intercept and Q is industry supply. Then, one firm, say firm 1, acquires an R&D technology that enables it to reduce marginal cost from c to $c - x$ by sinking the cost gx^2, where g is a positive constant. The game has two stages. Without merger firm 1 first chooses x and then all firms play a Cournot game. This is a familiar setup in the literature; see, e.g., d'Aspremont and Jacquemin (1988). With merger, firm 1 and m other firms merge into a new firm; then the new firm invests in R&D, and finally all firms play a Cournot game.

In the no-merger case, given x, firm 1 earns the net profit

$$\text{(1)} \qquad \pi_1 = [(a - n(c - x) + (n - 1)c]^2 / (n + 1)^2 - gx^2$$
$$= (a - c + nx)^2 / (n + 1)^2 - gx^2$$

while $(n - 1)$ non-R&D firms j ($\neq 1$) earn the symmetric profits

$$\text{(2)} \qquad \pi_j = (a - c - x)^2 / (n + 1)^2.$$

In stage one, firm 1 choose x to maximize the profit in (1). The first-order condition is

$$n(a - c + nx) / (n + 1)^2 - gx = 0,$$

which is arranged to yield the optimal investment x* in R&D:

$$\text{(3)} \qquad x^* = n(a - c) / [g(n + 1)^2 - n^2].$$

The second-order is satisfied if

$$\text{(4)} \qquad g > n^2 / (n + 1)^2.$$

The new marginal cost $c_1 = c - x_1^*$ is positive if

$$\text{(5)} \qquad g > n(n - 1 + a/c) / (n + 1)^2.$$

Given $a > c$, (5) implies (4).

Substituting from (3) into (1) yields the maximal profit for firm 1:

$$\pi_1^* = (a - c)^2[g^2(n + 1)^2 - gn^2] / [g(n + 1)^2 - n^2]^2$$
$$= g(a - c)^2 / [g(n + 1)^2 - n^2].$$

Substitution from (3) into (2) yields the equilibrium profit for a non-R&D firm j:

(6) $$\pi_j^* = (a - c)^2[g(n + 1) - n]^2 / [g(n + 1)^2 - n^2]^2$$

Firm j produces a positive quantity of output if

(7) $$g > n / (n + 1).$$

The left-hand side expressions in (5) and (7) are increasing in n and tends to 1 as n goes to infinity. Since we seek results that hold for any n, we take that $g \geq 1$.

Suppose firm 1 and m non-R&D firms merge, where $1 \leq m \leq n - 2$. Merger reduces the number of firms to $(n - m)$, including the new firm. The equilibrium outcome is identical to the one without merger, except that n is replaced by $n - m$. The new firm earns the profit

$$\pi^*(m) = (a - c)^2 g / [g(n - m + 1)^2 - (n - m)^2].$$

We compare this profit with the combined pre-merger profit $\pi_1^* + m\pi_j^*$, which equals

$$\pi_1^* + m\pi_j^* = (a - c)^2\{[g^2(n + 1)^2 - gn^2] + m[g(n + 1) - n]^2\}/[g(n + 1)^2 - n^2]^2.$$

If $\pi^*(m) > \pi_1^* + m\pi_j^*$, merger is profitable. Note that without R&D opportunities this merger is unprofitable unless $(m + 1)/n \geq 0.8$.

The difference $\pi^*(m) - (\pi_1^* + m\pi_j^*)$ generally depends on m, n, and g, so it is difficult to obtain simple results. However, evaluating the difference at $g = 1$ yields this strong result.

Proposition 1: At $g = 1$, merger by the R&D firm and any number m of non-R&D firms is profitable for any industry size n ($> m + 1$).

Proof. Substituting $g = 1$ yields

$$\pi^*(m) = (a - c)^2/(2n - 2m + 1)$$

$$\pi_1^* + m\pi_j^* = (a - c)^2(2n + 1 + m)/(2n + 1)^2.$$

Since $n > m$,

$$\pi_1^*(m) - (\pi_1^* + m\pi_j^*) = (a - c)^2 (2n + 2m + 1)\, m /[(2n - 2m + 1)(2n + 1)^2] > 0. \quad |$$

Is it possible that proposition 1 is driven by firm 1' cost advantage alone? To check this possibility, suppose that c_1 is exogenously given. Then, without merger firm 1 and m other firms have the combined profit of

$$\{[a + (n - 1)(c - c_1) - c_1]^2 + m(a - 2c + c_1)^2\} / (n + 1)^2.$$

If these firms merge into a new firm, there are (n – m) firms in the industry. If the new firm uses firm 1's superior technology in production, its profit is

$$[a + (n - m - 1)(c - c_1) - c_1]^2 / (n - m + 1)^2.$$

The question is whether, given a and c, there is c_1 ($< c$) that makes merger profitable for any m and for any n. If we write $c_1 = sc$ (where $0 \leq s \leq 1$), it is equivalent to the question whether one can always find s that satisfies this condition:

$$(n + 1)^2 [a + (n - m - 1)(1 - s)c - sc]^2$$
$$- (n - m + 1)^2 \{[a + (n - 1)(1 - s)c - sc]^2 + m(a - 2c + sc)^2\} > 0.$$

The left-hand side expression is quadratic in s and its graph is U-shaped. At $s = 1$ all firms are symmetric, so if $(m + 1)/n < 4/5$ the 80-percent threshold is not reached, and therefore the above inequality cannot hold. For such m and n, if the above inequality does not hold at $s = 0$, either, then there is no s that can satisfy it. We can show that there is no s satisfying the above inequality when $n = 3$ and $m = 1$. This counterexample proves that proposition 1 is not driven by the cost asymmetry alone.

The above analysis shows that there is a greater incentive for the low cost firm to merge when its marginal cost is endogenous (when there are R&D opportunities). This is due to three effects that arise from R&D opportunities. One is the R&D cost saving effect; merger allows the R&D firm to share the R&D cost with participating firms, making merger relatively more attractive. The other two are a little subtler, however. To understand them, note first that without merger the R&D firm's second-stage marginal cost is fixed at c_1 and hence it is identical to the model with exogenous marginal costs. Second, with or without R&D opportunities the new firm's profit is always greater than the low-cost firm's pre-merger profit. These two facts imply that with R&D opportunities, the prospect of a larger profit motivates the new firm to invest more in R&D than firm 1 does in the absence of merger. As a result the new firm has a lower cost. This increases the new firm's profit relative to the fixed cost case (*marginal cost cutting effect*). More importantly, recall that merger is not profitable in the standard Cournot model because it induces non-participating firms to expand output, forcing the new firm to contract output, and this effect is stronger, the more numerous outside firms are. With R&D however the new firm has a lower cost, which mitigates the effect of an output expansion by non-participating firms or even forces them to contract output (*output expansion effect*). This effect is also highlighted in Daughety (1990). In his model merger is profitable because while firms play a Cournot game, after merger the new firm is assumed to act as the Stackelberg leader and outside firms behave as the follower, so that outside firms tend to contract output instead of increase it. Because of these three effects, an R&D firm has a greater incentive to merge than a low-cost firm without R&D opportunities.

When g increases above 1, there is less investment in R&D and hence a smaller cost reduction so merger becomes less profitable. Eventually merger becomes profitable when g reaches a sufficiently high value.[3] For example, for $n = 10$, $\pi^*(m) - (\pi_1^* + m\pi_j^*)$ is still positive at any $m \leq 10$ when $g \leq 34$ but negative at $m = 4$ and 5 when $g = 35$. At $n = 20$, merger is still profitable at any m but becomes unprofitable at negative at $m = 10$, 11, and 12 when $g = 15$.

3. Multiple R&D firms

[3] It is straightforward to show that $\partial[\pi^*(m) - (\pi_1^* + m\pi_j^*)]/\partial g < 0$.

In this section we suppose that there are k R&D firms in the n-firm industry, where $2 \leq k \leq n - 1$, and we examine whether merger by all k R&D firms is profitable. Without merger, the equilibrium second-stage profit is

$$\pi_j = \frac{[a-(k+1)c+\sum_{i\in K} c_i]^2}{(n+1)^2}$$

if firm j is a non-R&D firm and

$$\pi_j = \frac{[a-nc_j+(n-k)c+\sum_{i\in K_{-j}} c_i]^2}{(n+1)^2} - g{x_j}^2$$

if it is an R&D firm, where K denotes the set of R&D firms, and K_{-j} is derived from K by deleting R&D firm j. In the first stage, all firms j ∈K simultaneously choose their investment levels x_j in R&D to maximize (7). The first-order condition is

$$\frac{n[a-n(c-x_j)+(n-k)c+\sum_{i\in K_{-j}} c_i]}{(n+1)^2} - gx_j = 0\,,$$

which can be solved for the symmetric equilibrium level of investment in R&D:

$$x^* = \frac{n(a-c)}{g(n+1)^2 - n(n-k+1)}\,.$$

Substituting this expression for x* into the profit expressions above yields the (symmetric) equilibrium profit for an R&D firm

$$\pi^*(k) = (a-c)^2 \frac{g^2(n+1)^2 - gn^2}{[g(n+1)^2 - n(n-k+1)]^2}$$

where we suppress $j \in K$.

Suppose that all k R&D firms merge into a new R&D firm prior to investing in R&D. Since it is the only firm capable of R&D in the industry that has n – k + 1 firms, we can use Eq. (6) from the preceding section to write the equilibrium profit to the new firm:

$$\hat{\pi}(k) = \frac{g(a-c)^2}{g(n-k+2)^2 - (n-k+1)^2}$$

The above analysis makes sense if $\hat{x}(k)$ and $\hat{\pi}_1(k)$ are positive, which is assured if

(8) $$g > (n-k+1)^2 / (n-k+2)^2 .$$

The right-hand side of (8) increases to the limit of 1 as n goes to infinity, so again (8) is satisfied for all n if we assume $g \geq 1$.

We now compare the profits $\hat{\pi}(k)$ and $k\pi^*(k)$. The sharpest result again obtains at g = 1,

Proposition 2: At g = 1, merger by all k R&D firms is profitable for all k and n.

Proof. At g = 1, $\hat{\pi}(k)$ - $k\pi^*(k) > 0$ if and only if

$$[(n+1)^2 - n(n-k+1)]^2 - k\,[(n-k+2)^2 - (n-k+1)^2][(n+1)^2 - n^2] > 0.$$

Letting n + 1 = q, we can write the left-hand side as

$$[q^2 - (q-1)(q-k)]^2 - k[(q-k+1)^2 - (q-k)^2][q^2 - (q-1)^2]$$

$$= [q^2 - (q-1)(q-k)]^2 - k(2q - 2k + 1)(2q - 1)$$

$$= (k-1)[(q^2(k-1) + k(2q-1)]$$

which is positive |

We now consider the case in which k firms already have lower marginal cost c_1 and ask when merger by k efficient firms is profitable. Without merger, each efficient firm earns the equilibrium profit

$$\pi_{NM} = [a - (n - k + 1)c_1 + (n - k)c]^2 / (n + 1)^2$$

If they merge, the new firm has the profit

$$\pi_M = [a - (n - k + 1)c_1 + (n - k)c]^2 / (n - k + 1)^2.$$

Thus, merger is profitable if and only if

$$1 / (n - k + 1)^2 - k / (n + 1)^2 > 0.$$

This is the same condition that a profitable merger satisfies in symmetric Cournot oligopoly, and hence holds only if $k/n \geq 0.8$. In contrast, merger by k R&D firms again generates the R&D cost-sharing effect. This time, since all k firms invest in R&D without merger, this effect is more powerful than in the model of the previous section. Also, merger generates the cost-reducing effect and the output expansion effect as in the preceding section. Again, R&D opportunities make merger more profitable. However, as g is increased, the efficiency of investment in R&D decreases so that at g high enough, merger ceases to be profitable.

4. Conclusions

This paper reconsiders the merger paradox in Cournot oligopoly, when firms have the opportunities to invest in cost-reducing R&D. Two scenarios are examined under the assumptions of linear demand and quadratic R&D cost. (1) With one R&D firm, there is a range of R&D technology over which merger by this R&D firm and any number of non-R&D firms is profitable. (2) With multiple R&D firms, there is a range of R&D technology over which merger by all R&D firms is profitable, regardless of the number of R& firms and the total number of firms in the industry.

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