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Network-Motivated Lending Decisions[*]

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Abstract

We theoretically and empirically demonstrate that monopolistic or collusive banks will keep lending to a loss-making firm at an interest rate lower than the prime rate if the firm is located in an influential position in an inter-firm supply network. An influential firm generates a positive externality, and its exit damages the sales in the supply network. To internalize this externality, the banks may forbear on debt collection and/or bail out such influential firms when the cost to support the loss-making influential company can be recouped by imposing high interest on less influential companies. The analytical model shows that such forbearance can improve welfare. Our empirical study, performed using a unique dataset containing information about inter-firm transactions, provides evidence for such network-motivated lending decisions. In particular, this effect is more clearly observed at less credit-worthy firms whose main bank is a regional bank. Notably, we observe that such banks are often dominant lenders in the local loan market, and most of their clientele do not have direct access to the stock and bond market.

Keyword: supply network, influence coefficient, centrality, forbearance, bailout

JEL Classification: C55, D57, G21, G32, L13, L14

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1 Introduction

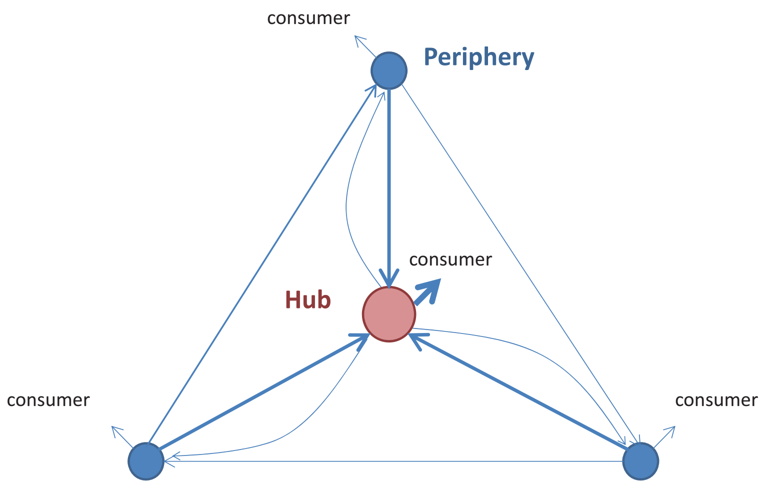
In economic downturns, we frequently observe governments and financial institutions rescuing "too big to fail" and "too connected to fail" companies. For example, the U.S. government bailed out the country's Big Three automakers by providing some 8 billion USD in the midst of the 2008 global financial crisis, despite heated controversy over this action. A news article reports, "The Big Three directly employ almost 250,000, [$\cdots$], not counting the vast network of suppliers and dealers whose businesses are intertwined. In all, administration officials estimate that the failure of the U.S. auto makers would cost the economy more than one million jobs".[1] It is also well recognized in the academic literature that Japanese major banks engaged in extensive *forbearance* or *zombie* lending in the 1990s (e.g., Sekine et al., 2003; Peek and Rosengren, 2005; Caballero et al., 2008). Forbearance or zombie lending refers to the behavior of a bank that keeps extending additional loans at a lower interest rate to under-performing or non-performing firms to which the bank has existing exposure.

In this paper, we theoretically and empirically demonstrate that profit-maximizing banks are undertaking forbearance lending. Moreover, our theory suggests that this can be welfare-improving. We argue that some recipient firms are influential in a supply network in the sense that its existence creates positive externalities for the total sales and profit of the network. Even when that firm is loss-making, forbearance may enhance welfare when it internalizes the externalities caused by the existence of this firm. A profit-maximizing bank that is a dominant financier in a region or an industrial group may have an incentive to undertake forbearance in order to internalize this externality. In particular, the bank can increase its profit by supporting a loss-making influential firm and recouping the cost by imposing higher interests on the other firms that depend on sales to the influential firm. We also conduct an empirical study that uses a unique dataset of inter-firm transactions. The empirical results are consistent with our theory.

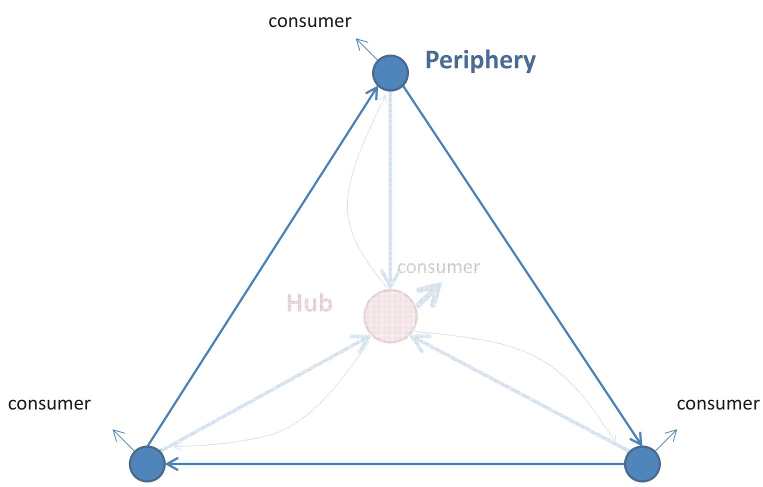
[1] "Detroit Gets Access To Bailout Funds," Dec. 13, 2008, *Wall Street Journal*.

Figure 1: Supply network as a directed and weighted network

(Note) Each node is a hub or peripheral firm. The direction of each arrow represents the direction of product sales and the thickness indicates the amounts of sales.



(a) Keeping the hub open



(b) Closing the hub

To illustrate the economic problem that we try to address in this study, let us consider the following example, illustrated in Figure 1. There are two types of companies. One type of company is a large company that heavily depends on intermediate inputs from other companies. We call these hub companies, and one is shown in panel (a) of Figure 1. The other type is small companies that are not as dependent on intermediate goods but are major providers of intermediate inputs; we call these periphery firms. The direction of the arrows in the figure indicates the direction of the flow of products, and the thicknesses indicate the amount of sales. The negative impact of the closure of the hub company propagates throughout the network by reducing sales. This can even trigger a chain of closures of peripheral companies, as is shown in panel (b) of the figure. Even when the hub company is loss-making, it may be welfare-improving to undertake forbearance for it. Moreover, when a bank is a dominant financier for all firms in this network, the bank may increase its profit by supporting the loss-making hub company and recouping the cost by imposing higher interest on the peripheral companies.

To formalize and generalize the insights obtained above, we construct a theoretical model of a supply network. We formulate the inter-firm supply network as an incomplete directed and weighted network of sales among oligopolistic firms offering differentiated products that are intermediate inputs as well as final products. This model is based on multi-sector general equilibrium models, such as Long and Plosser (1983), Horvath (2000), and Acemoglu et al. (2012). Under the assumption that each firm has to depend on external financing to pay its fixed costs, we identify the conditions under which a monopolistic bank or collusive banks will strategically extend a loan to a loss-making but influential company at a rate lower than the prime rate.

In our inter-firm supply network, each firm has both positive and negative externalities. The positive externality comes from the fact that the production of a firm induces demand for intermediate goods from suppliers, which propagates through the supply network. The extent of this externality is measured by the influence coefficient, which is well-known in industry-level input–output analysis and has been extended to firm-level analysis by Acemoglu et al. (2012). The negative externality is the business-stealing effect (Mankiw and Whinston, 1986) or the congestion effect (Caballero et al., 2008). The model shows that a monopolistic bank or collusive banks will engage in forbearance lending when the former outweighs the latter.

Our empirical study shows evidence for the occurrence of network-motivated lending decisions. We compute the influence coefficients of firms from a unique dataset, the *TSR Corporate Relationship Database*, which provides information about inter-firm transactions in Japan. We then examine the relationship between influence coefficients and interest payments.

TSR Corporate Relationship Database contains records of inter-firm transactions among about 650,000 firms in Japan. We assume that each supply network consists of firms that share a main bank. By using the data set above, we write the directed adjacency matrix of firms for each network. Our theoretical model indicates a spatial autoregressive model of sales with the weighting matrix being approximated by the adjacency matrix. We estimate the resulting spatial autoregressive model and compute the influence coefficients.

We then match the estimated influence coefficients with corporate financial data and examine the effects of the influence of a firm on the interest rates of offered loans. We find that the interest payments of influential firms are lower than those of the other firms, and significantly so in both the statistical and economic senses. This result holds after controlling for relevant observable characteristics of each company and each bank, and the tendency is more pronounced for less credit-worthy companies. Moreover, we find that the effect is more prevalent for less credit-worthy firms and those whose main bank is a regional bank. We note that, in Japan, regional banks tend to be a dominant lender in their loan market. These empirical results are consistent with our theory of network-motivated lending decisions.

This paper contributes to the literature in various ways. First, and most importantly, this paper points out the importance of firms' location in a supply network for the capital costs. In particular, we show that forbearance can occur as a result of internalizing externalities that run through the supply network. Second, our empirical study that uses inter-firm transaction data, including data from small businesses, is new in the corporate finance literature, as is merging the transaction data with the main bank information. This dataset enables us to observe the relationship between inter-firm supply networks and bank lending decisions, which has never been directly examined in the literature. This paper illustrates the usefulness of this type of data in the context of corporate finance.

This paper also highlights some policy implications. In particular, our finding implies that public bailouts could be economically efficient. We theoretically clarify the conditions under which a public bailout is welfare improving. Our analysis additionally suggests that information about the influence coefficients of companies in an economy is useful to quickly formulate an economically efficient bailout plan.

Related literature

Our study proposes a novel viewpoint for academic discussion about the mechanism by which a bank will choose to engage in forbearance or zombie lending. Existing theories on this subject (e.g., Dewatripont and Maskin, 1995; Berglöf and Roland, 1997; Bruche and Llobet, 2014) have focused on the one-to-one bank–firm relationship. For example, Dewatripont and Maskin (1995) shows that a bank tends to invest excessively in borrower-specific information or monitoring ability when the bank repeatedly lends to a specific firm. Relying on this excessive monitoring ability, which is a sunk cost, a bank tends to keep supplying funds to a firm to which outside banks would refuse lending. Since smaller and privately held firms are more dependent on relational lending (Cole et al., 2004), this theory implies that we are more likely to observe forbearance among those firms. However, empirical studies of extensive microdata about small business financing show that credit tightening, rather than forbearance, was prevalent among those bank-dependent small firms (Hosono, 2008; Ogawa, 2008; Sakai et al., 2010; Hamao et al., 2012).

Our explanation of forbearance is clearly different from those existing theories and is consistent with the observed phenomenon that bank-dependent small firms are not benefited by forbearance. Our focus is not on the one-to-one bank-firm relationship; instead, we consider the fact that a bank lends to thousands of firms interconnected through a supply network and can take into account the network effect. We can also show that forbearance can happen even when the target firm is loss-making. In contrast, existing theories do not indicate such a phenomenon because under those theories a bank may lend to a firm that is less profitable but will not lend to a loss-making firm.

The zombie lending in Japan in the 1990s and early 2000s (the so-called lost decade) has been investigated by, for example, Sekine et al. (2003), Peek and Rosengren (2005) and Caballero et al. (2008). However, they mainly consider the existence of zombie lending and the consequences of such lending, while we consider the mechanism that leads to zombie lending or forbearance. Since our dataset does not cover the same periods, our empirical results do not necessarily explain the events in the 1990s and early 2000s in Japan. Nonetheless, our theory is useful in understanding the motivation behind zombie lending generally. It is said that zombie lending in the lost decade in Japan occurred because the government induced the banks to extend loans to loss-making large companies.[2] Our theory indicates that such a policy may be of interest for governments because large firms are likely to have high influence in supply networks. Since our argument is based on a model with one period, our argument would be applicable only in the short run. Therefore, our argument implies that such a policy may be beneficial in the short run, but the results of, for example, Caballero et al. (2008) indicate that it may hurt the economy in the long run. The long-term welfare implications of network-motivated forbearance is an important topic for future research.

Analytical models of the supply network have already been proposed in the macroeconomics literature. However, studies within macroeconomics tend to focus on the impact of sector-specific shocks on aggregate variables. The list of related macroeconomic studies includes Long and Plosser (1983), and Horvath (2000) about the simulation of aggregate variables, Battiston et al. (2007) about firm distributions, Acemoglu et al. (2012) about the possibility of an idiosyncratic shock to be transformed into an aggregate shock, and Bigio and La'O (2013) about the aggregate impact of collateral constraints.[3] The present paper does not examine the effect of supply networks on aggregate macroeconomic variables; instead, it focuses on the behavior of a bank facing a supply network. Another other

[2] For example, about a loss-making giant retailer in Japan, "Daiei (pronounced die-ay) has been on artificial support for several years: despite sales that have fallen 25 percent since 1999, it has managed to borrow trillions more yen from its banks.... Daiei's major banks have been encouraged by politicians and government officials to keep it going at all cost. [... omitted ...] [The] Minister of Economy, Trade and Industry announced earlier this year that Daiei, which employs 96,000 people, was simply too big to be allowed to fail." ("They're Alive! They're Alive! Not!; Japan Hesitates to Put an End to its 'Zombie' Businesses," October 25, 2002, *New York Times*).

[3] Dupor (1999), in contrast, argues that the effect of the input–output structure on the aggregates might be minor.

important difference from the macroeconomic strand in the literature is that research in macroeconomics typically use the industry-level input–output table to empirically examine the network effect. We use a dataset of inter-firm transactions instead.

Analytical models to formulate the chain of inter-firm trade credit are also abundant, but the focus of such models is on aggregate volatility (Giesecke and Weber, 2006) and on the possibility of contagion through the chain of trade credit (Kiyotaki and Moore, 1997; Boissay, 2006). Among empirical studies on this point, several studies find significant effects of a liquidity shock, a bankruptcy, and other negative shocks of a firm, as reflected in stock returns, the yield spreads of bonds, and the supply of trade credits of its direct suppliers, customers, or competitors (Hertzel et al., 2008; Boissay and Gropp, 2013; Chen et al., 2013; Calvalho et al., 2014). However, none of these studies looks at the influence of a shock on the response of a bank. Moreover, we consider the higher-order influence of a shock as measured by the influence coefficient while existing studies, in contrast, examine only first-order impacts.

Network models have been popular in the banking literature. However, to the best of our knowledge, our paper is the first to examine the role of supply network on the lending decision of banks. The existing studies in the banking literature focus mainly on inter-bank networks and concern how a shock to a single bank is propagated to the entire network of banks (see Acemoglu et al., 2015; Allen and Gale, 2000; Billio et al., 2012; Castiglionesi and Navarro, 2011; Eisenberg and Noe, 2001; Elliot et al., 2014; Gai et al., 2011; Nier et al., 2007, among others), and a possible autonomous bailout to avoid such a contagion (see Leitner, 2005; Rogers and Veraart, 2013). While the present paper also belongs to the literature of banking, we consider networks of non-financial firms, not networks of banks.

Our paper is also related to development finance. For example, Morck (2009) discusses the possibility that business groups and financial transactions within such groups, which are often found in developing economies, contribute to economic development by complementing the underdeveloped financial sector. Business groups can be interpreted as a network of group firms and group financial institutions. The role of business groups as a financial institution is empirically examined in Gopalan et al. (2007). Our analysis, based on inter-firm supply networks, is readily applicable to this context.

Organization of the paper

The remaining part of this paper is organized as follows. We introduce a model of a supply network and derive the equilibrium in Section 2. Our main theoretical results are in Section 3. There, we show the possibility and the welfare implications of network-motivated forbearance. We specify the hypotheses to be tested in our empirical study in Section 4. Section 5 explains the dataset used in our empirical analysis. The estimation of the influence coefficient is explained in Section 6. Section 7 presents the results of our empirical analysis of the relationship between interest rate and influence coefficient. A robustness check of our empirical study is described in Section 8. Section 9 discusses potential caveats in interpreting the welfare implications of our results. Section 10 is the conclusion and comments on possible future research topics.

2 Theoretical analysis

In this section, we present a theoretical model of a supply network with the financial sector. The equilibrium in this model is also presented here.

2.1 Setup

Our model is a version of a one-period oligopoly model with a banking sector. We assume that production technology requires intermediate goods and that this creates the supply network. We also assume that firms require fixed costs to keep operating. The role of the bank is to finance the fixed costs.

The basic setup of the model is as follows. There are H households, indexed by h $(= 1, \ldots, H)$; n firms, indexed by i $(= 1, \ldots, n)$; and a monopolistic bank. There are $(n + 1)$ goods in the economy. These goods are differentiated. Good 0 is a pure intermediate good (i.e., not a consumption good). It is supplied from outside of the supply network in the model. Good i, for $0 < i \leq n$, is produced by firm i. The aggregate quantity and the nominal price of good i are denoted by x_i and p_i, respectively. The nominal price of good 0, p_0, is exogenously given. We assume that the banking sector is a monopoly for the time being to elucidate the logic of our model in the simplest manner. An economy with multiple banks is discussed later. More details of each economic agent are described below.

We assume the decision timing and the settlement scheme are as follows.

Time 1. The bank decides on which firms and which outside opportunities it will invest in. Those firms whose fixed costs are not financed by the bank exit from the economy. Which firms keep operating becomes common knowledge.

Time 2. Firms decide the production level and their demand for input. At the same time, households decide the product demand. A Walrasian auctioneer announces the prices that correspond with the demands and the supplies. All payments are settled by trade credit.

Time 3. Firm profits and outside investment outcomes are realized. All trade credits are cleared. The bank captures all the profits of the firms and obtains the return of the outside investment opportunity.

Let e_i $(i = 1, 2, \cdots, n)$ equal one if firm i operates and zero otherwise. We note that if $e_j = 0$ for some j, then good j is not supplied. We assume $e_0 = 1$ always, that is, the input supply from the outside of the network always exists.

2.1.1 Households

Households are utility maximizing, with the utility function of household h given by

$$U_h = \left(\sum_{j=1}^{n} c_{hj}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \tag{1}$$

where c_{hj} is the consumption of good j by household h, and θ is the elasticity of substitution. We assume that households are symmetric so that the value of θ does not vary across households. The budget constraint of household h is

$$\sum_{j=1}^{n} c_{hj} p_j \leq R_h, \tag{2}$$

where R_h is the nominal income of household h, which will be determined in the equilibrium (see Section 2.2.4). Note that the households also face the availability constraint: $(1 - e_j)c_{hj} = 0$.

2.1.2 Firms

Firms are profit maximizing and have the production function

$$x_i = \left(\sum_{j=0}^{n} w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \tag{3}$$

where θ is the elasticity of input substitution, w_{ij} is the "technological importance" of the input j for the production of firm i, and x_{ij} is the quantity of the input quantity from firm j into the production of firm i. We assume that the elasticity of input substitution θ is equal to θ in the utility function to simplify the analysis.

The supply network is described by $\{\{w_{ij}\}_{i=1}^{n}\}_{j=0}^{n}$. We assume that the supply network is rigid in the sense that w_{ij} does not change, even if a supplier of an intermediate product ends up closing.[4] This assumption also means that there is no free-entry of new firms into the market. We assume that $\{\{w_{ij}\}_{i=1}^{n}\}_{j=0}^{n}$ satisfies $0 \leq w_{ij} \leq 1$, $\forall\, i, j$; $w_{ii} = 0$; and $0 < \sum_{j=0}^{n} w_{ij} \leq 1$.[5] Note that since some firms may fail to operate, the supply network the firms actually face is $\{\{e_j w_{ij}\}_{i=1}^{n}\}_{j=0}$.

Each firm has to expend an exogenously given fixed cost F_i $(i = 1, 2, \cdots, n)$ to operate. The value of F_i is fixed in the real term but varies by firm. Each firm has to rely on external financing for this expense.[6] Those firms whose fixed costs are financed always operate even when the operation results in a loss; presumably, this is because of the limited liability for firm owners. Payments for fixed costs exit the economy.

[4]This assumption is more plausible in industries where the designs of input products or the contents of services are highly customized, information- or skill-intensive, and specific to each user. Automobiles, construction, and some types of retailers/wholesalers dealing in custom-made items are of this type.

[5]The value of $\sum_{j=0}^{n} w_{ij}$ represents the productivity level. Note also that the condition $\sum_{j=0}^{n} w_{ij} \leq 1$ guarantees the existence of the equilibrium price vector (11) as well as the existence of an equilibrium.

[6]This assumption is employed to make the analysis tractable and is a simplified version of the situation in which external finance is necessary. A literal interpretation of this assumption may be the upfront payment for obtaining or renting fixed assets. An alternative example may be from the case of the bail-out of the Big Three car companies. One of their most important problems was the cost of pensions owed to their retired employees. This pension cost must be financed in order for them to operate. However, it is not directly related to their output-level. For example, a news article reports that the Big Three car makers "saddled themselves with a cost structure in flush times that has proved unsustainable as their market share has eroded. They have made great strides of late in shedding legacy pension and health-care costs, but they took decades to do so." ("The Next Bailout: Detroit," August 22, 2008, *Wall Street Journal*).

2.1.3 Financial market

We assume that the financial market is a monopoly. Each household is endowed with a numéraire κ/H. They deposit this at the monopolistic bank because it is the only agent that permits investment in firms and other investment opportunities. The bank chooses which firms will be financed so as to maximize the total return from these investments. The bank captures the entire profits (and the losses as well) of firms that are financed by the bank. The bank also has an outside opportunity that can yield the real risk-free rate of return ρ (> 0), which we call the prime rate. The bank profit from the investments is shared by households. We assume that $\kappa \geq \sum_{i=1}^{n} F_i$, that is, the total deposit or loanable funds of the bank exceeds the total demand for funds.

2.2 Equilibrium

We apply backward induction to the model. That is, we first derive the equilibrium outcome in the input and product markets and then derive the optimal investment decision by the bank to produce the given product-market outcome.

2.2.1 Final demand for each product

The final demand function for each product is determined by the utility maximization of the households after observing the list of operating firms $\{e_i\}_{i=0}^{n}$. By solving the maximization problem of the utility in (1) with respect to c_{hi} ($i = 1, 2, \ldots, n$) under the budget constraint (2) and the availability constraint ($(1 - e_j)c_{hj} = 0$), we obtain the demand function of household h:

$$c_{hi} = \frac{e_i R_h}{p_c} \cdot \left(\frac{p_c}{p_i}\right)^{\theta}, \text{ where } p_c \equiv \left(\sum_{j=1}^{n} e_i p_j^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{4}$$

Note that p_c is understood as the consumer price index (CPI).

2.2.2 Intermediate demand for each product

The demand function for each product as an intermediate good is derived by solving the cost minimization problem of firms given their production levels. The problem for firm i

is

$$\min_{\{x_j\}_{j=0}^n} \sum_{j=0}^n p_j x_{ij}, \quad \text{s.t.,} \quad x_i = \left(\sum_{j=0}^n w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \quad \text{and } (1-e_j)x_{ij} = 0,$$

where the second constraint is the availability constraint. The usual cost minimization gives the demand of firm i for good j:

$$x_{ij} = \left(\frac{p^i}{p_j} \right)^\theta e_j w_{ji} x_i, \text{ where } p^i \equiv \left(\sum_{j=0}^n e_j w_{ij} p_j^{1-\theta} \right)^{\frac{1}{1-\theta}}, \tag{5}$$

for $j = 0, 1, \cdots, n$. We note that p^i is the average cost of producing good i because

$$\sum_{j=0}^n p_j x_{ij} = p^i x_i. \tag{6}$$

Similarly to CPI, p^i is understood as the producer price index (PPI).

2.2.3 Profit maximization by each firm

Each firm sets its price so as to maximize its profit since each firm produces a differentiated product. We assume that each firm ignores the impact of its pricing strategy on the price indexes (CPI, PPI): thus, firm i maximizes profit under the assumption that $\partial p_c/(\partial p_i) = 0$ and $\partial p^j/(\partial p_i) = 0, \ \forall j \neq i$. This assumption is common in models of monopolistic competition; it is sometimes referred to as the aggregate demand externality (Blanchard and Kiyotaki, 1987).

The profit maximization problem for firm i is

$$\max_{p_i} \quad (p_i - p^i)x_i, \tag{7}$$

under the demand function for firm i,

$$x_i = \sum_{h=1}^H c_{hi} + \sum_{j=1}^n x_{ji} = \frac{\sum_{h=1}^H R_h}{p_c} \cdot \left(\frac{p_c}{p_i} \right)^\theta + \sum_{j=1}^n \left(\frac{p^j}{p_i} \right)^\theta e_j w_{ji} x_j. \tag{8}$$

The first term and the second term in the demand function are the final demand (4) and the intermediate demand (5), respectively.

Solving the profit maximization problem, we obtain the price level and the profit of firm i. The first-order condition provides the price level:

$$p_i = \frac{\theta}{\theta - 1} p^i, \tag{9}$$

for each i $(= 1, 2, \cdots, n)$. Without loss of generality, we assume that (9) holds for firm i with $e_i = 0$. The coefficient $\theta/(\theta-1)$ is the mark-up. The (nominal) profit of firm i is

$$\pi_i = \frac{p_i x_i}{\theta}. \tag{10}$$

2.2.4 Equilibrium outcomes of the product market

We now describe the equilibrium level of price, sales, and household income in the product market. These equilibrium outcomes are uniquely determined.

The equilibrium price level is obtained by solving the system of simultaneous equations characterized by (9) (raising both sides to the power $1-\theta$) and the definition of PPI. Note that the system is linear on $p_i^{1-\theta}$. Therefore, a simple matrix calculation gives

$$\mathbf{p}_\theta = \left\{ \mathbf{I_n} - \left(\frac{\theta-1}{\theta}\right)^{\theta-1} \mathbf{W} \right\}^{-1} \mathbf{w_0} p_0^{1-\theta}, \tag{11}$$

where $\mathbf{p}_\theta \equiv (p_1^{1-\theta}, p_2^{1-\theta}, \cdots, p_n^{1-\theta})'$, $\mathbf{I}_n$ is the $n \times n$ identity matrix, $\mathbf{w_0} \equiv (w_{10}, w_{20}, \cdots, w_{n0})'$, and $\mathbf{W}$ is the $n \times n$ matrix whose (i, j) element is equal to $e_j w_{ij}$.[7] The price level is thus uniquely determined.

Next, we derive the sales in the equilibrium. The consumption level is determined by (4). Let $c_i = \sum_{h=1}^{H} c_{hi}$ be the total consumption of good i. Multiplying both sides of (8) by $p_i e_i$ gives the vector of total sales:

$$\mathbf{s} = \mathbf{f} + \mathbf{Q}\mathbf{s}, \tag{12}$$

where $\mathbf{s}$ (total sales) $\equiv (e_1 p_1 x_1, e_2 p_2 x_2, \cdots, e_n p_n x_n)'$; $\mathbf{Q}$ is the $n \times n$ matrix whose (i, j) element is $q_{ij} \equiv e_i w_{ji} p_i^{1-\theta} {p^j}^{\theta} / p_j$; and $\mathbf{f}$ (sales to consumers) $\equiv (e_1 p_1 c_1, e_2 p_2 c_2, \cdots, e_n p_n c_n)'$. By the assumptions on w_{ij} and the definition of p^i, the matrix $\mathbf{I_n} - \mathbf{Q}$ is invertible.[8]

[7] Note that the matrix inverse in (11) is well-defined because $((\theta-1)/\theta)^{\theta-1} < 1$ and the largest eigenvalue of $\mathbf{W}$ is less than 1 by the assumption that $0 < \sum_{j=0}^{n} w_{ij} \leq 1$.

[8] Let $\|\cdot\|_1$ denote the norm by Bowker (1947), defined as $\|\mathbf{Q}\|_1 \equiv \max_j \sum_{i=1}^{n} \|Q_{ij}\|$. Let λ be an eigenvalue of $\mathbf{Q}$. It is known that $\|\lambda\| \leq \|\mathbf{Q}\|$. $\|\mathbf{Q}\|_1 < 1$ holds because

$$\begin{aligned} \sum_{i=1}^{n} q_{ij} &= ({p^j}^{1-\theta} - w_{j0} p_0^{1-\theta}) \frac{{p^j}^{\theta}}{p_j} \\ &= \frac{p^j}{p_j} \left(1 - \frac{w_{j0} p_0^{1-\theta}}{\sum_{i=0}^{n} w_{ji} p_i^{1-\theta}}\right). \end{aligned}$$

The first term, which is the inverse of the mark-up rate, is smaller than one by (9). The second term is also smaller than 1. It follows that $\|\lambda\| \leq \|\mathbf{Q}\|_1 < 1$.

Therefore, the sales vector is uniquely determined and can be written as

$$\mathbf{s} = (\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{f} = \sum_{k=0}^{\infty} \mathbf{Q}^k \mathbf{f}. \tag{13}$$

We note that the sales vector $\mathbf{s}$ is conceptually similar to the Bonacich centrality (Bonacich, 1987).

The aggregate income is equal to the profit of the monopolistic bank, which consists of aggregate firm profit and the aggregate return from the outside investment opportunity. Thus, the real aggregate income is

$$\frac{\sum_{h=1}^{H} R_h}{p_c} = \frac{\sum_{i=1}^{n} e_i p_i x_i}{p_c \theta} + (\kappa - \sum_{i=1}^{n} e_i F_i)(1+\rho).$$

By (13) and (4), we obtain the closed form solution of the equilibrium household income:[9]

$$\frac{\sum_{h=1}^{H} R_h}{p_c} = \frac{\theta(1+\rho)(\kappa - \sum_{i=1}^{n} e_i F_i)}{\theta - p_c^{\theta-1} \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{p}_\theta}. \tag{14}$$

We now complete the characterization of the equilibrium in the product market. The price level is determined by (11). Given the price vector, the household income is determined by (14). The price level and the household income yield the consumption by (4). The sales (and the quantities of goods) are then determined by (13). Since the price level is determined uniquely, the other equilibrium outcomes are also unique.

2.2.5 Financial market

Lastly, we discuss the equilibrium of the financial market. The profit maximization problem of the monopolistic bank is to determine which firms the bank will finance. Since the profit of firm i in real terms is $x_i p_i/(\theta p_c)$ and the opportunity cost of financing the fixed costs of firm i is $(1+\rho)F_i$, the bank's profit maximization problem is

$$\max_{\{e_i\}_{i=1}^{n}} \sum_{i=1}^{n} e_i \left\{ \frac{p_i x_i}{\theta p_c} - (1+\rho)F_i \right\}. \tag{15}$$

[9] The details of the derivation are the following. Note that $\sum_{i=1}^{n} e_i p_i x_i = \mathbf{1}'\mathbf{s}$. By (13), we have $\mathbf{1}'\mathbf{s} = \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{f}$. By (4), the ith element of $\mathbf{f}$ is $e_i \sum_{h=1}^{H} R_h (p_c/p_i)^{\theta-1}$. We thus have the following equation:

$$\frac{\sum_{h=1}^{H} R_h}{p_c} = \frac{p_c^{\theta-1}}{\theta} \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{p}_\theta \frac{\sum_{h=1}^{H} R_h}{p_c} + (\kappa - \sum_{i=1}^{n} e_i F_i)(1+\rho).$$

Solving this equation provides the closed form solution of the household income.

Let $\{e_i^*\}_{i=1}^n$ be the solution to this problem. It is important to note that $\{p_i\}_{i=1}^n$ and $\{x_i\}_{i=1}^n$ depend on $\{e_i\}_{i=1}^n$, and the bank chooses $\{e_i^*\}_{i=1}^n$ with the knowledge that it affects the price and quantity levels in the product market equilibrium. We note that since the set of values that $\{e_i\}_{i=1}^n$ can take is finite, the bank's problem has a solution, although the solution is not guaranteed to be unique.

3 Network-motivated Forbearance for an Influential Firm

Given the above equilibrium, we now discuss the main result of this paper: the possibility that a profit maximizing bank may rationally undertake forbearance. We also show that firms with a strong influence on the aggregate profit through the supply network are likely to be the target of forbearance. We first define the influence coefficients of firms.

3.1 Influence vector

The concept of influence coefficient is useful in analyzing rational forbearance. The influence vector $\mathbf{v}$ is defined as

$$\mathbf{v} \equiv \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1} = \mathbf{1}'\sum_{k=0}^{\infty}\mathbf{Q}^k, \tag{16}$$

where $\mathbf{1}$ is the $n \times 1$ vector of ones. The ith element of $\mathbf{v}$, denoted by v_i, is called the *influence coefficient* of firm i. The vector $\mathbf{v}$ is a modified version of *the influence vector* (i.e., the vector of the influence coefficients) proposed by Acemoglu et al. (2012).

The influence coefficient represents the influence of firm i on the aggregate profit. This observation comes from the fact that, by (10), the aggregate profit is θ^{-1} times

$$\mathbf{1}'\mathbf{s} = \mathbf{1}'(\mathbf{I_n} - \mathbf{Q})^{-1}\mathbf{f} = \mathbf{v}'\mathbf{f}. \tag{17}$$

In words, the influence coefficient v_i indicates the magnitude of the influence on aggregate sales of the change in the sales of firm i to households, that is,

$$v_i = \frac{\Delta\text{Aggregare Sales}}{\Delta\text{Sales of firm } i \text{ to households.}}.$$

Higher values of v_i indicate that a negative shock to the final sales of firm i is more damaging to the aggregate profit than a negative shock to a firm j with $v_j < v_i$. It is important to recognize that the influence coefficient takes into account not only the first-order impact (i.e., the impact to the adjacent neighbors) but also all the higher-order impacts. Note that the matrix $\mathbf{I_n} - \mathbf{Q}$ is conceptually the same as the Leontief matrix in input–output analysis and the logic of input–output analysis applies here. Specifically, the first-order impact of an increase in the sales of firm i to households on the total sales of firm j is determined by the factor of the (j, i) element of $\mathbf{Q}$; the second-order impact is determined by the factor of the (j, i) element of $\mathbf{Q}^2$; and so forth.

3.2 Rational forbearance

We now discuss the possibility that the profit maximizing monopolistic bank may undertake forbearance for a loss-making but influential firm. Our argument is based on externalities working through the supply network. By the same logic as in Leontief input–output analysis, the level of sales to households of a firm with a higher influence coefficient has a greater impact on the levels of sales of the other firms and, thereby, on the aggregate profit. The monopolistic bank can fully take this positive externality into account to maximize the aggregate profit. The concept of the influence vector $\mathbf{v}$ is useful as a way to characterize the conditions in which this positive externality is large.

We first define forbearance in the context of our model.

Definition 1 (Forbearance) *We say a bank undertakes* ***forbearance*** *if it extends a loan to firm z despite that firm's real economic profit being negative; namely,*

$$e_z = 1 \text{ and } \frac{x_z p_z}{\theta p_c} - (1 + \rho)F_z < 0. \tag{18}$$

We then discuss the condition under which forbearance occurs. Let $\{\tilde{e}_i\}_{i=1}^n$ be the solution to (15) with the additional constraint $e_z = 0$. We indicate the equilibrium outcomes of x_i, p_i, and p_c under $\{e_i^*\}_{i=1}^n$ by the superscript $*$ and those under $\{\tilde{e}_i\}_{i=1}^n$ by a tilde. The condition for firm z to be financed, that is, the condition for forbearance to be provided to firm z, is

$$\sum_{i=1}^{n} e_i^* \left(\frac{x_i^* p_i^*}{\theta p_c^*} - (1 + \rho)F_i \right) \geq \sum_{i=1}^{n} \tilde{e}_i \left(\frac{\tilde{x}_i \tilde{p}_i}{\theta \tilde{p}_c} - (1 + \rho)F_i \right).$$

Using the influence vector, the condition can be written as

$$\sum_{i=1}^{n} e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1+\rho)F_i \right) \;\geq\; \sum_{i=1}^{n} \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1+\rho)F_i \right), \tag{19}$$

by (17). Rearranging this inequality so as to separate the terms of firm z and those of other firms, we obtain

$$\underbrace{\frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*}}_{\text{influence-coefficient effect}} \tag{20}$$

$$+ \underbrace{\sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1+\rho)F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1+\rho)F_i \right) \right]}_{\text{business-stealing effect/influence-enhancing effect by firm } z} \tag{21}$$

$$> \quad \underbrace{-\left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1+\rho)F_z \right)}_{\text{direct cost to support firm } z} \quad > 0. \tag{22}$$

The left-hand side of the inequality captures the externalities to the system-wide profit from keeping firm z open. The right-hand side is the direct cost to keep firm z open.

The term in (20) is the positive externality of firm z to the other firms. This term is always positive so long as at least two firms including firm z and another firm that supplies firm z operate at the optimum for the bank. The higher the influence coefficient v_z^* of firm z at the optimum is, the larger the propagation effect to the sales and profits of the other firms, and so the more beneficial it is for the bank to support firm z.

The term in (21) is the effect of the existence of firm z on the influence of the other firms. A part of this effect is referred to as the business-stealing effect (Mankiw and Whinston, 1986) or the congestion effect (Caballero et al., 2008). If the business-stealing effect of firm z is dominant, this part is negative. In contrast, this term could be positive if the existence of firm z increases the influence coefficient of the other firms by improving their connectivity. We discuss this effect in more detail in the next subsection.

The first term in (20) suggests that firm z is more likely to be the target of forbearance if it is more "influential" than the others; that is, when v_z^* is larger than v_i^* ($i \neq z$). This also implies that the larger the level of sales to households by firm z (i.e., $p_z^* c_z^*$), the more likely the firm is to be the target of forbearance. However, we acknowledge that the complicated interdependence of the various externalities in the two terms (20) and (21) through the equilibrium price (11) makes it difficult to show clear-cut propositions about this point.

If the bank undertakes network-motivated forbearance, then the bank clearly requires an interest rate less than the prime rate ρ, and perhaps even a negative rate. The interest rate for firm z is $x_z^* p_z^* / (\theta p_c^* F_z) - 1$. Under the condition (18), this interest rate is less than ρ. This point supports the empirical strategy of using the information about whether a firm borrows at a rate below the prime rate as an indicator for the existence of forbearance lending, following Caballero et al. (2008). Under the conditions of (20), (21), and (22), the cost of extending a loan to a loss-making firm at a rate less than the prime rate is covered by the higher interest payments from loans to other firms profiting by selling their products to the loss-making firm.

Moreover, in our setting, forbearance is welfare-improving. The social welfare in this case is the aggregate indirect utility of households minus the total cost of producing final products after netting out the intermediate inputs within the network. We can show that the aggregate indirect utility of households equals $\sum_{h=1}^{H} R_h/p_c$ from the households' optimization. Thus, the social welfare is equal to

$$\frac{\sum_{h=1}^{H} R_h}{p_c} - (1+\rho)\sum_{i=1}^{n} e_i F_i - \frac{p_0 \sum_{i=1}^{n} e_i x_{i0}}{p_c}. \tag{23}$$

It is easy to show that this is equivalent to the total real profit of firms in the network so long as good 0, the intermediate input from the outside of the network, is used only for production and not for consumption. The total real profit of firms within the network is[10]

$$\frac{\sum_{i=1}^{n} e_i(p_i x_i - p^i x_i)}{p_c} - (1+\rho)\sum_{i=1}^{n} e_i F_i = \frac{\sum_{h=1}^{H} R_h}{p_c} - \frac{p_0 \sum_{i=1}^{n} e_i x_{i0}}{p_c} - (1+\rho)\sum_{i=1}^{n} e_i F_i.$$

Clearly, this is equivalent to the social welfare (23). Thus, the profit maximizing bank behavior also maximizes the social welfare in our setup.

[10]This equality holds because

$$\begin{aligned}\sum_{i=1}^{n} e_i(p_i x_i - p^i x_i) = \sum_{i=1}^{n} e_i p_i x_i - \sum_{i=1}^{n}\sum_{j=0}^{n} e_i p_j x_{ij} &= \sum_{i=1}^{n} e_i p_i c_i + \sum_{i=1}^{n} p_i \sum_{j=1}^{n} e_i x_{ji} - \sum_{j=0}^{n} p_j \sum_{i=1}^{n} e_i x_{ij} \\ &= \sum_{h=1}^{H} R_h - p_0 \sum_{i=1}^{n} e_i x_{i0}.\end{aligned}$$

Proposition 1 (Network-motivated Forbearance) *The monopolistic bank can maximize its profit by undertaking forbearance for firm z when the inequality consisting of (20), (21), and (22) holds. The interest rate of a loan to firm z is below the prime rate, and that of at least a loan to the other firm is over the prime rate. Forbearance is welfare improving.*

3.3 Closure of a profitable firm

The bank may also take a strategy opposite to forbearance and decline to supply funds to a firm that is individually profitable but less influential. This argument is related to the business-stealing effect (Mankiw and Whinston, 1986) or the congestion effect (Caballero et al., 2008). When investors make investment decisions on the individual profitability basis, they may allow more firms to operate than the socially optimal level. This problem occurs when the economic value added is smaller than the individual profit of each firm since each firm does not care the extent to which their business steals sales from the competitors in the entry decision. A monopolistic bank, however, can take this into account in making its investment decision.

The explicit condition for this case to emerge is the opposite side of the condition (20), (21), and (22). That is, it occurs when

$$\begin{aligned} &\frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*} + \sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1+\rho) F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1+\rho) F_i \right) \right] \\ &< - \left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1+\rho) F_z \right) \leq 0. \end{aligned} \tag{24}$$

This condition holds when the business stealing effect dominates the positive externality. In other words, if the profit of firm z mainly comes from stealing the sales of the other firms, then the bank will refuse to finance the fixed costs of firm z to maximize its profit even though firm z is individually profitable. As before, because social welfare and bank profit coincide, it is welfare-improving to close a profitable firm when (24) holds.

Proposition 2 (Closure of a profitable firm) *The monopolistic bank can maximize its profit by refusing financing and closing firm z if the inequality (24) holds. Moreover, closing firm z is improves welfare when (24) holds.*

3.4 Decentralized financial market

In this section, we examine the case in which the financial market is decentralized. So far, we have considered the extreme case of a monopolistic bank to keep the exposition simpler. However, the financial market in the reality is more decentralized. We show that forbearance lending never happens in a perfectly competitive financial market. In contrast with this, we argue that in an oligopolistic financial market, forbearance can happen under either of the following two conditions. One condition is tacit collusion or coalition, and the other possibility is so-called relationship banking, which yields monopolistic power to the relational lender. Both generate the possibility for lenders to recoup the costs of forbearance lending.

3.4.1 Competitive financial market

It is clear that rational forbearance lending is impossible when the financial market is perfectly competitive. In order to undertake forbearance, the cost of the action must be financed by extracting profit from other firms. However, extracting the profit is impossible in a competitive financial market.

To make the argument formal, we consider the following scenario: multiple banks that pool numéraires deposited by households strategically quote the interest rate; they directly lend to each firm or take the outside opportunity; and no single bank can finance the entire part of the fixed cost of a firm. In this case, no bank is willing to lend the numéraire at a rate under ρ without any coordination since it is better for them to invest in the outside opportunity that yields the return ρ. Thus, there is no room for forbearance lending to emerge. This situation is conceptually the same situation as the credit freeze presented by Bebchuk and Goldstein (2011).

Note that even in this case, forbearance for a loss-making yet influential firm may improve welfare. This result has a policy implication. When the financial market is perfectly competitive, it is necessary for the government to intervene in the financial market to facilitate welfare improving forbearance. For example, in the 1990s in Japan, the government asked banks to engage in forbearance to help large yet unprofitable companies.[11]

[11] See footnote 2.

In the US in 2008, a public bailout of the Big Three automakers was conducted. Such policies may be justified from the viewpoint of our analysis.

3.4.2 Tacit coalition

When the financial market is oligopolistic, banks may be able to collude and engage in forbearance that is profitable and improves welfare.

We consider an arrangement where each of competing banks holds a share of the optimal portfolio that the monopolistic bank would have. If one of them declines to extend a loan to an influential but unprofitable firm z, this can lead to the closure of firm z and a resulting cascade of firm closures. If each bank profit is larger in the former case than in the latter, then each bank has an incentive to keep the coalition intact.

More precisely, suppose that bank b purchases the share s_{bi} of firm i by giving the numéraire of $s_{bi}F_i$, where $\sum_b s_{bi} = 1$, $0 < s_{bi} \leq 1$, $\forall\ b, i$ (i.e., no short-selling), and $\sum_i s_{bi}F_i \leq \kappa_b$ where κ_b is the amount of numéraires deposited at bank b (liquidity constraint of investors). The product market structure is the same as the case for monopolistic financial market. After a production cycle, the bank obtains $s_{bi}\pi_i$, where π_i is defined as in (10). We denote bank b's share of the optimal portfolio of the monopolistic bank by s_b^*.

We examine the Nash equilibrium of the game played by the banks. We consider the case where bank k deviates from this optimal portfolio s_k^* while the other banks hold the optimal portfolio s_b^* ($b \neq k$), and examine the conditions under which this is unprofitable for bank k. We denote this deviated portfolio by $\{s_{bi,-k}^*\}_{b,i}$, where $s_{bi,-k}^* = s_b^*$ if $b \neq k$ and

$$s_{bi,-k}^* \equiv \mathrm{argmax}_{s_{ki}} \sum_i e_i(s_{ki}) \left(\frac{p_i(s_{ki})x_i(s_{ki})}{\theta p_c(s_{ki})} - (1+\rho)F_i \right) s_{ki},$$

subject to $s_{bi} = s_b^*$ for $b \neq k$. If the deviation by bank k results in the closure of firm z, then the necessary and sufficient condition for bank k not to deviate from the optimal portfolio s_k^* is

$$\left[\sum_i e_i^* \left(\frac{x_i^* p_i^*}{\theta p_c^*} - (1+\rho)F_i \right) \right] s_k^* \geq \sum_i e_i^{-k} \left(\frac{x_{i,-k}^* p_{i,-k}^*}{\theta p_{c,-k}^*} - (1+\rho)F_i \right) s_{ki,-k}^*,$$

where the additional subscript $-k$ indicates "the value when k deviates from s_k^*." We can rewrite this condition by using the influence coefficient,

$$s_k^*\left(\frac{v_z^* p_z^* c_z^*}{\theta p_c^*} - \frac{p_z^* x_z^*}{\theta p_c^*}\right) + \sum_{i \neq z}\left\{e_i^* s_k^* \cdot \frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - \frac{v_{i,-k}^* p_{i,-k}^* c_{i,-k}^*}{\theta p_{c,-k}^*}\right\} - (1+\rho)\sum_{i \neq z}(e_i^* s_k^* - e_{i,-k}^* s_{ki,-k}^*)F_i$$
$$\geq -s_k^*\left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1+\rho)F_z\right) \geq 0,$$

where v_i^{-k} is the i th element of $\mathbf{e^{-k}}(\mathbf{I}-\mathbf{Q}(s^{-k}))^{-1}$ with $\mathbf{e^{-k}} \equiv (e_1^{-k}s_{k1}^{-k}, e_2^{-k}s_{k2}^{-k}, \cdots, e_n^{-k}s_{kn}^{-k})$, and $\mathbf{Q}(s^{-k})$ is the $\mathbf{Q}$ for firms operating under the portfolio $\{s_{bi}^{-k}, s_{ki}^{-k}\}$.

Thus, we can conclude again that firm z is more likely to be collusively supported when v_z^* is sufficiently higher than v_i^* $(i \neq z)$. This result is qualitatively the same as what we obtain in the case of a monopolistic financial market.

3.4.3 Relational lending

The other possible situation in which rational forbearance happens in an oligopolistic financial market is related to so-called relational lending. Suppose that a bank has a monopolistic power over some firms. In this case, the bank may have an incentive to undertake forbearance for a firm that influences the profits of those firms that are under relational lending with the bank.

A bank has a monopolistic power over some firms, for example, when that bank has a long-term relationship with those firms and has some informational advantages. It is widely recognized that a bank can earn a quasi-rent by maintaining lending relationships, by achieving the information advantage over rival banks, or by providing firm-specific value-adding services by making use of this information advantage (Sharpe, 1990; Rajan, 1992; Boot and Thakor, 2000). Many empirical studies provide evidence supportive of this possibility in the financing of small businesses (Degryse and van Cayseele, 2000; Ioannidou and Ongena, 2010), which are presumably peripheral in the supply network described here.

There is a possibility that the bank engages in network-motivated forbearance for a firm that has a strong influence on the profits of firms under relationship banking with the bank. Suppose that forbearance increases the profits of those small firms that are under relational banking with the bank. Since the bank has monopoly power over those firms, it can extract the benefits of forbearance from those small firms. If the excess return of the bank from relational lending to small businesses is large enough and the bank covers a large enough part of the financing of the supply network that is connected to an influential firm, then the bank has an ability to recoup the loss from forbearance toward an influential firm. In this way, forbearance is possible even when banks do not collude.

4 Hypothesis setting for the empirical study

Our model analysis provides us with several hypotheses that can be statistically tested on the relationship between the supply network and bank's lending decision. In this section, we present the hypotheses that are tested empirically in our study.

The analysis leading to Proposition 1 indicates that a firm with a high influence coefficient is likely to obtain forbearance. In our model, forbearance is interpreted as when a firm receives an interest rate lower than the prime rate.[12] We empirically test the following hypothesis.

Hypothesis 1 *The influence coefficient has a negative impact on the loan interest rate.*

This is our main hypothesis.

We also test whether a less credit-worthy firm can obtain a lower interest rate if it is influential. This hypothesis is related to the fact that the real financial market is unlikely to be monopolistic. Since it would be difficult for banks to extract the entire profits of firms in a non-monopolistic financial market, the interest rate for a profitable firm is unlikely to be proportional to the level of the profit. We expect that an influential firm

[12] We focus on interest rate in our analysis because other measures of bank's support may not be suitable for our analysis. For example, the effect of the influence coefficient on loan amount is ambiguous. Banks' support through loan amounts could take two forms. One is to keep lending at a lower rate, and the other is to discharge a firm from its obligation on overdue debts. Indeed, about 5.9 trillion JPY are reserved for possible loan losses out of 389 trillion JPY (1.5%) loans in total, and, additionally, about 398.5 billion JPY of loans (0.1%) were written off in 2006 in Japan (the author calculated these figures from the non-consolidated financial statements of major banks—including Mitsubishi UFJ, Sumitomo Mitsui, Mizuho, Risona, and Saitama Risona—and regional banks—including those who are members of the second association of regional banks. These lists are available from the website of the Japanese Bankers' Association). If an influential firm is more likely to be supported by a bank, then the change in loans outstanding should have a positive correlation with the influence coefficient in the former case, while it is negative in the latter case. Thus, the test using changes in loan amounts is expected to produce ambiguous results. In contrast, it is expected that the actual interest payment for loans have a negative correlation with the influence coefficient in either of the above cases.

obtains a lower interest rate when its profit is low (or negative). However, the interest rate for a profitable firm may not correspond to its profit. We thus have the following hypothesis.

Hypothesis 2 *The above effect is larger for less credit-worthy firms.*

To test Hypotheses 1 and 2, we estimate the following interest rate equation:

$$rate_i = b_0 + b_1 \cdot ln(v_i) + b_2 \cdot score_i + b_3 \cdot ln(v_i) \times score_i + \mathbf{b_4}'\mathbf{X}_i + \epsilon_i, \quad (25)$$

where $rate_i$ is the interest rate for a loan to firm i, $ln(v_i)$ is the natural logarithm of the influence coefficient of firm i, $score_i$ is the credit score used to measure the credit-worthiness of firm i (with higher scores meaning more credit-worthy), $\mathbf{X}_i$ is the column vector of control variables for firm i, ϵ_i is the error term, and bs are coefficients to be estimated. Hypothesis 1 predicts that b_1 is negative, and Hypothesis 2 predicts that b_3 is positive.

We also examine whether regional banks are more likely to engage in network-motivated forbearance.

Hypothesis 3 *The effect of the influence coefficient is stronger for firms with regional banks as their main bank.*

This hypothesis is established by the observation that regional banks in Japan are often a single dominant lender in a regional lending market and that they widely engage in relationship banking. Forbearance is more likely when a bank is dominant in a lending market, such that it lends to both hub companies and peripheral companies in the market. The branch networks of major banks are concentrated in metropolitan areas, overlapping with each other, and their sizes are similar.[13] Therefore, none of them is likely to be a single dominant lender in a metropolitan market. In contrast, regional banks are often a single dominant lender in a regional lending market.[14] For example, a single regional bank holds a market share of more than 40% in the lending market of 18 prefectures and

[13]Outstanding loans and bills discounted as of March 2006 at the largest three major banks—Mitsubishi UFJ Bank, Mizuho Bank (including Mizuho Corporate Bank), and Sumitomo Mitsui Bank—are 70 trillion JPY, 62 trillion JPY, and 52 trillion JPY, respectively.

[14]In the case of the U.S., Slovin et al. (1999) argue that regional banking markets are not contestable.

a market share of more than 30% in 34 prefectures out of 47 prefectures in Japan, as of March 2006.[15] Moreover, it is known that regional banks in Japan are more active in relationship banking than the major banks are (see, e.g., Uchida et al., 2008, 2012; Nemoto et al., 2011). Indeed, the ratio of companies listed on the stock exchange, which supposedly depend on arm's length financing and are less prone to be exploited by a main bank, is much higher for major banks.[16] The analysis in Section 3.4.3 indicates that banks engaging in relationship banking are more likely to undertake network-motivated forbearance. Thus, the interest reduction effect is expected to be more prominent among regional banks than among major banks. For Hypothesis 3, we split the sample according to the type of main bank.

Note that our model analysis suggests other hypotheses that are not tested in this paper. For example, Proposition 2 suggests that a firm with a smaller influence coefficient and with a larger business-stealing effect may be closed even when it is profitable. Another possible hypothesis is that forbearance is more likely in sectors where it is hard for firms to switch suppliers or clients because of relation-specific factors. This hypothesis is motivated by the observation that our theoretical results depend on the assumption that the supply network is rigid. We do not consider these hypotheses in our empirical analysis; instead, we leave them for future research.

5 Data

This section explains the datasets used in our empirical study. We use several different datasets on Japanese firms. The TSR Corporate Relationship Database provides information about inter-firm transactions, and it is used here to estimate the influence coefficients of firms. Detailed data on the characteristics of firms are obtained through the TSR financial Information Database and TSR Company Information Database.

[15] The market share information is from page 11 in *Gekkan Kin'yu Janaru, Zokan Go, Kin'yu Mappu 2007 Nen Ban* (Monthly Finance Journal, Extra Edition, Financial Map 2007 Edition), December 2006, Kin'yu Janaru Sha.

[16] The average ratio of listed companies in the largest three major banks (Mitsubishi UFJ, Sumitomo Mitsui, and Mizuho) is 22%, whereas that of regional banks is merely 3% in our dataset.

5.1 TSR Relationship Database

We use the Corporate Relationship Database (*TSR Kigyo Sokan File*) provided by the Tokyo Shoko Research (TSR) as of 2005 (hereafter, we call this dataset the *TSR Relationship Data*) to estimate the influence coefficient of each firm. This database contains the names and IDs of important corporate clients and suppliers up to the largest 24 for each company, including those in the process of a bankruptcy.[17] The dataset also contains the basic items in the financial statement from the prior three years, such as sales and profits, and other characteristics including the credit score provided by TSR, the number of employees, the names and IDs of the largest 10 lending banks, the head office address, and industry classification.

We use the sample comprising firms listed in the TSR Relationship Data for which the number of employees and positive sales in (Year, Year -1) are recorded. After dropping the observations whose latest accounting year is before August 2004, we obtain 651,913 observations.[18] We compute the value of the influence coefficients for 306,354 firms about which information on main banks is available and the main banks are not government-owned.

5.2 TSR Financial Information Database

We match the estimated influence coefficients with the financial data and the data on firms' characteristics to construct the dataset for our empirical study that tests the hypotheses about network-motivated lending. The financial data are collected from the TSR Financial Information Database (*TSR Zaimu Joho File*). The credit score provided by TSR and other corporate information are collected from the TSR Company Information Database as of 2006 (*TSR Kigyo Joho File*). We obtained detailed information about the balance sheet and the income statement in the accounting period of 12 months ending in any month in 2006 from 16,369 companies, which are chosen by random sampling stratified independently on employment-size class, capital-size class, and industry category. We

[17]We keep bankrupt companies if their sales are reported since, in that case, they are still operating with the aim of revival.

[18]We also conduct the whole analysis after dropping observations whose latest accounting year is before June 2004. We confirm that the results are hardly changed. Similarly, dropping observations whose latest accounting year is before September 2004 does not alter the results.

drop those without interest rate measures, which we define below (3,654 firms dropped), outliers in the top 1% with respect to the interest rate measure (128 firms), and those reporting negative or zero interest rates (413 firms). We also dropped observations whose influence coefficient in the common-main-bank network is not available due to the lack of TSR Relationship Data or because their main bank is a government-owned bank (3,130 firms) as well as those with infeasible ratios of tangible asset to total assets (54 firms). After dropping those lacking in the information required for our regression analysis (1,662 firms), we obtain 7,328 sample firms for our regression analysis.

6 Estimating the influence coefficient

In this section, we discuss how the influence coefficients are estimated from the TSR Relationship data. The dataset contains information about which firms are connected but does not provide the price levels of the products nor the magnitude of the trade between firms. The influence vector appearing in our theoretical model cannot, therefore, be computed directly. We first estimate a spatial autoregressive model of the supply network. We then compute the influence coefficient of each firm by using the estimates from the spatial autoregressive model.

6.1 Spatial autoregressive model

We first estimate the spatial autoregressive model of the supply network indicated by (12). To eliminate the firm fixed effects, the differenced version of (12) is considered:

$$\Delta \mathbf{s} = \mathbf{Q} \Delta \mathbf{s} + \Delta \mathbf{f}, \tag{26}$$

where Δ indicates the difference between the value in the latest accounting year and that in the previous accounting year. Note that while $\Delta \mathbf{s}$ (the difference in sales) can be observed in the data, $\mathbf{Q}$ and $\Delta \mathbf{f}$ cannot.

We consider the following approximation of $\mathbf{Q}$ and $\Delta \mathbf{f}$. We approximate $\mathbf{Q}$ by $\beta \mathbf{G}$, where β is a parameter to be estimated and $\mathbf{G}$ is the adjacency matrix for the supply network. We assume that firm i purchases a product from firm j if firm i identifies firm j

Table 1: Estimation results of the spatial autoregressive model

	Est. coef.	s.e.	
β	0.00197	0.0000494	***
Industry factor	yes		
Prefecture factor	yes		
Adj. R-squared	0.1451		
# of observations	652,280		

(note) Industry factor is controlled by the 95 sector dummies, which indicate one of the 2-digit "chu-bunrui" classifications of the Japanese standard industry classification, revised in 2002. Prefecture factor is controlled by 46 prefecture dummies (Hokkaido is the base class).

as an important supplier or firm j identifies firm i as an important client.[19] The (i, j)th element of the adjacency matrix $\mathbf{G}$ is $g_{ij} = 1$ when firm i sells its product to firm j and $g_{ij} = 0$ otherwise. Note that when $g_{ij} = 0$, it is also the case that q_{ij} (the (i, j)th element of $\mathbf{Q}$) is 0 since $q_{ij} = p_i^{1-\theta}(w_{ji}p^j)^\theta / p_j$. We examine alternative approximations of $\mathbf{Q}$ in Section 8. We assume that $\Delta\mathbf{f}$ is written as $\Delta\mathbf{f} = \gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \mathbf{u}$, where γ_I and γ_P are unknown coefficients, $\mathbf{Ind}$ is the matrix of industry dummies, $\mathbf{Pre}$ is the matrix of prefecture dummies, and $\mathbf{u}$ is the vector of an unobserved error term.[20]

Under these assumptions, we have the following spatial autoregressive model that is estimated by ordinary least squares (OLS):

$$\Delta\mathbf{s} = \beta\mathbf{G}\Delta\mathbf{s} + \gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \epsilon, \tag{27}$$

where ϵ is the sum of $\mathbf{u}$ and the approximation error of $\beta\mathbf{G}$. We estimate this model by using the sample of 652,280 observations from TSR Relationship Data. The estimation result is summarized in Table 1. The estimated β is positive and statistically significant, although the value is small. We comment on our use of OLS to estimate the spatial autoregressive models. It is known in the literature on spatial models that the OLS estimator is not guaranteed to be consistent (Kelejian and Prucha, 1998). The reason for this is that the regressor $\mathbf{Gs}$ and the error term ϵ are likely to be correlated, by construction. However, in our case, the correlation is likely to be small. Appendix A.1 provides a

[19] Note that while the dataset contains the information about the 24 most important corporate clients and suppliers, the observed degree of a firm, that is, the number of suppliers for a firm, can be larger than 24 by adopting our assumptions.

[20] The industries are categorized according to the two-digit Japan Standard Industry Classification (96 sectors). The number of prefectures in Japan is 47.

more detailed discussion. Note that Lee (2002) also examines the conditions in which the OLS estimator is consistent. However, that argument cannot be applied here and our discussion is different.

As a robustness check, we also estimate the parameters by use of the instrumental variables estimator, as suggested by Kelejian and Prucha (1998). For example, when β_1 is estimated, $\mathbf{Gs}$ is instrumented by $\mathbf{G} \times \mathbf{Ind}$ and $\mathbf{G} \times \mathbf{Pre}$. The resulting estimates are very imprecise, possibly because of the weakness of the instruments. Moreover, the resulting influence vector is very highly correlated with the one obtained by using the OLS estimate. Therefore, we decided to rely on OLS to estimate β values.

6.2 Influence coefficients

We now compute the influence coefficient of each firm. We measure the influence of a firm on the other firms that share the main bank. We construct the supply network for each bank and compute the influence coefficient of each firm in that supply network.

We primarily consider the influence of a firm on the profits of other firms in the same supply network, where each supply network consists of firms that share the same main bank. Let $\mathbf{G}^{(b)}$ be the adjacency matrix of the supply network that consists of firms whose main bank is bank b. Let $\mathbf{v}^{(b)}$ be the vector of influence coefficients in the supply network for bank b. The influence vector for the supply network determined by b is estimated by

$$\mathbf{v}^{(b)} = \mathbf{1}' \sum_{k=0}^{100} (\hat{\beta}\mathbf{G}^{(b)})^k, \tag{28}$$

where $\mathbf{1}$ is the vector of ones whose dimension is the same as $\mathbf{v}^{(b)}$ and $\hat{\beta}$ is the estimated coefficient in (27). Note that the influence vector is defined in (16), and it is $\mathbf{v} = \mathbf{1}' \sum_{k=0}^{\infty} \mathbf{Q}^k$. Our influence vector estimate is obtained by approximating the infinite series $\sum_{k=0}^{\infty} \mathbf{Q}^k$ by $\sum_{k=0}^{100} \mathbf{Q}^k$ and then replacing $\mathbf{Q}$ with $\hat{\beta}\mathbf{G}^{(b)}$.[21]

We argue that this definition of supply network and influence is appropriate for our analysis on forbearance. A bank does not have any incentive to undertake forbearance if the benefits of the forbearance are received by firms with which the bank does not have any relationship. An alternative argument may be that each bank may observe the supply

[21] We have verified the accuracy of the approximation of the infinite series by trying various truncation points.

Table 2: Descriptive statistics of the influence coefficient

# of obs.	mean	sd	min	p10	med	p75	p90	p95	p99	max
306,354	1.003	0.011	1.000	1.000	1.002	1.004	1.006	1.010	1.025	2.837

network only among firms for which the bank is the main bank. There are as many supply networks as the number of banks that act as a main bank since we construct a network for each main bank. We assume that no firm has multiple main banks, so that supply networks are mutually exclusive, to simplify the analysis.[22]

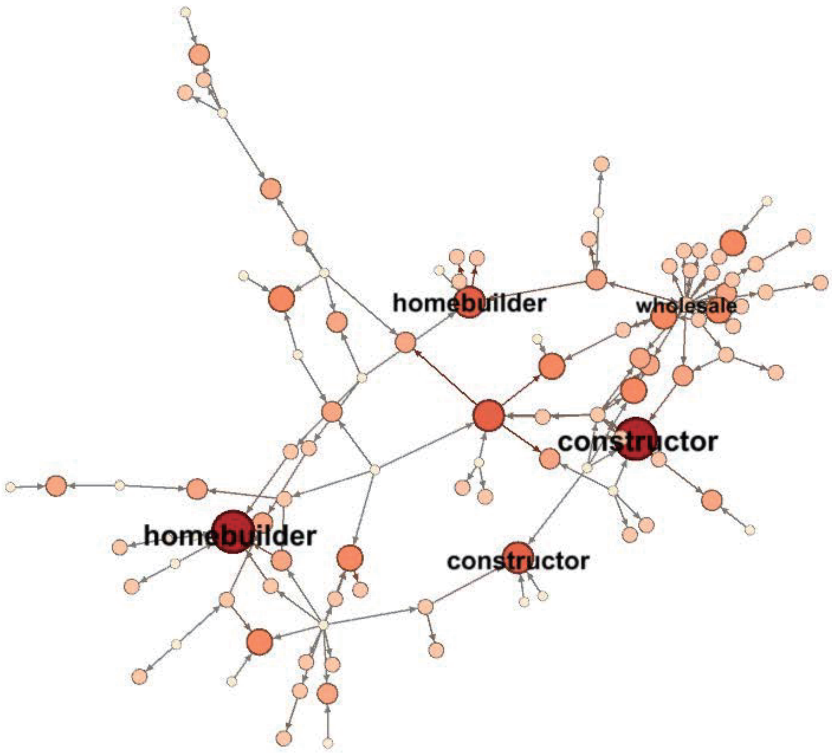
It is important to note that to estimate β we use the model of an economy-wide supply network while influence coefficients are computed for each bank-specific supply network, which is a sub-network of the economy-wide network. In other words, β is estimated with the adjacency matrix $\mathbf{G}$ defined in the economy-wide supply network, but $\mathbf{v}^{(b)}$ is computed with $\mathbf{G}^{(b)}$. There are two main reasons that we estimate the parameters by using the economy-wide supply network instead of the bank-level supply networks. First, for some banks, only a small number of firms choose it as their main bank, and therefore it is difficult to allow the parameters to be bank-specific. The other and more important reason is that since the product market covers the entire economy, not restricted to within firms who share the main bank, and $\mathbf{Q}$ is an outcome of the equilibrium in the product market, it would be more appropriate to use the information about the economy-wide supply network to estimate the parameter that determines $\mathbf{Q}$.

6.3 Descriptive statistics of the influence coefficient

Table 2 shows the descriptive statistics of the estimated influence coefficients of the supply networks of firms with a common main bank v, which is estimated by (28). The influence coefficients range from 1 to about 3. This means that a one unit increase in sales to households by a firm generates at most three units of increase in total sales, including intermediate inputs, within the common-main-bank network. The distribution of the influence coefficient is highly skewed to the left. More than half of firms have a factor equal to 1. Even at the 99th percentile, the factor is very close to 1. Most of the variation

[22] In the data, we observe which bank has had the most transactions for each firm and we define that bank as the main bank of that firm.

Figure 2: Example of a bank-specific supply network



is concentrate in the top one percent. This is plausible given the fact that large companies, which are more likely to be a major procurer from multiple suppliers, account for less than 1% of the total number of companies in Japan.[23]

Figure 2 is an example of the supply networks of firms with a common main bank. The size of each node indicates the level of the influence coefficient v. The largest ones are a construction company and a homebuilder ($v = 1.0119$). They purchase a wide variety of materials and services for constructions. In contrast, a construction-material wholesale company (upper right in the figure) has a relatively small influence coefficient

[23] *Establishment and Enterprise Census* (Ministry of Internal Affairs and Communications) in 2004 reports the number of small and medium-sized enterprises (SMEs) is 1,508,194, excluding sole proprietorships, in the non-agricultural sectors. The number of large enterprises is 11,793 in the non-agricultural sectors. The latter accounts for only 0.8% of the total number of enterprises. An enterprise is classified as an SME if its number of full-time employees is 300 persons or less, or its capital is 300 million JPY (about 3 million USD) or less. These thresholds are 100 persons and 100 million JPY for the wholesale sector, 50 persons and 50 million JPY for the retail and restaurant sectors, and 100 persons and 50 million JPY for the service sector.

($v = 1.002$) despite its high out-degree. This is because the wholesale company is not an important corporate client in the network but is, rather, an important supplier for the supply network. Our theory predicts that the interest cost for the construction company and a homebuilder is lower than that for less influential firms.

7 Interest rate regression

In this section, we present the results of our empirical study on the relationship between influence coefficients and interest rates of loans. This section provides our main empirical findings. We first explain the definitions and the characteristics of the variables used in the empirical study. We then examine the relationship between influence coefficients and interest rates.

7.1 Variables

We first explain the characteristics of the variables used in the regression analysis. The precise definitions of variables for our regression analysis are listed in Table 3.

7.1.1 Key variables

The key variable in our regression analysis are interest rate (which is the dependent variable), influence coefficient, and credit score.

We define the dependent variable $rate_i$ by using the items in the financial statement of each firm, following the existing literature (e.g., Caballero et al., 2008).

$$rate_i \equiv \frac{\text{firm } i\text{'s interest expense in the accounting period ending}}{\text{firm } i\text{'s total loans outstanding at the end of the previous accounting year}}.$$

The key independent variable is the influence coefficient $\hat{v}$. It is computed by the procedure documented in Section 6. We find that the using the natural logarithm yields a better fit for the model. Thus, we use $\ln(v)$ in our regression analysis.

The most important control variable is *score*. We use a normalized version of the credit score provided by TSR. TSR credit score (*Hyo Ten*) ranges from 0 (high default risk) to 100 (no default risk): a higher score indicates a higher credit-worthiness. The score is calculated on the basis of both quantitative and qualitative information including

Table 3: Definition of variables

Variable	Definition
rate	Interest costs(t) / total assets(t-1) $\times$ 100%.
ln(v)	Natural logarithm of the influence factor v.
score	Normalized credit score from Tokyo Shoko Research, original score / 50 - 1.
DISTRESS	A dummy variable equal to 1 if score <0, or zero otherwise.
LN(INT_COV)	Natural logarithm of (x+1), where x := EBITDA / (1+interest payment) if EBITDA $\geq$ 0, or 0 otherwise, and EBITDA = operating profit + depreciation.
LEVERAGE	Total assets / capital.
TANGIBLE	Tangible fixed assets / total assets.
CURRENT	Short-term assets / short-term liability.
PROFITABLE	EBITDA / sales
EBITDA_G	{EBITDA(t)$-$EBITDA(t-1)}/ total assets(t-1)
SALES_G	{sales(t)/sales(t-1)$-$1} $\times$ 100%
LN(SALES)	Natural logarithm of sales.
LN(FIRM AGE)	Natural logarithm of firm age in years.
LISTED	1 if the firm is listed on a stock market, 0 otherwise.
BOND_RATIO	Bonds outstanding / (bonds outstanding + loans outstanding). Set equal to 0 if the firm does not issue bonds and loans.
#LENDING_BKS	Number of lending banks. The maximum is 10 banks.
MAJOR_BK	1 if the main bank (the first one in the list of lending banks) is a major bank, 0 otherwise.
REGIONAL_BK	1 if the main bank (the first one in the list of lending banks) is a regional bank, 0 otherwise.
HI	Herfindahl index of the number of branches of banks (excluding government-owned ones) and Shinkin banks (larger cooperative banks) in the telephone area-code area where the head office of the firm is located. The index is calculated after excluding Shinkin banks if the capital of a firm is 900 million JPY or more.

manager's competence (20 points), growth potential (25 points), stability (45 points), and disclosure/reputation (10 points). TSR instructs that a score below 50 indicates an "alerting" situation.[24] We normalize it so that it equals 0 when the original score is at 50. That is, we define *score* by (original score)/50 - 1.

We also use an indicator variable, DISTRESS, which equals one when *score* is negative. The scatter plot of *score* and *rate* indicates that they are negatively correlated when *score* is positive, whereas they are positively correlated when *score* is negative. This is probably

[24]TSR instructs that a score between 30 and 49 indicates "Ichiou Keikai" (alerting provisionally), and that between 0-29 indicates "Keikai" (alerting).

because of the default of interest payments or the resulting rescheduling for repayments.[25] The variable DISTRESS is used to control for this effect.

7.1.2 Control variables

In addition, we consider three categories of control variables. The first set of control variables is related to the characteristics of each firm. The second set characterizes access to the financial market and the degree of competition in the financial market. We also consider the sectoral and regional dummies.

The first set of control variables on the characteristics of each firm minimizes the possibility that the influence coefficient acts as a proxy for other firm characteristics, that is, it controls for the omitted variable bias. For example, the number of suppliers to a credit-worthy firm could be larger than those less credit-worthy thanks to the lower probability of default. If so, the influence coefficient may reflect the credit-worthiness of the firm. We minimize this possibility by using these control variables. Following the existing empirical literature (e.g., Petersen and Rajan, 1994; Peek and Rosengren, 2005; Bharath et al., 2011), we include the following set of control variables related to firm characteristics: LN(INT_COV) (natural logarithm of the interest coverage ratio), LEVERAGE (ratio of total debts over total assets), TANGIBLE (ratio of collateralizable assets over total assets), CURRENT (current ratio), PROFITABLE (EBITDA / sales), EBITDA_G (growth of EBITDA / total assets), SALES_G (growth rate of sales), LN(SALES) (natural logarithm of sales), and LN(AGE) (natural logarithm of firm age). Many existing empirical studies include a dummy variable to indicate whether a firm is a corporation, but we do not use this variable since we have only one firm that is not a corporation in our sample.

The second set characterizes access to the financial market and the degree of competition there. These factors affect the influence coefficient as well as interest rates. A firm with direct access to the financial market tends to be more credit-worthy and larger, and so its main bank is more likely to be a large bank. The supply network among borrowers

[25] The non-performing loan ratio, i.e., the ratio of risk-management loans over total loans, is 1.9% for major banks, 4.6% for regional banks including the members of the second association of regional banks, and 7.8% for Shinkin banks (cooperative banks) in March 2006. The aggregate non-performing loan ratio is 4.1%. These numbers are calculated from the financial statement of each bank.

of a large bank connects a larger number of firms, and so the influence coefficient can theoretically be larger. Likewise, the market structure affects the size of the network and the influence coefficient of each firm in it. We avoid the possibility that the influence coefficient acts as a proxy for these factors by introducing the second set of control variables. The list of variables includes LISTED (a dummy variable to indicate whether a firm is listed on the stock exchange), BOND_RATIO (ratio of bond financing to total debts), #LENDING_BKS (the number of lending banks), MAJOR_BK (a dummy variable to indicate whether the main bank is one of the major banks), REGIONAL_BK (a dummy variable to indicate whether the main bank is one of the regional banks), and HI (Herfindahl index of bank branches in the area where the head office of the firm is located).

To control for unobservable effects in each industrial sector and region, we include the fixed effects of each industrial sector and region. The industrial sectors include the following 8 sectors: manufacturing (30.8% in the sample), construction (21.9%), real estate (2.9%), retail (9.4%), wholesale (18.6%), communications (2.3%), logistics (2.8%), and others (11.4%). The regions include the following 10 regions: Hokkaido (6.5%), Tohoku (10.9%), Kanto (27.4%), Koshinetsu (8.3%), Tokai (9.6%), Hokuriku (4.7%), Kansai (13.3%), Chugoku (7.3%), Shikoku (4.3%), Kyushu (6.8%), and Okinawa (0.8%).

7.1.3 Descriptive statistics

Table 4 shows the descriptive statistics of variables. Next, we discuss the basic characteristics of the firms in our sample, the degree of accessibility to the financial market and the characteristics of the financial market.

The dependent variable *rate* ranges from 0.0001% to about 13.6%. We do not repeat the discussion on the influence coefficient; instead, see Section 6.3. The mean of the dummy variable DISTRESS indicates that 15.6% of our sample firms are in the situation of distress or near to distress by our definition. The mean and median of LN(SALES) are both about 7.9, which means that the median or mean of sales of our sample firms is about 2.7 billion JPY (27 million USD at a rate of 100 JPY = 1 USD). The dataset also includes quite large firms in terms of sales, 30.9 billion JPY at the 90th percentile, and a

Table 4: Descriptive statistics of variables for interest rate regression

	N	mean	sd	min	p1	p10	p50	p90	p99	max
rate	7,328	2.301	1.514	0.000	0.164	0.868	2.012	3.900	8.205	13.591
ln(v)	7,328	0.011	0.033	0.000	0.000	0.000	0.004	0.020	0.130	0.979
score	7,328	0.135	0.153	-1.000	-0.200	-0.040	0.120	0.340	0.500	0.840
DISTRESS	7,328	0.156	0.363							
LN(INT_COV)	7,328	1.829	1.405	0.000	0.000	0.000	1.623	3.724	5.996	10.571
LEVERAGE	7,328	0.725	0.307	0.008	0.165	0.399	0.741	0.950	1.553	7.161
TANGIBLE	7,328	0.293	0.203	0.000	0.001	0.039	0.267	0.574	0.839	0.989
CURRENT	7,328	1.684	3.356	0.024	0.256	0.731	1.259	2.569	7.470	135.105
PROFITABLE	7,328	0.041	0.831	-46.771	-0.146	-0.008	0.027	0.110	0.345	52.643
EBITDA_G	7,328	0.016	0.888	-0.576	-0.156	-0.041	0.001	0.050	0.235	75.757
SALES_G	7,328	0.043	0.244	-0.959	-0.452	-0.147	0.023	0.234	0.759	6.749
LN(SALES)	7,328	7.905	1.835	2.059	4.399	5.696	7.734	10.340	12.936	16.221
LN(FIRM AGE)	7,328	3.578	0.570	-1.792	1.792	2.767	3.712	4.097	4.477	4.827
LISTED	7,328	0.119	0.324							
BOND_RATIO	7,328	0.061	0.154	0.000	0.000	0.000	0.000	0.225	0.801	1.000
#LENDING_BKS	7,328	4.674	2.304	1	1	2	4	8	10	10
MAJOR_BK	7,328	0.379	0.485							
REGIONAL_BK	7,328	0.539	0.499							
HI	7,328	0.182	0.109	0.050	0.050	0.050	0.164	0.324	0.510	1.000

maximum of 11.1 trillion JPY. The mean and median ages of firms in our data are about 36 and 41 years old, respectively.

The descriptive statistics indicate the importance of bank lending. The mean of the dummy variable LISTED indicates that about 12% of our sample firms are listed at a stock exchange. The ratio of firms issuing bonds (not listed in the table) is 22.8%. Given the fact that a company that is not listed in the stock exchange rarely issues a public bond, we can reasonably expect that only 12% of our sample firms have direct access to the financial market for their fundraising, and bank lending is still the most important external financing source for the remaining 88% of firms. More than 95% of firms obtain loans from multiple banks (#LENDING_BKS).

The most popular type of main bank is a regional bank. These operate on a prefecture-wide basis, including adjacent prefectures. The mean of REGIONAL_BK indicates that 53.9% of firms use a regional bank as their main bank. Some of these banks maintain the market largest share of the lending market in the prefecture where they are located. The major banks, which mainly operate in the metropolitan areas in Japan, are the second most popular type of a main bank. The mean of MAJOR_BK indicates that 37.9% of firms

Table 5: Correlation matrix of variables

		1	2	3	4	5	6	7	8	9	10
1	rate	1.000									
2	ln(v)	-0.089	1.000								
3	ln(v)× score	-0.089	0.875	1.000							
4	score	-0.312	0.239	0.304	1.000						
5	DISTRESS	0.193	-0.059	-0.083	-0.603	1.000					
6	score × DISTRESS	-0.113	0.040	0.065	0.560	-0.655	1.000				
7	LN(INT_COV)	-0.327	0.140	0.172	0.627	-0.325	0.230	1.000			
8	LEVERAGE	0.164	-0.059	-0.086	-0.485	0.388	-0.405	-0.359	1.000		
9	TANGIBLE	-0.031	-0.051	-0.041	-0.027	0.053	-0.031	-0.053	-0.010	1.000	
10	CURRENT	-0.013	-0.027	-0.012	0.015	-0.019	0.008	-0.005	-0.182	-0.061	1.000
11	PROFITABLE	-0.009	0.005	0.006	0.040	-0.038	0.035	0.104	-0.021	0.012	-0.006
12	EBITDA_G	0.027	-0.002	-0.001	-0.004	-0.002	-0.001	0.068	0.007	-0.006	-0.001
13	SALES_G	0.021	-0.005	0.006	0.052	-0.051	0.033	0.153	0.005	-0.042	-0.049
14	LN(SALES)	-0.266	0.416	0.340	0.585	-0.330	0.249	0.490	-0.169	-0.086	-0.075
15	LN(AGE)	-0.160	0.176	0.130	0.252	-0.139	0.129	0.130	-0.188	0.139	-0.015
16	LISTED	-0.165	0.303	0.252	0.326	-0.107	0.080	0.311	-0.198	-0.050	-0.010
17	BOND_RATIO	-0.091	0.151	0.170	0.232	-0.125	0.089	0.156	-0.122	-0.059	0.018
18	#LENDING_BKS	-0.089	0.135	0.076	0.203	-0.138	0.126	0.110	0.008	0.040	-0.041
19	MAJOR_BK	-0.155	0.107	0.111	0.239	-0.165	0.132	0.274	-0.115	-0.177	-0.001
20	REGIONAL_BK	0.101	-0.068	-0.084	-0.126	0.080	-0.054	-0.188	0.061	0.162	-0.027
21	HI	0.075	0.002	-0.026	-0.096	0.093	-0.081	-0.133	0.026	0.184	-0.005

		11	12	13	14	15	16	17	18	19	20
12	EBITDA_G	0.740	1.000								
13	SALES_G	0.048	0.030	1.000							
14	LN(SALES)	0.042	-0.009	0.068	1.000						
15	LN(AGE)	0.020	-0.002	-0.066	0.378	1.000					
16	LISTED	0.020	-0.004	-0.005	0.516	0.207	1.000				
17	BOND_RATIO	0.010	-0.006	-0.001	0.259	0.130	0.201	1.000			
18	#LENDING_BKS	0.018	-0.009	0.005	0.482	0.241	0.241	0.146	1.000		
19	MAJOR_BK	0.014	0.016	0.013	0.460	0.169	0.314	0.280	0.256	1.000	
20	REGIONAL_BK	-0.010	-0.014	-0.001	-0.326	-0.118	-0.250	-0.225	-0.196	-0.848	1.000
21	HI	-0.020	-0.015	-0.022	-0.241	-0.051	-0.121	-0.183	-0.174	-0.573	0.456

use a major bank as their main bank. The main banks of the other firms are cooperative financial institutions, which are smaller than regional banks in terms of asset size and operating area. The Herfindahl index indicates that more than half of firms are located in an area where Herfindahl index is less than 0.18, and the lending market is relatively competitive according to the U.S. regulatory standard; however, some firms are located in areas of highly concentrated banks, where the Herfindahl index is more than 0.5.

Table 5 is the matrix of correlation coefficients among variables to be used in the regression analysis. The table indicates that the influence coefficient has a negative correlation with *rate*, but at a smaller magnitude relative to the traditional indicators of financial soundness, such as *score*, LN(INT_COV), LN(SALES), and LN(AGE). The influence coefficients have positive correlations with LN(SALES). The correlation coefficient is about 0.42. This suggests that larger firms are more likely to have larger influence, but not always.

LISTED, too, has a negative correlation with *rate*. This may be because only those credit-worthy enough can be approved to be publicly traded in the stock exchange. The alternative explanation may be that banks may exploit firms that lack direct access to finance markets. The MAJOR_BK dummy also has a negative correlation with *rate*. This suggests that borrowers from major banks are more credit-worthy firms, and/or that the financing costs at major banks are lower than those at the other banks.

7.2 Econometric frameworks

We briefly restate our hypothesis tests here.

To test Hypotheses 1 and 2, we estimate the following interest rate equation:

$$rate_i = b_0 + b_1 \cdot ln(v_i) + b_2 \cdot score_i + b_3 \cdot ln(v_i) \times score_i + \mathbf{b_4}'\mathbf{X}_i + \epsilon_i, \tag{29}$$

where $rate_i$ is the interest rate for a loan to firm i, $ln(v_i)$ is the natural logarithm of the influence coefficient of firm i, $score_i$ is the credit score to measure the credit-worthiness of firm i, $\mathbf{X}_i$ is the column vector of control variables for firm i, ϵ_i is the error term, and b's are coefficients to be estimated. Hypothesis 1 predicts that b_1 is negative, and Hypothesis 2 predicts that b_3 is positive.

The model is estimated by OLS. We compute the standard errors, which are robust to heteroskedasticity and take into account the estimation error in the influence coefficients. Note that we do not observe the true influence coefficients but instead need to estimate them. They therefore suffer from measurement errors. Appendix 2 explains how to modify the standard errors to take into account the estimation error in the influence coefficients.

7.3 Baseline results

The regression results are supportive of our hypotheses. They indicate that a firm with a high influence coefficient receives a lower interest rate on average. Moreover, we find that the effect is more prominent for less credit-worthy firms.

Table 6 shows the results of the baseline regression of (29) by OLS with robust standard errors. Column (1) shows that the influence coefficient and *score* are negatively correlated with interest rates. Column (2) shows that the interaction term between the influence coefficient and *score* has a positive and significant coefficient, while each of their main

Table 6: Baseline regression on rate

	(1) coef. (s.e.)		(2) coef. (s.e.)		(3) coef. (s.e.)	
$\ln(v)$	-3.913	***	0.372		-1.681	*
	(1.113)		(0.410)		(0.972)	
$\ln(v)\times$ score	7.726	***			4.578	**
	(2.050)				(1.828)	
score	-2.879	***	-1.402	***	-1.511	***
	(0.132)		(0.186)		(0.194)	
DISTRESS			0.296	***	0.290	***
			(0.071)		(0.071)	
score × DISTRESS			2.539	***	2.598	***
			(0.636)		(0.635)	
LN(INT_COV)			-0.205	***	-0.205	***
			(0.016)		(0.016)	
LEVERAGE			-0.044		-0.049	
			(0.074)		(0.073)	
TANGIBLE			-0.268	***	-0.270	***
			(0.093)		(0.093)	
CURRENT			-0.009		-0.008	
			(0.005)		(0.005)	
PROFITABLE			-0.050	***	-0.050	***
			(0.018)		(0.018)	
EBITDA_G			0.097	***	0.098	***
			(0.013)		(0.013)	
SALES_G			0.359	***	0.355	***
			(0.098)		(0.098)	
LN(SALES)			-0.038	**	-0.031	*
			(0.016)		(0.016)	
LN(AGE)			-0.183	***	-0.180	***
			(0.038)		(0.038)	
LISTED			-0.038		-0.035	
			(0.050)		(0.050)	
BOND_RATIO			0.073		0.059	
			(0.124)		(0.124)	
#LENDING_BKS			0.017	**	0.018	**
			(0.008)		(0.008)	
MAJOR_BK			-0.207	**	-0.203	**
			(0.081)		(0.081)	
REGIONAL_BK			-0.084		-0.077	
			(0.071)		(0.071)	
HI			-0.018		0.002	
			(0.209)		(0.209)	
Industry factor	yes		yes		yes	
Region factor	yes		yes		yes	
N	7,355		7,328		7,328	
Adj. R-squared	0.123		0.160		0.160	

(Note) Estimated by OLS. Standard errors are adjusted to include the standard error of the estimated influence coefficients. The constant term, the coefficients of industry, and regional factors are omitted from the report. *, **, and ***, indicate that the estimated coefficient is statistically significant at the 10%, 5%, or 1% significance level, respectively.

Table 7: Marginal effects

At score =	d rate/d ln(v)	(s.e.)	
-0.2	-2.597	1.314	**
-0.1	-2.139	1.141	*
0	-1.681	0.972	*
0.1	-1.223	0.809	
0.2	-0.766	0.657	
0.3	-0.308	0.524	
0.4	0.150	0.430	
0.5	0.608	0.402	

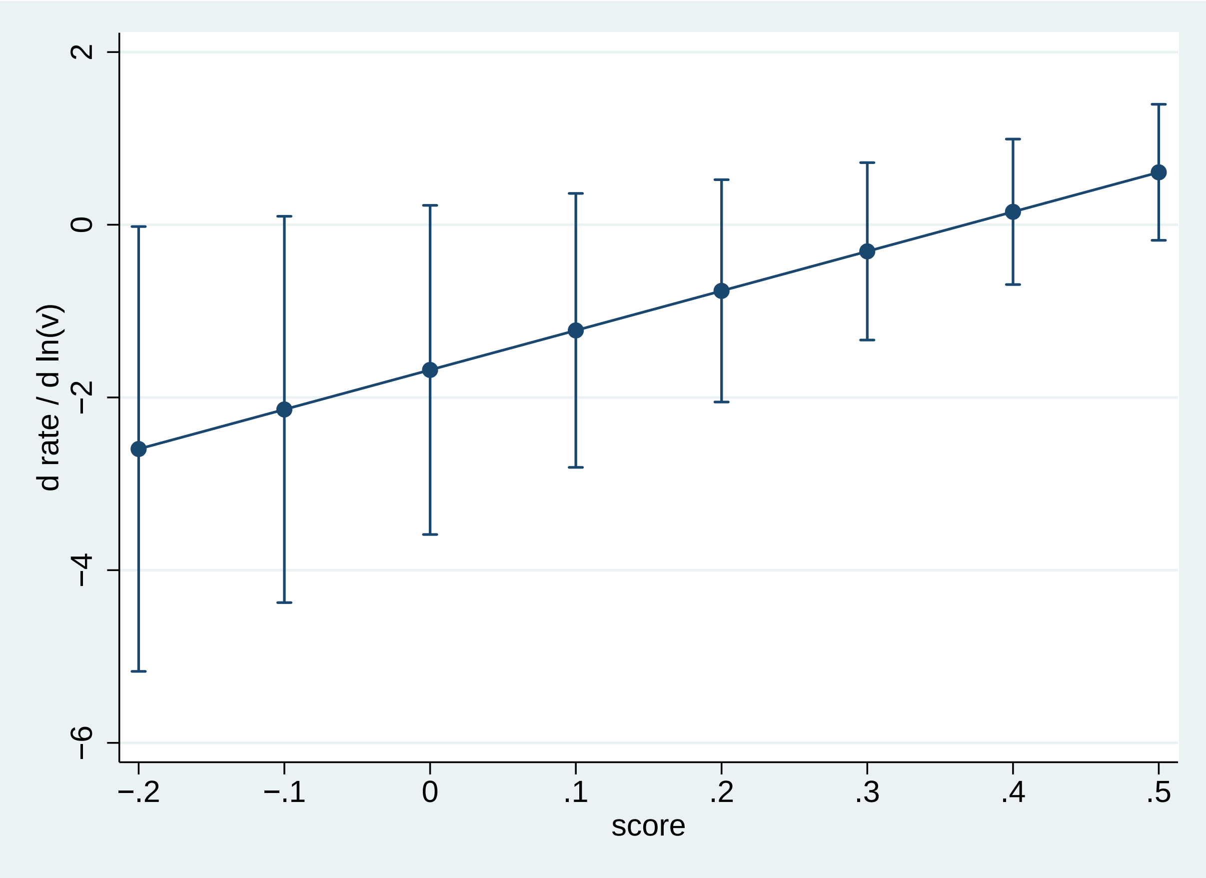
(Note) Variables except score are set at the sample mean. Calculated from the results in (3) in Table 6. *, **, and ***, indicate that the estimated coefficient is statistically significant at the 10%, 5%, or 1% significance level, respectively.

effects has a negative and significant coefficient. Thus, the regression analysis supports Hypotheses 1 and 2.

To gauge the economic importance of the effect of the influence coefficient, we compute the marginal effect of $ln(v)$ at each grid cell of *score* to *rate*, which is listed in Table 7 and illustrated in Figure 3. The interest reduction effect of the influence coefficient is statistically significant for firms with a score less than 0, that is, for distressed firms. This effect is not significant for financially sound firms, for which the main bank does not need to consider forbearance lending. The estimates indicate, for example, that the interest rate for a firm with an influence coefficient at the median is larger than that for a firm with an influence coefficient at the 90th percentile by about 3 basis points when their scores are 0. The difference is larger, at 21 basis points, when we compare with firms having an influence factor at the 99th percentile. The difference is even larger, at 164 basis points, when we compare with the firm having the maximum influence coefficient. Thus, the effect of the influence coefficient is more economically significant for firms with a higher influence coefficient.

Among the control variables, DISTRESS has a positive and significant coefficient, and the interaction term of *score* and DISTRESS is positive and significant. This result suggests that the interest rates for distressed firms are typically higher, but that more severely distressed firms cannot afford to do anything but postpone the repayment of agreed interest (note that we define *rate* on the basis of actual interest payments by each

Figure 3: Marginal effect on *rate* of the log of the each-bank influence coefficient



(Note) Vertical line segments indicate the 95% confidence intervals.

firm). Those financially sound and secure firms with higher LN(INT_COV), TANGIBLE, PROFITABLE, larger LN(SALES), and longer LN(AGE) pay lower interest. Rapidly growing firms with higher EBITDA_G and SALES_G pay higher interest, probably because of the risk implied by rapid growth. The interest cost is significantly lower when the main bank is a major bank (MAJOR_BK). This is because the funding costs for these banks are lower than at regional banks because of their size and creditworthiness.

7.4 Subsample regression: type of main bank

We estimate the model using each of three subsamples divided by the type of a main bank. Comparing the results from these three subsamples provides an empirical test of Hypothesis 3. In particular, we examine whether the effect of the influence coefficient is larger for firms with a regional bank as their main bank. Recall that we have surmised from our theoretical analysis that forbearance is more likely when a bank is dominant in

Table 8: Subsample regression: Type of main bank (dependent variable: *rate*)

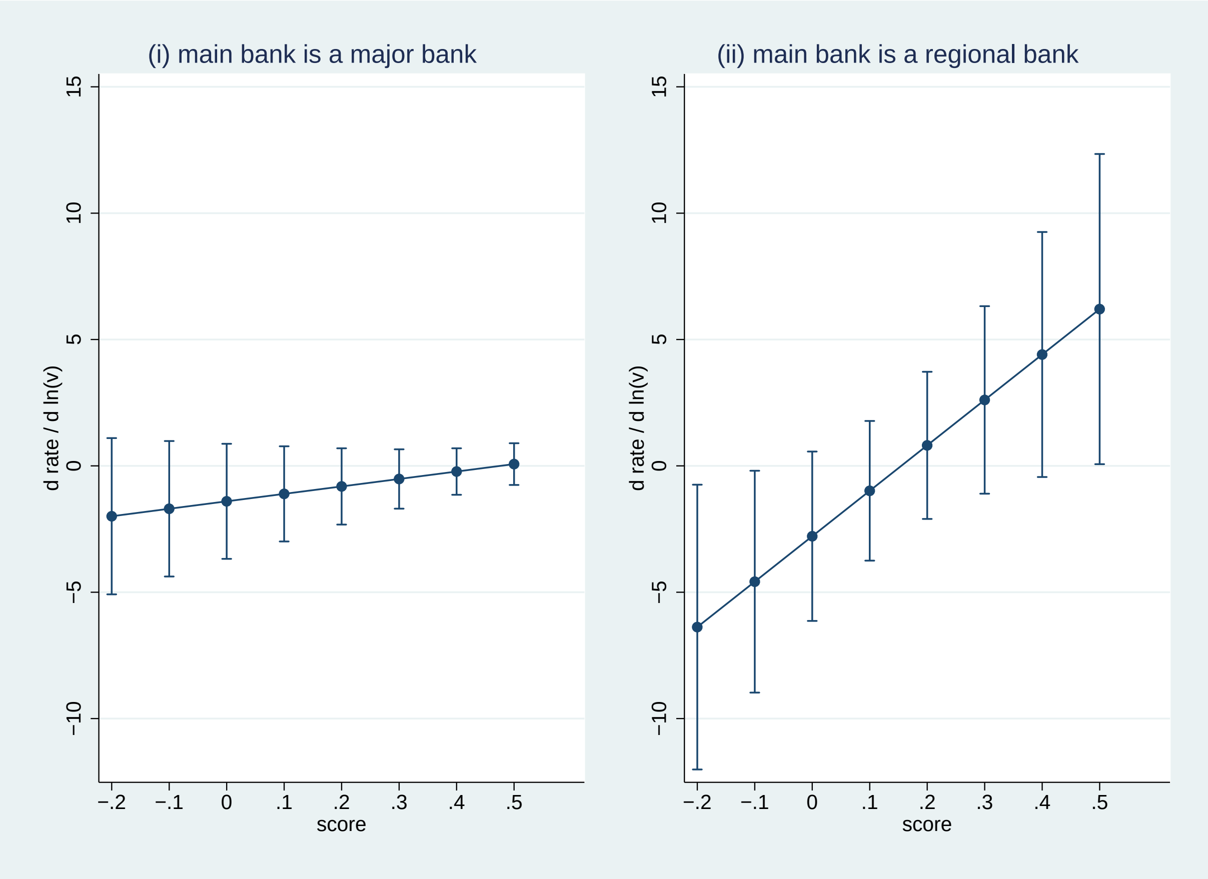
	(i) Major banks only			(ii) Regional banks only			(iii) Cooperative banks only	
	coef.	(s.e.)		coef.	(s.e.)		coef.	(s.e.)
$\ln(v)$	-1.403	(1.161)		-2.784	(1.710)		-24.167	(18.252)
$\ln(v)\times$ score	2.948	(2.188)		17.977	(7.604)	**	82.388	(153.353)
Controls	yes	yes		yes	yes		yes	yes
Region factor	yes	yes		yes	yes		yes	yes
Industry factor	yes	yes		yes	yes		yes	yes
Adj. R-squared	0.162			0.146			0.119	
N	2,780			3,949			599	
(Marginal Effect)								
	$\frac{d\,rate}{d\ln(v)}$	(s.e.)		$\frac{d\,rate}{d\ln(v)}$	(s.e.)		$\frac{d\,rate}{d\ln(v)}$	(s.e.)
at score = -0.2	-1.993	(1.577)		-6.380	(2.875)	**	-40.644	(44.694)
-0.1	-1.698	(1.368)		-4.582	(2.240)	**	-32.405	(30.498)
0	-1.403	(1.161)		-2.784	(1.710)		-24.167	(18.252)
0.1	-1.108	(0.961)		-0.987	(1.410)		-15.928	(14.370)
0.2	-0.814	(0.770)		0.811	(1.486)		-7.689	(23.456)
0.3	-0.519	(0.599)		2.609	(1.893)		0.550	(36.935)
0.4	-0.224	(0.469)		4.407	(2.473)	*	8.789	(51.464)
0.5	0.071	(0.421)		6.204	(3.131)	**	17.028	(66.357)

(note) Estimated by OLS. Standard errors are adjusted to include the standard error of the estimated influence coefficients. We drop the major bank dummy and the regional bank dummy from the set of control variables. The other variables are the same as in column (3) of Table 6. In the calculation of the marginal effects, control variables except score are all set at the sample mean. *, **, and ***, indicate that the estimated coefficient is statistically significant at the 10%, 5%, or 1% significance level, respectively.

its lending market and/or it actively engages in relationship banking. We thus expect that the interest reduction effect will be more prevalent among firms with a regional bank as their main bank.

Our empirical result supports this conjecture. We split the sample into three subsamples according to the type of main bank: major bank, regional bank, or cooperative bank. We estimate the baseline model for each of these subsamples. The results are listed in Table 8. The marginal effect of the influence coefficient is illustrated in Figure 4. In the figure, the estimate with cooperative banks is dropped because of the scale and statistical insignificance. The result shows that forbearance by the main bank is more clearly observed among regional banks (grouping (ii) in Table 8), while this tendency is not visible for the other types of banks. Thus, the results support Hypotheses 3.

Figure 4: Marginal effect on *rate* of $\ln(v)$, by main bank type



(Note) Vertical line segments indicate the 95% confidence interval.

8 Robustness check

In this section, we examine the robustness of our results. We first examine the effects of alternative specifications of the influence coefficients. We then use quantile regression to investigate whether our result is driven by outliers. Quantile regression analysis also provides valuable information about the heterogeneity of the effect of influence coefficient.

8.1 Alternative specification for influence coefficients

Since the influence coefficients are estimated and not directly observable, it is important to check whether our results are driven by the particular specification used in estimation. To do so, we consider an alternative procedure for estimating the influence coefficients. The result demonstrates that our results are robust in this regard.

The alternative methods for estimating influence coefficients are also based on the sales equation (12) but use a different specification of the matrix $\mathbf{Q}$. The alternative specification of $\mathbf{Q}$ is $\mathbf{Q}_s = \beta_s \mathbf{GS}$, where $\mathbf{S}$ is the $n \times n$ diagonal matrix whose ith diagonal element is the square root of firm i's credit score (as a percent of maximum, achieved by dividing the raw score by 100) provided by TSR as of 2005. In this way, $\mathbf{Q}_s$ reflects to some extent the possibility of firm defaults. We estimate the following model by OLS

$$\Delta \mathbf{s} = \beta_s \mathbf{GS} \Delta \mathbf{s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon.$$

We then compute

$$\hat{\mathbf{v}}_s^{(b)} = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta}_s \mathbf{G}^{(b)} \mathbf{S}^{(b)})^k, \tag{30}$$

where $\mathbf{S}^{(b)}$ is the submatrix of $\mathbf{S}$ induced by including only firms in bank b's network.

We find that the two types of influence coefficient estimates are similar and which estimate is used does not affect our main results for regression on interest rate. The first row in panel (a) of Table 9 shows the descriptive statistics of the estimated influence coefficients for the supply networks of firms with a common main bank v_s, which is estimated by (30). The underlying regression results are presented in panel (b). The two versions of influence coefficients, v and v_s, are highly correlated with each other. The correlation coefficients between v and v_s is 0.9969, as seen in panel (c). The empirical results are the same for estimation with the alternative version of the influence coefficient (column (1) in Table 10).

8.2 Economy-wide network

We also re-estimate the baseline model with the influence coefficient in the economy-wide supply network. We conduct this analysis to confirm that the effect of influence coefficients appears through the network-motivated lending decision, and not through some other mechanism that affects both the location of firms in the network and the interest rate. Our result showing that this is related to Hypothesis 3 indicates that influence coefficients defined in terms of bank-specific supply networks are more appropriate than economy-wide influence coefficients. This is because a bank is highly likely to be a dominant financier and be able to observe the influence of each firm in the supply network of its borrowers.

Table 9: Alternative influence coefficients

Variable	N	mean	sd	min	p10	p50	p75	p90	p95	p99	max
v_s	306,354	1.003	0.011	1.000	1.000	1.002	1.003	1.005	1.008	1.022	2.700
v_w	651,913	1.010	0.054	1.000	1.000	1.004	1.010	1.019	1.028	1.071	11.490
v_{ws}	651,913	1.008	0.051	1.000	1.000	1.003	1.008	1.015	1.023	1.062	11.161
$v - v_w$	306,354	0.011	0.069	0.000	0.000	0.004	0.011	0.022	0.034	0.093	9.690

(a) Description of alternative influence coefficients

	Estimate	s.e.	
β_s	0.00337	0.0000708	***
Industry factor	yes		
Prefecture factor	yes		
Adj. R-squared	0.1457		

(b) First-stage regression to calculate influence coefficient v_s (N = 651,913)

Variable	v	v_s	v_w
v_s	0.9969	1	
v_w	0.7481	0.7525	1
v_{ws}	0.7493	0.7573	0.9981

(c) Pairwise correlation coefficients among alternative influence coefficients

(Note for panel (b)) Industry factor is controlled by the 95 sector dummies, which indicate one 2-digit *chu-bunrui* classification. Prefecture factor is controlled by 46 prefecture dummies (Hokkaido is the base class).

In contrast, it is less likely that a bank will consider the influence of a borrower on the economy-wide supply network. We thus expect to observe a weak effect of the influence coefficient when it is defined in terms of the economy-wide supply network. Consistent with this prediction, we find a smaller effect from the economy-wide influence coefficients than from the bank-specific coefficients.

The economy-wide influence coefficients are computed similarly to (28) and (30), but we use the entire matrix $\mathbf{Q}$. For example, v_w, which is the economy-wide version of v, is

Table 10: Regression with the alternative influence coefficients (dependent variable: *rate*)

	(1)		(2)		(3)	
Influence coef. =	$\ln(v_s)$		$\ln(v_w)$		$\ln(v_{ws})$	
	coef.		coef.		coef.	
	(s.e.)		(s.e.)		(s.e.)	
Influence coef.	-1.918	*	-0.886	***	-0.962	***
	(1.009)		(0.226)		(0.240)	
Influence coef. × score	4.938	***	2.171	***	2.322	***
	(1.810)		(0.461)		(0.481)	
Controls	yes		yes		yes	
Industry factor	yes		yes		yes	
Region factor	yes		yes		yes	
N	7,328		9,134		9,134	
Adj. R-squared	0.160		0.156		0.156	

(Note) Estimated by OLS. Standard errors are adjusted to include the standard error of the estimated influence coefficients. The constant term, the coefficients of industry, and regional factors are omitted from the report. The control variables are the same as in column (3) of Table 6. *, **, and ***, indicate that the estimated coefficient is statistically significant at the 10%, 5%, or 1% significance level, respectively.

computed by

$$\hat{\mathbf{v}}_w = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta}\mathbf{G})^k,$$

where $\hat{\mathbf{v}}_w$ is the estimated vector of v_w for all the firms. Similarly, $\hat{\mathbf{v}}_{ws}$, which is the economy-wide version of $\hat{\mathbf{v}}_s$, is computed by

$$\hat{\mathbf{v}}_{ws} = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta}_s\mathbf{GS})^k.$$

Rows 2 and 3 in panel (a) of Table 9 show the descriptive statistics of the estimated influence coefficients of the economy-wide supply network, v_w and v_{sw} (panel (b)). Compared with the influence coefficients defined in terms of bank-specific networks, the economy-wide influence coefficients are larger and more variable. Their distribution is more strongly skewed.This result is natural because the number of firms in the economy-wide supply network is much larger and many firms have transactions with firms that do not share their main bank.

There is a considerable difference between the economy-wide influence coefficient and the bank-specific influence coefficient. The correlation coefficient between the alternative economy-wide influence coefficients and those defined in the bank-specific networks is about 0.75 (panel (c)), while the correlation coefficient between alternative economy-wide influence coefficients is almost one. The last row in panel (a) is the descriptive statistics of the difference between the bank-specific influence coefficient and the economy-wide influence coefficient. The difference is larger in the upper tail.

The results in columns (2) and (3) of Table 10 demonstrate that the economy-wide influence coefficients have a weak effect on average interest rate, as expected. The coefficient of the influence-coefficient term is negative and statistically significant, and the interaction term between the influence coefficient and *score* is positive and statistically significant, as seen in columns (2) and (3). The statistical significance is stronger in the economy-wide network due to the higher variation of the influence coefficient. However, the coefficient and the marginal effect are about half of the estimate obtained with using bank-specific networks. This result is consistent with our theoretical analysis.

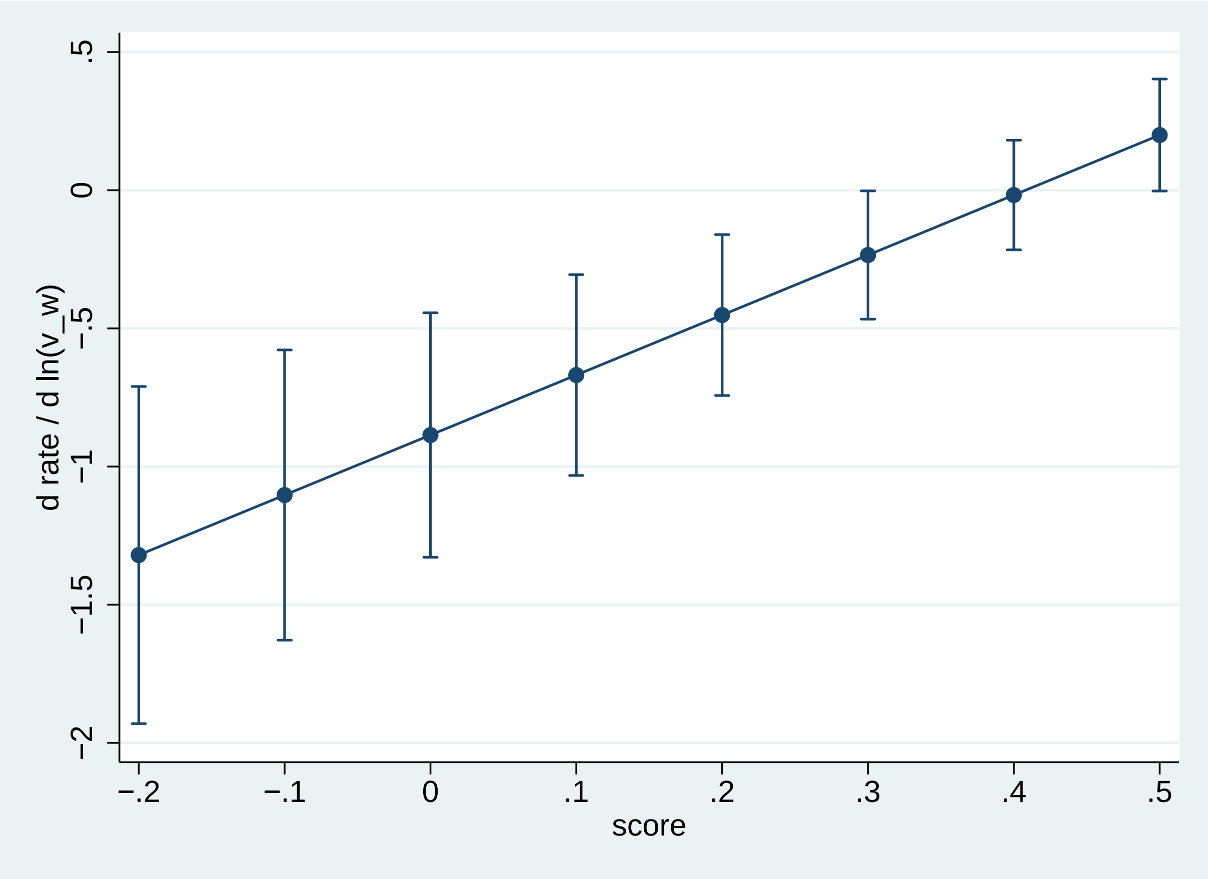
8.3 Quantile regression

We also estimate the model by performing quantile regressions. Doing so enables us to examine the effects of outliers. Moreover, the results of quantile regression provide more detailed information about the mechanism of the effect of influence coefficients.

In our main analysis, we drop outliers with respect to *rate* and those firms that report zero or negative interest payments. These treatments of outliers can introduce bias to the OLS estimate of coefficients. To avoid this problem, we estimated the baseline model by quantile regression for each decile of *rate*, using the dataset without dropping outliers.

The estimation result at each decile is listed in Table 11. The table shows that the forbearance motivation is more supported for observations around higher deciles, particularly at the 40th, 50th, 60th, 70th, and 80th percentiles. The results indicate that higher deciles of interest rate for firms with high influence coefficients are much larger than those for firms with low influence coefficients, and the difference between deciles of interest rate is larger for higher deciles. This suggests that the interest reduction effect of the influence coefficient is more significant for relatively less credit-worthy firms because a higher *rate*

Figure 5: Marginal effect on *rate* of log of economy-wide influence coefficients



(Note) Vertical line segments indicate 95% confidence intervals.

can be considered to indicate lower credit-worthiness.[26]

9 Discussions

In this section, we provide several caveats about deriving welfare implications from our analysis. We show that network-motivate forbearance improves welfare. However, our welfare analysis is inherently a short-run analysis. Another important problem is that we ignore the moral hazard problem to simplify the analysis.

First, as we note, our analysis is a short-run analysis. We assume that the technological importance of inputs w_{ij}, which is the primary determinant of the influence coefficient of each firm, is fixed. In other words, we do not allow for entries from outside of the given supply network. This assumption may be plausible in the short run. The analytical

[26]Since we have controlled for the effect of credit score and other observable characteristics, firms with a high interest rate in this context should be less credit-worthy in some unobservable way.

Table 11: Quantile regression (dependent variable: rate; N = 7,415)

Independent variable	Est. coef.	(s.e.)		Est. coef.	(s.e.)		Est. coef.	(s.e.)	
	10th percentile			20th percentile			30th percentile		
$\ln(v)$	-0.307	(1.068)		-0.673	(0.879)		-1.177	(0.787)	
$\ln(v)\times$ score	1.626	(2.066)		2.794	(1.699)		3.990	(1.522)	***
Controls	yes			yes			yes		
Region factor	yes			yes			yes		
Industry factor	yes			yes			yes		
Pseudo R^2	0.049			0.051			0.049		
	40th percentile			50th percentile			60th percentile		
$\ln(v)$	-2.185	(0.836)	***	-2.799	(0.845)	***	-3.138	(0.993)	***
$\ln(v)\times$score	5.940	(1.617)	***	6.973	(1.635)	***	7.563	(1.920)	***
Controls	yes			yes			yes		
Region factor	yes			yes			yes		
Industry factor	yes			yes			yes		
Pseudo R^2	0.047			0.043			0.038		
	70th percentile			80th percentile			90th percentile		
$\ln(v)$	-3.456	(1.276)	***	-3.538	(1.935)	*	-0.828	(4.152)	
$\ln(v)\times$score	7.947	(2.466)	***	7.575	(3.742)	**	3.187	(8.028)	
Controls	yes			yes			yes		
Region factor	yes			yes			yes		
Industry factor	yes			yes			yes		
Pseudo R^2	0.032			0.024			0.017		

(Note) The set of control variables is the same as that in column (3) of Table 6. The estimated coefficients of control variables, region factors, year factors, and the constant term are omitted from the report. *, **, and ***, indicate that the estimated coefficient is statistically significant at the 10%, 5%, or 1% significance level, respectively.

part of Caballero et al. (2008) adopts the assumption at the opposite extreme: that there are always potential entrants who are more efficient than the incumbent firms. That assumption is suitable for a long-run welfare analysis because the supply network will be flexible in the long run. Empirical studies are needed to clarify how rigid the supply network is.[27] Nonetheless, we conjecture that whether such efficient potential entrants exist depends on the type of industry and the economic environment at each time point. For example, if an industry requires the accumulation of relation-specific

[27]Known empirical results on this point are mixed. For example, Fukao and Kwon (2006) and Nishimura et al. (2005) find that less efficient companies increased their market shares and were less likely to exit in the 1990s in Japan. In contrast, Sakai et al. (2010) find that those that exited were less profitable than those that survived in the 1990s, from examining extensive microdata of small business lending in Japan. Calvalho et al. (2014) finds that a firm whose suppliers or corporate customers are damaged by a great earthquake is more likely to search and find a new supplier or a new customer. However, the economic significance of such adjustment in a circumstance without such an extreme shock could be smaller.

information and design to improve productivity or product quality, which is known to occur in the automotive and financial sectors, then potential entrants are not likely to be more efficient than incumbent firms. However, if an industry treats commodities that are less differentiated and do not require relation-specific investments, then potential entrants could be more efficient than incumbent firms. Thus, the applicability of our results on the welfare analysis will differ across economies and sectors. In any case, it is important to note that what we examine is short-run welfare, and the implications for long-run welfare could be different.

Second, while we discuss the welfare implication for an economy with rigid supply networks, there is some concern about whether an economy with rigid supply networks is efficient. It has been pointed out that economies in which specificity works as a barrier to entry, termed *sclerosis* in Caballero and Hammour (1998), is inherently inefficient. Our efficiency result for network-motivated forbearance says that forbearance could be better for the economic efficiency under the assumption that the economy had already been constructed and is full of relation-specific investments. Although it is clearly beyond the scope of the current paper, it would be interesting to examine the possibility that constructing the economy without such specificity from the beginning would be better for social welfare.

Third, in our setting, the capital cost of external finance is not related to the output level. The usual route from the monopolistic capital cost to welfare loss is through the reduction of output caused by high capital costs. However, this channel of welfare loss is shut down by making sales independent of capital costs in our setup to make the analysis tractable. If this route is taken into account, the ability of the bank to recoup the forbearance cost by imposing higher interest rates on loans to peripheral firms could be limited. This might reduce the chance of forbearance, and the welfare consequences of forbearance will be ambiguous under this scenario.

Fourth, we do not explicitly include the possible moral hazard for an influential company entailed by being considered too big to fail or too connected to fail. Such a company can expect a bailout by a bank or a government, and this assurance is likely to provide perverse incentives for managers and shareholders. This point might affect the welfare implications. In particular, influential firms might become more inefficient and thereby inflate the cost of forbearance. Even in this case, we conjecture that rational forbearance by a profit maximizing bank would improve welfare. However, this possibility makes it difficult to examine whether public bailouts improve welfare.

10 Concluding remarks

We showed that a bank acting as the dominant financier to an inter-firm supply network will be motivated to strategically support an influential producer in the network by forbearance lending, and we illustrated the conditions under which this behavior would emerge by using an oligopoly model of final and intermediate products with an incomplete supply network. Our analytical model shows that this network-motivated forbearance is an economically rational response by a bank. Our empirical study provides evidence for this motivation by statistically verifying that the interest rate of loans is lower for influential firms within the supply networks among borrowers of a bank, and this tendency is more pronounced for financially less sound firms, even after controlling for all other observable factors.

Our findings suggest that the terms of a financial contract can be affected by the importance of the contracting firm relative to other firms within the portfolio of a bank. Thus, we need to look at the relative position of each firm within the portfolio of a bank and also at the characteristics of individual firms and banks in examining the economic efficiency of loan contracts and the financial market. In addition, the shape of the supply network could be an important determinant of the risk characteristics of each bank portfolio.

As concluding remarks, we would like to comment on some of the abundant remaining research questions and directions. First, it would be interesting to extend our analysis to the case in which the supply network is flexible. We have assumed that the supply network is rigid, which implies that we examine the short-run effect. In the short run, it is hard to find a corporate client and to create a new link in the supply network. However, these strategic behaviors of banks could harm economic efficiency in the long run because, in the long run, a firm can find new corporate clients. Thus, the assumption acts to deter natural selection and crowd out potentially efficient new entrants, as is shown by Caballero et al. (2008). Evaluating the welfare implication of forbearance lending (or public bailouts) with taking into account both of these effects remains a challenging future research subject.

Second, cross-country comparisons or comparisons between bank-dominant markets and market-oriented markets would be useful. Our model shows that a bank that is dominant in some market is more likely to undertake network-motivated lending decisions. This finding highlights a novel viewpoint for discussing the efficient structure of the financial market in conjunction with the argument in the previous paragraph. In this paper, we found that network-motivated lending decisions were particularly notable in the regional financial markets of Japan. However, it has been argued that the US and UK are more market-oriented markets. Empirical research on these markets and comparison with our results would be interesting.

Lastly, our empirical study illustrates the usefulness of the inter-firm transaction database for financial researchers as well as for policy makers. In this study, we estimate the influence coefficient of each firm by making use of limited, but available, information on inter-firm transactions. More precise inter-firm transaction data would improve the accuracy of influence coefficients and ease computation of them. This sort of dataset enabled us to identify influential companies within a portfolio of a bank or within an economy, and we believe that this kind of dataset will be highly useful for regulators, banks and other institutional investors, and academic researchers.

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Appendix 1: The bias of the OLS estimator

We provide a justification for the use of the OLS estimator to estimate the spatial autoregressive models. We focus on the estimation of β_1, but a similar argument holds for the estimation of other parameters. We derive the formula for the bias of the OLS estimator and evaluate the bias in our sample. As a result, we find that the bias is likely to be small.

Recall that the model is

$$\Delta\mathbf{s} = \beta\mathbf{G}\Delta\mathbf{s} + \gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \epsilon. \tag{31}$$

We assume that **Ind** and **Pre** are exogenously given variables, so that they are uncorrelated with ϵ. In this section, we assume that ϵ is homoskedastic, making this assumption to evaluate the bias of the estimator. However, note that the standard errors reported in the main text are robust to heteroskedasticity.

The bias of the OLS estimator of β can be derived in the following way. Let $\tilde{\mathbf{X}}$ be the residual from the regression of $\mathbf{G}\Delta\mathbf{s}$ on **Ind** and **Pre**. The OLS estimator of β is

$$\hat{\beta} = \frac{\tilde{\mathbf{X}}'\Delta\mathbf{s}}{\tilde{\mathbf{X}}'\tilde{\mathbf{X}}} = \beta + \frac{\tilde{\mathbf{X}}'\epsilon}{\tilde{\mathbf{X}}'\tilde{\mathbf{X}}}. \tag{32}$$

Since **Ind** and **Pre** are assumed to be exogenous, $E(\tilde{\mathbf{X}}'\epsilon) = E((\mathbf{G}\Delta s)'\epsilon)$. The reduced-form equation for $\Delta\mathbf{s}$ is

$$\Delta\mathbf{s} = (\mathbf{I}_n - \beta\mathbf{G})^{-1}(\gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \epsilon). \tag{33}$$

Since the regressors are assumed to be exogenous, we have

$$E((\mathbf{G}\Delta s)'\epsilon) = E\left(\epsilon'\mathbf{G}(\mathbf{I}_n - \beta\mathbf{G})^{-1}\epsilon\right) = \sum_{k=0}^{\infty} E\left(\epsilon'\beta^k\mathbf{G}^{k+1}\epsilon\right). \tag{34}$$

Thus, the asymptotic bias of the OLS estimator is

$$\frac{\lim_{N\to\infty}\sum_{k=0}^{\infty} E\left(\epsilon'\beta^k\mathbf{G}^{k+1}\epsilon\right)/N}{\lim_{N\to\infty}\tilde{\mathbf{X}}'\tilde{\mathbf{X}}/N}. \tag{35}$$

The bias of the OLS estimator may be numerically evaluated. The value of $\tilde{\mathbf{X}}'\tilde{\mathbf{X}}$ can be computed from the data. Let $g_{ii}(k)$ be the ith element on the main diagonal of $\mathbf{G}^k$. Assume that ϵ is homoskedastic with variance σ^2. Then, we have

$$\sum_{k=0}^{\infty} E\left(\epsilon'\beta^k\mathbf{G}^{k+1}\epsilon\right) = \sigma^2\sum_{k=0}^{\infty}\beta^k\sum_{i=1}^{n} g_{ii}(k+1). \tag{36}$$

Since $g_{ii}(1) = 0$ by the definition of $\mathbf{G}$, the value at $k = 0$ does not contribute to the sum. Note that $g_{ii}(k)$ can be computed from the data. Thus, once we have the values of σ^2 and β, we can compute the bias by using the formula

$$\frac{\sigma^2\sum_{k=1}^{\infty}\beta^k\sum_{i=1}^{n} g_{ii}(k+1)}{\tilde{\mathbf{X}}'\tilde{\mathbf{X}}}. \tag{37}$$

We compute an approximated value of the bias in the data and find that the bias is likely to be small. To do so, we use the OLS estimator of β and the estimate σ^2 from the OLS estimation to evaluate $\sigma^2\sum_{k=0}^{\infty}\beta^k\sum_{i=1}^{n} g_{ii}(k+1)$. The infinite sum is truncated at $k = 2$. This truncation is justified because the value of β is small (recall that the OLS estimate is 0.00197). The values of $\sum_{i=1}^{n} g_{ii}(2)$ and $\sum_{i=1}^{n} g_{ii}(3)$ in our sample are 175,222 and 177,677, respectively. Using these numbers, the approximated value of the bias is 0.000000854. Since the OLS estimate is 0.00197 and the standard error is 0.0000494, we argue that the bias is small and so we may rely on the OLS estimator. Here, we use the values of σ^2 and β from the OLS estimation. However, the computed bias would be small even if different (reasonable) values of σ^2 and β were used.

Appendix 2: Standard errors when a regressor is generated

In this appendix, we provide the formula for the standard errors for the interest rate regression, taking into account the fact that the key regressor (influence coefficient) is estimated. We first briefly review the effect of a generated regressor in general linear regression frameworks. We then provide the formula for the specific case in which the generated regressor is influence coefficient.

General results

We first examine the effect of a generated regressor in linear regression models. In particular, we explain how to modify the asymptotic variance estimator for the OLS estimator. Note that this is a rather standard exercise in econometrics.

The model we analyze is the following linear regression model:

$$y_i = x_i' \beta + u_i. \tag{38}$$

However, x_i is not directly observed. We know that x_i can be written as $x_i = x_i(\gamma_0)$, where the function $x_i(\cdot)$ is known and γ_0 is estimable. For simplicity, we assume that γ_0 is a scalar. Let $\hat{\gamma}$ be an estimate of γ_0. We thus use a generated regressor $\hat{x}_i = x_i(\hat{\gamma})$ instead of x_i. The OLS estimator of β with $\hat{x}_i$ is

$$\hat{\beta} = \left(\sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \sum_{i=1}^{N} \hat{x}_i y_i. \tag{39}$$

We make the following assumptions about the generated regressor. We assume that $x_i(\cdot)$ is differentiable and that the mean value theorem for it, so that

$$x_i - \hat{x}_i = -\frac{\partial x_i}{\partial \gamma}(\tilde{\gamma})(\hat{\gamma} - \gamma_0), \tag{40}$$

where $\tilde{\gamma}$ is between $\hat{\gamma}$ and γ_0. We also assume that $\hat{\gamma}$ is asymptotically linear:

$$\hat{\gamma} - \gamma = \frac{1}{N^*} \sum_{j=1}^{N^*} \phi_j, \tag{41}$$

where ϕ_j has mean zero and finite variance. We note that here we allow γ to be estimated from a different sample than the sample used in the estimation of β. These samples are allowed to be overlapping or disjoint. Let N^* denote the sample size of the sample used in the estimation of γ; N^* may be different from N. We assume that $\lim_{N,N^*\to\infty}(N/N^*) = \kappa < \infty$.

The asymptotic distribution of the OLS estimator $\hat{\beta}$ depends on the estimation error in $\hat{\gamma}$. We observe that

$$y_i = \hat{x}_i'\beta + (x_i - \hat{x}_i)'\beta + u_i. \tag{42}$$

We therefore have the following expansion of $\hat{\beta}$:

$$\hat{\beta} = \beta + \left(\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \sum_{i=1}^{N} \hat{x}_i u_i + \left(\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \sum_{i=1}^{N} \hat{x}_i(x_i - \hat{x}_i)'\beta. \tag{43}$$

The second term in the right-hand side yields the usual asymptotic distribution of the OLS estimator. The third term depends on the estimation error in $\hat{\gamma}$. Using the assumption on $\hat{x}_i$, we have

$$\hat{\beta} - \beta = \left(\frac{1}{N}\sum_{i=1}^{N} \hat{x}_i\hat{x}_i\right)^{-1} \left(\frac{1}{N}\sum_{i=1}^{N} \hat{x}_i u_i - \frac{1}{N}\sum_{i=1}^{N} \hat{x}_i\beta'\frac{\partial x_i}{\partial\gamma}(\tilde{\gamma})\frac{1}{N^*}\sum_{j=1}^{N^*}\phi_j\right). \tag{44}$$

Let

$$A = E\left(x_i\beta'\frac{\partial x_i}{\partial\gamma}(\gamma_0)\right), \tag{45}$$

and

$$B = \text{plim}_{N\to\infty}\frac{1}{N}\sum_i x_i u_i \phi_i', \tag{46}$$

where $\sum_i$ in B is taken over the set of observations that appear in both the sample used for the estimation γ and that for β. From the expansion of $\hat{\beta}$, it is easy to derive the asymptotic distribution of $\hat{\beta}$, which is

$$\sqrt{N}(\hat{\beta} - \beta) \to_d N(0, V), \tag{47}$$

where

$$V = \left(E(x_i x_i')\right)^{-1}\left(E(u_i^2 x_i x_i') - BA' - AB' + A\kappa E(\phi_j\phi_j')A'\right)\left(E(x_i x_i')\right)^{-1}. \tag{48}$$

We note that when $\kappa = 0$ (i.e., N^* is much larger than N), the estimation error in the generated regressor does not affect the asymptotic distribution of $\hat{\beta}$.

The asymptotic variance can be estimated by

$$\hat{V} = \left(\frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{x}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2\hat{x}_i\hat{x}_i' - \hat{B}\hat{A}' - \hat{A}\hat{B}' + \hat{A}\frac{N}{N^*}\frac{1}{N^*}\sum_{j=1}^{N^*}\hat{\phi}_j\hat{\phi}_j'\hat{A}'\right)\left(\frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{x}_i\right)^{-1}, \tag{49}$$

where

$$\hat{A} = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i\hat{\beta}'\frac{\partial x_i}{\partial\gamma}(\hat{\gamma}) \tag{50}$$

and

$$\hat{B} = \frac{1}{N}\sum_{i}\hat{x}_i\hat{u}_i\hat{\phi}_i'. \tag{51}$$

In this, $\hat{\phi}_i$ is an estimate of ϕ_i.

When the influence coefficient is generated

In our application, the generated regressor is the value of the influence coefficient. This subsection provides the formula for ϕ_j and $x_i(\cdot)$ in our case.

Recall that the influence coefficient is computed from the OLS estimate of the model

$$\Delta\mathbf{s} = \beta\mathbf{G}\Delta\mathbf{s} + \gamma_I'\mathbf{Ind} + \gamma_P'\mathbf{Pre} + \epsilon. \tag{52}$$

Thus, in our setting, γ is β. Since x_i is the logarithm of the influence vector, we have $x_i(\beta) = \log((\sum_{k=0}^{\infty}(\gamma G')^k\mathbf{1})_i)$, where $(a)_i$ denotes the element of a corresponding to the ith firm.

The formula for ϕ_j and $\partial x_i/(\partial\gamma)$ can be derived easily in our setting. Let $\tilde{\mathbf{X}}$ be the residual from the regression of $\mathbf{G}\Delta\mathbf{s}$ on $\mathbf{Ind}$ and $\mathbf{Pre}$. The OLS estimator of β is

$$\hat{\beta} = \frac{\tilde{\mathbf{X}}'\Delta\mathbf{s}}{\tilde{\mathbf{X}}'\tilde{\mathbf{X}}} = \beta + \frac{\tilde{\mathbf{X}}'\epsilon}{\tilde{\mathbf{X}}'\tilde{\mathbf{X}}}. \tag{53}$$

Therefore, the formula for ϕ_j is

$$\phi_j = \left(\frac{1}{N^*}\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\tilde{\mathbf{X}}_j\epsilon_j, \tag{54}$$

where $\tilde{\mathbf{X}}_j$ is the jth element of vector $\tilde{\mathbf{X}}$. The formula for $\partial x_i/(\partial\gamma)$ can be computed directly by taking the derivative. This gives

$$\frac{\partial x_i}{\partial\gamma}(\gamma) = \frac{1}{\left(\sum_{k=0}^{\infty}(\gamma G')^k\mathbf{1}\right)_i}\left(\sum_{k=1}^{\infty}k\gamma^{k-1}(G')^k\mathbf{1}\right)_i. \tag{55}$$