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# Asymmetric information allocation to avoid coordination failure *

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**Abstract**

This study addresses optimal information allocation in team production. We present a unique implementation problem of desirable effort levels and show that, under certain conditions, it is optimal to asymmetrically inform the agents even if they are ex ante symmetric. The main intuition is that the asymmetric information allocation is effective in avoiding "bad" equilibria, that is, equilibria with coordination failure. This analysis provides an explanation as to why informing agents asymmetrically might be beneficial in improving the agents' coordination behaviors.

KEYWORDS: Moral hazard, Unique implementation, Asymmetric information

JEL Classification: D21, D23, D86

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# 1 Introduction

This paper examines an optimal information allocation among agents in an organization. In many types of organizations (such as firms, universities, and communities), agents are often asymmetrically informed. For example, certain information is often retained within a subset of managers in a company even if the information is relevant to the entire company. Managers of company-owned chain stores are typically exposed to more information through periodic meetings compared to franchisees. Explaining such phenomena requires an understanding of the key channels through which information structure affects organizational performance.

If agents engage in different tasks or have different characteristics, trivial situations would occur where an asymmetric information structure arises naturally. However, this paper establishes a novel channel between information allocation and implementation cost that implies that an asymmetric information structure among the agents is optimal, even when the agents are completely symmetric in terms of their characteristics and the tasks in which they engage. We interpret this channel as a source of "intrinsic" motivation for asymmetric information allocation in an organization.

Specifically, we consider the following team production model based on Holmstrom (1982).[1] Two agents engage in a single project, and each of the agents chooses a binary effort (high/low or work/shirk). The principal, a residual claimant, offers a bonus contract contingent on the binary outcome of the project (success/failure). The success probability of the project depends on the agents' total efforts and the binary state of the world (good/bad). We assume that the failure of the project is extremely hazardous (e.g., accidents in a nuclear power plant or loss of a brand's long-term reputation). In such a situation, we believe it is reasonable to require that the optimal contract admits a *unique* (Bayesian) equilibrium such that *every* agent chooses a desired

[1] We make simple assumptions to highlight the key insights, but we believe that similar results should hold in more general environments.

level of effort in *every* state.[2] As a direct predecessor, in the team production context but without uncertain state variables, Winter (2004) studies optimal bonus contracts that uniquely implement the desired effort choices and shows the optimality of an asymmetric bonus contract even if the agents are symmetric. This paper considers a similar model but with uncertain parameters in the production function. This introduces a novel dimension to the principal's design problem, namely, the allocation of information among the agents.[3]

The key observation is that asymmetric information allocation can significantly mitigate the agents' coordination failure. To provide a rough intuition, consider the following numerical example.

**Example 1.** Each agent earns, regardless of the state of the world, an expected payoff "3" if both agents work, "2" if a specific agent shirks but the other works, and "1" if he works but the other shirks. In the case where both agents shirk, the agent's expected payoff is "0" if the state is good, and "3" if the state is bad (see Table 1 below).[4]

| ("good") | work | shirk |
|---|---|---|
| work | $(5,5)$ | $(1,2)$ |
| shirk | $(2,1)$ | $(0,0)$ |

| ("bad") | work | shirk |
|---|---|---|
| work | $(5,5)$ | $(1,2)$ |
| shirk | $(2,1)$ | $(3,3)$ |

Each state is equally likely and, hence, if no agent is informed about the state, then "both work" and "both shirk" are equilibria, which is not desirable for the principal.[5] Informing both agents about the state is not

[2] It is a common practice in the literature to focus on the implementation of such a "high-effort" equilibrium to simplify the analysis. We consider the unique implementation of such a desired equilibrium given the concerns in the literature for multiple-equilibrium problems (e.g., Mookherjee (1984)). We discuss this in more detail in Section 2.

[3] We discuss how the asymmetry of bonus contracts and the asymmetry of information allocation interact in Section 5.1.

[4] One can interpret these numbers as the agents' expected utilities given an arbitrary bonus contract fixed.

[5] Of course, if the principal employs wishful thinking and believes that both will work,

desirable either because "both shirk" is again an equilibrium in the bad state.

Nevertheless, informing *just one agent* can eliminate this bad coordination. Specifically, suppose that only agent 2 is informed about the state, whereas agent 1 is not (but agent 1 knows that agent 2 knows the state, and so on). First, if the state is good, then it is dominant for agent 2 to work. *Given this*, it is now (iteratively) dominant for agent 1 to work, as illustrated in the following table:

| | work in both states | work only in good state |
|---|---|---|
| work | $(5,5)$ | $(3,\frac{5}{2})$ $(=(\frac{5+1}{2},\frac{5+2}{2}))$ |
| shirk | $(2,1)$ | $(\frac{5}{2},2)$ $(=(\frac{2+3}{2},\frac{1+3}{2}))$ |

Finally, given that agent 1 works (in any state), it is (iteratively) dominant for agent 2 to work even in the bad state. Therefore, the desired outcome that "both work in every state" is the unique strategy profile that survives iterative elimination of dominated strategies. □

The conclusion in this example may seem somewhat *ad hoc*. For example, a bonus contract that generates these numbers may not be the optimal contract. Nevertheless, the example illustrates an important observation that the bad coordination outcome (i.e., both shirking) becomes less sustainabale in an equilibrium by asymmetrically informing the agents.[6] The aim of

then the principal does not foresee a problem. However, in case project failure is hazardous, the principal may adopts a more pessimistic view.

[6]The problem of information disclosure by the principal is also studied in mechanism design, such as in Bergemann and Pesendorfer (2007) and Esó and Szentes (2007). Particularly, Bergemann and Pesendorfer (2007) show that asymmetrically informing bidders (agents) in auction may be optimal for the revenue-maximizing seller (principal). In their paper, more information implies that the auction outcome becomes more efficient, which itself induces more revenue through bidders' bidding behaviors, but it also implies that the bidders enjoy more information rent. Our paper shows that providing information asymmetrically can be useful in eliminating undesirable equilibria in a team production setting, and in this sense, our paper features a different aspect of providing information asymmetrically to the agents.

this paper is to show that this novel intuition holds in more general team production environments.

The paper is structured as follows. Section 2 introduces the model, and Section 3 studies the optimal bonus contract and information structure in a simple two-agent case with anonymous contracts. Sections 4 and 5 examine several extensions and generalizations of the base model in Section 3 with respect to the number of agents (Section 4.1), asymmetry in agents' efforts costs (Section 4.2), and elaborate contracts such as non-anonymous (or individual-specific) bonus contracts (Section 5.1) and stochastic information allocation (Section 5.2). Section 6 concludes the paper. Except for those in Section 3, all proofs are shown in the Appendix.

# 2 Model

We consider a team production model with one manager (a principal) and $n$ workers (agents) who engage in a project. Each worker $i \in \{1, \ldots, n\}$ simultaneously chooses an effort level $e_i \in \{0, 1\}$, which costs $ce_i$ for $c > 0$. The profit of the project is $y \in \{S, F\}$ ($S > F$), and $p_\theta(x)$ denotes the probability of success ($y = S$), which depends on the agents' total effort $x = \sum_i e_i$ and task environment $\theta \in \{H, L\}$. We assume that $p_\theta(x)$ is increasing in $x$ for any $\theta$. The probability that $\theta = H$ is $f \in (0, 1)$, and the probability that $\theta = L$ is $1 - f$. Let $p_\phi(x) = fp_H(x) + (1 - f)p_L(x)$ denote the mean success probability given $x$.

The marginal productivity of effort, denoted by $\Delta p_\theta(x) \equiv p_\theta(x) - p_\theta(x - 1)$, satisfies (i) $\Delta p_H(x) > \Delta p_L(x)$ for all $x$ and (ii) $\Delta p_\theta(x) > \Delta p_\theta(x-1)$ for all $x$ and $\theta$. The first condition requires that the marginal productivity is always higher in state $H$ than in state $L$, and the second condition requires that the agents' efforts are complementary and, therefore, the agents' collaboration is important for success.[7]

[7]This convexity in the production function is essential for the multiple-equilibrium problem, as observed by Winter (2004).

**Information structure** The principal can organize the information structure of the team. Except for Section 5.2, this simply refers to how many agents are informed of $\theta$ when they make effort choices.[8] Specifically, we compare the following three types of information structure.

- No information (NI): no agent observes $\theta$,
- Asymmetric information (AI): agent $i \in \{1, \ldots, m\}$ does not observe $\theta$, but agent $j \in \{m+1, \ldots, n\}$ observes $\theta$,
- Full information (FI): all agents observe $\theta$.

Let $s_i \in S_i = \{H, L, \phi\}$ represent agent $i$'s information about the state. More specifically, (i) if he is informed, then $s_i = \theta$ for each $\theta$, and (ii) if he is uninformed, then $s_i = \phi$ for each $\theta$. Hence, agent $i$'s strategy is to choose $e_i(s_i) \in \{0, 1\}$ for each $s_i$.

Once the principal chooses an information structure, we assume that this structure itself becomes common knowledge among the agents. We also assume that the principal commits to the information structure of the team without knowing the realization of $\theta$.[9]

**Contract** Except for Section 5.1, we consider *anonymous bonus contracts* where all the agents are paid symmetrically. Specifically, an anonymous bonus contract is represented by $b \geq 0$, where each agent is paid 0 if $y = F$,

[8] An essentially equivalent formulation is that the principal controls the cost of acquiring information on $\theta$. Specifically, the cost is set as zero for an agent whom the principal wants to inform, and it is set to infinity for an agent whom the principal does not want to inform. This assumption contrasts with the literature on information acquisition in moral hazard environments. For example, Andreoni (2006) and Kobayashi and Suehiro (2005) study models of endogenous leadership or leadership battles, where each agent chooses whether to acquire certain information on the environment to influence other agents as a leader.

[9] The results of the paper would not qualitatively change even if the bonus contract is contingent on the realized $\theta$ (i.e., $b$ becomes a function of $\theta$ rather than a scalar invariant in $\theta$), as long as the principal can commit to the information structure and the (possibly contingent) bonus contract before observing $\theta$.

and each agent is paid a bonus $b$ if $y = S$.[10] In Section 5.1, we examine how the results would change if the principal can offer different contracts to different agents.

Given bonus $b$ and the agents' effort profile $e = (e_1, \ldots, e_n) \in \{0,1\}^N$, the principal's payoff in state $\theta$ is $p_\theta(\sum_{i=1}^N e_i)(S - nb) + (1 - p_\theta(\sum_{i=1}^N e_i))F$. We assume that $S$ is much larger than $F$ so that the principal's objective is to implement the full effort strategy of the agents, that is, $(e_i(s_i))_{i,s_i}$ such that $e_i(s_i) = 1$ for every $i$ and $s_i$.

Given bonus $b$ and the agents' effort profile $e = (e_1, \ldots, e_n) \in \{0,1\}^N$, agent $i$'s payoff in state $\theta$ is $u_i(e, \theta; b) = bp_\theta(\sum_{j=1}^N e_j) - ce_i$. Thus, given the information structure, the agents' strategy profile $\mathbf{e} = (e_i(s_i))_{i,s_i}$ is a (pure-strategy Bayesian) *equilibrium* if (i) for each $i$ who is informed: for each $\theta \in \{H, L\}$ and $e'_i \in \{0,1\}$,

$$u_i(e_i(\theta), e_{-i}(s_{-i}), \theta; b) \geq u_i(e'_i, e_{-i}(s_{-i}), \theta; b),$$

and (ii) for each $i$ who is uninformed: for each $e'_i \in \{0,1\}$,

$$E[u_i(e_i(\phi), e_{-i}(s_{-i}), \theta; b)] \geq E[u_i(e'_i, e_{-i}(s_{-i}), \theta; b)].$$

**Benchmark: Full-effort strategy profile as one of the equilibria** We first derive the optimal contract that makes the full-effort strategy profile *one of the equilibria* and observe that the no-information scenario is the optimal information structure.

Under the no-information scenario, the full-effort strategy profile is an equilibrium if $bp_\phi(n) - c \geq bp_\phi(n-1)$ or, equivalently, $b \geq \frac{c}{\Delta p_\phi(n)}$. Thus, the optimal contract is $b = \frac{c}{\Delta p_\phi(n)}$. Under any other information structure, the bonus must be sufficiently high so that an informed agent works in the low

[10] Alternatively, we define an anonymous bonus contract by a pair $(b, w)$ where each agent is paid $w$ if $y = F$, and each agent is paid $b + w$ if $y = S$. When the agents are protected by limited liability so that a feasible contract must satisfy $w \geq 0$ and $b + w \geq 0$, then, as in the standard argument, we focus on bonus contracts with $w = 0$ without loss of generality to seek optimal contracts.

state, that is, $bp_L(n) - c \geq bp_L(n-1)$ or, equivalently, $b \geq \frac{c}{\Delta p_L(n)}$. Thus, the optimal contract is $b = \frac{c}{\Delta p_L(n)} > \frac{1}{\Delta p_\phi(n)}$. Therefore, the no-information scenario dominates any other information structure. Intuitively, this is because we must incentivize the agents based on the *average* state under the no-information structure, whereas under any other information structure we must incentivize informed agents for *every* state.

**Incentive-inducing contract** The contract that implements the full-effort strategy profile only as *one of the equilibria* implicitly assumes that the agents would play the best equilibrium in view of the principal, even if there are multiple equilibria given the contract. However, in case the failure of the project is extremely hazardous (e.g., accidents in a nuclear power plant, or loss of a brand's long-term reputation), the principal may not want to follow such a wishful thinking. Rather, it may be more reasonable to require that the full-effort strategy profile is a *unique* equilibrium.

Potential multiple equilibria have been an important topic in the literature.[11] According to Kreps (1990), the equilibrium played is determined by the "corporate culture."[12] Unless the principal can fully control the culture within an organization, undesirable equilibria may be selected. Given this concern, we consider the problem of making the desired equilibria a unique equilibrium, so that the desired outcome is "guaranteed", independent of the corporate culture.

There are several studies that derive the optimal mechanism that uniquely implements the desired effort choice. Ma (1988) shows that a mechanism with communication can eliminate undesirable equilibria without any additional cost if the agents' outputs are individually observed. However, in

[11] See, for example, Mookherjee (1984) for general moral hazard environments. See also Baliga and Sjostrom (1998) for the study of collusive behaviors in moral hazard environments.

[12] See Hermalin (2012) for a survey of the literature of corporate culture. In empirical studies, Cronqvist, Low, and Nilsson (2007) argue that corporate culture may be a key determinant of the long-term tendency of corporate policies and performance.

team-production models as in Holmstrom (1982), the agents' outputs are not individually observed (instead, only the aggregate output is observed). Therefore, we cannot directly apply the mechanism in Ma (1988).[13] Winter (2004) considers a team-production model without state uncertainty and derives the optimal bonus contracts that uniquely implement the full-effort strategy profile. In this sense, our approach is closest to that of Winter (2004), although we allow for state uncertainty.

Following Winter (2004), we assume that $b$ is an *incentive-inducing contract* (for the full-effort strategy profile) if (i) the full-effort strategy profile is an equilibrium given $b$, and (ii) any other strategy profile is not an equilibrium. Our goal is to identify the minimum (or the infimum) bonus that uniquely implements the full-effort strategy profile and to study how it varies among different information structures.

**Remark.** In some applications, the entity that engages in information allocation (principal) may have limited ability to design $b$. For example, consider research collaboration between two junior researchers (agents) working on a joint project, and a senior researcher (principal) engages in information allocation. In such a case, the senior researcher may not fully control $b$, the prizes for successful outcomes (such as publication or future promotion). As another example, consider multiple investors (agents) and a startup company (principal), where the startup has information concerning its potential for success. The startup may have limited ability to control $b$ through the design of financial contracts. In such cases, our results can be interpreted as providing the region of $b$ where unique implementation is possible (given each information structure). Our comparison of various information structures is completely valid if we say that one information structure is better than another if and only if the region of $b$, where unique implementation is

[13] Arya, Glover, and Hughes (1997) consider a variant of a team-production model where each agent has an option to quit, and an agent's output can be individually observed if the other agent quits. In this case, the authors obtain a mechanism that can approximate the second-best outcome.

possible in the first information structure, is larger than the region of $b$ in the second structure.

# 3 Baseline case

We first study the case where there are only two agents. In Sections 3.1, 3.2, and 3.3, we identify the cost of uniquely implementing the full-effort strategy profile under each information structure. Then, in Section 3.4, we examine the optimal information structure and perform comparative statics.

## 3.1 No information

In the no-information scenario where neither agent is informed, the optimal bonus contract that implements $\mathbf{e} = (1,1)$ as *one of the equilibria* is $b = \frac{c}{\Delta p_\phi(2)}$. However, under this contract, not only $\mathbf{e} = (1,1)$, but also other effort choices could be an equilibrium. In fact, if $i$ chooses $e_i = 0$, then it is (strictly) optimal for $j$ to choose $e_j = 0$ and, thus, $\mathbf{e} = (0,0)$ is another equilibrium given this contract.[14]

Therefore, the optimal bonus level that *uniquely* implements $\mathbf{e} = (1,1)$ must be strictly greater than $\frac{c}{\Delta p_\phi(2)}$. Specifically, for $\mathbf{e} = (0,0)$ to not be an equilibrium, we must have $b > \frac{c}{\Delta p_\phi(1)} (> \frac{c}{\Delta p_\phi(2)})$ so that an agent works even if the other agent does not work.

Now, given $b > \frac{c}{\Delta p_\phi(1)}$, a high effort is dominant for each agent and, hence, $\mathbf{e} = (0,0)$ and any other effort choice (except for $\mathbf{e} = (1,1)$) cannot be an equilibrium. Therefore, the optimal bonus level that uniquely implements $\mathbf{e} = (1,1)$ is $\frac{c}{\Delta p_\phi(1)}$.[15] Thus, we obtain the following.

[14]Note that the convexity of the success probability function, $p$, plays a key role in this argument.

[15]To be rigorous, for $\mathbf{e} = (0,0)$ not to be an equilibrium, we must set $b > \frac{c}{\Delta p_\phi(1)}$ rather than $b = \frac{c}{\Delta p_\phi(1)}$. For this openness issue, we follow Winter (2004) by defining the optimal bonus level as the infimum of those that uniquely implement the desired effort level.

**Proposition 1.** Under the no-information scenario, the optimal bonus is $b_{NI} = \frac{c}{\Delta p_\phi(1)}$.

## 3.2 Asymmetric information

We now consider the asymmetric-information scenario where only one of the agents (for example, agent 2) is informed and the other agent (agent 1) is not. The optimal bonus level is given in the following proposition.

**Proposition 2.** Under the asymmetric-information scenario, the optimal bonus is $b_{AI} = \max\left\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}, \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}\right\}$.

*Proof.* We first show the following lemma.

**Lemma 1.** Let $b$ be any bonus contract such that neither $(0,(0,0))$, $(0,(1,0))$, nor $(1,(1,0))$ are equilibria. Then, $b > \bar{b} = \max\left\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}, \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}\right\}$.

*Proof.* First, to prevent $(0,(0,0))$ from being an equilibrium, we must have either $b > \frac{c}{\Delta p_\phi(1)}$, $b > \frac{c}{\Delta p_H(1)}$, or $b > \frac{c}{\Delta p_L(1)}$. Because $\frac{c}{\Delta p_H(1)} < \frac{c}{\Delta p_\phi(1)}, \frac{c}{\Delta p_L(1)}$, we obtain $b > \frac{c}{\Delta p_H(1)}$ as its necessary condition.

Given $b > \frac{c}{\Delta p_H(1)}$, to prevent $(0,(1,0))$ from being an equilibrium, we must have either $b > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ or $b > \frac{c}{\Delta p_L(1)}$. Because $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} < \frac{c}{\Delta p_L(1)}$, we obtain $b > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ as its necessary condition.

Given $b > \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}\}$, to prevent $(1,(1,0))$ from being an equilibrium, we must have $b > \frac{c}{\Delta p_L(2)}$. □

Note that $b > \bar{b}$ is a necessary condition for uniquely implementing the full-effort profile $(1,(1,1))$. Now we show that, conversely, $(1,(1,1))$ is uniquely implemented by any $b$ such that $b > \bar{b}$. First, because $b > \frac{c}{\Delta p_H(1)}$, it is strictly dominant for the informed agent to make a high effort in state $H$. Given this, because $b > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, it is (iteratively) strictly dominant for the uninformed agent to make a high effort. Given this, because $b > \frac{c}{\Delta p_L(2)}$, it is (iteratively) strictly dominant for the informed agent to make a high effort even in state $L$. Therefore, $(1,(1,1))$ is the unique strategy profile that survives iterative elimination of strictly dominated strategies and, hence, it is a unique equilibrium. $\square$

## 3.3 Full information

Finally, we consider the case where both of the agents are informed. In this case, it is necessary to incentivize an agent to make a high effort in *any* state, even if the other agent does not work. Therefore, the bonus must be at least $\frac{c}{\Delta p_L(1)}$. Given such a bonus level, it is dominant for each agent to choose a high effort in any state. Hence, the full-effort strategy profile becomes the unique equilibrium. Thus, we obtain the following.

**Proposition 3.** Under the full-information scenario, the optimal bonus is $b_{FI} = \frac{c}{\Delta p_L(1)}$.

## 3.4 Optimal information structure

In this subsection, we compare the three information structures discussed above. Recall that $b_{NI} = \frac{c}{\Delta p_\phi(1)}$ is the bonus level that incentivizes an agent to work *in the average state* even if the other agent does not work, whereas $b_{FI} = \frac{c}{\Delta p_L(1)}$ is the bonus level that incentivizes an agent to work *in any state* even if the other agent does not work. We have $b_{NI} < b_{FI}$ and, hence, the no-information scenario is optimal compared to the full-information scenario.

Whether the no-information scenario is better than the asymmetric-information scenario, or vice versa, depends on the parameter values .

**Theorem 1.** The no-information scenario is always better than the full-information scenario. The asymmetric-information scenario is better than the no-information scenario if and only if

$$\frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \geq f.$$

*Proof.* We only prove the second statement. If $b_{AI} = \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, then the no-information scenario is never optimal because

$$f\Delta p_H(2) + (1-f)\Delta p_L(1) - \Delta p_\phi(1) \quad = \quad f[\Delta p_H(2) - \Delta p_H(1)] \geq 0.$$

Similarly, if $b_{AI} = \frac{c}{\Delta p_H(1)}$, then the no-information scenario is never optimal because

$$\Delta p_H(1) - \Delta p_\phi(1) \quad = \quad (1-f)[\Delta p_H(1) - \Delta p_L(1)] \geq 0.$$

If $b_{AI} = \frac{c}{\Delta p_L(2)}$, we have $b_{NI} \geq b_{AI}$ if and only if

$$\frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \geq f.$$

□

The proposition shows that, first, allocating information asymmetrically can be better than no information (and than full information). It also shows that no information can be better than asymmetric information *only if* $b_{AI} = \frac{c}{\Delta p_L(2)}$. If $b_{AI} \neq \frac{c}{\Delta p_L(2)}$, $b_{AI}$ is the greater of $\frac{c}{\Delta p_H(1)}$ and $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, but both are smaller than $b_{NI} = \frac{c}{\Delta p_\phi(1)}$.

In case $b_{AI} = \frac{c}{\Delta p_L(2)}$, $b_{AI}$ becomes higher than $b_{NI}$ (i.e., the no-information scenario becomes better) as (i) the "effort effect" on the production function measured by $\Delta p_L(2) - \Delta p_L(1)$ becomes smaller, and (ii) the "state effect" measured by $\Delta p_H(1) - \Delta p_L(1)$ becomes larger (see Figure 1). This is because (i) if the effort effect is smaller, which means that the complementarity of the agents' efforts in success probability is smaller, then the concern of potential coordination failure is smaller, and therefore, the benefit of asymmetric information allocation is smaller. On the other hand, (ii) if the state effect is larger, then it is costly to incentivize an informed agent in the low state, and therefore, informing no agent is likely to be better.

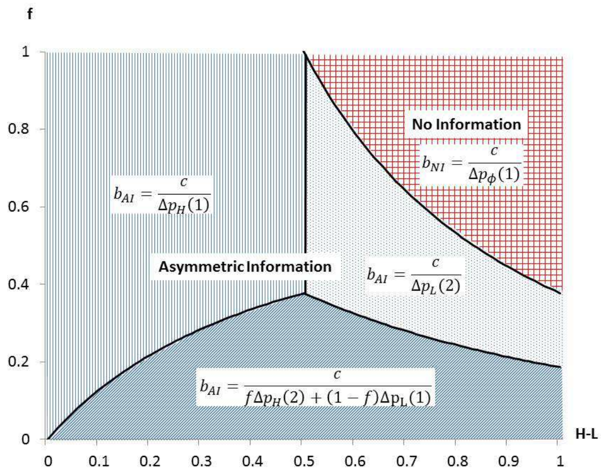


Figure 1: Optimal Information Structure
$(p_\theta(x) = \beta e^{x/2+\theta})$

# 4 Extensions

The basic model examined in the previous section is extended in two directions. In Section 4.1, we assume there are $n$ agents. The main conclusion remains similar, but we argue that with more agents, asymmetric information tends to be better than no information, in a certain sense. The main questions are: How many agents should be informed, and how does this optimal number of informed agents vary with the parameters? In Section 4.2, we consider a case where the agents have different effort costs. The analysis provides some insights for which agents should be informed under asymmetric information allocation. The analysis also shows how the optimal information structure varies with the cost difference across agents.

## 4.1 $n$-agent case

This subsection considers the model where a team consists of $n$ agents. Let $b_m$ denote the optimal bonus when $m(\leq n)$ agents are uninformed and $m-n$ agents are informed.

First, we observe that under the no-information and full-information scenarios, the optimal bonus has a similar property as in Section 3 in that the optimal bonus level is at exactly the level above which an agent is incentivized to work even if no agent works (in the average state and in every state, respectively).

**Proposition 4.** Under the no-information scenario, the optimal bonus is $b_n = \frac{c}{\Delta p_\phi(1)}$.

Similarly, under the full-information scenario, the optimal bonus is $b_0 = \frac{c}{\Delta p_L(1)}$.

Next, we provide the optimal bonus under the asymmetric-information scenario, i.e., with $1 \leq m \leq n-1$.

**Proposition 5.** Under the asymmetric-information scenario where $m$ agents are uninformed and $n-m$ agents are informed, the optimal bonus is $b_m = \max\{\frac{c}{\Delta p_L(m+1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}\}$.

Now, we consider how many agents should be informed or uninformed among $n$ agents. Let $m^* \in \{0,\dots,n\}$ denote the optimal number of uninformed agents (i.e., $b_{m^*} \leq b_m$ for any $m \neq m^*$). We provide some general properties concerning $m^*$.

**Theorem 2.** (i) $m^* \neq 0$. (ii) $b_m$ is quasi-convex in $m$ for $m = 1,\dots,n$. (iii) The no-information scenario is optimal if and only if

$$\frac{\Delta p_L(n) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \leq f.$$

The first statement shows that full information is never optimal. The second statement implies that the minimizer of $b_m$, $m^*$, smoothly varies with the parameter values. The second statement also implies that the no-information scenario (i.e., $m = 0$) is optimal if and only if $b_m$ is (globally) decreasing in $m$. Therefore, as a corollary of the second statement, we obtain the third statement, which is analogous to the condition in Theorem 1 in the base model. As the effort effect $\Delta p_L(n) - \Delta p_L(1)$ increases relative to the state effect $\Delta p_H(1) - \Delta p_L(1)$, asymmetric information becomes more advantageous. Notice that the effort-effect term $\Delta p_L(n) - \Delta p_L(1)$ is increasing in $n$, while the state-effect term $\Delta p_H(1) - \Delta p_L(1)$ is not. Therefore, adding more agents in the team (without changing the success probability function) makes asymmetric information more likely be optimal.

## 4.2 Asymmetric effort cost

In this subsection, we allow the agents to have different effort costs. The main objective of the analysis is to study which agent should be informed if the agents have different characteristics.

We consider the same model as that of the base model in Section 2, except that each agent $i$ incurs cost $c_i$ for $e_i = 1$. Assume $c_1 < c_2$, whereby agent 1 is more "productive" than agent 2. Because of this change, under the asymmetric-information scenario, we consider two cases separately: one where the productive agent 1 is informed, and the other where the unproductive agent 2 is informed.

We first consider the no-information scenario.

**Proposition 6.** Under the no-information scenario, the optimal bonus is

$$b_{NI} = \max\{\frac{c_1}{\Delta p_\phi(1)}, \frac{c_2}{\Delta p_\phi(2)}\}.$$

As long as $c_1$ and $c_2$ are close to each other, we have $b_{NI} = \frac{c_1}{\Delta p_\phi(1)}$, that is, the condition of avoiding the low effort equilibrium binds. However, when the cost difference becomes sufficiently large, the incentive condition for the non-productive agent starts to bind.

We omit the full-information case because the intuition is similar to the no-information case.

Next, we consider the asymmetric-information scenario. An interesting question is which agent (productive or unproductive) should be informed. The answer depends on the parameter values. Intuitively, it is better to inform the productive agent if the optimal bonus is determined by the informed agent's incentive conditions, and vice versa. Specifically, first assume that $c_1$ and $c_2$ are close to each other. Recall that, in the base model (with $c_1 = c_2 = c$), the optimal bonus under the asymmetric-information scenario $b_{AI}$ is the greatest of $\frac{c}{\Delta p_H(1)}$, $\frac{c}{\Delta p_L(2)}$, and $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$.

If $b_{AI} = \frac{c}{\Delta p_H(1)}$ or $\frac{c}{\Delta p_L(2)}$, this implies that the optimal bonus is determined by one of the incentive conditions of the *informed* agent. Therefore, if $c_1$ is (slightly) smaller than $c_2$, it is optimal to inform the productive agent, agent 1.

If $b_{AI} = \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, this implies that the optimal bonus is determined by the incentive condition of the *uninformed* agent. Therefore, if $c_1$ is (slightly) smaller than $c_2$, it is optimal to inform the unproductive agent, agent 2.

Therefore, we obtain the following result.

**Proposition 7.** Assume that the cost difference is sufficiently small so that $\frac{c_1}{c_2} \geq \gamma^*$, where $\gamma^*$ is the smaller of $\frac{\Delta p_\phi(1)}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ and $\frac{\Delta p_\phi(1)}{\min\{\Delta p_H(1),\Delta p_L(2)\}}$. Then, under the asymmetric-information scenario, $b_{AI}$ is the smaller of

$$b^1_{AI} = \max\{\frac{c_1}{\Delta p_H(1)}, \frac{c_1}{\Delta p_L(2)}, \frac{c_2}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}\},$$

where the productive agent 1 is informed, and

$$b^2_{AI} = \max\{\frac{c_2}{\Delta p_H(1)}, \frac{c_2}{\Delta p_L(2)}, \frac{c_1}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}\},$$

where the unproductive agent 2 is informed.

When the cost difference is sufficiently large to satisfy $\frac{c_1}{c_2} < \gamma^*$, the no-information scenario outperforms the asymmetric-information scenario (regardless of who is informed). Therefore, we do not derive the optimal bonus contract under asymmetric information in such a case.[16] The following result characterizes the optimal information structure.

**Theorem 3.** When $\frac{c_1}{c_2} < \gamma^*$, the no-information scenario is better than the asymmetric-information scenario.

When $\frac{c_1}{c_2} \geq \gamma^*$,[17] the no-information scenario is better than the asymmetric-information scenario if and only if

$$\frac{c_1}{\Delta p_\phi(1)} \geq \min\{b^1_{AI}, b^2_{AI}\}.$$

Again, when the cost difference is small, then the result is qualitatively similar to the base case (although the expressions are more complicated), whereas when the cost difference is large, then the potential advantage of asymmetric information is lost and, hence, a symmetric information structure is better.

# 5 Elaborate contracts

The previous sections only consider symmetric bonus contracts and deterministic information allocation. Although we believe that both features are reasonable restrictions in practice, it is theoretically interesting to investigate whether the main results under those restrictions still hold even if they are relaxed. The full characterization of the optimal contract and information allocation without any restriction is beyond the scope of the paper, and we leave this to future research.

[16]The intuition is roughly as follows. When the cost difference is large, the optimal bonus becomes one whereby (regardless of who is informed) the productive agent is incentivized to work in any state even if the unproductive agent does not work. In such a case, however, the no-information scenario becomes better than the asymmetric-information scenario (regardless of who is informed).

[17]Note that this implies $\frac{c_1}{\Delta p_\phi(1)} \geq \frac{c_2}{\Delta p_\phi(2)}$ and, hence, the optimal bonus under the no-information scenario is $b_{NI} = \frac{c_1}{\Delta p_\phi(1)}$.

## 5.1 Individual Bonus

In this subsection, we allow for the contracts that are not necessarily anonymous (which we refer to as *individual* bonus contracts). The main qualitative result stays the same: informing the agents asymmetrically can be optimal in minimizing the agency cost. However, the interaction of individual bonuses and information allocation adds a new dimension for the analysis. For example, we observe that asymmetric information allocation and asymmetric bonus schemes are "(imperfect) substitutes" for the principal in the sense that the set of parameters under which asymmetric information allocation is optimal becomes smaller under individual bonus contracts than under symmetric bonus contracts. The main objective of this subsection is to provide the intuition for this substitution effect.

An individual bonus contract is represented by a pair $(b_1, b_2) \in \mathbb{R}^2_+$ such that each agent $i$ is paid zero if $y = F$, and each agent is paid $b_i$ if $y = S$. We aim to characterize the optimal incentive-inducing contract by minimizing $b_1 + b_2$ among all contracts that uniquely implement the full-effort strategy profile.

If we allow for individual-specific bonus contracts, the optimal contract is typically asymmetric, as found by Winter (2004) in the context of team production without the state uncertainty. In the no-information scenario and full-information scenario in our model, the result directly applies as follows.

**Proposition 8.** Under the no-information scenario, the optimal bonus contract is

$$b_{NI} = (\frac{c}{\Delta p_\phi(2)}, \frac{c}{\Delta p_\phi(1)}) \text{ or } (\frac{c}{\Delta p_\phi(1)}, \frac{c}{\Delta p_\phi(2)}).$$

Similarly, under the full-information scenario, the optimal bonus contract is

$$b_{FI} = (\frac{c}{\Delta p_L(2)}, \frac{c}{\Delta p_L(1)}) \text{ or } (\frac{c}{\Delta p_L(1)}, \frac{c}{\Delta p_L(2)}).$$

To provide some intuition for the optimal bonus contract under the no-information scenario (and a similar intuition applies to the full-information case), consider a bonus contract that uniquely implements $\mathbf{e} = (1, 1)$. To prevent $\mathbf{e} = (0, 0)$ from being an equilibrium, such a contract must give $\frac{c}{\Delta p_\phi(1)}$ to one of the agents, for example, agent 1, so that agent 1 would work even if agent 2 does not work. As opposed to anonymous contracts, we do not need to give $\frac{c}{\Delta p_\phi(1)}$ to both of the agents. In fact, for agent 2, we only need to give $\frac{c}{\Delta p_\phi(2)}$ so that he would work if agent 1 works. Therefore, the optimal contract is as given in the proposition.

In the asymmetric-information scenario, the optimal individual-bonus contract is given as follows.

**Proposition 9.** Under the asymmetric-information scenario, the optimal bonus contract $b_{AI}$ is the one that minimizes the total bonus payment among

$$\begin{aligned}
&(\frac{c}{\Delta p_\phi(1)}, \frac{c}{\Delta p_L(2)}),\\
&(\frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}, \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}), \text{ and}\\
&(\frac{c}{\Delta p_\phi(2)}, \frac{c}{\Delta p_L(1)}).
\end{aligned}$$

Consider a bonus contract that uniquely implements $\mathbf{e} = (1, (1, 1))$. To prevent $\mathbf{e} = (0, (0, 0))$ from being an equilibrium, such a contract must give either (i) $\frac{c}{\Delta p_\phi(1)}$ to the uninformed agent or (ii) $\frac{c}{\Delta p_H(1)}$ to the informed agent.

In Case (i), working becomes dominant for the uninformed agent and, hence, we must give $\frac{c}{\Delta p_L(2)}$ to uniquely implement $\mathbf{e} = (1, (1, 1))$ because, then, the informed agent has an incentive to work in any state given that the uninformed agent works. The total bonus in such a contract is thus $\frac{c}{\Delta p_\phi(1)} + \frac{c}{\Delta p_L(2)}$.

In Case (ii), working becomes dominant for the informed agent in state $H$. To prevent $(0, (1, 0))$ from being an equilibrium, we must give either (ii-a) $\frac{c}{\Delta p_L(1)}$ to the informed agent so that working becomes dominant for the informed agent in any state, or (ii-b) $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ to the uninformed agent so that he has an incentive to work given that the informed agent works in state $H$.

In Case (ii-a), we must give $\frac{c}{\Delta p_\phi(2)}$ to uniquely implement $\mathbf{e} = (1, (1, 1))$ because, then, the uninformed agent has an incentive to work given that the informed agent works in any state. The total bonus in such a contract is, thus, $\frac{c}{\Delta p_\phi(2)} + \frac{c}{\Delta p_L(1)}$.

In Case (ii-b), we must give $\frac{c}{\Delta p_L(2)}$ to the informed agent so that the agent works in state $L$. Thus, in Case (ii-b), the bonus to the informed agent is $\max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}$ and, therefore, the total bonus in such a contract is $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} + \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}$.

Now, we compare the three information structures.

**Theorem 4.** The no-information scenario is better than the full-information scenario. The asymmetric-information scenario is better than the no-information scenario if and only if

$$\frac{1}{\Delta p_\phi(2)} + \frac{1}{\Delta p_\phi(1)} \geq \frac{1}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} + \frac{1}{\min\{\Delta p_L(2), \Delta p_H(1)\}}.$$

As in the anonymous-contract case, the no-information scenario is better than the asymmetric-information scenario when (i) the effort effect, measured by $\Delta p_\theta(2) - \Delta p_\theta(1)$, $\theta = H, L$, is sufficiently small (so that the concern for potential coordination failure is small) and (ii) the state effect, measured by $\Delta p_H(x) - \Delta p_L(x)$, $x = 1, 2$, is sufficiently large (so that the incentive cost for an informed agent is large).

However, with individual contracts, the no-information scenario is better in the opposite case as well, that is, when (i) the effort effect is sufficiently large and (ii) the state effect is sufficiently small (see Figure 2). The main reason for this difference is that, with individual contracts, we can save the implementation cost by making the bonus payment asymmetric. When the effort effect is large and the state effect is small, the amount of this cost saving is more significant under the no-information scenario than under the asymmetric-information scenario, which makes the total bonus payment smaller under the no-information scenario.

To see this more formally, recall that, with anonymous contracts, the optimal bonus level under the no-information scenario is $\frac{c}{\Delta p_\phi(1)}$, whereas under

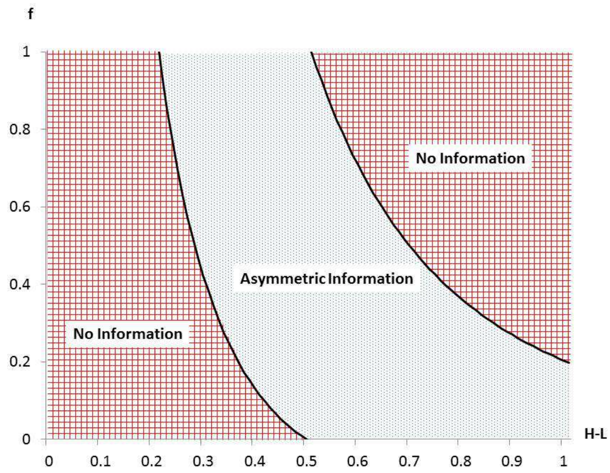


Figure 2: Optimal Information Structure
$(p_\theta(x) = \beta e^{x/2+\theta})$

the asymmetric-information scenario, the optimal bonus level is $\max\{\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L}$ Thus, the difference of the two terms converges to zero as the state effect goes to zero, that is, as $\Delta p_H(x) - \Delta p_L(x)$ goes to zero for each $x = 1, 2$. Now, if individual contracts are allowed, then, under the no-information scenario, we can save the total bonus payment compared to the anonymous-bonus case by

$$2\frac{c}{\Delta p_\phi(1)} - (\frac{c}{\Delta p_\phi(1)} + \frac{c}{\Delta p_\phi(2)}) = \frac{c}{\Delta p_\phi(1)} - \frac{c}{\Delta p_\phi(2)} > 0,$$

which does not vanish even if the state effect goes to zero (and it is larger when the effort effect is larger). On the other hand, under the asymmetric-information scenario, we can save the total bonus payment compared to the anonymous-bonus case by

$$2\max\{\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}$$
$$-\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} - \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\},$$

which converges to zero as the state effect goes to zero. Therefore, when the state effect is sufficiently small and the effort effect is sufficiently large, the no-information scenario becomes better.

## 5.2 Stochastic information allocation

In the previous sections, only "deterministic" information allocation is considered, that is, each agent is either perfectly informed of $\theta$ or not informed at all. However, at least theoretically, we can consider more general "stochastic" information allocations.

The complete characterization of the optimal stochastic information allocation is beyond the scope of this paper.[18] Instead, this subsection shows two partial characterization results. First, as soon as stochastic information allocation is allowed, the no-information scenario (in fact, any (possibly stochastic) "symmetric" information allocation) becomes suboptimal for *any* parameter values. Therefore, the optimal information allocation is necessarily asymmetric. Second, under certain parameter values, deterministic asymmetric information allocation (as in Section 3) is optimal, even if arbitrary stochastic information allocation is allowed.

In the two-agent setting considered in Section 3, a *stochastic information allocation* is denoted by $(S_1, S_2, \mu)$ where $S_i$ is an arbitrary finite set for $i = 1, 2$ and denotes the set of the signals agent $i$ receives (before choosing his effort level), and $\mu : \Theta \to \Delta(S_1 \times S_2)$ is such that, for each $\theta \in \Theta$ and $(s_1, s_2) \in S_1 \times S_2$, $\mu(s_1, s_2|\theta)$ represents the probability that each agent $i$ receives signal $s_i$ when the state is $\theta$. and $\mu$ is a joint probability distribution over $\Theta \times S_1 \times S_2$. In the most general setting, the principal can choose

[18]A potential challenge for the complete characterization is that the standard technique in the Bayesian persuasion literature (e.g., Kamenica and Gentzkow (2011)) is not directly applicable. Some results in Bayesian persuasion crucially depend on the assumption that receivers (agents) play the best equilibrium with respect to the sender (principal) if there are multiple equilibria, whereas in our paper the key driving force for optimal information allocation is the concern that the agents may play a "bad" equilibrium if there are multiple equilibria.

an arbitrary $S_i$ for each $i$ and arbitrary $\mu$ under the feasibility constraint that the marginal of $\mu$ over $\Theta$ coincides with the common prior. As special cases, the deterministic allocations considered in the previous sections are in this class. For example, the no-information scheme is identified by a singleton $S_i = \{\emptyset\}$ for every $i$, and $\mu$ coincides with the common prior. (Deterministic) asymmetric information is identified by $S_1 = \{\emptyset\}$, $S_2 = \Theta$, $\mu(\emptyset, H|H) = \mu(\emptyset, L|L) = 1$. However, in general, more complicated information allocations are allowed here.

Agent $i$'s strategy is given by $e_i : S_i \to \{0, 1\}$, and the full-effort strategy profile is $\mathbf{e} = (e_1, e_2)$ such that $e_i(s_i) = 1$ for all $i, s_i$.

Given a (symmetric) bonus contract $b \in \mathbb{R}_+$, a strategy profile $e$ is a (pure-strategy Bayesian) *equilibrium* if, for each $i$, $s_i \in S_i$ and $e_i \in \{0, 1\}$,

$$E[u_i(e_i(s_i), e_{-i}(s_{-i}), \theta; b)|s_i] \geq E[u_i(e_i, \mathbf{e}_{-i}(s_{-i}), \theta; b)|s_i],$$

where $E[\cdot|s_i]$ represents the conditional (on $s_i$) expected-value operator with respect to $s_{-i}$ and $\theta$ induced by $\mu$. We say that $b$ uniquely implements the full-effort strategy profile if the full-effort strategy profile is the unique equilibrium given $b$.

First, as soon as stochastic information allocation is allowed, any symmetric information allocation (including no information as a special case) becomes suboptimal for any parameter values. We say that $(S_1, S_2, \mu)$ is *symmetric* if (i) $S_1 = S_2$ and (ii) $s_1 \neq s_2$ implies $\mu(s_1, s_2|H) = \mu(s_1, s_2|L) = 0$. That is, the agents do not have asymmetric information in such an information allocation rule. The following result implies that the optimal information structure is necessarily asymmetric among the class of stochastic information allocation.

**Theorem 5.** There exist sn (asymmetric) stochastic information allocation that is strictly better than any symmetric information allocation.

Next, under certain parameter values, deterministic asymmetric information allocation (as in Section 3) is optimal, even if arbitrary stochastic information allocation is allowed.

**Theorem 6.** If $\Delta p_H(1) < \Delta p_L(2)$ and $\Delta p_H(1) < f\Delta p_H(2) + (1-f)p_L(1)$, then (deterministic) asymmetric information allocation is optimal.

Note that under the conditions in the statement, $b_{AI} = \frac{c}{\Delta p_H(1)}$. The conditions are satisfied if the effort effect is sufficiently large, that is, when the concern for potential coordination failure is large. In this case, we have shown in Section 3 that asymmetric information is better than no information (and full information). The result is strengthened because asymmetric information is proven to be better than any other, possibly stochastic, information allocation.

To provide some intuition, suppose, contrarily, that there exists a stochastic information allocation $(S_1, S_2, \mu)$ with a strictly smaller bonus $b < b_{AI}$ that uniquely implements the full-effort profile. As a necessary condition for this unique implementation, there must exist some agent $i$ and some signal $s_i \in S_i$ such that, even if the other agent works with probability zero, an agent still prefers to work. Therefore,

$$b(qp_H(1) + (1-q)p_L(1)) - c \geq b(qp_L(0) + (1-q)p_L(0)),$$

where $q$ is the conditional probability that the state is H given $i$ observes $s_i$. Or equivalently,

$$b \geq \frac{c}{q\Delta p_H(1) + (1-q)\Delta p_L(1)}.$$

Note that the right-hand side is decreasing in $q$ and coincides with $b_{AI}$ when $q = 1$. This contradicts $b < b_{AI}$. Therefore, we conclude that there is no stochastic information allocation that is strictly better than the deterministic asymmetric information allocation.

# 6 Conclusion

This paper considers a team-production model with state uncertainty. When the principal's goal is to uniquely implement desired effort choices, we show that, under certain conditions, asymmetrically informing the agents is the optimal information allocation. The main intuition is that by allocating information asymmetrically, it becomes less costly to avoid badly coordinated equilibria. As the degree of effort complementarity (called the effort effect in this paper) increases, asymmetric information allocation tends to improve. On the other hand, informing an agent is always costly in the sense that this agent must be incentivized even in a low state. As the state effect on the success probability function increases, asymmetric information allocation tends to worsen. Therefore, the optimal information allocation is determined by the relative magnitude of the two effects.

While we show the robustness of this main intuition in a number of extensions and generalizations, further research is necessary for a more comprehensive understanding of desirable information allocation in organizations. We believe that the analysis in this paper can serve as a useful benchmark for future research.

# A Proof of Proposition 4

Let $b$ represent any bonus contract such that $(0, \ldots, 0)$ is not an equilibrium. Then, we have $b > \frac{c}{\Delta p_\phi(1)}$. Note that this is a necessary condition for uniquely implementing the full-effort profile $(1, \ldots, 1)$.

Conversely, given any $b$ such that $b > \frac{c}{\Delta p_\phi(1)}$, it is strictly dominant for each agent to make a high effort. Therefore, the full-effort profile is the unique equilibrium.

# B Proof of Proposition 5

Let $(e_U, (e_{IH}, e_{IL}))$ represent a strategy profile such that all the uninformed agents play $e_U$, and all the informed agents play $e_{IH}$ in state $H$ and play $e_{IL}$ in state $L$. We first show the following lemma.

**Lemma 2.** Let $b$ be any bonus contract such that neither $(0,(0,0))$, $(0,(1,0))$, nor $(1,(1,0))$ are equilibria. Then, $b > \hat{b} = \max\{\frac{c}{\Delta p_L(m+1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}\}$.

*Proof.* First, to prevent $(0,(0,0))$ from being an equilibrium, we must have either $b > \frac{c}{\Delta p_\phi(1)}$, $b > \frac{c}{\Delta p_H(1)}$, or $b > \frac{c}{\Delta p_L(1)}$. Because $\frac{c}{\Delta p_H(1)} < \frac{c}{\Delta p_\phi(1)}, \frac{c}{\Delta p_L(1)}$, we obtain $b > \frac{c}{\Delta p_H(1)}$ as its necessary condition.

Given $b > \frac{c}{\Delta p_H(1)}$, to prevent $(0,(1,0))$ from being an equilibrium, we must have either $b > \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ or $b > \frac{c}{\Delta p_L(1)}$. Because $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)} < \frac{c}{\Delta p_L(1)}$, we obtain $b > \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ as its necessary condition.

Given $b > \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}\}$, to prevent $(1,(1,0))$ from being an equilibrium, we must have $b > \frac{c}{\Delta p_L(m+1)}$. □

Note that $b > \bar{b}$ is a necessary condition for uniquely implementing the full-effort profile $(1,(1,1))$. Now, we show that, conversely, $(1,(1,1))$ is uniquely implemented by any $b$ such that $b > \bar{b}$. First, because $b > \frac{c}{\Delta p_H(1)}$, it is strictly dominant for each informed agent to make a high effort in state $H$. Given this, because $b > \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$, it is (iteratively) strictly dominant for each uninformed agent to make a high effort. Given this, because $b > \frac{c}{\Delta p_L(m+1)}$, it is (iteratively) strictly dominant for each informed agent to make a high effort even in state $L$. Therefore, $(1,(1,1))$ is the unique strategy profile that survives iterative elimination of strictly dominated strategies and, hence, it is a unique equilibrium.

# C Proof of Theorem 2

The statement (i) is trivial. For the statement (ii), recall first that, for $m = 1,\dots,n-1$, $b_m = \max\{\frac{c}{\Delta p_L(m+1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}\}$. As $m$ increases, $\frac{c}{\Delta p_L(m+1)}$ decreases, $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ increases, and $\frac{c}{\Delta p_H(1)}$ stays constant. Therefore, $b_m$ is quasi-convex in $m$ for $m = 1,\dots,n-1$.

For the statement (iii), observe first that, for any $m = 1,\dots,n-1$, we have $b_n > \frac{c}{\Delta p_H(1)}$ and $b_n > \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$. Therefore, we have $b_n < b_m$ if and only if $b_n < \frac{c}{\Delta p_L(m+1)}$. Because $\frac{c}{\Delta p_L(m+1)}$ is decreasing in $m$, we have $b_n < b_m$ for all $m > 1$ if and only if $b_n < b_{n-1}$, or equivalently,

$$\frac{c}{\Delta p_\phi(1)} < \frac{c}{\Delta p_L(n)},$$

and hence we obtain the inequality as in the statement.

# D Proof of Proposition 6

First, for $\mathbf{e} = (0,0)$ not to be an equilibrium, we must have $b > \frac{c_i}{\Delta p_\phi(1)}$ at least for some $i$ so that the agent will make a high effort even if the other agent does not. Given $c_1 < c_2$, we obtain $b > \frac{c_1}{\Delta p_\phi(1)}$.

Also, for $\mathbf{e} = (1,1)$ to be an equilibrium, we must have $b \geq \max_i \frac{c_i}{\Delta p_\phi(2)}$. Given $c_1 < c_2$, we obtain $b \geq \frac{c_2}{\Delta p_\phi(2)}$. Therefore, we obtain $b > b_{NI}$ as a necessary condition for uniquely implementing $\mathbf{e} = (1,1)$.

Conversely, $b > b_{NI}$ is also sufficient for unique implementation. First, $e_1 = 1$ is strictly dominant for agent 1 because $b > \frac{c_1}{\Delta p_\phi(1)}$. Given this, $e_2 = 1$ is (iteratively) strictly dominant for agent 2 because $b > \frac{c_2}{\Delta p_\phi(2)}$.

# E Proof of Proposition 7

We only consider the case with $\frac{1}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} \geq \max\{\frac{1}{\Delta p_H(1)}, \frac{1}{\Delta p_L(2)}\}$. The other case with $\frac{1}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} < \max\{\frac{1}{\Delta p_H(1)}, \frac{1}{\Delta p_L(2)}\}$ is similar and, hence, the proof is omitted.

We have

$$b^1_{AI} = \frac{c_2}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \geq b^2_{AI}.$$

As in the base case in Section 3, if $b > b^2_{AI}$, then the full-effort strategy profile is the unique equilibrium when agent 2 is informed: first, it is strictly dominant for (informed) agent 2 to make a high effort in the high state, even if agent 1 does not work; given this, a high effort is (iteratively) strictly dominant for (uninformed) agent 1; given this, a high effort in any state is (iteratively) strictly dominant for agent 2.

Conversely, $b > b^2_{AI}$ is also necessary to uniquely implement the full-effort strategy profile when agent 2 is informed. First, to prevent $\mathbf{e} = (0,(0,0))$ from being an equilibrium, the least costly way is to incentivize (informed) agent 2 in the high state as long as $\frac{c_1}{\Delta p_\phi(1)} \geq \frac{c_2}{\Delta p_H(1)}$. This last inequality is satisfied because $\frac{c_1}{c_2} \geq \gamma^* = \max\{\frac{\Delta p_\phi(1)}{\Delta p_H(1)}, \frac{\Delta p_\phi(1)}{\Delta p_L(2)}\}$. Therefore, we must have $b > \frac{c_2}{\Delta p_H(1)}$.

Second, to prevent $\mathbf{e} = (0,(1,0))$ from being an equilibrium, the least costly way is to incentivize (uninformed) agent 1. Therefore, we must have $b > \frac{c_1}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$.

Finally, to prevent $\mathbf{e} = (1,(1,0))$ from being an equilibrium, agent 2 must be incentivized in the low state. Therefore, $b > \frac{c_2}{\Delta p_L(2)}$. These three inequalities amount to $b > b^2_{AI}$.

# F Proof of Theorem 3

The case with $\frac{c_1}{c_2} \geq \gamma^*$ is straightforward and, hence, we omit the proof. Therefore, in the following, we assume $\frac{c_1}{c_2} < \gamma^*$.

Moreover, we only consider the case with $\frac{1}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} \geq \max\{\frac{1}{\Delta p_H(1)}, \frac{1}{\Delta p_L(2)}\}$, which implies $\frac{c_1}{c_2} < \gamma^* = \max\{\frac{\Delta p_\phi(1)}{\Delta p_H(1)}, \frac{\Delta p_\phi(1)}{\Delta p_L(2)}\}$. The other case is similar and, hence, the proof for that case is omitted.

**Lemma 3.** Whenever agent 1 is informed, any bonus contract $b$ that uniquely implements the full-effort strategy profile $\mathbf{e} = ((1,1),1)$ must satisfy $b > b_{NI} = \max\{\frac{c_1}{\Delta p_\phi(1)}, \frac{c_2}{\Delta p_\phi(2)}\}$, and, therefore, the no-information scenario is better than informing agent 1.

*Proof.* (of the lemma) First, we consider the case with $\frac{c_1}{\Delta p_L(1)} \geq \frac{c_2}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$.

In this case, to prevent $\mathbf{e} = ((1,0),0)$ from being an equilibrium, the least costly way is to incentivize agent 2 by setting $b > \frac{c_2}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$. As in the base case, to prevent $\mathbf{e} = ((0,0),0)$ and $\mathbf{e} = ((1,0),1)$ from being equilibria, the least costly way is to incentivize agent 1 by setting

$b > \max\{\frac{c_1}{\Delta p_H(1)}, \frac{c_1}{\Delta p_L(2)}\}$. These inequalities imply

$$\begin{aligned} b &> b_{AI}^1 \\ &= \frac{c_2}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \\ &> \max\{\frac{c_2}{\Delta p_H(1)}, \frac{c_2}{\Delta p_L(2)}\}, \end{aligned}$$

where the equality and the last inequality is because of our current assumption that $\frac{1}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} \geq \max\{\frac{1}{\Delta p_H(1)}, \frac{1}{\Delta p_L(2)}\}$. This immediately implies $b > \frac{c_2}{\Delta p_\phi(2)}$ and, moreover, together with $\frac{c_1}{c_2} < \gamma^*$, implies $b > \frac{c_1}{\Delta p_\phi(1)}$. Therefore, we have $b > \max\{\frac{c_2}{\Delta p_\phi(2)}, \frac{c_1}{\Delta p_\phi(1)}\} = b_{NI}$.

Next, we consider the opposite case with $\frac{c_1}{\Delta p_L(1)} < \frac{c_2}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$. In this case, to prevent $\mathbf{e} = ((1,0),0)$ from being an equilibrium, the least costly way is to incentivize agent 1 in the low state by setting $b > \frac{c_1}{\Delta p_L(1)}$. To prevent $\mathbf{e} = ((1,1),0)$ from being an equilibrium, we must have $b > \frac{c_2}{\Delta p_\phi(2)}$. These two inequalities imply $b > \max\{\frac{c_2}{\Delta p_\phi(2)}, \frac{c_1}{\Delta p_\phi(1)}\} = b_{NI}$. □

**Lemma 4.** Whenever agent 2 is informed, any bonus contract $b$ that uniquely implements the full-effort strategy profile $\mathbf{e} = (1,(1,1))$ must satisfy $b > b_{NI} = \max\{\frac{c_1}{\Delta p_\phi(1)}, \frac{c_2}{\Delta p_\phi(2)}\}$, and, therefore, the no-information scenario is better than informing agent 2.

*Proof.* (of the lemma)

To prevent $\mathbf{e} = (1,(1,0))$ from being an equilibrium, we must have $b > \frac{c_2}{\Delta p_L(2)}$.

First, we consider the case with $\frac{1}{\Delta p_H(1)} < \frac{1}{\Delta p_L(2)}$ (hence, $\gamma^* = \frac{\Delta p_\phi(1)}{\Delta p_L(2)}$). In this case, $\frac{c_1}{c_2} < \gamma^*$ implies $\frac{c_2}{\Delta p_L(2)} > \frac{c_1}{\Delta p_\phi(1)}$. Therefore, $b > \frac{c_2}{\Delta p_L(2)}$ directly implies $b > \max\{\frac{c_1}{\Delta p_\phi(1)}, \frac{c_2}{\Delta p_\phi(2)}\} = b_{NI}$.

Second, we consider the case with $\frac{1}{\Delta p_H(1)} \geq \frac{1}{\Delta p_L(2)}$ (hence, $\gamma^* = \frac{\Delta p_\phi(1)}{\Delta p_H(1)}$). Because $b > \frac{c_2}{\Delta p_L(2)}$ implies $b > \frac{c_2}{\Delta p_\phi(2)}$, it suffices to show $b > \frac{c_1}{\Delta p_\phi(1)}$. To show this, note that $\frac{c_1}{c_2} < \gamma^*$ implies $\frac{c_1}{\Delta p_\phi(1)} < \frac{c_2}{\Delta p_H(1)}$. Therefore, to prevent $\mathbf{e} = (0,(0,0))$ from being an equilibrium, the least costly way is to incentivize agent 1 by setting $b > \frac{c_1}{\Delta p_\phi(1)}$, which is our desired inequality. □

# G Proof of Proposition 8

We first show the following lemma.

**Lemma 5.** Let $b$ be any bonus contract such that neither $(0,0)$, $(1,0)$, nor $(0,1)$ are equilibria. Then, either (i) $b_1 > \frac{c}{\Delta p_\phi(1)}$ and $b_2 > \frac{c}{\Delta p_\phi(2)}$, or (ii) $b_1 > \frac{c}{\Delta p_\phi(2)}$ and $b_2 > \frac{c}{\Delta p_\phi(1)}$.

*Proof.* Let $b$ be any bonus contract such that $(0,0)$ are not equilibria. Then, we must have $b_i > \frac{c}{\Delta p_\phi(1)}$ for some agent $i$. Given any $b = (b_i, b_j)$ such that $b_i > \frac{c}{\Delta p_\phi(1)}$, to prevent $\mathbf{e} = (e_i, e_j) = (1,0)$ from being an equilibrium, we must have $b_i > \frac{c}{\Delta p_\phi(1)}$ for $j \neq i$. □

Note that this is a necessary condition for uniquely implementing the full-effort profile $(1,1)$. Conversely, consider any $b$ such that $b_1 > \frac{c}{\Delta p_\phi(1)}$ and $b_2 > \frac{c}{\Delta p_\phi(2)}$. First, it is strictly dominant for agent 1 to make a high effort. Then, given this, it is (iteratively) strictly dominant for agent 2 to make a high effort. Therefore, the full-effort profile is the unique equilibrium. The case where $b$ satisfies $b_1 > \frac{c}{\Delta p_\phi(2)}$ and $b_2 > \frac{c}{\Delta p_\phi(1)}$ is analogous, and so we omit this case.

# H Proof of Proposition 9

We first show the following lemma.

**Lemma 6.** Let $b$ be any bonus contract such that neither $(0,(0,0))$, $(0,(1,0))$, $(1,(1,0))$, nor $(0,(1,1))$ are equilibria. Then, either (I) $b_1 > \frac{c}{\Delta p_\phi(1)}$ and $b_2 > \frac{c}{\Delta p_L(2)}$, (II) $b_1 > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ and $b_2 > \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}$, or (III) $b_1 > \frac{c}{\Delta p_\phi(2)}$ and $b_2 > \frac{c}{\Delta p_L(1)}$.

*Proof.* For any $b$ such that $(0,(0,0))$ is not an equilibrium, we must have either (i) $b_1 > \frac{c}{\Delta p_\phi(1)}$, or (ii) $b_2 > \frac{c}{\Delta p_H(1)}$.

First, consider any $b$ such that $b_1 > \frac{c}{\Delta p_\phi(1)}$. To prevent $(1,(1,0))$ from being an equilibrium, we must have $b_2 > \frac{c}{\Delta p_L(2)}$. Hence, we obtain Case (I).

Second, consider any $b$ such that $b_2 > \frac{c}{\Delta p_H(1)}$. To prevent $(0,(1,0))$ from being an equilibrium, we must have either (ii-a) $b_1 > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ or (ii-b) $b_2 > \frac{c}{\Delta p_L(1)}$.

Consider any $b$ such that $b_2 > \frac{c}{\Delta p_H(1)}$ and $b_1 > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$. To prevent $(1,(1,0))$ from being an equilibrium, we must have $b_2 > \frac{c}{\Delta p_L(2)}$. Hence, we obtain Case (II).

Finally, consider any $b$ such that ($b_2 > \frac{c}{\Delta p_H(1)}$ and) $b_2 > \frac{c}{\Delta p_L(1)}$. To prevent $(0,(1,1))$ from being an equilibrium, we must have $b_1 > \frac{c}{\Delta p_\phi(2)}$. Hence, we obtain Case (III). □

Note that this is a necessary condition for uniquely implementing the full-effort profile $(1,1)$.

We now show that the converse is true. First, consider any $b$ such that $b_1 > \frac{c}{\Delta p_\phi(1)}$ and $b_2 > \frac{c}{\Delta p_L(2)}$. It is strictly dominant for the uninformed agent to make a high effort. Given this, it is (iteratively) strictly dominant for the informed agent to make a high effort for any state. Therefore, $(1,(1,1))$ is a unique equilibrium.

Second, consider any $b$ such that $b_1 > \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ and $b_2 > \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\}$. Because $b_2 > \frac{c}{\Delta p_H(1)}$, it is strictly dominant for the informed agent to make a high effort in state $H$. Given this, it is (iteratively) strictly dominant for the uninformed agent to make a high effort. Given this, because $b_2 > \frac{c}{\Delta p_L(2)}$, it is (iteratively) strictly dominant for the informed agent to make a high effort in state $L$. Therefore, $(1,(1,1))$ is a unique equilibrium.

Third, consider any $b$ such that $b_1 > \frac{c}{\Delta p_\phi(2)}$ and $b_2 > \frac{c}{\Delta p_L(1)}$. It is strictly dominant for the informed agent to make a high effort in any state. Given this, it is (iteratively) strictly dominant for the uninformed agent to make a high effort. Therefore, $(1,(1,1))$ is a unique equilibrium.

# I Proof of Theorem 4

The first statement is trivial, hence, we omit its proof. To prove the second statement, first, assume

$$\frac{1}{\Delta p_\phi(2)} + \frac{1}{\Delta p_\phi(1)} \geq \frac{1}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} + \frac{1}{\min\{\Delta p_L(2), \Delta p_H(1)\}}.$$

Then,

$$\begin{aligned} b_{1,AI} + b_{2,AI} &\leq \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} + \frac{c}{\min\{\Delta p_L(2), \Delta p_H(1)\}} \\ &\leq \frac{c}{\Delta p_\phi(2)} + \frac{c}{\Delta p_\phi(1)} \\ &= b_{1,NI} + b_{2,NI}. \end{aligned}$$

Conversely, assume

$$\frac{1}{\Delta p_\phi(2)} + \frac{1}{\Delta p_\phi(1)} < \frac{1}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} + \frac{1}{\min\{\Delta p_L(2), \Delta p_H(1)\}}.$$

If $b_{AI} = (\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}, \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}\})$, then the no-information scenario is better.

If $b_{AI} = (\frac{c}{\Delta p_\phi(1)}, \frac{c}{\Delta p_L(2)})$, then the no-information scenario is better because

$$(b_{1,AI} + b_{2,AI} - (b_{1,NI} + b_{1,NI}) = \frac{f(\Delta p_H(2) - \Delta p_L(2))}{\Delta p_\phi(2)\Delta p_L(2)} \geq 0.$$

Finally, if $b_{AI} = (\frac{c}{\Delta p_\phi(2)}, \frac{c}{\Delta p_L(1)})$, then the no-information scenario is better because

$$(b_{1,AI} + b_{2,AI}) - (b_{1,NI} + b_{2,NI}) = \frac{f(\Delta p_H(1) - \Delta p_L(1))}{\Delta p_\phi(1)\Delta p_L(1)} \geq 0.$$

# J Proof of Theorem 5

The proof is composed of two steps. First, we show that the no-information scenario is better than any (possibly stochastic) symmetric information allocation. Next, we show that there is a stochastic (asymmetric) information allocation that is strictly better than the no-information scenario.

**Lemma 7.** Let $b < b_{NI} = \frac{c}{\Delta p_\phi(1)}$. Given the bonus contract $b$ and any symmetric information allocation $(S, S, \mu)$, there exists a Bayesian equilibrium $\mathbf{e} = ((e_1(s_1))_{s_1 \in S}, (e_2(s_2))_{s_2 \in S})$, $s \in S$ and $\theta$ such that (i) $e_1(s) = e_2(s) = 0$ and (ii) $\mu(s, s, \theta) > 0$ (i.e., unique implementation is not achieved).

*Proof.* Suppose not. Without loss of generality, we assume that for each $s$, there is some $\theta$ such that $\mu(s, s, \theta) > 0$. Then, for any $s \in S$, we have

$$b\frac{f\mu(s,s|H)p_H(1) + (1-f)\mu(s,s|L)p_L(1)}{f\mu(s,s|H) + (1-f)\mu(s,s|L)} - c \geq b\frac{f\mu(s,s|H)p_H(0) + (1-f)\mu(s,s|L)p_L(0)}{f\mu(s,s|H) + (1-f)\mu(s,s|L)},$$

or equivalently,

$$b(f\mu(s,s|H)\Delta p_H(1) + (1-f)\mu(s,s|L)\Delta p_L(1)) > c(f\mu(s,s|H) + (1-f)\mu(s,s|L)).$$

Summing up each side across all $s$, because $\sum_s \mu(s,s|H) = \sum_s \mu(s,s|L) = 1$, we have

$$b(f\Delta p_H(1) + (1-f)\Delta p_L(1)) > c,$$

or equivalently, $b > \frac{c}{\Delta p_H(1)} = b_{NI}$, which contradicts $b < b_{NI}$. □

**Lemma 8.** There exists a stochastic information allocation $(S_1, S_2, \mu)$ that is strictly better than the no-information scenario.

*Proof.* Recall that under the no-information scenario, the optimal bonus is $b_{NI} = \frac{c}{\Delta p_\phi(1)}$. We now construct a stochastic information allocation $(S_1, S_2, \mu)$ that achieves a strictly lower bonus level.

Let $\varepsilon \in (0, 1)$. Define $S_1 = \{\emptyset\}$, $S_2 = \{Leak, Not\}$, $\mu(\emptyset, Leak|H) = \varepsilon$, $\mu(\emptyset, Not|H) = f(1-\varepsilon)$, and $\mu(\emptyset, Not|L) = 1 - f$. The idea is that agent 2 would be informed ("leaked") with some probability $\varepsilon$ if the state is $H$.

Let $b^*$ denote the optimal bonus under this $(S_1, S_2, \mu)$. First, to prevent $\mathbf{e} = (0, (0, 0))$ from being an equilibrium, $b^*$ must be at least as high as

$$\frac{c}{\Delta p_H(1)} \; (< b_{NI}),$$

so that agent 2, given the leak, has an incentive to work even if agent 1 does not work. Next, to prevent $\mathbf{e} = (0, (1, 0))$ from being an equilibrium, $b^*$ must be at least as high as

$$\frac{c}{f\varepsilon\Delta p_H(2) + f(1-\varepsilon)\Delta p_H(1) + (1-f)\Delta p_L(2)} \ (< b_{NI}),$$

so that agent 1 has an incentive to work if agent 2 works at least when the leak occurs. Finally, to prevent $\mathbf{e} = (1, (1, 0))$ from being an equilibrium, $b^*$ must be at least as high as

$$\frac{c}{\frac{f(1-\varepsilon)}{1-f\varepsilon}\Delta p_H(2) - \frac{1-f}{1-f\varepsilon}\Delta p_L(2)},$$

so that agent 2 without the leak has an incentive to work if agent 1 definitely works. Observe that when $\varepsilon = 0$, this reduces to $\frac{c}{\Delta p_\phi(2)}$ $(< b_{NI})$. Therefore, by continuity, there exists some $\varepsilon^* > 0$ such that, for any $\varepsilon \in (0, \varepsilon^*)$,

$$\frac{c}{\frac{f(1-\varepsilon)}{1-f\varepsilon}\Delta p_H(2) - \frac{1-f}{1-f\varepsilon}\Delta p_L(2)} < b_{NI}.$$

We fix one such $\varepsilon$ in the following.

Conversely, as in the deterministic asymmetric information allocation, as long as $b^*$ is higher than any of these three terms, the full-effort strategy profile is the only equilibrium outcome (obtained by iterative elimination of dominated strategies). Therefore, we conclude $b^* < b_{NI}$. $\square$

# K Proof of Theorem 6

Suppose that under information allocation $(S_1, S_2, \mu)$, bonus $b$ uniquely implements the full-effort strategy profile. Then, because the "no effort" strategy profile $\mathbf{e}^0 = (e_1^0, e_2^0)$ (i.e., $e_i^0(s_i) = 0$ for all $i, s_i$) is not an equilibrium, there exists some $i$ and $s_i$ such that agent $i$ has an incentive to work even if the other agent works with probability zero. Hence,

$$b(hp_H(2) + (1-h)p_L(2)) - c \geq b(hp_H(1) + (1-h)p_L(1)),$$

where $h$ is agent $i$'s conditional belief on $\theta = H$ given the signal $s_i$. Because $h \leq 1$, this implies that $b \geq \frac{c}{\Delta p_H(1)}$.

Recall that under the deterministic asymmetric-information allocation, the optimal bonus level that uniquely implements the full-effort strategy profile is $b_{AI} = \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}, \frac{c}{f\Delta p_H(2)+(1-f)p_L(1)}\}$, which reduces to $b_{AI} = \frac{c}{\Delta p_H(1)}$ by the assumption in the statement. This implies that the deterministic asymmetric-information allocation is optimal among all stochastic information allocations.

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