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CAPACITY CONSTRAINT, MERGER PARADOX AND WELFARE-IMPROVING PRO-MERGER POLICY*

Baomin Dong**

School of Economics, Henan University
Henan 475000, China
baomindong@vip.163.com

Guixia Guo

Institute of International Economy, University of International Business and Economics
Beijing 100029, China
guoguixia@gmail.com

Xiaolin Qian

Faculty of Business Administration, University of Macau
Macau, China
xiaolinqian@umac.mo

Frank Yong Wang

School of International Trade and Economics, University of International Business and Economics
Beijing 100029, China
wangyong9110@gmail.com

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** Corresponding author.

Abstract

In this paper, we show that the "Merger Paradox" (Salant, Switzer and Reynolds, 1983) is mitigated when capacity constraint is considered. This is because outside firms who do not participate in a merger cannot expand their output beyond their existing capacity, and therefore, Stigler type of free riding is alleviated. When overcapacity is socially costly, it is also shown that a pro-merger fiscal policy may discourage ex ante capacity investment and hence alleviate overcapacity, if capacity building is not too costly. Furthermore, it can be shown that the optimal pro-merger subsidy is always welfare improving when it discourages capacity building.

Keywords: capacity constraint, merger paradox, overcapacity
JEL Classification Codes: C72, D24, L41

I. *Introduction*

Mergers are of significant competition policy concern. According to the UN's World Investment Report 2011, M&A rose by 36% in value in the year of 2010 over 2009. Despite voluminous anecdotal evidence and empirical importance of mergers, the incentives behind horizontal mergers are still partially understood. Salant, Switzer and Reynolds (1983) (hereafter SSR) show that in a standard Cournot setting, merger will be profitable only when more than 80% of firms take part in. This is now referred to as the "merger paradox"[1] . The main assumption is that the so-called "outsiders", i.e., those who do not take part in the merger, could expand their output immediately in response to a merger. As a result, the newly merged firm, or "insiders", is vulnerable to such free riding behavior unless sufficiently enough number of the firms merge at one time.

In fact, Stigler (1950) points out that "...the major difficulty in forming a merger is that it is more profitable to be outside than to be a participant". The SSR results have triggered a flurry of studies offering justifications for profitable mergers that are otherwise unprofitable in SSR. For instance, Deneckere and Davidson (1985) show that mergers are always beneficial for the merging firms in the Bertrand setting, although the outsiders gain more than the insiders. Perry and Porter (1985) specify a cost function in which a scarce asset (capital), owned by all separated firms, is necessary for production. They then assume that the quantity setting oligopolists behave as a Stakelberg group with respect to the competitive fringe and find that due to both structural and behavioral reasons, more mergers are profitable relative to the SSR case. Kwoka (1989) extends Perry and Porter (1985)'s behavioral analysis using simpler settings where pre- and post- merger costs are unchanged. Similar results are obtained and mergers are more likely to be profitable in more competitive environment as defined by the conjectural variation. However, the results depend heavily on the assumptions of values of conjectural variation, which lacks empirical evidence.

Daughety (1990) shows in a standard Stackelberg oligopoly model that without cost

[1] Pepall et al (1999) defined the "merger paradox" as the difficulty in constructing "a simple economic model in which there are sizable profitability gains for the firms participating in a horizontal merger that is not a merger to monopoly".

synergy, leader-generating mergers in close-to-symmetric industries with a small number of leaders can be both privately profitable and socially desirable. The intuition is straightforward since Stackelberg leaders tend to "over-produce" relative to the followers. When there is information asymmetry between the antitrust authority and the firms concerning the cost reduction synergy from mergers, Cheung (1992) shows that output-reducing mergers should be banned to prevent firms from misrepresenting their cost-savings, even though some of these mergers can be welfare-enhancing. Moreover, without cost synergy considerations, if the merging firms' combined market share exceeds 50%, other firms are unable to make further profitable mergers.

While existing literature on horizontal mergers has offered good insights from different angles, few are based on well-documented stylized facts or even anecdotal evidence. But it is noticed that production capacity constraint is robust in many industries and affects mergers from both structural and behavioral aspects. It is shown in Kreps and Scheinkman (1983) that if capacity is constrained, then the firms in the industry tend to be Cournot types. The empirical study based on United Kingdom panel data by Haskel and Martin (1994) supports this argument.

Capacity constraint is notable for its theoretical application in price competition and collusion (Davidson and Deneckere, 1990). Capacity building is a double-edged sword as ambitious capacity investment is costly at present but its future usage is unpredictable. As a consequence, capacity investment by individual firms appears to be shortsighted. Therefore, capacity acquisition has been a major reason for M&As in many industries, such as airlines, natural resource related, and some manufacturing industries.

Although capacity constraint is not a new concept, its incorporation into the modeling of merger decisions has not been fully investigated[2] . In the current paper, we study firms' incentive to merge when production capacity is introduced. We show that when the capacity is either exogenous or endogenous, the "merger paradox" disappears or becomes less paradoxical, in the sense that the minimum number of the firms to participate a profitable merger is significantly smaller than that in SSR (1983), or equivalently, some mergers that are otherwise non-profitable in SSR (1983) model are now profitable.

Thus, capacity constraint justifies mergers. When capacity is required for production, firms may not want to invest too much in capacity initially as it can be gained later through mergers. On the other hand, the output will be truncated at full capacity if they are not involved in a merger, and are thus unable to free ride on the merged firms.

We then apply our model to study the overcapacity problem in China. In several industries in China such as steel, aluminum, cement, chemical, oil refining, and equipment manufacturing, massive overcapacity has been developed in the recent years[3]. Firms build up excess capacity for production that exceeds the actual market demand. The rise of overcapacity problem in China may be due to economic cyclical fluctuations, information coordination failure, or maybe government policy oriented. Cyclical fluctuations in macroeconomic conditions may lead to

[2] There are empirical findings that mergers occur in declining industries as a device to rationalize capacity building (Dutz, 1989).

[3] European Union Chamber of Commerce in China (EUCCC) 2009, Overcapacity in China: causes impacts and recommendations. available at
http://www.rolandberger.com/media/publications/2009-12-01-rbsc-pub-Overcapacity_in_China.html.

excess supply cyclically, because production is not as flexibly adjustable as market demand. Some Chinese economists point out that another reason for overcapacity problem is that firms generally have common knowledge on the next prosperous new industry (due to favorable industrial policies, or mature technologies, etc.), and thus herd into the same industry but do not have enough information about how many firms are already in (Lin, 2007; Lin, et al, 2010). Other studies stress the role of local government policy in exacerbating this problem. Since Chinese local government officials are generally evaluated and promoted according to their local economic performance (like local GDP), they have an incentive to provide subsidies for a particular industry that contributes to local GDP directly or indirectly. This may lead to overcapacity in this industry (Geng, et al, 2011). No matter for what reason, overcapacity problem results in inefficient capacity utilization and high stock expenditure[4]. Therefore, concerns over the low profitability and even trade tensions due to overcapacity have been placed on the top agenda of the Chinese government recently. One of the measures that the Ministry of Industry and Information Technology (MIIT) released to encourage controlling and eliminating redundant capacity, is the tax reduction for firms who conduct mergers and acquisitions[5]. While overcapacity is not uncommon worldwide, using pro-merger policies to curb overcapacity is innovative.

We apply our basic model to study the impact of a pro-merger policy on the incentives of *ex ante* capacity investment as well as social welfare. It is found that a pro-merger policy can mitigate *ex ante* overcapacity and may be welfare improving. Intuitively, a tax break or subsidy on the merging parties may constrain firms' *ex ante* incentives on capacity investment, because they expect more mergers to occur, and thus outsiders will bear too much redundant capacity if they do not initially adjust capacity downward. On the other hand, knowing that the government will bailout, firms may want to enlarge capacity initially (relative to the *laissez faire* policy) because savings from the tax reduction will partially or fully cover the capacity building cost for insiders. Therefore, a well-designed pro-merger policy mitigates overcapacity from a social welfare perspective.

The rest of the paper is organized as follows. Section II explains the "merger paradox" in a model with exogenous capacity constraint. Section III endogenizes firms' capacity investment decisions. Section IV investigates the welfare effect of the pro-merger policy and its role in curbing overcapacity, and Section V concludes.

II. *Exogenous Capacity Constraint and the "Merger Paradox"*

In SSR (1983), firms can immediately adjust their capacity to realize the long-run equilibrium output once a merger occurs. However, as is evidenced by empirical papers, say, Kim and Lee (2001), that in reality, capital and labor, as two fundamental production factors, cannot be fully adjusted to the long-run equilibrium level instantly, so that firms generally face a capacity constraint. In this section, we examine whether the "merger paradox" can be at least partially resolved with exogenously given capacity constraint.

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[4] IMF Staffs, 2012. IMF Country Report No. 12/195 People's Republic of China, 2012 Article IV Consultation. Available at http://www.imf.org/external/pubs/ft/scr/2012/cr12195.pdf

[5] The plan calls upon the top ten steel producers to increase their capacity by means of mergers and acquisitions.

Consider an industry with n identical Cournot oligopolistic firms producing a homogenous product. Firms have the same production capacity of $\bar{q}\geq 0$ before merger, which is assumed to be exogenous for the moment. Suppose the marginal production cost is $c>0$ if $q_i\leq\bar{q}$, but is infinitely high for additional units produced beyond its present capacity. For this reason, $\bar{q}$ is the "capacity constraint". Therefore, firm i's production cost function is,

$$C_i(q_i)=\begin{cases} cq_i, \ if \ q_i\leq\bar{q} \\ \infty, \ if \ q_i>\bar{q} \end{cases}, \ i=1,...,n. \tag{1}$$

Firms face a linear inverse demand function, i.e.,

$$p=a-b\sum_{i=1}^{n}q_i, \ (a>c, \ b>0), \tag{2}$$

where p is the market price, and q_i is the output of firm i.

The best response functions of firm i when $q_i\leq\bar{q}$, is readily available from a standard Cournot model, i.e.,

$$q_i=\frac{a-c}{2b}-\frac{1}{2}\sum_{j=1,\, j\neq i}^{n}q_j, \ i=1,...,n, \tag{3}$$

from which it can be noticed that if there is no capacity constraint, in equilibrium firm i expands its output in response to other firms' reduction in output. More generally, when the aggregate output of insiders decreases, the aggregate output of outsiders increases. Accordingly, SSR (1983) attributes the "merger paradox" to the phenomenon that insiders may be hurt by such an increase in the aggregate output of outsiders, unless the number of insiders is sufficiently large (so that free riders are sufficiently few). In what follows, we will show that if outsiders face capacity constraints and insiders can successfully gain capacity through merger, outsiders cannot expand their output to such a degree that insiders find merger non-profitable. Thus mergers may be profitable even when a smaller number of firms merge.

Suppose now $m\in[1,n]$ firms merge, so that there are m insiders, and all firms stay independent if $m=1$. The merged firm behaves as a multi-plant Cournot firm but with production capacity $m\bar{q}$, competing with the remaining $(n-m)$ independent firms. Denote by $\Pi^{NC}(n,m)$ and $\Pi^{C}(n,m)$ the aggregate profit of the m merged firms in an industry with n firms before and after merger respectively, and q_c the output of the merged firm. Therefore, a merger is profitable if and only if

$$\Pi^{C}(n,m)\geq\Pi^{NC}(n,m). \tag{4}$$

Clearly, whether a merger is profitable depends on the values of $\bar{q}$ and m, and firms' incentives to merge are summarized in the following proposition[6]. For the ease of exposition, the proof of this proposition is detailed in text, and that of all the remaining lemmas and

[6] It is usually the case where the firms outside the merger (or outsiders) free ride a merger by making a larger equilibrium profit than that of the firms inside the merger (or insiders). However, to maintain the consistency between our research and the merger paradox literature, throughout the paper, unless otherwise specified, the term "profitable" refers to the case where an insider earns a larger profit relative to the case with no merger.

propositions are relegated to the Appendix.

Proposition1 Incentive to merge. *An m-firm merger is profitable if (1) $m > m^*$; or (2) $m \leq m^*$ and $\bar{q} \leq \bar{q}^*$, where*

$$m^* \equiv \frac{2n+3-\sqrt{4n+5}}{2},\ \bar{q}^* \equiv \frac{(n+1-2\sqrt{m})(a-c)}{(n+1)(n-m)b}. \tag{5}$$

Proof: If firms are not capacity constrained, the standard Cournot model predicts that the equilibrium output of firm i before and after merger is $\frac{a-c}{(n+1)b}$ and $\frac{a-c}{(n-m+2)b}$, respectively. We examine profitable mergers in four cases.

Case 1: $\bar{q} \geq \frac{a-c}{(n-m+2)b}$, so that no firm is capacity constrained before and after merger. This reduces to the SSR (1983) case. Easily,

$$\Pi^{NC}(n,m) = \frac{m(a-c)^2}{b(n+1)^2},\ \Pi^{C}(n,m) = \frac{(a-c)^2}{b(n-m+2)^2} \tag{6}$$

and the m-firm merger is profitable if and only if

$$n \geq m \geq \frac{2n+3-\sqrt{4n+5}}{2} \equiv m^* \tag{7}$$

where m^*/n is at least 80%, degenerating to SSR (1983)'s result.

Case 2: $0 \leq \bar{q} < \frac{a-c}{(n+m)b}$, so that all firms are capacity constrained before and after merger. This condition is obtained as follows.

If $\bar{q} < \frac{a-c}{(n+1)b}$, then firms are capacity constrained before merger. Since the equilibrium output of an individual firm increases when the number of firms in the industry decreases due to merger, outsiders are still capacity constrained after merger. Hence, they can only produce $\bar{q}$. Then the merged firm chooses its output by maximizing its aggregate profit, and thus

$$q_c = \frac{a-c-b(n-m)\bar{q}}{2b} \tag{8}$$

which exceeds its total capacity $m\bar{q}$ if and only if $\bar{q} < \frac{a-c}{(n+m)b} < \frac{a-c}{(n+1)b}$. Therefore, if $\bar{q} < \frac{a-c}{(n+m)b}$, the merged firm can only supply $m\bar{q}$ to the market. Obviously, the m merged firms are indifferent whether merged or not:

$$\Pi^{C}(n,m) = \Pi^{NC}(n,m) = m(a-c-nb\bar{q})\bar{q} > 0 \tag{9}$$

Case 3: $\frac{a-c}{(n+m)b} \leq \bar{q} < \frac{a-c}{(n+1)b}$, so that all firms are capacity constrained before merger, and only insiders are not constrained after merger. In this case, outsiders produce $\bar{q}$, and the aggregate production of the merged firms is q_c as defined in (8), with $q_c \leq m\bar{q}$. Therefore,

$$\Pi^{NC}(n,m)=m(a-c-nb\bar{q})\bar{q},\ \Pi^{C}(n,m)=\frac{[a-c-b(n-m)\bar{q}]^2}{4b} \tag{10}$$

and the m-firm merger is profitable if and only if

$$[a-c-b(n+m)\bar{q}]^2 \geq 0 \tag{11}$$

which always holds. Therefore, mergers are always profitable when $\frac{a-c}{(n+m)b} \leq \bar{q} < \frac{a-c}{(n+1)b}$.

Case 4: $\frac{a-c}{(n+1)b} \leq \bar{q} < \frac{a-c}{(n-m+2)b}$, so that all firms are not capacity constrained before merger, and only outsiders are capacity constrained after merger. In this case,

$$\Pi^{NC}(n,m)=\frac{m(a-c)^2}{b(n+1)^2},\ \Pi^{C}(n,m)=\frac{[a-c-b(n-m)\bar{q}]^2}{4b} \tag{12}$$

and the m-firm merger is profitable if and only if

$$\bar{q} \leq \frac{(n+1-2\sqrt{m})(a-c)}{(n+1)(n-m)b} \equiv \bar{q}^* \text{ or } \bar{q} \geq \frac{(n+1+2\sqrt{m})(a-c)}{(n+1)(n-m)b} \tag{13}$$

Because $\frac{(n+1+2\sqrt{m})(a-c)}{(n+1)(n-m)b} > \frac{a-c}{(n-m+2)b}$, it is impossible for $\bar{q} \geq \frac{(n+1+2\sqrt{m})(a-c)}{(n+1)(n-m)b}$ to hold. What remains is to compare $\bar{q}^*$ and $\frac{a-c}{(n-m+2)b}$. With some algebra, it is found that when $m > m^*$, mergers are always profitable if $\frac{a-c}{(n+1)b} \leq \bar{q} < \frac{a-c}{(n-m+2)b}$; when $m \leq m^*$, mergers are profitable if $\frac{a-c}{(n+1)b} \leq \bar{q} \leq \bar{q}^*$, and nonprofitable if $\bar{q}^* < \bar{q} < \frac{a-c}{(n-m+2)b}$.

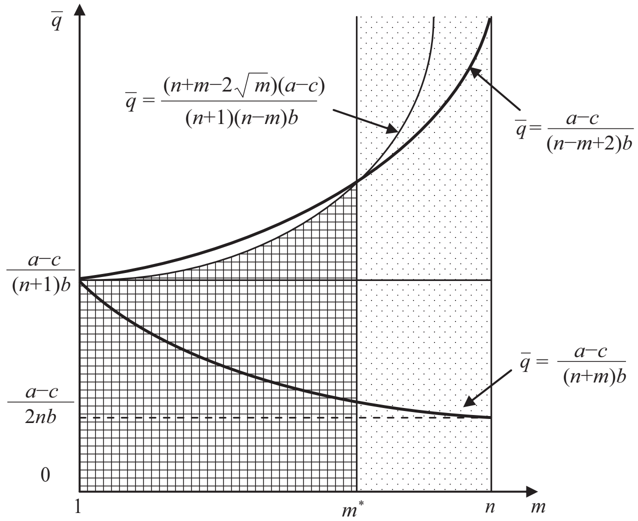
Summarizing these four cases, an m-firm merger is non-profitable only when $m < m^*$ and $\bar{q} > \bar{q}^*$. ■

The result of Proposition 1 is heuristically depicted in Figure 1[7]. In Figure 1, the blank area represents non-profitable mergers, and all the remaining areas denote profitable mergers. Notice that profitable mergers fall into two areas: (1) the dotted area of $m > m^*$, which is precisely the conventional SSR (1983)'s finding; and (2) the gridded area of $m \leq m^*$ and $\bar{q} \leq \bar{q}^*$, which becomes profitable once capacity constraint is introduced.

From Proposition 1 and Figure 1, the "merger paradox" of SSR (1983) is partially resolved when capacity constraint is considered: a merger is profitable not only when sufficiently more firms take part in, but also when the capacity constraint is sufficiently stringent. The intuition is that in Cournot competition among firms that are not capacity constrained, the profitability of any given merger depends on the interaction between two opposite forces: the merging firms internalize the competition amongst themselves, which benefits them, and the non-merging

[7] The figure is heuristic because m and n are integers rather than continuous.

FIGURE 1. INCENTIVE TO MERGE WITH EXOGENOUS CAPACITY CONSTRAINT



firms free ride on the reduced competition by competing more aggressively, which hurts the merged firm. However, once the firms are capacity constrained, merger constitutes an additional competitive advantage of insiders by relaxing their capacity constraint (Case 3), and/or rendering outsiders' capacity constrained (Case 4). Therefore, capacity constraint helps to mitigate the outsiders' incentive of free riding.

Proposition 2 *The threshold merger size* m^* *increases in* n; *the threshold capacity* $\bar{q}^*$ *increases in the merge size* m, *but decreases in* n.

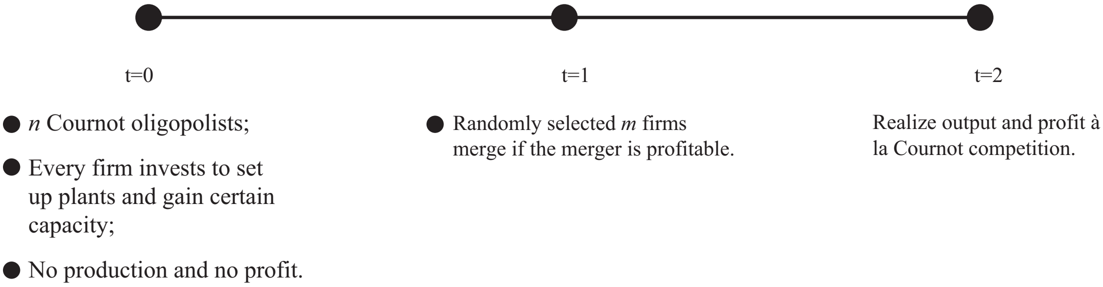
Intuitively, merger is more profitable if more firms merge or if the capacity constraint is more restrictive. For this reason, as n increases, m^* and $\bar{q}^*$ move in opposite directions: merger size should be larger, and capacity constraint should be more stringent. Therefore, merger size and capacity constraint can be considered as substitutes in increasing merger profitability. That is why capacity constraint can be less stringent as long as merger size increases.

III. *Endogenizing the Capacity Investment Decision*

In this section, we extend Section II by endogenizing firms' capacity decision and examine whether the "merger paradox" can still be partially resolved in this case. The timeline is shown in Figure 2.

At date 0, firms build up capacity at unit cost $r>0$ for production at date 1, and denote firm i's built capacity and output by $\bar{q}_{i0}$ and q_{i2} $(i=1,2,...,n)$, respectively. Since capacity investment is costly, it is never desirable for a firm to expand capacity larger than the expected

FIGURE 2. TIMELINE



output level. Suppose merger occurs at date 1, when randomly selected m firms merge[8]. Then at date 2, the firms produce with their built capacity and compete *à la* Cournot.

Denote by $\pi^I_{i2}(\bar{q}_{i0},\bar{q}_{-i0})$ and $\pi^O_{i2}(\bar{q}_{i0},\bar{q}_{-i0})$ the profit of firm i as an insider or an outsider, respectively. We now study the endogenous capacity investment decision.

At $t=0$, firm i maximizes its expected profit at $t=2$ to determine $\bar{q}^*_{i0}$,

$$\max_{\bar{q}_{i0}} \mathrm{E}[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\frac{m}{n}\cdot\pi^I_{i2}(\bar{q}_{i0},\bar{q}_{-i0})+(1-\frac{m}{n})\cdot\pi^O_{i2}(\bar{q}_{i0},\bar{q}_{-i0})-r\bar{q}_{i0} \tag{14}$$

from which we solve for the equilibrium capacity. Before we move on to Lemma 1, it is necessary to define three possible scenarios, which will be used in its proof (in the appendix) and throughout this paper:

- Scenario 1: Firm i is not capacity constrained after merger at $t=1$, no matter as an insider or an outsider.
- Scenario 2: Firm i is not capacity constrained after merger at $t=1$ as an insider, but is constrained as an outsider.
- Scenario 3: Firm i is capacity constrained after merger at $t=1$, no matter as an insider or as an outsider.

Having analyzed the three scenarios in sequence, the equilibrium capacity investment of an individual firm is summarized in Lemma 1 and then illustrated in Figure 3.

Lemma 1 *Firm i's equilibrium capacity investment at $t=0$ is:*

- If $m>\frac{1+\sqrt{1+8n}}{4}$,

[8] We have to point out that the analysis in Sections II and III basically follows the same model. In Section II, we only consider the profit of an m-firm merger, but here in Section III, we have to study the profit of an individual firm in order to find its optimal capacity investment. Therefore, here we assume that all firms are ex-ante homogenous and have the same probability of being an insider. In this sense, the merger decision of an individual firm is exogenous, and only the capacity decision is endogenous. Thanks to an anonymous referee for pointing this out.

$$\bar{q}_{i0}^{*}=\begin{cases}\dfrac{a-c}{b(n-m+2)}, & \text{if } r\leq\dfrac{(n-m)(a-c)}{2n(n-m+2)}\\ \dfrac{(n-m)(a-c)-2nr}{b(n-m)(n-m+1)}, & \text{if } \dfrac{(n-m)(a-c)}{2n(n-m+2)}<r\leq\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}\\ \dfrac{a-c}{(n+m)b}, & \text{if } \dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}<r\leq\dfrac{(m-1)(a-c)}{n+m}\\ \dfrac{a-c-r}{b(n+1)}, & \text{if } \dfrac{(m-1)(a-c)}{n+m}<r\leq a-c\\ 0, & \text{if } r>a-c\end{cases} \tag{15}$$

- If $\dfrac{n(n+3)-\sqrt{n^4+4n^2+4n}}{2n-1}<m\leq\dfrac{1+\sqrt{1+8n}}{4}$,

$$\bar{q}_{i0}^{*}=\begin{cases}\dfrac{a-c}{b(n-m+2)}, & \text{if } r\leq\dfrac{(n-m)(a-c)}{2n(n-m+2)}\\ \dfrac{(n-m)(a-c)-2nr}{b(n-m)(n-m+1)}, & \text{if } \dfrac{(n-m)(a-c)}{2n(n-m+2)}<r\leq\dfrac{(m-1)(a-c)}{n+m}\\ \dfrac{a-c-r}{b(n+1)}, & \text{if } \dfrac{(m-1)(a-c)}{n+m}<r\leq a-c\\ 0, & \text{if } r>a-c\end{cases} \tag{16}$$

- If $0<m\leq\dfrac{n(n+3)-\sqrt{n^4+4n^2+4n}}{2n-1}$,

$$\bar{q}_{i0}^{*}=\begin{cases}\dfrac{a-c}{b(n-m+2)}, & \text{if } r\leq\dfrac{(m-1)(a-c)}{n+m}\\ \dfrac{a-c-r}{b(n+1)}, & \text{if } \dfrac{(m-1)(a-c)}{n+m}<r\leq a-c\\ 0, & \text{if } r>a-c.\end{cases} \tag{17}$$

It is clear from Figure 3 that the equilibrium capacity investment weakly decreases in the unit investment cost r.

Figure 4 depicts the incentives for firms to build up capacity at $t=0$. In the dotted area of Figure 4, firms have the highest incentive to build up capacity, i.e. $\bar{q}_{i0}^{*}=\dfrac{a-c}{b(n-m+2)}$, due to the relatively low investment cost. Thus in this area, no firm is capacity constrained after merger. In the gridded areas (both loosely and densely gridded areas), investment cost is a little bit higher, and in this case, initially firms invest less and prefer to be capacity constrained as an outsider. This is the area where merger acts as a vehicle for insiders to eliminate the capacity constraint they face as an independent firm. If the investment cost further increases into the blank area, firms have the lowest incentive to invest in capacity, so that all firms would rather be capacity constrained after merger. These observations are summarized in Proposition 3.

FIGURE 3. OPTIMAL CAPACITY INVESTMENT

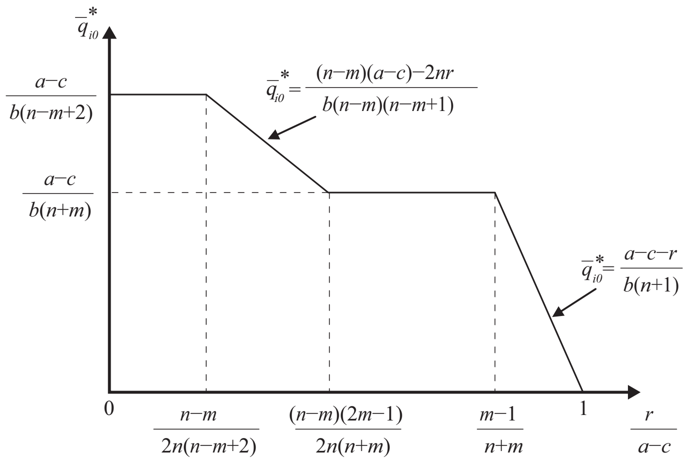
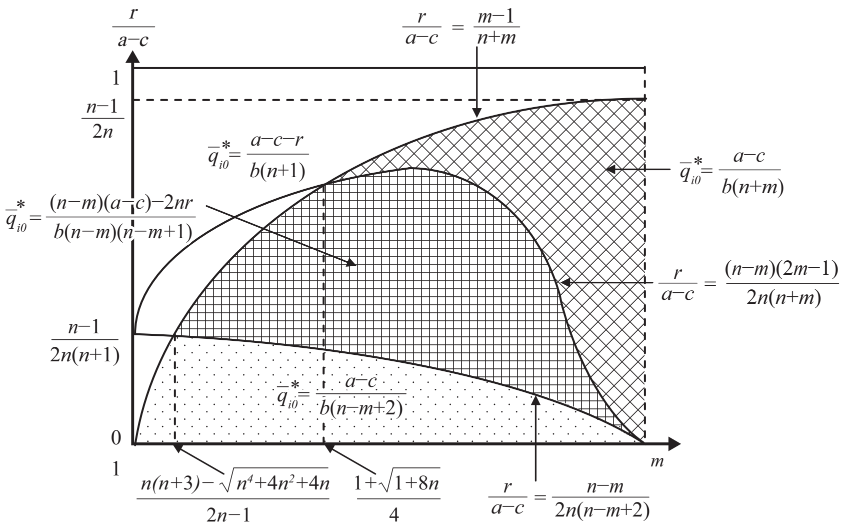


FIGURE 4. INCENTIVE OF CAPACITY INVESTMENT



Proposition 3 *Merger functions as a vehicle of eliminating inside firm's capacity constraint when*

$$m \geq \frac{n(n+3)-\sqrt{n^4+4n^2+4n}}{2n-1}, \textit{ and } \frac{(n-m)(a-c)}{2n(n-m+2)} < r \leq \frac{(m-1)(a-c)}{n+m}. \tag{18}$$

Intuitively, on the one hand, the possibility of being capacity constrained after merger enhances the attractiveness of merger; and on the other hand, firms still have an incentive to free ride the reduction of competition due to merger. The positive effect increases in the merger size, since firms contribute their individual capacity to the merged firm. But the negative effect also increases in the merger size, and is undermined by the fact that the probability of being an insider rises when the merger size becomes larger. Therefore, merger effectively relaxes individual firm's capacity constraint only when the merger size is sufficiently large, and certainly, only when the investment cost is not too high.

The analytical solution to the critical merger size m^* for the merger to be profitable is very complex in the general case. To gain more intuition, we construct a numerical example and compare with SSR (1983) in the appendix after proofs of all the lemmas and propositions. The numerical example illustrates that with fairly moderate parameter values, the equilibrium merger size can be often as low as two, in contrast to 80% in SSR (1983). Therefore, for the endogenous capacity choice case, the minimum proportion of merging firms that necessitates a profitable merger is significantly smaller than SSR (1983)'s result. Even when the capacity investment decision is endogenized, the "merger paradox" can still be significantly resolved.

IV. *Pro-Merger Policy and the Overcapacity Problem*

As mentioned in the Introduction, pro-merger policies have been adopted by China to curb the overcapacity problem in recent years, by means of reductions in value added taxes, etc. The rationale for such policies is not straightforward. In particular, there are two countervailing effects. On the one hand, a pro-merger policy may discourage firms from *ex ante* capacity buildup, because they expect more mergers to occur, and should correspondingly set their capacity smaller to avoid additional redundant capacity if they become outsiders. On the other hand, a subsidy to mergers may encourage capacity buildup since such tax reduction or subsidy may make an otherwise non-profitable merger profitable, thus firms may want to enlarge capacity initially (relative to the *laissez faire* policy). Since both overcapacity and fiscal expenditure are socially costly, one needs to weigh the pros and cons. In this section, we examine the impact of the pro-merger policy on overcapacity and its welfare consequences.

Suppose the regulator offers the merged firm a subsidy $s \in (0,1)$ for each unit of output, and it is publicly announced before firms make capacity decisions[9]. With such a pro-merger policy, we will show that the regulator expects firms to have a higher incentive to merge and a lower incentive to build up capacity. To rule out uninteresting cases where too much subsidy reverses the market mechanism, assume $s \leq \bar{s} \equiv \frac{(m-1)(a-c)}{n+1}$[10].

[9] This can be equivalently interpreted as the policy that charges a unit tax $s \in (0,1)$ for each outsider, while the merged firm can be offered tax exemption. A unit tax, proportional to firm's output, is somewhat not as usual as corporate tax, which is proportional to firm's gross revenue. However, it has been shown in Wang and Zhao (2009) that a unit tax and an ad valorem tax are welfare equivalent for non-differentiated oligopolists, as in our model. For computational convenience and also to keep our model specifications and results of previous sections unchanged, we use unit subsidy rather than unit tax or ad valorem tax in this section.

[10] The equilibrium outputs of the merged firm and the outsiders are $\frac{a-c+s}{(n+m)b}$ and $\frac{a-c-s}{b(n-m+2)}$, respectively. A merger that renders outsiders capacity constrained but insiders not requires $\frac{a-c+s}{(n+m)b} < \frac{a-c-s}{b(n-m+2)}$, from which the condition is obtained.

Lemma 2 *With the pro-merger policy, firm i's equilibrium capacity investment at* $t=0$ *is:*

- *If* $m>\dfrac{1+\sqrt{1+8n}}{4}$,

$$\bar{q}_{i0}^*=\begin{cases}\dfrac{a-c-s}{b(n-m+2)}, & \text{if } r\le\dfrac{(n-m)(a-c-s)}{2n(n-m+2)}\\ \dfrac{(n-m)(a-c-s)-2nr}{b(n-m)(n-m+1)}, & \text{if } \dfrac{(n-m)(a-c-s)}{2n(n-m+2)}<r\le\dfrac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}\\ \dfrac{a-c+s}{(n+m)b}, & \text{if } \dfrac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}<r\le\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\\ \dfrac{n(a-c-r)+ms}{bn(n+1)}, & \text{if } \dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\le\dfrac{n(a-c)+ms}{n}\\ 0 & \text{if } r>\dfrac{n(a-c)+ms}{n}\end{cases} \tag{19}$$

- *If* $\widehat{m}<m\le\dfrac{1+\sqrt{1+8n}}{4}$,

$$\bar{q}_{i0}^*=\begin{cases}\dfrac{a-c-s}{b(n-m+2)}, & \text{if } r\le\dfrac{(n-m)(a-c-s)}{2n(n-m+2)}\\ \dfrac{(n-m)(a-c-s)-2nr}{b(n-m)(n-m+1)}, & \text{if } \dfrac{(n-m)(a-c-s)}{2n(n-m+2)}<r\le\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\\ \dfrac{n(a-c-r)+ms}{bn(n+1)}, & \text{if } \dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\le\dfrac{n(a-c)+ms}{n}\\ 0 & \text{if } r>\dfrac{n(a-c)+ms}{n}\end{cases} \tag{20}$$

- *If* $0<m\le\widehat{m}$,

$$\bar{q}_{i0}^*=\begin{cases}\dfrac{a-c-s}{b(n-m+2)}, & \text{if } r\le\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\\ \dfrac{n(a-c-r)+ms}{bn(n+1)}, & \text{if } \dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\le\dfrac{n(a-c)+ms}{n}\\ 0 & \text{if } r>\dfrac{n(a-c)+ms}{n}\end{cases} \tag{21}$$

where $\widehat{m}$ *is implicitly determined by*

$$\frac{(n-m)(a-c-s)}{2n(n-m+2)}=\frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}. \tag{22}$$

Based on the optimal capacity investment decision, we examine firms' incentive to build up capacity. The following proposition can be observed from the expressions of $\bar{q}_{i0}^*$, and thus its proof is omitted.

Proposition 4 *With the pro-merger policy, firms have a lower incentive to invest in capacity if and only if*

$$r \leq \min\left\{\frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}, \frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\right\}. \quad (23)$$

Intuitively, when capacity investment is not very costly (i.e., (23) holds), firms prefer mergers to relax the capacity constraint since it is not too costly for insiders not to be capacity constrained. Therefore, the policy to encourage merger reduces firms' *ex ante* incentive of capacity buildup. But when the capacity investment is so costly that even the capacity constraints of insiders are also binding, the pro-merger subsidy instead encourages capacity buildup, because under this policy, firms' equilibrium output increases and the output is just equal to its capacity.

More importantly, from the proof of Lemma 2, it can be found that (23) is always satisfied in the case when neither insiders nor outsiders are capacity constrained after merger. Therefore, in this case, the pro-merger subsidy encourages merger, while it reduces firms' incentive to build up capacity. This is because when the merged firm is given a per unit favorable treatment, it will behave like a more efficient firm by expanding output more than other firms do. In an asymmetric Cournot game, it is well known that less efficient firms reduce their output relative to the symmetric case. Hence, as an optimal response, firms will build up less capacity *ex ante* if such a pro-merger treatment is provided. This resembles much of the government's efforts in curbing overcapacity phenomenon in several industries in China. Although the immediate consequence of a pro-merger policy increases the degree of industry concentration, it alleviates firm's *ex ante* incentive to invest in excessive capacity.

Next, we examine whether such a policy is socially desirable. To study the welfare effect of the pro-merger subsidy, define the social welfare W as follows:

$$W \equiv n\cdot \mathrm{E}[\pi_{i2}(\bar{q}^*_{i0}, \bar{q}^*_{-i0})] + CS_2 - sq^*_{c2} \quad (24)$$

where CS_2 is the consumer surplus at $t=2$, q^*_{c2} is the output of the merged firm, and sq^*_{c2} is the total subsidy the government pays. With the linear demand function, consumer surplus can be readily written as

$$CS_2 = \frac{1}{2}b\cdot(Q^*_2)^2 \quad (25)$$

where Q^*_2 is the total output of all the firms at $t=2$. The regulator is benevolent in the sense that it chooses the optimal merger subsidy to maximize the social welfare.

The welfare effect and the optimal subsidy are then shown in Proposition 5.

Proposition 5 *Under the conditions of Proposition 4, the optimal pro-merger policy is welfare enhancing. The optimal pro-merger subsidy is*

- *If* $m > \frac{1+\sqrt{1+8n}}{4}$,

$$s^*=\begin{cases}\frac{(m-1)(a-c)}{n+1}, & if\ r\le\frac{(n-m)(a-c)}{2n(n+1)}\\ \min\left\{\frac{m(a-c)-(m+n)r}{n},\frac{(m-1)(a-c)}{n+1}\right\}, & if\ \frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s^*]}{2n(n+m)}<r\le\frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s^*}{n(n+m)}\\ \min\left\{\frac{a-c-r}{m},\frac{(m-1)(a-c)}{n+1}\right\}, & if\ \frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s^*}{n(n+m)}<r\le\frac{n(a-c)+ms^*}{n}\\ 0 & if\ r>a-c\end{cases}\tag{26}$$

• *If* $0<m\le\frac{1+\sqrt{1+8n}}{4}$,

$$s^*=\begin{cases}\frac{(m-1)(a-c)}{n+1}, & if\ r\le\frac{m(m-1)(a-c)}{n(n+1)}\\ \min\left\{\frac{a-c-r}{m},\frac{(m-1)(a-c)}{n+1}\right\}, & if\ \frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s^*}{n(n+m)}<r\le\frac{n(a-c)+ms^*}{n}\\ 0 & if\ r>a-c.\end{cases}\tag{27}$$

Proposition 5 shows that the pro-merger subsidy is welfare improving when this policy helps to restrict firms' incentive to build up capacity. Actually the reason for this welfare increment is well understood in the industrial organization literature. For instance, Lahiri and Ono (2004) find that in an asymmetric oligopoly industry, granting cost efficient firms with R&D subsidies and making them further more asymmetric, tends to increase social welfare. Similar logic applies here. To understand why the pro-merger policy is always welfare improving, it is useful to break social welfare into firms' profits, consumer surplus and the cost of subsidy. As evidenced by the appended proof of Proposition 5, on the one hand, in all subcases, the pro-merger subsidy increases the profit of merged firm, since they invest less in capacity (under the conditions of Proposition 4) but can produce more. On the other hand, the profit of the outsiders is lower because a subsidy to the insiders entails the outsiders a cost disadvantage. However, aggregate profit of the industry still increases since the profit of the insiders increases more than the profit decrease of the outsiders. Also from the appended proof of Proposition 5, total output of the industry increases with the subsidy s, so that market price decreases due to such a direct subsidy to the merged firm. Therefore, consumer surplus rises. In an asymmetric oligopoly industry like the post-merger industry in our model, the increases in producer and consumer surpluses dominate the cost of subsidy, and thus social welfare is improved.

Investigating the optimal subsidy s^*, it is interesting to notice the complementarity of market and the government. The comparative statics is summarized in the following corollary without proofs.

Corollary 1 *The optimal pro-merger subsidy s^* decreases with unit capacity investment cost r and number of firms n.*

Intuitively, when the market functions towards more profitable mergers, there is less room for the government to stand in. If building up capacity is more costly, or if the degree of market competition increases to render staying outside less profitable, it is more attractive for each individual firm to utilize merger either as an instrument to save capacity investment cost, or as a vehicle to enhance profit. Therefore, the regulator is less willing to encourage mergers with the "visible hand".

V. *Conclusion*

This paper has considered capacity constraint and studied SSR (1983)'s "merger paradox" in a context with production capacity. We show that the capacity plays a crucial role in merger decisions in a symmetric Cournot model, in which firms must build up capacity before production. SSR (1983)'s "merger paradox" is alleviated or even disappears if capacity constraint is introduced, either exogenously or endogenously. In particular, an outside firm who do not participate in a merger, cannot expand its production freely in response to a merger due to capacity constraint. Thus its free riding effect is mitigated. On the other hand, the merged firm may not be capacity constrained or subject to less severe constraint, making a merger more profitable than SSR (1983) case, *ceteris paribus*.

This paper has also shown that when capacity investment is not too costly, a unit tax reduction or subsidy always mitigates overcapacity by discouraging *ex ante* capacity investment. Interestingly, in this case, the optimal pro-merger subsidy helps increase social welfare. Future research may consider the endogenous merger decision of an individual firm, merger waves or allow multiple mergers to occur simultaneously.

Appendix

Proof of Proposition 2

Proof: With some algebra, it can be derived that

$$\frac{dm^*}{dn}=1-\frac{1}{\sqrt{4n+5}}>0 \tag{28}$$

$$\frac{\partial\bar{q}^*}{\partial m}=\frac{(\sqrt{m}-1)(n-\sqrt{m})(a-c)}{\sqrt{m}(n+1)(n-m)^2\cdot b}>0 \tag{29}$$

$$\frac{\partial\bar{q}^*}{\partial n}=-\frac{\left[(n+1)(\sqrt{m}-1)^2+(n-m)(n+1-2\sqrt{m})\right]\cdot(a-c)}{(n+1)^2(n-m)^2\cdot b}<0 \tag{30}$$

■

Proof of Lemma 1

Proof: To determine $\bar{q}_{i0}$, consider the three possible scenarios as defined in Section III, depending on whether firm i will be capacity constrained after merger at $t=1$ as an insider or an outsider.

Scenario 1: If firm i is not capacity constrained after merger at $t=1$, no matter as an insider or an outsider, i.e. $\bar{q}_{i0}\geq\frac{a-c}{b(n-m+2)}$. In this case,

$$\pi^I_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=\frac{(a-c)^2}{mb(n-m+2)^2},\ \pi^O_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=\frac{(a-c)^2}{b(n-m+2)^2} \tag{31}$$

inserting which into (14), monotonically decreasing in $\bar{q}_{i0}$,

$$\bar{q}_{i0}=\frac{a-c}{b(n-m+2)} \tag{32}$$

and

$$E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\begin{cases}\dfrac{a-c}{b(n-m+2)}\cdot\left[\dfrac{(a-c)(n-m+1)}{n(n-m+2)}-r\right], & \text{if } r<\dfrac{(a-c)(n-m+1)}{n(n-m+2)}\\ 0, & \text{if } r\geq\dfrac{(a-c)(n-m+1)}{n(n-m+2)}\end{cases} \tag{33}$$

Scenario 2: If firm i is not capacity constrained after merger at $t=1$ as an insider, but is constrained as an outsider, i.e. $\frac{a-c}{(n+m)b}\leq\bar{q}_{i0}<\frac{a-c}{b(n-m+2)}$. In this case, with probability $\frac{m}{n}$, firm i is an insider and not capacity constrained, and there are $(n-m)$ outsiders producing $\bar{q}_{j0}$ each. Therefore, the merged firm as a whole produces

$$q_{c2}=\frac{a-c-b(n-m)\bar{q}_{j0}}{2b} \tag{34}$$

and thus

$$\pi^{I}_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=\frac{[a-c-b(n-m)\bar{q}_{j0}]^2}{4mb} \tag{35}$$

With probability $(1-\frac{m}{n})$, firm i is an outsider producing $\bar{q}_{i0}$, and there are $(n-m-1)$ other outsiders producing $\bar{q}_{j0}$ each, as well as an m-firm merger producing q_c. Therefore, the merged firm produces

$$q_{c2}=\frac{a-c-b(n-m-1)\bar{q}_{j0}-b\bar{q}_{i0}}{2b} \tag{36}$$

and

$$\pi^{O}_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=\frac{1}{2}\bar{q}_{i0}\cdot[a-c-b(n-m-1)\bar{q}_{j0}-b\bar{q}_{i0}] \tag{37}$$

Inserting (35) and (37) into (14),

$$\bar{q}_{i0}=\begin{cases}\dfrac{a-c}{b(n-m+2)}, & \text{if } r\leq\dfrac{(n-m)(a-c)}{2n(n-m+2)}\\ \dfrac{(n-m)(a-c)-2nr}{b(n-m)(n-m+1)}, & \text{if } \dfrac{(n-m)(a-c)}{2n(n-m+2)}<r\leq\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}\\ \dfrac{a-c}{(n+m)b}, & \text{if } r>\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}\end{cases} \tag{38}$$

and thus

$$E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\begin{cases}\dfrac{a-c}{b(n-m+2)}\cdot\left[\dfrac{(a-c)(n-m+1)}{n(n-m+2)}-r\right], & \text{if } r\leq\dfrac{(n-m)(a-c)}{2n(n-m+2)}\\ \dfrac{n(n-m+2)}{b(n-m)(n-m+1)^2}\cdot\left[r-\dfrac{(n-m)(a-c)}{2n(n-m+2)}\right]^2+\dfrac{(a-c)^2}{2nb(n-m+2)}, & \text{if } \dfrac{(n-m)(a-c)}{2n(n-m+2)}<r\leq\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}\\ \dfrac{a-c}{(n+m)b}\cdot\left[\dfrac{m(a-c)}{n+m}-r\right], & \text{if } r>\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}\end{cases} \tag{39}$$

Scenario 3: If firm i is capacity constrained after merger at $t=1$, no matter as an insider or as an outsider, i.e. $0\leq\bar{q}_{i0}<\dfrac{a-c}{(n+m)b}$. In this case,

$$\pi^I_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=\pi^O_{i2}(\bar{q}_{i0},\bar{q}_{-i0})=E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=(a-c-r-b\bar{q}_{i0}-b\sum_{j\neq i,\, j=1}^{n}\bar{q}_{j0})\cdot\bar{q}_{i0} \tag{40}$$

Therefore,

$$\bar{q}_{i0}=\begin{cases}\dfrac{a-c}{(n+m)b}, & \text{if } r\leq\dfrac{(m-1)(a-c)}{n+m}\\ \dfrac{a-c-r}{b(n+1)}, & \text{if } \dfrac{(m-1)(a-c)}{n+m}<r\leq a-c\\ 0, & \text{if } r>a-c\end{cases} \tag{41}$$

and thus

$$E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\begin{cases}\dfrac{a-c}{(n+m)b}\cdot\left[\dfrac{m(a-c)}{n+m}-r\right], & \text{if } r\leq\dfrac{(m-1)(a-c)}{n+m}\\ \dfrac{(a-c-r)^2}{b(n+1)^2}, & \text{if } \dfrac{(m-1)(a-c)}{n+m}<r\leq a-c\\ 0, & \text{if } r>a-c\end{cases} \tag{42}$$

Comparing these three scenarios with tedious algebra, it is found that

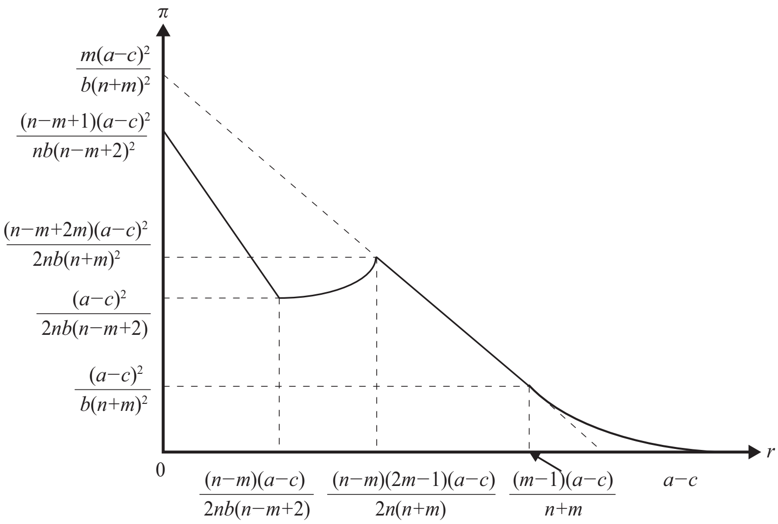
- If $m>\dfrac{1+\sqrt{1+8n}}{4}$, then $\dfrac{(m-1)(a-c)}{n+m}>\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}$, and thus the optimal capacity investment $\bar{q}^*_{i0}$ is as in (15). The equilibrium expected profit in this case is shown in Figure A.1.
- If $\dfrac{n(n+3)-\sqrt{n^4+4n^2+4n}}{2n-1}<m\leq\dfrac{1+\sqrt{1+8n}}{4}$, then $\dfrac{(n-m)(a-c)}{2n(n-m+2)}<\dfrac{(m-1)(a-c)}{n+m}\leq\dfrac{(n-m)(2m-1)(a-c)}{2n(n+m)}$, and thus the optimal capacity investment $\bar{q}^*_{i0}$ is as in (16). The expected profit can be similarly drawn and is omitted here.
- If $0<m\leq\dfrac{n(n+3)-\sqrt{n^4+4n^2+4n}}{2n-1}$, then $\dfrac{(m-1)(a-c)}{n+m}\leq\dfrac{(n-m)(a-c)}{2n(n-m+2)}$, and thus the optimal capacity investment $\bar{q}^*_{i0}$ is as in (17). The expected profit can be similarly drawn and is omitted here.

■

Proof of Lemma 2

Proof: Similarly to the proof of Lemma 1, to determine $\bar{q}_{i0}$, consider the three possible scenarios as

FIGURE A.1. EQUILIBRIUM EXPECTED PROFIT WHEN m IS RELATIVELY LARGE



defined in Section III, depending on whether firm i will be capacity constrained after merger at $t=1$ as an insider or an outsider.

Scenario 1: If firm i is not capacity constrained after merger at $t=1$, no matter as an insider or an outsider, i.e. $\bar{q}_{i0} \geq \frac{a-c-s}{b(n-m+2)}$. In this case, suppose the merged firm as a whole produces q_{c2} and each outsider's output is q_i. With some algebra,

$$q_{c2}=\frac{a-c+(n-m+1)s}{b(n-m+2)},\ q_i=\frac{a-c-s}{b(n-m+2)} \tag{43}$$

Therefore,

$$\pi_{i2}^I(\bar{q}_{i0},\bar{q}_{-i0})=\frac{[a-c+(n-m+1)s]^2}{mb(n-m+2)^2},\ \pi_{i2}^O(\bar{q}_{i0},\bar{q}_{-i0})=\frac{(a-c-s)^2}{b(n-m+2)^2} \tag{44}$$

inserting which into (14), monotonically decreasing in $\bar{q}_{i0}$,

$$\bar{q}_{i0}=\frac{a-c-s}{b(n-m+2)} \tag{45}$$

and

$$E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\begin{cases}\frac{a-c-s}{b(n-m+2)}\cdot\left[\frac{(n-m+1)(a-c)+(n-m+3)s}{n(n-m+2)}+\frac{(n-m+2)s^2}{n(a-c-s)}-r\right], & \text{if } r<\frac{(n-m+1)(a-c)+(n-m+3)s}{n(n-m+2)}+\frac{(n-m+2)s^2}{n(a-c-s)}\\ 0, & \text{if } r\geq\frac{(n-m+1)(a-c)+(n-m+3)s}{n(n-m+2)}+\frac{(n-m+2)s^2}{n(a-c-s)}\end{cases} \tag{46}$$

Scenario 2: If firm i is not capacity constrained after merger at $t=1$ as an insider, but is constrained as an outsider, i.e. $\frac{a-c+s}{(n+m)b} \leq \bar{q}_{i0} < \frac{a-c-s}{b(n-m+2)}$. In this case, with probability $\frac{m}{n}$, firm i is an insider and not capacity constrained, and there are $(n-m)$ outsiders producing $\bar{q}_{j0}$ each. Therefore, the merged firm as a whole produces

$$q_{c2} = \frac{a-c+s-b(n-m)\bar{q}_{j0}}{2b} \tag{47}$$

and thus

$$\pi_{i2}^{I}(\bar{q}_{i0}, \bar{q}_{-i0}) = \frac{[a-c+s-b(n-m)\bar{q}_{j0}]^2}{4mb} \tag{48}$$

With probability $(1-\frac{m}{n})$, firm i is an outsider producing $\bar{q}_{i0}$, and there are $(n-m-1)$ other outsiders each producing $\bar{q}_{j0}$, as well as an m-firm merger producing q_c. Therefore, the merged firm produces

$$q_{c2} = \frac{a-c+s-b(n-m-1)\bar{q}_{j0} - b\bar{q}_{i0}}{2b} \tag{49}$$

and

$$\pi_{i2}^{O}(\bar{q}_{i0}, \bar{q}_{-i0}) = \frac{1}{2}\bar{q}_{i0} \cdot [a-c-s-b(n-m-1)\bar{q}_{j0} - b\bar{q}_{i0}] \tag{50}$$

Inserting (48) and (50) into (14),

$$\bar{q}_{i0} = \begin{cases} \frac{a-c-s}{b(n-m+2)}, & \text{if } r \leq \frac{(n-m)(a-c-s)}{2n(n-m+2)} \\ \frac{(n-m)(a-c-s)-2nr}{b(n-m)(n-m+1)}, & \text{if } \frac{(n-m)(a-c-s)}{2n(n-m+2)} < r \leq \frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)} \\ \frac{a-c+s}{(n+m)b}, & \text{if } r > \frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)} \end{cases} \tag{51}$$

and thus

$$E[\pi_{i2}(\bar{q}_{i0}, \bar{q}_{-i0})] = \begin{cases} \frac{a-c-s}{b(n-m+2)} \cdot \left[\frac{(n-m+1)(a-c)+(n-m+3)s}{n(n-m+2)} + \frac{(n-m+2)s^2}{n(a-c-s)} - r\right], & \text{if } r \leq \frac{(n-m)(a-c-s)}{2n(n-m+2)} \\ \frac{(n-m)(a-c-s)-2nr}{b(n-m)(n-m+1)} \cdot \\ \left\{\frac{(n-m)[a-c+(2n-2m+1)s+2nr]^2}{4n(n-m+1)[(n-m)(a-c-s)-2nr]} + \frac{(n-m)(a-c-s+2nr)}{2n(n-m+1)} - r\right\}, & \text{if } \frac{(n-m)(a-c-s)}{2n(n-m+2)} < r \leq \frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)} \\ \frac{a-c+s}{(n+m)b} \cdot \left[\frac{nm(a-c)+(m^2-n^2+mn)s}{n(n+m)} - r\right], & \text{if } r > \frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)} \end{cases} \tag{52}$$

Scenario 3: If firm i is capacity constrained after merger at $t=1$, no matter as an insider or as an outsider, i.e. $0 \leq \bar{q}_{i0} < \frac{a-c+s}{(n+m)b}$. Following similar procedures,

$$\bar{q}_{i0}=\begin{cases}\dfrac{a-c+s}{(n+m)b}, & \text{if } r\leq\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\\ \dfrac{n(a-c-r)+ms}{bn(n+1)}, & \text{if } \dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\leq\dfrac{n(a-c)+ms}{n}\\ 0, & \text{if } r>\dfrac{n(a-c)+ms}{n}\end{cases} \tag{53}$$

and thus

$$E[\pi_{i2}(\bar{q}_{i0},\bar{q}_{-i0})]=\begin{cases}\dfrac{a-c+s}{(n+m)b}\cdot\left[\dfrac{nm(a-c)+(m^2-n^2+mn)s}{n(n+m)}-r\right], & \text{if } r\leq\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\\ \dfrac{[n(a-c-r)+ms]^2}{n^2(n+1)^2b}, & \text{if } \dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\leq\dfrac{n(a-c)+ms}{n}\\ 0, & \text{if } r>\dfrac{n(a-c)+ms}{n}\end{cases} \tag{54}$$

Comparing these three scenarios, it is found that

- If $m>\dfrac{1+\sqrt{1+8n}}{4}$, then $\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}>\dfrac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}$, and thus the optimal capacity investment $\bar{q}_{i0}^*$ is as in (19).
- If $\hat{m}<m\leq\dfrac{1+\sqrt{1+8n}}{4}$, where $\hat{m}$ is implicitly determined by $\dfrac{(n-m)(a-c-s)}{2n(n-m+2)}=\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}$. Then $\dfrac{(n-m)(a-c-s)}{2n(n-m+2)}<\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\leq\dfrac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}$, and thus the optimal capacity investment $\bar{q}_{i0}^*$ is as in (20).
- If $0<m\leq\hat{m}$, then $\dfrac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}\leq\dfrac{(n-m)(a-c-s)}{2n(n-m+2)}$, and thus the optimal capacity investment $\bar{q}_{i0}^*$ is as in (21).

■

Proof of Proposition 5

Proof: Since the equilibrium $\bar{q}_{i0}^*$ varies in three different scenarios as defined in Section III, we calculate the social welfare in sequence.

Scenario 1: In this case,

$$Q_2^*=\frac{(n-m+1)(a-c)+s}{b(n-m+2)},\ q_{c2}^*=\frac{a-c+(n-m+1)s}{b(n-m+2)} \tag{55}$$

and thus the social welfare is

$$W=\frac{1}{2b(n-m+2)^2}\cdot\{-s^2+2[a-c+nr(n-m+2)]s+\text{Constant}\} \tag{56}$$

where the constant is equal to $(n-m+1)(n-m+3)(a-c)^2-2nr(n-m+2)(a-c)$. Denote the solution to (56) as $\hat{s}$, then $\hat{s}=a-c+nr(n-m+2)$. It can be shown that $\hat{s}>\bar{s}$. Since $0\leq s\leq\bar{s}$, and $W'(s)>0$ for all $s\in[0,\hat{s}]$, $s^*=\bar{s}=\dfrac{(m-1)(a-c)}{n+1}$, and $W(s^*)>W(0)$ always holds.

Scenario 2: We consider three subcases for this scenario.

- If $r\leq\frac{(n-m)(a-c-s)}{2n(n-m+2)}$, then the equilibrium $\bar{q}^*_{i0}$ is as in Scenario 1, and thus all the analysis of s^* and W is the same.
- If $\frac{(n-m)(a-c-s)}{2n(n-m+2)}<r\leq\frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}$,

$$Q^*_2=\frac{(2n-2m+1)(a-c)+s-2nr}{2b(n-m+1)},\ q^*_{c2}=\frac{a-c+(2n-2m+1)s-2nr}{2b(n-m+1)} \tag{57}$$

and thus the social welfare is

$$W=\frac{1}{8b(n-m)(n-m+1)^2}\cdot\{-(n-m)s^2+2(n-m)[a-c+2nr(2n-2m+1)]s+\text{Constant}\} \tag{58}$$

where the constant is equal to $4n^2r^2(3n-3m+4)-4nr(a-c)(n-m)(2n-2m+3)+(n-m)(a-c)^2[6(n-m)+(2n-2m+1)^2]$. Denote the solution to (58) as $\hat{s}$, then $\hat{s}=a-c+2nr(2n-2m+1)$. It can be shown that $\hat{s}>\bar{s}$. Since $0\leq s\leq\bar{s}$, and $W'(s)>0$ for all $s\in[0,\hat{s}]$, $s^*=\bar{s}=\frac{(m-1)(a-c)}{n+1}$, and $W(s^*)>W(0)$ for sure.

- If $r>\frac{(n-m)\cdot[(2m-1)(a-c)-(2n+1)s]}{2n(n+m)}$,

$$Q^*_2=\frac{n(a-c+s)}{b(n+m)},\ q^*_{c2}=\frac{m(a-c+s)}{b(n+m)} \tag{59}$$

and thus the social welfare is

$$W=\frac{1}{2b(n+m)^2}\cdot\{-n^2s^2+2n[m(a-c)-(m+n)r]s+\text{Constant}\} \tag{60}$$

where the constant is equal to $(a-c)[(n^2+2mn)(a-c)-2nr(n+m)]$. Denote the solution to (60) as $\hat{s}$, then $\hat{s}=\frac{m(a-c)-(m+n)r}{n}$. Since $0\leq s\leq\bar{s}$, then $s^*=\min\{\hat{s},\bar{s}\}$. Because $W'(s)>0$ for all $s\in[0,\hat{s}]$, $W(s^*)>W(0)$ holds unconditionally.

Scenario 3: Similarly, we consider three subcases of this scenario.

- If $r\leq\frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}$, then the equilibrium $\bar{q}^*_{i0}$ is as in the third subcase of Scenario 2, and thus all the analysis of s^* and W is the same.
- If $\frac{n(m-1)(a-c)+(mn+m^2-n^2-n)s}{n(n+m)}<r\leq\frac{n(a-c)+ms}{n}$,

$$Q^*_2=\frac{n(a-c-r)+ms}{b(n+1)},\ q^*_{c2}=\frac{m[n(a-c-r)+ms]}{bn(n+1)} \tag{61}$$

and thus the social welfare is

$$W=\frac{1}{2b(n+1)^2}\cdot\{-m^2s^2+2m(a-c-r)s+Constant\} \tag{62}$$

where the constant is equal to $n(n+2)(a-c-r)^2$. Denote the solution to (6) as $\hat{s}$, then $\hat{s}=\frac{a-c-r}{m}$. Since $0\leq s\leq\bar{s}$, then $s^*=\min\{\hat{s},\bar{s}\}$. Because $W'(s)>0$ for all $s\in[0,\hat{s}]$, $W(s^*)>W(0)$ always holds.

- If $r>\frac{n(a-c)+ms}{n}$, then capacity building is too costly for firms to survive, so there is no need to provide subsidy, and $s^*=0$.

Rewriting the conditions of r by substituting s^*, and summarizing all the above analysis, the optimal subsidy is as in Proposition 5, and in any scenario, the optimal pro-merger subsidy is welfare enhancing. ■

A Numerical Example of Profitable Mergers

Let $a=1$ and $c=0$. Denote the expected profit of an individual inside firm with merger and without merger as $E[\pi_{i2}^{C}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]$ and $E[\pi_{i2}^{NC}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]$, respectively.

- **1.** If $r\leq\frac{n-m}{2n(n-m+2)}$, then $\bar{q}_{i0}^*=\frac{1}{b(n-m+2)}$, and

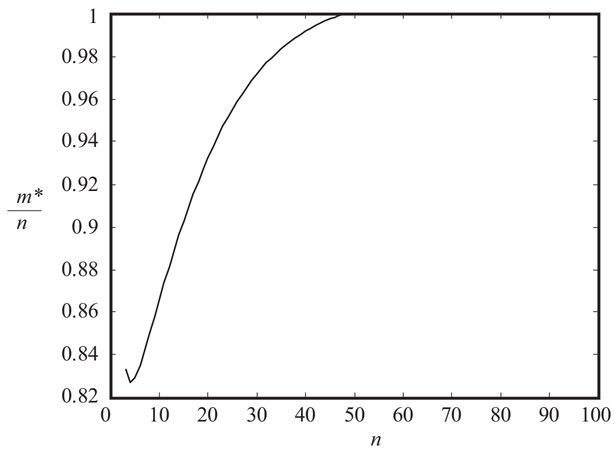
$$E[\pi_{i2}^{C}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]=\frac{1}{mb(n-m+2)^2}-\frac{r}{b(n-m+2)},\ E[\pi_{i2}^{NC}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]=\frac{1}{b(n+1)^2}-\frac{r}{b(n+1)} \tag{63}$$

Then it can be shown that $E[\pi_{i2}^{C}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]>E[\pi_{i2}^{NC}(\bar{q}_{i0}^*,\bar{q}_{-i0}^*)]$ iff $m>m^*$, where

$$m^*=\frac{3-n^2r-2r-3nr+2n-\sqrt{(3-n^2r-2r-3nr+2n)^2-4(n+1)^2(1-r-nr)}}{2(1-r-nr)}$$

In Figure A.2, we depict $\frac{m^*}{n}$ when $r=0.01$, from which $\frac{m^*}{n}>80\%$. Also, as a special case, when $r=0$, clearly this reduces to SSR(1983), as has been analyzed in Section II.

FIGURE A.2. THE MINIMUM SIZE OF A PROFITABLE MERGER WHEN $r=0.01$



• **2.** If $\frac{n-m}{2n(n-m+2)}<r\leq\frac{m(n-m)}{2n(n+1)}$, then $\bar{q}_{i0}^{*}=\frac{n-m-2nr}{b(n-m)(n-m+1)}$, and this case arises only when $m>\frac{1+\sqrt{1+8n}}{4}$. In this case,

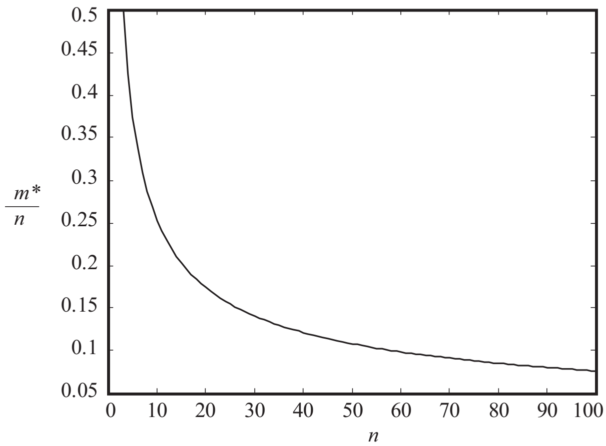
$$E[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{(1+2nr)^2}{4mb(n-m+1)^2}-r\cdot\frac{n-m-2nr}{b(n-m)(n-m+1)},\ E[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{1}{b(n+1)^2}-\frac{r}{b(n+1)} \tag{64}$$

Then it can be shown that $E[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]>E[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]$ as long as $m>\frac{1+\sqrt{1+8n}}{4}$. Therefore, when $\frac{n-m}{2n(n-m+2)}<r\leq\frac{m(n-m)}{2n(n+1)}$, merger is profitable when

$$m>m^{*}=\frac{1+\sqrt{1+8n}}{4} \tag{65}$$

and it can be shown in Figure A.3 that $\frac{1+\sqrt{1+8n}}{4n}<0.5$. Thus, in this example, for $n<5$, a profitable merger may only require two firms. In this case, insiders are not capacity constrained, while outsiders are. From Figure A.3, it can be seen that when merger can function as an instrument to reduce capacity restriction, the proportion of firms necessary for a profitable merger is significantly smaller than the 80% result of SSR(1983).

FIGURE A.3. THE THE MINIMUM SIZE OF A PROFITABLE MERGER WHEN $\frac{n-m}{2n(n-m+2)}<r\leq\frac{m(n-m)}{2n(n+1)}$



• **3.** If $\frac{m(n-m)}{2n(n+1)}<r\leq\frac{(n-m)(2m-1)}{2n(n+m)}$, then $\bar{q}_{i0}^{*}=\frac{n-m-2nr}{b(n-m)(n-m+1)}$, and

$$E[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{(1+2nr)^2}{4mb(n-m+1)^2}-r\cdot\frac{n-m-2nr}{b(n-m)(n-m+1)},\ E[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{(1-r)^2}{(n+1)^2b} \tag{66}$$

Then it can be shown that $\mathrm{E}[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]<\mathrm{E}[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]$ holds for any value of m.

- **4.** If $\frac{(n-m)(2m-1)}{2n(n+m)}<r\leq\frac{m-1}{n+m}$, then $\bar{q}_{i0}^{*}=\frac{1}{b(n+m)}$, and

$$\mathrm{E}[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{m}{(n+m)^2 b}-\frac{r}{(n+m)b},\ \mathrm{E}[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{(1-r)^2}{(n+1)^2 b} \quad (67)$$

Then it can be shown that $\mathrm{E}[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]<\mathrm{E}[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]$ holds for any value of m.

- **5.** If $r>\frac{m-1}{n+m}$, then $\mathrm{E}[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\mathrm{E}[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\frac{(1-r)^2}{(n+1)^2 b}$ when $\frac{m-1}{n+m}<r\leq 1$; or $\mathrm{E}[\pi_{i2}^{C}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=\mathrm{E}[\pi_{i2}^{NC}(\bar{q}_{i0}^{*},\bar{q}_{-i0}^{*})]=0$ when $r>1$. Then it is clear that in this case merger is never strictly profitable.

Summarizing difference scenarios in this numerical example, we find that when $a=1$, $c=0$, merger is profitable if

$$r\leq\frac{n-m}{2n(n-m+2)},\ m>\frac{3-n^2r-2r-3nr+2n-\sqrt{(3-n^2r-2r-3nr+2n)^2-4(n+1)^2(1-r-nr)}}{2(1-r-nr)} \quad (68)$$

or

$$\frac{n-m}{2n(n-m+2)}<r\leq\frac{m(n-m)}{2n(n+1)},\ m>\frac{1+\sqrt{1+8n}}{4} \quad (69)$$

It can be seen from the example that for the endogenous capacity choice case, the minimum proportion of merging firms that necessitates a profitable merger is significantly smaller than SSR(1983)'s result. ■

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