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The Advantage of Dual Discrimination in Lottery Contest Games¹

by

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Abstract

In a simple class of contest games, the designer can combine two types of discrimination: a change of the contestants' prize valuations subject to a balanced-budget constraint (direct discrimination), as well as a bias of the impact of their efforts (structural discrimination). Applying dual discrimination, the designer reduces (increases) the higher (lower) prize value up to a minimal (maximal) level, but suitably increases (reduces) the corresponding prize share. Our main result establishes that in some cases this dual discrimination is advantageous and can yield almost the maximal possible efforts - the highest valuation of the contested prize. The efforts in our setting can therefore be larger than those obtained under alternative contests with optimal structural discrimination. This is true in particular with respect to the optimally biased simple N -player lottery, Franke et al. (2013). In contrast to the main findings in Franke et al. (2014a, 2014b), in our setting, efforts under the simple lottery are not necessarily smaller than those under an optimally biased N -player all-pay auction. Finally, the exclusion principle noted in Baye et al. (1999) – the elimination of the strongest player - is not valid under dual discrimination.

JEL Classification: D70, D72, D74, D78

Keywords: contest design, dual discrimination, direct discrimination, balanced-budget-constraint, structural discrimination.

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1. Introduction

In the vast contest literature that has numerous applications (internal labor market tournaments, promotional competitions, R&D races, rent-seeking, political and public policy competitions, litigation and sports), the most commonly assumed contest success function (CSF) is the simple lottery proposed by Tullock (1980), see Konrad (2009) and references therein. In two-player contests, for $x_1 \geq 0$, $x_2 \geq 0$, and $\delta > 0$, this simple logit functions take the form:

$$(1) \quad p_1(x_1, x_2) = \begin{cases} \frac{x_1}{x_1 + \delta x_2} & \text{if } x_1 + x_2 > 0 \\ 0.5 & \text{if } x_1 = x_2 = 0 \end{cases}$$

Usually, x_1 and x_2 are interpreted as the contestants' efforts. However, p_1 has two possible interpretations. It can be interpreted as contestant 1's winning probability of an indivisible prize or as his share in a divisible prize. In turn, the winning probability of contestant 2 or his share in the prize is equal to $p_2 = 1 - p_1$. Henceforth, we use the second interpretation, as in Corchon and Dahm (2010), Franke et al. (2013), Lee and Lee (2012), Warneryd (1998). Nevertheless, although under this interpretation there is no uncertainty in the model and the contestants compete on the certain shares of a divisible prize, we preserve the terms "contest" and "CSF". The asymmetry between the impact of the contestants' efforts is captured by the parameter $\delta > 0$.

In our extended setting, we do enable the contest designer to control δ , as first suggested in Lien (1986, 1990) and later by Clark and Riis (2000). This means that the designer can apply *structural discrimination* that affects the contestants' shares in the contested prize (the same efforts may yield different shares, depending on the value of this parameter). By (1), a reduction in δ increases the bias in favor of contestant 1, who is assumed to be, with no loss of generality, the more motivated contestant (the one with the higher prize valuation). Furthermore, $0 < \delta < 1$ ($\delta > 1$) implies a bias in favor of contestant 1 (contestant 2). When $\delta = 1$ the contest is fair. The empirical relevance of such discrimination in contests with a logit CSF is thoroughly discussed in Epstein et al. (2011) and Franke (2012). Franke et al. (2013) have justified structural discrimination on the grounds that it lends itself to a very appealing competitive-market interpretation. For a recent survey of discrimination in contests, see Mealem and Nitzan (2016).

Fang (2002) considered the unbiased simple class of N -player lottery contests

assuming that the discretionary power of the contest designer is restricted to the exclusion of specific players. He showed that the exclusion principle, established in Baye et al. (1993) for an all-pay auction framework with complete information, is not valid in an unbiased lottery framework. Franke et al. (2013) extend this result to the biased lottery contest showing that it is never optimal for the designer to discourage strong contestants from participating in order to enhance competitive pressure among the remaining weaker contestants. Moreover, they have pointed out the existence of an additional inclusion principle: some weak contestants, who are inactive in the unbiased case of Fang (2002), are encouraged to become active. This implies that in their setting of structural discrimination, the designer will endogenously induce a more leveled playing field in comparison to the unbiased contest setting. This enhances competition and, in turn, increases the contestants' exerted efforts. However, as long as $N > 2$, it is not optimal for the designer to completely level the playing ground so some weak contestants may still remain inactive, although at least three will always be active.

More recently, Franke et al. (2014a, 2014b) have shown that an optimally biased all-pay auction contest with structural discrimination and an optimal Head-starts all-pay auction always dominate the optimally biased lottery contest; it yields larger efforts. This is in contrast to the outcome of the comparison between the unbiased versions of these contest models where the (unbiased) all-pay auction can yield less efforts when the exclusion principle applies (it is effort-enhancing to exclude the player with the highest prize valuation from participation, but the two active weaker contestants may expend less efforts than all the active players in the lottery contest). Franke et al. (2014a) show that when the designer can apply structural discrimination, the exclusion principle of the all-pay auction becomes obsolete. The designer will always bias the all-pay auction such that the two strongest players are active and, moreover, compete on equal terms (the strongest player is not excluded, but his effectiveness is sufficiently weakened). All other players choose to be inactive. Applying a less extreme discrimination in the lottery contest induces more contestants (at least three) to participate. But the effect of increased competitiveness due to a higher number of active contestants cannot offset the effect of reduced competitiveness due to a less extreme discrimination and, consequently, the optimally biased all-pay auction yields larger efforts than the optimally biased simple lottery. When the designer applies head starts-discrimination, Franke et al. (2014b) have

shown that the total efforts of N contestants, $N \geq 2$, are larger in an all-pay auction than in a simple lottery.

In our contest environment, the designer's ability to discriminate is enriched in comparison to Franke et al. (2013, 2014a, 2014b). In addition to structural discrimination, i.e., the control of δ , the contest designer can affect the contestants' incentives by directly changing their rewards in case of winning the contest, thereby increasing or decreasing the gap between their prize valuations. In other words, the designer can manipulate the size of the divisible prize. Such a policy is usually based on a "give and take" mechanism in case of winning, which is henceforth referred to as *direct discrimination*. This form of discrimination has been recently introduced and studied in Mealem and Nitzan (2014, 2016), focusing on its comparative application in an all-pay-auction relative to a logit CSF, disregarding the possibility of structural discrimination.

A crucial element in this second type of discrimination is the balanced-budget constraint faced by the contest designer. This constraint, which limits the design of the optimal tax schedule, implies that when one contestant's winning a share of the prize is subjected to a positive tax, the share of the prize won by the other contestant must be subjected to a negative tax, viz., the granting of a subsidy. The tax scheme consists then of two numbers (negative and positive) that are added to the contestants' initial valuations of the divisible prize. These numbers need to satisfy the requirement that the designer's net expenditures are equal to zero in equilibrium. Of course, whether the constraint is satisfied or not depends both on the applied structural and direct discrimination; the former determining the contestants' shares in the prize and the latter the actual modified values of the prize.

The dual discrimination setting significantly changes the results obtained in Franke et al. (2013, 2014a, 2014b). Most importantly, in a simple lottery with dual discrimination, the maximal efforts can be increased to almost the initially highest prize valuation.² These efforts are larger than those obtained in Fang (2002), where discrimination is not allowed, and larger than the efforts obtained in Franke et al. (2013, 2014a) where only structural discrimination is allowed (by Theorem 3.4 in Franke et al. (2014a), the efforts in the optimally biased simple lottery are less than

² This is implied by our main result, assuming that the upper bound of the net value of the prize approaches infinity and the lower bound of the net value of the prize approaches zero.

the average of the two highest prize valuations). In our simple lottery contest with dual discrimination, total efforts can be larger than those obtained in any optimally biased contest game and, in particular, in an N -player all-pay auction. This is in contrast to the main results in Franke et al. (2014a, 2014b).

Finally note that in a simple lottery contest with dual discrimination, even when $N > 2$, the maximal efforts can be increased to almost the initially highest prize valuation, where only two contestants are active in equilibrium; one of them must be the contestant with the highest prize valuation, but the second active player can be any of the other contestants. This result differs from the result (Theorem 4.6) obtained in Franke et al. (2013), where at least three contestants are active in the equilibrium of the simple N -player contest with just structural discrimination. Furthermore, the exclusion principle of Baye et al. (1993) is not necessarily valid in our extended contest because the strongest player can be induced to be always active.

Interestingly, in the extreme case of our setting (see footnote 1), the individual with the lower prize valuation is offered the illusion of competing on a very large prize, albeit only a very small share of it can be won. The expected value of his prize is nevertheless positive and in fact, almost equal to the initial prize valuation of his rival, the individual with the higher prize valuation. The existence of effective incentives that induce participation in the contest together with the existence of an extreme illusion that results in efforts incurred by the individual with the lower prize valuation is a distinctive interesting feature of our contest. This feature is manifested in the examples presented in the next section.

2. Illustration of direct discrimination

Our model of dual discrimination in a simple lottery contest is of particular relevance in certain applications. To illustrate the plausibility of direct discrimination with a balanced-budget constraint in contests, we present two applications. In the first one, the contest designer typically engages in a certain activity (some well-defined task or project) restricted to a certain budget. Although the budget is earmarked only for this activity, it can be used to manipulate and affect the incentives of the contestants (the contractors) who compete for the outsourced project. But a designer who engages in such manipulations and in particular, in discrimination, must satisfy the contest balanced-budget constraint that we assume in order to ensure the overall budget constraint is satisfied:

1. *Municipal projects.* A municipal authority is conducting a tender for a divisible project such as urban development including development of a sewage system, roads, sidewalks and gardening. Two companies compete for a share in the project. The municipal authority is restricted to a budget allocated, for example, by the federal government. Although the budget is allocated only to urban development, it can be used to influence the incentives of the competing contestants by applying the two possible modes of discrimination. In order to satisfy the overall budget constraint, a designer who resorts to structural and direct discrimination must also satisfy the assumed balanced budget constraint.

The next application describes a situation where the balanced-budget constraint is due to a different reason. The constraint is no longer related to a fixed budget which is at the disposal of the designer for the purpose of carrying out a particular project. It is due to the fact that the two competing contestants are (at least partly) controlled by a parent company. The parent company may not prevent competition between its two subsidiaries by custom or by the law. However, despite the existing competition, the parent company still has the ability to enforce some overall financial discipline as well as the power to ensure that the designer's strength in manipulating the companies is limited. The control of the parent company on its two subsidiaries and its power in dealing with the designer, given the conflict of interests among them, explains its success in enforcing the balanced-budget constraint:

2. *Portfolio distribution between two investment houses.* In the capital market, the commission rate charged by an investment house is usually inversely related to the size of the customer's investment. Suppose that the average commission rate in the industry is \bar{z} and that a large client (e.g., a pension fund of some large employer) is interested in distributing its portfolio between two investment houses that are subsidiaries of some parent company. Despite the affinity among the two investment houses, they compete in the market.³ The first investment house has an established reputation while the second smaller house is relatively less known. Given the importance of the large customer, the preservation of the reputation of the first investment house (contestant 1) implies that it assigns a higher value to

² Two such competing investment houses, Psagot and Ofek, were subsidiaries of the leading Bank in Israel, Bank Leumi. Another example of two such companies is Gadish and Tagmulim, that were two subsidiaries of another major Israeli bank, Bank Hapoalim.

the investment of the large customer (the employer's pension fund). Often, such a pension fund prefers to invest a large share of its portfolio in the reputable and usually larger investment house and this enables obtaining a commission rate lower than \bar{z} . This implies that the pension fund actually "taxes" the larger and more reputable investment house relative to \bar{z} . On the other hand, the investment house with the lower reputation usually receives the smaller share of the portfolio. However, the commission rate they charge are higher than \bar{z} . The balanced-budget constraint is satisfied because of the market forces; The pension fund is interested in reducing the commission rate, and the parent company of the two investment houses is interested in increasing the commission rate.

3. The setting

In our contest there are two risk-neutral contestants,⁴ the high and low benefit contestants, 1 and 2. With no loss of generality, the initial prize valuations of the contestants, n_1 and n_2 , satisfy the inequality $n_1 \geq n_2$ or $k = \frac{n_1}{n_2} \geq 1$ and that the contest designer is assumed to have full knowledge of the contestants' prize valuations. In two-player contests, for $x_1 \geq 0$, $x_2 \geq 0$ and $\delta > 0$, the simple logit functions take the form:

$$(1) \quad p_1(x_1, x_2) = \begin{cases} \frac{x_1}{x_1 + \delta x_2} & \text{if } x_1 + x_2 > 0 \\ 0.5 & \text{if } x_1 = x_2 = 0 \end{cases}$$

and therefore $p_2 = 1 - p_1$.

Direct discrimination via differential taxation of the contested prize that affects the contestants' actual prize valuations, n_1 and n_2 , is a pair of (positive or negative) amounts, ε_1 and ε_2 that changes the prize valuations to $(n_1 + \varepsilon_1)$ and $(n_2 + \varepsilon_2)$ where $0 < \underline{n} \leq n_i + \varepsilon_i \leq \bar{n} < \infty$ (the lower and upper bounds of the contestants' actual net prize are the parameters \underline{n} and \bar{n}). We also assume that the contest designer faces an ex ante expected balanced-budget constraint, that is, ε_1 and ε_2 must also satisfy the requirement that:

³ The case of multiple contestants is dealt with later on.

$$(2) \quad p_1 \varepsilon_1 + p_2 \varepsilon_2 = 0.$$

Given the contestants' fixed prize valuations and the CSF, the function that specifies the contestants' winning probability given their efforts, $p_i(x_1, x_2)$, the expected net payoff (surplus) of contestant i is:

$$(3) \quad E(u_i) = p_i(x_1, x_2)(n_i + \varepsilon_i) - x_i \quad (i=1,2).$$

In the optimal contest design setting, the objective function of the contest designer is:

$$(4) \quad G = x_1 + x_2.$$

The designer selects δ , ε_1 and ε_2 . In this case, the two contestants maximize their net payoffs:

$$(5) \quad E(u_1) = \frac{x_1}{x_1 + \delta x_2} (n_1 + \varepsilon_1) - x_1 \quad \text{and} \quad E(u_2) = \frac{\delta x_2}{x_1 + \delta x_2} (n_2 + \varepsilon_2) - x_2.$$

If the designer selects $(\varepsilon_1, \varepsilon_2) = (0,0)$, then the optimal δ is $\delta = \frac{n_1}{n_2}$, and the

corresponding efforts are equal to $G = 0.25(n_1 + n_2)$, see Epstein et al. (2013).

Therefore, in the following, we assume $(\varepsilon_1, \varepsilon_2) \neq (0,0)$ and from (2) we get that $\varepsilon_1 \varepsilon_2 < 0$. Let

$$(6) \quad d = \frac{\left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2} \right)}{\delta}.$$

By the first order conditions (maximization of (5)),

$$(7) \quad x_1^* = \frac{d(n_1 + \varepsilon_1)}{(d+1)^2} \quad \text{and} \quad x_2^* = \frac{d(n_2 + \varepsilon_2)}{(d+1)^2}$$

and, therefore,

$$(8) \quad G = x_1^* + x_2^* = \frac{d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)^2}$$

$$(9) \quad p_1 = \frac{d}{d+1} \quad \text{and} \quad p_2 = \frac{1}{d+1}$$

and the balanced-budget constraint (2) takes the form

$$(10) \quad p_1 \varepsilon_1 + p_2 \varepsilon_2 = \frac{d}{d+1} \varepsilon_1 + \frac{1}{d+1} \varepsilon_2 = 0$$

or

$$(11) \quad d\varepsilon_1 + \varepsilon_2 = 0$$

By the balanced-budget constraint (11), $d = -\frac{\varepsilon_2}{\varepsilon_1}$. Substituting $d = -\frac{\varepsilon_2}{\varepsilon_1}$ in (7) we get

that $G = x_1^* + x_2^* = -\frac{\varepsilon_1 \varepsilon_2 (n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)^2}$. By (6) and $d = -\frac{\varepsilon_2}{\varepsilon_1}$, we get that in

equilibrium,

$$(12) \quad \delta = -\left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}\right) \frac{\varepsilon_1}{\varepsilon_2}$$

and the positive expected net payoffs are:

$$(13) \quad u_1^* = \frac{(n_1 + \varepsilon_1)d^2}{(d+1)^2} \text{ and } u_2^* = \frac{(n_2 + \varepsilon_2)}{(d+1)^2}.$$

Therefore, the designer objective function is:

$$(14) \quad G(\varepsilon_1, \varepsilon_2) = -\frac{\varepsilon_1 \varepsilon_2 (n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)^2}$$

and his optimal strategy $(\varepsilon_1^*, \varepsilon_2^*)$ solves:

$$(15) \quad \underset{(\varepsilon_1, \varepsilon_2) \in C}{\text{Max}} G(\varepsilon_1, \varepsilon_2)$$

where

$$(16) \quad C = \left\{ (\varepsilon_1, \varepsilon_2) : \underline{n} \leq n_1 + \varepsilon_1 \leq \bar{n}, \underline{n} \leq n_2 + \varepsilon_2 \leq \bar{n}, \varepsilon_1 \varepsilon_2 < 0 \right\}$$

Proposition 1

Assume that $n_1 > n_2$, $\underline{n} > 0$ is sufficiently close to zero, and \bar{n} is sufficiently large.

Then,

$$\begin{aligned} G(\varepsilon_1^*, \varepsilon_2^*) &= G(\underline{n} - n_1, \bar{n} - n_2) = \frac{(\bar{n} + \underline{n})(n_1 - \underline{n})(\bar{n} - n_2)}{(\bar{n} - \underline{n} + n_1 - n_2)^2}, \\ x_1^* &= \frac{\underline{n}(n_1 - \underline{n})(\bar{n} - n_2)}{(\bar{n} - \underline{n} + n_1 - n_2)^2}, \quad x_2^* = \frac{\bar{n}(n_1 - \underline{n})(\bar{n} - n_2)}{(\bar{n} - \underline{n} + n_1 - n_2)^2}, \\ u_1^* &= \frac{\underline{n}(\bar{n} - n_2)^2}{(\bar{n} - \underline{n} + n_1 - n_2)^2}, \quad u_2^* = \frac{\bar{n}(\underline{n} - n_1)^2}{(\bar{n} - \underline{n} + n_1 - n_2)^2}. \end{aligned}$$

To prove the above, let

$$(17) \quad C_1 = \left\{ (\varepsilon_1, \varepsilon_2) : \underline{n} \leq n_1 + \varepsilon_1 \leq \bar{n}, \underline{n} \leq n_2 + \varepsilon_2 \leq \bar{n}, \varepsilon_1 < 0 < \varepsilon_2 \right\}$$

and

$$(18) \quad C_2 = \left\{ (\varepsilon_1, \varepsilon_2) : \underline{n} \leq n_1 + \varepsilon_1 \leq \bar{n}, \underline{n} \leq n_2 + \varepsilon_2 \leq \bar{n}, \varepsilon_2 < 0 < \varepsilon_1 \right\}.$$

Note that $C = C_1 \cup C_2$.

Lemma 1

Assume that $n_1 > n_2$, $\underline{n} > 0$ is sufficiently close to zero, and \bar{n} is sufficiently large.

Then,

(a) $\arg \underset{(\varepsilon_1, \varepsilon_2) \in C_1}{\text{Max}} G(\varepsilon_1, \varepsilon_2) = \left\{ \left(\underline{n} - n_1, \bar{n} - n_2 \right) \right\}$

(b)

$$\arg \underset{(\varepsilon_1, \varepsilon_2) \in C_2}{\text{Max}} G(\varepsilon_1, \varepsilon_2) = \begin{cases} \left\{ \left(\bar{n} - n_1, \underline{n} - n_2 \right) \right\} & \text{if } n_1 \leq 2n_2 - 3\underline{n} \\ \subseteq \left\{ \left(\bar{n} - n_1, \underline{n} - n_2 \right), \left(\frac{(n_1 + \underline{n})(n_2 - \underline{n})}{n_1 - 2n_2 + 3\underline{n}}, \underline{n} - n_2 \right) \right\} & \text{if } n_1 > 2n_2 - 3\underline{n} \end{cases}$$

Proof of Lemma 1

We first show (a). Notice that

$$(19) \quad \begin{aligned} \frac{\partial G}{\partial \varepsilon_1} &= \frac{\varepsilon_2 \{ \varepsilon_1 (n + 3\varepsilon_2) + \varepsilon_2 (n + \varepsilon_2) \}}{(\varepsilon_1 - \varepsilon_2)^3} \\ \frac{\partial G}{\partial \varepsilon_2} &= \frac{\varepsilon_1 \{ \varepsilon_2 (n + 3\varepsilon_1) + \varepsilon_1 (n + \varepsilon_1) \}}{(\varepsilon_2 - \varepsilon_1)^3} \end{aligned}$$

where $n = n_1 + n_2$. Thus for $\varepsilon_1 < 0 < \varepsilon_2$,

$$(20) \quad \begin{aligned} \frac{\partial G}{\partial \varepsilon_1} < 0 &\Leftrightarrow \varepsilon_1 (n + 3\varepsilon_2) > -\varepsilon_2 (n + \varepsilon_2) \\ \frac{\partial G}{\partial \varepsilon_2} > 0 &\Leftrightarrow \varepsilon_2 (n + 3\varepsilon_1) < -\varepsilon_1 (n + \varepsilon_1) \end{aligned}$$

Therefore, for $\varepsilon_2 > 0$,

$$(21) \quad \arg \underset{\varepsilon_1 \leq 0}{\text{Max}} G(\varepsilon_1, \varepsilon_2) = \max \left\{ \left(\underline{n} - n_1, -\frac{\varepsilon_2 (n + \varepsilon_2)}{n + 3\varepsilon_2} \right) \right\}$$

and for $\varepsilon_1 < 0$,

$$(22) \quad \arg \underset{\varepsilon_2 \geq 0}{\text{Max}} G(\varepsilon_1, \varepsilon_2) = \begin{cases} \min \left\{ \left(\bar{n} - n_2, -\frac{\varepsilon_1 (n + \varepsilon_1)}{n + 3\varepsilon_1} \right) \right\} & \text{if } n + 3\varepsilon_1 > 0 \\ \bar{n} - n_2 & \text{if } n + 3\varepsilon_1 \leq 0 \end{cases}$$

Let $(\bar{\varepsilon}_1, \bar{\varepsilon}_2) \in \arg \underset{(\varepsilon_1, \varepsilon_2) \in C_1}{\text{Max}} G(\varepsilon_1, \varepsilon_2)$. Then

$$(23) \quad \bar{\varepsilon}_1 = \max \left\{ \underline{n} - n_1, -\frac{\bar{\varepsilon}_2(n + \bar{\varepsilon}_2)}{n + 3\bar{\varepsilon}_2} \right\} = \underline{n} - n_1$$

because if $\bar{\varepsilon}_2 = \bar{n} - n_2$, then

$$(24) \quad (\underline{n} - n_1) - \left\{ -\frac{\bar{\varepsilon}_2(n + \bar{\varepsilon}_2)}{n + 3\bar{\varepsilon}_2} \right\} = \frac{(\bar{n} - n_2)(\bar{n} + n_1)}{3\bar{n} - 2n_2 + n_1} + \underline{n} - n_1 > 0$$

when \bar{n} is sufficiently large and if $\bar{\varepsilon}_2 = -\frac{\bar{\varepsilon}_1(n + \bar{\varepsilon}_1)}{n + 3\bar{\varepsilon}_1}$ and $\bar{\varepsilon}_1 = -\frac{\bar{\varepsilon}_2(n + \bar{\varepsilon}_2)}{n + 3\bar{\varepsilon}_2}$, then

$$(\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (0, 0) \quad \text{or} \quad (-0.5n, -0.5n) \quad . \quad \text{Therefore} \quad (\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (\underline{n} - n_1, \bar{n} - n_2) \quad \text{or} \\ \left(\underline{n} - n_1, -\frac{\bar{\varepsilon}_1(n + \bar{\varepsilon}_1)}{n + 3\bar{\varepsilon}_1} \right). \quad \text{However, because}$$

$$(25) \quad n + 3\bar{\varepsilon}_1 = n_2 - 2n_1 + 3\underline{n} < 0$$

when \underline{n} is sufficiently close to zero, we must have $(\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (\underline{n} - n_1, \bar{n} - n_2)$.

We next show (b). Let $(\bar{\varepsilon}_1, \bar{\varepsilon}_2) \in \arg \max_{(\varepsilon_1, \varepsilon_2) \in C_2} G(\varepsilon_1, \varepsilon_2)$. By a similar argument,

$$(\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (\bar{n} - n_1, \underline{n} - n_2) \quad \text{or} \quad \left(-\frac{\bar{\varepsilon}_2(n + \bar{\varepsilon}_2)}{n + 3\bar{\varepsilon}_2}, \underline{n} - n_2 \right), \quad \text{but the latter solution is also}$$

possible if $n + 3\bar{\varepsilon}_2 = n_1 - 2n_2 + 3\underline{n} > 0$. **Q.E.D**

Proof of Proposition 1

Since $n_1 > n_2$, we have

$$(26) \quad G(\underline{n} - n_1, \bar{n} - n_2) - G(\bar{n} - n_1, \underline{n} - n_2) = \frac{(n_1 - n_2)(\bar{n}^2 - \underline{n}^2)(\bar{n} + \underline{n} - n_1 - n_2)^2}{(\bar{n} - \underline{n} + n_1 - n_2)^2(\bar{n} - \underline{n} - n_1 + n_2)^2} > 0$$

If $n_1 > 2n_2 - 3\underline{n}$, then we can verify that

$$(27) \quad \lim_{\underline{n} \rightarrow \infty} G(\underline{n} - n_1, \bar{n} - n_2) - G\left(\frac{(n_1 + \underline{n})(n_2 - \underline{n})}{n_1 - 2n_2 + 3\underline{n}}, \underline{n} - n_2\right) \\ = \frac{\underline{n}(2n_1 + 4n_2) - 9\underline{n}^2 + n_1(3n_1 - 4n_2)}{8\underline{n} + 4n_1 - 4n_2} > \frac{-\underline{n}(7n_1 - 4n_2) - 9\underline{n}^2 + 2n_1n_2}{8\underline{n} + 4n_1 - 4n_2} > 0$$

when \underline{n} is sufficiently small. Therefore, we must have $G(\varepsilon_1^*, \varepsilon_2^*) = G(\underline{n} - n_1, \bar{n} - n_2)$

when \underline{n} is sufficiently small and \bar{n} is sufficiently large. **Q.E.D**

In the next proposition, we obtain a sufficient condition for the values of \bar{n} and \underline{n} .

Proposition 2

Assume that $n_1 > n_2$, $\bar{n} = n_1 r$, $\underline{n} = \frac{n_2}{r}$, and $r > 4.5$. Then,

$$G(\varepsilon_1^*, \varepsilon_2^*) = G(\underline{n} - n_1, \bar{n} - n_2) = \frac{n_2 + n_1 r^2}{(1+r)^2},$$

$$x_1^* = \frac{n_2}{(1+r)^2}, \quad x_2^* = \frac{n_1 r^2}{(1+r)^2},$$

$$u_1^* = \frac{n_2 r}{(1+r)^2}, \quad u_2^* = \frac{n_1 r}{(1+r)^2}.$$

Proof of Proposition 2

We show that $r > 4.5$ suffices the assumption that $\underline{n} > 0$ is sufficiently close to zero and \bar{n} is sufficiently large.

We first consider the proof of Lemma 1, where (24) and (25) use this assumption. The left-hand side of (24) is reduced to

$$(\underline{n} - n_1) - \left\{ -\frac{\bar{\varepsilon}_2(n + \bar{\varepsilon}_2)}{n + 3\bar{\varepsilon}_2} \right\} = \frac{(n_1 r - n_2)(n_1 r^2 - 2n_1 r - n_1 + 2n_2)}{r[(3r+1)n_1 - 2n_2]}$$

which is positive if $r > 4.5$. The left-hand side of (25) is reduced to

$$n + 3\bar{\varepsilon}_1 = (r+3)\frac{n_2}{r} - 2n_1$$

which is negative if $r > 4.5$.

We next consider the proof of Proposition 1. It is enough to show that if

$n_1 > 2n_2 - 3\underline{n} = (2r-3)\frac{n_2}{r}$ and $r > 4.5$ then

$$G(\underline{n} - n_1, \bar{n} - n_2) - G\left(\frac{(n_1 + \underline{n})(n_2 - \underline{n})}{n_1 - 2n_2 + 3\underline{n}}, \underline{n} - n_2\right) > 0.$$

The left-hand side is reduced to

$$\frac{(r-1)}{4r(r+1)^2[r(n_1 - n_2) + 2n_2]} \cdot f(r)$$

where

$$f(r) = r^3 n_1 (3n_1 - 4n_2) + r^2 n_1 (n_1 + 2n_2) + r n_2 (2n_1 - 5n_2) + n_2^2$$

Note that $n_1 > (2r - 3)\frac{n_2}{r}$ and $r > 4.5$ imply

$$3n_1 - 4n_2 > (2r - 9)\frac{n_2}{r} > 0$$

Thus, $f''(r) > 0$ and

$$f'(r) > f'(1) = 11n_1^2 - 6n_1n_2 - 5n_2^2 > 0$$

because $n_1 > n_2$. Consequently,

$$f(r) > f(1) = 4(n_1^2 - n_2^2) > 0 \text{ if } r > 4.5.$$

Q.E.D

It is straightforward to verify that comparative statics with respect to the parameter r yields the following results: $\frac{\partial x_1^*}{\partial r} = -\frac{2n_2}{(1+r)^3} < 0$, $\frac{\partial x_2^*}{\partial r} = \frac{2n_1r}{(1+r)^3} > 0$ and

$$\frac{\partial G}{\partial r} = \frac{2(n_1r - n_2)}{(1+r)^3} > 0. \text{ The latter effect is due to the dominance of the positive effect}$$

of a change in r on the efforts of player 1 relative to its negative effect on the efforts of player 2. Also note that the equilibrium efforts are increasing in the prize valuations of the two players, however these efforts are smaller than n_1 as long as $n_1 > n_2$ and r is finite.

3. The N – player contest

With dual discrimination total efforts cannot exceed the prize valuation of contestant 1. That is,

Proposition 3

Under dual discrimination, in equilibrium, $n_1 \geq G$.

Proof of Proposition 3

In equilibrium, the net payoff of every contestant under dual discrimination cannot be negative, since otherwise he can improve his situation and secure a zero net payoff by not taking part in the contest. In other words, for every contestant i , $u_i = p_i(n_i + \varepsilon_i) - x_i \geq 0$. Summing over all the contestants, we get that

$$\sum_{j=1}^N u_j = \sum_{j=1}^N [p_j(n_j + \varepsilon_j) - x_j] \geq 0 \text{ or } \sum_{j=1}^N (p_j n_j) + \sum_{j=1}^N (p_j \varepsilon_j) - \sum_{j=1}^N x_j \geq 0. \text{ Since } G = \sum_{j=1}^N x_j$$

and $\sum_{j=1}^N (p_j \varepsilon_j) = 0$, $\sum_{j=1}^N (p_j n_j) \geq G$. Since $n_1 \geq n_2 \geq \dots \geq n_N$ and $\sum_{j=1}^N p_j = 1$, it must be

true that $n_1 \geq \sum_{j=1}^N (p_j n_j) \geq G$.⁵ **Q.E.D.**

Proposition 4

If $N = 2$ and $n_1 > n_2$, then the contest designer can always set r such that the equilibrium efforts are sufficiently close to n_1 . In this case $x_2^* \rightarrow n_1 - \underline{n}$, $u_1^* \rightarrow \underline{n}$ and $u_2^* \rightarrow 0$.

Proof of Proposition 4

It can be verified that $G(\varepsilon_1^*, \varepsilon_2^*) = \frac{n_2 + n_1 r^2}{(1+r)^2} < n_1$ and $\lim_{r \rightarrow \infty} G = n_1$. Since G is continuous and monotone in r , there exists r such that G is sufficiently close to n_1 .

Q.E.D

By Proposition 3, we have shown that the contestants cannot be induced to exert larger efforts than n_1 . Therefore, Proposition 4 can be extended to the case of any number of contestants N . In the more general multi-player contest, the designer has to reduce the stakes of $N - 2$ contestants to zero, making sure that contestant 1 with the highest stake is not included among them. That is,

⁵ This result is also valid for unbiased contests or for contests with alternative modes of discrimination, when discrimination is through modifying the efforts. The explanation is that in equilibrium, the net payoff of every contestant cannot be negative, since otherwise he can improve his situation and secure a zero net payoff by not taking part in the contest. In other words, for every contestant i ,

$u_i = p_i n_i - x_i \geq 0$. Summing over all the contestants, we get that $\sum_{j=1}^N u_j = \sum_{j=1}^N (p_j n_j - x_j) \geq 0$ or

$\sum_{j=1}^N (p_j n_j) \geq \underbrace{\sum_{j=1}^N x_j}_G$. Since $n_1 \geq n_2 \geq \dots \geq n_N$ and $\sum_{j=1}^N p_j = 1$, it must be true that $n_1 \geq \sum_{j=1}^N (p_j n_j) \geq G$.

Corollary 1

Given any number of contestants N , such that $\infty > \bar{n} \geq n_1 \geq n_2, \dots, \geq n_N \geq \underline{n} > 0$, the contest designer can always set r such that the equilibrium efforts are sufficiently close to n_1 .

Proof of Corollary 1

The proof is based on the following simple three-stage strategy that the designer applies:

1. *Stage 1*: The designer selects a contestant $j \in \{2, \dots, N\}$.
2. *Stage 2*: For any contestant $i \neq \{1, j\}$, the designer chooses $\varepsilon_i = -n_i$. That is, he reduces the initial prize valuations of $N - 2$ players to zero
3. *Stage 3*: Applying the dual discrimination strategy with respect to the two contestants 1 and j , according to Proposition 4, the designer can induce efforts that are almost equal to n_1 .

Q.E.D.

The explanation of our results is based on the following idea. First, assume without loss of generality that $j = 2$. On the one hand, the designer applies direct discrimination in favor of contestant 2 by reducing the stake of contestant 1 (the contestant with the initially higher prize value) to \underline{n} by choosing ε_1 and increases the stake of contestant 2 (the contestant with the initially lower prize value) to \bar{n} by choosing ε_2 . On the other hand, in order to satisfy the balanced-budget constraint, the designer must create an appropriate bias in favor of contestant 1 by selecting δ , such that the balanced-budget constraint (10) satisfies equation (12):

$$(12) \quad \delta = - \left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2} \right) \frac{\varepsilon_1}{\varepsilon_2}$$

Propositions 1 and 2 imply that dual discrimination, that is, $\varepsilon_1 = \underline{n} - n_1$ and $\varepsilon_2 = \bar{n} - n_2$, is an effective strategy for increasing efforts. These efforts can be increased almost up to n_1 , the initial higher prize valuation of contestant 1, provided that $r \rightarrow \infty$.

Propositions 1 and 2 imply that when the designer applies the two modes of discrimination, each type has a positive “added value” that enhances the exertion of

efforts relative to the situation where the designer resorts to just structural discrimination (Franke et al. (2013, 2014a, 2014b)). That is, the two modes of discrimination are supportive or “complementing” - their combination can yield larger efforts than those obtained by just structural discrimination. The advantage of combining these two types of discrimination relative to the use of just structural discrimination is due to the distinctive features of the contribution of each of these modes of discrimination to the exerted efforts as described below.

- (i) Direct discrimination sufficiently increases the initially lower prize valuation while sufficiently reducing the initially higher prize valuation. This increases the sum of the contestants’ prize valuations to infinity when $r \rightarrow \infty$ and makes the ‘income effect’ (associated with a scheme that increases the sum of the final stakes from $(n_1 + n_2)$ to $(n_1 + \varepsilon_1 + n_2 + \varepsilon_2) = \bar{n} + \underline{n}$) of this mode of discrimination the dominant effect.⁶
- (ii) The maximal possible increase in the sum of the contestants’ prize valuations is not the result of direct discrimination alone. It is rendered possible by structural discrimination that makes sure that the balanced-budget constraint is satisfied. Specifically, structural discrimination counterbalances the above ‘income effect’ by almost completely favorably discriminating contestant 1, ensuring that his prize share converges to 1. The moderating effect described in (ii) is necessary to attain the maximal efforts. While structural discrimination has a ‘second order’ effect on efforts that moderates the income effect of direct discrimination, it also enables the dominance of this ‘first order’ income effect on efforts described in (i), namely, the increase in efforts due to the increase in the sum of the contestants’ prize valuations. The dominance of the effect of direct discrimination means that the more extreme this mode of discrimination, the higher the total efforts and this requires the extremity of structural discrimination.

Corollary 1 implies, in particular, that dual discrimination can be advantageous relative to just structural discrimination in the 2-player all-pay auction or the simple lottery contest.

⁶ For a clarification of the meaning of the income effect associated with direct discrimination, see the discussion following Proposition 2 in Mealem and Nitzan (2014).

Corollary 2

If $N = 2$ and $n_1 > n_2$, then there exists r such that dual discrimination yields efforts that are larger than the efforts obtained in the optimally biased 2-player all-pay auction (simple lottery) contest with structural discrimination. That is,

$$\frac{n_2 + n_1 r^2}{(1+r)^2} > 0.5(n_1 + n_2) > 0.25(n_1 + n_2) \text{ or } r > \frac{k+1 + \sqrt{2(k^2+1)}}{k-1}.$$

Note that our results imply that under the assumption that \bar{n} is sufficiently large and \underline{n} is sufficiently close to zero ($r \rightarrow \infty$), the combination of the two modes of discrimination results in an outcome which is practically reasonable; the amount transferred between the contestants is finite. When the actual payment to contestant 2 (the tax taken from contestant 1) is equal to $p_2 \varepsilon_2 \rightarrow n_1$, since δ converges to zero, that is, p_2 converges to zero, it seems that contestant 2 has no incentive to compete. But this is not the case because he plays a crucial role in yielding the almost maximal possible efforts n_1 . The designer sufficiently increases his prize valuation and by that induces him to exert efforts that are almost equal to n_1 .

In the first example of a *municipal project* (see the examples discussed in section 2), despite the fact that both modes of discrimination can be extreme, their combined use still results in a balanced effect. The designer promises the small company a very large value in case it receives the entire project. The designer, by structural discrimination, ensures that the small company's share in the project is sufficiently small, such that the large reputable company wins almost the entire project. In this extreme and most effective case from the designer's point of view, the large company transfers the small company a reasonable finite amount which is almost equal to its initial valuation of the entire project.

In the second example of *portfolio distribution between two investment houses* in the extreme case the tax-subsidy transfer between the two investment houses under the optimal dual discrimination strategy is finite. In particular, by Proposition 1, it is equal almost to n_1 , the value for investment house 1 of getting the entire portfolio of the large customer. Again, this result is plausible not because of the good will of the customer, but because it ensures him that the two investment houses exert almost the maximal efforts.

5. Conclusion

The enrichment of the “box of tools” of the contest designer, that is, allowing him to exercise dual discrimination instead of just structural discrimination clearly *matters*. We have shown that both modes of discrimination are effective and therefore will be used by the designer possibly yielding almost the largest possible efforts that are equal to the initially highest prize valuation, when $r \rightarrow \infty$. When the extent of discrimination is sufficiently high, the efforts in our setting are larger than those obtained in a biased lottery contest allowing just structural discrimination, Franke et al. (2013, 2014a, 2014b), While in the latter context, at least three players are active, under dual discrimination, there are just two active contestants; the contestant with the highest prize valuation and another contestant who can be any of the other contestants. The exclusion principle noted in Baye et al. (1999) the elimination of the strongest player - is thus not valid, not only in the unbiased lottery contest without discrimination, as shown by Fang (2002), but also in our biased lottery contest with dual discrimination. The inclusion principle is, however, valid, as in Franke et al. (2013); a weak contestant, who is inactive in the unbiased case of Fang (2002), can be encouraged to become active. Finally, under our biased lottery contest with dual discrimination, efforts can be larger than those under any alternative contest game and, in particular, an all-pay auction. This is in contrast to the main results in Franke et al. (2014a, 2014b), where efforts in the optimally biased N -player all-pay auction contest with structural discrimination and an optimal Head-starts all-pay auction are strictly larger than those obtained in the optimally biased simple lottery contest.

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