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Abstract

The paper shows that taking inventory control out of the hands of retailers and assigning it to an intermediary increases the value of a supply chain when demand volatility is high. This is because an intermediary can help solve two incentive problems associated with retailers' inventory control and thereby improve the intertemporal allocation of inventory. Adding an intermediary as a new link in a supply chain is also shown to reduce total inventory, to make shipments from the manufacturer less frequent and more variable in size, as well as to reduce social welfare.

JEL classification: L11, L12, L22, L81

Keywords: intermediation, inventory, demand volatility, supply chain

1 Introduction

The optimal control of inventory is one of the greatest challenges faced by firms in a supply chain. Consider, for instance, a supply chain, in which a manufacturer distributes goods through retailers who hold inventory because they have to order and take delivery of goods before observing the state of demand and selling to consumers. A key problem in this setting, as explained by Krishnan and Winter (2007), is that the retailers' incentives to hold inventory are generally not aligned with the manufacturer's interests. Hence the challenge is how to solve such incentive problems so that the supply chain's value can be maximized.

In the current paper, we offer two main insights into this issue. First, we identify two specific incentive problems associated with retailers' inventory control that arise from the intertemporal allocation of inventory. Second, we show how these incentive problems may be mitigated by taking inventory control out of the hands of retailers and assigning it to an intermediary. In particular, we derive conditions under which adding an intermediary in the supply chain to control inventory raises the chain's value. Our model of intermediation based on inventory control allows us then to derive predictions about the use of intermediaries in supply chains and about the associated pattern of shipments and inventories, as well as to examine the normative implications of using intermediaries to control inventory.

Studying the role of intermediaries in controlling inventory is relevant and interesting, not least because we observe a general trend toward shifting inventory control from retailers to intermediaries in many retail industries. Three developments are particularly noteworthy in this respect, namely 'drop-shipping', 'inventory consignment' and 'vendor-managed inventory'. Drop-shipping is an arrangement where (mostly internet) retailers forward buyers' orders to a wholesaler who then ships the product from its own inventory; this arrangement allows retailers to avoid holding any inventory while providing a large choice of varieties to consumers.[1] Inventory consignment allows an upstream firm to own inventories held by downstream firms while vendor-managed inventory (VMI), an increasingly popular arrangement thanks in part to electronic sales and inventory tracking technologies, allows an upstream firm to control these inventories.[2]

[1] See Randall, Netessine and Rudi (2006) for a recent comparison of drop shipping with the classic case of retailer-controlled inventory. Companies engaged in this practice include Alliance (www.aent.com), the largest distributor of home entertainment audio and video in the US; Baker and Taylor (www.baker-taylor.com), a leading distributor of books, videos and music products; ChemPoint (chempoint.com) in the chemical industry, and Garden.com in the garden supply retail industry; see Netessine and Rudi, 2004. Retailers such as Staples are both traditional retailers holding inventories and engage in drop-shipping for out-of-stock items; see Randall et al. 2002.

[2] See Govindam (2013) for a survey, and Mateen and Chatterjee (2015). Wal-Mart pioneered VMI with Procter and Gamble in the early 1980s; early adopters include KMart

These practices are part of a more general trend to take inventory control out of the hands of downstream firms and to deliver goods to them 'just in time'. This trend is reflected in the growing use of third party logistics (3PL) providers and their increasingly important role in handling inventory. Warehousing and distribution activities represented 23.7% of the US 3PL market of $158 billion (3PL gross revenue) in 2014, and the importance of these activities is likely to increase in the future.[3] Not only is the 3PL market expected to grow at an annual rate of 4.4% between 2015 and 2022,[4] but the number of supply chains wanting 3PL providers to deal with their warehousing and distribution needs has been growing as well year after year.[5] In effect 3PL providers have made themselves indispensable to supply chains, transforming themselves from being used on demand to establishing stable and multi-year contractual arrangements with them. As a result the 3PL industry has experienced significant consolidations, especially concerning the most successful 'value-added warehouse distribution providers' (Excel, UPS SCS, Kenco, Genco, Jacobson and DSC) which were growing especially fast in the early 2000s (Foster and Armstrong, 2004) and which have now been largely absorbed by large 3PL providers.[6]

To construct a theory of intermediation focusing on the incentive issues associated with inventory control one needs to put aside a variety of other aspects that have undoubtedly also contributed to the success of drop-shipping, inventory consignment, VMI and, more generally, the inventory management activities of intermediaries like the 3PL providers. In particular, we ignore economies of scale in inventory management that might give intermediaries an advantage over retailers because they pool products from different firms. We also ignore advantages coming from complementarities between inventory management and other specialized services such as transportation, as well as technological advantages that intermediaries may have, for instance, in inventory tracking technology that may lower their variable

and JCPenney. Procter and Gamble now uses this practice worldwide; see datalliance.com. Some of the intermediaries mentioned in the previous footnote also use VMI.

[3] Of course, domestic and international transportation management still represents the main activity of logistics providers with 66% of US 3PL's gross revenue; see http://www.3plogistics.com/3pl-market-info-resources/3pl-market-information/us-3pl-market-size-estimates/. On the size of 3PL's market, see also World Bank (2014).

[4] See https://www.gminsights.com/industry-analysis/third-party-logistics-3pl-market-size.

[5] For instance, 25% of users wanted their 3PL providers to handle inventory management in 2016 compared to 21% in 2012, while order management and fulfillment represented respectively 19% in 2016 and 14% in 2012; see Third-Party Logistics Study 2012 and 2016. For the 2016 report, see https://www.kornferry.com/media/sidebar_downloads/2016_3PL_study.pdf.

[6] Excel has become DHL Supply Chain in 2016; Genco has been absorbed by FedEx in 2015; Jacobson was absorbed by Norbert Dentressangle in 2014 which in turn was absorbed by XPO Logistics in 2015; Kenco, a North American provider, has partnered with Hermes, a large European logistics provider for its international business.

costs of managing inventory relative to retailers. Each of these features has the potential of reinforcing the role played by intermediaries. By ignoring them we choose to stack the model against intermediation in order to isolate how intermediaries may help solve the incentive issues associated with inventory control.

The model used for this purpose is a standard model of a supply chain, in which a manufacturer distributes goods through retailers. What is new is that the manufacturer has to decide whether to assign inventory control to retailers or to an intermediary. A key aspect of our theory rests on the assumption that the intermediary possesses market power relative to retailers. This is consistent with Spulber (1999) who argues that any theory of intermediation requires intermediaries to have market power relative to other market participants so that they can set prices and balance supply and demand across time by holding inventory. Balancing demand and supply by standing ready to buy goods and sell them at different points in time, is exactly the role we assign to the intermediary. We model the asymmetry in market power between retailers and intermediary in the simplest possible way, namely by assuming that retailers are perfectly competitive, whereas the intermediary is a monopolist.[7]

We show that market power gives the intermediary two advantages over competitive retailers in controlling inventory. The first advantage has to do with the intertemporal allocation of inventory. Competitive retailers allocate inventory so that today's retail price is equalized with tomorrow's expected retail price. We refer to this as *intertemporal market integration.* An intermediary with market power, by contrast, allocates inventory to equalize today's marginal revenue with tomorrow's expected marginal revenue. This *intertemporal market segmentation* implies that retail prices can adjust more readily when demand conditions differ across periods; that is, it improves intertemporal price discrimination.

The second advantage of the intermediary arises when goods can be reordered. Competitive retailers are ready to sell inventory carried over from the past at any positive retail price, simply because the cost of these units has already been sunk. This means that the residual demand faced by the manufacturer today is reduced and so is the manufacturer's price. In effect, the manufacturer competes with the inventory carried over by retailers from the past. To limit this *inventory competition* the manufacturer would have to keep shipments to retailers small in each period, but by doing so he runs the risk of losing sales due to stockouts. We show that by assigning inventory control to an intermediary the manufacturer may limit *inventory competition*, while at the same time avoiding stockouts.

Simply put then, the intermediary's incentives to control inventory are

[7] See Deneckere et al. (1996) for an influential paper that adopts a similar vertical structure without intermediation to study incentive issues in inventory control.

better aligned on two counts with the interests of the manufacturer. Our model predicts that the intertemporal misallocation associated with *intertemporal market integration* and with *inventory competition* becomes worse the higher is the variance of demand. Thus, from the manufacturer's point of view an intermediary is especially useful in markets where final demand is very volatile. We also show that, consistent with the intermediary's role in reducing *inventory competition*, a shift in inventory control from retailers to the intermediary reduces total inventory holdings and decreases social welfare.

By constructing a theory of intermediation based on inventory control our paper directly contributes to the market microstructure literature that seeks to understand the role of intermediaries in market clearing. While this literature has recognized the role that inventory plays in helping intermediaries match supply and demand intertemporally, it does not examine the incentive problems that inventory control entails.[8] Studying these incentive problems is the domain of the literature on vertical control in industrial organization and the management literature on supply chain coordination.[9] In particular, Krishnan and Winter (2007) and Deneckere et al. (1996) explain that the price system generically fails to align retailers' incentives with those of the manufacturer as soon as inventory control is involved. Using a vertically integrated supply chain as benchmark, and thus one in which all incentive problems are solved, the approach of Krishnan and Winter (2007, 2010) is to identify contract forms, such as vertical price controls and buy-back policies, that would permit a decentralized supply chain to achieve the vertically integrated solution. Importantly, the structure of the supply chain is held fixed by assuming that inventory can only be held by retailers. Thus this literature does not examine how, within a supply chain, an intermediary may help solve incentive problems associated with inventory.

Our paper differs from the literature on at least two counts: first, the intertemporal incentive problems we study are different from those examined in the papers above. In Krishnan and Winter (2007) and Deneckere et al. (1996) inventory perishes after one period, so intertemporal problems do not even arise. In Krishnan and Winter (2010) inventory does not perish so quickly, which allows them to explicitly study intertemporal incentives associated with retailer controlled inventory. But the incentive problems in their paper arise from the assumption that the level of inventory held by a retailer directly raises consumer demand, because a big inventory signals

[8] See Spulber (1999) for a survey. The optimal control of inventory in a supply chain is a classic problem of both economics and management science. The analysis of optimal order policies and inventory levels goes back to the seminal contribution of Arrow, Harris and Marshak (1951). Clark and Scarf (1960) were the first to establish an optimal inventory policy in a multi-echelon supply chain.

[9] See Krishnan and Winter (2011) for a synthesis of the theory of contracts in supply chains.

to consumers that the retailer is more likely to have the product in stock; hence retailers may hold too little inventory from the manufacturer's point of view. In our model, by contrast, the manufacturer is mostly concerned about retailers holding, if anything, too much inventory, both because *intertemporal market integration* implies they may carry more inventory forward than would be required for optimal price discrimination, and because unsold goods create *inventory competition*.

Second, rather than holding the structure of the supply chain fixed and looking for contracts that achieve the vertically integrated solution, we ask what happens if inventory control is passed from retailers to an intermediary. By showing that assigning inventory control to an intermediary may raise the aggregate profit of the supply chain, our model may explain why, increasingly, intermediaries act as an important link in supply chains, especially if, in addition to inventory control, they offer complementary services such as transportation and logistics coordination.

One of the intertemporal incentive problems identified by our paper, namely the inventory competition problem, is closer in spirit to that faced by a storable goods monopolist (Dundine et al., 2006): if consumers anticipate an increase in future demand and thus higher future prices, they stockpile goods today, which reduces future residual demand and forces the monopolist to cut future prices.[10] The consequences of strategic consumer behavior like this for supply chains have been investigated by a number of papers in the management science literature (see Krishnan and Winter, 2011, for a discussion). But, unlike in our paper, their focus is neither on the intertemporal incentive issues involved in inventory control, nor on the potential benefits of shifting inventory control from retailers to an intermediary.

The rest of the paper is organized as follows. In Section 2 we present our model and the benchmark equilibrium for the case in which inventory is controlled by retailers. We go through two versions of the benchmark case. In the first, restricted version retailers order and take possession of goods only once and then allocate goods across time. This restricted version allows us to highlight the problem of *intertemporal market integration*. In the second version, we allow retailers to order goods in each period which leads to the problem of *inventory competition*. We mainly use the restricted version as a device to distinguish the two problems highlighted by the analysis: intertemporal market integration and inventory competition. In Section 3 inventory control is passed to an intermediary. In Section 4 we show how the intermediary helps solve these problems and what this implies for the use of intermediaries, order and inventory patterns, as well as social welfare.

[10] Notice also the connection with the durable goods monopoly problem (Bulow, 1982) where the monopolist has an incentive to cut price in the future, once consumers with the highest willingness to pay have been served.

Section 5 contains conclusions, and the Appendix collects the proofs of our results.

2 Model and Benchmark Cases

Consider a manufacturer producing and selling goods before demand is known. The manufacturer receives orders from and ships products either directly to competitive retailers (in the absence of intermediation), or to a single intermediary. Once demand has been revealed, the retailers then sell to consumers if they hold the products, or in the case of intermediation, the intermediary sells to retailers who then sell to consumers. This highlights two key differences between the case with and without intermediation. First, in the absence of intermediation, the retailers hold inventories, that is to say the units received from the manufacturer and stored before they are sold; otherwise inventory is held by the intermediary. Second, intermediation involves a single inventory holder, whereas in the absence of intermediation, inventories are held by many retailers.

There are two sales periods, $t = 1, 2$, which means that the product under consideration loses its value after two periods. Demand at time $t = 1, 2$ is given by the linear inverse demand function: $p_t = A - s_t + \varepsilon_t$, where p_t is the retail price and s_t denotes final sales. The random variables $\varepsilon_t \in [-d, d]$ are intertemporally independent and uniformly distributed with density $1/2d$.

The order of moves when the manufacturer sells directly to retailers is as follows. At the beginning of period 1, the manufacturer announces a producer price P_1, retailers order and take possession of q_1 units of goods before demand in period 1 is known; then period-one demand is revealed and the retailers sell $s_1 \leq q_1$ in period 1, holding unsold units in inventory for period 2. In period 2, the manufacturer sets producer price P_2, and retailers order quantity q_2, again before period-two demand is known.[11] Demand in period 2 is then revealed and retailers sell $s_2 \leq q_2 + (q_1 - s_1)$.

When dealing with an intermediary the manufacturer may use two-part tariffs, consisting of a producer price, P_t, and a fixed payment (or transfer), T_t for $t = 1, 2$.[12] The timing of moves is then as follows: at the beginning of period 1 the manufacturer sets a two-part tariff (P_1, T_1), the intermediary orders and takes possession of quantity q_1. Demand in period 1 is then revealed, the intermediary chooses wholesale price w_1, retailers purchase from the intermediary and sell to consumers a quantity $s_1 \leq q_1$. In period 2 the manufacturer chooses the two-part tariff (P_2, T_2), and the intermediary orders a quantity q_2. Then demand in period 2 is revealed, the intermediary

[11] We also consider the restricted case with a single order in period 1 covering both sales periods; see below.

[12] In principle the manufacturer could also use two-part tariffs when it sells directly to retailers, as could the intermediary in its dealings with retailers. But perfect competition in retailing implies that the transfer in each case would be equal to zero in equilibrium.

sets wholesale price w_2, retailers order and sell $s_2 \leq q_2 + (q_1 - s_1)$. Notice our implicit assumption that the intermediary only ships to retailers in each period exactly what they sell, i.e., s_t in period $t = 1, 2$. Any inventory of unsold goods in period 1, $q_1 - s_1$, is thus held by the intermediary. This feature of the model mirrors the arrangements under drop-shipping and vendor-managed inventory, where retailers do not exercise any control over inventory.[13]

Our assumptions about pricing strategies (two-part tariffs when the manufacturer deals with the intermediary, simple unit prices otherwise) imply that there is no double marginalization and that the entire expected profit generated by the supply chain goes to the manufacturer. They also imply that the manufacturer has nothing to gain from having more than one intermediary.[14]

Our assumptions about the production and distribution technologies are as simple as possible. The manufacturer incurs a constant unit cost of production c. We let c_w denote the per-unit cost of intermediation. The marginal cost of retailers is normalized to zero, as is the cost of holding inventory. There is also no discounting between periods. All market participants are risk neutral. Hence importantly, intermediation is not associated with economies of scale and even involves a cost disadvantage relative to direct sales to retailers.

We also assume $A > c + c_w$, and that the demand shock is not too big, $d \leq \bar{d}$, so that equilibrium prices and quantities in each period are always non-negative in all the environments considered in the analysis.[15] Importantly, the latter assumption implies that in equilibrium all inventory remaining in period 2 is being sold at a positive price and thus that destructive competition (Deneckere et al., 1996, 1997), another source of (static) incentive problems arising from retailer controlled inventory, is assumed away. Allowing for these additional problems would obscure the intertermporal incentive issues that we focus on.

We now consider the equilibrium in which inventory is controlled by retailers. We find it useful to start the analysis with the restricted case in

[13] One way to think further about intermediation which is implicit in the timing of deliveries is that the intermediary is physically located closer to the retailers than it is to the manufacturer.

[14] The assumption of perfectly competitive retailers is also not overly restrictive in the sense that one can view the competitive outcome as the limit of a a sequential game among oligopolistic retailers as the number of retailers gets large. In stage one retailers order inventory, each taking the quantity of the other retailers as given (Cournot competition). In stage two, after observing the true realization of demand, retailers simultaneously announce retail prices. The outcome of the subgame perfect equilibrium of this game converges to the perfectly competitive outcome as the number of retailers goes to infinity. See Tirole (1988), ch. 5 and the references cited there for the relevant convergence results.

[15] Of course, $\bar{d}$ depends on the other parameters of the model, A, c and c_w, in a way that is specific to each environment; see Appendix 1.

which retailers make a single order and take delivery at the beginning of period 1 only, and then analyze the case where retailers are free to order and to receive delivery at the start of each period. Investigating these two cases separately allows us to isolate the two incentive issues that arise when retailers manage inventory, namely *intertemporal market integration* and *inventory competition*.

2.1 Intertemporal Market Integration

Consider first the case in which retailers may order goods only once. Thus at the beginning of period 1 the manufacturer announces a producer price P. Retailers order and take possession of q units of goods before demand in period 1 is known; then period-one demand is revealed and the retailers sell $s_1 \leq q$, holding unsold units in inventory for period 2. In period 2, retailers sell $s_2 \leq q - s_1$.

To derive the equilibrium suppose the demand shock ε_1 has been observed by retailers in period 1. Retailers then sell in period 1 as long as the first period retail price, p_1, exceeds the expected second-period retail price, $E(p_2)$; otherwise, they hold on to goods for sale in period 2, where all remaining inventory is sold. Hence, in equilibrium, $p_1 = E(p_2)$, or $A - s_1 + \varepsilon_1 = E(A - s_2 + \varepsilon_2) = A - s_2$. This is what we mean by retailers engaging in *intertemporal market integration*.

Given that $s_1 + s_2 = q$, we have $s_1 = (q + \varepsilon_1)/2$ and $s_2 = (q - \varepsilon_1)/2$. The first-period retail price and expected second-period retail price are then both equal to $p(q, \varepsilon_1) = \frac{2A-q+\varepsilon_1}{2}$. This has the following implications for the manufacturer's expected equilibrium output, prices and profits, where '$\sim$' denotes equilibrium values and the superscript 'r' refers to retailer controlled inventory:

Proposition 1 *Suppose inventory is controlled by retailers and there is no re-ordering. Intertemporal market integration implies that the producer price and the expected retail prices in both periods are equal to the static monopoly price,* $\tilde{P}^r = E(\tilde{p}^r) = \frac{(A+c)}{2}$*. The manufacturer's equilibrium output,* $\tilde{q}^r = A - c$*, and expected equilibrium profit,* $E(\tilde{\pi}^r) = \frac{(A-c)^2}{2}$*, are equal to the sum over two periods of the static monopoly outputs, respectively static monopoly profits.*

Proof: see Appendix 2.

Competition among retailers ensures that retailers equalize first-period retail price with second-period expected retail price. Thus with a single order to cover both sales periods and inventory controlled by retailers, the manufacturer completely gives up control of the intertemporal allocation of the units he sells and there is nothing he can do to exploit demand differences across periods: the market in period one and two are ex ante identical from the manufacturer's point of view. Not surprisingly then the outcome is equivalent to the static monopoly situation.

2.2 Inventory Competition

Consider now the case where retailers are free to order goods in each period. The first thing we show is that it makes a difference for the producer price and expected retail price in period 2 whether retailers carry unsold goods into the period ($s_1 < q_1$), or whether they have sold all their initial inventory ($s_1 = q_1$). In the former case the manufacturer faces *inventory competition* and is forced to reduce producer price P_2 below the static monopoly price. In particular, we can show:

Proposition 2 *Suppose inventory is controlled by retailers. The second-period producer price and expected retail price are lower than the static monopoly price if there is inventory competition, and is equal to the static monopoly price otherwise. Specifically*

$$P_2 = E(p_2) = \begin{cases} \frac{A+c}{2} - \frac{(q_1 - s_1)}{2} & \text{if } s_1 < q_1, \\ \frac{A+c}{2} & \text{otherwise.} \end{cases}$$

Proof: see Appendix 2.

How does the manufacturer optimally reduce *inventory competition*? To understand this notice how much retailers with a given amount of inventory in period 1 choose to sell in period 1 and how much inventory they keep for period 2, after they have observed ε_1. As we know from the previous subsection, retailers engage in intertemporal market integration by selling a quantity s_1 so that the retail price in period 1 equals the expected retail price in period 2, that is, $p_1 = E(p_2)$, or

$$p_1 = A - s_1 + \varepsilon_1 = \frac{A + c - q_1 + s_1}{2} = E_2(p_2). \tag{1}$$

Given the level of q_1 held by retailers, (1) offers two possible outcomes for s_1. Either $s_1 < q_1$ so that there is inventory competition in period 2, or $s_1 = q_1$ meaning that retailers stock out and do not carry any unsold goods into period 2. Which solution is relevant depends on the value of ε_1. Specifically,

$$s_1 = \begin{cases} \frac{A-c+2\varepsilon_1}{3} + \frac{1}{3}q_1 < q_1 & \text{if } \varepsilon_1 < \hat{\varepsilon}_1, \\ q_1 & \text{otherwise,} \end{cases} \tag{2}$$

where

$$\hat{\varepsilon}_1 = q_1 - \frac{A-c}{2}. \tag{3}$$

This shows how the manufacturer may reduce the likelihood of inventory competition, namely by reducing q_1, which decreases $\hat{\varepsilon}_1$. But obviously there is a trade-off, since by reducing q_1 the manufacturer not only limits inventory competition, but also raises the likelihood that a stockout occurs and sales are lost. The following proposition shows how the manufacturer optimally deals with this trade-off, where '$\smile$' denotes equilibrium values:

Proposition 3 *Suppose inventory is controlled by retailers. The manufacturer reduces inventory competition by selling in period 1 more than the one-period static monopoly output, namely* $\breve{q}_1^r = \frac{(A-c)}{2} + \frac{d}{5}$*, and by incurring a 40% probability that retailers stock out.*

Proof: See Appendix 2.

To understand the intuition behind the manufacturer's choice of q_1, it is useful to rewrite the manufacturer's first-order condition derived in Appendix 2 (see (14)) as follows:

$$2\left(A-c-2q_1\right)+7\int_{\hat{\varepsilon}_1}^{d}\left(A-c-2q_1+\frac{8}{7}\varepsilon_1\right)\frac{1}{2d}d\varepsilon_1=0. \tag{4}$$

The first term represents the maximization condition for expected profits assuming there is no possibility of a stockout. In this situation the unsold inventory carried by retailers into period 2 causes inventory competition, leading to a lower second-period producer price and to lower retail prices. This term would be equal to zero at the static monopoly output of $q_1 = (A-c)/2$. That is, if there were no possibility of a stockout, the manufacturer would simply deliver the unconstrained optimal monopoly quantity for period 1, because shipping so little reduces inventory competition. The second term reflects the adjustment in the maximization condition that has to be made to account for the possibility of a stockout. Here it is clearly seen that the static monopoly output is insufficient to maximize profit. Instead the manufacturer would want to choose a higher output in period 1. The probability of a stockout being less than 50% is then the by-product of shipping more than the static monopoly quantity in period 1.

For period 2, however, we can show that the expected output is smaller than the static monopoly output. Overall, we obtain the following result for the manufacturer's expected total output and total profit:

Proposition 4 *Suppose inventory is controlled by retailers. The manufacturer's total expected equilibrium output,* $E(\breve{q}^r) = \breve{q}_1^r + E(\breve{q}_2^r) = (A-c) + \frac{2d}{25}$*, and expected equilibrium profit,* $E\left(\breve{\pi}^r\right) = \frac{(A-c)^2}{2} + \frac{d^2}{25}$*, exceed the sum over two periods of the static monopoly output, respectively static monopoly profit.*

Proof: See Appendix 2.

Re-ordering allows the manufacturer to set P_2 after observing the demand in period 1. Since $P_2 = E(p_2) = p_1$ due to retailers' competition, the manufacturer's price in period 2 is different than his period 1 price which satisfies $P_1 = E(p_1)$. This means that the two periods are not the same for the manufacturer, leading to intertemporal price discrimination and higher expected profits than the sum over two periods of the static monopoly profits.

This section has shown that, when retailers control inventory, intertemporal market integration and inventory competition place specific constraints on a manufacturer in a two-period environment. In the next Section, we consider the case where the manufacturer uses an intermediary with the mandate to control inventory and to respond to orders from the retailers.

3 Inventory Control by an Intermediary

Like in our previous analysis, it is useful to start with the restricted case where the intermediary may order only once at the beginning of period 1 (no-re-ordering case) before turning to the unrestricted case where the intermediary is free to order in both periods (re-ordering case).

3.1 The No-Re-ordering Case

In this restricted case the manufacturer sets a two-part tariff (P, T) at the beginning of period 1, and the intermediary orders and takes possession of quantity q. Demand in period 1 is then revealed, the intermediary chooses wholesale price w_1, retailers purchase from the intermediary and sell to consumers a quantity $s_1 \leq q$. In period 2 the intermediary sets wholesale price w_2, retailers order from the intermediary and sell $s_2 \leq q - s_1$.

The key difference compared to the case of retailer controlled inventory lies in the way the intermediary allocates q across the two periods. To see this consider period 2. After the realization of ε_2 has been revealed and given a wholesale price w_2, retailers order and then sell an amount s_2 so that the second-period retail price equals the marginal cost faced by retailers; that is $(A - s_2 + \varepsilon_2) = w_2$. The intermediary's expected period-two revenue is thus equal to $E\left[(A - s_2 + \varepsilon_2)\, s_2\right] = (A - s_2)\, s_2$, and the expected marginal revenue is $E\left(MR_2\right) = A - 2s_2$.

Similarly, in period 1, after ε_1 has been revealed, the intermediary sets a wholesale price w_1 and retailers purchase and sell quantity s_1, such that the first-period retail price equals w_1, $(A - s_1 + \varepsilon_1) = w_1$. The intermediary's revenue hence is equal to $(A - s_1 + \varepsilon_1)\, s_1$, and the corresponding marginal revenue is $MR_1 = A - 2s_1 + \varepsilon_1$.

In period 1, given the revealed demand shock ε_1, the intermediary allocates output across periods until the marginal revenue in period 1 is equal to expected marginal revenue in period 2, that is until

$$A - 2s_1 + \varepsilon_1 = A - 2\left(q - s_1\right). \tag{5}$$

This is what we mean by *intertemporal market segmentation*: by equalizing marginal revenues, the intermediary generally causes retail prices to differ across periods. In other words, the intermediary engages in intertemporal price discrimination independently of re-ordering. This has the following consequences for the manufacturer's equilibrium output and expected profit:

Proposition 5 *Suppose inventory is controlled by an intermediary and there is no re-ordering. Intertemporal market segmentation implies that the manufacturer's equilibrium output,* $\tilde{q}^i = A - c - c_w$*, is equal to the sum over two periods of the static monopoly output, and expected equilibrium profit,* $E\left(\tilde{\pi}^i\right) = \frac{(A-c-c_w)^2}{2} + \frac{d^2}{24}$*, exceeds the sum of static monopoly profits.*

Proof: see Appendix 2.

Several comments are in order. First, the fact that the manufacturer's equilibrium output is the same (at least net of c_w) with and without intermediation when a single order is involved is not surprising. The intermediary is exactly in the same *ex ante* position as the manufacturer, and the two-part tariff ensures there is no inefficiency associated with the vertical structure. Second, the manufacturer's expected profit is greater with intermediation (again at least if c_w is low enough) because the intermediary is able to price discriminate across periods.[16] Third, the benefit from price discrimination increases with d and thus with the variance of demand. To see why this is so recall how retailers would allocate goods intertemporally. After observing demand in period 1, they would sell the quantity that equalizes the first-period consumer price with the expected second-period consumer price. When demand turns out to be low in period 1, this intertemporal market integration implies that retailers sell very little and thus hold a large inventory of goods for period 2; but when first-period demand is high, retailers sell a lot and hold very little inventory for period 2. In fact, when retailers control the inventory, first-period sales differ from expected second-period sales by the amount of the demand shock: $s_1 - s_2 = \varepsilon_1$. By contrast, when an intermediary controls the inventory, sales in period 1 differ from expected sales in period 2 by only half the amount of the shock: $s_1 - s_2 = \frac{1}{2}\varepsilon_1$ (see (5)). Hence the intertemporal misallocation of inventory when it is controlled by retailers increases with the size of the demand shock; a bigger d implies that big demand shocks are more likely to occur.

3.2 The Re-ordering Case

In this subsection we want to show that by using an intermediary to control inventory the manufacturer is better able to suppress *inventory competition.*

[16] With linear demand the comparison is also similar to the one between uniform pricing and price discrimination when all markets (here both periods) are served: total expected output is the same with and without price discrimination while expected profits are not. See Tirole (1988).

To see why inventory competition is a potential concern, notice that the intermediary in period 2 can sell any inventory left over from period 1, $q_1 - s_1$. If it does not order any additional goods, it can sell these units at an expected profit margin of $[A - (q_1 - s_1) - c_w]$ and thus guarantee itself a profit in period 2 of at least

$$\pi^{out} \equiv [A - (q_1 - s_1) - c_w](q_1 - s_1). \tag{6}$$

As a result the manufacturer has to either reduce the transfer it charges the intermediary in period 2 by this amount, or else decrease its second-period producer price.

The manufacturer's optimal solution for this problem is to ship exactly the same quantity in period 1 as in the case of no re-ordering. That is, it ships in period 1 the static monopoly output for two periods. The reason for this result, as we show in the proof of the following proposition, lies in the fact that the intertemporal arbitrage condition that governs the intermediary's allocation of first-period orders between periods 1 and 2 is the same as (5), so that the manufacturer can trust the intermediary to correctly allocate goods across periods:

Proposition 6 *Suppose inventory is controlled by an intermediary. The manufacturer optimally sells in period 1 the same quantity as without re-ordering, namely $\breve{q}_1^i = \tilde{q}^i = A - c - c_w$. There is hence no possibility of a stockout.*

Proof: see Appendix 2.

Now that we have determined how much the manufacturer will ship in period 1, we can determine how much will be shipped in equilibrium in period 2. Because expected sales in period 2 are equal to $\frac{A-c-c_w}{2}$ and the optimal allocation between periods 1 and 2 dictates that $s_1 = \frac{q_1}{2} + \frac{\varepsilon_1}{4}$ (see (18) in Appendix 2), then

$$\begin{aligned} q_2 &= s_2 - (q_1 - s_1) \\ &= \frac{A - c - c_w}{2} - (q_1 - s_1) \\ &= \frac{\varepsilon_1}{4}. \end{aligned} \tag{7}$$

It follows that $q_2 > 0$, precisely if $\varepsilon_1 > 0$. That is, goods are ordered in period 2 only if demand in the first period turns out to be greater than expected. When $\varepsilon_1 < 0$, the intermediary does not order any goods in period 2 and is content with the initial order. Accordingly, the expected second-period shipment by the manufacturer is only

$$E\left(\breve{q}_2^i\right) = \int_0^d \left[\frac{\varepsilon_1}{4}\right] \frac{1}{2d} d\varepsilon_1 = \frac{d}{16}. \tag{8}$$

The manufacturer's total expected output and total expected profit can now be computed. They are as follows:

Proposition 7 *Suppose inventory is controlled by an intermediary. The manufacturer's equilibrium output,* $E\left(\breve{q}^i\right) \equiv \breve{q}_1^i + E(\breve{q}_2^i) = A - c - c_w + \frac{d}{16}$, *is greater than the sum over two periods of the static monopoly output, and the expected equilibrium profit,* $E\left(\breve{\pi}^i\right) = \frac{(A-c-c_w)^2}{2} + \frac{5d^2}{96}$, *exceeds the sum of static monopoly profits.*

Proof: see Appendix 2.

Clearly, an intermediary has the potential to add more value to the supply chain when he engages in intertemporal market segmentation and when he controls inventory competition.

4 Implications

We are now in a position to draw several implications from the analysis by comparing the equilibrium where retailers control inventories with the equilibrium where an intermediary does so. We focus most of our attention on the cases where agents have the ability to place orders in every period.

The first implication concerns the circumstances under which the manufacturer would use an intermediary. Recall that in the restricted case where re-ordering is not allowed the profit associated with intermediation is increasing in the variance of demand and thus d, simply reflecting the fact that intertemporal market segmentation and therefore price discrimination become more important the greater is the potential for demand differences across periods. A comparison of Propositions 1 and 5 reveals that the manufacturer would prefer inventory to be controlled by an intermediary if d is sufficiently big to compensate for the resource cost of intermediation.

The following proposition shows that this result also holds in the more general case where we allow intermediaries and retailers to order goods in both periods (Propositions 4 and 7). We hence obtain the following implication:

Proposition 8 *The manufacturer uses an intermediary to control inventory if the variance of demand (and thus d) is sufficiently big.*

Thus whether through the ability to price discriminate or to control inventory competition, the opportunity cost of not using an intermediary is higher for the manufacturer the bigger is the variance of demand. To see why, consider the impact of a big negative demand shock on inventory competition. In this case retailers are likely to carry a lot of unsold inventory into the second period, even if the manufacturer only shipped a small quantity in period 1, thus exposing the manufacturer to inventory competition and forcing him to cut the producer price. In the case of a big positive demand shock the manufacturer faces another problem, namely that retailers are likely to stock out and sales are lost.

Another implication concerns the total expected output of the manufacturer and hence total inventory:

Proposition 9 *Total expected output and hence total inventory is smaller when inventory is controlled by an intermediary rather than retailers.*

Notice that this result holds, even if $c_w = 0$. The reason has to do with how the manufacturer deals with the problem of intertemporal competition. In the case of retailer-controlled inventory the manufacturer wants to keep first-period output down so that retailers do not carry too much inventory into period 2. But the less it ships, the bigger is the risk of a stockout. In the case of intermediary-controlled inventory the manufacturer is able to keep overall output small without running the risk of a stockout simply by shipping a bigger quantity in period 1, namely the unconstrained optimal monopoly quantity for both periods, and shipping additional units in period 2 only if demand in period 1 is higher than expected.

Clearly the different strategy the manufacturer employs to limit inventory competition when inventory is controlled by an intermediary rather than retailers also has implications for the pattern of per-period shipments by the manufacturer. We can show:

Proposition 10 *If inventory is controlled by an intermediary rather than retailers, then (i) per-period shipments by the manufacturer occur on average less frequently, and (ii) the variation in the size of expected shipments over the two periods is greater.*

As explained above, half of the time there is no shipment between the manufacturer and the intermediary in period 2. By contrast, when retailers control inventory, second period shipments by the manufacturer, as is easily ascertained, are positive for every realization of ε_1.

A simple comparison of shipments in period 1 and expected shipments in period 2 proves (ii). Also notice that the intermediary's expected shipments to retailers are the same in both periods, as retailers' sales satisfy $E(s_1) = E(s_2) = \frac{1}{2}(A - c - c_w)$. An interesting implication of Proposition 10 is thus that the intermediary smooths the shipments to the retailers as compared to those between the manufacturer and the retailers, or to those between the manufacturer and the intermediary.

By itself, being able to engage in intertemporal market segmentation reduces expected consumer surplus but it does not reduce expected social welfare when there is no impact on expected output (i.e. for $\tilde{q}^r = \tilde{q}^i$ when $c_w = 0$). However, since an intermediary reduces expected output even for

$c_w = 0$ when it engages in inventory control (i.e., $E(\breve{q}^i) < E(\breve{q}^r)$), it is primarily through this effect that the use of an intermediary has an anti-competitive effect and negative impacts on both expected consumer surplus and expected social welfare. Indeed, we show that, even if $c_w = 0$,

Proposition 11 *Inventory control by an intermediary reduces expected consumer surplus and expected social welfare.*

Proof: See Appendix 2.

5 Conclusions

This paper shows that shifting inventory control from retailers to an intermediary, thereby adding a link in a supply chain, may be an optimal strategy to follow for manufacturers in an environment in which orders have to be placed before demand is known. This is the case even if adding an intermediary is costly and decreases the overall volume of sales.

The reason is that an intermediary brings two advantages to inventory control, both of which stem from better incentives to allocate inventory over time. First, an intermediary can help a manufacturer price discriminate across periods by intertemporally segmenting markets. Second, he can play a role in reducing inventory competition, precisely because an intermediary is able to segment markets intertemporally and can hence be trusted to allocate inventory optimally. These advantages are shown to be especially big in markets where demand is very volatile.

A number of implications flow from the analysis. The first one is that using intermediaries to control inventory tends to be anticompetitive because it reduces total sales and thus total inventory. This is because an intermediary takes over the manufacturer's monopoly position once orders have been shipped by the manufacturer.

The second implication of the analysis is that the shipments between the manufacturer and the intermediary tend to be less frequent and their sizes more volatile than those between the manufacturer and the retailers. This is interesting because it says that the lumpiness and volatility of shipments may have a lot to do with who the buyer is and his role in the supply chain and not only, as it is usually assumed, with product characteristics or the existence of fixed costs per shipment (see, for instance, Hornok and Koren, 2015 and Kropf and Sauré, 2014).

In fact the analysis reveals that the use of intermediation and shipment lumpiness and volatility often go hand in hand but without clear-cut causal links. On the one hand, when shipment lumpiness and volatility come from exogenous constraints (as implicitly assumed in the restricted no-reordering case), our results show that an intermediary may still increase the value of a supply chain, although less so than when no such exogenous constraint exists. On the other hand, when re-ordering is possible and intermediation is optimal, shipments to an intermediary may still be as lumpy and as volatile, this time because the optimal shipments and their timing dictate it. Hence, intermediation and shipment lumpiness/volatility tend to go together whether by choice or by constraint.[17]

Showing that intermediation is able to enhance the value of a supply chain does not imply that intermediation is necessarily the best tool to do so. In fact it can be shown that intermediation does not succeed in achieving maximal vertical value. And since the literature on the theory of contracts in supply chains shows that vertical restraints and other policies can help achieving, at least in principle, maximal vertical value, one could conclude that adding intermediation as a new link in a supply chain may well be a second-best tool. This suggests that the choice between vertical contracting arrangements and intermediation very much depends on the specific conditions under which supply chains operate. In that regard the rapid growth of 3PL firms noted in the introduction, especially their increased global reach as warehousing and distribution providers, as well as other new arrangements aimed at shifting inventory control away from retailers, demonstrate that intermediation might be especially useful in international markets.[18] This should not come as a surprise. Global supply chains deal with long distance, multiple markets, different legal jurisdictions and cultural environments, all of which likely increase the cost of vertical contracting as compared to the cost of delegating inventory control and associated decisions to an intermediary.

Although it is well beyond the scope of the present article, our results are potentially testable, especially in an international trade context as shipments, product characteristics, and the buyer's identity are often recorded. But it is interesting to note that some of our theoretical predictions are consistent with the empirical results about drop-shipping provided by Randall, Netessine and Rudi (2006). The authors compare drop-shipping to the more traditional arrangement where retailers hold their own inventories. This is a similar structure to ours in so far as the drop-shipping arrangement corresponds to the case where an intermediary takes over inventory control from retailers. The authors find empirical evidence that traditional retailers who manage their own inventories face lower demand uncertainty than the retailers that rely on drop-shippers to control inventory. This is consistent with our result that using intermediaries to control inventories is optimal

[17] Choice and constraint can obviously operate both at the same time as is likely the case for Welspun, India's biggest manufacturer of towels, when it ships 40-50 containers of towels at once to its warehouse in the US, a voyage taking at least 22 days to reach its destination and when the shipment stays for another 20-25 days as inventory before retailers' orders arrive (Economist, 2015).

[18] Interestingly, wholesale drop-shipping and other innovative wholesaling practices often have an important international component; see PRWeb, 2006, 2012 for examples.

when there is high demand uncertainty.[19] They also find that the greater the number of retailers, the greater the use of drop-shipping. Although our retailers are perfectly competitive and thus we have no particular result on that dimension, it is interesting to note that the fundamental reason why intermediaries might be needed is because retailers, as price takers, do not have the same incentives as a manufacturer or as an intermediary. In that sense this empirical finding is also consistent with our theoretical results.

6 Appendix 1

The upper limit $\bar{d}$ is determined as follows. First, consider the case of *inventory control by retailers and no re-ordering.* In equilibrium, $q = A - c$. Thus $s_1 = \frac{q+\varepsilon_1}{2} = \frac{A-c+\varepsilon_1}{2}$ and $s_2 = \frac{A-c-\varepsilon_1}{2}$. Thus, s_1 and s_2 are positive when $d < A - c$. It is easy to establish that p_1 and p_2 are both positive if $d < \frac{A+c}{3}$.

Second, consider the case of *inventory control by retailers with re-ordering.* In equilibrium, $q_1 = \frac{1}{2}(A-c) + \frac{1}{5}d$. Thus $s_1 = \frac{A-c+2\varepsilon_1}{3} + \frac{1}{3}q_1 = \frac{1}{2}(A-c) + \frac{2}{3}\varepsilon_1 + \frac{1}{15}d$ and $s_2 = \frac{A-c+(q_1-s_1)}{2}$. We know that if $\varepsilon_1 < \frac{1}{5}d$ (see Appendix 2), then $q_1 > s_1$; otherwise there is stockout. It is apparent then that s_2 is always larger than zero. To ensure $s_1 > 0$, we require $d < \frac{5}{6}(A-c)$. Along with $d < \frac{A+c}{3}$, this condition guarantees that q_2, p_1 and p_2 are positive irrespective of ε_1.

Third, consider the case of *inventory control by an intermediary and no re-ordering.* In equilibrium, $q = A - (c + c_w)$. Thus $s_1 = \frac{2[A-(c+c_w)]+\varepsilon_1}{4}$ and $s_2 = \frac{2[A-(c+c_w)]-\varepsilon_1}{4}$. Thus, to make sure $s_1, s_2 > 0$, we need $d < 2[A - c - c_w]$. An intermediary controlling inventory also controls how much to sell in the second period. In particular, one needs to make sure that all the inventory from period 1, $q - s_1 = \frac{2[A-(c+c_w)]-\varepsilon_1}{4}$ is sold in period 2. This implies $MR_2 = A - 2\frac{2[A-(c+c_w)]-\varepsilon_1}{4} + \varepsilon_2 > 0$ and thus that $d < \frac{2(c+c_w)}{3}$. It can then be checked that this condition also makes sure that MR_1, p_1 and p_2 are positive as long as $s_1, s_2 > 0$.

Finally, the last case is *inventory control by intermediary with re-ordering.* In equilibrium, $q_1 = A - c - c_w$, $s_1 = \frac{A-c-c_w}{2} + \frac{\varepsilon_1}{4}$. If $\varepsilon_1 > 0$, then $q_2 = \frac{\varepsilon_1}{4} > 0$; otherwise $q_2 = 0$. $s_2 = q_2 + (q_1 - s_1) = q_2 + \frac{A-c-c_w}{2} - \frac{\varepsilon_1}{4}$ is obviously always larger than zero. To make sure $s_1 > 0$, we need $d < 2[A - c - c_w]$, which also guarantees that there is no stockout in the first period (i.e., $q_1 > s_1$). It can also be checked that for the marginal revenue to be greater than zero in each period, then $MR_2 = A - 2s_2 + \varepsilon_2 > 0$ requires $d < \frac{2(c+c_w)}{3}$, which in turn makes sure that $MR_1 > 0$, $p_1 > 0$ and $p_2 > 0$ as long as s_1,$s_2 > 0$.

[19] Belavina and Girotra (2012) argue that intermediaries help firms adapt to a volatile environment even if they are much larger than the intermediaries they typically use.

It follows that

$$d < \overline{d} = \min\left\{\frac{2\left(c + c_w\right)}{3}, 2\left(A - c - c_w\right), \frac{5}{6}\left(A - c\right), \frac{A + c}{3}\right\} \tag{9}$$

for all the prices and quantities to be positive.

7 Appendix 2

7.1 Proof of Proposition 1

Consider how much retailers order before observing ε_1. Given perfect competition the equilibrium orders satisfy that the expected retail price $Ep(q, \varepsilon_1)$ is equal to the marginal cost faced by retailers, namely the producer price P,

$$\int_{-d}^{d} \frac{(2A - q + \varepsilon_1)}{2} \frac{1}{2d} d\varepsilon_1 = P.$$

The manufacturer maximizes expected profit $E\left[(P - c)q\right]$. Solving the corresponding first-order condition,

$$\int_{-d}^{d} \frac{2A - 2q + \varepsilon_1}{2} \frac{1}{2d} d\varepsilon_1 - c = 0,$$

yields the desired result for expected output. The result for expected prices and profit follows immediately.

7.2 Proof of Proposition 2

In period 2 retailers sell all of the products on hand, because they have already paid for these goods and, since $d \leq \bar{d}$, the retail price is positive; hence $s_2 = q_2 + q_1 - s_1$. Retailers order goods in period 2 until the expected consumer price in period 2 equals the producer price P_2:

$$E\left(A - s_2 + \varepsilon_2\right) = A - q_2 - q_1 + s_1 = P_2. \tag{10}$$

The manufacturer chooses P_2, respectively q_2, that maximizes its period-2 expected profit $(P_2 - c)q_2$. This expected profit is maximized for $q_2 = \left(A - c - q_1 + s_1\right)/2$. Using this output in (10) gives the result.

7.3 Proof of Proposition 3

Being perfectly competitive, retailers order goods in period 1 until the expected retail price is equal to marginal cost, which in this case is the producer price P_1:

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{A + c - q_1 + s_1}{2} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^{d} \left(A - q_1 + \varepsilon_1\right) \frac{1}{2d} d\varepsilon_1 = P_1. \tag{11}$$

The first term in (11) is the expected retail price in period 1 if there is no stockout that we know from (1); the second term is the expected retail price in case of a stockout. Substituting for s_1, we can rewrite (11) as

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{2A + c + \varepsilon_1 - q_1}{3} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^{d} (A + \varepsilon_1 - q_1) \frac{1}{2d} d\varepsilon_1 = P_1.$$

The manufacturer chooses q_1 to maximize total expected profit over the two periods, which is given by

$$\begin{aligned} &(P_1 - c)q_1 + (P_2(q_1) - c)q_2(q_1) = \\ &\quad \int_{-d}^{\hat{\varepsilon}_1} \left[\frac{2(A-c) + \varepsilon_1 - q_1}{3} q_1 + \frac{(2(A-c) + \varepsilon_1 - q_1)^2}{9} \right] \frac{1}{2d} d\varepsilon_1 \\ &\quad\quad + \int_{\hat{\varepsilon}_1}^{d} \left[(A - c + \varepsilon_1 - q_1)\, q_1 + \frac{(A-c)^2}{4} \right] \frac{1}{2d} d\varepsilon_1. \end{aligned} \tag{12}$$

Using the Leibniz Rule, we can write the first-order condition associated with (12) as

$$\int_{-d}^{\hat{\varepsilon}_1} \frac{2(A-c) + \varepsilon_1 - 4q_1}{9} \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^{d} (A - c + \varepsilon_1 - 2q_1) \frac{1}{2d} d\varepsilon_1 + X = 0 \tag{13}$$

where

$$\begin{aligned} X &= \left[\frac{2(A-c) + \hat{\varepsilon}_1 - q_1}{3} q_1 + \frac{(2(A-c) + \hat{\varepsilon}_1 - q_1)^2}{9} \right] \frac{d\hat{\varepsilon}_1}{dq_1} \\ &\quad - \left[(A - c + \hat{\varepsilon}_1 - q_1)\, q_1 + \frac{(A-c)^2}{4} \right] \frac{d\hat{\varepsilon}_1}{dq_1}. \end{aligned}$$

Noting that $\frac{d\hat{\varepsilon}_1}{dq_1} = 1$ and substituting for $\hat{\varepsilon}_1$ from (3) it is easily shown that $X = 0$, so that we can rewrite (13) as

$$\begin{aligned} &\int_{-d}^{d} \frac{2(A-c) + \varepsilon_1 - 4q_1}{9} \frac{1}{2d} d\varepsilon_1 \\ &\quad + \int_{\hat{\varepsilon}_1}^{d} \left[(A - c + \varepsilon_1 - 2q_1) - \frac{2(A-c) + \varepsilon_1 - 4q_1}{9} \right] \frac{1}{2d} d\varepsilon_1 = 0, \end{aligned} \tag{14}$$

which, after simplification, becomes the first-order condition (4) in the main body of the paper.

After substitution for $\hat{\varepsilon}_1$ and integration we can rewrite (14) to obtain

$$(A - c + 4d - 2q_1)(5(A - c) + 2d - 10q_1) = 0.$$

There are two solutions: $q_1 = \frac{1}{2}(A - c) + \frac{1}{5}d$ and $q_1 = \frac{1}{2}(A - c) + 2d$. Since we require $d > \hat{\varepsilon}_1 = q_1 - \frac{1}{2}(A - c)$, only the first solution is valid. Hence

$$\breve{q}_1^r = \frac{1}{2}(A - c) + \frac{1}{5}d.$$

Using this level of output we obtain $\hat{\varepsilon}_1 = \frac{1}{5}d$. The probability of a stockout can then be computed as $\frac{d - \frac{1}{5}d}{2d} = 0.4$.

7.4 Proof of Proposition 4

The expected second-period output can be computed using $q_2 = \frac{A - c - (q_1 - s_1)}{2}$ if $q_1 > s_1$, and $q_2 = \frac{A-c}{2}$ if there is a stockout. This implies:

$$\begin{aligned} E(\breve{q}_2^r) &= \int_{-d}^{\hat{\varepsilon}_1} \left[\frac{2(A - c) - q_1 + \varepsilon_1}{3}\right] \frac{1}{2d} d\varepsilon_1 + \int_{\hat{\varepsilon}_1}^{d} \left(\frac{A - c}{2}\right) \frac{1}{2d} d\varepsilon_1 \quad (15) \\ &= \frac{1}{2}(A - c) - \frac{3}{25}d. \quad (16) \end{aligned}$$

Total expected output is thus given by $\breve{q}_1^r + E(\breve{q}_2^r) = \frac{1}{2}(A - c) + \frac{2}{25}d$.

The manufacturer's total expected profit can be computed as

$$\int_{-d}^{\hat{\varepsilon}_1} \left[\frac{2(A - c) + \varepsilon_1 - \left(\frac{1}{2}(A - c) + \frac{1}{5}d\right)}{3} \left(\frac{1}{2}(A - c) + \frac{1}{5}d\right)\right] \frac{1}{2d} d\varepsilon_1$$

$$+ \int_{-d}^{\hat{\varepsilon}_1} \left[\frac{\left(2(A - c) + \varepsilon_1 - \left(\frac{1}{2}(A - c) + \frac{1}{5}d\right)\right)^2}{9}\right] \frac{1}{2d} d\varepsilon_1$$

$$+ \int_{\hat{\varepsilon}_1}^{d} \left[\left(A - c + \varepsilon_1 - \left(\frac{1}{2}(A - c) + \frac{1}{5}d\right)\right)\left(\frac{1}{2}(A - c) + \frac{1}{5}d\right) + \frac{(A - c)^2}{4}\right] \frac{1}{2d} d\varepsilon_1,$$

which simplifies to

$$E(\pi^r) = E(\tilde{\pi}^r) + \frac{d^2}{25}. \quad (17)$$

7.5 Proof of Proposition 5

Using $s_1 + s_2 = q$ in (5), we obtain $s_1 = (2q + \varepsilon_1)/4$ and $s_2 = (2q - \varepsilon_1)/4$. After observing ε_1, the total expected revenue of the intermediary, $E(R) = w_1 s_1 + w_2 s_2$, is given by

$$\begin{aligned} E(R) &= (A - s_1 + \varepsilon_1) s_1 + (A - s_2) s_2, \\ &= (A - q)q + \frac{1}{8} (2q + \varepsilon_1)^2 . \end{aligned}$$

Prior to observing ε_1, the intermediary chooses q to maximize

$$\int_{-d}^{d} \left((A - q)q + \frac{1}{8} (2q + \varepsilon_1)^2 \right) \frac{1}{2d} d\varepsilon_1 - (P + c_w) q - T.$$

From the first-order condition,

$$\int_{-d}^{d} \left(A - q + \frac{\varepsilon_1}{2} \right) \frac{1}{2d} d\varepsilon_1 - (P + c_w) = 0,$$

we obtain the optimal order quantity $q = A - (P + c_w)$ and the total expected profit of the intermediary of

$$\frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{24} - T.$$

The manufacturer optimally sets $P = c$, and extracts the intermediary's profit through the transfer T. This yields the manufacturer's expected output $\tilde{q}^i$ and the expected profit $E\left(\tilde{\pi}^i\right)$.

7.6 Proof of Proposition 6

We have to distinguish between two cases: one where $q_2 > 0$ and one where $q_2 = 0$. Suppose first that $q_2 > 0$. In period 2 after ε_2 has been revealed and given a wholesale price w_2, retailers sell an amount s_2 such that the retail price in period 2 equals their marginal cost, that is $(A - s_2 + \varepsilon_2) = w_2$. The intermediary has to order goods in period 2 before observing ε_2. Using $s_2 = q_2 + (q_1 - s_1)$, the intermediary's expected profit maximization problem in period 2 can be written as

$$\max_{q_2} (A - q_2 - q_1 + s_1 - c_w)(q_2 + q_1 - s_1) - P_2 q_2 - T_2,$$

Solving the respective first-order condition, $A - 2q_2 - 2q_1 + 2s_1 = P_2 + c_w$, yields as solution a quantity

$$q_2 = \frac{A - P_2 - c_w}{2} - (q_1 - s_1),$$

and an expected retail price equal to

$$E(p_2) = \frac{A + P_2 + c_w}{2}.$$

The intermediary's expected profit in period 2 can then be calculated as

$$\begin{aligned}
\pi_2^{int} &= \left(\frac{A + P_2 + c_w}{2} - c_w\right)\frac{A - P_2 - c_w}{2} \\
&\quad - P_2\left[\frac{A - P_2 - c_w}{2} - (q_1 - s_1)\right] - T_2 \\
&= \frac{(A - P_2 - c_w)^2}{4} + P_2(q_1 - s_1) - T_2.
\end{aligned}$$

The manufacturer chooses (P_2, T_2) such that T_2 extracts the intermediary's expected period-2 profit net of the intermediary's outside option, π^{out}, and P_2 maximizes

$$\max_{P_2}\left\{(P_2 - c)\left[\frac{A - P_2 - c_w}{2} - (q_1 - s_1)\right] \right.$$
$$\left. + \frac{(A - P_2 - c_w)^2}{4} + P_2(q_1 - s_1) - \pi^{out}\right\}.$$

From the corresponding first-order condition we obtain the manufacturer's optimal choice $P_2 = c$.

In period 1, after ε_1 has been revealed, the intermediary sets a wholesale price w_1 and retailers purchase and sell quantity s_1, such that $(A - s_1 + \varepsilon_1) = w_1$. The intermediary's revenue hence is equal to $(A - s_1 + \varepsilon_1)\, s_1$ and the corresponding marginal profit in period one is $MR_1 - c_w = A - 2s_1 + \varepsilon_1 - c_w$.

The intermediary's inter-temporal arbitrage condition is that $MR_1 - c_w = \frac{dOut}{d(q_1 - s_1)}$, that is, a unit sold in period 2 instead of period 1 has to yield the same marginal profit as one sold in period 1, where the intermediary's profit in period 2 is determined by its outside option, as the rest is extracted by the manufacturer through T_2. We hence have

$$A - 2s_1 + \varepsilon_1 = A - 2(q_1 - s_1), \tag{18}$$

which is identical to (5) if $q_1 = \tilde{q}^i$.

The intermediary's total expected profit is given by

$$E\left\{(A - s_1 + \varepsilon_1 - c_w)\, s_1 - P_1 q_1 - (T_1 - \pi^{out})\right\},$$

where we note that the manufacturer can extract π^{out} in period 1. Using (18) to obtain s_1 and substituting for π^{out} from (6), we can rewrite the intermediary's expected profit as

$$E\left\{\left(A + \frac{\varepsilon_1}{2} - \frac{q_1}{2} - c_w\right) q_1 + \frac{\varepsilon_1^2}{8} - P_1 q_1 - T_1\right\}.$$

The intermediary maximizes this profit by ordering a quantity equal to $q_1 = A - P_1 - c_w$ and earns an expected profit of

$$E\left\{\frac{(A-P_1-c_w)^2}{2}+\frac{\varepsilon_1}{2}(A-P_1-c_w)+\frac{\varepsilon_1^2}{8}-T_1\right\}.$$

It is now straightforward to show that the manufacturer chooses a two-part tariff such that $P_1 = c$ and T_1 extracts the intermediary's profit. Thus the quantity shipped to the intermediary by the manufacturer in period 1 is the same as without re-ordering, namely $\breve{q}_1^i = A - c - c_w$.

Next suppose that $q_2 = 0$. In this case we know from the proof of the previous proposition that $\breve{q}_1^i = \tilde{q}^i = A - c - c_w$.

7.7 Proof of Proposition 7

The manufacturer's expected profit with intermediation and re-ordering can be written as the sum of the expected profit in period 1 and in period 2 and thus as

$$\begin{aligned}\int_{-d}^{d}\{\frac{(A-c-c_w)^2}{2}+(A-c-c_w)\frac{\varepsilon_1}{2}+\frac{\varepsilon_1^2}{8}\}\frac{1}{2d}d\epsilon_1 \\ +\int_0^d\{(\frac{(A-c-c_w)^2}{4}+c(q_1-s_1)-\pi^{out}\}\frac{1}{2d}d\epsilon_1,\end{aligned}$$

where the first line corresponds to the expected total revenue of the intermediary in period 1 and the second line corresponds to the total expected net revenue of the intermediary given that an order is placed only if the realization of demand in period 1 is above average. Using $s_1 = \frac{\breve{q}_1^i}{2} + \frac{\varepsilon_1}{4}$, $\breve{q}_1^i = A - c - c_w$, (6) yields the desired result.

7.8 Proof of Proposition 11

With retailer-controlled inventory consumer surplus is given by:

$$\begin{aligned}CS^r &= \frac{1}{2}E\left[(A-p_1)\,s_1\right]+\frac{1}{2}E\left[(A-p_2)\,s_2\right] \\ &= \frac{1}{2}E\left[(s_1-\varepsilon_1)\,s_1\right]+\frac{1}{2}E\left[(s_2)^2\right]\end{aligned}$$

where

$$\begin{aligned}
& \frac{1}{2}E\left[\left(s_1-\varepsilon_1\right)s_1\right] \\
= & \frac{1}{2}\int\limits_{-d}^{\frac{1}{5}d}\left(\frac{A-c-\varepsilon_1}{3}+\frac{\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d\right)}{3}\right) \\
& \times\left(\frac{A-c+2\varepsilon_1}{3}+\frac{\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d\right)}{3}\right)\frac{1}{2d}d\varepsilon_1 \\
& +\frac{1}{2}\int\limits_{\frac{1}{5}d}^{d}\left[\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d-\varepsilon_1\right)\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d\right)\right]\frac{1}{2d}d\varepsilon_1 \\
= & \frac{1}{2}\int\limits_{-d}^{\frac{1}{5}d}\left[\left(\frac{15(A-c)+2d-10\varepsilon_1}{30}\right)\left(\frac{15(A-c)+2d+20\varepsilon_1}{30}\right)\right]\frac{1}{2d}d\varepsilon_1 \\
& +\frac{1}{2}\int\limits_{\frac{1}{5}d}^{d}\left[\left(\frac{5(A-c)+2d-10\varepsilon_1}{10}\right)\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d\right)\right]\frac{1}{2d}d\varepsilon_1 \\
= & \frac{1}{8}\left(A-c\right)^2-\frac{1}{50}d\left(A-c\right)-\frac{9}{250}d^2,
\end{aligned}$$

and

$$\begin{aligned}
& \frac{1}{2}E\left[\left(s_2\right)^2\right] \\
= & \frac{1}{2}\int\limits_{-d}^{\frac{1}{5}d}\left(\frac{A-c+\frac{2}{3}\left(\frac{1}{2}\left(A-c\right)+\frac{1}{5}d\right)-\frac{A-c+2\varepsilon_1}{3}}{2}\right)^2\frac{1}{2d}d\varepsilon_1 \\
& +\frac{1}{2}\int\limits_{\frac{1}{5}d}^{d}\left(\frac{A-c}{2}\right)^2\frac{1}{2d}d\varepsilon_1 \\
= & \frac{1}{2}\int\limits_{-d}^{\frac{1}{5}d}\left(\frac{15(A-c)+2d-10\varepsilon_1}{30}\right)^2\frac{1}{2d}d\varepsilon_1+\frac{1}{2}\int\limits_{\frac{1}{5}d}^{d}\left(\frac{A-c}{2}\right)^2\frac{1}{2d}d\varepsilon_1 \\
= & \frac{1}{8}\left(A-c\right)^2+\frac{3}{50}d(A-c)+\frac{2}{125}d^2
\end{aligned}$$

Hence we can rewrite consumer surplus as

$$\begin{aligned}
CS^r &= \frac{1}{8}\left(A-c\right)^2+\frac{1}{8}\left(A-c\right)^2-\frac{1}{50}d\left(A-c\right)+\frac{3}{50}d(A-c)-\frac{9}{250}d^2+\frac{2}{125}d^2 \\
&= \frac{1}{4}\left(A-c\right)^2+\frac{1}{25}\left(A-c\right)d-\frac{1}{50}d^2
\end{aligned}$$

Now consider inventory control by an intermediary and assume that $c_w = 0$. Consumer surplus is then given by

$$CS^i = \frac{1}{2}E\left[\left(s_1 - \varepsilon_1\right)s_1\right] + \frac{1}{2}E\left[\left(s_2\right)^2\right]$$

where

$$\begin{aligned}
\frac{1}{2}E\left[\left(s_1 - \varepsilon_1\right)s_1\right] &= \frac{1}{2}\int_{-d}^{d}\left(\frac{A-c}{2} - \frac{3\varepsilon_1}{4}\right)\left(\frac{A-c}{2} + \frac{\varepsilon_1}{4}\right)\frac{1}{2d}d\varepsilon_1 \\
&= \frac{1}{8}\left(A-c\right)^2 - \frac{1}{32}d^2
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2}E\left[\left(s_2\right)^2\right] &= \frac{1}{2}\int_{-d}^{0}\left(\frac{A-c}{2} - \frac{\varepsilon_1}{4}\right)^2\frac{1}{2d}d\varepsilon_1 + \frac{1}{2}\int_{0}^{d}\left(\frac{A-c}{2}\right)^2\frac{1}{2d}d\varepsilon_1 \\
&= \frac{1}{8}\left(A-c\right)^2 + \frac{1}{32}d(A-c) + \frac{1}{192}d^2
\end{aligned}$$

Hence

$$CS^i = \frac{1}{4}\left(A-c\right)^2 + \frac{1}{32}d(A-c) - \frac{5}{192}d^2.$$

We first can show that for $c_w = 0$ consumer surplus when retailers control inventory exceeds consumer surplus when inventory is controlled by an intermediary

$$CS^r - CS^i = \frac{1}{25}\left(A-c\right)d - \frac{1}{50}d^2 - \left(\frac{1}{32}d(A-c) - \frac{5}{192}d^2\right) > 0.$$

Now we compute social welfare.

$$\begin{aligned}
SW^r &= CS^r + E\left(\breve{\pi}^r\right) = \frac{1}{4}\left(A-c\right)^2 + \frac{1}{25}\left(A-c\right)d - \frac{1}{50}d^2 + \frac{\left(A-c\right)^2}{2} + \frac{d^2}{25} \\
&= \frac{3}{4}\left(A-c\right)^2 + \frac{1}{25}d(A-c) + \frac{1}{50}d^2
\end{aligned}$$

$$\begin{aligned}
SW^i &= CS^i + E\left(\breve{\pi}^i\right) = \frac{1}{4}\left(A-c\right)^2 + \frac{1}{32}d(A-c) - \frac{5}{192}d^2 + \frac{\left(A-c\right)^2}{2} + \frac{5d^2}{96} \\
&= \frac{3}{4}\left(A-c\right)^2 + \frac{1}{32}d(A-c) + \frac{5}{192}d^2
\end{aligned}$$

$$\begin{aligned}
SW^r - SW^i &= \frac{3}{4}\left(A-c\right)^2 + \frac{1}{25}d(A-c) + \frac{1}{50}d^2 \\
&\quad - \left(\frac{3}{4}\left(A-c\right)^2 + \frac{1}{32}d(A-c) + \frac{5}{192}d^2\right) \\
&= \frac{7}{800}d(A-c) - \frac{29}{4800}d^2 = \frac{d\left[42(A-c) - 29d\right]}{4800} > 0
\end{aligned}$$

We thus obtain $SW^r > SW^i$ for all admissible d even if $c_w = 0$.

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