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Product Customization in the Spokes Model*

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Abstract

We use a spokes model to analyze firms' customization incentives when facing the choices of standard and niche products. Products at or near the end of the spokes are customized products, while products near the origin are more standardized products that cater to the taste of many consumers. Our results indicate that although monopolist always offers the standard product, if a firm anticipates entry, it may choose to stake claim to a customized product. For low transportation costs, the early entrant chooses the standard product. But this equilibrium is characterized by aggressive pricing behavior.

JEL Classification: L11, L13.

Keywords: product differentiation, product customization, entry, spatial oligopoly.

1 Introduction

We use a spokes model of customization to show that niche markets are not just for small entrants but also for incumbents. We analyse firms' incentives to customize instead of choosing the standard product in a spokes model, where the standard product (at the center of spoke) appeals to many while a customized product (away from the end of spokes) appeals to a specific group of consumers. We argue that the construct of the spokes model has a natural interpretation for the trade-off between offering the standard product versus a niche product. Without relying on cost of customization, we offer a rational for customization driven by consumer preferences.

In our model, the incumbent and entrant choose a pair of product (location) and price sequentially. We allow firms to choose the same location and show that pure strategy

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equilibrium exists. When choosing the optimal location and price, the incumbent faces the trade-off: locating at the centre is close to more consumers, but such a lucrative location attracts entry. In anticipation of entry, the incumbent has to price aggressively if it were to offer the standard product. We characterize two pure strategy equilibria. For sufficiently large transportation costs, the first mover (incumbent) chooses a customized product and the second mover (entrant) chooses the standard product. Our result says that for incumbents anticipating new entrants into the market, the profit maximizing strategy may be to offer the niche market product. This is due to the new entrant's ability to offer the same product and undercut the incumbent's price. In this case, if the incumbent chooses to offer the standard product, in order to render price undercutting unprofitable, it must commit to a too low price which would make offering the customized product more attractive. For small transportation costs, an early entrant chooses to offer the standard product and the second mover offers the customized product. For this case, the market features aggressive pricing behavior from the incumbent to prevent price undercutting. We note that there is no minimum differentiation even with linear transportation costs.

When a firm has an innovative product, it is the first in the market but expects an entrant to follow. This is particularly true when the start-up establishes a new market that is attractive to other firms, including established firms (Markides and Geroski, 2005). Federal Express started the overnight delivery service soon followed by established UPS. Gourmet potato chips started with small local companies but now Frit-Lay for instance has its line of gourmet chips. Norman, Pepall and Richards (2009) identified how patents can be designed to protect these innovative firms who are doomed to be followed by an established firm. Firms can also undertake actions to protect its incumbency. One example which has proved to be particularly effective in network industries is by increasing consumer switching cost (Farrell and Klemperer, 2007). Mizuno and Okumura (2014) show that the possibility of collaborating with the incumbents may affect the entrant's locaiton choice. Our paper suggests that one strategy firms can adopt is produt design, i.e., the choice of product location. Although catering for mass consumers near the centre may be more profitable absent entry, offering the niche product may be the profit maximizing strategy when facing the possibility of entry.

Our model resembles the spokes model employed by Chen and Riordan (2007) to model spatial monopolistic competition. In Chen and Riordan (2007), consumers are uniformly distributed on a N spokes network, with each spoke representing a product variety. There are n firms, $n \leq N$, each locating at the end point of a spoke. Consumers desire at most two varieties. If the spoke where the consumer is located has a local firm at the end of the spoke, the consumer would desire this local brand. All other brands are of equal distance to the consumer and are bought each with probability $\frac{1}{N-1}$. Chen and Riordan fix firms' locations at the end of the spokes and analyse only price competition.

We analyse a spokes model with three spokes. The results extend qualitatively to the general case of N spokes with the restriction that the number of firms is less than the number of spokes. Consumers are uniformly distributed on the spokes structure and demand at most one unit of the product. The consumer purchases from the firm whose product gives him the highest non-negative net surplus. Firms enter sequentially and compete by choosing locations as well as prices. The common origin of the spokes represents the standard product since there are mass consumers. By moving further away from the centre, the degree of customization increases. If a firm caters for one specific niche market by choosing a location further away from the centre along one specific spoke, the product becomes less attractive to consumers who prefer other types of customization (located on other spokes). One example of this type of product is general sports shoes which appeal to most sports needs and specialized sports shoes such as running shoes and tennis shoes which cater for more specific markets. While firms' locations are fixed at the end point of the spokes in Chen and Riordan, it is an important choice variable in our model.

With the assumption that one firm can only offer one product, we show that a monopoly firm always offers the standard product. For duopolists, there are two possible equilibrium configurations. The first mover may offer a niche product or the standard product in equilibrium, depending on the transportation costs. For sufficiently large transportation costs, the first mover offers a customized product while the second offers the standard product. When the transportation cost is low, we have the equilibrium where the first mover chooses the standard product. But this location choice has to be coupled with aggressive pricing.

There is a large literature on product differentiation and a growing literature on product customization. We only provide a brief literature review with emphasis on papers more related to ours. One line of research interprets customization by adjusting the transportation costs. By reducing the transportation costs, firms customize the products for consumers. The trade-offs are essentially on the cost of customization versus the benefit of higher consumer valuation, not on serving the standard market versus the niche market, as in our analysis (see for example, Hendel and de Figueiredo, 1997). Literature on "delivered pricing" also has a similar flavour (examples include Beckman (1976), Thisse and Vives (1988), and Von Ungern-Sternberg (1988)). Other papers which also emphasizes on the cost of customization include Alptekinoğlu and Corbett (2008), Dewan et al. (2003), Mendelson and Parkturk (2005), and Xia and Rajagopalan (2009).

A paper with similar interpretation of standard versus customized products to ours is Doraszelski and Draganska (2006). They model two types of consumers, A and B, each preferring one type of good. Apart from the two customized goods A and B, there is also a general good available, S. Given the same price, a type A of consumer would prefer good A to good S, and good B is the least preferred option. Consumers have preferences for customization catered to their needs. If it is not available or too expensive, the standard product would be a better fit than the customization aimed for other type of consumers. With cost of customization, Doraszelski and Draganska look at multi-product firms and product line competition with their main results driven by the cost of setting up a new product.

We now turn to the comparison between our results and those from standard Hotelling model. d'Aspremont *et al.* (1979) argue that the principle of minimum differentiation in Hotelling (1929) is invalid if firms also compete in prices. With a two stage example where firms choose locations first and then compete in prices, they show that maximal differentiation is obtained by assuming quadratic transportation cost. Note that with simultaneous price and location pair choices, no Nash equilibrium exists. In this paper, we keep the assumption of linear transportation. To solve the existence problem, we let firms choose the price and location pair sequentially, as in Prescott and Visscher (1977). But we do not allow for price revision. We allow for the possibility that the new entrant can locate at exactly the same location as the incumbent. In our model, we do not have minimum differentiation, nor maximum differentiation. Firms keep some distance among each other, but not the maximal distance.

The rest of the paper is organized as follows. Section 2 presents our model set up. Section 3 analyses the monopoly equilibrium. Section 4 presents the duopoly analysis. The final section contains the concluding remarks.

2 The Model

Consumers are located uniformly along three rays with a common origin in the centre. Each ray is of length 1, and represents a different type of customization (different attribute of the product). With a common origin, by locating closer to the centre, a firm sells a product which appeals to more customers. Along any given ray, the degree of customization increases as the distance from the central point increases. By specializing in one attribute of the product, the product becomes less attractive to consumers who value other attributes. Consumers and products are identified by both the ray and the distance from the origin. A firm i is identified by $i = (r_i, i)$, where r_i indicates the ray and i is the distance of this firm from the center. Consumer i is the consumer whose ideal product (most preferred product) is offered by firm i. For two points $x_1 = (r_{x_1}, x_1)$ and $x_2 = (r_{x_2}, x_2)$, the distance $\Delta(x_1, x_2)$ is defined by,

Definition 1

$$\Delta (x_1, x_2) = \begin{cases} |x_1 - x_2| & \text{if } r_{x_1} = r_{x_2} \\ x_1 + x_2 & \text{if } r_{x_1} \neq r_{x_2} \end{cases}$$

The product space is illustrated in Figure 1.

It can be verified that $\Delta (x_1, x_2)$ is a metric in this product space. We can apply the standard analysis of product differentiation. Consumer $t = (r_t, t)$'s valuation of a product $x = (r_x, x)$ with price p_x is $v_t (x, p_x)$:

$$v_t (x, p_x) = V - \tau \Delta (t, x) - p_x, \tag{1}$$

where V is some inherent value from consumption of one unit of the ideal product, and τ measures the marginal disutility of moving a unit distance away from the ideal product. A

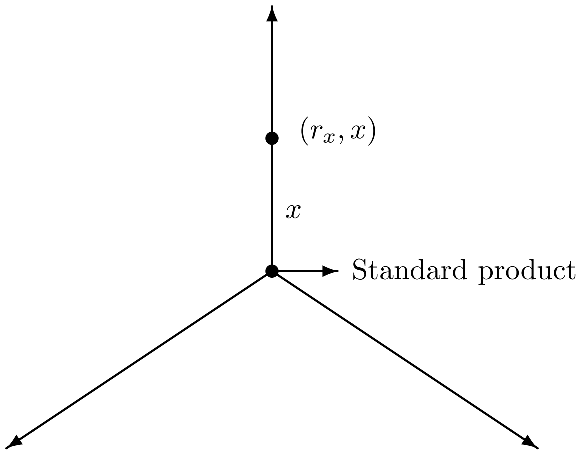

Figure 1: Product space.

consumer buys one unit of product if and only if the valuation is non-negative. In the case of duopoly, consumers buy the product that yields the highest non-negative valuation. We assume no cost of production and customization.

For duopoly analysis, we consider sequential entry of firms. The first mover chooses a pair of location and price. The follower observes the first mover's choices and make its price and location choice. We assume that repositioning and price revision is prohibitively costly for firms.

3 Monopoly

We first determine the optimal location and price for a monopolist without entry threat. We define some notations which will be used throughout the paper. For any given (r_x, x, p_x), the marginal consumers $\bar{t} = (r_{\bar{t}}, \bar{t})$ and $\underline{t} = (r_{\underline{t}}, \underline{t})$ are those who satisfy $v_t(x, p_x) = 0$ or $\Delta(t, x) = \frac{V - p_x}{\tau}$. We define $\bar{t}$ to be the marginal consumer located at r_x with $\bar{t} > x$. The other marginal consumer $r_{\underline{t}}$ could be either located at r_x or located at other rays depending on x and p_x.[1]

For any given $x \geq 0$ and p_x, we have

$$\bar{t}_x = \frac{V - p_x}{\tau} + x.$$

For $r_{\underline{t}} = r_x$, $\underline{t}_x$ is defined by

$$\underline{t}_x = x - \frac{V - p_x}{\tau}$$

and for $r_{\underline{t}} \neq r_x$, $\underline{t}_x$ is defined by

$$\underline{t}_x = \frac{V - p_x}{\tau} - x.$$

[1] There is no disction between $\bar{t}$ and $\underline{t}$ when the firm locates at the centre.

Assumption 1 $\frac{V}{2} \leq \tau \leq \frac{3V}{2}$.

Assumption 1 ensures that the transportation cost is sufficiently high such that a monopolist locating at the centre finds it optimal to charge unconstrained monopoly price. That is, given the central location, its marginal consumers locate within the spoke structure. The transportaion cost is also sufficiently low such that two firms cannot be fitted on one ray and both charge the monopoly price without competing with each other.

Proposition 1 *A monopolist should sell the standard product. That is, choose $x = 0$.*

Proof. The monopolist solves the problem

$$\max_{p_m, m} D(p_m, m)\, p_m.$$

We characterise the optimal monopoly pricing and corresponding profit for given m below.

$$p_m = \begin{cases} \frac{3V - m\tau}{6} \\ \frac{V}{2} \end{cases}, \quad \pi_m = \begin{cases} \frac{(3V - m\tau)^2}{12\tau} & m \leq \frac{(3-\sqrt{6})V}{\tau} \\ \frac{V^2}{2\tau} & m > \frac{(3-\sqrt{6})V}{\tau} \end{cases}.$$

When product is very specialized (large m), it is sold only to one type (spoke) of consumers. With more standard product (small m), it is sold to all types of consumers. Both price and profit are decreasing in m in this case. Any customization for a single type comes at the cost of becoming less attractive for the other two types (rays). Monopoly price is $\frac{V}{2}$, demand by each type is $\frac{V}{2\tau}$ and total demand is $\frac{3V}{2\tau}$. ■

4 Duopoly Competition

Firm x first chooses location and price, (r_x, x, p_x), and upon observing x's choices, firm y chooses its location and price (r_y, y, p_y). We solve the game backwards starting with firm y's decisions to get the subgame perfect Nash equilibrium. Before doing so, we present the following remarks to facilitate the discussion of equilibrium.

Given that firm y can always locate at the same position as firm x and undercut p_x marginally, we have:

Remark 1 *In equilibrium, $\pi_y \geq \pi_x$.*

For most cases, $\pi_y [r_y = r_x, y = x, p_y = p_x]$ is strictly greater than $\pi_x [x, p_x]$ since firm y faces no other competitor by locating exactly at the same position as firm x and undercutting p_x marginally.

For any given (r_x, x, p_x), firm y's best response can be either choosing a different location from firm x and optimize accordingly or choosing the same location as firm x with the pricing constraint $p_y \leq p_x$. In solving the model, we first formulate firm y's local best reponse while it chooses a different location from firm x and work backwards to get firm x's local optimal choice. At this local optimal choice, we than examine whether or not firm y has the incentive to instead locating at the same position as firm x and undercut p_x marginally. If the answer is yes, then firm x needs to price such that firm y would not find it profitable to locate and the same position. In this case, we say that *the price undercutting constraint is binding* in equilibrium for firm x.

From Assumption 1, we have the following result.

Remark 2 *For $r_i = r_j$ and $i > j$, firm i locate at the location such that $\bar{t}_i = 1$.*

Assumption 1 ensures that the ray is not long enough to accommodate two local monopolists. For $\bar{t}_i < 1$ and $\underline{t}_i > \bar{t}_j$, the two firms are not in competition. From Proposition 1, at least one firm would have the incentive to either lower its price or move closer to the competitor to expand demand. For $\bar{t}_i > 1$, with any given price, π_i increases by moving closer to the centre. For $\bar{t}_i < 1$ and $\underline{t}_i < \bar{t}_j$, with any given price, π_i increases by moving closer to the end of the spoke. Thus it is never an equilibrium to choose a price and location combination such that $\bar{t}_i \neq 1$.

4.1 The follower's decision

The complete characterization of firm y's best response requires the specification of (r_y, y, p_y) given any possible combinations of (r_x, x, p_x). Instead of going through all different possibilities, we argue first that some location configurations would never arise in equilibrium. The following is a list of all possible location confugurations:

1. $x = 0$ (the incumbent chooses the standard product):
   (a) $y > 0$ (the entrant chooses a niche product).
   (b) $y = 0$ (the entrant locates at the sam position as the incumbent and also chooses the standard product).
2. $x > 0$ (the incumbent chooses a niche product):
   (a) $r_y \neq r_x$ and $y > 0$ (the entrant chooses a niche product on a different ray).
   (b) $r_y = r_x$ and $y > x$ (the entrant chooses a most customized niche product on the same ray).
   (c) $r_y = r_x$ and $x \geq y \geq 0$ (the entrant either chooses a less customized niche product on the same ray or offer the standard product).

We denote by t_{xy} the relavant marginal consumer indifferent between buying from the two firms, i.e., $v_{t_{xy}}(x, p_x) = v_{t_{xy}}(y, p_y) > 0$. We first make the following observation.

Remark 3 *When $x > 0$, it is never an equilibrium for firm y to choose (a) $r_y \neq r_x$ and $y > 0$ and (b) $r_y = r_x$ and $y > x$.*

Proof. (a) $r_y \neq r_x$ and $y > 0$ is never an equilibrium: If the consumer at the centre buys from y. By moving towards 0 while holding p_y constant, the loss of demand from r_y is compensated by gain of demand from the third spoke, r_i, $i \neq x \neq y$ by exactly the same amount. Therefore, total demand from $r_i \neq r_x$ and r_y remains the same while demand from r_x increases. Thus, firm y always has the incentive to move to $y = 0$, and it is never optimal to choose $y > 0$ and $r_y \neq r_x$. This situation is depcited in Figure 2. Similarly, if the consumer at the centre buys from x. Firm x should move to the centre. Given the mass consumer close to the origin, it is never an equilibrium that this consumer does not buy. Thus, $x > 0$, $r_y \neq r_x$, and $y > 0$ is never an equilibrium. (b) $r_y = r_x$ and $y > x$ is never an equilibrium. With Assumption 1, $\underline{t}_y < \bar{t}_x$, firm y is always better off locating on an empty ray with $r_y \neq r_x$. ■

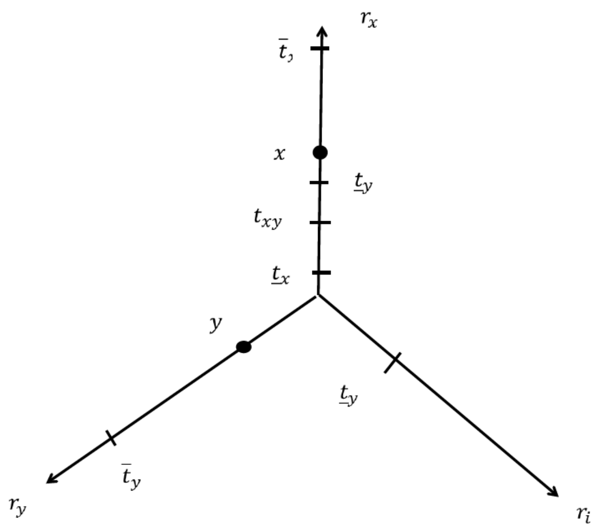

Figure 2: $r_y \neq r_x$ and $y > 0$

Thus we only need to analyze case 1 and case 2(c).

4.2 The subgame with $x = 0$ (Case 1)

When firm x locates at the center, firm y can either choose to locate off the center and set its price optimally given p_x or it can choose to locate at the center. If firm y locates at the centre, its optimal strategy is to undercut p_x marginally whenever $p_x \leq \frac{V}{2}$. For $p_x > \frac{V}{2}$, the optimal $p_y = \frac{V}{2}$.

For the case $y > 0$, it is never an equilibrium if D_y comes from the other two rays $r_i \neq r_y$. Similar to the argument given in Remark 3, if consumer located at the centre buys from firm y, firm y always has the incentive to move close to the centre and choose $y^* = 0$. Thus, for the case $y > 0$, we only look at the case that D_y comes only from r_y.

To characterise D_y, we define two critical p_y levels. Let $\overline{p}_1$ be the price such that $\bar{t}_y = 1$, and p_2 be the price such that $\underline{t}_y = \bar{t}_x$:

$$\bar{t}_y = \frac{V - p_1}{\tau} + y = 1 \Leftrightarrow \overline{p}_1 \equiv V - \tau\,(1 - y)\,. \tag{2}$$

$$y - \frac{V - p_2}{\tau} = \frac{V - p_x}{\tau} \Leftrightarrow p_2 \equiv 2V - y\tau - p_x. \tag{3}$$

From Assumption 1 and Remark 2, there is not enough space for two local monopolists on a single ray. In equilibrium, we must have $p_y = \overline{p}_1 \leq p_2$. This gives $y = \frac{\tau - V + p_y}{\tau}$. The optimisation problem is thus

$$\max_{p_y} (1 - t_{xy})\, p_y = \frac{V + \tau + p_x - 2p_y}{2\tau} p_y. \tag{4}$$

The FOC gives the optimal price and location,

$$p_y = \frac{V + \tau + p_x}{4} \text{ and } y = \frac{5\tau - 3V + p_x}{4\tau}. \tag{5}$$

The resulting profit is $\pi_y = \frac{(V+\tau+p_x)^2}{16\tau}$. This is interior if $p_y \leq p_2$ or $p_x \leq \frac{5}{3}V - \tau$.

4.3 The subgame $r_y = r_x$ and $x \geq y \geq 0$ (Case 2(c))

Firm y again always can choose to locate at $y = x$ and undercut p_x marginally. We show in this section that for this subgame where the two firms choose the same ray or the standard product, firm y either locates at $y = 0$ or $y = x$. We first discuss the possible demand configuration for firm y.

Let $\underline{p}_1$ be the price for firm y such that $\underline{t}_y = 1$:

$$\underline{p}_1 = V - \tau\,(1 + y)\,.$$

With Assumption 1, we know that $V\,(t_{xy}) \geq 0$. Thus demand for firm y is $D_y = 2\underline{t}_y + t_{xy}$ for $p_y \geq \underline{p}_1$ and $D_y = 2 + t_{xy}$ for $p_y < \underline{p}_1$.

Lemma 1 *For the case $x > y \geq 0$, the optimal location is $y^* = 0$ with $p_y = \frac{4V + p_x + x\tau}{10}$ for $p_x \geq 6V - 10\tau - x\tau$ and $p_y = \underline{p}_1 = V - \tau$ for $p_x < 6V - 10\tau - x\tau$.*

For the case $x = y > 0$, the local best-reponsemust is for firm y to undercut p_x marginally.

Proof. We first show that if firm y does not choose to locate at the same position as firm x, the local optimal location is $y = 0$. For large enough p_x, $p_y > \underline{p}_1$ and firm y does not fully cover the other two rays where $r_i \neq r_x$. For small enough p_x, the local optimal $p_y = \underline{p}_1$. See the appendix for the details. ■

4.4 Equilibrium

With the analysis of y's local best best responses in the two cases, $x = 0$ and $x > 0$, we now look at firm x's optimal pricing and location in the first stage. First, we note that given the mass consumer around the centre, the price undercutting constraint is always binding if firm x chooses $x = 0$. Thus, if firm x wishes to choose $x = 0$, it needs to revise its price downwards such that firm y has no incentive to choose $y = x = 0$. Firm x must price low enough so that firm y is indifferent between choosing the same location as firm x and choosing a cutomized product according to its local best response.

Lemma 2 *For $x = 0$, firm x's local optimal pricing is always constrained by firm y's incentive to locate at $y = x = 0$ and undercut p_x marginally. Firm x prices such that $\pi_y [y = x = 0, p_y = p_x] \leq \pi_y [x = 0, y > 0, p_y]$.*

Proof. The incentive to locate at $x = 0$ is due to the mass consumer around the centre. The constrained p_x required to persuade firm y not to locate at $y = x = 0$ is presented in the appendix. ■

For the case, $x \geq y \geq 0$ and $r_x = r_y$, we only need to consider the case that firm x's demand D_x comes from r_x in equilibrium. From Remark 2, we have $p_x = V - \tau (1 - x)$. The following lemma establishes that for large enough τ, if $x > 0$, firm y prefers locating at $y = 0$ and serving the mass consumers instead of locating at the same position as firm x.

Lemma 3 *If $x > 0$, for large enough τ ($\tau > \frac{23}{29}V$), $y^* = 0$ and the price undercutting constraint is not binding.*

Proof. For the proof, compare y's profits for $y = 0$ and $y = x$ according to local best responses in Lemma 2. We show that for large enough τ, $y = 0$ gives higher profit. See the appendix for the details of firms' equilibrium locations and profits. ■

For any given price, the central position is more profitable given the mass consumers, and thus the price undercutting constraint binds for $x = 0$. For sufficiently large τ, firm x is better off locating off the centre by choosing a sufficiently customized product (away from 0). Combining the results in the previous two lemmas and comparing the profit levels, we have the following propositions.

Proposition 2 *For sufficiently large τ ($\tau > \frac{23}{29}V$), the equilibrium locations are $x = \frac{47\tau - 23V}{36\tau} > 0$ and $y = 0$ with $p_x = \frac{13V + 11V}{36}$ and $p_y = \frac{67V + 29\tau}{180}$. Firm x's product becomes more customized, charges a higher price and earns higher profits as τ increases. Firm y's price and profits also increase as τ increases.*

Proof. See the appendix. ■

Note that $p_x > p_y$. The customized product charges a higher price. In particular, p_x is greater than the monopoly price. For this parameter range ($\tau > \frac{23}{29}V$), p_y is greater than the monopoly price as well.

The previous result says that for sufficiently large x, firm y has no incentive to choose $y = x > 0$ and undercuts p_x. In equilibrium, firm x is better off offering the customized product than choosing the standard product and having to price low enough to prevent y's incentive of locating at the centre. By locating at $y = x$ and undercutting p_x, firm y does not face any competition. The trade-off is that firm y needs to be further away from the mass consumers at the centre. For x very close to 0, firm y has strong incentive to choose $y = x$ and the price undercutting constraint binds in equilibrium for firm x. As τ gets smaller, the consumers are more mobile, the distance x required for firm x not having to be constrained by the price undercutting constraint increases. Indeed, as shown in the proof for Lemma 3, for small enough τ, the price undercutting constraint binds even when $x > 0$. The next proposition shows that for small enough τ and when firm x's pricing is distorted by the price undercutting constraint both when $x = 0$ and when $x > 0$, firm x is better off locating at the centre and serving the mass consumers.

Proposition 3 *For small τ ($\tau < \frac{113}{209}V$), there exists some parameter range such that $x = 0$ and $y > 0$ is an equilibrium configuration.*

Proof. For τ small, the price undercutting constraint is binding both when $x = 0$ and when $x > 0$. We note that in equilibrium, firm y gets higher profit than x by locating at $y = x$ and charging $p_y = p_x$ since firm y does not face any competitor and thus have higher demand compared to firm x. This discrepancy is bigger when $x > 0$ and τ is small since D_y comes from 3 rays while D_x only comes from 1 ray with $y = 0$. This hurts firm x's profitability. When τ is sufficiently small, firm x gets higher profit locating at the centre. See the appendix for details. ■

However, since the centre features mass consumers, the price undercutting constraint is always binding. Thus when x locates at the centre, the equilibrium is characterized by low prices. In this equilibrium, $p_x = (23\tau - V) - \sqrt{48\tau(11\tau - V)}$ is less than the monopoly price. The resulting $p_y = 6\tau - \sqrt{3\tau(11\tau - V)}$. We have $p_y > p_x$. The customized product charges a higher price than the standard product. For this parameter range, p_y is less than the monopoly price. Due to the first mover's aggressive pricing behavior, both products' prices are lower than the monopoly price.

5 Conclusion

We employ a new product space specification to study firms' incentives to customize and pricing behavior. In our model, the product space gives a natural interpretation of standard or general versus customized products. Our results indicate that in a sequential move game, whether or not the first mover offers the standard product depends on the transportation cost. For high enough transportation cost, the first mover customizes while the subsequent entrant offers the standard product. For small transportation cost, the first mover offers the standard product while the follower customizes. But the first mover has to price aggressively in this equilibrium. In both equilibria, the price for the customized product is higher than the price for the standard product. This is entirely due to the product space without assuming higher willingness to pay from consumers for the niche product. For high transportation costs, both firms charge prices higher than the monopoly price. For small transportation costs, the first mover has to price aggressively and in equilibrium, both firms price lower than the monopoly price. We show that transportation costs not only affects firms pricing behavior, it also affects firms' product design (location choices).

References

[1] Alptekinoğlu, A. and C. J. Corbett (2008). Mass customization vs. mass production: variety and price competition, *Manufacturing Service Operation Management*, **10** (2): 204 - 17.

[2] Beckman, M. (1976). Spatial price policies revisited, *Bell Journal of Economics*, **7**(2): 619–630.

[3] Chen, Y. and M. H. Riordan (2007). Price and variety in the spokes model, *The Economic Journal*, **117** (522): 897 - 921.

[4] Chen, Y. and M. H. Riordan (2008). Price-increasing competition, *Rand Journal of Economics*, **39** (4): 1042 - 58.

[5] d'Aspremont, C., J. J. Gabszewicz, and J. F. Thisse (1979). On Hotelling's "stability in competition," *Econometrica*, **47** (5): 1145 - 50.

[6] Dewan, R., B. Jing, and A. Seidmann (2003). Product Customization and Price Competition on the Internet, *Management Science*, **49** (8): 1055 - 70.

[7] Doraszelski, U. and M. Draganska (2006). Market segmentation strategies of multi-product firms, *Journal of Industrial Economics*, **LIV** (1): 125 - 49.

[8] Farrell, J. and P. Klemperer, 2007. "Coordination and Lock-In: Competition with Switching Costs and Network Effects," in M. Armstrong and R. H. Porter eds. *Handbook of Industrial Organization* Volume 3, North-Holland.

[9] Hendel, I. and J. N. de Figueiredo (1997). Product differentiation and endogenous disutility, *International Journal of Industrial Organization*, **16**: 63-79.

[10] Hotelling, H. (1929). Stability in competition, *Economic Journal*, **39**: 41 - 57.

[11] Markides, C.C. and Geroski, P.A. (2005). *Fast Second: How Smart Companies Bypass Radical Innovation to Enter and Dominate New Markets*, London, Jossey-Bass Inc Pub.

[12] Mendelson, H. and A. K. Parkturk (2005). Product-line competition: customization vs. proliferation, *Management Science*, **54** (12): 2039-53.

[13] Mizuno, K. and Y. Okumura (2014). Strategic Location Choice and Network Formation for Entry, *Journal of Institutional and Theoretical Economics* (JITE), **170** (2): 201-224.

[14] Norman, G., L. Pepall, and D. Richards (2011). 'Innovation, Fast Seconds, and Patent Policy' Northwestern University School of Law, Searle Center on Law, Regulation, and Economic Growth Working paper: 2011-033.

[15] Prescott, E. C. and M. Visscher (1977). Sequential location among firms with foresight, *The Bell Journal of Economics*, **8** (2): 378-93.

[16] Thisse, J.-F. and X. Vives (1988). On the strategic choice of spatial price policy. *American Economic. Review*, **78** (1): 122 - 37.

[17] Von Ungern-Sternberg, T. (1988). Monopolistic competition and general purpose products. *Review of Economic Studies*, **55**: 231-246.

[18] Xia, N. and S. Rajagopalan (2009). Standard vs. custom products: variety, lead time, and price competition, *Marketing Science*, **28** (5): 887 - 900.

6 Appendix

Proof. of Lemma 1

We show that firm y never has incentives to charge $p_y < \underline{p}_1$. For $p_y \geq \underline{p}_1$, for $y \neq x$, it is always profit enhancing for y to move closer to 0. For any given price, the gain of demand from the other two rays is greater than the loss in demand from r_x. Thus, $y^* = 0$. The argument can be formalized as the following. For any $x > y \geq 0$, $t_{xy} = \frac{\tau y + \tau x + p_x - p_y}{2\tau}$ and $t_{\underline{y}} = \frac{V - p_y}{\tau} - y$ with the restriction $y \geq 0$.

$$\pi_y = \left(t_{xy} + 2t_{\underline{y}}\right) p_y = \frac{1}{2\tau}\left(4V + p_x - 5p_y + x\tau - 3y\tau\right) p_y. \tag{6}$$

It is then immediate that the optimal location is $y^* = 0$. Maximizing gives

$$p_y = \frac{4V + p_x + x\tau}{10} \text{ and } \pi_y = \frac{\left(4V + p_x + x\tau\right)^2}{40\tau}. \tag{7}$$

The restriction $p_y \geq \underline{p}_1$ requires $p_x \geq 6V - 10\tau - x\tau$.

For $p_x < 6V - 10\tau - x\tau$ and $p_y < \underline{p}_1$, the demand configuration is $2 + t_{xy}$. The optimization problem is

$$\max_{p_y} \left(2 + \frac{\tau y + \tau x + p_x - p_y}{2\tau}\right) p_y \tag{8}$$

subject to $y \leq x$, $y \geq 0$, $p_y \leq V - \tau\left(1 + y\right)$, and $p_y \geq 0$.

The Lagrangian gives

$$\max_{\{y, p_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}} \left(2 + \frac{\tau y + \tau x + p_x - p_y}{2\tau}\right) p_y + \lambda_1\left(x - y\right) + \lambda_2 y + \lambda_3\left(V - \tau\left(1 + y\right) - p_y\right) + \lambda_4 p_y \tag{9}$$

From Kuhn and Tucker conditions, the critical points satisfy

$$\frac{1}{2}p_y - \lambda_1 + \lambda_2 - \lambda_3\tau = 0 \tag{10}$$

$$\frac{-1}{2\tau}p_y + \left(2 + \frac{\tau y + \tau x + p_x - p_y}{2\tau}\right) - \lambda_3 + \lambda_4 = 0 \tag{11}$$

$$\lambda_1 \geq 0,\ x - y \geq 0,\ \lambda_1 (x - y) = 0. \tag{12}$$

$$\lambda_2 \geq 0,\ y \geq 0,\ \lambda_2 y = 0 \tag{13}$$

$$\lambda_3 \geq 0,\ V - \tau(1 + y) - p_y \geq 0,\ \lambda_2 (V + \tau(1 + y) - p_y) = 0. \tag{14}$$

$$\lambda_4 \geq 0,\ p_y \geq 0,\ \lambda_4 p_y = 0. \tag{15}$$

For $y = x$, y's local best response is to price slightly below p_x. We discuss the case $y \neq x$ here. Thus, we have $\lambda_1 = 0$. We know that for profit maximization, we have to have $p_y > 0$. Thus $\lambda_4 = 0$. Given this, it can be shown that the solution would be $\{\lambda_2 > 0, \lambda_3 > 0\}$ with $y = 0$ and $p_y = V - \tau$. Thus, for $y \neq x$, the local optimal location is always $y = 0$ with the local optimal price constrained by $\underline{p}_1$ for small enough p_x. ■

Proof. of Lemma 2: For the location $(x = 0, y > 0)$, the price p_1 such that $\bar{t}_x = 1$ is $p_1 = V - \tau$. It is easy to show that it is never optimal for firm x to price $p_x < p_1$. Firm y's best responses are contained in Section4.2.

Firm x solves

$$\max_{p_x} \pi_x = \frac{4V + y\tau - 5p_x + p_y}{2\tau} p_x = \frac{7Vp_x + 3\tau p_x - 9p_x^2}{4\tau}. \tag{16}$$

The FOC gives the local maximizer $p_x^* = \frac{7V+3\tau}{18}$ with $\pi_x^* = \frac{1}{144\tau}(7V + 3\tau)^2$. The resulting $p_y^* = \frac{25V+21\tau}{72}$, $y^* = \frac{93\tau-47V}{72\tau}$, and

$$\pi_y [x = 0, y > 0] = \frac{(V + \tau + p_x)^2}{16\tau} = \frac{(25V + 21\tau)^2}{5184\tau}. \tag{17}$$

If $y = x = 0$,

$$\pi_y \left[y = 0, p_y = \frac{7V + 3\tau}{18}\right] = 3\frac{V - p_y}{\tau}(p_y) = \frac{(7V + 3\tau)(11V - 3\tau)}{108\tau}. \tag{18}$$

Given Assumption 1, $\pi_y [x = 0, y > 0] < \pi_y \left[y = 0, p_y = \frac{7V+3\tau}{18}\right]$. Since if y chooses $y = x = 0$ and undercuts marginally, y faces no other competitor with profit equal to $3\frac{V-p_x}{\tau}(p_x)$. On the other hand, if $y > x = 0$, the resulting profit is $\frac{(V+\tau+p_x)^2}{16\tau}$. To eliminate y's incentive to undercut, x needs to price such that

$$3\frac{V - p_x}{\tau}(p_x) \leq \frac{(V + \tau + p_x)^2}{16\tau}. \tag{19}$$

The undercutting proof price requires

$$p_x \leq \frac{-\tau + 23V - \sqrt{48(2V - \tau)(5V + \tau)}}{49}.$$

■

Proof. of Lemma 3:

Case 1: For $p_x \leq 6V - 10\tau - x\tau$, from Lemma 1, we have $p_y^* = V - \tau$ and $y^* = 0$. Given Remark 2, $x = 1 - \frac{V-p_x}{\tau}$. Firm x solves

$$\max_{p_x} (1 - t_{xy}) p_x = \frac{1}{\tau} (V - p_x) p_x. \tag{20}$$

Optimizing gives $p_x^* = \frac{V}{2}$ and $x^* = \frac{(2\tau - V)}{2\tau}$. This price is interior if $\tau \leq \frac{6}{11}V$. The resulting $\pi_y [y = 0] = \left(2 + \frac{(2\tau-V)}{2\tau}\right)(V - \tau) = \frac{(V-\tau)(6\tau-V)}{2\tau}$.

If locating at $y = x$ with $p_y = p_x$,

$$\pi_y [y = x, p_y = p_x] = \frac{V(2V - \tau)}{2\tau} > \pi_y [y = 0]. \tag{21}$$

Thus, firm y has incentive to locate at $y = x$ and undercut p_x marginally. In order to persuade firm y not to locate at the same position, firm x needs to price such that

$$\left(1 + 2\left(\frac{V - p_x}{\tau} - x\right)\right) p_x \leq \left(2 + \frac{p_x - (V - \tau) + \tau x}{2\tau}\right)(V - \tau). \tag{22}$$

We first show that even with this price cutting constraint binding, in equlibrium, Remark 2 sitll holds. Firm x's optimization problem is

$$\max_{\{x, p_x\}} (1 - t_{xy}) p_x = \left(1 - \frac{p_x - (V - \tau) + \tau x}{2\tau}\right) p_x \tag{23}$$

subject to $x \geq 1 - \frac{V-p_x}{\tau}$ and $\left(1 + 2\left(\frac{V-p_x}{\tau} - x\right)\right) p_x \leq \left(2 + \frac{p_x-(V-\tau)+\tau x}{2\tau}\right)(V - \tau)$.

The Lagrangian is

$$\begin{aligned}&\left(1 - \frac{p_x - (V - \tau) + \tau x}{2\tau}\right) p_x + \lambda_1 \left(x - 1 + \frac{V - p_x}{\tau}\right)\\ &+\lambda_2 \left(\left(2 + \frac{p_x - (V - \tau) + \tau x}{2\tau}\right)(V - \tau) - \left(1 + 2\left(\frac{V - p_x}{\tau} - x\right)\right) p_x\right).\end{aligned} \tag{24}$$

We know that the second constraint is binding in equilibrium with $\lambda_2 > 0$. To determine the optimal location x, we want to know if the first constraint is binding.

The FOCs are

$$\left(-\frac{1}{2}\right) p_x + \lambda_1 + \lambda_2 \left(\frac{1}{2}(V - \tau) + 2p_x\right) = 0$$

$$\left(-\frac{1}{2\tau}\right) p_x + \left(1 - \frac{p_x - (V - \tau) + \tau x}{2\tau}\right) - \lambda_1 \frac{1}{\tau} + \lambda_2 \left(-\frac{1}{2\tau}(3V + 3\tau - 8p_x - 4x\tau)\right) = 0$$

$$\lambda_1 \left(x - 1 + \frac{V - p_x}{\tau}\right) = 0$$

$$\lambda_2 > 0,\ \left(2 + \frac{p_x - (V - \tau) + \tau x}{2\tau}\right)(V - \tau) - \left(1 + 2\left(\frac{V - p_x}{\tau} - x\right)\right) p_x = 0.$$

Note that we do not place the restriction that $\lambda_1 \geq 0$ since for $x < 1 - \frac{V-p_x}{\tau}$, the demand is defined by $\bar{t}_x$ instead of 1 and the profit function is different. Suppose $\lambda_1 = 0$. We have

$$\left(-\frac{1}{2}\right) p_x + \lambda_2 \left(\frac{1}{2}(V - \tau) + 2p_x\right) = 0$$

$$\left(-\frac{1}{2\tau}\right)p_x + \left(1 - \frac{p_x - (V-\tau) + \tau x}{2\tau}\right) + \lambda_2\left(-\frac{1}{2\tau}\left(3V + 3\tau - 8p_x - 4x\tau\right)\right) = 0$$

$$\left(2 + \frac{p_x - (V-\tau) + \tau x}{2\tau}\right)(V-\tau) - \left(1 + 2\left(\frac{V-p_x}{\tau} - x\right)\right)p_x = 0.$$

From the first equation, we have $\lambda_2 = \frac{p_x}{V-\tau+4p_x}$. Substitute this λ_2 into the second equation:

$$x = -\frac{1}{V\tau - \tau^2}\left(\tau^2 + Vp_x - 3\tau p_x - V^2\right)$$

From the third equation,

$$x = \frac{\left(V^2 - 6V\tau + 3Vp_x + 5\tau^2 + 3\tau p_x - 4p_x^2\right)}{\tau\left(V - \tau + 4p_x\right)}$$

The above two equations together gives

$$4p_x^2 + 2\left(V-\tau\right)p_x + 3\left(V-\tau\right)^2 = 0$$

This cannot be true with any $p_x \geq 0$. Thus, in equilibrium, it must be the case that $\lambda_1 > 0$ and $x = 1 - \frac{V-p_x}{\tau}$. The other three FOCs are

$$\left(-\frac{1}{2}\right)p_x + \lambda_1 + \lambda_2\left(\frac{1}{2}\left(V-\tau\right) + 2p_x\right) = 0 \tag{25}$$

$$\left(-\frac{1}{2\tau}\right)p_x + \frac{1}{\tau}\left(V - p_x\right) - \lambda_1\frac{1}{\tau} + \lambda_2\left(\frac{1}{2\tau}\left(\tau - 7V + 12p_x\right)\right) = 0 \tag{26}$$

$$\lambda_2 > 0,\ \left(3\tau - V + p_x\right)\left(V-\tau\right) + \left(\tau - 4V + 4p_x\right)p_x = 0. \tag{27}$$

The third equation gives

$$p_x = \frac{3V \pm \sqrt{48\tau^2 - 64V\tau + 25V^2}}{8}.$$

It is easy to see that to satisfy the price undercutting constraint, we need $p_x = \frac{3V - \sqrt{48\tau^2 - 64V\tau + 25V^2}}{8}$ with $\lambda_1 = \frac{(2\tau - V)\sqrt{48\tau^2 - 64V\tau + 25V^2} - V(5V - 2\tau)}{16\sqrt{48\tau^2 - 64V\tau + 25V^2}}$ and $\lambda_2 = \frac{\sqrt{48\tau^2 - 64V\tau + 25V^2} + V}{4\sqrt{48\tau^2 - 64V\tau + 25V^2}} > 0$.

Case 2: For $p_x \geq 6V - 10\tau - x\tau$, given firm y's local best responses $y = 0$ and $p_y = \frac{4V + \tau x + p_x}{10}$ with $\bar{t}_x = 1$, firm x's optimisation problem is

$$\max_{p_x}\left(1 - t_{xy}\right)p_x = \frac{\left(13V + 11\tau - 18p_x\right)p_x}{20\tau}. \tag{28}$$

The FOC gives

$$p_x = \frac{13V + 11\tau}{36},\ x = \frac{47\tau - 23V}{36\tau} \text{ and } \pi_x = \frac{\left(13V + 11\tau\right)^2}{1440\tau}. \tag{29}$$

The solution is interior if $\tau \geq \frac{113}{209}V \approx 0.540V$. From firm y's best responses:

$$y = 0,\ p_y = \frac{67V + 29\tau}{180} \text{ and } \pi_y = \frac{\left(67V + 29\tau\right)^2}{12\,960\tau}. \tag{30}$$

If y locates at $y = x$ instead, we first determine the location of marginal consumers $\underline{t}_y$. With (x, p_x), the consumer locating at 0 gets utility $V(0) = \frac{(23V - 29\tau)}{18}$. The consumer purchases if $V(0) \geq 0$ or $\tau \leq \frac{23}{29}V \approx 0.793V$.

For $\frac{113}{209}V \leq \tau \leq \frac{23}{29}V$, for $y = x$ and $p_y \leq p_x$, firm y's optimisation problem is

$$\max_{p_y} \left(1 + 2\underline{t}_y\right) p_y = \left(1 + 2\left(\frac{V - p_y}{\tau} - \frac{47\tau - 23V}{36\tau}\right)\right) p_y. \tag{31}$$

The FOC gives $p_y = \frac{(59V - 29\tau)}{72}$. The price is constrained by p_x if $\frac{(59V - 29\tau)}{72} \geq \frac{13V + 11\tau}{36}$ or if $\tau \leq \frac{11}{17}V \approx 0.647V$. Thus, for $\frac{113}{209}V \leq \tau \leq \frac{11}{17}V$,

$$\pi_y\left(y = x, p_y = p_x\right) = (23V - 20\tau)\,\frac{13V + 11\tau}{324\tau}.$$

Firm y gets higher profit locating at the centre if $\frac{(67V + 29\tau)^2}{12\,960\tau} \geq (23V - 20\tau)\,\frac{13V + 11\tau}{324\tau}$. Or if $\tau \geq \frac{-2083 + 180\sqrt{2357}}{9641}V > \frac{11}{17}V$. Thus for $\frac{113}{209}V \leq \tau \leq \frac{11}{17}V$, firm y would like to locate at $y = x$.

For $\frac{11}{17}V \leq \tau \leq \frac{23}{29}V$,

$$\pi_y\left[y = x\right] = \frac{1}{2592\tau}\left(29\tau - 59V\right)^2 > \pi_y\left[y = 0\right]. \tag{32}$$

For $\tau \geq \frac{23}{29}V$, $V\left[0, p_x, x\right] < 0$ and $r_{\underline{t}_y} = r_y$ with $\underline{t}_y = y - \frac{V - p_y}{\tau}$. Firm y maximizes

$$\pi_y\left[y = x\right] = \left(1 - \underline{t}_y\right) p_y = \left(1 - \left(\frac{47\tau - 23V}{36\tau} - \frac{V - p_y}{\tau}\right)\right) p_y. \tag{33}$$

The FOC gives $p_y = \frac{(59V - 11\tau)}{72}$. This price is constrained by p_x if $\frac{(59V - 11\tau)}{72} \geq \frac{13V + 11\tau}{36}$. Or if $\tau \leq V$.

For $\frac{23}{29}V \leq \tau \leq V$,

$$\pi_y\left(y = x, p_y = p_x\right) = (23V - 11\tau)\,\frac{13V + 11\tau}{648\tau}. \tag{34}$$

Firm y gets higher profit locating at the centre if $\frac{(67V + 29\tau)^2}{12\,960\tau} \geq \frac{1}{648}\left(23V - 11\tau\right)\frac{13V + 11\tau}{\tau}$. This holds for the relevant parameter range.

For $\tau \geq V$,

$$\pi_y\left(y = x, p_y = \frac{59V - 11\tau}{72}\right) = \frac{1}{5184\tau}\left(11\tau - 59V\right)^2. \tag{35}$$

Firm y gets higher profit locating at the centre if $\frac{(67V + 29\tau)^2}{12\,960\tau} \geq \frac{1}{5184\tau}\left(11\tau - 59V\right)^2$. This holds given $\tau \geq \frac{23}{29}V$.

Thus, for sufficiently large τ, firm y prefers to locate at the centre and the price undercutting constraint is not binding. With small τ, firm x's pricing and location are distorted by the price undercutting constraint. ■

Proof. of Proposition 2: Consider $\tau > \frac{23}{29}V$. From Lemma 3, for this parameter range, if x locates off the centre with $\pi_x = \frac{(13V + 11\tau)^2}{1440\tau}$, $y^* = 0$. We show that in this case, locating off the centre gives higher profit than locating at the centre with the price undercutting proof constraint.

For $\frac{11}{21}V \leq \tau \leq \frac{3+\sqrt{6}}{6}V \approx 0.908V$, locating at the centre with $p_x = (23\tau - V) - \sqrt{48\tau(11\tau - V)}$ gives

$$\begin{aligned}\pi_x &= (2 + t_{xy})\left((23\tau - V) - \sqrt{48\tau(11\tau - V)}\right) \\ &< 3\left((23\tau - V) - \sqrt{48\tau(11\tau - V)}\right).\end{aligned}$$

In the given parameter range, this p_x is decreasing in τ. Since the price is constrained, higher price gives higher profit. We find the upper bound for the profit $\pi_x[x = 0]$ by using the smallest possible τ. Let $\tau = \frac{23}{29}V$, the undercutting-proof price is

$$p_x = \left(\frac{500}{29} - \sqrt{\frac{247\,296}{841}}\right)V < 0.094V.$$

$$\frac{(13V + 11\tau)^2}{1440\tau} \geq 3\left(\frac{94}{1000}\right)V$$

holds for the given τ range and locating off the center gives higher profit.

For $\frac{3+\sqrt{6}}{6}V \leq \tau \leq \frac{11}{9}V$ and $\tau \geq \frac{63-\sqrt{297}}{54}V$, locating at the centre gives $p_x = \frac{-\tau+23V-\sqrt{48(2V-\tau)(5V+\tau)}}{49}$ and $\pi_x = \frac{(V+\tau+p_x)^2}{16\tau}$. This price is increasing in τ. Let $\tau = \frac{11}{9}V$.

$$p_x = \frac{\frac{196}{9} - \sqrt{\frac{6272}{27}}}{49}V < 0.14V.$$

Locating off the centre gives higher profit if $\frac{(13V+11\tau)^2}{1440\tau} \geq \frac{\left(V+\tau+\frac{14}{100}V\right)^2}{16\tau}$. This holds for sure.

For $\frac{11}{9}V < \tau \leq \frac{63+\sqrt{297}}{54}V < \frac{149}{100}V$, locating at the centre gives $p_x = \frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5}$ and $\pi_x = 3\frac{V-p_x}{\tau}p_x$. The price is increasing in τ if $\tau \leq \frac{3}{2}$. Let $\tau = \frac{149}{100}V$,

$$p_x = \frac{\frac{249}{100} - \sqrt{\frac{12\,503}{5000}}}{5}V < \frac{19}{100}V.$$

$$\frac{(13V + 11\tau)^2}{1440\tau} \geq 3\frac{V - \left(\frac{19}{100}V\right)}{\tau}\left(\frac{19}{100}V\right)$$

holds given $\tau \geq \frac{11}{9}V$.

Therefore, for $\tau \geq \frac{23}{29}V$, firm x gets higher profit locating off the centre. ■

Proof. of Proposition 3: Consider $\tau \leq \frac{113}{209}V$. From discussion in Section 4.2 and Lemma 1, the price undercutting constraint is binding for both $x > 0$ and $x = 0$. For $x > 0$, we have $x = 1 - \frac{V-p_x}{\tau}$, $p_x = \frac{3V-\sqrt{48\tau^2-64V\tau+25V^2}}{8}$, $y = 0$, and $p_y = V - \tau$. The price level $\frac{3V+\sqrt{48\tau^2-64V\tau+25V^2}}{8}$ is decreasing in τ in this parameter range. Thus, we use a smallest τ, $\tau = \frac{1}{2}V$, to establish the upper bound for the profit level off the centre. This gives price

$$p_x = \frac{3-\sqrt{5}}{8}V < \frac{96}{1000}V$$

The resulting profit in this equilibrium is

$$\pi_x[x>0] = \frac{1}{\tau}\left(V - \frac{96}{1000}V\right)\frac{96}{1000}V.$$

If locating at the centre, we have $p_x = (23\tau - V) - \sqrt{48\tau(11\tau - V)}$. This price is decreasing in τ in the relevant parameter range. Thus, we use $\tau = \frac{113}{209}V$ to establish the lower bound for $\pi_x[x=0]$. This gives

$$p_x = \left(\frac{2390}{209} - \sqrt{\frac{509\,856}{3971}}\right)V > \frac{1}{10}V$$

with profit

$$\pi_x[x=0] = (2 + t_{xy})\,p_y = -\frac{1}{4}\left(\frac{1}{10}V\right)\frac{V - 11\tau + \left(\frac{1}{10}V\right)}{\tau}.$$

Locating at the centre gives higher profit if

$$-\frac{1}{4}\left(\frac{1}{10}V\right)\frac{V - 11\tau + \left(\frac{1}{10}V\right)}{\tau} \geq \frac{1}{\tau}\left(V - \frac{96}{1000}V\right)\frac{96}{1000}V$$

Or if $\tau > \frac{28\,571}{68\,750}V$. This holds. ■