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Abstract

In a sequential auction, we analyze whether selling a stochastically more valuable good in the first or second auction generates more revenue and welfare. One of the buyers is a global bidder who enjoys synergy if she wins both goods. The others are local bidders interested in one specific good. After deriving the equilibrium, we show that there are cases in which selling the less valuable good in the first auction generates higher revenue and/or welfare. We also show the impact of inefficient allocations on revenue.

JEL Codes: D44, D82

Keywords: Sequential Auctions, Inefficiencies, Multi-dimensional values, Simulations

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1 Introduction

There are auctions in which one good is more valuable than the other one, such as spectrum license auctions (e.g., Meng and Gunay, 2017). In some of these auctions, there are local bidders bidding only for one specific good, and global bidders bidding for all goods (e.g. Krishna and Rosenthal, 1996). Global bidders enjoy synergies if they win all goods.

In this paper, we analyze whether selling a stochastically more valuable good in the first or second auction generates higher revenue and/or welfare in a second-price sequential auction. By using simulation methods, we find cases where selling the more valuable good first might result in less revenue and/or welfare. We also show how the inefficient allocations affect the revenue in these auctions.

In the literature, it has been assumed that the goods are either stochastically equivalent (Krishna and Rosenthal, 1996; Branco, 1997), or the second good becomes more valuable to the winner of the first auction (Jeitschko and Wolfstetter, 2002; De Silva 2005; Leufkens et. al., 2010), and thus, the order of selling goods has no impact on revenue and welfare. One exception is Benoit and Krishna (2001). They find that selling the more valuable good first generates more revenue in a model with budget constrained (global) bidders in a complete information game.

2 The Model

Two goods, A and B, are sold in a second-price sequential auction. The goods has zero value to the seller. There is one risk-neutral global bidder, G, who bids for both goods, and enjoy a synergy of $\theta > 0$ if wins both goods.[1] There are also $N_i > 0$ risk neutral local bidders bidding for good $i = A, B$. $N_i + 1$ independent draws from the distribution function F_i determines the private valuation, v_{ki}, for each bidder, $k = G, 1, 2.., N_i$, and $i = A, B$. The distribution function F_i, $i = A, B$, has a twice differentiable density function $f_i > 0$ on the

[1]Assuming one global bidder when a bidder has *multi-dimensional* valuations is not uncommon in the literature (see Meng and Gunay, 2017 and the references therein) since the equilibrium strategy for multiple global bidders have not been calculated unless they have single types.

interval $(0, 1]$ with $f_i(0) \geq 0$.

Definition: We say that A is stochastically more valuable than B if (i) $F_A(x) \leq F_B(x)$ and (ii) $[F_A(x)]^{N_A} \leq [F_B(x)]^{N_B}$ for all x.

The definition guarantees that, $[F_A(x)]^{N_A+1} \leq [F_B(x)]^{N_B+1}$, the expected value of the first highest order statistics for A is greater than for B.

We use symmetric subgame perfect Bayesian equilibrium like Leufkens et. al., (2010). The equilibrium strategy for local bidders is bidding their valuations truthfully in both auctions (in weakly undominated strategies). The global bidder's equilibrium strategy in the second auction is bidding her marginal valuation truthfully; hence, she bids $v_{Gj} + \theta$ if won good i in the first auction, and bid v_{Gj} otherwise, where $i, j = A, B$ and $i \neq j$.

To derive the global bidder's equilibrium strategy in the first auction for good i, we have to maximize her payoff given the sequential rationality. Let $p_i = max\{v_{ki}\}, k = 1, 2.., N_i$ denote the maximum valuation of local bidders for good $i = A, B$. Then, the distribution function for p_i is $G_i(.) = [F_i(.)]^{N_i}$ for $i = A, B$. The expected payoff for the global bidder when she bids p is

$$Max_p \int_0^p (v_{Gi} - p_i) dG_i(p_i) + Pr(p > p_i) \int_0^{min\{v_{Gj}+\theta,1\}} (v_{Gj} + \theta - p_j) dG_j(p_j) \\ + Pr(p < p_i) \int_0^{v_{Gj}} (v_{Gj} - p_j) dG_j(p_j) \quad (1)$$

The first integral is the expected profit from winning i in the first auction, second is the expected profit from winning j after winning i, and the third is the expected profit from winning j after losing i.

Equation 2 is the first order condition, and it gives the equilibrium bidding price which we denote as p_{ij} when good i is auctioned first, and j second. The global bidder compares two payoffs when he decides on p_{ij}; the first one is the expected payoff from winning the first auction, and the second one is the expected payoff from losing the first auction. If the first expected payoff is higher, she should increase p_{ij} until both payoffs are equal.

$$\underbrace{(v_{Gi}-p_{ij})+\int_0^{min\{v_j+\theta,1\}}(v_{Gj}+\theta-p_j)dG_j(p_j)}_{\text{Expected profit from winning the first auction}} = \underbrace{\int_0^{v_j}(v_{Gj}-p_j)dG_j(p_j)}_{\text{and losing the first auction}} \quad (2)$$

By using integration by parts and equation 2, we derive the global bidder's equilibrium bid.[2]

Proposition 1 *The global bidder's equilibrium bid, p_{ij} in the first auction for good i is*

$$a)\ \textit{If } v_{Gj}+\theta<1, \textit{then}\ \ p_{ij}(v_{Gi},v_{Gj},N_j) \;=\; v_{Gi}+\int_{v_{Gj}}^{v_{Gj}+\theta}G_j(p,N_j)dp$$

$$b)\ \textit{If } v_{Gj}+\theta\geq 1, \textit{then}\ \ p_{ij}(v_{Gi},v_{Gj},N_j) \;=\; v_{Gi}+(v_{Gj}+\theta-1)+\int_{v_{Gj}}^{1}G_j(p,N_j)dp$$

We have some observations based on proposition 1. First, we have $p_{ij}\neq p_{ji}$ in our model. This is different than the aforementioned papers in the literature (except the complete information model of Benoit and Krishna, 2001) since we use multi-dimensional types and different number of bidders on each auction. Second, the global bidder's equilibrium incentives are conditioned on winning or losing the first auction, and its continuation payoff; hence, p_{ij} is a function of the number of second-auction local bidders but not the number of first-auction local bidders. Third, the global bidder bids over her stand-alone valuation which exposes her to the ex-post loss as well known in the literature (e.g. Krishna and Rosenthal, 1996). Fourth, the global bidder does not bid above the possible highest marginal valuation for license A. That is, $p_{ij}<v_{Gi}+\theta$ holds since $G_i(x)<1$. Fifth, when $v_{GA}+v_{GB}+\theta\geq 2$, the global bidder bids above 1 in both auctions, and wins both goods.[3]

Next, we show that the global bidder is expected to bid higher in the first auction if the more valuable good is auctioned first.

Proposition 2 *Assume that good A is stochastically more valuable good, and $v_{GA}>v_{GB}$. Then $p_{AB}\geq p_{BA}$.*

[2]See the discussion paper of Gunay and Meng (2017) for the proof.

[3]Since this implies $v_{Gj}+\theta\geq 2-v_{Gi}\geq 1$, we have to use part b of proposition 1. But then $p_{ij}\geq v_{Gi}+(v_{Gj}+\theta-1)\geq 1$, which guarantees that global bidder wins the first auction.

While the global bidder's bid is higher in the AB auction where more valuable A is sold first compared to the BA auction where B is sold first, we show that this does not guarantee a higher expected revenue in the AB auction in the next section.

3 Simulations

Since we have multi-unit valuations, calculating ex-ante revenue, welfare, and probability of inefficient allocations are extremely difficult; hence, we use simulations like Krishna and Rosenthal (1996) to compare the AB and BA auctions.[4] We draw 50000 valuations for each bidder in MATLAB, and calculate the equilibrium bidding prices, p_{AB} and p_{BA} with the help of proposition 1. Table 1 shows all possible outcomes, and the corresponding revenue and welfare in AB and BA auctions which helps us in calculating them ex-post. The table shows that there are four different types of inefficiency. Two of them are the global bidder winning one or both goods with an ex-post loss (rows 2 and 4 in Table 1); one of them is the global bidder winning (one good) inefficiently with a profit (row 6), and the last one is the local bidders winning both goods inefficiently (row 8). There cannot be an inefficient outcome in which the global bidder wins the first good and loses the second one with profit. In the next proposition, we prove this.

Proposition 3 *There is no inefficient outcome in an ij auction such that global bidder wins i with profit but loses j.*

	License i won by	License j won by	Global bidder makes	Allocation is	Revenue is	Welfare is
1.	Global Bidder	Global Bidder	Profit	Efficient	$p_i + p_j$	$v_{Gi} + v_{Gj} + \theta$
2.	Global Bidder	Global Bidder	Loss	Inefficient	$p_i + p_j$	$v_{Gi} + v_{Gj} + \theta$
3.	Global Bidder	Local Bidder j	Profit	Efficient	$p_i + v_j + \theta$	$v_{Gi} + p_j$
4.	Global Bidder	Local Bidder j	Loss	Inefficient	$p_i + v_j + \theta$	$v_{Gi} + p_j$
5.	Local Bidder i	Global Bidder	Profit	Efficient	$p^*_{ij} + p_j$	$p_i + v_{Gj}$
6.	Local Bidder i	Global Bidder	Profit	Inefficient	$p^*_{ij} + p_j$	$p_i + v_{Gj}$
7.	Local Bidder i	Local Bidder j	Zero Profit	Efficient	$p^*_{ij} + v_j$	$p_i + p_j$
8.	Local Bidder i	Local Bidder j	Zero Profit	Inefficient	$p^*_{ij} + v_j$	$p_i + p_j$

Table 1: All possible outcomes in an ij auction when $N_i = N_j = 1$

[4]Krishna and Rosenthal (1996) use simulations except for uniform distribution but they have single type for the global bidder.

	Synergy with $\theta = 0.2$ $N_A = N_B = 1$	Synergy with $\theta = 0.5$ $N_A = N_B = 1$	Synergy with $\theta = 0.8$ $N_A = N_B = 1$	Synergy with $\theta = 0.2$ $N_A = N_B = 2$	Synergy with $\theta = 0.5$ $N_A = N_B = 2$	Synergy with $\theta = 0.8$ $N_A = N_B = 2$
$F_A =$ **Beta Distribution with** $\alpha = 3$ **and** $\beta = 1$; F_B **=Uniform Distribution**						
$p_{AB}^{Exp.Loss}$	4.5060 %	6.3200 %	3.6480 %	4.6300 %	7.7980 %	5.6860 %
$p_{AB}^{Ineff.(GW)}$	2.8760 %	1.7920 %	0.3680 %	2.8260 %	1.8820 %	0.3780 %
$p_{AB}^{Ineff.}$	8.0320 %	9.3060 %	4.5400 %	8.4380 %	12.3580 %	7.8860 %
$p_{BA}^{Exp.Loss}$	2.0560 %	2.3340 %	1.5620 %	1.8080 %	2.4240 %	1.9720 %
$p_{BA}^{Ineff.(GW)}$	1.5600 %	1.2320 %	0.5860 %	1.1220 %	1.0380 %	0.6120 %
$p_{BA}^{Ineff.}$	4.0020 %	4.3080 %	2.6760 %	3.5280 %	4.8760 %	3.7120 %
$\triangle R\%$	0.1447 %	0.7650 %	0.6961 %	-0.1020 %	-0.2140 %	0.0981 %
$\triangle SW\%$	-0.1325 %	-0.3432 %	-0.1847 %	-0.1369 %	-0.4394 %	-0.3155 %
F_A **=Uniform distribution;** $F_B =$ **Beta Distribution with** $\alpha = 1$ **and** $\beta = 3$						
$p_{AB}^{Exp.Loss}$	2.9080 %	3.6200 %	2.3720 %	2.8700 %	5.2860 %	4.5160 %
$p_{AB}^{Ineff.(GW)}$	1.8920 %	1.7380 %	0.7560 %	1.7320 %	2.1180 %	1.1640 %
$p_{AB}^{Ineff.}$	5.5460 %	6.4820 %	3.7680 %	5.6820 %	6.9220 %	7.7240 %
$p_{BA}^{Exp.Loss}$	4.5060 %	6.1260 %	3.5200 %	3.5220 %	6.6720 %	5.7500 %
$p_{BA}^{Ineff.(GW)}$	2.8960 %	1.7700 %	0.3400 %	2.6720 %	2.1760 %	0.4740 %
$p_{BA}^{Ineff.}$	8.1080 %	9.2380 %	4.3920 %	7.0700 %	11.6740 %	8.4120 %
$\triangle R\%$	0.2639 %	0.2127 %	-0.1359 %	0.2451 %	1.0171 %	0.6351 %
$\triangle SW\%$	0.1081 %	0.2520 %	0.1179 %	0.0608 %	0.1343 %	0.1197 %

$p_{ij}^{Exp.Loss}$=100*(Sum of outcomes in row 2 and 4 in Table 1)/50000. Probability of the global bidder having ex-post loss.
$p_{ij}^{Inef.(GW)}$=100*(Outcomes in row 6 in Table 1)/50000. Probability of the global bidder winning inefficiently with profit.
$p_{ij}^{Ineff.}$=100*(Sum of outcomes in row 2,4,6, and 8 in Table 1)/50000. Probability of all inefficient allocations.
$\triangle R\%$ =$100 * (R_{AB} - R_{BA})/R_{BA}$. Percentage change in revenue between AB and BA auctions.
$\triangle SW\%$ =$100 * (SW_{AB} - SW_{BA})/SW_{BA}$. Percentage change in social welfare (SW) between AB and BA auctions.

Table 2: Simulation Results

Simulations are done for three different synergy levels of 0.2, 0.5 and 0.8 for $N_A = N_B = 1$ and then $N_A = N_B = 2$.

Simulation results are summarized in Table 2. The main observation is that there are cases where selling more valuable good A first might generate less revenue and/or welfare compared to selling it second.

Another observation is that whenever $\triangle R\% < 0$ we have $p_{BA}^{Exp.Loss} > p_{AB}^{Exp.Loss}$ and/or $p_{BA}^{Inef.(GW)} < p_{AB}^{Inef.(GW)}$. In the next two propositions, we link these types of inefficient allocations to the revenue of AB and BA auctions. Let R_{ij} denote the (ex-post) revenue in the ij auction with $i, j = A, B$ and $i \neq j$.

Proposition 4 *Consider a set of valuations v_{GA}, v_{GB}, p_i, and p_j. If the global bidder wins* *one or both good(s) with an ex-post loss in an ij auction, then $R_{ij} \geq R_{ji}$.*

Proposition 4 states that, as $p_{ij}^{Exp.Loss}$ increases, the revenue difference $R_{ij} - R_{ji}$ (weakly) increases.

Proposition 5 *Consider a set of valuations v_{GA}, v_{GB}, p_i, and p_j. If the global bidder wins one good with profit and the allocation is inefficient in an ij auction, then $R_{ij} \leq R_{ji}$.*

Proposition 5 states that as the difference $p_{ij}^{Inef.(GW)}$ decreases, the revenue difference $R_{ij} - R_{ji}$ (weakly) increases.

4 Conclusion

The main message of the simulations is that selling more valuable good first does not necessarily generate higher revenue and/or welfare compared to selling it second. While the models are not directly comparable, this is different than Benoit and Krishna (2001) who show that selling more valuable good generate higher revenue.

It is clear that choosing which good to sell first has an impact on revenue, welfare, and probability of inefficient allocations. The revenue difference was more than 1 per cent in some of our examples.

5 Appendix-Proofs

Proof of proposition 1

We take the derivative of equation 1 by using the fact that $Pr(p > p_i) = G_i(p)$. After equating the derivative to zero, and cancelling $g(p)$ from the equation, we get equation 2

$$p \equiv p_{ij} = v_{Gi} + \int_0^{min\{v_j+\theta,1\}} (v_j + \theta - p_j) dG_j(p_j) - \int_0^{v_j} (v_{Gj} - p_j) dG_j(p_j)$$

We can re-write this by using integration by parts, by letting $dv = dG_j(p_j)$ in both integrals. When $v_{Gj} + \theta < 1$,

$$p_{ij} = v_{Gi} + \int_0^{v_{Gj}+\theta} G_j(p_j)dp_j - \int_0^{v_{Gj}} G_j(p_j)dp_j \quad = \quad v_{Gi} + \int_{v_{Gj}}^{v_{Gj}+\theta} G_j(p)dp$$

When $v_{Gj} + \theta > 1$, the proof is similar, and omitted.

Finally, we show that the SOC is satisfied when $v_{Gj} + \theta < 1$.

$FOC = [v_{Gi} - p + \int_0^{min\{v_j+\theta,1\}}(v_j + \theta - p_j)dG_j(p_j) - \int_0^{v_j}(v_{Gj} - p_j)dG_j(p_j)]g_i(p)$
$= [v_{Gi} - p + \int_0^{v_j+\theta} G_j d(p_j) - \int_0^{v_j} G_j d(p_j)]g_i(p)$

$SOC = [v_{Gi} - p + \int_0^{v_j+\theta} G_j d(p_j) - \int_0^{v_j} G_j d(p_j)]g_i'(p) - g_i(p) < 0$, since $FOC = v_{Gi} - p + \int_0^{v_j+\theta} G_j d(p_j) - \int_0^{v_j} G_j d(p_j) = 0$. Since there is a unique equilibrium p_{ij}, and SOC is negative at p_{ij}, SOC is satisfied.

When $v_{Gj} + \theta > 1$, the proof is similar, and omitted.

■

Proof of proposition 2: First, assume that $v_{GA} + \theta < 1$. Then, by using proposition 1,

$$p_{AB} - p_{BA} = v_{GA} - v_{GB} + \int_{v_{GB}}^{v_{GB}+\theta} G_B(p)dp - \int_{v_{GA}}^{v_{GA}+\theta} G_A(p)dp \; >$$
$$v_{GA} - v_{GB} + \int_{v_{GA}}^{v_{GB}+\theta} G_A(p)dp - \int_{v_{GA}}^{v_{GA}+\theta} G_A(p)dp \; = \; v_{GA} - v_{GB} - \int_{v_{GB}+\theta}^{v_{GA}+\theta} G_A(p)dp$$
$$\geq \; v_{GA} - v_{GB} - \int_{v_{GB}+\theta}^{v_{GA}+\theta} 1dp \; = 0 \; \Rightarrow \; p_{AB} \; > \; p_{BA}$$

The first inequality is written by using $G_A \leq G_B$ since A is stochastically more valuable and $v_{GA} < v_{GB}$ by assumption. The last inequality is written since $G_A(.) \leq 1$

Second, assume that $v_{GB} + \theta < 1 < v_{GA} + \theta$. Then, by using proposition 1,

$$p_{AB} - p_{BA} = v_{GA} + \int_{v_{GB}}^{v_{GB}+\theta} G_B(p)dp - [v_{GB} + v_{GA} + \theta - 1 + \int_{v_{GA}}^{1} G_A(p)dp] \; =$$
$$\int_{v_{GB}}^{v_{GB}+\theta} G_B(p)dp + \int_{v_{GB}+\theta}^{1} 1dp - \int_{v_{GA}}^{1} G_A(p)dp \; > \; \int_{v_{GB}}^{1} G_A(p)dp - \int_{v_{GA}}^{1} G_A(p)dp$$
$$= \; \int_{v_{GB}}^{v_{GA}} G_A(p)dp \; > 0 \; \Rightarrow \; p_{AB} \; > \; p_{BA}$$

The first inequality is written by using $G_A \leq G_B$ and $G_A \leq 1$. The third case, $v_{GB}+\theta > 1$ is similar.

■

Proof of proposition 3:

If the global bidder wins i with profit but loses j in an ij auction,

$$p_{ij} > v_{Gi} > p_i \ \ and \ \ p_j > v_{Gj} + \theta \tag{3}$$

should hold. Revenue in the ij auction is $R_{ij} = p_i + v_{Gj} + \theta$. There is no inefficient outcome since the winner of license i with $v_{Gi} > p_i$ and the winner of license j with valuation p_j such that $p_j > v_{Gj} + \theta$, and $v_{Gi} + p_j > v_{Gi} + v_{Gj} + \theta$.

■

Proof of Proposition 4: First we prove this when the global bidder wins one good and makes an ex-post loss. For this to happen, in the ij auction,

$$p_{ij} > p_i > v_{Gi} \ \ and \ \ v_{Gj} + \theta < p_j \tag{4}$$

should hold. Revenue in the ij auction is $R_{ij} = p_i + v_{Gj} + \theta$.

If instead, ji auction is conducted with these valuations, global bidder will lose j since $p_{ji} < v_{Gj} + \theta < p_j$ since bidding price can never be above the marginal valuation $v_{Gj} + \theta$ and $v_{Gj} + \theta < p_j$ by equation 4. The global bidder will also lose license i since $p_i > v_{Gi}$ by equation 4. Hence,

$$R_{ji} = p_{ji} + v_{Gi} < R_{ij} = p_i + v_{Gj} + \theta$$

Second, we prove this when the global bidder wins both goods and makes an ex-post loss.

$$p_{ij} > p_i > v_{Gi} \ \ and \ \ v_{Gj} + \theta > p_j \tag{5}$$

should hold. Revenue in the ij auction is $R_{ij} = p_i + p_j$. i) Assume that $p_{ji} < p_j$. By equation 5, $p_i > v_{Gi}$, hence, local bidders win both goods. $R_{ji} = p_{ji} + v_{Gi} < p_i + p_j = R_{ij}$.

ii) Now assume that $p_{ji} > p_j$. The global bidder wins j auction and hence, bids $v_{Gi} + \theta$ in the i auction and wins it since $v_{Gi} + \theta > p_{ij} > p_i$. The last inequality is from equation 5. Hence, $R_{ij} = R_{ji}$. ■

Proof of Proposition 5: In ij auction, local bidder wins i, global bidder wins j, and this is an inefficient allocation; hence,

$$p_i > p_{ij} \ \ and \ \ v_{Gj} > p_j \ \ and \ \ p_i + v_{Gj} < v_{Gi} + \theta + v_{Gj} \tag{6}$$

We have $R_{ij} = p_{ij} + p_j$. If this is sold in ji auction, global bidder wins j auction since $p_{ji} > v_{Gj} > p_j$ and i auction since $v_{Gi} + \theta > p_i$ by equation 6. Hence, $R_{ji} = p_j + p_i > p_j + p_{ij} = R_{ij}$ by equation 6.

6 Appendix-B

	License i won by	License j won by	Global bidder makes	Allocation is	Revenue is
1.	Global Bidder	Global Bidder	Profit	Efficient	$v_{1i} + v_{1j}$
2.	Global Bidder	Global Bidder	Loss	Inefficient	$v_{1i} + v_{1j}$
3.	Global Bidder	Local Bidder j	Profit	Efficient	$v_{1i} + v_{Gj} + \theta$
4.	Global Bidder	Local Bidder j	Loss	Inefficient	$v_{1i} + v_{Gj} + \theta$
5.	Global Bidder	Local Bidder j	Profit	Efficient	$v_{1i} + v_{2j}$
6.	Global Bidder	Local Bidder j	Loss	Inefficient	$v_{1i} + v_{2j}$
7.	Local Bidder i	Global Bidder	Profit	Efficient	$p_{ij} + v_{1j}$
8.	Local Bidder i	Global Bidder	Profit	Inefficient	$p_{ij} + v_{1j}$
9.	Local Bidder i	Global Bidder	Profit	Efficient	$v_{2i} + v_{1j}$
10.	Local Bidder i	Global Bidder	Profit	Inefficient	$v_{2i} + v_{1j}$
11.	Local Bidder i	Local Bidder j	Zero Profit	Efficient	$p_{ij} + v_{Gj}$
12.	Local Bidder i	Local Bidder j	Zero Profit	Inefficient	$p_{ij} + v_{Gj}$
13.	Local Bidder i	Local Bidder j	Zero Profit	Efficient	$p_{ij} + v_{2j}$
14.	Local Bidder i	Local Bidder j	Zero Profit	Inefficient	$p_{ij} + v_{2j}$
15.	Local Bidder i	Local Bidder j	Zero Profit	Efficient	$v_{2i} + v_{Gj}$
16.	Local Bidder i	Local Bidder j	Zero Profit	Inefficient	$v_{2i} + v_{Gj}$
17.	Local Bidder i	Local Bidder j	Zero Profit	Efficient	$v_{2i} + v_{2j}$
18.	Local Bidder i	Local Bidder j	Zero Profit	Inefficient	$v_{2i} + v_{2j}$

Table 3: All possible outcomes when $N_A = N_B = 2$

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References

Benoit J.P., Krishna V., 2001. Multiple-object auctions with budget constrained bidders. Review of Economic Studies 68(1), 155-179.

Branco F., 1997. Sequential auctions with synergies: An example. Economics Letters 54(2), 159-163.

Chakraborty, I., 2004. Multi-unit auctions with synergy. Economics Bulletin 4, 1-14.

De Silva, D.G., 2005. Synergies in recurring procurement auctions: an empirical investigation. Economic Inquiry 43(1),55-66.

Jeitschko, T.D., Wolfstetter, E., 2002. Scale Economies and the Dynamics of Recurring Auctions. Economic Inquiry 40(3), 403-414.

Krishna, V., Rosenthal, R.W., 1996. Simultaneous auctions with synergies. Games and Economic Behavior 17, 1-31.

Leufkens, K., R Peeters,R., Vermeulen,D., 2010. Sequential auctions with synergies: The paradox of positive synergies. Economics Letters 109(3),139-141.

Meng, X., Gunay, H., 2017. Exposure problem in multi-unit auctions. International Journal of Industrial Organization 52, 165-187.