The Social Value of Public Information with Convex Costs of Information Acquisition[*]

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Abstract

In a beauty contest framework, welfare can decrease with public information if the precision of private information is exogenous, whereas welfare necessarily increases with public information if the precision is endogenous with linear costs of information acquisition. The purpose of this paper is to reconcile these results by considering nonlinear costs of information acquisition. The main result of this paper is a necessary and sufficient condition for welfare to increase with public information. Using it, we show that costs of information acquisition are linear if and only if welfare necessarily increases with public information. Thus, welfare can decrease with public information for any strictly convex costs. This is because convex costs mitigate the so-called crowding-out effect of public information on private information, thereby making the social value of public information with endogenous precision closer to that with exogenous precision.

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1 Introduction

In multi-agent situations, more information is not necessarily valuable. A notable example is the anti-transparency result in a beauty contest game of Morris and Shin (2002). They demonstrate that welfare can decrease with public information if agents have access to sufficiently precise private information.

The anti-transparency result of Morris and Shin (2002) has prompted considerable debates.[1] In particular, Colombo and Femminis (2008) reach the opposite conclusion by allowing agents to choose the precision of private information given that of public information.[2] They show that welfare necessarily increases with public information if the cost of the precision is linear and the marginal cost of private information exceeds that of public information. In their result, the crowding-out effect of public information on private information has an essential role: an increase in the precision of public information reduces the incentives for acquisition of private information and delivers substantial cost savings enough to compensate any decrease in the expected payoffs.

The purpose of this paper is to reconcile the anti-transparency result with exogenous precision and the pro-transparency result with endogenous precision by considering general nonlinear costs in the model of Colombo and Femminis (2008).

The main result of this paper is a necessary and sufficient condition for welfare to increase with public information. Using it, we show that costs of information acquisition are linear if and only if welfare necessarily increases with public information. Thus, welfare can decrease with public information for any strictly convex costs. This is because convex costs mitigate the crowding-out effect of public information, thereby making the social value of public information with endogenous precision closer to that with exogenous precision.

In a more general framework, Colombo et al. (2014) compare the social value of public information with endogenous precision and that with exogenous precision. They give a sufficient condition guaranteeing that the former is positive whenever the latter is positive based upon a comparison between the equilibrium and the socially optimal strategy profile, which is their focus. As an example, they show that the model of Colombo and Femminis (2008) with nonlinear costs satisfies the condition. We complement their result in the following respects. Firstly, we give a full characterization of the social value of public information in the model of Colombo and Femminis (2008) with nonlinear costs. Secondly, we show that convexity of costs plays an essential role in determining the social value of public information through the strength of the crowding-out effect.

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[1] See Angeletos and Pavan (2004, 2007), Hellwig (2005), Svensson (2006), Cornand and Heinemann (2008), and James and Lawler (2011), among others.

[2] Information acquisition in a beauty contest framework is also studied by Wong (2008), Hellwig and Veldkamp (2009), and Myatt and Wallace (2012).

2 The model

There is a continuum of agents indexed by $i \in [0,1]$. Agent i chooses an action $a_i \in \mathbb{R}$, and we write a for the action profile. The payoff function of agent i is

$$u_i(a,\theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}), \tag{1}$$

where θ is the state, r is a constant with $0 < r < 1$, and

$$L_i = \int_0^1 (a_j - a_i)^2 dj,\ \bar{L} = \int_0^1 L_j dj.$$

Agent i aims to minimize the weighted mean of the squared error of his action from the state and that from an opponent's action. This payoff function is characterized by two conditions: (i) the best response is the weighted mean of the state and the average action of the opponents; (ii) the total ex ante expected payoff is proportional to $-\int E[(a_i - \theta)^2]di$.

Agent i observes a public signal $y = \theta + \eta$ and a private signal $x_i = \theta + \varepsilon_i$, where θ, η, and ε_i are independently and normally distributed with

$$E[\theta] = \bar{\theta},\ E[\eta] = E[\varepsilon_i] = 0,\ \mathrm{var}[\theta] = 1/\alpha_\theta,\ \mathrm{var}[\eta] = 1/\alpha_y,\ \mathrm{var}[\varepsilon_i] = 1/\beta_i,$$

and ε_i and ε_j are independent for $i \neq j$. We refer to α_y and β_i as the precision of public information and that of private information, respectively. We write $\alpha = \alpha_\theta + \alpha_y$.

We consider a two-stage game with information acquisition.[3] In the first stage, agent i chooses the precision of private information β_i. In the second stage, he observes public and private signals and chooses an action a_i. The second-stage game when $\beta_i = \beta_j$ for all $i, j \in [0,1]$ is the beauty contest game of Morris and Shin (2002).

The cost of choosing β_i in the first stage is $C(\beta_i) \geq 0$, where $C : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and convex; that is, $C'(\beta_i) > 0$ and $C''(\beta_i) \geq 0$ for all $\beta_i \geq 0$. The net payoff function of agent i is

$$u_i(a,\theta) - C(\beta_i).$$

Colombo and Femminis (2008) restrict attention to a linear cost function, but we consider all convex cost functions. They incorporate the cost of public information as well, but we focus on that of private information because our concern is the effects of public information on agents' information acquisition. This follows Colombo et al. (2014), who study information acquisition in symmetric linear quadratic Gaussian games of Angeletos and Pavan (2007). Our model is a special case.

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[3] The earliest papers on information acquisition in linear quadratic Gaussian games are Li et al. (1987) and Vives (1988), who study Cournot games.

3 The equilibrium

We consider a symmetric equilibrium. To characterize it and obtain the expected payoff, we utilize two known results. The following result on the second stage is due to Morris and Shin (2002).

Lemma 1. *In the unique equilibrium of the second-stage game when* $\beta_i = \beta$ *for all* $i \in [0,1]$*, agent i's action is*

$$a_i = \lambda E[\theta|y] + (1-\lambda)x_i,$$

where $\lambda = \alpha/(\alpha + (1-r)\beta)$ *and* $E[\theta|y] = (\alpha_\theta \bar{\theta} + \alpha_y y)/(\alpha_\theta + \alpha_y)$*. The total ex ante expected payoff in this equilibrium is*

$$F(\alpha,\beta) \equiv -(1-r)\int E[(a_i - \theta)^2]di = -(1-r)(\lambda^2/\alpha + (1-\lambda)^2/\beta). \tag{2}$$

Morris and Shin (2002) show that $dF(\alpha,\beta)/d\alpha_y = \partial F(\alpha,\beta)/\partial\alpha < 0$ if and only if $r > 1/2$ and $\alpha < (2r-1)(1-r)\beta$. That is, when the precision of private information is exogenous, welfare can decrease with public information if $r > 1/2$. As explained by Morris and Shin (2002), a strategic complementarity induces players' overreaction to public information relative to the efficiency benchmark, which can reduce welfare.

The following result on the first stage is due to Colombo and Femminis (2008).

Lemma 2. *When all the opponents follow the equilibrium strategy in the second-stage game given in Lemma 1, player i's marginal benefit of the precision* β_i *in the first stage evaluated at* $\beta_i = \beta$ *is*

$$\left.\frac{d}{d\beta_i}E[u_i(a,\theta)]\right|_{\beta_i=\beta} = (1-\lambda)^2/\beta^2 = (1-r)^2/(\alpha+(1-r)\beta)^2.$$

By this lemma, the first order condition for the equilibrium precision is

$$(1-r)^2/(\alpha+(1-r)\beta)^2 = C'(\beta). \tag{3}$$

Note that the marginal benefit is strictly decreasing in α and β, whereas the marginal cost is increasing in β (see Figure 1). The equilibrium precision is the unique value of β solving (3) if $C'(0) \leq (1-r)^2/\alpha^2$ and it is zero if $C'(0) > (1-r)^2/\alpha^2$. We denote the equilibrium precision by $\phi(\alpha)$ as a function of $\alpha = \alpha_\theta + \alpha_y$. Let $\overline{\beta} \equiv \phi(0) = \sup_{\alpha>0}\phi(\alpha)$ be the supremum of the equilibrium precision, which equals the unique solution of $\beta\sqrt{C'(\beta)} = 1$.

For example, suppose that $C(\beta) = c\beta$ with $c > 0$. Then, $\phi(\alpha) = 1/\sqrt{c} - \alpha/(1-r)$ if $c \leq (1-r)^2/\alpha^2$, as shown by Colombo and Femminis (2008).[4] Only in the linear case, we can obtain $\phi(\alpha)$ in such a simple form.

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[4] Li et al. (1987) and Vives (1988) also obtain a similar formula in Cournot games.

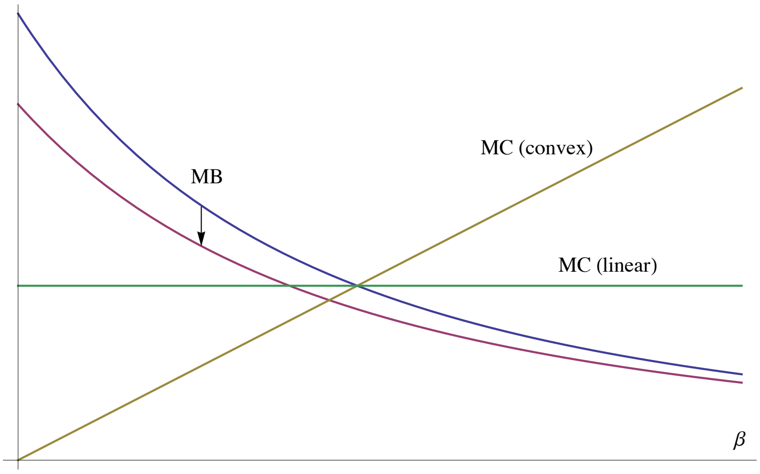

Figure 1: The marginal benefit curve and the marginal cost curve. An increase in α shifts the marginal benefit curve down, by which the equilibrium precision decreases.

By (3) and Figure 1, an increase in the precision of public information results in a decrease in the precision of private information as shown by Colombo and Femminis (2008) and Colombo et al. (2014); that is, $\phi'(\alpha) < 0$. We refer to this effect as the crowding-out effect of public information on private information.[5]

4 Main results

We write the total ex ante expected net payoff as a function of the precision of public information

$$W(\alpha_y) \equiv F(\alpha_\theta + \alpha_y, \phi(\alpha_\theta + \alpha_y)) - C(\phi(\alpha_\theta + \alpha_y)),$$

which is our measure of welfare. Colombo and Femminis (2008) show that if $C(\beta) = c\beta$ with $c > 0$ then $W'(\alpha_y) = c$. Thus, welfare necessarily increases with public information if the cost is linear.

In the nonlinear case, it is difficult to directly calculate $W(\alpha_y)$ because we do not have $\phi(\alpha)$ in a closed form. Thus, Colombo et al. (2014) use an implicit function to represent $W(\alpha_y)$. Rather than using an implicit function, we use the inverse function

$$\psi(\beta) \equiv \phi^{-1}(\beta) = (1-r)(1/\sqrt{C'(\beta)} - \beta) \tag{4}$$

and represent $W(\alpha_y)$ as a function of β by plugging the above into (2):

$$F(\psi(\beta), \beta) - C(\beta) = r\beta C'(\beta) - \sqrt{C'(\beta)} - C(\beta). \tag{5}$$

This direct representation of welfare leads us to the following main result of this paper.

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[5] This effect is also found in Wong (2008), Hellwig and Veldkamp (2009), and Myatt and Wallace (2012).

Proposition 1. *Let $\underline{\beta}$ be the unique solution of $\beta\sqrt{C'(\beta)} = 1/2$. If $\alpha_\theta + \alpha_y \geq \psi(\underline{\beta})$, then $W'(\alpha_y) > 0$. If $\alpha_\theta + \alpha_y < \psi(\underline{\beta})$, then, for $\beta = \phi(\alpha_\theta + \alpha_y)$,*

$$W'(\alpha_y) \gtreqless 0 \;\Leftrightarrow\; r \lesseqgtr R(\beta) \equiv \frac{C'(\beta) + C''(\beta)/(2\sqrt{C'(\beta)})}{C'(\beta) + C''(\beta)\beta}. \tag{6}$$

In the latter case, $R(\beta) \leq 1$, where the equality holds if and only if $C''(\beta) = 0$.

Proof. Note that $\underline{\beta} < \overline{\beta}$ because $\overline{\beta}$ is the unique solution of $\beta\sqrt{C'(\beta)} = 1$. Let $\beta = \phi(\alpha_\theta + \alpha_y)$. Because

$$W'(\alpha_y) = \frac{\partial F}{\partial \alpha} + \left(\frac{\partial F}{\partial \beta} - C'\right)\phi' = \frac{1}{\psi'}\left(\frac{\partial F}{\partial \alpha}\psi' + \frac{\partial F}{\partial \beta} - C'\right) = \frac{1}{\psi'}\frac{d}{d\beta}(F(\psi(\beta),\beta) - C(\beta)),$$

the sign of $W'(\alpha_y)$ is opposite to that of $d(F(\psi(\beta),\beta) - C(\beta))/d\beta$. By (5), we have

$$\frac{d}{d\beta}(F(\psi(\beta),\beta) - C(\beta)) = -(1-r)C'(\beta) + C''(\beta)\left(r\beta - \frac{1}{2\sqrt{C'(\beta)}}\right).$$

By rearranging the above, we obtain (6) for all $\beta < \overline{\beta}$. If $\alpha_\theta + \alpha_y \geq \psi(\underline{\beta})$, then $\beta \leq \underline{\beta}$ and thus $\beta \leq 1/(2\sqrt{C'(\beta)})$, which implies that $R(\beta) \geq 1$ and thus $W'(\alpha_y) > 0$ because $r < 1$. If $\alpha_\theta + \alpha_y < \psi(\underline{\beta})$, then $\beta > \underline{\beta}$ and thus $\beta > 1/(2\sqrt{C'(\beta)})$, which implies the last statement. □

Proposition 1 says that welfare increases with public information if $\alpha \geq \psi(\underline{\beta})$ or if $\alpha < \psi(\underline{\beta})$ and $r < R(\beta)$ but decreases if $\alpha < \psi(\underline{\beta})$ and $r > R(\beta)$. In particular, if $C''(\beta) = 0$, then $r < R(\beta) = 1$. That is, if the cost is linear, then welfare necessarily increases with public information for all $\alpha > 0$ and $r \in (0,1)$, as shown by Colombo and Femminis (2008). We can say more than this: welfare increases with public information for all $\alpha > 0$ and $r \in (0,1)$ if and only if the cost is linear on $(\underline{\beta},\overline{\beta})$.

Corollary 2. *$W'(\alpha_y) > 0$ for all $\alpha > 0$ and $r \in (0,1)$ if and only if $C''(\beta) = 0$ for all $\beta \in (\underline{\beta},\overline{\beta})$.*

Proof. If $C''(\beta) = 0$ for all $\beta \in (\underline{\beta},\overline{\beta})$, then $r < R(\beta) = 1$ and thus $W'(\alpha_y) > 0$ for all $\alpha > 0$ and $r \in (0,1)$ by Proposition 1. If $W'(\alpha_y) > 0$ for all $\alpha > 0$ and $r \in (0,1)$, then $r < R(\beta) \leq 1$ for all $\beta \in (\underline{\beta},\overline{\beta})$ and $r \in (0,1)$ by Proposition 1, and thus $R(\beta) = 1$. This implies that $C''(\beta) = 0$. □

The "only if" part implies that, for any strictly convex costs, welfare can decrease with public information. To illustrate it, suppose that $C''(\overline{\beta}) > 0$ and

$$1 > r > R(\overline{\beta}) = \left(C''(\overline{\beta})/(2C'(\overline{\beta})) + \sqrt{C'(\overline{\beta})}\right) / \left(C''(\overline{\beta})/C'(\overline{\beta}) + \sqrt{C'(\overline{\beta})}\right). \tag{7}$$

Then, welfare decreases with public information if α is sufficiently small.

Corollary 3. *Suppose that* $C''(\overline{\beta}) > 0$ *and* $r > R(\overline{\beta})$. *Let* $\beta^* < \overline{\beta}$ *be the maximum value of* β *solving* $r = R(\beta)$. *If* $\alpha < \psi(\beta^*)$ *then* $W'(\alpha_y) < 0$. *Suppose, in addition, that* β^* *is the unique solution of* $r = R(\beta)$. *Then,* $W'(\alpha_y) < 0$ *if and only if* $\alpha < \psi(\beta^*)$.

Proof. If $\psi(\overline{\beta}) = 0 < \alpha = \psi(\beta) < \psi(\beta^*)$, then $\beta^* < \beta < \overline{\beta}$ and thus $r > R(\beta)$ by the definition of β^*. Therefore, $W'(\alpha_y) < 0$ by Proposition 1. Suppose that β^* is the unique solution of $r = R(\beta)$. Then, $r > R(\beta)$ if and only if $\beta > \beta^*$ because $r < R(\underline{\beta}) = 1$. Therefore, if $W'(\alpha_y) < 0$ then $r > R(\phi(\alpha))$ by Proposition 1 and thus $\phi(\alpha) > \beta^*$, i.e., $\alpha < \psi(\beta^*)$. □

By the latter half of the corollary, when $R(\beta)$ is strictly decreasing in β and $r > R(\overline{\beta})$, welfare decreases with public information if and only if $\alpha < \psi(\beta^*)$. For example, consider an isoelastic cost function $C(\beta) = c\beta^\rho/\rho$ with $\rho > 1$, where

$$R(\beta) = \left(1 + (\rho - 1)\beta^{-(\rho+1)/2}/(2\sqrt{c})\right)/\rho$$

is strictly decreasing in β. By calculating $R(\overline{\beta})$ and $\psi(\beta^*)$, we obtain the following result.

Corollary 4. *Suppose that* $C(\beta) = c\beta^\rho/\rho$ *with* $\rho > 1$ *and* $r > (\rho + 1)/(2\rho)$. *Then,* $W'(\alpha_y) < 0$ *if and only if*

$$\alpha < \frac{(1 - r)(2\rho r - (\rho + 1))}{c^{\frac{1}{\rho+1}}(\rho - 1)^{\frac{\rho-1}{\rho+1}}(2\rho r - 2)^{\frac{2}{\rho+1}}}.$$

This corollary says that welfare can decrease with public information if $r > (\rho + 1)/(2\rho)$. Moreover, the threshold $(\rho + 1)/(2\rho)$ decreases to $1/2$ as ρ goes to infinity. This is because $R(\overline{\beta})$ can be arbitrarily close to $1/2$ if $C''(\overline{\beta})/C'(\overline{\beta})$ is large enough by (7), where $C''(\overline{\beta})/C'(\overline{\beta})$ is analogous to the Arrow-Pratt measure of risk aversion. Recall that, when the precision of private information is exogenous, welfare can decrease with public information if $r > 1/2$. Therefore, convex costs make the social value of public information with endogenous precision arbitrarily close to that with exogenous precision.

To explain the role of convex costs in the above result, we show that convex costs mitigate the crowding-out effect of public information measured by $\phi'(\alpha)$.

Proposition 5. *Let* $\phi_C(\alpha)$ *be the equilibrium precision with strictly convex costs and let* $\phi_L(\alpha)$ *be the equilibrium precision with linear costs. If* $\phi_C(\alpha) = \phi_L(\alpha) > 0$, *then* $\phi'_L(\alpha) < \phi'_C(\alpha) < 0$.

Proof. Let ψ_C and ψ_L be the inverse functions of ϕ_C and ϕ_L, respectively. For $\beta = \phi_C(\alpha) = \phi_L(\alpha)$, $1/\phi'_C(\alpha) = \psi'_C(\beta) = -(1 - r)(1 + C''(\beta)(C'(\beta))^{-3/2}/2) < -(1 - r) = \psi'_L(\beta) = 1/\phi'_L(\beta) < 0$. Thus, $\phi'_L(\alpha) < \phi'_C(\alpha) < 0$. □

Recall that the equilibrium precision is determined by the intersection of the downward sloping marginal benefit curve and the upward sloping marginal cost curve (see Figure 1). An increase in the precision of public information shifts the marginal benefit curve down, by which the equilibrium precision decreases. Clearly, the decrease is greater when the marginal cost curve is flat, i.e., the cost is linear.

Proposition 5 explains the following role of convex costs. Because

$$W'(\alpha_y) = \frac{\partial F}{\partial \alpha} + \left(\frac{\partial F}{\partial \beta} - C' \right) \phi',$$

the welfare effects of public information consist of a direct effect $\partial F / \partial \alpha$ and an indirect effect $(\partial F / \partial \beta - C')\phi'$ through the crowding-out effect ϕ'. If the cost is strictly convex, the crowding-out effect is small, thus making the direct effect dominant. In this case, the total effect can be negative because the direct effect can be negative. In contrast, if the cost is linear, the crowding-out effect is large, thus making the indirect effect dominant. In this case, the total effect is positive because, whenever the direct effect is negative, the indirect effect is positive due to substantial cost savings.

In fact, it holds that

$$\left. \frac{\partial F}{\partial \alpha} \right|_{\alpha = \psi(\beta)} < 0 \;\Leftrightarrow\; \beta \sqrt{C'(\beta)} > 1/(2r) > 1/2, \tag{8}$$

$$\left. \left(\frac{\partial F}{\partial \beta} - C' \right) \phi' \right|_{\alpha = \psi(\beta)} > 0 \;\Leftrightarrow\; \beta \sqrt{C'(\beta)} > 1/2 \;\Leftrightarrow\; \alpha_\theta + \alpha_y < \psi(\underline{\beta}) \tag{9}$$

by (2) and (4).[6] Thus, by Proposition 1, the total effect can be negative only when the direct effect is negative, but whenever the direct effect is negative, the indirect effect is positive. Convex costs reduce the positive indirect effect, thus making the total effect negative, whereas linear costs do not. In this sense, convexity plays an essential role in determining the social value of public information through the strength of the crowding-out effect.

Remark 1. The above discussion on the role of convex costs does not depend upon the specific setting of the beauty contest model. Thus, Proposition 5 may be relevant also in the general model of Colombo et al. (2014).

Remark 2. Colombo et al. (2014) compare the total effect and the direct effect in their general model. They give a sufficient condition guaranteeing that the total effect is positive whenever the direct effect is positive and show that the beauty contest model satisfies the condition, which is confirmed by (8), (9), and Proposition 1.

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[6] $\partial F / \partial \alpha = C'(\beta) \left(1 - 2r\beta\sqrt{C'(\beta)} \right) / (1 - r)$ and $(\partial F / \partial \beta - C')\phi' = rC'(\beta) \left(1 - 2\beta\sqrt{C'(\beta)} \right) \phi'$.

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