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A Note on the Possible Advantage of Size Flexibility in Committees

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A note on the possible advantage of size flexibility in committees

by

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Abstract

This note analyzes the possible advantage of size flexibility in decision-making bodies facing an uncertain dichotomous choice. We find that under constant size variability, application of the unanimity rule might be more desirable than the simple majority rule, yielding higher average performance. In contrast, and as is well known, the latter rule is always the superior one, given a fixed number of decision makers with identical decisional competence.

Keywords: obligatory or flexible committee size, unanimity rule, simple majority rule.

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1. **Introduction**

The size of decision-making bodies (court, committee, jury) is often fixed and mandatory. The objective of this note is to examine the plausibility of such a restriction under two commonly used collective decision rules: the simple majority and the unanimity rules. Our main results establish that in the uncertain dichotomous choice setting, such a restriction is advantageous under the simple majority rule, but it is not plausible under the unanimity rule, especially in relatively large decision-making bodies that are naturally more vulnerable to nonattendance.

In countries where the jury system is applied, the collective decision making rule is typically the unanimity rule. Unanimous support by all voters is required in order to select a certain alternative: in our case conviction. Otherwise, the other alternative (commonly the status quo or acquittal) is chosen.[1] In contrast, in many legal systems the collective decision making rule is the simple majority rule and the number of decision makers (the judges) is fixed.

The objective of this note is to show that in the former case the number of jurors need not be fixed. Flexibility in the jury size might be advantageous, that is, the average probability of making a correct jury decision is higher provided that size variability is constant. This result is not valid in decision-making bodies such as courts, expert committees or board of managers where the applied decision rule is the simple majority rule. Under this alternative decision-making setting, a fixed mandatory number of decision makers is superior because it yields a higher probability of making the correct decision. Notice that practically, size flexibility might also be more plausible in large decision-making bodies such as juries relative to courts or relatively small expert committees. Furthermore, since the jury members are not payed professionals like judges or experts, it might be more sensible to allow them increased flexibility in attendance when fulfilling their voluntary decision-making task.

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[1] Earlier studies focusing on the unanimity (hierarchy) rule include Ben-Yashar and Danziger (2016), Ben-Yashar and Nitzan (1998, 2001), Sah and Stiglitz (1988).

2. The model

Consider a group N_{n+k} with an odd number of members, *n+k*. The group confronts two alternatives, 1 and -1, one of which is correct and therefore better for all voters.[2] As is common in decision problems, the identity of the better alternative is unknown. Every voter selects one of the two alternatives, and an aggregation mechanism is applied to select the collectively preferred alternative. Each voter chooses the correct alternative with probability p, which reflects his competence. We assume independent decisional competencies and that $½<p<1$ .By the unanimity rule, one of the alternatives (hereafter 1) is the selected outcome if and only if it is chosen by all voters (otherwise the other alternative, -1, is chosen). Let T_n denote the probability that a group consisting of n voters associated with decisional competence p chooses the correct alternative under the unanimity rule

$$T_n = \frac{1}{2}p^n + \frac{1}{2}\left(1-(1-p)^n\right) .$$

Let T_{n+k} and T_{n-k} denote the probability that groups consisting of $n+k$ and $n-k$ voters , $n-k>1$, choose, respectively, the correct alternative under the unanimity rule.

3. Result

Assuming constant size variability, that is, the number of decision makers is larger or smaller by k relative to n , we obtain the following result.

Theorem:

If n-k>1 , then $\frac{T_{n+k}+T_{n-k}}{2} \geq T_n$

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[2] Earlier studies of two alternative models include Baharad and Ben-Yashar (2009), Ben-Yashar and Nitzan (2014), Berend and Sapir (2005), Feld and Grofman (1984) and Young (1988).

Proof:

$$\frac{1}{4}(p^{n+k}+p^{n-k}+2-(1-p)^{n+k}-(1-p)^{n-k})\geq\frac{1}{2}(p^{n}+1-(1-p)^{n})\Leftrightarrow$$

$$p^{n+k}+p^{n-k}+2-(1-p)^{n+k}-(1-p)^{n-k}\geq 2p^{n}+2-2(1-p)^{n}\Leftrightarrow$$

$$(1-p)^{n}-(1-p)^{n+k}+(1-p)^{n}-(1-p)^{n-k}\geq p^{n}(1-p^{k})+p^{n}(1-p^{-k})\Leftrightarrow$$

$$(1-p)^{n}(1-(1-p)^{k})+(1-p)^{n}(1-(1-p)^{-k}\geq p^{n}(1-p^{k})+p^{n}(1-p^{-k})\Leftrightarrow$$

$$(1-p)^{n}(1-(1-p)^{k}+1-(1-p)^{-k})\geq p^{n}(1-p^{k}+1-p^{-k})\Leftrightarrow$$

$$\frac{2(1-p)^{k}-(1-p)^{2k}-1}{(1-p)^{k}}\geq\frac{p^{n}}{(1-p)^{n}}\frac{2p^{k}-p^{2k}-1}{p^{k}}\Leftrightarrow$$

$$-\frac{\left(1-(1-p)^{k}\right)^{2}}{(1-p)^{k}}\geq-\frac{p^{n}}{(1-p)^{n}}\frac{\left(1-p^{k}\right)^{2}}{p^{k}}\Leftrightarrow$$

$$\left(\frac{1-(1-p)^{k}}{1-p^{k}}\right)^{2}\leq\left(\frac{p}{1-p}\right)^{n-k}\Leftrightarrow$$

$$\left(\frac{1-(1-p)^{k}}{1-p^{k}}\right)^{2}\leq\left(\frac{p}{1-p}\right)^{n-k}\Leftrightarrow\left(\frac{1-(1-p)^{k}}{1-p^{k}}\right)^{2}\left(\frac{1-p}{p}\right)^{2}\leq\left(\frac{p}{1-p}\right)^{n-k-2}\Leftrightarrow$$

$$\left(\frac{1-(1-p)^{k}}{1-p^{k}}\frac{1-p}{p}\right)^{2}\leq\left(\frac{p}{1-p}\right)^{n-k-2}$$

Since the right hand side term is no less than 1 if n-k>1, to complete the proof we need to show that the left hand side term is not larger than 1. That is, we need to prove that

$$\left(\frac{1-(1-p)^k}{1-p^k}\frac{1-p}{p}\right)^2 \le 1 \Leftrightarrow \frac{1-(1-p)^k}{1-p^k}\frac{1-p}{p} \le 1 \Leftrightarrow 1-p-(1-p)^{k+1} \le p-p^{k+1} \Leftrightarrow$$

$$p^{k+1}-(1-p)^{k+1} \le 2p-1$$

This inequality is proved by induction.

If k=1, $p^2-(1-p)^2 \le 2p-1 \Leftrightarrow 2p-1 \le 2p-1$, which establishes the inequality for this case.

If k=2 , $p^3-(1-p)^3 \le 2p-1 \Leftrightarrow$

$$(2p-1)(p^2+p(1-p)+(1-p)^2) \le 2p-1 \Leftrightarrow p+1-2p+p^2 \le 1 \Leftrightarrow p^2+1-p \le 1$$

, which establishes the inequality for this case.

By induction, assume that for some k

$p^k-(1-p)^k \le 2p-1 \Leftrightarrow p^k \le 2p-1+(1-p)^k$

For k+1 we prove that

$$p^{k+1}-(1-p)^{k+1} \le 2p-1 \Leftrightarrow pp^k-(1-p)(1-p)^k \le 2p-1 \Leftrightarrow p^k \le \frac{2p-1}{p}+\frac{(1-p)^{k+1}}{p}.$$

Since

$$\frac{2p-1}{p}+\frac{(1-p)^{k+1}}{p} \ge 2p-1+(1-p)^k \Leftrightarrow (2p-1)(1-p) \ge (1-p)^k(p-1+p) \Leftrightarrow 1 \ge (1-p)^{k-1}$$

, the proof is complete.

Q.E.D.

Note that, if n-k=1, which implies that one of the groups consists of just one member, the result is not valid.

Under simple majority, the result is reversed. The probability of making a correct decision under the simple majority rule increases with the number of decision makers, but in a decreasing rate. Therefore, in this case, a fixed group size is superior to constant variable group size because performance in the former case is larger than the average performance in the latter case, see Baharad, Ben-Yashar and Patal (2017).

4. Conclusion

Our results imply that the rigidity of the decision-making systems resorting to the unanimity rule (notably the jury system) is questionable. That is, mandatory participation is disadvantageous because it reduces performance relative to a flexible system that allows constant size variability. In contrast, flexible decision-making systems resorting to the majority rule are dubious. That is, non-mandatory attendance is disadvantageous because it reduces performance relative to a rigid system that does not allow constant size variability.

An alternative "dual" application of the results is the following. If the size of the group is fixed, then it makes sense to apply the simple majority rule because it results in a higher performance. However, under constant size variability, applying the unanimity rule might be more desirable than applying the simple majority rule, which is clearly the superior rule given a fixed number of decision makers with identical decisional competence. In other words, it is useful to recognize that under variable size flexibility a modified version of Condorcet Jury Theorem need not be valid.

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