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Abstract: The empirical finding by Trefler (2004, AER) and others that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the Melitz (2003, Econometrica) model. But it is actually evidence against the Melitz model. Segerstrom and Sugita (2015, JEEA) showed that under very general assumptions, the multi-industry Melitz model predicts that productivity increases more strongly in non-liberalized industries than in liberalized industries. This disconnect between theory and evidence we call the Melitz-Trefler Puzzle. This paper presents a solution to the Melitz-Trefler puzzle, a new model consistent with the Trefler finding.

JEL classification: F12, F13.

Keywords: Trade liberalization, firm heterogeneity, industrial productivity.

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1 Introduction

In the last decade, the empirical trade literature have established a new mechanism of gains from trade. Trade liberalization improves industrial productivity by shifting resources from less productive to more productive firms within industries. For instance, by investigating the impact of the Canada-USA free trade agreement on Canadian manufacturing industries, Trefler (2004) found that industrial productivity increased more strongly in liberalized industries that experienced large Canadian tariff cuts than in non-liberalized industries, and that the rise in industrial productivity was mainly due to the shift of resources from less productive to more productive firms. Similar productivity gains through intra-industry reallocation in liberalized industries are also observed in other large liberalization episodes (e.g. Pavcnik 2002, for Chile; Eslava, Haltiwanger, Kugler and Kugler, 2012, for Colombia; Nataraji, 2011, for India).

The empirical finding by Trefler (2004) and others that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the seminal model by Melitz (2003) on intra-industry reallocation due to trade liberalization. Virtually all recently published survey papers by leading scholars cite Trefler (2004) as evidence for the Melitz model (Bernard, Jensen, Redding, and Schott, 2007, 2012; Helpman, 2011; Redding, 2011; Melitz and Trefler, 2012). In addition to survey papers, empirical studies on intra-industry reallocation following trade liberalization judge whether their findings support Melitz (2003) or not based on the same belief (e.g. Eslava et al., 2013; Fernandes, 2007; Harrison et al., 2013; Nataraj, 2011; Sivadasan, 2009). When they observe that the increase in industrial productivity (or the exit of low productivity firms) is greater in liberalized industries than in non-liberalized industries, they regard their findings as support for the Melitz model.

This conventional wisdom is wrong. The Trefler finding is actually evidence against the Melitz model. In Segerstrom and Sugita (2015a), we show that under very general assumptions, a multi-industry version of the Melitz model predicts the opposite relationship that industrial productivity increases more strongly in non-liberalized industries than in liberalized industries. When a country like Canada opens up to trade in some industries but not others, the Melitz model implies that productivity increases more strongly in the Canadian industries that did not experience tariff cuts. This disconnect between theory and evidence we call the Melitz-Trefler Puzzle.

In this paper, we present a solution to the Melitz-Trefler Puzzle. We present a new model of international trade with two countries and two differentiated good sectors (or industries), and then study what happens when country 1 opens up to trade in industry A but not industry B. We show that this unilateral trade liberalization by country 1 causes productivity to increase more strongly in the liberalized industry A than in the non-liberalized industry B, consistent with the evidence in Trefler (2004) and other previously-mentioned papers. As Segerstrom and Sugita (2015b) show, trade liberalization has two effects in the Melitz model with two countries and two industries, a *competitiveness effect* that contributes to lowering productivity in the liberalized industry and a *wage effect* that contributes to raising productivity in both liberalized and non-liberalized industries. In the new model, trade liberalization still has the same two effects but they both go in the opposite direction. The *competitiveness effect* of trade liberalization contributes to raising productivity in the liberalized industry (Theorem 1) and the *wage effect* of trade liberalization contributes to lowering productivity in both liberalized and non-liberalized industries (Theorem 2). It is possible to write down a trade model with opposite properties compared to the Melitz model.

The basic structure of the new model is the same as the Melitz model with two industries and two countries. All consumers have the same two tier utility function where the upper tier is Cobb-Douglas and the lower tier is CES. Labor is the only factor of production and workers in each country earn the competitive wage rate. Firms are risk neutral and maximize expected profits. In each time period, there is a fixed cost of entry and an endogenously determined measure of firms choose to enter in each country and sector. Each firm then independently draws its productivity from a Pareto distribution. A firm incurs a fixed "marketing" cost to sell to domestic consumers and incurs an even larger fixed cost to sell to foreign consumers, so only those firms with productivity levels exceeding a threshold value choose to produce for the domestic market and only those firms with productivity levels exceeding a higher threshold value choose to export. In addition to the fixed costs of serving domestic and foreign markets, there are also iceberg trade costs associated with shipping products across countries.

Compared to the Melitz model, the key new assumption concerns the fixed cost of entry. We assume that individual firms take this fixed cost of entry as given but at the aggregate level, entry costs go up as more firms choose to enter. With this new assumption, we are in effect assuming that there are decreasing returns to research and development (R&D) at the sector level: when R&D input (entry costs) is doubled, R&D output (new varieties) less than doubles. In contrast, Melitz (2003) assumed that there are constant returns to R&D at the sector level: when R&D input is doubled, R&D output doubles. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the sector level (e.g., Kortum 1993; Jones 2009).

Although the Melitz model cannot explain the Trefler finding, this model does have other attractive properties that have been confirmed in many empirical studies. For example, a recent survey paper by Redding (2011) mentions two other facts as empirical motivations for the Melitz model: (1) exporters are larger and more productive than non-exporters; (2) entry and exit simultaneously occur within the same industry even without trade liberalization. The new model continues to predict these two facts.

The Melitz model also predicts the Home Market effect, which has received empirical support (e.g., Davis and Weinstein, 2003; Hanson and Xiang, 2004) and plays an important role in the New Economic Geography literature. With a moderate degree of decreasing returns to R&D, the new model predicts both the Home Market effect and the Trefler finding.

The current paper is related to previous studies of trade liberalization using versions of the Melitz model. Demidova and Rodriguez-Clare (2009, 2013), Felbermayr, Jung, and Larch (2013) and Ossa (2011) analyze unilateral trade liberalization in models with one Melitz industry. Bernard, Redding, and Schott (2007) and Okubo (2009) analyze symmetric multilateral liberalization in models with multiple Melitz industries and endogenous factor prices. Arkolakis, Costinot, and Rodriguez-Clare (2012) derive a formula by which one can calculate the the welfare effect of trade liberalization in a multi-industry Melitz model. Segerstrom and Sugita (2015a) derive the Melitz model's implication for difference-in-differences estimates of the impact of tariff cuts on industrial productivity. While these studies maintain the constant returns to R&D assumption as in the Melitz model, our paper is the first to introduce the decreasing returns to R&D assumption in this literature. We find that constant returns to R&D, which is assumed for analytical convenience, is not innocuous. In this class of models, the impacts of trade liberalization on resource reallocation, productivity and welfare crucially depend on the degree of returns to scale in R&D.

The degree of returns to scale in R&D has played an important role in R&D-based endogenous growth models. First generation models such as Grossman and Helpman (1991) assumed constant returns to R&D and as a result, these models have the scale effect property that a larger economy grows faster. Because this scale effect property is clearly at odds with the empirical evidence, second generation models weakened the degree of returns to scale in R&D (e.g., Jones, 1995; Segerstrom, 1998). This paper shares the same spirit with this literature: assuming decreasing returns to R&D also solves a puzzle in international trade.

The rest of the paper is organized as follows. In section 2, we present the model and our main results. In section 3, we discuss intuition and other predictions of the model. In section 4, we offer some concluding comments and there is an Appendix where calculations that we did to solve the model are presented in more detail.

2 The Model

2.1 Setting

Consider two countries, 1 and 2, with two differentiated goods sectors (or industries), A and B. Throughout the paper, subscripts i and j denote countries ($i, j \in \{1, 2\}$) and subscript s denotes sectors ($s \in \{A, B\}$). Though the model has infinitely many periods, there is no means for saving over periods. Following Melitz (2003), we focus on a stationary steady state equilibrium where aggregate variables do not change over time and omit notation for time periods.

The representative consumer in country i has a two-tier (Cobb-Douglas plus CES) utility function:

$$U_i \equiv C_{iA}^{\alpha_A} C_{iB}^{\alpha_B} \qquad \text{where} \qquad C_{is} \equiv \left[\int_{\omega \in \Omega_{is}} q_{is}(\omega)^{\rho} \, d\omega \right]^{1/\rho} \text{ and } \alpha_A + \alpha_B = 1.$$

In the utility equation, $q_{is}(\omega)$ is country i's consumption of a product variety ω produced in sector s, Ω_{is} is the set of available varieties in sector s and ρ measures the degree of product differentiation. We assume that products within a sector are closer substitutes than products across sectors, which implies that the within-sector elasticity of substitution $\sigma \equiv 1/(1-\rho)$ satisfies $\sigma > 1$. Given that $\alpha_A + \alpha_B = 1$, α_s represents the share of consumer expenditure on sector s products.

Country i is endowed with L_i units of labor as the only factor of production. Labor is inelastically supplied and workers in country i earn the competitive wage rate w_i. We measure all prices relative to the price of labor in country 2 by setting $w_2 = 1$.

Firms are risk neutral and maximize expected profits. In each time period, the measure M_{ise} of firms choose to enter in country i and sector s. Each firm uses f_{ise} units of labor to enter and incurs the fixed entry cost $w_i f_{ise}$. Each firm then independently draws its productivity φ from a Pareto distribution. The cumulative distribution function $G(\varphi)$ and the corresponding density function $g(\varphi) = G'(\varphi)$ are given by $G(\varphi) = 1 - (b/\varphi)^{\theta}$ and $g(\varphi) = \theta b^{\theta}/\varphi^{\theta+1}$ for $\varphi \in [b, \infty)$, where $\theta > 0$ and $b > 0$ are the shape and scale parameters of the distribution. We assume that $\theta > \sigma - 1$ to guarantee that expected profits are finite.

A firm with productivity φ uses $1/\varphi$ units of labor to produce one unit of output and has constant marginal cost w_i/φ in country i. This firm must use f_{ij} units of domestic labor and incur the fixed "marketing" cost $w_i f_{ij}$ to sell in country j. Denoting $f_{ii} = f_d$ and $f_{ij} = f_x$ for $i \neq j$, we assume that exporting require higher fixed costs than local selling ($f_x > f_d$). There are also iceberg trade costs associated with shipping products across countries: a firm that exports from country i to country $j \neq i$ in sector s needs to ship $\tau_{ijs} > 1$ units of a product in order for one unit to arrive at the foreign destination (if $j = i$, then $\tau_{iis} = 1$).

Decreasing Returns to R&D So far, the model is a two-industry version of Melitz (2003) with a Cobb-Douglas upper-tier utility function and a Pareto distribution. The key new assumption concerns the fixed cost of entry $w_i f_{ise}$. We assume that individual firms take f_{ise} as given but at the aggregate level, entry costs satisfy

$$f_{ise} = F \cdot M_{ise}^{\zeta} \qquad \text{where} \qquad \zeta > 0, \tag{1}$$

that is, entry costs go up as more firms choose to enter.

Since M_{ise} is the number of firms that enter and $F \cdot M_{ise}^{\zeta}$ is the labor used per firm, the total labor used for R&D in country i and sector s is $L_{ise} \equiv F \cdot M_{ise}^{1+\zeta}$. Solving this expression for M_{ise} yields $M_{ise} = (L_{ise}/F)^{1/(1+\zeta)}$, where M_{ise} can be thought of as the flow of new products developed by researchers and L_{ise} is the sector level of R&D labor. By assuming that $\zeta > 0$, we obtain decreasing returns to R&D at the sector level: when R&D input L_{ise} is doubled, R&D output M_{ise} less than doubles. Melitz (2003) assumed that $\zeta = 0$. This implies constant returns to R&D at the sector level: when R&D input L_{ise} is doubled, R&D output M_{ise} doubles. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the sector level. The patents per R&D worker ratio has declined for most time of the 20th century (Griliches, 1994). This trend holds across countries (Evenson, 1984) and across industries (Kortum, 1993). A more recent study by Jones (2009) confirms the decreasing returns to R&D using microdata on US patents and innovators. According to Kortum (1993), point estimates of $1/(1 + \zeta)$ lie between 0.1 and 0.6, which corresponds to ζ values between 0.66 and 9. The Melitz model case where $\zeta = 0$ is outside the range of empirical estimates.

There are two reasons for decreasing returns to R&D. One reason is that the duplication and overlap of research at a point of time decreases the research output per researcher (the duplication effect). Another reason is that as an industry matures, innovation becomes harder and needs more inputs (the fishing out effect). We focus on the first effect for simplicity.[1]

2.2 Equilibrium Conditions

A firm in country i and sector s with productivity φ sets a profit-maximizing price $p_{ijs}(\varphi)$ for goods it sells to country j. This firm earns revenue $r_{ijs}(\varphi)$ and gross profits $r_{ijs}(\varphi)/\sigma$ from selling to country j.

[1] An alternative formulation is $f_{ise} = F M_{ise}^{\zeta} M_{is}^{\varsigma}$. The mass of actively operating firms M_{is} expresses the amount of past successful innovation and parameter $\varsigma > 0$ captures the decreasing returns to R&D due to the fishing out effect. With this formulation, our main results continue to hold but the calculations become more complex. These results can be obtained from the authors upon request.

Solving the consumer optimization and profit maximization problems yields

$$p_{ijs}(\varphi) = \frac{w_i \tau_{ijs}}{\rho \varphi} \qquad \text{and} \qquad r_{ijs}(\varphi) = \alpha_s w_j L_j \left(\frac{p_{ijs}(\varphi)}{P_{js}} \right)^{1-\sigma}, \tag{2}$$

where P_{js} is the price index. Each firm charges a fixed markup over its marginal cost $w_i \tau_{ijs}/\varphi$.

Because of the fixed marketing costs, there exist productivity cut-off levels φ^*_{ijs} such that only firms with $\varphi \geq \varphi^*_{ijs}$ sell products from country i to country j in sector s. We solve the model for an equilibrium where both countries produce both goods A and B, and the more productive firms export ($\varphi^*_{iis} < \varphi^*_{ijs}$). Firms with $\varphi \geq \varphi^*_{ijs}$ export and sell domestically, firms with $\varphi \in [\varphi^*_{iis}, \varphi^*_{ijs})$ only sell domestically and firms with $\varphi < \varphi^*_{iis}$ exit. A firm with cut-off productivity φ^*_{ijs} just breaks even from selling to country j:

$$\frac{r_{ijs}\left(\varphi^*_{ijs}\right)}{\sigma} = \frac{\alpha_s w_j L_j}{\sigma} \left(\frac{p_{ijs}(\varphi^*_{ijs})}{P_{js}} \right)^{1-\sigma} = w_i f_{ij}, \tag{3}$$

where $P_{js} \equiv \left[\sum_{i=1,2} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} M_{is} \mu_{is}(\varphi) d\varphi \right]^{1/(1-\sigma)}$ is the price index for sector s products in country j, M_{is} is the mass of actively operating firms in country i and sector s, and $\mu_{is}(\varphi) = g(\varphi)/[1 - G(\varphi^*_{iis})]$ is the equilibrium productivity density function for country i and sector s.

In each period, there is an exogenous probability δ with which actively operating firms in country i and sector s die and exit. In a stationary steady state equilibrium, the mass of actively operating firms M_{is} and the mass of entrants M_{ise} in country i and sector s satisfy

$$\left[1 - G\left(\varphi^*_{iis}\right)\right] M_{ise} = \delta M_{is}, \tag{4}$$

that is, firm entry in each time period is matched by firm exit.

From (2) and (3), the cut-off productivity levels of domestic and foreign firms in country j are related as follows:

$$\varphi^*_{ijs} = \tau_{ijs} \left(\frac{f_{ij}}{f_{jj}} \right)^{1/(\sigma-1)} \left(\frac{w_i}{w_j} \right)^{1/\rho} \varphi^*_{jjs}. \tag{5}$$

This equation shows that the cut-off productivity levels of domestic and foreign firms in country j would be the same if it were not for differences in trade costs and labor costs. Let ϕ_{ijs} denote the ratio of the expected profit of an entrant in country i from selling to country j in sector s to that captured by an entrant in country j from selling to country j. Using (2), (3), (4), and (5), the relative expected profit simplifies to:

$$\phi_{ijs} \equiv \frac{\delta^{-1} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij} \right] g(\varphi) d\varphi}{\delta^{-1} \int_{\varphi_{jjs}^*}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma} - w_j f_{jj} \right] g(\varphi) d\varphi} = \frac{1}{\tau_{ijs}^{\theta}} \left(\frac{f_{jj}}{f_{ij}} \right)^{(\theta - \sigma + 1)/(\sigma - 1)} \left(\frac{w_j}{w_i} \right)^{(\theta - \rho)/\rho}. \tag{6}$$

Variable ϕ_{ijs} is an index summarizing the degree of country i's market access to country j in sector s. Since $\theta > \sigma - 1$ and $(\theta - \rho)/\rho > \theta$, it decreases in variable trade costs τ_{ijs}, relative marketing costs f_{ij}/f_{jj}, and the relative wage w_i/w_j. As export barriers τ_{ijs} or f_{ij} increase to infinity, the market access index ϕ_{ijs} converges to zero.

Using the equilibrium price (2), the cutoff conditions (5) and the relative expected profit (6), the price index can be rewritten as

$$P_{is}^{1-\sigma} = \eta\, p_{iis} \left(\varphi_{iis}^* \right)^{1-\sigma} \left(\frac{b}{\varphi_{iis}^*} \right)^{\theta} \left(\frac{M_{ise}}{\delta} + \phi_{jis} \frac{M_{jse}}{\delta} \right) \tag{7}$$

where $\eta \equiv \theta / (\theta - \sigma + 1) > 0$. To understand equation (7), consider first autarky with $\phi_{jis} = 0$. Then, from (4), it becomes that $P_{is}^{1-\sigma} = \eta\, p_{iis} \left(\varphi_{iis}^* \right)^{1-\sigma} M_{is}$. The price index depends on the mass of domestic varieties and the distribution of prices. Under the Pareto distribution, the latter is summarized by the highest price set by the least productive firms on the market. In the open economy with $\phi_{jis} > 0$, the price index also depends on the mass of foreign varieties (M_{jse}/δ) and the degree of their market access (ϕ_{jis}).

Substituting the price index (7) into the cutoff condition (3), we obtain

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\delta \left(\theta - \sigma + 1 \right)} \frac{\sigma f_d}{\alpha_s L_1} \left(M_{1se} + \phi_{21s} M_{2se} \right). \tag{8}$$

The domestic productivity cutoff φ_{11s}^* rises if and only if $(M_{1se} + \phi_{21s} M_{2se})$ rises. If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ increasing, more firms are entering and competition is becoming tougher in country 1 and sector s. With tougher competition, firms need to have a higher productivity level to survive, so the domestic productivity cutoff φ_{11s}^* increases, and it follows that industrial productivity Φ_{1s}^L rises. If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ decreasing, then fewer firms enter, competition becomes less tough, lower productivity firms can now survive and industrial productivity falls. Equation (8) implies that, for determining how trade liberalization impacts the domestic productivity cut-off and industrial productivity, it is sufficient to consider how the mass of entrants in both countries and country 2's market access index ϕ_{21s} change.

A convenient property of the model with the Cobb-Douglas upper tier utility and the Pareto distribution is that we can solve for the mass of entrants M_{ise} as a function of the wage w_1 and trade costs τ_{ijs}. First, free entry implies that the expected profits from entry must equal the cost of entry:

$$\frac{1}{\delta}\sum_{j=1,2}\int_{\varphi_{ijs}^*}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]g(\varphi)d\varphi = w_i f_{ise}. \tag{9}$$

Following Melitz (2003) and Demidova (2008), equation (9) can be rewritten as

$$\frac{1}{\delta}\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\sum_{j=1,2} f_{ij}\left(\frac{b}{\varphi_{ijs}^*}\right)^{\theta} = f_{ise}. \tag{10}$$

Second, equation (10) implies that the total fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s:

$$w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi_{ijs}^*}^{\infty} f_{ij}M_{is}\mu_{is}(\varphi)\,d\varphi\right) = w_i M_{ise}\left(\frac{\theta f_{ise}}{\sigma-1}\right). \tag{11}$$

Third, the free entry condition (9) implies that the total fixed costs are equal to the total gross profits in each country i and sector s, that is,

$$w_i M_{ise}\left(\frac{\theta f_{ise}}{\sigma-1}\right) = \frac{1}{\sigma}\sum_{j=1,2} R_{ijs} \tag{12}$$

where $R_{ijs} \equiv \int_{\varphi_{ijs}^*}^{\infty} r_{ijs}(\varphi)M_{is}\mu_{is}(\varphi)d\varphi$ is the total revenue associated with shipments from country i to country j in sector s. Fourth, from (2), (4), and (7), the total revenue R_{ijs} can be rewritten as

$$R_{ijs} = \alpha_s w_j L_j\left(\frac{M_{ise}\phi_{ijs}}{\sum_{k=1,2} M_{kse}\phi_{kjs}}\right). \tag{13}$$

Substituting (13) into (12), we obtain

$$\sum_{j=1,2}\alpha_s w_j L_j\left(\frac{\phi_{ijs}}{\sum_{k=1,2} M_{kse}\phi_{kjs}}\right) = w_i f_{ise}\left(\frac{\theta}{\rho}\right) \text{ for } i=1,2. \tag{14}$$

Since f_{ise} is a function of M_{ise} and ϕ_{ijs} is a function of τ_{ijs} and w_1, it is possible to express the mass of entrants $M_{ise}(\tau_{12s},\tau_{21s},w_1)$ as a function of variable trade costs and the country 1 relative wage. Then, from (5) and (8), we obtain the domestic and export productivity cutoffs as functions of variable trade costs and the country 1 relative wage.

The labor market clearing condition for country 1 determines the wage w_1. Free entry implies that wage payments to labor equal total revenue in each country i and sector s, that is, $w_i L_{is} = \sum_{j=1,2} R_{ijs}$, where L_{is} is labor demand in country i and sector s. From (1) and (12), this leads to

$$L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = M_{ise} \left(\frac{\sigma\theta}{\sigma - 1} \right) f_{ise} = M_{ise}^{1+\zeta} \left(\frac{\theta F}{\rho} \right). \tag{15}$$

Notice that labor demand L_{is} depends only on the mass of entrants M_{ise} and not on any cut-off productivity levels φ_{ijs}^*. The country 1 labor supply is given by L_1 so the requirement that labor supply equal labor demand

$$L_1 = \left(\frac{\theta F}{\rho} \right) \sum_{s=A,B} M_{1se} \left(\tau_{12s}, \tau_{21s}, w_1 \right)^{1+\zeta}. \tag{16}$$

determines the equilibrium wage rate w_1 given the trade costs (τ_{12s}, τ_{21s}).

Following Segerstrom and Sugita (2015b), we consider two measures of industrial labor productivity. The first measure is the real industrial output per unit of labor: $\Phi_{1s}^L \equiv \left(\sum_{j=1,2} R_{1js} \right) / \left(\tilde{P}_{1s} L_{1s} \right)$. In this definition, the price deflater $\tilde{P}_{1s} \equiv \int_{\varphi_{11s}^*}^{\infty} p_{11s}(\varphi)\, \mu_{1s}(\varphi) d\varphi$ is the simple average of prices set by domestic firms at the factory gate and aims to resemble the industrial product price index, which is used for the calculation of the real industrial output.[2] This measure is widely used in empirical studies (e.g. Trefler, 2004). The second measure is industrial labor productivity calculated using the theoretically consistent "exact" price index P_{1s} that we derived earlier: $\Phi_{1s}^W \equiv \left(\sum_{j=1,2} R_{1js} \right) / (P_{1s} L_{1s})$. This measure is motivated by thinking about consumer welfare. Consider the representative consumer in country 1 who supplies one unit of labor. Since her utility satisfies $U_1 = \left(\alpha_A \Phi_{1A}^W \right)^{\alpha_A} \left(\alpha_B \Phi_{1B}^W \right)^{\alpha_B}$, Φ_{1A}^W and Φ_{1B}^W are the productivity measures for industries A and B that are directly relevant for calculating consumer welfare U_1. From (2), (3) and (15), the productivity measures satisfy

$$\Phi_{1s}^L = \left(\frac{\theta + 1}{\theta} \right) \rho \varphi_{11s}^* \qquad \text{and} \qquad \Phi_{1s}^W = \left(\frac{\alpha_s L_1}{\sigma f_{11}} \right)^{1/(\sigma-1)} \rho \varphi_{11s}^*. \tag{17}$$

Thus, these two measures are increasing functions of the domestic productivity cut-off φ_{11s}^*.

2.3 The Effects of a Small Change in Trade Costs

We now compute the effects of a small change in trade costs τ_{ijs}. We assume that countries and sectors are initially symmetric before trade liberalization with one exception: we allow the fraction α_A of consumer expenditure on sector A products to differ from the fraction α_B of consumer expenditure on sector B products. Thus, the derivatives that we calculate are evaluated at a "symmetric" equilibrium

[2] The term $\sum_{j=1,2} R_{1js}$ is the total revenue of firms in country 1 and sector s. Dividing by the price index $\tilde{P}_{1s}$ gives a measure of the real output of sector s. Then dividing by the number of workers L_{1s} gives a measure of real output per worker.

where $M_{1se} = M_{2se}$ and $\phi_{ijs} = \phi$ hold. The market access index ϕ takes a value between 0 (autarky) and 1 (free trade).

Taking logs of both sides and then totally differentiating (6), we obtain

$$d \ln \phi_{21s} = -\theta \, d \ln \tau_{21s} + \left(\frac{\theta}{\rho} - 1 \right) d \ln w_1. \tag{18}$$

A decrease in country 1's import barrier ($\tau_{21s} \downarrow$) or an increase in the relative wage of country 1 ($w_1 \uparrow$) improve country 2's market access to country 1 ($\phi_{21s} \uparrow$), given that $\theta > \rho > 0$.

Writing out (14) yields a system of 2 linear equations that can be solved using Cramer's Rule. Taking logs of both sides and differentiating the solution equations, and then evaluating the resulting derivatives at the symmetric equilibrium, we obtain

$$\begin{aligned} d \ln M_{1se} &= \iota_\tau \, d \ln \tau_{21s} - \iota_\tau \, d \ln \tau_{12s} - \iota_w \, d \ln w_1 - \iota_1 \, d \ln f_{1se} + \iota_2 \, d \ln f_{2se} \\ d \ln M_{2se} &= -\iota_\tau \, d \ln \tau_{21s} + \iota_\tau \, d \ln \tau_{12s} + \iota_w \, d \ln w_1 + \iota_2 \, d \ln f_{1se} - \iota_1 \, d \ln f_{2se}, \end{aligned} \tag{19}$$

where

$$\iota_\tau \equiv \frac{\phi\theta}{(1-\phi)^2} > 0, \;\; \iota_w \equiv \frac{\phi \left[2\theta - \rho (1-\phi)\right]}{\rho (1-\phi)^2} > 0, \;\; \iota_1 \equiv \frac{1+\phi^2}{(1-\phi)^2} > 0 \;\text{ and }\; \iota_2 \equiv \frac{2\phi}{(1-\phi)^2} > 0.$$

Increases in the wage ($w_1 \uparrow$), export barriers ($\tau_{12s} \uparrow$) or domestic entry costs ($f_{1se} \uparrow$) discourage entry ($M_{1se} \downarrow$), while increases in import barriers ($\tau_{21s} \uparrow$) or foreign entry costs ($f_{2se} \uparrow$) encourage entry ($M_{1se} \uparrow$). Since entry costs are endogenous, substituting $d \ln f_{ise} = \zeta \, d \ln M_{ise}$ into (19), we obtain

$$\begin{aligned} d \ln M_{1se} &= \varepsilon_\tau \, d \ln \tau_{21s} - \varepsilon_\tau \, d \ln \tau_{12s} - \varepsilon_w \, d \ln w_1 \\ d \ln M_{2se} &= -\varepsilon_\tau \, d \ln \tau_{21s} + \varepsilon_\tau \, d \ln \tau_{12s} + \varepsilon_w \, d \ln w_1 \end{aligned} \tag{20}$$

where

$$\varepsilon_\tau \equiv \frac{\phi\theta}{(1-\phi)^2 + \zeta (1+\phi)^2} > 0 \;\text{ and }\; \varepsilon_w \equiv \frac{\phi \left[2\theta - \rho (1-\phi)\right]}{\rho \left[(1-\phi)^2 + \zeta (1+\phi)^2\right]} > 0.$$

Since both ε_τ and ε_w are decreasing in ζ, we can see that decreasing returns to R&D makes entry less responsive to changes in trade costs and the wage. To understand why this is happening, it suffices to recall that for firms in country i and sector s, the cost of entry is $w_i F M_{ise}^{\zeta}$. When $\zeta = 0$ (the Melitz model case), the cost of entry does not depend on the mass of entering firms M_{ise} but when $\zeta > 0$, the cost of entry goes up when M_{ise} increases and the cost of entry goes down when M_{ise} decreases. So in a sector where trade liberalization encourages more entry, as more firms enter, the cost of entry goes up, which serves to discourage further entry. And in a sector where trade liberalization leads to less entry, as less firms enter, the cost of entry goes down, which serves to make entry more attractive. As ζ increases, we get less adjustment in the up direction because the cost of entry is going up and we get less adjustment in the down direction because the cost of entry is going down.

Taking logs and then differentiating (8) and (17), we obtain that changes in industrial productivity Φ_{1s}^{k} and domestic productivity cutoffs φ_{11s}^{*} are proportional to the change in $M_{1se} + \phi_{21s} M_{2se}$:

$$d \ln \Phi_{1s}^{k=L,W} = d \ln \varphi_{11s}^{*} = \frac{1}{\theta} d \ln \left(M_{1se} + \phi_{21s} M_{2se} \right). \tag{21}$$

Using (6), (20) and (21), we obtain our key equation:

$$d \ln \Phi_{1s}^{k=L,W} = d \ln \varphi_{11s}^{*} = \gamma_1 \, d \ln \tau_{21s} - \gamma_2 \, d \ln \tau_{12s} - \gamma_3 \, d \ln w_1 \tag{22}$$

where

$$\gamma_1 \equiv \frac{\phi \left[\phi - \lambda \left(\zeta \right) \right]}{1 - \phi^2}, \; \gamma_2 \equiv \frac{\phi \left[1 - \lambda \left(\zeta \right) \right]}{1 - \phi^2} > 0, \; \gamma_3 \equiv \frac{\phi}{\beta \left(1 - \phi^2 \right)} \left[\frac{\theta (1 + \phi)}{2\theta - \rho \left(1 - \phi \right)} - \lambda \left(\zeta \right) \right],$$
$$\lambda \left(\zeta \right) \equiv \frac{\zeta \left(1 + \phi \right)^2}{\left(1 - \phi \right)^2 + \zeta \left(1 + \phi \right)^2} \in (0, 1) \;\; \text{and} \;\; \beta \equiv \frac{\rho \theta}{2\theta - \rho \left(1 - \phi \right)} > 0.$$

Segerstrom and Sugita (2015a) derive a similar equation to (22) for the Melitz model with $\zeta = 0$ and find that γ_1, γ_2, and γ_3 are all strictly positive. When $\zeta > 0$, γ_1, γ_2, and γ_3 include an additional term $\lambda \left(\zeta \right)$. Since $\lambda \left(\zeta \right)$ is positive and smaller than one, the sign of γ_2 is always positive. Since $\lambda \left(\zeta \right)$ is increasing in ζ, the signs of γ_1 and γ_3 are ambiguous and become negative if ζ is sufficiently large. Straightforward calculations lead to our main theorem about the sign of γ_1:

Theorem 1. *(1) There exists a positive threshold* $\zeta_1 \equiv \frac{\phi(1-\phi)}{(1+\phi)^2} > 0$ *such that* $\gamma_1 > 0$ *if* $\zeta < \zeta_1$ *and* $\gamma_1 < 0$ *if* $\zeta > \zeta_1$*; (2)* $\zeta_1 \leq 1/8$ *holds for all* $\phi \in (0, 1)$.

Segerstrom and Sugita (2015b) analyze unilateral trade liberalization by country 1 ($d \ln \tau_{21s} < d \ln \tau_{12s} = 0$) and decompose the impact on industrial productivity in country 1 into two effects, the competitiveness effect and the wage effect. In their terminology, $\gamma_1 \, d \ln \tau_{21s}$ in (22) expresses the competitiveness effect, while $-\gamma_3 \, d \ln w_1$ expresses the wage effect. For the unilateral trade liberalization that they study, the middle term $-\gamma_2 \, d \ln \tau_{12s}$ equals zero. Theorem 1 implies that as the decreasing returns to R&D becomes stronger ($\zeta \uparrow$), the competitiveness effect becomes weaker ($\gamma_1 \downarrow$) and eventually takes the opposite sign ($\gamma_1 < 0$). The threshold level ζ_1 for the decreasing returns to R&D parameter ζ is bounded above by 1/8. This is a small degree of decreasing returns to R&D when compared with estimates of ζ ranging from 0.66 to 9 reported in Kortum (1993). Even a small degree of decreasing returns to R&D is sufficient for flipping the sign of the competitiveness effect.

To understand the intuition for Theorem 1, consider how the entrant index $M_{1se} + \phi_{21s} M_{2se}$ changes when country 1 unilaterally opens up to trade in industry s and the country 1 relative wage w_1 is held fixed ($d \ln \tau_{21s} < d \ln \tau_{12s} = d \ln w_1 = 0$). From (18) and (20), country 2's market access rises ($\tau_{21s} \downarrow \Rightarrow \phi_{21s} \uparrow$), the mass of entrants in country 2 M_{2se} increases ($\tau_{21s} \downarrow \Rightarrow M_{2se} \uparrow$), and the mass of entrants in country 1 decreases ($\tau_{21s} \downarrow \Rightarrow M_{1se} \downarrow$). The first two effects increase $M_{1se} + \phi_{21s} M_{2se}$, while the last effect decreases it. When $\zeta = 0$ (the Melitz model case), M_{1se} falls so much that it offsets the increase in $\phi_{21s} M_{2se}$ and $M_{1se} + \phi_{21s} M_{2se}$ falls. As we have seen, when ζ increases, entry becomes less responsive to changes in trade costs. On the other hand, equation (18) with $d \ln w_1 = 0$ implies that the increase in country 2's market access ϕ_{12s} does not depend on the size of ζ but just on the size of parameter θ: $d \ln \phi_{21s} = -\theta \, d \ln \tau_{21s}$. Therefore, as ζ increases, the dominant change eventually becomes the increase in ϕ_{12s}, so $M_{1se} + \phi_{21s} M_{2se}$ rises.

Theorem 1 offers a solution to the Melitz-Trefler puzzle. When country 1 opens up to trade in industry A but not in industry B ($d \ln \tau_{21A} < d \ln \tau_{21B} = d \ln \tau_{12A} = d \ln \tau_{12B} = 0$), it follows from (22) that

$$
\begin{aligned}
d \ln \Phi_{1A}^{k} - d \ln \Phi_{1B}^{k} &= (\gamma_1 \, d \ln \tau_{21A} - \gamma_3 \, d \ln w_1) - (-\gamma_3 \, d \ln w_1) \\
&= \gamma_1 \, d \ln \tau_{21A}.
\end{aligned}
$$

That is, the competitiveness effect of trade liberalization is equal to the difference-in-differences change in productivity between liberalized and non-liberalized industries in the liberalizing country. The Melitz model with $\zeta = 0$ predicts that $\gamma_1 > 0$, that is, productivity rises more strongly in non-liberalized industries than in liberalized industries ($d \ln \tau_{21A} < 0 \Rightarrow d \ln \Phi_{1A}^{k} < \ln \Phi_{1B}^{k}$). This is the exact opposite of the Trefler finding ($d \ln \tau_{21A} < 0 \Rightarrow d \ln \Phi_{1A}^{k} > \ln \Phi_{1B}^{k}$). On the other hand, when ζ is sufficiently greater than zero, the current model predicts $\gamma_1 < 0$, which is consistent with the Trefler finding.

Corollary 1. *When country 1 opens up to trade in industry A but not in industry B, productivity increases more strongly in the liberalized industry A than in the non-liberalized industry B if* $\zeta > \zeta_1$. *Productivity increases more strongly in the non-liberalized industry B than in the liberalized industry A if* $\zeta < \zeta_1$.

The decreasing returns to R&D also affects the wage effect of trade liberalization $-\gamma_3 \, d \ln w_1$. To determine the size of the wage effect, we need to solve for the wage change from the labor market clearing condition. Taking logs of both sides and then differentiating (16) and substituting using (20), we obtain

$$d \ln w_1 = \beta \sum_{s=A,B} \alpha_s \left(d \ln \tau_{21s} - d \ln \tau_{12s} \right). \tag{23}$$

Notice that the wage change does not depend on the size of ζ, so the decreasing returns to R&D affects the wage effect only through the size of γ_3. Straightforward calculations lead to our second theorem about the sign of γ_3:

Theorem 2. *(1) There exists a positive threshold* $\zeta_3 \equiv \frac{\theta(1-\phi)}{(\theta-\rho)(1+\phi)} > 0$ *such that* $\gamma_3 > 0$ *if* $\zeta < \zeta_3$ *and* $\gamma_3 < 0$ *if* $\zeta > \zeta_3$*; (2)* $\zeta_3/\zeta_1 = \left(1 + \frac{1}{\phi}\right)\left(1 + \frac{\rho}{\theta-\rho}\right) > 1$.

As the decreasing returns to R&D becomes stronger starting from $\zeta = 0$, γ_3 is initially positive, decreases and eventually turns negative. To understand the intuition for Theorem 2, suppose that country 1's wage exogenously increases while trade costs are held fixed ($d \ln w_1 > d \ln \tau_{12s} = d \ln \tau_{21s} = 0$), and consider how the entry index $M_{1se} + \phi_{21s} M_{2se}$ changes. From (18) and (20), country 2's market access rises ($w_1 \uparrow \Rightarrow \phi_{21s} \uparrow$), the mass of entrants in country 2 increases ($w_1 \uparrow \Rightarrow M_{2se} \uparrow$), and the mass of entrants in country 1 decreases ($w_1 \uparrow \Rightarrow M_{1se} \downarrow$). The first two effects increase $M_{1se} + \phi_{21s} M_{2se}$, while the last effect decreases it. When $\zeta = 0$ and γ_3 is positive (the Melitz model case), M_{1se} falls so much that it offsets the increase in $\phi_{21s} M_{2se}$ and $M_{1se} + \phi_{21s} M_{2se}$ falls. On the other hand, when ζ increases from zero, the adjustment of entrants becomes smaller, while the increase in ϕ_{12s} remains the same. Therefore, as ζ increases, the dominant change eventually becomes the increase in ϕ_{12s}, so $M_{1se} + \phi_{21s} M_{2se}$ rises and γ_3 becomes negative.

The case where $\gamma_3 < 0$ seems to be more intuitive. When the domestic wage w_1 exogenously rises, one should expect the lowest productivity firms to exit and the domestic productivity cutoff to rise. However, the Melitz model with $\zeta = 0$ actually predicts the opposite: when the domestic wage increases, the domestic productivity cutoff falls ($w_1 \uparrow \Rightarrow \varphi^*_{11s} \downarrow$ when $\gamma_3 > 0$). The current model predicts that the domestic productivity cutoff rises when $\zeta > \zeta_3$ ($w_1 \uparrow \Rightarrow \varphi^*_{11s} \uparrow$ when $\gamma_3 < 0$). Again, introducing decreasing returns to R&D makes the model more intuitive.

Corollary 2. *When the domestic wage exogenously rises, the domestic productivity cutoffs and industrial productivity rise if* $\zeta > \zeta_3$ *and fall if* $\zeta < \zeta_3$.

The case where $\gamma_3 < 0$ is also consistent with empirical studies on the effect of exchange rate appreciation on firm exit. Since the wage of country 2 is normalized to one, the wage of country 1 represents the relative wage of country 1. An appreciation of the real exchange rate is a shock increasing the relative wage of a country. Several empirical studies have found that the exit probability of low productivity firms rises during periods of real exchange rate appreciation, such as Baggs, Beaulie, and

Fung (2008) and Tomlin and Fung (2015) for Canada and Ekholm, Moxnes and Ulltveit-Moe (2012, Table 9 in Appendix) for Norway.

Substituting the wage change (23) into (22), we obtain the total impact of trade liberalization on industrial productivity in sector A in country 1:

$$\begin{aligned} d\ln\Phi_{1A}^{k} &= -\xi_{1A}\, d\ln\tau_{21A} - \xi_{2A}\, d\ln\tau_{12A} - \xi_{3A}\left(d\ln\tau_{21B} - d\ln\tau_{12B}\right) \\ \text{where } \xi_{1A} &\equiv \gamma_3\beta\alpha_A - \gamma_1,\ \xi_{2A} \equiv \gamma_2 - \gamma_3\beta\alpha_A \ \text{ and } \ \xi_{3A} \equiv \gamma_3\beta\left(1-\alpha_A\right). \end{aligned} \tag{24}$$

The signs of ξ_{1A}, ξ_{2A} and ξ_{3A} depend on five parameters γ_1, γ_2, γ_3, α_A and β. Letting $\bar{\alpha}(\zeta) \equiv \gamma_1/\left(\beta\gamma_3\right) = \left[2\theta - \rho\left(1-\phi\right)\right]\left(\zeta_1 - \zeta\right)/\left[\left(\theta-\rho\right)\left(\zeta_3 - \zeta\right)\right]$, straightforward calculations lead to the following theorem:

Theorem 3. *(1)* $\xi_{1A} < 0$ *if* $\zeta < \zeta_1$ *and* $\alpha_A < \bar{\alpha}(\zeta)$*; (2)* $\xi_{1A} > 0$ *if* $\zeta < \zeta_1$ *and* $\alpha_A > \bar{\alpha}(\zeta)$ *or* $\zeta \geq \zeta_1$*; (3)* $\xi_{2A} > 0$*; (4)* $\xi_{3A} > 0$ *if* $\zeta < \zeta_3$*; and (5)* $\xi_{3A} < 0$ *if* $\zeta > \zeta_3$.

Theorem 3 implies that the impact of trade liberalization on industrial productivity crucially depends on the decreasing returns to R&D parameter ζ and the size of the liberalizing industry α_A. Figure 1 is drawn based on Theorem 3 and shows how the signs of ξ_{1A} and ξ_{3A} depend on ζ and α_A. When the degree of the decreasing returns to R&D is sufficiently small (Area I in Figure 1), as in the Melitz model (when $\zeta = 0$), unilateral trade liberalization reduces the productivity of the liberalized industry when the liberalized industry is small ($\tau_{21A} \downarrow \Rightarrow \Phi_{1A}^{k} \downarrow$ when $\xi_{1A} < 0$). However, with just a slight degree of decreasing returns to R&D (Areas II and III where $\zeta_1 \leq 1/8$), unilateral trade liberalization raises the productivity of the liberalized industry ($\tau_{21A} \downarrow \Rightarrow \Phi_{1A}^{k} \uparrow$ when $\xi_{1A} > 0$). The impact on the non-liberalized industry also depends on the degree of the decreasing returns to R&D ($\tau_{21B} \downarrow \Rightarrow \Phi_{1A}^{k} \uparrow$ when $\xi_{3A} > 0$ and $\tau_{21B} \downarrow \Rightarrow \Phi_{1A}^{k} \downarrow$ when $\xi_{3A} < 0$). Unilateral trade liberalization raises the productivity of the non-liberalized industry when the degree of decreasing returns to R&D is small (Areas I and II) but reduces it when the degree of decreasing returns to R&D is sufficiently large (Area III). Interestingly, trade liberalization by foreign countries always raises the productivity of the liberalized industry in the domestic country ($\tau_{12A} \downarrow \Rightarrow \Phi_{1A}^{k} \uparrow$ given $\xi_{2A} > 0$), but its impact on the non-liberalized industry depend on the degree of decreasing returns to R&D ($\tau_{12B} \downarrow \Rightarrow \Phi_{1A}^{k} \downarrow$ if $\xi_{3A} > 0$, and $\tau_{12B} \downarrow \Rightarrow \Phi_{1A}^{k} \uparrow$ if $\xi_{3A} < 0$).

Using Theorem 3, we can analyze the types of trade liberalization that previous studies analyze. First, we consider the symmetric trade liberalization that Melitz (2003) analyzes. Suppose country 1 and country 2 symmetrically liberalize ($d\ln\tau_{21s} = d\ln\tau_{12s} = d\ln\tau_s < 0$) in a single industry s. Since symmetric trade liberalization keeps countries symmetric, the wage continues to be $w_1 = 1$. Thus,

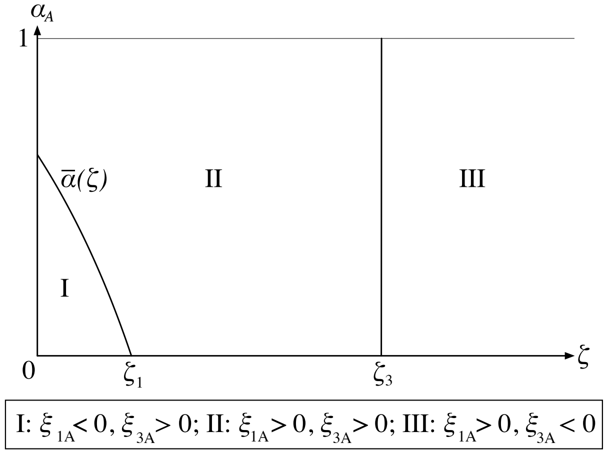

Figure 1: The signs of ξ_{1A} and ξ_{3A}

equation (22) leads to

$$d \ln \Phi_{1s}^k = (\gamma_1 - \gamma_2)\ d \ln \tau_s = -\frac{\phi}{1+\phi}\, d \ln \tau_s > 0,$$

so symmetric trade liberalization raises the productivity of the liberalized industry and does not affect the productivity of the non-liberalized industry. Second, we consider unilateral trade liberalization by country 1 that is uniform across industries ($d \ln \tau_{21A} = d \ln \tau_{21B} = d \ln \tau < d \ln \tau_{12A} = d \ln \tau_{12B} = 0$). Then, equation (24) leads to

$$d \ln \Phi_{1A}^k = d \ln \Phi_{1B}^k = -\frac{\phi\,(\theta + \rho\phi)}{(1+\phi)\,[2\theta - \rho\,(1-\phi)]} d \ln \tau > 0.$$

Thus, unilateral and uniform trade liberalization always raises productivity in the liberalizing country. This is consistent with previous studies on unilateral trade liberalization in the Melitz model with one industry such as Demidova and Rodriguez-Clare (2009, 2013) and Felbermayr, Jung, and Larch (2013).

3 Discussion

3.1 Intuition from the Free Entry Condition

Another way to understand the intuition behind Theorems 1 and 2 is to investigate the free entry condition (10). The condition for entrants in country 1 and sector A can be written as follows:

$$\underbrace{f_{1Ae} = F\, M_{1Ae}^{\zeta}}_{\text{Entry R\&D Costs}} = \underbrace{\frac{kf_{11}}{\varphi_{11A}^{*\theta}}}_{\substack{\text{Expected} \\ \text{Domestic Profit}}} + \underbrace{\frac{kf_{12}}{\varphi_{12A}^{*\theta}}}_{\substack{\text{Expected} \\ \text{Export Profit}}} \tag{25}$$

where $k \equiv b^{\theta} (\sigma - 1) / [\delta (\theta - \sigma + 1)]$ is constant. Roughly speaking, the left hand side in (25) represents entry R&D costs, while the right hand side represents the expected profit from entry. The expected profit from entry consist of expected domestic profit (the first term) and expected export profit (the second term). The expected domestic profit is decreasing in the domestic productivity cutoff φ_{11A}^{*}, while the expected export profit is decreasing in the export productivity cutoff φ_{12A}^{*}.

When $\zeta = 0$ (the Melitz model case), entry R&D costs in (25) are constant. This means that entry must yield the same expected profit (before and after trade liberalization) to cover the R&D entry costs: otherwise, no firm enters and the number of active firms becomes zero in a steady state. When the domestic productivity cutoff rises, the expected domestic profit falls, since fewer firms can survive in the domestic market. Then, the export productivity cutoff must fall and the expected export profit must rise enough to keep total expected profit constant. Notice that the reverse is also true. When the export productivity cutoff falls and the expected export profit rises, the domestic productivity cutoff must rise and the expected domestic profit must fall enough to keep total expected profit constant. φ_{11A}^{*} and φ_{12A}^{*} move in opposite directions to keep total expected profit constant.

First, consider the competitiveness effect $\gamma_1 > 0$ when $\zeta = 0$. Suppose the import tariff by country 1 τ_{21A} falls and the wage w_1 is held fixed. The fall in country 1's import tariff makes exporting by country 2 firms more profitable, so the country 2 export productivity cutoff φ_{21A}^{*} decreases. Since entry R&D costs in country 2 do not change, the domestic productivity cutoff φ_{22A}^{*} in country 2 must rise so that the expected domestic profit for country 2 firms falls. When the wage w_1 is held fixed, an increase in the domestic productivity cutoff φ_{22A}^{*} in country 2 implies an increase in the export productivity cutoff φ_{12A}^{*} in country 1 [see the productivity cutoff condition (5)] because selling to country 2 becomes less profitable for country 1 firms as well as for country 2 firms. Since the expected export profit for country 1 firms falls, the domestic productivity cutoff φ_{11A}^{*} in country 1 must fall so that the expected domestic profit increases enough to cover the entry R&D costs ($\tau_{21A} \downarrow, w_1$ fixed $\Rightarrow \varphi^*_{11A} \downarrow, \Phi^k_{1A} \downarrow$).

Next, consider the wage effect $\gamma_3 > 0$ when $\zeta = 0$. An exogenous decrease in country 1's wage w_1 increases the expected export profit of country 1 firms. Thus, the domestic productivity cutoff φ^*_{11A} must rise so that the domestic expected profit decreases enough to cover the constant entry R&D costs ($w_1 \downarrow \Rightarrow \varphi^*_{11A} \uparrow, \Phi^k_{1A} \uparrow$).

The assumption of decreasing returns to entry R&D ($\zeta > 0$) weakens the above-mentioned adjustment mechanisms in two ways. First, when the import tariff τ_{21A} falls, the mass of country 1 entrants M_{1Ae} falls so that entry costs f_{1Ae} fall in country 1. Therefore, the expected domestic profit does not have to increase when the export productivity cutoff rises. Second, the mass of entrants in country 2 M_{2Ae} rises and entry costs rise in country 2. This also means that the expected domestic profit in country 2 does not have to fall.

3.2 The Welfare Effect

The utility of the representative consumer in country 1, $U_1 = \left(\alpha_A \Phi^W_{1A}\right)^{\alpha_A} \left(\alpha_B \Phi^W_{1B}\right)^{\alpha_B}$, is an increasing function of productivity in both industries, Φ^W_{1A} and Φ^W_{1B}. Therefore, the welfare effect of trade liberalization depends on how productivity in both industries change. In this section, we solve for how welfare changes.

Taking logs of both sides and differentiating the consumer utility function U_1, and then substituting for the productivity changes from (24), we obtain the welfare change:

$$d \ln U_1 = - \sum_{s=A,B} \alpha_s \left(\kappa_1 \, d \ln \tau_{21s} + \kappa_2 \, d \ln \tau_{12s}\right), \tag{26}$$

$$\text{where} \quad \kappa_1 \equiv \frac{\phi\left(\theta + \rho\phi\right)}{\left(1+\phi\right)\left[2\theta - \rho\left(1-\phi\right)\right]} > 0 \quad \text{and} \quad \kappa_2 \equiv \frac{\phi\left(\theta - \rho\right)}{\left(1+\phi\right)\left[2\theta - \rho\left(1-\phi\right)\right]} > 0.$$

Both domestic and foreign trade liberalization cause domestic welfare to increase ($\tau_{ijs} \downarrow \Rightarrow U_1 \uparrow$). Interestingly, the welfare effect does not depend on the decreasing returns to R&D parameter ζ. This means that the welfare effect does not depend on whether productivity goes up or down in the liberalized industry. Even when the productivity of the liberalized industry falls, consumer welfare rises thanks to the productivity gain in the non-liberalized industry. We have established

Theorem 4. *For all $\zeta \geq 0$, unilateral trade liberalization by country 1 in industry A leads to consumer welfare increasing in both countries ($\tau_{21A} \downarrow \Rightarrow U_1 \uparrow, U_2 \uparrow$).*

3.3 The Welfare Effect When Industries Are Asymmetric

When industries are asymmetric, the welfare effect of trade liberalization depends on the degree of decreasing returns to R&D. To see this, considers a case of asymmetric industries that Ossa (2011) analyzed. Suppose now that industry B produces a homogenous numeraire good with constant returns to scale technology, there is costless trade in this good and perfect competition prevails. Then, industry B fixes the wage ($w_1 = w_2 = 1$) and using (22), the welfare change from trade liberalization in industry A becomes

$$d\ln U_1 = \alpha_A \, d\ln \Phi_{1A}^{W} = \alpha_A \left[\gamma_1 \, d\ln \tau_{21A} - \gamma_2 \, d\ln \tau_{12A}\right] \tag{27}$$

In the case of $\zeta = 0$, Ossa (2011) showed that unilateral trade liberalization monotonically decreases the welfare of the liberalizing country ($\tau_{21A} \downarrow \Rightarrow U_1 \downarrow$) and thus the optimal tariff is infinite. Equation (27) shows this result comes from $\gamma_1 > 0$. As Theorem 1 shows, the sign of γ_1 changes when the degree of decreasing returns to R&D is increased. When $\zeta > \zeta_1$ and $\gamma_1 < 0$, unilateral trade liberalization increases the welfare of the liberalizing country ($\tau_{21A} \downarrow \Rightarrow U_1 \uparrow$). This is because unilateral trade liberalization raises productivity in the liberalizing industry as Trefler (2004) and many empirical studies observe.

3.4 Other "Melitz" Predictions

Although the Melitz model cannot explain the Trefler finding, this model does have other attractive properties that have been confirmed in many empirical studies. For example, a recent survey paper by Redding (2011) mentions two other facts as empirical motivations for the Melitz model: (1) exporters are larger and more productive than non-exporters; (2) entry and exit simultaneously occur within the same industry even without trade liberalization. This section shows that the new model continues to predict these and other facts that the Melitz model predicts.

Selection into Exporting A large number of empirical studies shows that within industries, firm productivity is positively correlated with the probability that the firm exports (e.g. Bernard and Jensen, 1995, 1999) and the number of markets to which the firm exports (e.g. Eaton, Kortum, and Kramarz, 2011). Eaton, Kortum, and Kramarz (2011) show that the Melitz model (with idiosyncratic trade costs and fixed entry) successfully predicts these cross-sectional facts. The new model also predicts these facts since firm behavior after entry is exactly the same as in the Melitz model.

Simultaneous Entry and Exit Another fact emphasized by Redding (2011) is that firm entry and exit simultaneously occur within industries even without trade liberalization. This fact is robustly found in the industrial organization literature and motivates the seminal model by Hopenhayn (1992) with random productivity draws following free entry and probabilistic exit. Similar to the Melitz model, the current model features random productivity draws following free entry and probabilistic exit, so it can predict simultaneous entry and exit.

Home Market Effect Our solution to the Melitz-Trefler Puzzle is to introduce decreasing returns to R&D into a model featuring increasing returns to scale in production. This could change the model's properties that are based on the increasing returns to scale in production. As an extension of the Krugman (1980) model, the Melitz model is known to predict the Home Market effect: a country with larger population creates net exports of goods with increasing returns to scale in production. The Home Market effect receives empirical support (e.g. Davis and Weinstein, 2003; Hanson and Xiang, 2004) and plays an important role in the New Economic Geography literature. Does introducing the decreasing returns to R&D have to eliminate the Home Market effect?

To answer this question, we consider the model with fixed wages, following a standard model of the Home Market effect by Helpman and Krugman (1985) and Ossa (2011). Suppose that industry B produces a homogenous numeraire good with constant returns to scale technology, there is costless trade in this good and perfect competition prevails. Then, industry B fixes the wage ($w_1 = w_2 = 1$). Suppose that the two countries are initially symmetric and that the population of country 1 increases ($d \ln L_1 > d \ln L_2 = 0$). Then, we analyze whether the net export of country 1 in industry A, $R_{12A} - R_{21A}$, becomes positive or negative. If it becomes positive, we conclude that the model predicts the Home Market effect.

Solving the system of linear equations (14) using Cramer's Rule, taking logs of both sides and then differentiating, we obtain

$$d \ln M_{1se} = \varepsilon_{1L}\, d \ln L_1 \;\text{ and }\; d \ln M_{2se} = -\varepsilon_{2L}\, d \ln L_1,$$

where

$$\varepsilon_{1L} \equiv \frac{1 - \phi + \zeta (1 + \phi)}{(1 + \zeta)\left[(1 - \phi)^2 + \zeta (1 + \phi)^2\right]} \;\text{ and}$$
$$\varepsilon_{2L} \equiv \frac{\phi \left[1 - \phi - \zeta (1 + \phi)\right]}{(1 + \zeta)\left[(1 - \phi)^2 + \zeta (1 + \phi)^2\right]}.$$

Using this and equation (13), we obtain

$$\frac{d \ln (R_{12A}/R_{21A})}{d \ln L_1} = \frac{2}{1 + \phi} (\varepsilon_{1L} + \varepsilon_{2L}) - 1.$$

Since $R_{12A} = R_{21A}$ initially holds, the net export of country 1 in industry A, $R_{12A} - R_{21A}$, becomes positive if and only if $d\ln\left(R_{12A}/R_{21A}\right)/d\ln L_1 > 0$. Straightforward calculations lead to the following theorem:

Theorem 5. *There exists a positive threshold* $\zeta_H \equiv (1-\phi)/(1+\phi) > \zeta_1$ *such that the model predicts the Home Market effect if and only if* $\zeta < \zeta_H$.

Theorem 5 implies that only a strong degree of decreasing returns to R&D eliminates the Home Market effect. For a moderate degree of decreasing returns to R&D, $\zeta \in (\zeta_1, \zeta_H)$, the model predicts both the Home Market effect and the Trefler finding. Another implication of Theorem 5 is that the Home Market effect is not the cause of the Melitz-Trefler Puzzle.

4 Conclusion

In this paper, we present a new model on how trade liberalization reallocates resources across and within industries. When one country opens up to trade in some industries but not others, the new model predicts that productivity increases more strongly in liberalized industries than in non-liberalized industries. Productivity unambiguously rises in the liberalized industries and falls in the non-liberalized industries. In contrast, the Melitz model has opposite properties. When one country opens up to trade in some industries but not others, the Melitz model predicts that productivity increases more strongly in non-liberalized industries than in liberalized industries. Productivity unambiguously rises in the non-liberalized industries and can fall in the liberalized industries. What drives our new results is one new assumption: we introduce decreasing returns to R&D into an otherwise standard Melitz model.

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Appendix: Solving The Model (Not for Publication)

In this Appendix, calculations that we did to solve the model are presented in more detail.

Consumers

First, we solve the within-sector consumer optimization problem

$$\max_{q_{is}(\cdot)} C_{is} \equiv \left[\int_{\omega\in\Omega_{is}} q_{is}(\omega)^{\rho}\, d\omega\right]^{1/\rho} \qquad \text{s.t.} \qquad \int_{\omega\in\Omega_{is}} p_{is}(\omega) q_{is}(\omega)\, d\omega = E_{is}$$

where $q_{is}(\omega)$ is quantity demanded for variety ω in country i and sector s, $p_{is}(\omega)$ is the price of variety ω and E_{is} is consumer expenditure on sector s products. This problem of maximizing a CES utility function subject to a budget constraint can be rewritten as the optimal control problem

$$\max_{q_{is}(\cdot)} \int_{\omega\in\Omega_{is}} q_{is}(\omega)^{\rho}\, d\omega \qquad \text{s.t.} \qquad \dot{y}_{is}(\omega) = p_{is}(\omega) q_{is}(\omega),\ y_{is}(0) = 0,\ y_{is}(+\infty) = E_{is}$$

where $y_{is}(\omega)$ is a new state variable and $\dot{y}_{is}(\omega)$ is the derivative of y_{is} with respect to ω. The Hamiltonian function for this optimal control problem is

$$H = q_{is}(\omega)^{\rho} + \xi(\omega) p_{is}(\omega) q_{is}(\omega)$$

where $\xi(\omega)$ is the costate variable. The costate equation $\frac{\partial H}{\partial y_{is}} = 0 = -\dot{\xi}(\omega)$ implies that $\xi(\omega)$ is constant across ω. $\frac{\partial H}{\partial q_{is}} = \rho q_{is}(\omega)^{\rho-1} + \xi \cdot p_{is}(\omega) = 0$ implies that

$$q_{is}(\omega) = \left(\frac{\rho}{-\xi \cdot p_{is}(\omega)}\right)^{1/(1-\rho)}.$$

Substituting this back into the budget constraint yields

$$\begin{aligned} E_{is} &= \int_{\omega\in\Omega_{is}} p_{is}(\omega) q_{is}(\omega)\, d\omega = \int_{\omega\in\Omega_{is}} p_{is}(\omega) \left(\frac{\rho}{-\xi \cdot p_{is}(\omega)}\right)^{1/(1-\rho)} d\omega \\ &= \left(\frac{\rho}{-\xi}\right)^{1/(1-\rho)} \int_{\omega\in\Omega_{is}} p_{is}(\omega)^{\frac{1-\rho-1}{1-\rho}}\, d\omega. \end{aligned}$$

Now $\sigma \equiv \frac{1}{1-\rho}$ implies that $1-\sigma = \frac{1-\rho-1}{1-\rho} = \frac{-\rho}{1-\rho}$, so

$$\frac{E_{is}}{\int_{\omega\in\Omega_{is}} p_{is}(\omega)^{1-\sigma} d\omega} = \left(\frac{\rho}{-\xi}\right)^{1/(1-\rho)}.$$

It immediately follows that the consumer demand function is

$$q_{is}(\omega) = \frac{p_{is}(\omega)^{-\sigma} E_{is}}{P_{is}^{1-\sigma}} \qquad (A.1)$$

where $P_{is} \equiv \left[\int_{\omega\in\Omega_{is}} p_{is}(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is the price index for country i and sector s. Substituting this consumer demand function back into the CES utility function yields

$$C_{is} = \left[\int_{\omega\in\Omega_{is}} q_{is}(\omega)^{\rho}\, d\omega\right]^{1/\rho} = \left[\int_{\omega\in\Omega_{is}} \frac{p_{is}(\omega)^{-\sigma\rho} E_{is}^{\rho}}{P_{is}^{(1-\sigma)\rho}}\, d\omega\right]^{1/\rho} = \frac{E_{is}}{P_{is}^{1-\sigma}} \left[\int_{\omega\in\Omega_{is}} p_{is}(\omega)^{-\sigma\rho}\, d\omega\right]^{1/\rho}.$$

Taking into account that $-\sigma\rho = \frac{-\rho}{1-\rho} = 1-\sigma$, the CES utility can be simplified further to

$$C_{is} = \frac{E_{is}}{P_{is}^{1-\sigma}} \left[\int_{\omega\in\Omega_{is}} p_{is}(\omega)^{1-\sigma}\, d\omega\right]^{1/\rho} = \frac{E_{is}}{P_{is}^{1-\sigma}} \left[P_{is}^{1-\sigma}\right]^{1/\rho} = \frac{E_{is}}{P_{is}^{1-\sigma}} P_{is}^{-\sigma} = \frac{E_{is}}{P_{is}}.$$

Thus, we can write the across-sector consumer optimization problem as

$$\max_{E_{iA},E_{iB}} U_i \equiv C_{iA}^{\alpha_A} C_{iB}^{\alpha_B} = \left(\frac{E_{iA}}{P_{iA}}\right)^{\alpha_A} \left(\frac{E_{iB}}{P_{iB}}\right)^{\alpha_B} \qquad \text{s.t.} \qquad E_{iA} + E_{iB} = E_i$$

where E_i is consumer expenditure on products in both sectors combined. The solution to this problem is $E_{iA} = \alpha_A E_i$ and $E_{iB} = \alpha_B E_i$.

In country i, workers earn the wage rate w_i and total labor supply is L_i, so total wage income that can be spent on products produced in both sectors is $w_i L_i$. Given free entry, there are no profits earned from entering markets, so consumers spend exactly what they earn in wage income. It follows that

$$E_{is} = \alpha_s w_i L_i. \qquad (A.2)$$

Firms

Given (A.1) and (A.2), a firm with productivity φ from country i earns revenue $r_{ijs}(\varphi)$ from selling to country j in sector s, where

$$r_{ijs}(\varphi) = p_{ijs}(\varphi) \cdot q_{ijs}(\varphi) = p_{ijs}(\varphi) \cdot \frac{p_{ijs}(\omega)^{-\sigma} E_{js}}{P_{js}^{1-\sigma}} = \alpha_s w_j L_j \left(\frac{p_{ijs}(\varphi)}{P_{js}}\right)^{1-\sigma}. \qquad (2a)$$

This firm earns gross profits $\pi_{ijs}(\varphi)$ from selling to country j in sector s (not including fixed costs). It follows that

$$\begin{aligned}\pi_{ijs}(\varphi) &= r_{ijs}(\varphi) - \frac{w_i \tau_{ijs}}{\varphi} q_{ijs}(\varphi) \\ &= \frac{\alpha_s w_j L_j \, p_{ijs}(\varphi)^{1-\sigma}}{P_{js}^{1-\sigma}} - \frac{w_i \tau_{ijs}}{\varphi} \frac{\alpha_s w_j L_j \, p_{ijs}(\varphi)^{-\sigma}}{P_{js}^{1-\sigma}}.\end{aligned}$$

We obtain the price that maximizes gross profits by solving the first order condition

$$\begin{aligned}\frac{\partial \pi_{ijs}(\varphi)}{\partial p_{ijs}(\varphi)} &= \frac{(1-\sigma)\alpha_s w_j L_j \, p_{ijs}(\varphi)^{-\sigma}}{P_{js}^{1-\sigma}} + \frac{w_i \tau_{ijs} \alpha_s w_j L_j \sigma p_{ijs}(\varphi)^{-\sigma-1}}{\varphi P_{js}^{1-\sigma}} \\ &= \frac{\alpha_s w_j L_j \, p_{ijs}(\varphi)^{-\sigma}}{P_{js}^{1-\sigma}} \left[1 - \sigma + \frac{w_i \tau_{ijs} \sigma}{\varphi p_{ijs}(\varphi)}\right] = 0\end{aligned}$$

which yields $\sigma - 1 = \frac{w_i \tau_{ijs} \sigma}{\varphi p_{ijs}(\varphi)}$. Taking into account that $\frac{\sigma}{\sigma-1} = \frac{1}{1-\rho} / \frac{1-(1-\rho)}{1-\rho} = \frac{1}{\rho}$, we obtain the profit-maximizing price

$$p_{ijs}(\varphi) = \frac{w_i \tau_{ijs}}{\rho \varphi}. \tag{2b}$$

Substituting $\rho \, p_{ijs}(\varphi) = w_i \tau_{ijs} / \varphi$ back into gross profits, we obtain

$$\begin{aligned}\pi_{ijs}(\varphi) &= r_{ijs}(\varphi) - \frac{w_i \tau_{ijs}}{\varphi} q_{ijs}(\varphi) \\ &= r_{ijs}(\varphi) - \rho \, p_{ijs}(\varphi) q_{ijs}(\varphi) \\ &= r_{ijs}(\varphi) \left[1 - \rho\right] \\ &= \frac{r_{ijs}(\varphi)}{\sigma}\end{aligned}$$

since $\sigma = \frac{1}{1-\rho}$ implies that $1 - \rho = \frac{1}{\sigma}$. A firm from country i and sector s needs to have a productivity $\varphi \geq \varphi^*_{ijs}$ to justify paying the fixed "marketing" cost $w_i f_{ij}$ of serving the country j market. Thus φ^*_{ijs} is determined by the cut-off productivity condition

$$\frac{r_{ijs}(\varphi^*_{ijs})}{\sigma} = \frac{\alpha_s w_j L_j}{\sigma} \left(\frac{p_{ijs}(\varphi^*_{ijs})}{P_{js}}\right)^{1-\sigma} = w_i f_{ij}. \tag{3}$$

The Price Index

Next we solve for the value of the price index P_{js} for country j and sector s. Given the Pareto distribution function $G(\varphi) \equiv 1 - (b/\varphi)^\theta$, let $g(\varphi) \equiv G'(\varphi) = b^\theta \theta \varphi^{-\theta-1}$ denote the corresponding productivity density function. Let $\mu_{is}(\varphi)$ denote the equilibrium productivity density function for country i and sector s. Since only firms with productivity $\varphi \geq \varphi^*_{iis}$ produce in equilibrium, firm exit is uncorrelated with productivity and $\varphi^*_{iis} < \varphi^*_{ijs}$, the equilibrium productivity density function is given by

$$\mu_{is}(\varphi) \equiv \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*_{iis})} & \text{if } \varphi \geq \varphi^*_{iis} \\ 0 & \text{otherwise.} \end{cases}$$

In deriving this equation, we have used Bayes' rule for calculating conditional probabilities, which states that $P(A|B) = P(A \cap B)/P(B)$.

Using $P_{is} \equiv \left[\int_{\omega \in \Omega_{is}} p_{is}(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, the price index P_{js} for country j and sector s satisfies

$$P_{js}^{1-\sigma} = \int_{\varphi^*_{jjs}}^{\infty} p_{jjs}(\varphi)^{1-\sigma} M_{js} \mu_{js}(\varphi)\, d\varphi + \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} M_{is} \mu_{is}(\varphi)\, d\varphi.$$

It follows that the price index P_{js} satisfies

$$P_{js} = \left[\sum_{i=1,2} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} M_{is} \mu_{is}(\varphi)\, d\varphi\right]^{1/(1-\sigma)}.$$

Comparing Cut-off Productivity Levels

Comparing the cut-off productivity levels of domestic firms and foreign firms in country j, we find that

$$\begin{aligned} \frac{w_i f_{ij}}{w_j f_{jj}} &= \frac{r_{ijs}(\varphi^*_{ijs})/\sigma}{r_{jjs}(\varphi^*_{jjs})/\sigma} \\ &= \frac{\alpha_s w_j L_j \left(p_{ijs}(\varphi^*_{ijs})/P_{js}\right)^{1-\sigma}}{\alpha_s w_j L_j \left(p_{jjs}(\varphi^*_{jjs})/P_{js}\right)^{1-\sigma}} \text{ from (2a)} \\ &= \frac{\left(w_i \tau_{ijs}/\rho\varphi^*_{ijs}\right)^{1-\sigma}}{\left(w_j \tau_{jjs}/\rho\varphi^*_{jjs}\right)^{1-\sigma}} \text{ from (2b)} \\ &= \left(\frac{w_i \tau_{ijs} \varphi^*_{jjs}}{w_j \varphi^*_{ijs}}\right)^{1-\sigma}. \end{aligned}$$

Rearranging terms yields

$$\begin{aligned}\left(\frac{\varphi_{jjs}^*}{\varphi_{ijs}^*}\right)^{1-\sigma} &= \tau_{ijs}^{\sigma-1}\frac{f_{ij}}{f_{jj}}\left(\frac{w_i}{w_j}\right)^{\sigma} \\ \frac{\varphi_{ijs}^*}{\varphi_{jjs}^*} &= \left[\tau_{ijs}^{\sigma-1}\frac{f_{ij}}{f_{jj}}\left(\frac{w_i}{w_j}\right)^{\sigma}\right]^{1/(\sigma-1)}\end{aligned}$$

and it follows that

$$\varphi_{ijs}^* = \tau_{ijs}\left(\frac{f_{ij}}{f_{jj}}\right)^{1/(\sigma-1)}\left(\frac{w_i}{w_j}\right)^{1/\rho}\varphi_{jjs}^*. \tag{5}$$

The Market Access Index

In each time period, there is free entry by firms in each sector s and country i. Let $\bar{\pi}_{is}$ denote the average profits across all domestic firms in country i and sector s (including the fixed marketing costs). Let $\bar{v}_{is} \equiv \sum_{t=0}^{\infty}(1-\delta)^t\bar{\pi}_{is} = \bar{\pi}_{is}/\delta$ denote the present value of average profit flows in country i and sector s, taking into account the rate δ at which firms exit in each time period. The average profits across all domestic firms (exporters and non-exporters) is given by

$$\begin{aligned}\bar{\pi}_{is} &= \frac{1}{M_{is}}\left\{\int_{\varphi_{iis}^*}^{\infty}[\pi_{iis}(\varphi) - w_i f_{ii}]M_{is}\mu_{is}(\varphi)\,d\varphi + \int_{\varphi_{ijs}^*}^{\infty}[\pi_{ijs}(\varphi) - w_i f_{ij}]M_{is}\mu_{is}(\varphi)\,d\varphi\right\} \\ &= \int_{\varphi_{iis}^*}^{\infty}\left[\frac{r_{iis}(\varphi)}{\sigma} - w_i f_{ii}\right]\frac{g(\varphi)}{1-G(\varphi_{iis}^*)}\,d\varphi + \int_{\varphi_{ijs}^*}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij}\right]\frac{g(\varphi)}{1-G(\varphi_{iis}^*)}\,d\varphi\end{aligned}$$

and rearranging yields

$$[1-G(\varphi_{iis}^*)]\,\bar{\pi}_{is} = \int_{\varphi_{iis}^*}^{\infty}\left[\frac{r_{iis}(\varphi)}{\sigma} - w_i f_{ii}\right]g(\varphi)\,d\varphi + \int_{\varphi_{ijs}^*}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij}\right]g(\varphi)\,d\varphi.$$

To evaluate the integrals, next note that from (2a) and (2b),

$$\frac{r_{ijs}(\varphi)}{r_{ijs}(\varphi_{ijs}^*)} = \frac{(\alpha_s w_j L_j)\,p_{ijs}(\varphi)^{1-\sigma}/P_{js}^{1-\sigma}}{(\alpha_s w_j L_j)\,p_{ijs}(\varphi_{ijs}^*)^{1-\sigma}/P_{js}^{1-\sigma}} = \left(\frac{p_{ijs}(\varphi)}{p_{ijs}(\varphi_{ijs}^*)}\right)^{1-\sigma} = \left(\frac{w_i\tau_{ijs}}{\rho\varphi}\frac{\rho\varphi_{ijs}^*}{w_i\tau_{ijs}}\right)^{1-\sigma} = \left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1}.$$

Using the cut-off productivity condition, it follows that

$$\frac{r_{ijs}(\varphi)}{\sigma} = \frac{r_{ijs}(\varphi_{ijs}^*)}{\sigma}\left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1} = \frac{\sigma w_i f_{ij}}{\sigma}\left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1} = w_i f_{ij}\left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1} \tag{A.3}$$

and

$$\int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij}\right] g(\varphi)\, d\varphi = \int_{\varphi^*_{ijs}}^{\infty} \left[w_i f_{ij} \left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma-1} - w_i f_{ij}\right] g(\varphi)\, d\varphi$$

$$= w_i f_{ij} \int_{\varphi^*_{ijs}}^{\infty} \left[\left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma-1} - 1\right] g(\varphi)\, d\varphi$$

$$= w_i f_{ij} J(\varphi^*_{ijs}), \tag{A.4}$$

where the function $J(\cdot)$ is given by

$$\begin{aligned} J(x) &\equiv \int_x^{\infty} \left[\left(\frac{\varphi}{x}\right)^{\sigma-1} - 1\right] g(\varphi)\, d\varphi \\ &= \int_x^{\infty} \left(\frac{\varphi}{x}\right)^{\sigma-1} b^{\theta}\theta\varphi^{-\theta-1}\, d\varphi - [1 - G(x)] \\ &= b^{\theta}\theta x^{1-\sigma} \int_x^{\infty} \varphi^{\sigma-1-\theta-1}\, d\varphi - \left(\frac{b}{x}\right)^{\theta} \\ &= b^{\theta}\theta x^{1-\sigma} \frac{x^{\sigma-1-\theta}}{\theta-\sigma+1} - \left(\frac{b}{x}\right)^{\theta} \\ &= \frac{\theta - (\theta-\sigma+1)}{\theta-\sigma+1} \left(\frac{b}{x}\right)^{\theta} \\ &= \frac{\sigma-1}{\theta-\sigma+1} \left(\frac{b}{x}\right)^{\theta}. \end{aligned} \tag{A.5}$$

We assume that $\theta > \sigma - 1$ to guarantee that expected profits are finite. From the previous argument, it also follows that

$$\int_x^{\infty} \left(\frac{\varphi}{x}\right)^{\sigma-1} g(\varphi)\, d\varphi = \eta \left(\frac{b}{x}\right)^{\theta} \qquad \text{where} \qquad \eta \equiv \frac{\theta}{\theta-\sigma+1} > 0. \tag{A.6}$$

The expected profit of an entrant in country i from selling to country j in sector s (after the entrant has paid the entry cost $w_i f_{ise}$) is

$$\frac{[1 - G(\varphi^*_{iis})]}{\delta} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ij}\right] \frac{g(\varphi)}{1 - G(\varphi^*_{iis})}\, d\varphi = \delta^{-1} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij}\right] g(\varphi)\, d\varphi.$$

The expected profit of an entrant in country j from selling to country j in sector s (after the entrant has paid the entry cost $w_j f_{jse}$) is

$$\frac{\left[1-G(\varphi^*_{jjs})\right]}{\delta}\int_{\varphi^*_{jjs}}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma}-w_j f_{jj}\right]\frac{g(\varphi)}{1-G(\varphi^*_{jjs})}\,d\varphi=\delta^{-1}\int_{\varphi^*_{jjs}}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma}-w_j f_{jj}\right]g(\varphi)\,d\varphi.$$

Thus the expected profit of an entrant in country i from selling to country j in sector s relative to that captured by an entrant in country j from selling to country j (or the relative expected profit) is given by

$$\begin{aligned}
\phi_{ijs} &\equiv \frac{\delta^{-1}\int_{\varphi^*_{ijs}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]g(\varphi)\,d\varphi}{\delta^{-1}\int_{\varphi^*_{jjs}}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma}-w_j f_{jj}\right]g(\varphi)\,d\varphi}\\
&= \frac{w_i f_{ij} J(\varphi^*_{ijs})}{w_j f_{jj} J(\varphi^*_{jjs})}\text{ from (A.4)}\\
&= \frac{w_i f_{ij}\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta}}{w_j f_{jj}\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta}}\text{ from (A.5)}\\
&= \frac{w_i f_{ij}}{w_j f_{jj}}\left(\frac{\varphi^*_{jjs}}{\varphi^*_{ijs}}\right)^{\theta}\\
&= \frac{w_i f_{ij}}{w_j f_{jj}}\left[\frac{1}{\tau_{ijs}}\left(\frac{f_{jj}}{f_{ij}}\right)^{1/(\sigma-1)}\left(\frac{w_j}{w_i}\right)^{1/\rho}\right]^{\theta}\text{ from (5)}
\end{aligned}$$

or

$$\phi_{ijs}=\frac{1}{\tau^{\theta}_{ijs}}\left(\frac{f_{jj}}{f_{ij}}\right)^{(\theta-\sigma+1)/(\sigma-1)}\left(\frac{w_j}{w_i}\right)^{(\theta-\rho)/\rho}. \tag{6}$$

Variable ϕ_{ijs} is an index summarizing the degree of country i's market access to country j in sector s.

Note that $\sigma=\frac{1}{1-\rho}$ implies that $\sigma-1=\frac{1}{1-\rho}-\frac{1-\rho}{1-\rho}=\frac{\rho}{1-\rho}$ and thus the assumption $\theta>\sigma-1$ implies that $\theta>\frac{\rho}{1-\rho}$. Rearranging yields $\theta-\rho\theta>\rho$ or $\theta-\rho>\theta\rho>0$.

The Domestic Productivity Cutoff

From firm's pricing (2) and the cutoff condition (5), we obtain

$$\begin{aligned}
\frac{p_{ijs}(\varphi^*_{ijs})}{p_{jjs}(\varphi^*_{jjs})} &= \frac{w_i\tau_{ijs}}{w_j}\left(\frac{\varphi^*_{jjs}}{\varphi^*_{ijs}}\right)\\
&= \frac{w_i\tau_{ijs}}{w_j}\left(\frac{1}{\tau_{ijs}}\left(\frac{f_{ij}}{f_{jj}}\right)^{1/(1-\sigma)}\left(\frac{w_i}{w_j}\right)^{-1/\rho}\right)\\
&= \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right)^{1/(1-\sigma)}
\end{aligned}$$

since $1-\sigma = \frac{-\rho}{1-\rho}$ implies that $\frac{1}{1-\sigma} = \frac{\rho-1}{\rho}$. Using this result, we can evaluate the price integral

$$\begin{aligned}
\int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} g(\varphi)\, d\varphi &= \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi^*_{ijs})^{1-\sigma} \left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma-1} g(\varphi)\, d\varphi \\
&= p_{ijs}(\varphi^*_{ijs})^{1-\sigma} \int_{\varphi^*_{ijs}}^{\infty} \left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma-1} g(\varphi)\, d\varphi \\
&= p_{jjs}\left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right) \eta \left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta} \\
&= \eta\, p_{jjs}\left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right) \left(\frac{b}{\tau_{ijs}\left(f_{ij}/f_{jj}\right)^{1/(\sigma-1)} \left(w_i/w_j\right)^{1/\rho} \varphi^*_{jjs}}\right)^{\theta} \\
&= \eta\, p_{jjs}\left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\tau_{ijs}^{-\theta} \left(\frac{f_{ij}}{f_{jj}}\right)^{-(\theta-\sigma+1)/(\sigma-1)} \left(\frac{w_j}{w_i}\right)^{(\theta-\rho)/\rho}\right) \left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta} \\
&= \eta\, p_{jjs}\left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta} \phi_{ijs} \qquad \text{(A.7)}
\end{aligned}$$

Substituting this back into the price index, we obtain

$$\begin{aligned}
P_{js}^{1-\sigma} &= \sum_{i=1,2} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma}\, M_{is}\mu_{is}(\varphi) d\varphi \\
&= \sum_{i=1,2} \frac{M_{ise}}{\delta} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} g(\varphi)\, d\varphi \\
&= \eta\, p_{jjs}\left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta} \sum_{k=1,2} \frac{M_{kse}}{\delta} \phi_{kjs}.
\end{aligned}$$

Changing indexes and noting that $\phi_{iis} = 1$ yields

$$P_{is}^{1-\sigma} = \eta\, p_{iis}\left(\varphi^*_{iis}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{iis}}\right)^{\theta} \left(\frac{M_{ise}}{\delta} + \phi_{jis}\frac{M_{jse}}{\delta}\right). \qquad (7)$$

In the special case of autarky ($\phi_{jis} = 0$), this equation simplifies to

$$\begin{aligned}
P_{is}^{1-\sigma} &= \eta\, p_{iis}\left(\varphi^*_{iis}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{iis}}\right)^{\theta} \frac{M_{ise}}{\delta} \\
&= \eta\, p_{iis}\left(\varphi^*_{iis}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{iis}}\right)^{\theta} \frac{M_{is}}{1-G(\varphi^*_{iis})} \\
&= \eta\, p_{iis}\left(\varphi^*_{iis}\right)^{1-\sigma} M_{is}.
\end{aligned}$$

Using these results, the cutoff condition (3) for country 1 can be written as

$$\begin{aligned}
\frac{r_{11s}(\varphi_{11s}^{*})}{\sigma} &= w_1 f_d \\
\frac{\alpha_s w_1 L_1}{\sigma}\left(\frac{p_{11s}(\varphi_{11s}^{*})}{P_{1s}}\right)^{1-\sigma} &= w_1 f_d \\
\frac{\alpha_s L_1}{\sigma}\left[(\eta/\delta)\,(b/\varphi_{11s}^{*})^{\theta}\,(M_{1se}+\phi_{21s}M_{2se})\right]^{-1} &= f_d.
\end{aligned}$$

Rearranging terms then yields

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\delta\,(\theta-\sigma+1)}\frac{\sigma f_d}{\alpha_s L_1}\,(M_{1se}+\phi_{21s}M_{2se})\,. \tag{8}$$

Free Entry

Free entry implies that the probability of successful entry times the expected profits earned from successful entry must equal the cost of entry, that is, $\text{Prob.}(\varphi \geq \varphi_{iis}^{*})\bar{v}_{is} = w_i f_{ise}$ or $[1-G(\varphi_{iis}^{*})]\bar{\pi}_{is}/\delta = w_i f_{ise}$. It follows that

$$[1-G(\varphi_{iis}^{*})]\,\bar{\pi}_{is} = \int_{\varphi_{iis}^{*}}^{\infty}\left[\frac{r_{iis}(\varphi)}{\sigma}-w_i f_{ii}\right]g(\varphi)\,d\varphi + \int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]g(\varphi)\,d\varphi = \delta w_i f_{ise}.$$

Thus we obtain

$$\frac{1}{\delta}\sum_{j=1,2}\int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]g(\varphi)\,d\varphi = w_i f_{ise}. \tag{9}$$

Making substitutions and rearranging terms, it follows that

$$\begin{aligned}
\sum_{j=1,2}\int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]g(\varphi)\,d\varphi &= \delta w_i f_{ise} \\
\sum_{j=1,2} w_i f_{ij} J(\varphi_{ijs}^{*}) &= \delta w_i f_{ise} \quad \text{from (A.4)} \\
\sum_{j=1,2} f_{ij} J(\varphi_{ijs}^{*}) &= \delta f_{ise} \\
\sum_{j=1,2} f_{ij}\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta} &= \delta f_{ise} \quad \text{from (A.5)}
\end{aligned}$$

and rearranging yields the free entry condition

$$\frac{1}{\delta}\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\sum_{j=1,2} f_{ij}\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta} = f_{ise}. \tag{10}$$

Labor Demand

We use a three step argument to solve for labor demand.

First, we show that the fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s.

$$\begin{aligned}
w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty} f_{ij}M_{is}\mu_{is}(\varphi)\,d\varphi\right) &= w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty} f_{ij}\frac{M_{ise}}{\delta}g(\varphi)\,d\varphi\right) \text{ from (4)}\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\sum_{j=1,2}f_{ij}[1-G(\varphi^*_{ijs})]\right)\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\sum_{j=1,2}f_{ij}\left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta}\right)\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\delta f_{ise}\left(\frac{\theta-\sigma+1}{\sigma-1}\right)\right) \text{ from (10)}\\
&= w_iM_{ise}f_{ise}\left(\frac{\sigma-1+\theta-\sigma+1}{\sigma-1}\right)
\end{aligned}$$

from which it follows that

$$w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty} f_{ij}M_{is}\mu_{is}(\varphi)\,d\varphi\right)=w_iM_{ise}\left(\frac{\theta f_{ise}}{\sigma-1}\right). \tag{11}$$

Second, we show that the fixed costs are equal to the gross profits in each country i and sector s.

From the free entry condition (9), we obtain

$$
\begin{aligned}
\delta w_i f_{ise} &= \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij} \right] g(\varphi)\, d\varphi \\
w_i \left(\delta f_{ise} + \sum_{j=1,2} f_{ij}[1 - G(\varphi^*_{ijs})] \right) &= \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \frac{r_{ijs}(\varphi)}{\sigma} g(\varphi)\, d\varphi \\
w_i \left(M_{ise} f_{ise} + \frac{M_{ise}}{\delta} \sum_{j=1,2} f_{ij}[1 - G(\varphi^*_{ijs})] \right) &= \frac{M_{ise}}{\delta} \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \frac{r_{ijs}(\varphi)}{\sigma} g(\varphi)\, d\varphi \\
w_i M_{ise} \left(\frac{\theta f_{ise}}{\sigma - 1} \right) &= \frac{M_{is}}{1 - G(\varphi^*_{iis})} \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \frac{r_{ijs}(\varphi)}{\sigma} g(\varphi)\, d\varphi \ \text{ from (11)} \\
&= \frac{1}{\sigma} \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) d\varphi \\
&= \frac{1}{\sigma} \sum_{j=1,2} R_{ijs} \qquad (12)
\end{aligned}
$$

where $R_{ijs} \equiv \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) d\varphi$ is the total revenue associated with shipments from country i to country j in sector s.

Third, we show that the wage payments to labor equals the total revenue in each country i and sector s. Let L_{is} denote labor demand by all firms in country i and sector s. Firms use labor for market entry, for the production of goods sold to domestic consumers and for the production of goods sold to foreign consumers. Taking into account both the marginal and fixed costs of production, we obtain

$$
\begin{aligned}
w_i L_{is} &= w_i M_{ise} f_{ise} + w_i \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \left[f_{ij} + q_{ijs}(\varphi) \frac{\tau_{ijs}}{\varphi} \right] M_{is} \mu_{is}(\varphi)\, d\varphi \\
&= w_i \left(M_{ise} f_{ise} + \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} f_{ij} M_{is} \mu_{is}(\varphi)\, d\varphi \right) + \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} q_{ijs}(\varphi) \frac{w_i \tau_{ijs}}{\rho \varphi} \rho M_{is} \mu_{is}(\varphi)\, d\varphi \\
&= w_i M_{ise} \left(\frac{\theta f_{ise}}{\sigma - 1} \right) + \rho \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi)\, d\varphi \ \text{ from (2) and (11)} \\
&= \frac{1}{\sigma} \sum_{j=1,2} R_{ijs} + \rho \sum_{j=1,2} R_{ijs} \ \text{ from (12)} \\
&= (1 - \rho + \rho) \sum_{j=1,2} R_{ijs} \\
&= \sum_{j=1,2} R_{ijs}. \qquad (A.8)
\end{aligned}
$$

Total Revenue

$$
\begin{aligned}
R_{ijs} &\equiv \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi)\, d\varphi \\
&= \frac{M_{is}}{1-G(\varphi^*_{iis})} \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) g(\varphi)\, d\varphi \\
&= \frac{[1-G(\varphi^*_{iis})]M_{ise}}{\delta[1-G(\varphi^*_{iis})]} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi) q_{ijs}(\varphi) g(\varphi)\, d\varphi \quad \text{from (4)} \\
&= \frac{M_{ise}}{\delta} \int_{\varphi^*_{ijs}}^{\infty} \frac{p_{ijs}(\varphi)^{1-\sigma} \alpha_s w_j L_j}{P_{js}^{1-\sigma}} g(\varphi)\, d\varphi \quad \text{from (2)} \\
&= \frac{\alpha_s w_j L_j}{P_{js}^{1-\sigma}} \frac{M_{ise}}{\delta} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} g(\varphi)\, d\varphi \\
&= \alpha_s w_j L_j \frac{M_{ise}}{\delta} \frac{\eta\, p_{jjs} \left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta} \phi_{ijs}}{\eta\, p_{jjs} \left(\varphi^*_{jjs}\right)^{1-\sigma} \left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta} \sum_{k=1,2} \frac{M_{kse}}{\delta} \phi_{kjs}} \quad \text{from (A.7) and (7)} \\
&= \alpha_s w_j L_j \left(\frac{M_{ise} \phi_{ijs}}{\sum_{k=1,2} M_{kse} \phi_{kjs}} \right). \qquad (13)
\end{aligned}
$$

The Labor Market Equilibrium

Equations (A.8) and (12) imply that

$$
L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = \frac{1}{w_i} w_i M_{ise} \left(\frac{\sigma\theta}{\sigma-1} \right) f_{ise} = M_{ise} \left(\frac{\theta f_{ise}}{\rho} \right) \qquad (15a)
$$

and it immediately follows from (1) that

$$
L_{is} = M_{ise}^{1+\zeta} \left(\frac{\theta F}{\rho} \right). \qquad (15b)
$$

Notice that labor demand L_{is} depends only on the mass of entrants M_{ise} and not on any cut-off productivity levels φ^*_{ijs}. The country 1 labor supply is given by L_1 so the requirement that labor supply equals labor demand

$$
L_1 = \sum_{s=A,B} L_{1s} = \left(\frac{\theta F}{\rho} \right) \sum_{s=A,B} M_{1se} \left(\tau_{12s}, \tau_{21s}, w_1 \right)^{1+\zeta} \qquad (16)
$$

determines the equilibrium wage rate w_1 given the trade costs (τ_{12s}, τ_{21s}).

Industrial Productivity

The first measure of industrial productivity is industrial labor productivity:

$$\Phi_{1s}^{L} \equiv \frac{\sum_{j=1,2} R_{1js}}{\tilde{P}_{1s} L_{1s}} \text{ where } \tilde{P}_{1s} = \int_{\varphi_{11s}^{*}}^{\infty} p_{11s}(\varphi)\, \mu_{1s}(\varphi) d\varphi.$$

From $w_1 L_{1s} = \sum_{j=1,2} R_{1js}$ and

$$\begin{aligned}
\tilde{P}_{1s} &= \int_{\varphi_{11s}^{*}}^{\infty} \left(\frac{w_1}{\rho\varphi}\right) \frac{g(\varphi)}{1 - G(\varphi_{11s}^{*})} d\varphi \\
&= \frac{w_1}{\rho \left(b/\varphi_{11s}^{*}\right)^{\theta}} \int_{\varphi_{11s}^{*}}^{\infty} \frac{\theta b^{\theta}}{\varphi^{\theta+2}} d\varphi \\
&= \frac{w_1 \theta \varphi_{11s}^{*\theta}}{\rho} \left[\frac{-\varphi_{11s}^{*-(\theta+2)+1}}{-(\theta+2)+1}\right] \\
&= \frac{w_1}{\rho \varphi_{11s}^{*}} \left(\frac{\theta}{\theta+1}\right),
\end{aligned}$$

industrial labor productivity becomes

$$\Phi_{1s}^{L} \equiv \frac{\sum_{j=1,2} R_{1js}}{\tilde{P}_{1s} L_{1s}} = \frac{w_1}{\tilde{P}_{1s}} = w_1 / \left[\frac{w_1}{\rho\varphi_{11s}^{*}} \left(\frac{\theta}{\theta+1}\right)\right]$$

or

$$\Phi_{1s}^{L} = \left(\frac{\theta+1}{\theta}\right) \rho \varphi_{11s}^{*}. \tag{17a}$$

The second measure of industrial productivity is industrial labor productivity calculated using a theoretically consistent "exact" price index:

$$\Phi_{1s}^{W} \equiv \frac{\sum_{j=1,2} R_{1js}}{P_{1s} L_{1s}}.$$

Starting from the cut-off productivity condition (3),

$$\begin{aligned}
\frac{r_{11s}(\varphi^*_{11s})}{\sigma} &= w_1 f_{11} \\
\alpha_s w_1 L_1 \frac{p_{11s}(\varphi^*_{11s})^{1-\sigma}}{P_{1s}^{1-\sigma}} &= \sigma w_1 f_{11} \text{ from (2)} \\
\alpha_s w_1 L_1 \left(\frac{w_1 \tau_{11s}}{\rho\varphi^*_{11s} P_{1s}}\right)^{1-\sigma} &= \sigma w_1 f_{11} \text{ from (2)} \\
\left(\frac{w_1}{P_{1s}}\right)^{1-\sigma} &= \frac{\sigma f_{11}}{\alpha_s L_1}(\rho\varphi^*_{11s})^{1-\sigma} \\
\frac{w_1}{P_{1s}} &= \left(\frac{\sigma f_{11}}{\alpha_s L_1}\right)^{1/(1-\sigma)} \rho\varphi^*_{11s}
\end{aligned}$$

and then using $w_1 L_{1s} = \sum_{j=1,2} R_{1js}$, we obtain

$$\Phi^W_{1s} \equiv \frac{\sum_{j=1,2} R_{1js}}{P_{1s}L_{1s}} = \frac{w_1}{P_{1s}} = \left(\frac{\alpha_s L_1}{\sigma f_{11}}\right)^{1/(\sigma-1)} \rho\varphi^*_{11s}. \tag{17b}$$

Finally, we derive the welfare formula for the representative consumer in country 1 who supplies one unit of labor. Since her income is w_1, her aggregate consumption over varieties in sector s is

$$C_{1s} = \frac{\alpha_s w_1}{P_{1s}}.$$

From the utility function U_1 and $\Phi^W_{1s} = w_1/P_{1s}$, her utility can be written as:

$$\begin{aligned}
U_1 &= \left(\frac{\alpha_A w_1}{P_{1A}}\right)^{\alpha_A} \left(\frac{\alpha_B w_1}{P_{1B}}\right)^{\alpha_B} \\
&= \left(\alpha_A \Phi^W_{1A}\right)^{\alpha_A} \left(\alpha_B \Phi^W_{1B}\right)^{\alpha_B}.
\end{aligned}$$

The effects of a small change in trade costs

We now compute the effects of a small change in trade costs τ_{ijs}. We assume that countries and sectors are initially symmetric before trade liberalization with one exception: we allow the fraction α_A of consumer expenditure on sector A products to differ from the fraction α_B of consumer expenditure on sector B products. Thus, the derivatives that we calculate are evaluated at a "symmetric" equilibrium where $M_{1se} = M_{2se}$ and $\phi_{ijs} = \phi$ hold. The market access index ϕ takes a value between 0 (autarky) and 1 (free trade).

Starting with the equation $\phi_{ijs} = \tau_{ijs}^{-\theta}\left(\frac{f_{jj}}{f_{ij}}\right)^{(\theta-\sigma+1)/(\sigma-1)}\left(\frac{w_j}{w_i}\right)^{(\theta-\rho)/\rho}$, taking logs of both sides and then totally differentiating yields

$$\begin{aligned} d\ln\phi_{12s} &= -\theta\, d\ln\tau_{12s} - \left(\frac{\theta}{\rho}-1\right) d\ln w_1 \\ d\ln\phi_{21s} &= -\theta\, d\ln\tau_{21s} + \left(\frac{\theta}{\rho}-1\right) d\ln w_1. \end{aligned} \tag{18}$$

Since $\phi_{ijs} = \tau_{ijs}^{-\theta}\left(\frac{f_{jj}}{f_{ij}}\right)^{(\theta-\sigma+1)/(\sigma-1)}\left(\frac{w_j}{w_i}\right)^{(\theta-\rho)/\rho}$ implies that $\phi_{iis}=1$, equations (14) for $i=1,2$

$$\sum_{j=1,2}\alpha_s w_j L_j\left(\frac{\phi_{ijs}}{\sum_{k=1,2}M_{kse}\phi_{kjs}}\right) = w_i f_{ise}\left(\frac{\theta}{\rho}\right)$$

can be written out as

$$\begin{aligned} \frac{\alpha_s w_1 L_1}{M_{1se}+M_{2se}\phi_{21s}} + \frac{\alpha_s L_2}{M_{1se}\phi_{12s}+M_{2se}}\phi_{12s} &= \left(\frac{\theta}{\rho}\right) w_1 f_{1se} \\ \frac{\alpha_s w_1 L_1}{M_{1se}+M_{2se}\phi_{21s}}\phi_{21s} + \frac{\alpha_s L_2}{M_{1se}\phi_{12s}+M_{2se}} &= \left(\frac{\theta}{\rho}\right) f_{2se}. \end{aligned}$$

Written in matrix form, this systems of linear equations become

$$\begin{pmatrix} 1 & \phi_{12s} \\ \phi_{21s} & 1 \end{pmatrix}\begin{pmatrix} \alpha_s w_1 L_1/\left(M_{1se}+M_{2se}\phi_{21s}\right) \\ \alpha_s L_2/\left(M_{1se}\phi_{12s}+M_{2se}\right)\end{pmatrix} = \left(\frac{\theta}{\rho}\right)\begin{pmatrix} w_1 f_{1se} \\ f_{2se}\end{pmatrix}.$$

Solving using Cramer's Rule yields

$$\begin{aligned} \frac{\alpha_s w_1 L_1}{M_{1se}+M_{2se}\phi_{21s}} &= \frac{\theta}{\rho}\left(\frac{w_1 f_{1se}-\phi_{12s}f_{2se}}{1-\phi_{12s}\phi_{21s}}\right) \\ \frac{\alpha_s L_2}{M_{1se}\phi_{12s}+M_{2se}} &= \frac{\theta}{\rho}\left(\frac{f_{2se}-\phi_{21s}w_1 f_{1se}}{1-\phi_{12s}\phi_{21s}}\right) \end{aligned}$$

where

$$1-\phi_{12s}\phi_{21s} = 1-\left(\tau_{12s}\tau_{21s}\right)^{-\theta}\left(\frac{f_x}{f_d}\right)^{-2(\theta-\sigma+1)/(\sigma-1)} > 0$$

since $\tau_{12s}\tau_{21s} > 1$, $f_x > f_d$, and $\theta-\sigma+1 > 0$. For these equations to make sense, we need

$$\frac{1}{\phi_{12s}} > \frac{f_{2se}}{w_1 f_{1se}} > \phi_{21s},$$

which is satisfied in the current case of symmetric countries and sectors. The above equations can be written as

$$\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)(M_{1se}+M_{2se}\phi_{21s})=\frac{\rho\alpha_s L_1}{\theta}(1-\phi_{12s}\phi_{21s})$$
$$(f_{2se}-\phi_{21s}w_1f_{1se})(M_{1se}\phi_{12s}+M_{2se})=\frac{\rho\alpha_s L_2}{\theta}(1-\phi_{12s}\phi_{21s})\,. \tag{A.9}$$

Taking logs of both sides and then totally differentiating these equations leads to

$$d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)+d\ln(M_{1se}+M_{2se}\phi_{21s})=d\ln(1-\phi_{12s}\phi_{21s})$$
$$d\ln(f_{2se}-\phi_{21s}w_1f_{1se})+d\ln(M_{1se}\phi_{12s}+M_{2se})=d\ln(1-\phi_{12s}\phi_{21s})\,. \tag{A.10}$$

Since countries and sectors are symmetric before trade liberalization, it follows that $\phi_{ijs}=\phi$, $w_1=1$, $M_{1se}=M_{2se}$ and $f_{1se}=f_{2se}$. Using this symmetry and (18), the terms in (A.10) are obtained as follows:

$$\begin{aligned}d\ln(1-\phi_{12s}\phi_{21s})&=\frac{1}{1-\phi_{12s}\phi_{21s}}(-\phi_{12s}d\phi_{21s}-\phi_{21s}d\phi_{12s})\\&=-\frac{\phi_{12s}\phi_{21s}}{1-\phi_{12s}\phi_{21s}}(d\ln\phi_{12s}+d\ln\phi_{21s})\\&=\frac{\phi^2\theta}{1-\phi^2}(d\ln\tau_{12s}+d\ln\tau_{21s})\,,\end{aligned}$$

$$\begin{aligned}d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)&=\frac{f_{1se}}{f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}}d\ln f_{1se}-\frac{\frac{\phi_{12s}}{w_1}f_{2se}}{f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}}(d\ln f_{2se}+d\ln\phi_{12s}-d\ln w_1)\\&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}(d\ln f_{2se}+d\ln\phi_{12s}-d\ln w_1)\\&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}\left(d\ln f_{2se}-\theta\,d\ln\tau_{12s}-\frac{\theta}{\rho}d\ln w_1\right)\\&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}d\ln f_{2se}+\frac{\phi\theta}{1-\phi}d\ln\tau_{12s}+\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1,\end{aligned}$$

$$
\begin{aligned}
d\ln\left(M_{1se}+M_{2se}\phi_{21s}\right) &= \frac{M_{1se}}{M_{1se}+M_{2se}\phi_{21s}}d\ln M_{1se}+\frac{M_{2se}\phi_{21s}}{M_{1se}+M_{2se}\phi_{21s}}\left(d\ln M_{2se}+d\ln\phi_{21s}\right)\\
&= \frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}\left(d\ln M_{2se}+d\ln\phi_{21s}\right),\\
&= \frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}\left(d\ln M_{2se}-\theta d\ln\tau_{21s}+\left(\frac{\theta}{\rho}-1\right)d\ln w_1\right)\\
&= \frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}d\ln M_{2se}-\frac{\phi\theta}{1+\phi}d\ln\tau_{21s}+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1,
\end{aligned}
$$

$$
\begin{aligned}
d\ln\left(f_{2se}-\phi_{21s}w_1f_{1se}\right) &= \frac{f_{2se}}{f_{2se}-\phi_{21s}w_1f_{1se}}d\ln f_{2se}-\frac{\phi_{21s}w_1f_{1se}}{f_{2se}-\phi_{21s}w_1f_{1se}}\left(d\ln\phi_{21s}+d\ln w_1+d\ln f_{1se}\right)\\
&= \frac{1}{1-\phi}d\ln f_{2se}-\frac{\phi}{1-\phi}\left(-\theta\, d\ln\tau_{21s}+\left(\frac{\theta}{\rho}-1\right)d\ln w_1+d\ln w_1+d\ln f_{1se}\right)\\
&= \frac{1}{1-\phi}d\ln f_{2se}-\frac{\phi}{1-\phi}d\ln f_{1se}+\frac{\phi\theta}{1-\phi}d\ln\tau_{21s}-\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1,
\end{aligned}
$$

$$
\begin{aligned}
d\ln\left(M_{1se}\phi_{12s}+M_{2se}\right) &= \frac{M_{1se}\phi_{12s}}{M_{1se}\phi_{12s}+M_{2se}}\left(d\ln M_{1se}+d\ln\phi_{12s}\right)+\frac{M_{2se}}{M_{1se}\phi_{12s}+M_{2se}}d\ln M_{2se}\\
&= \frac{\phi}{1+\phi}\left(d\ln M_{1se}-\theta\, d\ln\tau_{12s}-\left(\frac{\theta}{\rho}-1\right)d\ln w_1\right)+\frac{1}{1+\phi}d\ln M_{2se}\\
&= \frac{1}{1+\phi}d\ln M_{2se}+\frac{\phi}{1+\phi}d\ln M_{1se}-\frac{\phi\theta}{1+\phi}d\ln\tau_{12s}-\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1.
\end{aligned}
$$

Now substituting into the equation

$$
d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)+d\ln\left(M_{1se}+M_{2se}\phi_{21s}\right)=d\ln\left(1-\phi_{12s}\phi_{21s}\right),
$$

we obtain

$$
\begin{aligned}
&\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}d\ln f_{2se}+\frac{\phi\theta}{1-\phi}d\ln\tau_{12s}+\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1\\
&+\frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}d\ln M_{2se}-\frac{\phi\theta}{1+\phi}d\ln\tau_{21s}+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1\\
&\qquad=\frac{\phi^2\theta}{1-\phi^2}\left(d\ln\tau_{12s}+d\ln\tau_{21s}\right)
\end{aligned}
$$

and rearranging terms yields

$$\begin{aligned}\frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}d\ln M_{2se}=&-\left(\frac{\phi\theta}{1-\phi}-\frac{\phi^2\theta}{1-\phi^2}\right)d\ln\tau_{12s}+\left(\frac{\phi\theta}{1+\phi}+\frac{\phi^2\theta}{1-\phi^2}\right)d\ln\tau_{21s}\\&-\left[\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right]d\ln w_1\\&-\frac{1}{1-\phi}d\ln f_{1se}+\frac{\phi}{1-\phi}d\ln f_{2se}.\end{aligned}$$

This equation can be written more compactly as

$$\lambda_d\,d\ln M_{1se}+\lambda_f\,d\ln M_{2se}=-\nu_\tau\,d\ln\tau_{12s}+\nu_\tau\,d\ln\tau_{21s}-\nu_w\,d\ln w_1-\nu_d\,d\ln f_{1se}+\nu_f\,d\ln f_{2se}$$

where $\lambda_d\equiv 1/(1+\phi)$, $\lambda_f\equiv\phi/(1+\phi)$, $\nu_d\equiv 1/(1-\phi)$, $\nu_f\equiv\phi/(1-\phi)$

$$\nu_\tau\equiv\frac{\phi\theta}{1-\phi}-\frac{\phi^2\theta}{1-\phi^2}=\frac{\phi\theta(1+\phi)-\phi^2\theta}{(1-\phi)(1+\phi)}=\frac{\phi\theta}{1-\phi^2}=\frac{\phi\theta(1-\phi)+\phi^2\theta}{(1-\phi)(1+\phi)}=\frac{\phi\theta}{1+\phi}+\frac{\phi^2\theta}{1-\phi^2}$$

and

$$\nu_w\equiv\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)=\frac{\phi(1+\phi)+\phi(1-\phi)}{(1-\phi)(1+\phi)}\frac{\theta}{\rho}-\frac{\phi}{1+\phi}=\frac{\phi}{1+\phi}\left[\frac{2\theta}{\rho\,(1-\phi)}-1\right].$$

Note that $\theta-\rho>\theta\rho>0$ implies that $2\theta>2\rho>\rho(1-\phi)$, so $\frac{2\theta}{\rho(1-\phi)}-1>0$. Next, substituting into the equation

$$d\ln\left(f_{2se}-\phi_{21s}w_1f_{1se}\right)+d\ln\left(M_{1se}\phi_{12s}+M_{2se}\right)=d\ln\left(1-\phi_{12s}\phi_{21s}\right),$$

we obtain

$$\begin{aligned}&\frac{1}{1-\phi}d\ln f_{2se}-\frac{\phi}{1-\phi}d\ln f_{1se}+\frac{\phi\theta}{1-\phi}d\ln\tau_{21s}-\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1\\&+\frac{1}{1+\phi}d\ln M_{2se}+\frac{\phi}{1+\phi}d\ln M_{1se}-\frac{\phi\theta}{1+\phi}d\ln\tau_{12s}-\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1\\&=\frac{\phi^2\theta}{1-\phi^2}\left(d\ln\tau_{12s}+d\ln\tau_{21s}\right)\end{aligned}$$

and rearranging terms yields

$$\begin{aligned}\frac{\phi}{1+\phi} d\ln M_{1se} + \frac{1}{1+\phi} d\ln M_{2se} = & \left(\frac{\phi\theta}{1+\phi} + \frac{\phi^2\theta}{1-\phi^2}\right) d\ln\tau_{12s} - \left(\frac{\phi\theta}{1-\phi} - \frac{\phi^2\theta}{1-\phi^2}\right) d\ln\tau_{21s} \\ & + \left[\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right) + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho} - 1\right)\right] d\ln w_1 \\ & + \frac{\phi}{1-\phi} d\ln f_{1se} - \frac{1}{1-\phi} d\ln f_{2se}.\end{aligned}$$

This equation can be written more compactly as

$$\lambda_f\, d\ln M_{1se} + \lambda_d\, d\ln M_{2se} = \nu_\tau\, d\ln\tau_{12s} - \nu_\tau\, d\ln\tau_{21s} + \nu_w\, d\ln w_1 + \nu_f\, d\ln f_{1se} - \nu_d\, d\ln f_{2se}.$$

The two equations

$$\begin{aligned}\lambda_d\, d\ln M_{1se} + \lambda_f\, d\ln M_{2se} &= -\nu_\tau\, d\ln\tau_{12s} + \nu_\tau\, d\ln\tau_{21s} - \nu_w\, d\ln w_1 - \nu_d\, d\ln f_{1se} + \nu_f\, d\ln f_{2se} \\ \lambda_f\, d\ln M_{1se} + \lambda_d\, d\ln M_{2se} &= \nu_\tau\, d\ln\tau_{12s} - \nu_\tau\, d\ln\tau_{21s} + \nu_w\, d\ln w_1 + \nu_f\, d\ln f_{1se} - \nu_d\, d\ln f_{2se}\end{aligned}$$

can be written in matrix form as:

$$\begin{aligned}\frac{1}{1+\phi}\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix} = & -\frac{\phi\theta}{1-\phi^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{12s} + \frac{\phi\theta}{1-\phi^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{21s} \\ & - \frac{\phi}{1+\phi}\left(\frac{2\theta}{\rho(1-\phi)} - 1\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln w_1 \\ & - \frac{1}{1-\phi}\begin{pmatrix} 1 \\ -\phi \end{pmatrix} d\ln f_{1se} + \frac{1}{1-\phi}\begin{pmatrix} \phi \\ -1 \end{pmatrix} d\ln f_{2se}.\end{aligned}$$

Since

$$(1+\phi)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}^{-1} = \frac{1+\phi}{1-\phi^2}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix},$$

$$(1+\phi)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1+\phi}{1-\phi}\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$(1+\phi)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ -\phi \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -\phi \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1+\phi^2 \\ -2\phi \end{pmatrix},$$

and

$$(1+\phi)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}^{-1}\begin{pmatrix} \phi \\ -1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} \phi \\ -1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 2\phi \\ -(1+\phi^2) \end{pmatrix},$$

we obtain

$$\begin{aligned}
\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix} = & -\frac{\phi\theta}{(1-\phi)^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{12s} + \frac{\phi\theta}{(1-\phi)^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{21s} \\
& - \frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho\,(1-\phi)} - 1\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln w_1 \\
& - \frac{1}{(1-\phi)^2}\begin{pmatrix} 1+\phi^2 \\ -2\phi \end{pmatrix} d\ln f_{1se} + \frac{1}{(1-\phi)^2}\begin{pmatrix} 2\phi \\ -(1+\phi^2) \end{pmatrix} d\ln f_{2se}.
\end{aligned}$$

Defining

$$\iota_\tau \equiv \frac{\phi\theta}{(1-\phi)^2},\ \iota_w \equiv \frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho\,(1-\phi)} - 1\right),\ \iota_1 \equiv \frac{1+\phi^2}{(1-\phi)^2} \text{ and } \iota_2 \equiv \frac{2\phi}{(1-\phi)^2},$$

the system of equations can be written out as

$$\begin{aligned}
d\ln M_{1se} &= \iota_\tau\, d\ln\tau_{21s} - \iota_\tau\, d\ln\tau_{12s} - \iota_w\, d\ln w_1 - \iota_1\, d\ln f_{1se} + \iota_2\, d\ln f_{2se} \\
d\ln M_{2se} &= -\iota_\tau\, d\ln\tau_{21s} + \iota_\tau\, d\ln\tau_{12s} + \iota_w\, d\ln w_1 + \iota_2\, d\ln f_{1se} - \iota_1\, d\ln f_{2se}.
\end{aligned} \tag{19}$$

This system of equations can be further simplified by using $f_{ise} = F \cdot M_{ise}^{\zeta}$. From $d\ln f_{ise} = \zeta\, d\ln M_{ise}$,

$$\begin{pmatrix} 1+\zeta\iota_1 & -\zeta\iota_2 \\ -\zeta\iota_2 & 1+\zeta\iota_1 \end{pmatrix}\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix} = \iota_\tau\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{21s} - \iota_\tau\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln\tau_{12s} - \iota_w\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln w_1.$$

Using the result that $(1+\zeta\iota_1)^2 - (\zeta\iota_2)^2 = 1 + 2\zeta\iota_1 + \zeta^2(\iota_1^2 - \iota_2^2) = 1 + \zeta(\iota_1 - \iota_2) + \zeta(\iota_1 + \iota_2) +$

$\zeta^2(\iota_1-\iota_2)(\iota_1+\iota_2)=[1+\zeta(\iota_1-\iota_2)]\,[1+\zeta(\iota_1+\iota_2)]$,

$$\begin{aligned}\begin{pmatrix}1+\zeta\iota_1 & -\zeta\iota_2\\ -\zeta\iota_2 & 1+\zeta\iota_1\end{pmatrix}^{-1}\begin{pmatrix}1\\-1\end{pmatrix} &= \frac{1}{(1+\zeta\iota_1)^2-(\zeta\iota_2)^2}\begin{pmatrix}1+\zeta\iota_1 & \zeta\iota_2\\ \zeta\iota_2 & 1+\zeta\iota_1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\\ &= \frac{1+\zeta(\iota_1-\iota_2)}{[1+\zeta(\iota_1-\iota_2)]\,[1+\zeta(\iota_1+\iota_2)]}\begin{pmatrix}1\\-1\end{pmatrix}\\ &= \frac{1}{1+\zeta(\iota_1+\iota_2)}\begin{pmatrix}1\\-1\end{pmatrix}\\ &= \frac{1}{1+\zeta(1+2\phi+\phi^2)/(1-\phi)^2}\begin{pmatrix}1\\-1\end{pmatrix}\\ &= \frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\begin{pmatrix}1\\-1\end{pmatrix}.\end{aligned}$$

It follows that

$$\begin{aligned} d\ln M_{1se} &= \varepsilon_\tau\, d\ln\tau_{21s}-\varepsilon_\tau\, d\ln\tau_{12s}-\varepsilon_w\, d\ln w_1\\ d\ln M_{2se} &= -\varepsilon_\tau\, d\ln\tau_{21s}+\varepsilon_\tau\, d\ln\tau_{12s}+\varepsilon_w\, d\ln w_1 \end{aligned}\tag{20}$$

where

$$\varepsilon_\tau \equiv \frac{(1-\phi)^2\iota_\tau}{(1-\phi)^2+\zeta\,(1+\phi)^2}=\frac{(1-\phi)^2\phi\theta/(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}=\frac{\phi\theta}{(1-\phi)^2+\zeta\,(1+\phi)^2}$$

and

$$\varepsilon_w \equiv \frac{(1-\phi)^2\iota_w}{(1-\phi)^2+\zeta\,(1+\phi)^2}=\frac{(1-\phi)^2\frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho(1-\phi)}-1\right)}{(1-\phi)^2+\zeta\,(1+\phi)^2}=\frac{\phi\,[2\theta-\rho\,(1-\phi)]}{\rho\left[(1-\phi)^2+\zeta\,(1+\phi)^2\right]}.$$

The two measures of industrial labor productivity

$$\begin{aligned}\Phi^L_{1s} &\equiv \frac{\sum_{j=1,2}R_{1js}}{\tilde{P}_{1s}L_{1s}}=\left(\frac{\theta+1}{\theta}\right)\rho\varphi^*_{11s}\\ \Phi^W_{1s} &\equiv \frac{\sum_{j=1,2}R_{1js}}{P_{1s}L_{1s}}=\left(\frac{\alpha_s L_1}{\sigma f_{11}}\right)^{1/(\sigma-1)}\rho\varphi^*_{11s}\end{aligned}$$

imply that

$$d\ln\Phi^{k=L,W}_{1s}=d\ln\varphi^*_{11s}.\tag{21a}$$

Taking logs of both sides and then totally differentiating

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\delta(\theta-\sigma+1)} \frac{\sigma f_d}{\alpha_s L_1} \left(M_{1se} + \phi_{21s} M_{2se}\right)$$

yields

$$\begin{aligned}
\theta\, d\ln\varphi_{11s}^{*} &= d\ln\left(M_{1se}+\phi_{21s}M_{2se}\right) && \text{(21b)}\\
&= \frac{1}{1+\phi} d\ln M_{1se} + \frac{\phi}{1+\phi} d\ln M_{2se} - \frac{\phi\theta}{1+\phi} d\ln\tau_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right) d\ln w_1\\
&= \frac{1}{1+\phi}\left[\varepsilon_\tau\, d\ln\tau_{21s} - \varepsilon_\tau\, d\ln\tau_{12s} - \varepsilon_w\, d\ln w_1\right]\\
&\quad + \frac{\phi}{1+\phi}\left[-\varepsilon_\tau\, d\ln\tau_{21s} + \varepsilon_\tau\, d\ln\tau_{12s} + \varepsilon_w\, d\ln w_1\right]\\
&\quad - \frac{\phi\theta}{1+\phi}\, d\ln\tau_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right) d\ln w_1\\
&= \left[\frac{1-\phi}{1+\phi}\,\varepsilon_\tau - \frac{\phi\theta}{1+\phi}\right] d\ln\tau_{21s} - \left[\frac{1-\phi}{1+\phi}\,\varepsilon_\tau\right] d\ln\tau_{12s}\\
&\quad - \left[\frac{1-\phi}{1+\phi}\,\varepsilon_w - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] d\ln w_1.
\end{aligned}$$

Denote

$$\lambda(\zeta) \equiv \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} \in (0,1) \quad \text{and} \quad \beta \equiv \frac{\rho\theta}{2\theta-\rho(1-\phi)} > 0.$$

Then, the above equation becomes

$$d\ln\varphi_{11s}^{*} = \gamma_1\, d\ln\tau_{21s} - \gamma_2\, d\ln\tau_{12s} - \gamma_3\, d\ln w_1 \tag{22}$$

where

$$
\begin{aligned}
\gamma_1 &\equiv \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\,\varepsilon_\tau - \frac{\phi\theta}{1+\phi}\right] \\
&= \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\,\frac{\phi\theta}{(1-\phi)^2+\zeta\,(1+\phi)^2} - \frac{\phi\theta}{1+\phi}\right] \\
&= \frac{\phi}{1+\phi}\left[\frac{(1-\phi)}{(1-\phi)^2+\zeta\,(1+\phi)^2} - 1\right] \\
&= \frac{\phi}{(1+\phi)\,(1-\phi)}\left[\frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2} - (1-\phi)\right] \\
&= \frac{\phi}{1-\phi^2}\left[\phi - \left(1-\frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right)\right] \\
&= \frac{\phi}{1-\phi^2}\left[\phi - \frac{\zeta\,(1+\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right] \\
&= \frac{\phi}{1-\phi^2}\left[\phi-\lambda\,(\zeta)\right],
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2 &\equiv \frac{1-\phi}{\theta\,(1+\phi)}\,\varepsilon_\tau \\
&= \frac{1-\phi}{\theta\,(1+\phi)}\left[\frac{\phi\theta}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right] \\
&= \frac{1-\phi}{(1+\phi)}\left[\frac{\phi}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right] \\
&= \frac{\phi}{(1-\phi)\,(1+\phi)}\left[\frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right] \\
&= \frac{\phi}{1-\phi^2}\left[1-\left(1-\frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right)\right] \\
&= \frac{\phi}{1-\phi^2}\left[1-\frac{\zeta\,(1+\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right] \\
&= \frac{\phi}{1-\phi^2}\left[1-\lambda\,(\zeta)\right] > 0
\end{aligned}
$$

and

$$\begin{aligned}
\gamma_3 &\equiv \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\varepsilon_w - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] \\
&= \frac{1}{\theta(1+\phi)}\left[(1-\phi)\frac{\phi(2\theta-\rho(1-\phi))}{\rho\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} - \phi\left(\frac{\theta-\rho}{\rho}\right)\right] \\
&= \frac{\phi(2\theta-\rho(1-\phi))}{\rho\theta(1+\phi)}\left[\frac{1-\phi}{(1-\phi)^2+\zeta(1+\phi)^2} - \frac{\theta-\rho}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi(2\theta-\rho(1-\phi))}{\rho\theta(1+\phi)(1-\phi)}\left[\frac{(1-\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} - \frac{(\theta-\rho)(1-\phi)}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi}{\beta(1-\phi^2)}\left[1-\frac{\theta-\theta\phi-\rho(1-\phi)}{2\theta-\rho(1-\phi)} - \left(1-\frac{(1-\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2}\right)\right] \\
&= \frac{\phi}{\beta(1-\phi^2)}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2}\right] \\
&= \frac{\phi}{\beta(1-\phi^2)}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \lambda(\zeta)\right].
\end{aligned}$$

Proofs for Theorems 1 and 2 We are ready to determine the sign of γ_1,

$$\begin{aligned}
&\gamma_1 < 0 \\
&\Leftrightarrow \frac{\phi}{1-\phi^2}\left[\phi-\lambda(\zeta)\right] < 0 \\
&\Leftrightarrow \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} > \phi \\
&\Leftrightarrow \zeta(1+\phi)^2 > \phi(1-\phi)^2 + \phi\zeta(1+\phi)^2 \\
&\Leftrightarrow \zeta(1+\phi)^2(1-\phi) > \phi(1-\phi)^2 \\
&\Leftrightarrow \zeta > \zeta_1 \equiv \frac{\phi(1-\phi)}{(1+\phi)^2},
\end{aligned}$$

and the sign of γ_3,

$$\begin{aligned}
&\gamma_3 < 0 \\
&\Leftrightarrow \frac{\phi}{\beta\left(1-\phi^2\right)}\left[\frac{\theta\left(1+\phi\right)}{2\theta-\rho\left(1-\phi\right)}-\lambda\left(\zeta\right)\right]<0 \\
&\Leftrightarrow \frac{\zeta\left(1+\phi\right)^2}{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2} > \frac{\theta\left(1+\phi\right)}{2\theta-\rho\left(1-\phi\right)} \\
&\Leftrightarrow \zeta\left(1+\phi\right)^2\left[2\theta-\rho\left(1-\phi\right)-\theta\left(1+\phi\right)\right] > \theta\left(1-\phi\right)^2\left(1+\phi\right) \\
&\Leftrightarrow \zeta\left(1+\phi\right)^2\left(\theta-\rho\right)\left(1-\phi\right) > \theta\left(1-\phi\right)^2\left(1+\phi\right) \\
&\Leftrightarrow \zeta\left(1+\phi\right)\left(\theta-\rho\right) > \theta\left(1-\phi\right) \\
&\Leftrightarrow \zeta > \zeta_3 \equiv \frac{\theta\left(1-\phi\right)}{\left(\theta-\rho\right)\left(1+\phi\right)}.
\end{aligned}$$

A comparison of ζ_1 and ζ_3 leads to

$$\begin{aligned}
\frac{\zeta_3}{\zeta_1} &= \frac{\theta\left(1-\phi\right)}{\left(\theta-\rho\right)\left(1+\phi\right)}\,\frac{\left(1+\phi\right)^2}{\phi\left(1-\phi\right)} \\
&= \left(\frac{1+\phi}{\phi}\right)\left(\frac{\theta}{\theta-\rho}\right) \\
&= \left(1+\frac{1}{\phi}\right)\left(1+\frac{\rho}{\theta-\rho}\right) > 1.
\end{aligned}$$

To determine the maximum value of $\zeta_1 \equiv \phi\left(1-\phi\right)/\left(1+\phi\right)^2$, we take the derivative of $\ln\zeta_1 = \ln\phi + \ln\left(1-\phi\right) - 2\ln(1+\phi)$:

$$\begin{aligned}
\frac{d\ln\zeta_1(\phi)}{d\phi} &= \frac{1}{\phi}-\frac{1}{1-\phi}-\frac{2}{1+\phi} \\
&= \frac{\left(1-\phi\right)\left(1+\phi\right)-\phi\left(1+\phi\right)-2\phi(1-\phi)}{\phi(1-\phi)(1+\phi)} \\
&= \frac{1-\phi^2-\phi-\phi^2-2\phi+2\phi^2}{\phi(1-\phi^2)} \\
&= \frac{1-3\phi}{\phi(1-\phi^2)}.
\end{aligned}$$

Note that the derivative is positive for $\phi < 1/3$ and negative for $1/3 < \phi < 1$, so the second order condition is satisfied and the maximum value of ζ_1 occurs when $\phi = 1/3$. Since $\zeta_1(\phi) \equiv \phi\left(1-\phi\right)/\left(1+\phi\right)^2$,

$$\zeta_1(1/3) = \frac{\frac{1}{3}\left(1-\frac{1}{3}\right)}{\left(1+\frac{1}{3}\right)^2} = \frac{\frac{1}{3}\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)^2} = \frac{2}{16} = \frac{1}{8}.$$

Therefore, ζ_1 takes the maximum value $1/8$ at $\phi = 1/3$.

Wage change

Suppose that trade costs change in both sector A and sector B. Starting with the labor market clearing condition (16)

$$L_1 = L_{1A} + L_{1B} = \left(\frac{\theta F}{\rho}\right)\left(M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}\right),$$

first taking logs of both sides

$$\ln L_1 = \ln\left(\frac{\theta F}{\rho}\right) + \ln\left(M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}\right)$$

and then differentiating yields

$$\begin{aligned}
0 &= \frac{1}{M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}}\left[(1+\zeta)M_{1Ae}^{\zeta}\, dM_{1Ae} + (1+\zeta)M_{1Be}^{\zeta}\, dM_{1Be}\right] \\
&= (1+\zeta)\left[\frac{M_{1Ae}^{1+\zeta}}{M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}}\, d\ln M_{1Ae} + \frac{M_{1Be}^{1+\zeta}}{M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}}\, d\ln M_{1Be}\right] \\
&= (1+\zeta)\left[\frac{L_{1A}}{L_1}\, d\ln M_{1Ae} + \frac{L_{1B}}{L_1}\, d\ln M_{1Be}\right].
\end{aligned}$$

It follows from (20) that

$$0 = \frac{L_{1A}}{L_1}\left(\varepsilon_\tau\, d\ln\tau_{21A} - \varepsilon_\tau\, d\ln\tau_{12A} - \varepsilon_w\, d\ln w_1\right) + \frac{L_{1B}}{L_1}\left(\varepsilon_\tau\, d\ln\tau_{21B} - \varepsilon_\tau\, d\ln\tau_{12B} - \varepsilon_w\, d\ln w_1\right)$$

and rearranging terms yields

$$\begin{aligned}
\frac{L_{1A}}{L_1}\varepsilon_\tau\left(d\ln\tau_{21A} - d\ln\tau_{12A}\right) + \frac{L_{1B}}{L_1}\varepsilon_\tau\left(d\ln\tau_{21B} - d\ln\tau_{12B}\right) &= \frac{L_{1A}}{L_1}\varepsilon_w\, d\ln w_1 + \frac{L_{1B}}{L_1}\varepsilon_w\, d\ln w_1 \\
\varepsilon_\tau \sum_{s=A,B}\frac{L_{1s}}{L_1}\left(d\ln\tau_{21s} - d\ln\tau_{12s}\right) &= \varepsilon_w\, d\ln w_1. \qquad \text{(A.11)}
\end{aligned}$$

Since countries are initially symmetric before trade liberalization, it holds that $f_{1se} = f_{2se} = f_{se}$, $M_{1se} = M_{2se} = M_{se}$, $\phi_{12s} = \phi_{21s} = \phi$, $L_1 = L_2 = L$ and $w_1 = 1$. Thus, two equations in (A.9)

$$\begin{aligned}
\left(f_{1se} - \frac{\phi_{12s}}{w_1}f_{2se}\right)\left(M_{1se} + M_{2se}\phi_{21s}\right) &= \frac{\rho\alpha_s L_1}{\theta}(1 - \phi_{12s}\phi_{21s}) \\
\left(f_{2se} - \phi_{21s}w_1 f_{1se}\right)\left(M_{1se}\phi_{12s} + M_{2se}\right) &= \frac{\rho\alpha_s L_2}{\theta}\left(1 - \phi_{12s}\phi_{21s}\right)
\end{aligned}$$

becomes one equation

$$\left(f_{se}-\phi f_{se}\right)\left(M_{se}+M_{se}\phi\right)=\frac{\rho\alpha_s L}{\theta}(1-\phi^2).$$

Further simplifying using $f_{ise}=F\cdot M_{ise}^{\zeta}$, this equation becomes

$$\begin{aligned}
M_{se}f_{se}\left(1-\phi\right)\left(1+\phi\right)&=\frac{\rho\alpha_s L}{\theta}(1-\phi^2)\\
M_{se}f_{se}&=\frac{\rho\alpha_s L}{\theta}\\
\frac{\theta}{\rho}FM_{se}^{1+\zeta}&=\alpha_s L
\end{aligned}$$

Since $L_{is}=\frac{\theta}{\rho}FM_{ise}^{1+\zeta}$ from (15), $\alpha_s=L_{is}/L$ holds. Using this and rearranging equation (A.11), we obtain

$$\begin{aligned}
d\ln w_1&=\frac{\varepsilon_\tau}{\varepsilon_w}\sum_{s=A,B}\frac{L_{1s}}{L_1}\left(d\ln\tau_{21s}-d\ln\tau_{12s}\right)\\
&=\frac{\phi\theta}{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2}\frac{\rho\left[\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right]}{\phi\left[2\theta-\rho\left(1-\phi\right)\right]}\sum_{s=A,B}\alpha_s\left(d\ln\tau_{21s}-d\ln\tau_{12s}\right)\\
&=\frac{\theta\rho}{\left[2\theta-\rho\left(1-\phi\right)\right]}\sum_{s=A,B}\alpha_s\left(d\ln\tau_{21s}-d\ln\tau_{12s}\right)\\
&=\beta\sum_{s=A,B}\alpha_s\left(d\ln\tau_{21s}-d\ln\tau_{12s}\right)
\end{aligned}\tag{23}$$

Thus, the wage change does not depend on the size of ζ.

Substituting the wage change (23) into (22), we obtain the total impact of trade liberalization on industrial productivity in sector A in country 1:

$$\begin{aligned}
d\ln\Phi_{1A}^k&=\gamma_1\,d\ln\tau_{21A}-\gamma_2\,d\ln\tau_{12A}-\gamma_3\,d\ln w_1\\
&=\gamma_1\,d\ln\tau_{21A}-\gamma_2\,d\ln\tau_{12A}-\gamma_3\,\beta\left[\alpha_A\left(d\ln\tau_{21A}-d\ln\tau_{12A}\right)+\alpha_B\left(d\ln\tau_{21B}-d\ln\tau_{12B}\right)\right]\\
&=-\left(\gamma_3\beta\alpha_A-\gamma_1\right)d\ln\tau_{21A}-\left(\gamma_2-\gamma_3\beta\alpha_A\right)d\ln\tau_{12A}-\left(\gamma_3\,\beta\alpha_B\right)\left(d\ln\tau_{21B}-d\ln\tau_{12B}\right)\\
&=-\xi_{1A}\,d\ln\tau_{21A}-\xi_{2A}\,d\ln\tau_{12A}-\xi_{3A}\left(d\ln\tau_{21B}-d\ln\tau_{12B}\right)
\end{aligned}\tag{24}$$

where $\xi_{1A}\equiv\gamma_3\beta\alpha_A-\gamma_1$, $\xi_{2A}\equiv\gamma_2-\gamma_3\beta\alpha_A$, and $\xi_{3A}\equiv\gamma_3\beta\left(1-\alpha_A\right)$.

Proof for Theorem 3

First, we introduce two new terms $\kappa_1 \equiv \gamma_3\beta - \gamma_1$ and $\kappa_2 \equiv \gamma_2 - \gamma_3\beta$ that will be useful for the proof of Theorem 3 and for later welfare analysis. Both terms are strictly positive:

$$
\begin{aligned}
\kappa_1 &\equiv \gamma_3\beta - \gamma_1 \\
&= \frac{\phi}{1-\phi^2}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \lambda(\zeta)\right] - \frac{\phi\left[\phi-\lambda(\zeta)\right]}{1-\phi^2} \\
&= \frac{\phi}{1-\phi^2}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \phi\right] \\
&= \frac{\phi}{1-\phi^2}\left[\frac{\theta+\theta\phi-\phi 2\theta+\phi\rho(1-\phi)}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi}{(1-\phi)(1+\phi)}\left[\frac{\theta(1-\phi)+\rho\phi(1-\phi)}{2\theta-\rho(1+\phi)}\right] \\
&= \frac{\phi(\theta+\rho\phi)}{(1+\phi)\left[2\theta-\rho(1-\phi)\right]} > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\kappa_2 &\equiv \gamma_2 - \gamma_3\beta \\
&= \frac{\phi\left[1-\lambda(\zeta)\right]}{1-\phi^2} - \frac{\phi}{1-\phi^2}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \lambda(\zeta)\right] \\
&= \frac{\phi}{1-\phi^2}\left[1-\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi}{1-\phi^2}\left[\frac{2\theta-\rho(1-\phi)-\theta-\theta\phi}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi}{(1-\phi)(1+\phi)}\left[\frac{\theta(1-\phi)-\rho(1-\phi)}{2\theta-\rho(1-\phi)}\right] \\
&= \frac{\phi(\theta-\rho)}{(1+\phi)\left[2\theta-\rho(1-\phi)\right]} > 0.
\end{aligned}
$$

(1) If $\zeta < \zeta_1$, then $\gamma_1 > 0$ and $\gamma_3 > 0$ from Theorems 1 and 2. Then

$$
\begin{aligned}
&\xi_{1A} > 0 \\
&\Leftrightarrow \gamma_3\beta\alpha_A - \gamma_1 > 0 \\
&\Leftrightarrow \gamma_3\beta\alpha_A > \gamma_1 \\
&\Leftrightarrow \alpha_A > \bar{\alpha}(\zeta) \equiv \frac{\gamma_1}{\beta\gamma_3}
\end{aligned}
$$

It follows that $\xi_{1A} > 0$ if $\alpha_A > \bar{\alpha}(\zeta)$ and $\xi_{1A} < 0$ if $\alpha_A < \bar{\alpha}(\zeta)$. Next note that $\bar{\alpha}(\zeta)$ can be written as

$$
\begin{aligned}
\bar{\alpha}(\zeta) \equiv \frac{\gamma_1}{\beta\gamma_3} &= \left[\frac{\phi\left[\phi-\lambda(\zeta)\right]}{1-\phi^2}\right] / \left[\frac{\phi}{1-\phi^2}\left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)}-\lambda(\zeta)\right]\right] \\
&= \left[\phi - \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2}\right] / \left[\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2}\right]
\end{aligned}
$$

Using

$$
\begin{aligned}
\phi - \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} &= \frac{\phi(1-\phi)^2-\zeta(1-\phi)(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} \\
&= \frac{(1-\phi)\left[\phi(1-\phi)-\zeta(1+\phi)^2\right]}{(1-\phi)^2+\zeta(1+\phi)^2}
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\theta(1+\phi)}{2\theta-\rho(1-\phi)} - \frac{\zeta(1+\phi)^2}{(1-\phi)^2+\zeta(1+\phi)^2} \\
&= \frac{(1+\phi)\left[\theta(1-\phi)^2+\theta\zeta(1+\phi)^2-2\theta\zeta(1+\phi)+\rho(1-\phi)\zeta(1+\phi)\right]}{\left[2\theta-\rho(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} \\
&= \frac{(1+\phi)\left[\theta(1-\phi)^2+\theta\zeta(1+\phi)\{1+\phi-2\}+\rho(1-\phi)\zeta(1+\phi)\right]}{\left[2\theta-\rho(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} \\
&= \frac{(1+\phi)\left[\theta(1-\phi)^2-\zeta\theta(1-\phi)(1+\phi)+\zeta\rho(1-\phi)(1+\phi)\right]}{\left[2\theta-\rho(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} \\
&= \frac{(1+\phi)(1-\phi)\left[\theta(1-\phi)-\zeta(\theta-\rho)(1+\phi)\right]}{\left[2\theta-\rho(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right]},
\end{aligned}
$$

we obtain

$$
\begin{aligned}
\bar{\alpha}(\zeta) &= \left[\frac{(1-\phi)\left[\phi(1-\phi)-\zeta(1+\phi)^2\right]}{(1-\phi)^2+\zeta(1+\phi)^2} \right] \Big/ \left[\frac{(1+\phi)(1-\phi)\left[\theta(1-\phi)-\zeta(\theta-\rho)(1+\phi)\right]}{\left[2\theta-\rho(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} \right] \\
&\equiv \frac{\left[2\theta-\rho(1-\phi)\right]\left[\phi(1-\phi)-\zeta(1+\phi)^2\right]}{(1+\phi)\left[\theta(1-\phi)-\zeta(\theta-\rho)(1+\phi)\right]} \\
&= \frac{(1+\phi)^2\left[2\theta-\rho(1-\phi)\right]\left[\frac{\phi(1-\phi)}{(1+\phi)^2}-\zeta\right]}{(1+\phi)^2(\theta-\rho)\left[\frac{\theta(1-\phi)}{(1+\phi)(\theta-\rho)}-\zeta\right]} \\
&= \frac{\left[2\theta-\rho(1-\phi)\right](\zeta_1-\zeta)}{(\theta-\rho)(\zeta_3-\zeta)}.
\end{aligned}
$$

It immediately follows that $\bar{\alpha}(\zeta_1)=0$ and the derivative of the $\bar{\alpha}$ function is

$$
\begin{aligned}
\bar{\alpha}'(\zeta) &= \frac{\left[2\theta-\rho(1-\phi)\right]}{(\theta-\rho)} \frac{\left[-(\zeta_3-\zeta)+(\zeta_1-\zeta)\right]}{(\zeta_3-\zeta)^2} \\
&= -\frac{\left[2\theta-\rho(1-\phi)\right]}{(\theta-\rho)} \frac{(\zeta_3-\zeta_1)}{(\zeta_3-\zeta)^2} < 0 \ \text{ for } \ \zeta < \zeta_1.
\end{aligned}
$$

Next note that

$$
\begin{aligned}
\bar{\alpha}(0) &\equiv \frac{\left[2\theta-\rho(1-\phi)\right]\zeta_1}{(\theta-\rho)\zeta_3} \\
&= \frac{\left[2\theta-\rho(1-\phi)\right]}{(\theta-\rho)} \frac{\phi(1-\phi)}{(1+\phi)^2} \frac{(\theta-\rho)(1+\phi)}{\theta(1-\phi)} \\
&= \frac{\phi\left[2\theta-\rho(1-\phi)\right]}{\theta(1+\phi)} > 0
\end{aligned}
$$

and

$$
\begin{aligned}
1-\bar{\alpha}(0) &= \frac{\theta(1+\phi)-\phi\left[2\theta-\rho(1-\phi)\right]}{\theta(1+\phi)} \\
&= \frac{\theta-\theta\phi+\phi\rho(1-\phi)}{\theta(1+\phi)} \\
&= \frac{(\theta+\phi\rho)(1-\phi)}{\theta(1+\phi)} > 0,
\end{aligned}
$$

so $\bar{\alpha}(0)$ lies between 0 and 1. If $\zeta_3 > \zeta > \zeta_1$, then $\gamma_1 < 0$ and $\gamma_3 > 0$, so $\alpha_A > 0 > \bar{\alpha}(\zeta)$ and $\xi_{1A} > 0$. If $\zeta > \zeta_3$, then $\gamma_1 < 0$ and $\gamma_3 < 0$, so

$$
\xi_{1A} \equiv \gamma_3\beta\alpha_A - \gamma_1 \geq \gamma_3\beta - \gamma_1 \equiv \kappa_1 = \frac{\phi(\theta+\rho\phi)}{(1+\phi)\left[2\theta-\rho(1-\phi)\right]} > 0.
$$

(2) If $\zeta \geq \zeta_3$ and $\gamma_3 \leq 0$, then $\xi_{2A} \equiv \gamma_2 - \gamma_3 \beta \alpha_A > 0$. If $\zeta < \zeta_3$ and $\gamma_3 > 0$, then

$$\xi_{2A} \equiv \gamma_2 - \gamma_3 \beta \alpha_A \geq \gamma_2 - \gamma_3 \beta \equiv \kappa_2 = \frac{\phi\,(\theta - \rho)}{(1+\phi)\,[2\theta - \rho\,(1-\phi)]} > 0.$$

Thus $\xi_{2A} > 0$ holds in general.

(3) The sign of $\xi_{3A} \equiv \gamma_3 \beta\,(1-\alpha_A)$ is the same as the sign of γ_3, so $\xi_{3A} > 0$ if $\zeta < \zeta_3$ and $\xi_{3A} < 0$ if $\zeta > \zeta_3$.

Types of Trade Liberalization

First, we consider the symmetric trade liberalization that Melitz (2003) analyzes. Suppose country 1 and country 2 symmetrically liberalize ($d \ln \tau_{21s} = d \ln \tau_{12s} = d \ln \tau_s < 0$) in a single industry s. Since symmetric trade liberalization keeps countries symmetric, the wage continues to be $w_1 = 1$. Thus, equation (22) leads to

$$\begin{aligned}
d \ln \Phi_{1s}^{k} &= (\gamma_1 - \gamma_2)\; d \ln \tau_s \\
&= \left(\frac{\phi\,[\phi - \lambda\,(\zeta)]}{1-\phi^2} - \frac{\phi\,[1 - \lambda\,(\zeta)]}{1-\phi^2} \right) d \ln \tau_s \\
&= \frac{\phi\,(\phi - 1)}{(1-\phi)(1+\phi)}\, d \ln \tau_s \\
&= -\frac{\phi}{1+\phi}\, d \ln \tau_s > 0.
\end{aligned}$$

Second, we consider unilateral trade liberalization by country 1 that is uniform across industries ($d \ln \tau_{21A} = d \ln \tau_{21B} = d \ln \tau < d \ln \tau_{12A} = d \ln \tau_{12B} = 0$). Then, equation (24) leads to

$$\begin{aligned}
d \ln \Phi_{1A}^{k} &= -\xi_{1A}\, d \ln \tau - \xi_{3A}\, d \ln \tau \\
&= -(\gamma_3 \beta \alpha_A - \gamma_1 + \gamma_3 \beta\,(1-\alpha_A))\, d \ln \tau \\
&= -(\gamma_3 \beta - \gamma_1)\, d \ln \tau \\
&= -\kappa_1\, d \ln \tau \\
&= -\frac{\phi\,(\theta + \rho\phi)}{(1+\phi)\,[2\theta - \rho\,(1-\phi)]}\, d \ln \tau > 0.
\end{aligned}$$

The Welfare Effect

First, consider the symmetric industries case that we have analyzed so far. The utility of the representative consumer in country 1 is $U_1 = \left(\alpha_A \Phi_{1A}^{W}\right)^{\alpha_A} \left(\alpha_B \Phi_{1B}^{W}\right)^{\alpha_B}$. From equation (24), the productivity change for country 1 and industry A is

$$d\ln\Phi_{1A}^{W}=-(\gamma_3\beta\alpha_A-\gamma_1)\,d\ln\tau_{21A}-(\gamma_2-\gamma_3\beta\alpha_A)\,d\ln\tau_{12A}-(\gamma_3\beta\alpha_B)\,(d\ln\tau_{21B}-d\ln\tau_{12B})$$

and the corresponding expression for industry B is

$$d\ln\Phi_{1B}^{W}=-(\gamma_3\beta\alpha_B-\gamma_1)\,d\ln\tau_{21B}-(\gamma_2-\gamma_3\beta\alpha_B)\,d\ln\tau_{12B}-(\gamma_3\beta\alpha_A)\,(d\ln\tau_{21A}-d\ln\tau_{12A})\,.$$

Taking logs of both sides and differentiating the consumer utility function U_1, and then substituting for the productivity changes, we obtain the welfare change:

$$\begin{aligned}
d\ln U_1 &= \alpha_A\,d\ln\Phi_{1A}^{W}+\alpha_B\,d\ln\Phi_{1B}^{W}\\
&= -\alpha_A(\gamma_3\beta\alpha_A-\gamma_1)\,d\ln\tau_{21A}-\alpha_A(\gamma_2-\gamma_3\beta\alpha_A)\,d\ln\tau_{12A}-\alpha_A\gamma_3\beta\alpha_B\,(d\ln\tau_{21B}-d\ln\tau_{12B})\\
&\quad -\alpha_B(\gamma_3\beta\alpha_B-\gamma_1)\,d\ln\tau_{21B}-\alpha_B(\gamma_2-\gamma_3\beta\alpha_B)\,d\ln\tau_{12B}-\alpha_B\gamma_3\beta\alpha_A\,(d\ln\tau_{21A}-d\ln\tau_{12A})\\
&= -\alpha_A(\gamma_3\beta\alpha_A+\gamma_3\beta\alpha_B-\gamma_1)\,d\ln\tau_{21A}-\alpha_A(\gamma_2-\gamma_3\beta\alpha_A-\gamma_3\beta\alpha_B)\,d\ln\tau_{12A}\\
&\quad -\alpha_B(\gamma_3\beta\alpha_B+\gamma_3\beta\alpha_A-\gamma_1)\,d\ln\tau_{21B}-\alpha_B(\gamma_2-\gamma_3\beta\alpha_B-\gamma_3\beta\alpha_A)\,d\ln\tau_{12B}\\
&= -\alpha_A(\gamma_3\beta-\gamma_1)\,d\ln\tau_{21A}-\alpha_A(\gamma_2-\gamma_3\beta)\,d\ln\tau_{12A}\\
&\quad -\alpha_B(\gamma_3\beta-\gamma_1)\,d\ln\tau_{21B}-\alpha_B(\gamma_2-\gamma_3\beta)\,d\ln\tau_{12B}\\
&= -\sum_{s=A,B}\alpha_s\,(\kappa_1\,d\ln\tau_{21s}+\kappa_2\,d\ln\tau_{12s})
\end{aligned}\tag{26}$$

where

$$\kappa_1\equiv\gamma_3\beta-\gamma_1=\frac{\phi\,(\theta+\rho\phi)}{(1+\phi)\,[2\theta-\rho\,(1-\phi)]}>0$$

and

$$\kappa_2\equiv\gamma_2-\gamma_3\beta=\frac{\phi\,(\theta-\rho)}{(1+\phi)\,[2\theta-\rho\,(1-\phi)]}>0.$$

Both domestic and foreign trade liberalization cause domestic welfare to increase ($\tau_{ijs}\downarrow\Rightarrow U_1\uparrow$).

Home Market Effect

The system of equations (A.9) for industry A is

$$\begin{aligned}
\left(f_{1Ae}-\frac{\phi_{12A}}{w_1}f_{2se}\right)(M_{1Ae}+M_{2Ae}\phi_{21A}) &= \frac{\rho\alpha_A L_1}{\theta}(1-\phi_{12A}\phi_{21A})\\
(f_{2Ae}-\phi_{21A}w_1 f_{1Ae})\,(M_{1Ae}\phi_{12A}+M_{2sA}) &= \frac{\rho\alpha_A L_2}{\theta}\,(1-\phi_{12A}\phi_{21A})\,.
\end{aligned}$$

Taking into account that $f_{iAe} = F\, M_{iAe}^{\zeta}$, $w_1 = w_2 = 1$ and $\phi_{12s} = \phi_{21s} = \phi$, this system of equations becomes

$$\begin{aligned}\left(M_{1Ae}^{\zeta} - \phi M_{2Ae}^{\zeta}\right)(M_{1Ae} + M_{2Ae}\phi) &= \frac{\rho\alpha_A L_1}{\theta F}(1-\phi^2)\\ \left(M_{2Ae}^{\zeta} - \phi M_{1Ae}^{\zeta}\right)(M_{1Ae}\phi + M_{2Ae}) &= \frac{\rho\alpha_A L_2}{\theta F}(1-\phi^2).\end{aligned}$$

Note that ϕ does not change when L_1 increases, since it is only a function of trade costs and the relative wage. Taking logs of both sides and then differentiating this system of equations, we obtain

$$\begin{aligned}d\ln\left(M_{1Ae}^{\zeta} - \phi M_{2Ae}^{\zeta}\right) + d\ln(M_{1Ae} + M_{2Ae}\phi) &= d\ln L_1\\ d\ln\left(M_{2Ae}^{\zeta} - \phi M_{1Ae}^{\zeta}\right) + d\ln(M_{1Ae}\phi + M_{2Ae}) &= 0.\end{aligned}$$

Since

$$\begin{aligned}d\ln\left(M_{1Ae}^{\zeta} - \phi M_{2Ae}^{\zeta}\right) &= \frac{M_{1Ae}^{\zeta}}{M_{1Ae}^{\zeta} - \phi M_{2Ae}^{\zeta}}\,\zeta\, d\ln M_{1Ae} - \frac{\phi M_{2Ae}^{\zeta}}{M_{1Ae}^{\zeta} - \phi M_{2Ae}^{\zeta}}\,\zeta\, d\ln M_{2Ae}\\ &= \frac{\zeta}{1-\phi}\, d\ln M_{1Ae} - \frac{\zeta\phi}{1-\phi}\, d\ln M_{2Ae}\\ d\ln(M_{1Ae} + M_{2Ae}\phi) &= \frac{M_{1Ae}}{M_{1Ae} + M_{2Ae}\phi}\, d\ln M_{1Ae} + \frac{\phi M_{2Ae}}{M_{1Ae} + M_{2Ae}\phi}\, d\ln M_{2Ae}\\ &= \frac{1}{1+\phi}\, d\ln M_{1Ae} + \frac{\phi}{1+\phi}\, d\ln M_{2Ae}\\ d\ln\left(M_{2Ae}^{\zeta} - \phi M_{1Ae}^{\zeta}\right) &= \frac{M_{2Ae}^{\zeta}}{M_{2Ae}^{\zeta} - \phi M_{1Ae}^{\zeta}}\,\zeta\, d\ln M_{2Ae} - \frac{\phi M_{1Ae}^{\zeta}}{M_{2Ae}^{\zeta} - \phi M_{1Ae}^{\zeta}}\,\zeta\, d\ln M_{1Ae}\\ &= \frac{\zeta}{1-\phi}\, d\ln M_{2Ae} - \frac{\zeta\phi}{1-\phi}\, d\ln M_{1Ae}\\ d\ln(M_{1Ae}\phi + M_{2Ae}) &= \frac{M_{1Ae}\phi}{M_{1Ae}\phi + M_{2Ae}}\, d\ln M_{1Ae} + \frac{M_{2Ae}}{M_{1Ae}\phi + M_{2Ae}}\, d\ln M_{2Ae}\\ &= \frac{\phi}{1+\phi}\, d\ln M_{1Ae} + \frac{1}{1+\phi}\, d\ln M_{2Ae},\end{aligned}$$

we have

$$\begin{aligned}\left(\frac{1}{1+\phi} + \frac{\zeta}{1-\phi}\right) d\ln M_{1Ae} + \left(\frac{\phi}{1+\phi} - \frac{\zeta\phi}{1-\phi}\right) d\ln M_{2Ae} &= d\ln L_1\\ \left(\frac{\phi}{1+\phi} - \frac{\zeta\phi}{1-\phi}\right) d\ln M_{1Ae} + \left(\frac{1}{1+\phi} + \frac{\zeta}{1-\phi}\right) d\ln M_{2Ae} &= 0.\end{aligned}$$

Since

$$\frac{1}{1+\phi}+\frac{\zeta}{1-\phi}=\frac{(1-\phi)+\zeta(1+\phi)}{(1+\phi)(1-\phi)}$$

and

$$\frac{\phi}{1+\phi}-\frac{\zeta\phi}{1-\phi}=\frac{\phi\{(1-\phi)-\zeta(1+\phi)\}}{(1+\phi)(1-\phi)}$$

the system of equations can be written in matrix form as

$$\begin{pmatrix} (1-\phi)+\zeta(1+\phi) & \phi\{(1-\phi)-\zeta(1+\phi)\} \\ \phi\{(1-\phi)-\zeta(1+\phi)\} & (1-\phi)+\zeta(1+\phi) \end{pmatrix}\begin{pmatrix} d\ln M_{1Ae} \\ d\ln M_{2Ae} \end{pmatrix}=\begin{pmatrix} 1-\phi^2 \\ 0 \end{pmatrix} d\ln L_1.$$

The determinant of the matrix on the left hand side becomes

$$\begin{aligned}
&\{(1-\phi)+\zeta(1+\phi)\}^2-\phi^2\{(1-\phi)-\zeta(1+\phi)\}^2 \\
&=[(1-\phi)+\zeta(1+\phi)-\phi\{(1-\phi)-\zeta(1+\phi)\}][(1-\phi)+\zeta(1+\phi)+\phi\{(1-\phi)-\zeta(1+\phi)\}] \\
&=\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}[(1-\phi)(1+\phi)+\zeta(1+\phi)(1-\phi)] \\
&=\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}\left(1-\phi^2\right)(1+\zeta)>0
\end{aligned}$$

Using Cramer's law, we obtain

$$\begin{aligned}
\frac{d\ln M_{1Ae}}{d\ln L_1} &= \frac{\left(1-\phi^2\right)\{(1-\phi)+\zeta(1+\phi)\}}{(1-\phi^2)(1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}} \\
&= \frac{(1-\phi)+\zeta(1+\phi)}{(1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}} \equiv \varepsilon_{1L}
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\ln M_{2Ae}}{d\ln L_1} &= -\frac{\left(1-\phi^2\right)\phi\{(1-\phi)-\zeta(1+\phi)\}}{(1-\phi^2)(1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}} \\
&= -\frac{\phi\{(1-\phi)-\zeta(1+\phi)\}}{(1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}} \equiv -\varepsilon_{2L}.
\end{aligned}$$

From (13),

$$R_{ijs}=\alpha_s w_j L_j\left(\frac{M_{ise}\phi_{ijs}}{\sum_{k=1,2}M_{kse}\phi_{kjs}}\right),$$

it follows that

$$R_{12A}=\alpha_A L_2\left(\frac{M_{1Ae}\phi}{M_{1Ae}\phi+M_{2Ae}}\right),$$

$$R_{21A} = \alpha_A L_1 \left(\frac{M_{2Ae}\phi}{M_{1Ae} + M_{2Ae}\phi} \right),$$

and

$$\frac{R_{12A}}{R_{21A}} = \frac{L_2}{L_1} \left(\frac{M_{1Ae}}{M_{2Ae}} \right) \left(\frac{M_{1Ae} + M_{2Ae}\phi}{M_{1Ae}\phi + M_{2Ae}} \right).$$

Taking logs of both sides and then differentiating, we obtain

$$d \ln \left(\frac{R_{12A}}{R_{21A}} \right) = - d \ln L_1 + d \ln \left(\frac{M_{1Ae}}{M_{2Ae}} \right) + d \ln (M_{1Ae} + M_{2Ae}\phi) - d \ln (M_{1Ae}\phi + M_{2Ae}).$$

Using the following relationships

$$\begin{aligned}
d \ln \left(\frac{M_{1Ae}}{M_{2Ae}} \right) &= d \ln M_{1Ae} - d \ln M_{2Ae}, \\
d \ln (M_{1Ae} + M_{2Ae}\phi) &= \frac{M_{1Ae}}{M_{1Ae} + M_{2Ae}\phi} \, d \ln M_{1Ae} + \frac{M_{2Ae}\phi}{M_{1Ae} + M_{2Ae}\phi} \, d \ln M_{2Ae} \\
&= \frac{1}{1+\phi} \, d \ln M_{1Ae} + \frac{\phi}{1+\phi} \, d \ln M_{2Ae}, \\
d \ln (M_{1Ae}\phi + M_{2Ae}) &= \frac{M_{1Ae}\phi}{M_{1Ae}\phi + M_{2Ae}} \, d \ln M_{1Ae} + \frac{M_{2Ae}}{M_{1Ae}\phi + M_{2Ae}} \, d \ln M_{2Ae} \\
&= \frac{\phi}{1+\phi} \, d \ln M_{1Ae} + \frac{1}{1+\phi} \, d \ln M_{2Ae},
\end{aligned}$$

we obtain

$$\begin{aligned}
d \ln (R_{12A}/R_{21A}) &= - d \ln L_1 + \left(1 + \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \right) d \ln M_{1Ae} - \left(1 - \frac{\phi}{1+\phi} + \frac{1}{1+\phi} \right) d \ln M_{2Ae} \\
&= - d \ln L_1 + \frac{2}{1+\phi} (d \ln M_{1Ae} - d \ln M_{2Ae})
\end{aligned}$$

and

$$\begin{aligned}
\frac{d \ln (R_{12A}/R_{21A})}{d \ln L_1} &= -1 + \frac{2}{1+\phi} (\varepsilon_{1L} + \varepsilon_{2L}) \\
&= -1 + \frac{2 \left[(1-\phi) + \zeta (1+\phi) + \phi \left\{ (1-\phi) - \zeta (1+\phi) \right\} \right]}{(1+\phi)(1+\zeta) \left\{ (1-\phi)^2 + \zeta (1+\phi)^2 \right\}} \\
&= -1 + \frac{2 \left[(1-\phi)(1+\phi) + \zeta (1+\phi)(1-\phi) \right]}{(1+\phi)(1+\zeta) \left\{ (1-\phi)^2 + \zeta (1+\phi)^2 \right\}} \\
&= -1 + \frac{2 (1-\phi)}{(1-\phi)^2 + \zeta (1+\phi)^2} > 0
\end{aligned}$$

if and only if

$$
\begin{aligned}
2\,(1-\phi) &> (1-\phi)^2 + \zeta\,(1+\phi)^2 \\
2\,(1-\phi) - (1-\phi)^2 &> \zeta\,(1+\phi)^2 \\
(1-\phi)\,\{2-(1-\phi)\} &> \zeta\,(1+\phi)^2 \\
(1-\phi) &> \zeta\,(1+\phi) \\
\zeta_H \equiv \frac{1-\phi}{1+\phi} &> \zeta.
\end{aligned}
$$

Notice that $R_{12A}/R_{21A} = 1$ initially holds. Thus, the model predicts the Home market effect if and only if $d\ln(R_{12A}/R_{21A})/d\ln L_1 > 0$. Since

$$
\zeta_H - \zeta_1 = \frac{(1-\phi)}{(1+\phi)} - \frac{\phi\,(1-\phi)}{(1+\phi)^2} = \frac{(1-\phi)\,\{1+\phi-\phi\}}{(1+\phi)^2} = \frac{1-\phi}{(1+\phi)^2} > 0,
$$

the model predicts the Home Market effect and the Trefler finding if $\zeta_1 < \zeta < \zeta_H$.